RELATIVISTIC INTERSTELLAR SPACE FLIGHT

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ABSTRACT

A physical model for the interstellar ram-jet is proposed and closely investigated in this paper. An attempt is made to consider the problem with the utmost optimism, taking care to preserve a maximum of generality. First, it is shown that a static "slowly-varying" magnetic field is the most likely candidate for funnelling the galactic matter into the reactor. Then the full consequences of this magnetic field are worked out in terms of the equation of motion for the interstellar ram-jet and the restrictions that are put on its motion both at low velocities and in the relativistic limit. At the high velocities, there is a severe limitation put on the motion of the ram-jet by the minimum mass theorem of Kash.[7] The result is that the vehicle can accelerate at 1 g only up to a certain "cutoff" velocity; subsequent motion must obey a different equation of motion which predicts a linear increase in the time-dilution factor with the proper time. In the case of deuterium as a galactic fuel this cutoff is seen to be well below relativistic velocities and the rate of increase of the time dilution factor negligible.

INTRODUCTION

"Was this the face that launched
a thousand ships,
And burnt the topless towers
of Ilium?
Sweet Helen, make me immortal
with a kiss.
Her lips suck forth my soul
See, where it flies!"[1]

It has long been speculated that there is life in the universe other than what we know on earth. This would seem a feat of the imagination: a victory over the ideology of day-to-day life which holds man maternally bound to concerns here and now. Man's sensitivity is aroused by its object, the surroundings; and now that he has turned to regions in space far removed from home - where possibly he can find civilizations much more advanced than his own - it is no surprise that he will want to reach out and touch them for himself. In seeking to extend man's experience beyond the sphere of his native planet, Western Civilization, at least, is ever inviting an encore to the greater moments in its past, in love as it is with the Promise of Grace and the "face that launched a thousand ships."

There have been a few investigations into the realization of this dream. The most optimistic conclusions were drawn by R. W. Bussard[2] and a bold perspective given by Carl Sagan[3] a short time later. It is the belief of a few of us that the laws of physics impose severe constraints on the interstellar ram-jet of Bussard - severe enough, in fact, to raise the time-scale of interstellar flight hopelessly beyond a human life-time. I wish to undertake here a further development of the model proposed by Bussard. An attempt is made to impose on the assumptions and calculations the most <u>optimistic</u> point of view taking care not to ignore the existing well-known phenomena.

Since the expected distances between civilizations in our galaxy are of the order of a thousand light years,[3] it has been pointed out that any interstellar communication to be made within a human life-time requires vehicles that can attain near-optic velocities in the order of years with maximum time-dilation factors ($\gamma = (1 - \frac{v^2}{c^2})^{-1/2}$ much greater than unity. The momentous problems that are encountered in any such vehicle that carries its own propellant have been well-worked out.[4,5] It was not until Bussard's proposal that there was any optimism associated with the subject. The main feature of his star-ship is that it does not carry its own fuel. It collects matter from the interstellar medium and burns it in a fusion reactor, producing enough energy somehow to give the reaction products exhaust velocities sufficient to accelerate the vehicle to near-optic velocities. Admittedly, the process by which we convert the reaction energy into a directed backward impulse on the particles in the exhaust gases is unclear. It is a matter for further investigation by anyone who will pursue it.

Yet another problem is that of constructing the fusion reactor to do the job. If we use the hydrogen fusion chain as our energy source, it has been pointed out to me[6] that the length of time necessary for the reaction to go to completion in any reactor we can design may well allow the radiation from the plasma to carry off more energy than we can get from the reaction itself. To prevent this energy loss would require a means of reabsorbing the radiation and converting it, too, into a directed impulse exerted on the exhaust jet. I do not intend to deal with these problems here. I would like to point out, however, the staggering challenge and the proliferation of difficulties in the subject, if interstellar travel should ever become of immediate interest to physicists and to our society.

In order to achieve near-optic velocities in a matter of years rather than centuries ship-time it is necessary to obtain a proper vehicle acceleration of the order of earth's gravity. It is also sufficient, since our passengers will not be able to live under conditions of acceleration much in excess of this figure. Bussard has demonstrated that, with the use of the proton fusion chain, we can achieve a proper acceleration of 1 g only if the ratio of the total vehicle mass to the cross-section of interstellar matter swept out and confined as fuel by the ram-jet is less than 10^{-8} (grams/cm^2) per unit reactive-nucleon number-density in the interstellar medium. Due to the low interstellar number-densities and the relatively high densities of structural materials known to us, it is easy to see that our vehicle will have to take on a physical size much smaller than the size of space from which it takes its fuel — one of the critical departures that this ram-jet takes from its familiar counterpart in the earth's atmosphere.

This theoretical speculation is supported further by the fact that the densities of fuel required in a fusion reactor are many orders of magnitude greater than the density of nucleons in interstellar space. Accordingly, my purpose in this paper is to explore the <u>confinement</u> of large volumes of tenuous interstellar matter into the <u>comparatively</u> small volume of our reaction chamber. The methods proposed by Bussard and others[2,3] include electromagnetic lenses and static electromagnetic fields. These, of course, require that the interstellar gas be ionized: we simply shine a beam of light in the forward direction to ionize the medium ahead of the vehicle. This is done with very little cost as it takes only a few electron volts to ionize each atom while we get millions of electron volts from each nucleus that reacts. I make the assumption all along that our reactor has a perfect efficiency. Of the methods proposed, I have found that a slowly-varying magnetic field is the only suitable means of confining the ionized interstellar gas. I have eliminated the possibility of lenses on the premise that our vehicle's structure must be much smaller than the volume of interstellar gas we want to confine. The size of the lens represents the amount of interstellar gas that it effectively confines and the field source in a lens must be at least as big as the lens itself. I would hesitate to consider lenses until we find structural materials much lighter than ones in use today.

THE MAGNETIC FIELD

Interstellar space is abundant in hydrogen gas with a number density n of order unity. We have reason to suspect that deuterium occurs much less in interstellar hydrogen than it does in the terrestrial species. But I will adopt the isotopic abundance observed on earth, one deuterium per eight thousand hydrogen nucleii, as an upper limit for the interstellar abundance of deuterium. The interstellar gas has a temperature of about ten degrees (°K) and there is a magnetic field of about a microgauss which, at equilibrium, represents an energy density of the same order of magnitude as the thermal energy density. When ionized, the hydrogen gas breaks up into electrons whose characteristic radius of gyration is about 2×10^5 cm and protons with a radius of gyration $\sim 10^7$ cm. The centers of gyration of all the particles have trajectories which are, in general, parallel to the field lines.

Any magnetic field that we set up around our ram-jet as it accelerates to near-optic velocities in the interstellar medium will have to meet some very rigid requirements if it is to effectively localize within a very small compartment large volumes of the gas. Remember that, in the proper frame of the ram-ject, the interstellar medium is travelling backwards at the same velocity the ship is moving through space. First, the radius of gyration which represents an uncertainty in position of our fuel particles, the protons or deuterons, must be reduced from $\sim 10^7$ cm to the dimensions of our reactor. Lastly, the trajectories of the centers of gyration must form a bundle permeating the volume of interstellar matter which we hope to compress into the reactor. In order to do this, the field we create must penetrate the interstellar medium over large distances. At the same time it must not possess discontinuities large enough to cause any discontinuities in the motion of the interstellar gas such as shock waves, for this would result not only in a momentum loss to the surroundings, but a severe radiation loss as well. In other words, the field associated with the ram-jet must be "slowly-varying" so that the field the particles see does not vary significantly in one period of gyration of their motion. In such a field the particle motion may be described by certain "adiabatic" invariants, one of which is the magnetic flux through the orbit.

