22.312 ENGINEERING OF NUCLEAR REACTORS

Due November 16, 2007 by 12:00 pm

TAKE HOME QUIZ 2 (SOLUTION)

Problem 1 (45%) – Effect of internal cooling on fuel temperatures

i) *Solid pellet*
For the solid pellet the maximum temperature, T_{max}, is given by Eq. 8-62 in T&K (for constant thermal conductivity):

$$T_{\max} = T_{fo} + \frac{q'}{4\pi k_f} \approx 1761°\text{C}$$

Where T_{fo}=700°C.

ii) *Annular pellet with external cooling*
The maximum temperature is given by Eq. 8-67 in T&K (for constant thermal conductivity):

$$T_{\max} = T_{fo} + \frac{q'}{4\pi k_f}\left[1 - \frac{\ln(R_{fo}/R_{fi})^2}{(R_{fo}/R_{fi})^2 - 1}\right] \approx 1579°\text{C}$$

Where R_{fi} and R_{fo} are the inner and outer radii of the pellet.

iii) *Annular pellet with external and internal cooling*
To calculate the maximum temperature we must solve the heat conduction equation:

$$\frac{k_f}{r}\frac{d}{dr}(r\frac{dT}{dr}) + q''' = 0 \qquad (1)$$

Integrating Eq. (1) twice we get:

$$k_f T(r) + q'''\frac{r^2}{4} + c_1 \ln r + c_2 = 0 \qquad (2)$$

Where the constant of integration c_1 and c_2 can be found from the boundary conditions:

$$T = T_{fo} \text{ at } r = R_{fi} \qquad (3)$$
$$T = T_{fo} \text{ at } r = R_{fo} \qquad (4)$$

Then we have:

$$c_1 = -\frac{q'''}{4}\frac{R_{fo}^2 - R_{fi}^2}{\ln(R_{fo}/R_{fi})} \tag{5}$$

$$c_2 = -k_f T_{fo} - \frac{q'''}{4}R_{fi}^2 + \frac{q'''}{4}\frac{R_{fo}^2 - R_{fi}^2}{\ln(R_{fo}/R_{fi})}\ln(R_{fi}) \tag{6}$$

Substituting Eqs. (5) and (6) into Eq. (2) we get the temperature distribution in the fuel:

$$T(r) = T_{fo} + \frac{q'''}{4k_f}\left[R_{fi}^2 - r^2 + \frac{R_{fo}^2 - R_{fi}^2}{\ln(R_{fo}/R_{fi})}\ln(r/R_{fi})\right] \tag{7}$$

To find the radial location, r_{max}, at which the temperature has a maximum, we can set the derivative of Eq. (7) equal to zero and solve for r, which gives:

$$r_{max} = \sqrt{\frac{R_{fo}^2 - R_{fi}^2}{\ln(R_{fo}/R_{fi})^2}} \approx 5.97 \text{ mm} \tag{8}$$

Substituting Eq. (8) into (7) and noting that $q' = q'''\pi\left(R_{fo}^2 - R_{fi}^2\right)$, we get:

$$T_{max} = T_{fo} + \frac{q'}{4\pi k_f}\left[\frac{R_{fi}^2 - r_{max}^2}{R_{fo}^2 - R_{fi}^2} + \frac{\ln(r_{max}/R_{fi})}{\ln(R_{fo}/R_{fi})}\right] \approx 793°\text{C} \tag{9}$$

Notice the large reduction in maximum fuel temperature attainable with simultaneous internal and external cooling.

iv) The heat flux at any given location is given by Fourier's law:

$$q'' = -k_f\frac{dT}{dr} = \text{(from Eq. 7)} = \frac{q'}{4\pi}\left[\frac{2r}{R_{fo}^2 - R_{fi}^2} + \frac{1}{r\ln(R_{fo}/R_{fi})}\right] \tag{10}$$

Thus, setting r = R_{fi} and r = R_{fo} in Eq. (10), we find the heat flux at the inner and outer surface of the pellet to be about -567.9 and +504.3 kW/m^2, respectively. Note that the heat flux is negative at the inner surface (heat going in the negative radial direction) and positive at the outer surface (heat going in the positive radial direction). You may also want to verify that the following equality holds:

$$|q_i''|2\pi R_{fi} + q_o''2\pi R_{fo} = q'$$

v) Advantages: lower fuel temperature for given linear power, thus lower fission gas release, lower thermal expansion, lower swelling, lower stored energy during accidents

Drawbacks: harder to fabricate, requires two claddings, needs careful coolant split between channel inside and outside fuel

Problem 2 (55%) – Flow-levitated control rod in a PWR

i) There four forces acting on the control rod slug, i.e., rod weight, pressure on the top and bottom faces of the rod, shear stress due to flow friction. The force balance is as follows:

$$\rho_{CR}\frac{\pi}{4}D^2Lg+\frac{\pi}{4}D^2P_{out}=\frac{\pi}{4}D^2P_{in}+\tau\,\pi DL \tag{11}$$