Let us suppose the field has symmetry about an axis parallel to the velocity of the ram-jet. Further let us assume there is a region near the axis of this field in which the lines diverge <u>very slowly</u> away from the axis and that all the field lines in this region eventually lead into the mouth of the reactor. There is a boundary, a plane normal to the axis of symmetry, far in front of the ship, beyond which the interstellar fields dominate. Within this region, however, the interstellar fields dominate. Within this region, however, the fields of the ram-jet dominate and the particle-orbits will preserve their flux linkage. For the sake of simplicity and optimism, I will neglect the magnetic field outside this region. I will be content to discover the restrictions imposed on the interstellar ram-jet by the part of the field that is necessary for confining the protons (or deuterons) to the reactor.

If $p_\perp$ is the particle momentum transverse to the field lines, a is the radius of gyration and B is the field strength, we can write two quantities proportional to the flux linkage that are invariants of the motion:

$$p^2{}_\perp/B = \text{constant}$$

$$Ba^2 = \text{constant}$$

We can now express $p_\perp$ and a as explicit functions of the magnetic intensity B:

$$p_\perp(B) = \frac{B}{B_o}\, p_{\perp o} = \frac{mkT\,B}{B_o}$$

$$a(B) = \frac{B_o}{B}\, a_o = \frac{m_{c^2}kT}{e^2 B_o B}$$

where $p_{\perp o} = \sqrt{mkT}$ is the rms value of the transverse momentum due to thermal motion beyond the fields of the ship and, $a_o = p_{\perp_o} c/eB_o$ is the corresponding radius of gyration, both of which quantities are the same in the proper frame of the ship as they are in the local galactic rest-frame. B_o is the local galactic field.

Notice that at the same time the radius of gyration is reduced, the particle momentum is diverted more and more transverse to the field lines as the particles spiral into stronger fields on their way to the reactor. If the field is strong enough, eventually all the momentum will be converted into transverse motion. At this point, the particle motion will simply be mirrored back along the same field lines it came in on and will never reach the reactor. Obviously this will result in a momentum loss to the surroundings, a problem I will deal with in a later section. Suppose we do not mind reflecting all the particles that have an initial transverse momentum greater than F times the rms thermal momentum. Assuming that our ram-jet is travelling much faster than the thermal velocities of the interstellar gas, the particle momentum initially transverse to the axis is much less than its momentum parallel in the ship's proper frame. I then arrive at the following maximum of field intensity which occurs near the mouth of the reactor

$$B_m \doteq B_o \frac{p^2}{F^2 p^2_{\perp_o}} = B_o \frac{m c^2}{kT} \frac{\gamma^2 \beta^2}{F^2}$$

where the $\gamma = (1 - \frac{v^2}{c^2})^{-1/2}$ and $\beta = v/c$ refer to the motion of the ram-jet.

Since the interstellar gas obeys Boltzmann Statistics, the field of particles approaching the ship has a two-dimensional Boltzmann momentum distribution in the transverse direction. An integration over this distribution gives e^{-F^2} as the fraction of all particles of the type under consideration that are mirrored by the magnetic field. In effect, the ram-jet undergoes an elastic collision with a fraction e^{-F^2} of these particles while the rest, a fraction $1 - e^{-F^2}$, make it past the point of maximum field intensity at the reactor.

Not all of the particles that make it past the high field point necessarily go into the reactor, however. This is because not all have a gyration radius small enough to confine them within the dimensions of the reactor intake. If our reactor has an intake cross-section $\sigma = \pi d^2$ then all particles that have an initial gyration radius smaller than Na_o can make it into the reaction chamber (if they are not reflected) where N is given by the conservation of magnetic flux through the particle orbit:

$$Na_o = \frac{B_m}{B_o} d$$

$$N^2 = \frac{\sigma}{\pi} \left(\frac{kT}{eB_o}\right)^2 \frac{\gamma^2 \beta^2}{F^2}$$

This corresponds to a fraction e^{-N^2} of particles that can't be confined to the reactor by our magnetic field and, even if they manage to penetrate the magnetic field, will simply join the exhaust jet out the back. To get the total fraction of all the gas particles that will be confined to the reactor, one simply chooses the smaller of the two quantities given above: the fraction that are not reflected or the fraction whose radius of gyration is small enough to be reduced to the dimensions of the reactor by the magnetic field.

$$I = \text{Intake Fraction} = \begin{cases} 1 - e^{-N^2}, & N < F \\ 1 - e^{-F^2}, & F < N \end{cases}$$

Due to the isotropic momentum distribution in the interstellar gas, it is clear that the ram-jet must start out at velocities much larger than the thermal velocities of the gas particles. This is because the magnetic field ratio B_m/B_o that is required to confine the gas sufficiently is very large; the gas particles will be reflected, and there will be a great loss of momentum to the surroundings, unless their momentum in the ship's proper frame is directed largely backwards along the axis. Thus, in order to use interstellar hydrogen as a fuel we must first attain velocities higher than about 30 km/sec. It is evident, at least, that there is a definite lower limit on the velocities at which we can launch the ram-jet. This is a great departure from its counterpart in the earth's atmosphere which initiates its fuel intake by pressure gradients (suction). It makes use of the high pressures in the earth's atmosphere which unfortunately, is not available to us out in space.

It is now possible to derive an explicit form for the magnetic field B as a function of the position z along the symmetry axis. We simply require that the field does not change appreciably over one cycle of a proton gyration:

$$(1) \qquad \epsilon B = 2\pi \frac{v_{\parallel}}{v_{\perp}} a \frac{dB}{dz}$$

$$= 2\pi\gamma\beta \frac{m_p c^2}{e} \left(1 - \frac{kTB/B_0}{m_p c^2 \gamma^2 \beta^2}\right)^{1/2} \frac{1}{B} \frac{dB}{dz}$$

$$(\epsilon << 1)$$

I have included the fact that the total momentum and energy of the particles are also constants of the motion (neglecting radiation):

$$p = \sqrt{p_{\parallel}^2 + p_{\perp}^2} \sim p_{\parallel_0} \sim m_p c \gamma\beta$$

ϵ is the fraction of field variation we allow the proton to see in each of its gyrations, a number presumably much less than unity if the assumption of adiabatic invariance is to remain valid. Because of the dependence on mass of the expression above, the variation in field during one gyration is much greater for protons than for electrons. Any field $B(z)$ which satisfies the above conditions appropriate for the adiabatic invariance of proton gyrations will certainly do so for the electrons. In the case that the proton orbits are not appreciably oblique to the field lines

$$\cos\theta_p = \sqrt{1 - \frac{p_\perp^2}{p^2}} \sim 1$$

we can find a very simple integral for the functional dependence of the field:

$$B(z) \cong \left(1 + z\,\frac{\epsilon\, e B_m}{2\pi\gamma_c\beta\, m_p}\right)^{-1} = \frac{B_o\left(\frac{m_p c^2}{kT}\right)\frac{\gamma^2\beta^2}{F_o^2}}{1 + z\epsilon\left(\frac{eB_o}{2\pi kT}\right)\frac{\gamma\beta}{F_p^2}}$$

This shows that the particle orbits are confined roughly to the surfaces of paraboloids centered on the field lines.

The intake cross-section, A, of the interstellar matter confined by the magnetic field is determined by the number of field lines found suitable as trajectories for the particles. Now the quantity, F, proportional to the highest initial transverse momentum of particles admitted into the reactor, is good only for particles located very close to the axis where the field lines are nearly parallel to the motion of

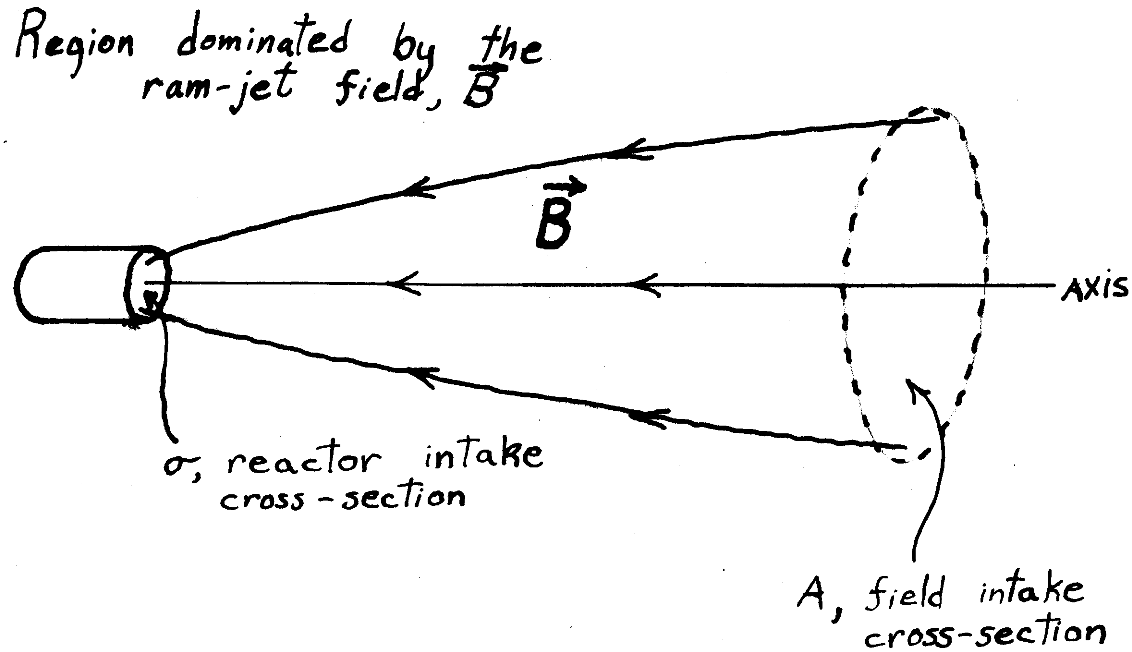


Figure 1. Diagram of the idealized field configuration used to confine the interstellar plasma, as seen in the proper frame of the ram-jet. In particular I have illustrated the bundle of field lines used as trajectories for the particle gyrations.

the ship. Far away from the axis the angle that the lines make with the relative velocity of the interstellar gas throws some of the initial backward momentum of the incident particles into the transverse motion. As it turns out, the curvature of the lines does not appreciably affect the motion of the particles. This is because all the contributions the curvature makes to the transverse momentum during a single gyration will cancel out since the curvature stays appreciably the same and the phases of the contributions all cancel during one cycle of the motion. In any case, we must make sure that the total fraction of particles reflected is not altered significantly by the convergence of the field lines. The requirement, then, that I must put on the field is that the amount of backward particle momentum it initially throws into the particle gyrations is much less than F times the rms value of the thermal momentum. I will write the radius of the effective intake cross-section as Ma_e, the largest distance off-axis at which the field lines obey the condition stated above. From this we can also deduce that the size of the bundle anywhere along the axis is M times the radius of gyration of the protons at that point. The shift in the quantity F for particles at the edge of the intake area is $\Delta F_p = -\epsilon M/4\pi$ for protons and $\Delta F_e = -\frac{m_e}{m_p}\epsilon M/4\pi$ for electrons. The conditions on M is thus given

$$\Delta F_p << F_p \qquad \frac{\epsilon M}{4\pi} << F_p = \frac{m_p}{m_e} F_e$$

$$\Delta F_e << F_e$$

Notice that if we allow $M = 4\pi F/\epsilon$, we are reflecting <u>all</u> the p articles from the outside edge of our intake cross-section. This situation can be shown fatal to our fraction of intake. It also results in a large fraction of particles that are reflected by the field. But we are only interested in orders of magnitude here and since the situation is aleviated if we decrease $M = 4\pi F/\epsilon$ by an order of magnitude

I will be content to use this upper limit as a guess. The intake-cross-section, then, is approximated according to the preceeding discussion:

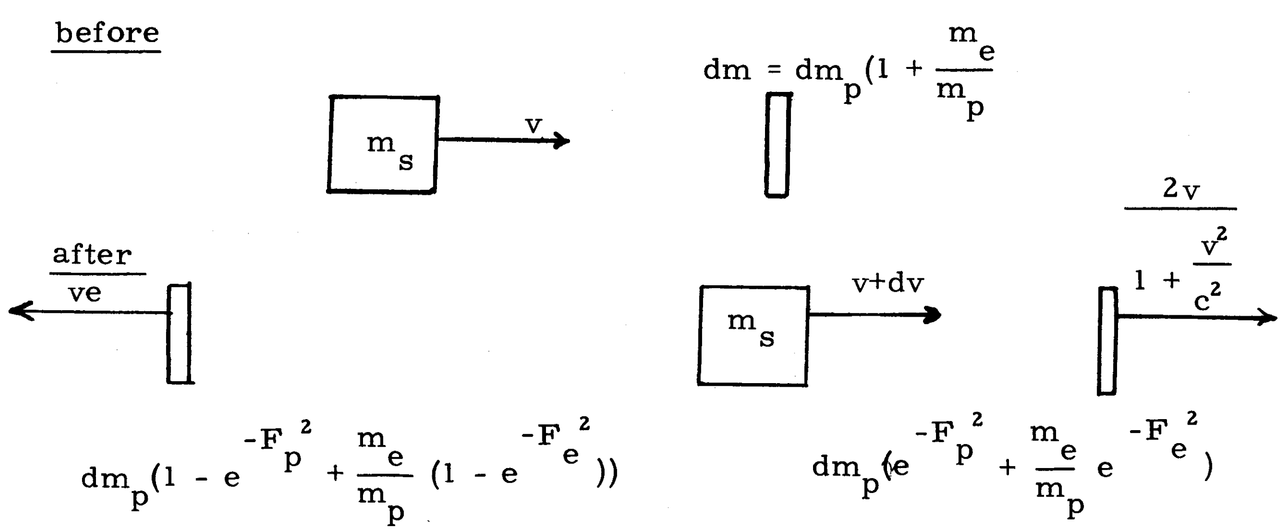
$$A = \pi a_o^2 M^2 = 16 \pi^3 \frac{F^2}{\epsilon^2} \frac{m_p c^2 kT}{e^2 B_o^2}$$

Since it is unclear exactly how small we intend to allow $\epsilon << 1$ to become, it is hard to ascribe any real upper limit on the intake cross-section as yet. Obviously, the longer we stretch our magnetic field along the axis, the less abruptly the field lines will converge toward the axis and the larger our effective intake can become. In a later section I will develop a better defined (and more lenient) upper limit for the intake cross-section. This will be done using the minimum mass theorem of Kash.[7]

EQUATION OF MOTION

In considering how the ram-jet and its magnetic field will interact with the interstellar gas, I have attempted to outline some of the difficulties that are involved in trying to localize large volumes of the gas inside a comparably small volume. There would, in fact, be no difficulty if there was no time limit on this process; stars, themselves, are examples of localized galactic matter and are the result of billions of years of condensation and gravitational compression. The process we are talking about, however, must be a very fast one - fast enough to enable us to compress a beam of relativistic particles $\sim 10^{7}$ cm thick into a beam about the size of our reaction chamber. The statements made so far are preliminary; I have laid the framework from which will evolve the form adopted by the equation of motion and more general limitations that arise from a realistic interpretation of the ram-jet.