Where ρ_{CR}=2500 kg/m^3 is the control rod density, P_{out} is the pressure immediately above the slug, P_{in} is the pressure immediately before the slug, and τ is the shear stress on the lateral surface of the slug, which can be calculated from knowledge of the friction factor, f, as:

$$\tau=\frac{f}{8}\frac{G^2}{\rho} \tag{12}$$

Where f=0.017, ρ=730 kg/m^3 is the coolant density and G is the (unknown) mass flux in the annulus between the control rod and guide tube. Substituting Eq. (12) into Eq. (11) and re-arranging the various terms, we get:

$$\rho_{CR}Lg=P_{in}-P_{out}+f\frac{L}{D}\frac{G^2}{2\rho} \tag{13}$$

The pressure difference P_{in}-P_{out} can be found form the momentum equation for the coolant in the annulus:

$$P_{in}-P_{out}=f\frac{L}{D_e}\frac{G^2}{2\rho}+(K_{in}+K_{out})\frac{G^2}{2\rho}+\rho Lg \tag{14}$$

Where it was assumed that the acceleration pressure change is negligible because the fluid is incompressible and there is no net change in flow area; D_e=D_o-D=0.6 cm is the annulus equivalent diameter, K_{in}=0.25 and K_{out}=1.0 are the form loss coefficient for the annulus entrance and exit, respectively. Substituting Eq. (14) into Eq. (13) and solving for G, we get:

$$G=\sqrt{\frac{2\rho(\rho_{CR}-\rho)Lg}{f\frac{L}{D_e}+f\frac{L}{D}+K_{in}+K_{out}}}\approx 2511\ \text{kg/m}^2\text{s} \tag{15}$$

Then the mass flow rate in the guide tube is $\dot{m} = G\frac{\pi}{4}(D_o^2 - D^2) \approx 0.54$ kg/s. Note that the Reynolds number in the annulus is Re=GD$_e$/μ≈167,000, therefore the assumption of turbulent flow is accurate. Finally, it is interesting to note that friction contributes to levitation of the control rod slug in two ways, i.e., it creates the shear stress applied to the lateral area of the control rod, and it decreases the pressure P_{out} applied to the top face of the control rod. This explains why the friction factor is present twice in Eq. (15).

ii) Because the volumetric heat generation rate is zero, the temperature is constant along the radial coordinate within the control rod. That is, the solution of the heat conduction equation within the control rod is T(r)=T_o, where T_o is the temperature at the surface of the control rod. Because the heat flux and the coolant bulk temperature vary along the axial direction, T_o will also vary axially, and must be found from knowledge of the surface heat flux distribution and heat transfer coefficient in the annulus. The heat transfer coefficient in this case can be found from the Dittus-Boelter correlation, as the fluid is water (Pr≈0.884), the flow is turbulent ($G = \dot{m} / \left[\frac{\pi}{4}(D_o^2 - D^2)\right]$=1384 kg/m^{2}s, Re=GD$_e$/μ=92267) and entry region effects can be neglected (L/D$_e$>>40). Thus, Nu=0.023Re$^{0.8}$Pr$^{0.4}$≈205, and h=Nu·k/D$_e$≈19.2 kW/m^{2}K. The heat flux is:

$$q''(z) = \frac{\pi}{2} q''_{av} \cos(\pi z / L)$$

where q''_{av}=80 kW/m^2 and the origin of the axial coordinate z was taken at the control rod midplane. Therefore, the linear power profile is:

$$q'(z) = q''(z)\pi D = \frac{\pi^2 D}{2} q''_{av} \cos(\pi z / L) = q'_o \cos(\pi z / L) \tag{16}$$

where $q'_o \equiv \frac{\pi^2 D}{2} q''_{av}$=7.89 kW/m is the linear power at the midplane. Set in these terms, the problem is very similar to the calculation of the maximum outer temperature of the cladding in a fuel rod. The solution to this problem is in Section IV.B of Chapter 13 of the T&K textbook. Therefore, the location at which the surface temperature has a maximum, z_{max}, can be found directly from Eq. (13-25b) in T&K:

$$z_{\max} = \frac{L}{\pi} \tan^{-1}\left[\frac{DLh}{\dot{m}c_p}\right] \approx 0.837 \text{ m} \tag{17}$$

And the maximum surface temperature $T_{o,max}$ can be found substituting Eq. (17) into Eq. (13-22) in T&K:

$$T_{o,\max} = T_{in} + q'_o \left[\frac{L}{\pi \dot{m} c_p} \left(\sin(\pi z_{\max} / L) + 1 \right) + \frac{1}{\pi D h} \cos(\pi z_{\max} / L) \right] \approx 298.3°\text{C}$$

Where T_{in}=284°C and the extrapolation length L_e was assumed equal to the physical length of the control rod, L.