Figure 2. A diagram illustrating the kinematical problem of the ram-jet travel through and reacting with the interstellar medium.



The kinematical diagram above displays the interaction of the ram-jet of mass m_s with an increment of the interstellar gas containing a proton mass $dm_p = A\rho_o c\beta\gamma dt_s$ and by electrical neutrality an electron mass $dm_e = \frac{m_e}{m_p} dm_p$. dt_s is the corresponding increment of time spent in the ship's proper frame. α, $\sim .007$ in the case of the proton fusion chain, is the fraction of the rest-mass available for conversion into energy in the fusion reactor. The parameters F_e, F_p are simply related to the maximum field intensity according to:

$$\frac{B_m}{B_o} = \frac{m_p \gamma^2 c^2 \beta^2}{kTF_p^2} = \frac{m_e \gamma^2 c^2 \beta^2}{kTF_e^2}$$

$\rho_o = nm_p$ is the density of the galactic fuel. I have also included the radiation (dp, dE) by the ionized gas in the magnetic field which, by the axial symmetry of the field, must carry a total momentum directed parallel to the motion of the ship. Invoking the conservation of energy, we obtain the relationship:

$$E_{before} = m_s \gamma c^2 + dm_p \left(1 + \frac{m_e}{m_p}\right) c^2$$

$$= m_s c^2 (\gamma + d\gamma) + dE + dm_p c^2 \left[\left(1 - e^{-F_p^2}\right) - \alpha I + \frac{m_e}{m_p}\left(1 - e^{-F_e^2}\right)\right] \gamma_e + dm_p c^2 \left(e^{-F_p^2} + \frac{m_e}{m_p} e^{-F_e^2}\right) \frac{1 + \beta^2}{1 - \beta^2}$$

$$= E_{after}$$

where the ram-jet velocity v and the exhaust velocity v_e determine the quantities:

$$\beta = \frac{v}{c} \qquad \gamma = \left(1 - \beta^2\right)^{-\frac{1}{2}}$$

$$\beta_o = v_e/c \qquad \gamma_e = \left(1 - \beta_e^{\,2}\right)^{-\frac{1}{2}}$$

Similarly, the conservation of momentum yields

$$p_{before} = m_s \gamma \beta c$$

$$= m_s c \ \gamma\beta + \frac{d\gamma}{\beta} + dp - dm_p c \ (1 - e^{-F_p^{\,2}}) - \alpha I$$

$$+ \frac{m_e}{m_p}(1 - e^{-F_e^{\,2}}) \beta_e \gamma_e + 2dm_p c \left(e^{-F_p^{\,2}} + \frac{m_e}{m_p} e^{-F_e^{\,2}}\right) \gamma\beta = p_{after}$$

Next we define the energy and momentum radiated per unit rest mass of the protons

$$\frac{dE}{d(m_p c^2)} = E \qquad \frac{d_p}{d(m_p c^2)} = p$$

and our conservation laws become:

$$-m_s \frac{d\gamma}{dm_p} + (1 + \frac{m_e}{m_p}) - \gamma^2 (1 + \beta^2) \left(e^{-F_p^{\,2}} + \frac{m_e}{m_p} e^{-F_e^{\,2}}\right) - E$$

$$= \left[(1 - e^{-F_p^{\,2}}) + \frac{m_e}{m_p} (1 - e^{-F_e^{\,2}}) - \alpha I\right] \gamma_e$$

$$-\frac{m_s}{\beta}\frac{d\gamma}{dm_p} - 2\left(e^{-F_p^2} + \frac{m_e}{m_p}e^{-F_e^2}\right)\beta\gamma^2 - p$$

$$= \left[(1 - e^{-F_p^2}) + \frac{m_e}{m_p}(1 - e^{-F_e^2}) - \alpha I\right]\beta_e\gamma_e$$

The exhaust velocity dependence can be eliminated and the two equations above combined to give a first-order second degree differential equation for the ram-jet velocity. The resulting equation of motion is:

$$\frac{d\beta}{dt_s} = \frac{Ap_oc}{m_s}\left(1 + e^{-F_p^2} + \frac{m_e}{m_p} + \frac{m_e}{m_p}e^{-F_e^2} - E + \frac{p}{\beta}\right)\Bigg(-1 + \Bigg\{1 + \Bigg(\left(1 + \frac{m_e}{m_p}\right)\frac{2I\alpha - 2E}{\gamma^2\beta^2}$$

$$+ \left(e^{-F_p^2} + \frac{m_e}{m_p}e^{-F_e^2}\right)\left(\frac{4p}{\beta} + \frac{2E(1+\beta^2)}{\beta^2} - 2\left(1 + \frac{m_e}{m_p}\right) - \frac{2\alpha I}{\gamma^2\beta^2}\right)$$

$$+ \frac{E^2 - p^2 - \alpha^2 I^2}{\gamma^2\beta^2}\Bigg) \times \left(1 + \frac{m_e}{m_p} + e^{-F_p^2} + \frac{m_e}{m_p}e^{-F_e^2} - E + \frac{p}{\beta}\right)^{-2}\Bigg\}^{1/2}$$

It is instructive to look at the restrictions this equation puts on our magnetic field if we are to have acceleration: $\frac{d\beta}{dt_s} \gg 0$. Notice that if F_p and F_e are large numbers, then the condition that we have acceleration is simply $E < \alpha I$. Since the functional dependence of I on the velocity is unknown at this point and can be deduced only from a detailed knowledge of the ram-jet characteristics σ, given the velocity dependence of the F's, I will use the fact that $I < 1$ and obtain a more emphatic condition on the specific radiation $E \lessdot \alpha$. Actually, at high velocities the fraction I of particles used as fuel will become very close to unity. Notice also in the equation of motion that the radiation retards the motion much less at the low velocities $\beta \lll 1$. It turns out that the magnitude E of the radiation itself is much larger at relativistic velocities. Consequently, I will consider radiation effects only in the relativistic limit and the assumption $I \simeq 1$ becomes valid.

Without the radiation ($E = p = 0$) it is easy to see that large F_e and F_p are favorable to the acceleration of the ram-jet. Finding the lower limit on the F's which allows acceleration, I will be able to find the restrictions we must put on the magnetic field. At the same time, however, we must have a magnetic field ($\frac{B_m}{B_o} = \frac{mc^2\gamma^2\beta^2}{kTF^2}$) large enough to confine the particle gyrations to the dimensions of the reactor. In other words, we want the fraction of particles that make it into the reactor:

$$I = \begin{cases} 1 - e^{-F_p^2}, & F_p^2 < N^2 = \sigma\left(\frac{eB_o}{kT}\right)^2 \frac{\gamma^2\beta^2}{F_p^2} \\ 1 - e^{-N^2}, & F_p^2 > N^2 = \sigma\left(\frac{eB_o}{kT}\right)^2 \frac{\gamma^2\beta^2}{F_p^2} \end{cases}$$

as large as possible without our magnetic field being so large as to reflect all the particles and retard the motion of the ram-jet. The solution to this problem is to find out how small F_p can be without destroying the acceleration. Our determination of F_p will yield I as a function of velocity. This will, further, determine the equation of motion which tells us how soon the ship will reach its destination.

During the launch of our vehicle ($\beta \ll 1$), the electrons have no effect on the motion of the ram-jet. We need only worry about the protons. With $F_p \lesssim 1$ we have $F_e \lesssim \frac{m_e}{m_p}$ and it is easy to see that effectively all the electrons are reflected whenever an appreciable fraction of the protons are reflected. The condition for acceleration in this approximation becomes

$$F_p^{\,2} > \sinh^{-1} \frac{\gamma\beta}{\alpha}$$

and the effective equation of motion is:

$$\frac{d\beta}{dt_s} = \frac{A\rho_o c}{m_s} \begin{cases} \sqrt{2\alpha I\beta} \;, & \beta^2 \ll \sqrt{\alpha} \\ (1 - \beta^2)\alpha I \;, & \beta^2 \gg \sqrt{\alpha} \end{cases}$$

The integration of this equation to obtain the elapsed time Δt_s in the proper frame of the ram-jet during acceleration to near-optic velocities can be done only after finding a value for the intake cross-section A. In a later section I will find a definite upper limit for A which is a function of the velocity. This will give us a lower limit for the elapsed ship-time Δt_s.

The motion of the ram-jet goes according to the conditions stated above until the time-dilation factor, γ, reaches about 8 . It is at this point that the electrons can no longer be ignored and the magnetic field must be lowered in order to prevent reflecting them. The following condition is imposed on F_p by the equation of motion if we are to have acceleration beyond this point:

$$F_p^2 = \frac{m_p}{m_e} F_e^2 < \frac{m_p}{m_e} \log\left(\frac{\gamma^2 - 3/2}{m_p\, \alpha/2m_e}\right)$$

Depending on the size σ of the reactor intake, this may reduce the fraction I of fuel consumption and drag out further the process of acceleration.

A significant fraction $I \sim 1 - e^{-1}$ $(N \sim 1)$ of fuel consumption is reached sometime during the acceleration of the ram-jet; subsequently, F_p can take on a new functional form:

$$F_p^2 = \sigma\left(\frac{eB_o}{kT}\right)^2 \gamma^2\beta^2 = \frac{m_p}{m_e} F_e^2$$

This means that, beyond this point, our magnetic field is held constant and has a maximum intensity ratio:

$$\frac{B_m}{B_o} = \frac{m_p c^2 kT}{(eB_o)^2\, \sigma}$$

This relationship will help to interpret the calculation of the radiation and the magnetic field energy, quantities that become of interest in the relativistic limit.

RADIATION

As the ram-jet approaches closer to the speed of light, the conflict between momentum loss to the interstellar gas and the low proportion of fuel confined to the reactor becomes less of a problem. However, the synchrotron radiation emitted by the electrons trapped in the magnetic field becomes significant and will impose certain restrictions on the features of the ram-jet. The bremsstrahlung emitted by the electron-ion collisions is considered here, but is found to be negligible compared to the synchrotron and will not play any role in describing our model for interstellar flight.

The lorentz-invariant form of the radiation from a moving charge is given by:[8]

$$\frac{dU}{dt} = -\frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dp_\mu}{d\tau}\right)^2$$

For electrons gyrating in a magnetic field this can be rewritten:

$$\frac{dU}{dt} = -\frac{2}{3}\frac{e^2}{m^2c^3}\gamma^2\dot{p}^2$$

From the dependence on mass we see that much more energy is radiated by the electrons ($m_e << m_p$) than by the ions, a fact made even clearer when we notice that the electrons are deflected much more strongly by the magnetic field ($\dot{p}_e << \dot{p}_p$) while the heavier ions tend to pass through it unaffected. In order to calculate the radiation in the proper frame of the ram-jet we can make the following identifications from what has been said before:

$$\gamma^2 \dot{p}e^2 = \gamma^2 m_e^2 \frac{v_e^4}{a_e^2} = \frac{kT(eB_o)^2}{m_e c^2} \left(\frac{B}{B_o}\right)^3$$

$$dt = \frac{dz}{v_{11}} = \frac{dz}{c\beta \, (1 - kTB/B_o m_e c^2 \gamma^2 \beta^2)^{1/2}}$$

where I have transformed increments in the proper time dt into elements of motion dz along the axis of symmetry. Increments of energy radiated by an electron are then given by:

$$dE'_{rad.} = \frac{2}{3} \frac{e^2 kT(eB_o)^2}{(m_e c^2)^3 \beta} \frac{(B/B_o)^3}{\left(1 - \frac{kT\,B/B_o}{m_e c^2 \gamma^2 \beta^2}\right)^{1/2}} dz$$

To get the momentum radiated on the average by each electron, we simply take the projection of each element dp' of momentum radiated on the axis of symmetry. We know that the radiation pattern of a relativistic particle in a magnetic field is pointed mostly in the direction of motion of the particle and, in our case, has an angular width of approximately $1/\gamma$ where $\gamma = (1 - \beta^2)^{-1/2}$ is the time-dilution factor. In the relativistic limit ($\gamma \to \infty$) this tells us that all the momentum radiated is thrown parallel to the motion of the electron. In fact, since the radiation is thrown out the back with the exhaust, the deterrent effect of the radiation loss is mollified somewhat. Using the relationship between momentum and energy for the photon, $p = E/c$, we can write out the axial component of the momentum radiated by the electron:

$$dp'_z = -\frac{dE'}{c}\left(1 - \frac{kTB/B_o}{m_e c^2 \gamma^2 \beta^2}\right)^{1/2}$$

The transformation of the four-momentum dp'_μ then gives the energy radiated in the local galactic rest-frame in terms of the momentum and energy radiated in the ship's proper frame:

$$dE_{rad} = \gamma\left(dE' + c\beta\, dp_z\right)$$

$$= \frac{2}{3}\frac{e^2 kT (eB_o)^2}{(m_e c^2)^3 \beta}\left(\frac{B}{B_o}\right)^3 \left[\frac{1}{\left(1 - \frac{kTB/B_o}{m_e c^2 \gamma^2 \beta^2}\right)^{1/2}} - \beta\right] dz$$

We can transform increments of distance dz along the axis into changes, dB, in the magnetic field intensity experienced by the electron in the ship's proper frame. This is done by referring to the earlier section where I found the differential relationship that must be satisfied by the magnetic field. I will assume that a very insignificant fraction of the electrons are being reflected, which is valid in the relativistic limit.

For the vast majority of the electrons I can write:

$$\cos\theta_e = 1 - \frac{kTB/B_o}{m_e c^2\gamma^2\beta^2} \sim 1 - \frac{1}{2}\,\frac{kT\,B/B_o}{m_e c^2\gamma^2\beta^2}$$

and the radiation is finally:

$$dE = \frac{4\pi e^3 kTm_p c^2\gamma^2 BdB}{3(m_e c^2)^3\epsilon\, B_o}\left(1 - \beta + \frac{kT\,B/B_o}{2m_e c^2\gamma^2\beta^2}\right)$$

An integration over the range of the field intensity yields an approximation to the energy radiated by each electron. Since, by charge conservation, there is a proton for every electron, we also have calculated the energy radiated for each proton (or ion) that we encounter. The energy thus radiated per unit proton mass is then given by:

$$E^{Synch} \cong \frac{\pi r_e}{\epsilon}\left(\frac{eB_o}{kT}\right)\frac{\gamma^4\beta^4}{F_e^{\,4}}\left(\frac{1}{3} + \frac{2}{9}\,\frac{\gamma^2}{F_e^{\,2}}\right)$$

To get an idea of the relative magnitude of the Bremsstrahlung radiation, an equivalent expression for the specific radiation is found to be:

$$E^{Bremm} \quad \frac{32\,\pi}{3 \times 137}\;\frac{r_e^{\,2} m_e c^2}{eB_o}\;\frac{n}{\epsilon}\sqrt{\frac{m_e}{m_p}}\left(\frac{\gamma^4}{8F_e^{\,4}} + \frac{m_e}{m_p}\,\frac{\gamma^6}{24F_e^{\,6}}\right)\log\frac{\gamma^2}{2F_e^{\,2}}$$

where I have assumed $\gamma^2\beta^2/2F_e^{\,2} >> 1$. The ratio of the Bremsstrahlung to the synchrotron is approximately:

$$\frac{E^{Bremm}}{E^{Synch}} \approx \frac{kT_n}{B_o^{\,2}} \sqrt{\frac{m_e}{m_p}} \; \frac{32\,\pi}{3 \times 137} \; \log \frac{\gamma^2}{2F_e^{\,2}}$$

Remember that the thermal and magnetic energy densities, nkT and $B_o^{\,2}/8\pi$, are nearly the same in all of interstellar space as well as intergalactic space. This means that

$$\frac{kTn}{B_o^{\,2}} \approx \frac{1}{8\pi}$$

or

$$\frac{E^{Bremm}}{E^{Synch}} \approx \frac{4}{3 \times 137} \sqrt{\frac{m_e}{m_p}} \; \log \frac{\gamma^2}{2F_e^{\,2}}$$

If, in the relativistic limit, we want to confine an appreciable fraction of the galactic matter to the reaction chamber, then we must have:

$$\frac{\gamma^2}{2F_e^{\,2}} \sim \frac{m_p}{m_e} \; \frac{\pi}{2\,\sigma} \; \left(\frac{kT}{eB_o}\right)^2$$

It is easy to see that for any reasonable size σ we can adopt for the reactor's intake cross-section, the value of the logarithm will never be enough to bring this ratio up to unity

$$\frac{E^{Bremm}}{E^{Synch}} << 1.$$

Both in interstellar and intergalactic space-flight, the synchrotron mechanism will dominate the energy loss of the ram-jet in its relativistic limit.

As seen earlier in the equation of motion, the condition that must be satisfied by the specific radiation in order that there be acceleration in relativistic limit ($\beta \sim 1$; $F_e, F_p >> 1$) can be expressed $E^{Synch} < \alpha$. Using the expression given above for $\frac{\gamma^2 \beta^2}{F_e^2}$, this yields us a minimum size σ for the reactor intake:

$$\frac{\sigma^2}{1 + \frac{m_p}{m_e} \left(\frac{kT}{eB_o}\right) \frac{1}{\sigma}} > \frac{\pi r_e}{3\alpha} \left(\frac{kT}{eB_o}\right)^3 \left(\frac{m_p}{m_e}\right)^2$$

where I have included the fact that ϵ is small, $\epsilon < 1$. At most, this is a very modest result in the case of interstellar flight where $B_o \sim 10^{-6}$ gauss. It merely requires σ to be larger than a small fraction of a square centimeter. However, the results for an intergalactic ram-jet could prove to be more challenging.

RECAPITULATION

So far, there have been no significant quantitative restrictions put on the ram-jet. I have demonstrated that in the early stages of flight there is a difficulty in not being able to confine very much of the interstellar gas to the reactor intake. This slows down the acceleration at the beginning. Later on, when we must lower the magnetic field to allow the electrons to pass through the reactor, we are in danger of again not being able to confine the protons (or deuterons) sufficiently. This latter problem is not considered here. In the sections following, I will investigate the expected length of time spent during the early stages of flight. The figure arrived at will be a lower limit. More important, however, is the upper limit to the velocity we can attain and still support the desired acceleration of 1 g. This is due to the finite strength of construction materials and the fact that the faster we want to travel, the more force we must exert on the interstellar gas particles to confine them to the reactor. If we maintain a constant intake cross-section - which implies constant acceleration at high velocities - at some point during the acceleration the pressure of the gas against the magnetic field will be sufficient to deform (and burst) the structure we use to contain the field sources. The only way to prevent this is to reduce the intake cross-section, A, and the acceleration, thus reducing the volume of gas confined by the field and the stress the field delivers to the structure.

FIELD ENERGY

In connection with the minimum mass theorem of Kash[7] the most significant calculation I will perform is to determine the total magnetic field energy. This is done simply by integrating the magnetic energy density over the region occupied by the field:

$$E_{mag} = \frac{1}{8\pi} \int A(z) B^2(z) dz$$

A(z) is the cross-section of the bundle of field lines being used as particle-trajectories and B(z) is the field intensity at position z along the axis. The absence of magnetic charge tells us that the divergence of the field must vanish. As a result, the product A(z)B(z) is the same at all positions on the axis. Our expression for the field energy can be simplified

$$E_{mag} = \frac{A_o B_o}{8\pi} \int B(z) dz$$

where A_o is the fuel intake cross-section and B_o is the local galactic field. Using the differential relationship for the magnetic field, the integral over the distance along the axis can be converted into an integral over field intensity:

$$E_{mag} = \frac{A_o B_o \gamma\beta m_p c^2}{4e \ \epsilon} \int \cos\theta_p \ \frac{dB}{B}$$

For near-optic velocities, the proton trajectories are undeflected and very close to parallel to the axis ($\cos\theta_p \sim 1$). In this case the field energy can be written:

$$E_{mag} = \frac{A_o B_o m_p c^2}{4e} \frac{\gamma\beta}{\epsilon} \log \frac{B_m}{B_o} \qquad (\beta \sim 1)$$

The requirement that we must put on the ram-jet as a consequence of the above calculation is that the structure which confines the field sources be strong enough to counterbalance the forces exerted on the sources by the fields they create. This means that the rate of field momentum flow into the structure must be equal to the capacity of the structure to absorb it. Writing it in terms of the electromagnetic and material stress tensors S, σ our requirement is:

$$\frac{d}{dx_i} \left(S_{ij} + \sigma_{ij} \right) = 0$$

or

$$\int x_i \frac{d}{dx_i} \left(S_{ij} + \sigma_{ij} \right) d^3x = 0$$

An integration by parts yields:

$$- \int (\mathrm{Tr}S)\, d^3x = \int (\mathrm{Tr}\,\sigma)\, d^3x$$

The left side of the equation above is the magnetic field energy. Assuming a homogeneous density ρ of the structural material used and a maximum tensile strength $(\sigma_{ii})_{max}$ we can find the minimum mass[7] of the structure required to support the field:

$$m_{st} > \frac{\rho}{(\sigma_{ii})_{max}} E_{mag}$$

or

$$m_{st} > \frac{A_o B_o m_p c^2 \gamma\beta}{4e\,\epsilon} \frac{\rho}{(\sigma_{ii})_{max}} \log \frac{B_m}{B_o}$$

This certainly requires a minimum mass for the ram-jet as a whole, $m_s > m_{st}$. Rearranging this relationship, we see that the minimum mass theorem has led us to a condition on the velocity attainable by the ram-jet

$$\gamma\beta << \frac{4e}{B_o m_p c^2} \frac{(\sigma_{ii})_{max}}{\rho} \frac{m_s}{A_o} \frac{1}{\log \frac{B_m}{B_o}}$$

where I have used the fact that the magnetic field does not change appreciably over one cycle of the proton gyrations ($\epsilon << 1$).

If the magnetic field created around the ram-jet is to perturb the interstellar medium enough to confine the particle motions, then it must be significantly larger than the galactic fields. Although we saw earlier that the field ratio was given by

$$\frac{B_m}{B_o} = \frac{m_p c^2 kT}{(eB_o)^2 \sigma} ,$$

a very large number for reasonable values of σ, I will be content to choose $B_m/B_o \sim e$ as a significant field ratio. The most optimistic interpretation of the minimum mass theorem ($\epsilon = 1$, $B_m/B_o = e$) yields:

$$\gamma\beta < \frac{4e}{B_o m_p c^2} \left(\frac{m_s}{A}\right) \frac{(\sigma_{ii})_{max}}{\rho}$$

In order to interpret this equation, we must find some reasonable range of values for m_s/A_o. Remembering that we want the proper acceleration, a_s, to be of order 1 g. we can find the condition we want on m_s/A_o from the equation of motion for relativistic velocities ($\beta >> \alpha$):

$$a_s = c\gamma^2 \frac{d\beta}{dt_s}$$

$$+ \frac{A}{m_s} n\alpha m_p c^2 \sim 1 \text{ g.}$$

or

$$\frac{m_s}{A} \approx \frac{n\alpha m_p c^2}{g}$$

This allows us to state a cutoff velocity in terms of the properties of the structural materials ($(\sigma_{ii})_{max}$, ρ); the energy yield and local number density of the fuel (α, n); and the local galactic field intensity B_o:

$$(\gamma\beta) = \frac{4e}{g}\left(\frac{n\alpha}{B_o}\right)\frac{(\sigma_{ii})_{max}}{\rho}$$

For typical conditions of field and number density in the interstellar medium, this cutoff takes on the following form in the case of the proton and deuterium fusion chains:

$$(\gamma\beta)_{c,\ \text{proton}} \approx 1.3 \times 10^{-9} \frac{(\sigma_{ii})_{max}}{\rho}$$

$$(\gamma\beta)_{c,\ \text{deut.}} = 2.3 \times 10^{-14} \frac{(\sigma_{ii})_{max}}{\rho}$$

I have used the terrestrial isotopic abundance of deuterium, one deuterium for every eight thousand protons, as an optimistic guess (an upper limit, at least) of its abundance in interstellar hydrogen. To get an idea of the type of material strength needed to achieve the desired relativistic velocities at an acceleration of 1 g., I have constructed a table of velocities attainable at 1 g. by an interstellar ram-jet for several well-known materials.

Proton Fusion $(\gamma\beta)$	Deuterium $(\gamma\beta)$	Structural Material	$\frac{(\sigma_{ii})_{max}}{\rho}$ [9]
.82	$.145 \times 10^{-4}$	Aluminum	$.063 \times 10^{10}$
3.5	$.62 \times 10^{-4}$	Stainless Steel	$.27 \times 10^{10}$
70.5	12.4×10^{-4}	Patented Steel	5.4×10^{10}
195	34.5×10^{-4}	Diamond	15.5×10^{10}
55	9.7×10^{-4}	Silica	4.2×10^{10}
65	11.5×10^{-4}	Copper	5.0×10^{10}

Table 1: List of cutoff velocities of ram-jet accelerating at 1 g. in a typical interstellar medium.

Now, I do not mean to say that acceleration is impossible beyond this cutoff. I simply mean that theram-jet cannot continue to accelerate at 1 g. past this point. It must decrease its acceleration - that is, its intake cross-section - sufficiently to compensate for the added strain on the magnetic field with the increasing velocity. The proper acceleration must decrease with velocity according to

$$a_s(\beta) = \frac{(\sigma_{ii})_{max}}{\rho} \left(\frac{n\alpha}{B_o}\right) \frac{e}{(\gamma\beta)}$$

and the corresponding equation of motion will be:

$$\frac{d\beta}{dts} = \frac{(\sigma_{ii})_{max}}{\rho} \frac{n\alpha}{B_o} \frac{e}{c} \frac{1}{\gamma^3\beta}$$

We have a simpler form for the rate of change of the time-dilation factor:

$$\frac{d\gamma}{dts} = \frac{(\sigma_{ii})_{max}}{\rho} \frac{n\alpha}{B_o} \frac{e}{c}$$

In other words, past the point where we reach the limit of strain that our structural material can withstand, the time-dilution factor may increase linearly with the proper time:

$$\Delta\gamma = \frac{(\sigma_{ii})_{max}}{\rho} \frac{n\alpha}{B_o} \frac{e}{g} \Delta t_s$$

where Δt_s is measured approximately in years. In the case of the proton and deuterium fusion reactions

$$(\Delta\gamma)_{proton} = \frac{(\sigma_{ii})_{max}}{\rho} \quad 3.5 \times 10^{-9} \quad \Delta t_s$$

$$(\Delta\gamma)_{deut.} = \frac{(\sigma_{ii})_{max}}{\rho} \quad 6.2 \times 10^{-14} \quad \Delta t_s$$

Clearly this linear increase in the time-dilation factor is equivalent in the relativistic case ($\beta \sim 1$) to a constant acceleration in the non-relativistic case($\beta << 1$): in both cases the distance travelled by the vehicle in a given period of proper time will increase linearly with time. The rate of increase is seen to depend on the strength of the structural material. For the materials given in the table above, it is seen in the case of deuterium fusion that the yearly rate of increase of the time-dilution is negligible. With the proton fusion, however, there is more hope. But it must be remembered that I have made some assumptions along the way which throw an optimistic bias on all the results I obtain at this point. Both $(\gamma\beta)c$ and $d\gamma/dts$ must be multiplied by $\dfrac{\epsilon}{\log \dfrac{B_m}{B_o}}$

where $\epsilon << 1$ and B_m/B_o is a very large number!

If we want to travel to another galaxy (e.g. M31) the situation will not improve. The cutoff for $\gamma\beta$ and the subsequent rate of increase of the time-dilation factor γ are both proportional to n/B_o.

Now, the temperature of intergalactic gas is higher than the interstellar gas as a result of the cosmological condensation of the galaxies out of the intergalactic matter. If we assume equipartition of energy between the thermal motion and the magnetic fields, then n/B_o will be even smaller for intergalactic matter, and more stringent conditions, in fact, will be imposed on the ram-jet.

In order to get the minimum time required to attain near-optic velocities, it is necessary to look back at the equation of motion:

$$\frac{d\beta}{dts} = \frac{A_o}{m_s} nm_p c \begin{cases} \sqrt{2\alpha I}\,\beta, & \beta << \sqrt{\alpha} \\ \dfrac{\alpha I}{\gamma^2}, & \beta >> \sqrt{\alpha} \end{cases}$$

Most of the time spent in acceleration is at the beginning. Just as in the case of the familiar atmospheric ram-jet, the faster it moves the faster it can accelerate. We get a definite minimum, if not a good estimate, for the time spent in the early stages of flight if we integrate the equation of motion over only the low velocities:

$$(I = 1 - e^{-N^2} \sim N^2).$$

$$\Delta t_s > \int_{\beta_o^o}^{\sim 1} \frac{m_s}{A} \frac{kT\,F_p}{nm_p c\sqrt{2\,\alpha\sigma}} \frac{d\beta}{\gamma\beta^2}$$

The minimum mass theorem for low velocities is the same as it was in the relativistic limit. This is because most of the particles that make it into the reactor never attain transverse momentum comparable to their total momentum. Therefore, the minimum mass theorem yields:

$$\Delta t_s \gtrsim \int_{\beta_o}^{o} \frac{kTc}{4ne^2} \frac{1}{\alpha^{3/4}\sqrt{\sigma}} \frac{\rho}{\sigma_{ii}} \log \frac{B_m}{B_o} \frac{d\beta}{\sqrt{\beta}}$$

Using the field ratio for low velocities:

$$\frac{B_m}{B_o} (\beta) \cong \frac{m_p c^2 \sqrt{\alpha}}{2kT} \gamma\beta$$

This integral can be performed directly to give the time Δt_s in years:

$$\Delta t_s \gtrsim \frac{kTg}{n \alpha^{3/4} e^2} \frac{1}{\sqrt{\sigma}} \frac{\rho}{(\sigma_{ii})_{max}}$$

In the case of the hydrogen and deuterium as fuels for our fusion reactor, we obtain the following lower limits for Δt_s:

$$\Delta t_s^{prot.} \gtrsim .24 \times 10^{10} \frac{1}{\sqrt{\sigma}} \frac{\rho}{(\sigma_{ii})_{max}}$$

$$\Delta t_s^{deut.} \gtrsim \frac{10^{14}}{\sqrt{\sigma}} \frac{\rho}{(\sigma_{ii})_{max}}$$

In the case of the proton fusion chain, it is apparent that Δt_s is less than a year for most reasonable reactor sizes ($\sigma \gtrsim 1$). But in the case of using deuterium as a galactic fuel, our reactor size must be $\sim 10^7 cm^2$ before the initial acceleration time is within a human lifetime. This implies reactor dimensions of the order of a hundred meters.

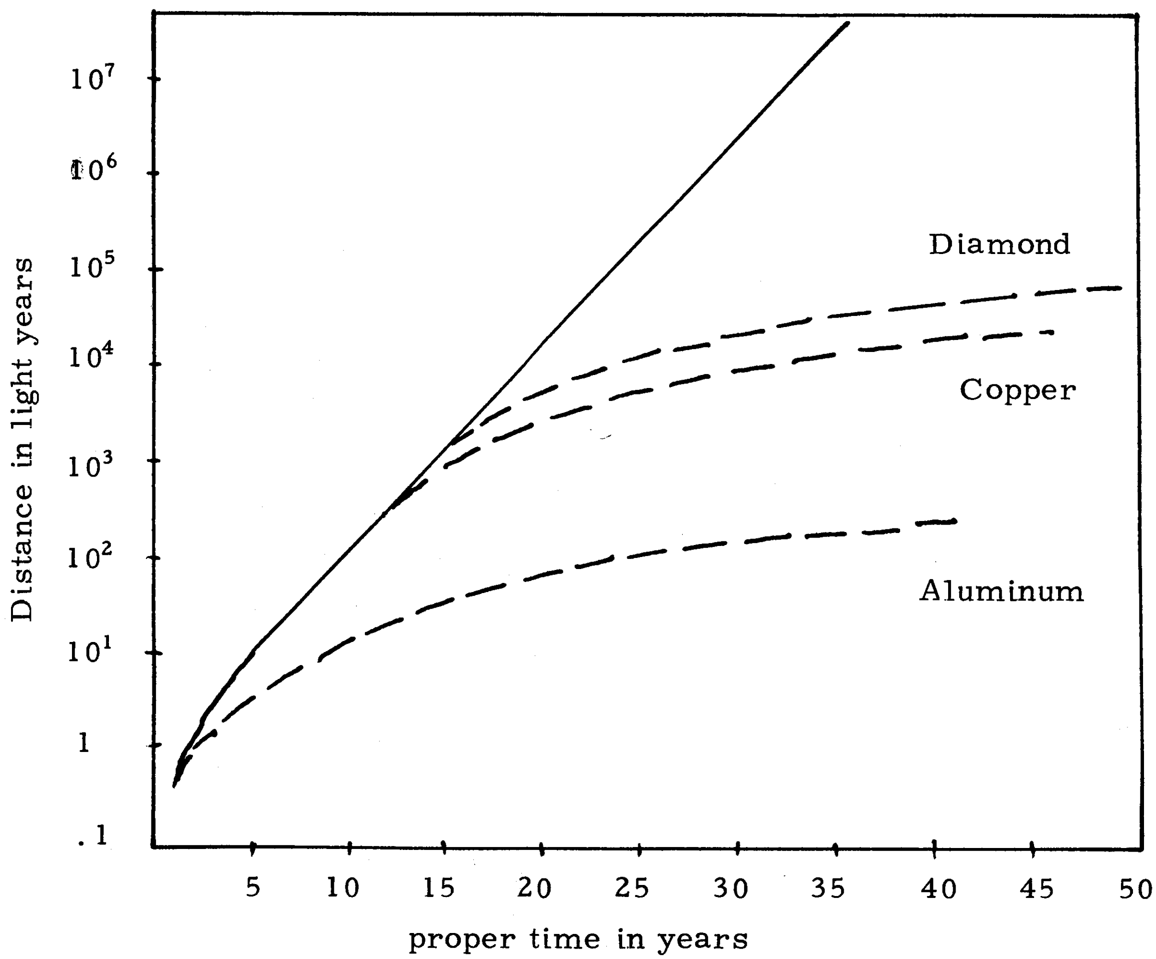


Figure 3. The performance curve of the proton ram-jet in a typical interstellar medium. The solid line is the result for constant proper acceleration at 1 g and the dotted lines represent the most optimistic interpretation of the minimum mass theorem for several of the structural materials considered in this paper.

CONCLUSION

I have restricted the discussion to the consequences of using a magnetic field to funnel the interstellar matter into the reactor. Of all the features that have become apparent, the singular restriction that has been put on the ram-jet is that it cannot accelerate indefinitely at 1 g. There is a velocity beyond which any increase in the time-dilation must be linear with the time elapsed on board the ram-jet. The results seem to indicate that there is doubtful hope for the proton ram-jet. Deuterium, on the basis of its scarcity, has been shown unfavorable as a fuel if we are to accomplish our purpose: namely, to reduce interstellar flight to within a human life-time.

Surely one does not have to look far to see more problems that lie ahead for the ram-jet. We anticipate velocities very close to the speed of light ($\gamma >> 1$). Regardless of the mechanism used to confine the interstellar gas, it must be pointed out that we can allow very little turbulence in the beam of particles that interact with the ram-jet. In the case of the magnetic field, turbulence would result from any imperfections (discontinuities) in the field. If the relative kinetic energy of the particles ever become comparable to their incident energy in the ship's proper frame then, depending on the particles we want to use as fuel and the velocity at which we are travelling, we may lose energy through annihilation of the particles. For example, in the case of mildly relativistic velocities, we may easily expect turbulence to give particles relative energies of several Mev. The energy loss through fission processes would eliminate deuterium entirely as a fuel. In the case of highly relativistic velocities, we can expect particle energies in the proper frame of the ram-jet to be several Bev and we run the risk of annihilating the protons themselves. Should turbulence become appreciable in the relativistic limit ($\gamma >> 1$) we can expect to lose a lot of momentum and energy to uncharged particle and γ-ray production. It may also be pointed out that such particle annihilation would scatter all the reaction products in the direction of the ram-jet itself. This would require a heavy radiation shield to protect the crew.

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