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**COMPLEX SYSTEMS: A REVIEW**

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**Seth Lloyd**

MIT Engineering Systems Division,  
MIT Department of Mechanical Engineering  
Santa Fe Institute, New England Complex Systems Institute

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## Complex systems: a review

Seth Lloyd

MIT Engineering Systems Division, MIT Department of Mechanical Engineering  
Santa Fe Institute, New England Complex Systems Institute

Engineers have worked on complex systems ever since engineering began. But the sciences of complexity have come to their own in the last few decades. Hoping to find common threads that weave their disciplines together, researchers from the fields of physics, biology, chemistry, math, computer science, economics, anthropology, linguistics, *et al.* have banded together to try to develop unifying frameworks for understanding complex systems. This paper reports on successes and failures of these efforts.

The Latin *complexus* comes from the Greek  $\pi\lambda\epsilon'\kappa\omega$ , to plait or twine. A complex system is woven out of many parts. The sciences of complexity try to understand the patterns of the weave. The orator Cicero used *complexus* to describe an intricate rhetorical argument, while the bawdy playwright Plautus preferred to use *complexus* to describe intertwined limbs in a sexual embrace. To make sense of the current debate on complexity, Plautus's meaning is more helpful.

The sciences of complexity promiscuously embrace almost every subject that calls itself science, and a few that do not. A search of the MIT library data base reveals 'complexity' in anthropology, biology, chemistry, computer science, cosmology, dentistry, design, economics, ethnography, functional analysis, geology, historical studies, housing, immunology, information theory, Islamic architecture, Japanese calligraphy, knapsack problems, linguistics, material science, mathematics, music, numismatics, operations research, philosophy,

physics, portfolio management, quantum computers, radiology, statistics, telecommunications, theology, ultrathin films, urban planning, vibrational failure, water pollution, wavelets, X-ray diffraction, ytterbium spectra, and zoology, to name but a few out of thousands of references. It is just not possible for the same mathematical techniques to apply rigorously to all these subjects: in some fields ‘scientific’ approaches to the study of complexity are indeed devoid of concrete results. Like Don Juan, the sciences of complexity sometimes simply strike out.

In some fields, however, a systematic approach to studying complexity is not only successful, but is the only possible way to succeed. Just what does a scientific study of complex systems have to offer? Compared with conventional sciences, the sciences of complexity under consideration emphasize distinctive methods and questions. 1) They focus on information: how do complex systems get information and what do they do with it? 2) They use detailed computer models for hypothesis testing and generation: how do computerized neurons behave when wired together in a chunk of artificial brain called a neural net? When artificial stock brokers buy and sell artificial stocks does the resulting market exhibit booms and busts? 3) They emphasize emergent properties: how do the laws of chemistry arise from the laws of physics, or the laws of biology from the laws of chemistry? In general, how do complex, specific laws arise from simple, generic ones? The techniques developed for studying complex systems are useful at the boundaries between conventional fields, where well-understood laws like chemical laws give rise to well-documented phenomena like life in a way that no one fully understands. Even when the parts of a system are perfectly understood, when woven together they can exhibit behavior that is too intricate and involved to be easily understood. In such cases, often the only recourse is to create an information-based model for the system and simulate it on a computer. For some fields, the systematic study of complexity is essential.

The goals of the sciences of complexity are hardly new. As noted above, engineers have been laboring with problems of complexity ever since the first system was engineered (it is rare to find a simple engineered system). In the realm of the sciences, Aristotle’s *Physics* (from the Greek  $\phi\upsilon\sigma\iota\kappa\acute{\omicron}\varsigma$ , begetting or becoming) can be regarded as an abortive attempt to understand the laws of emergence. Montesquieu’s *Spirit of the Laws* or Comte’s *Positivism*, Poisson’s probabilistic analyses of the fairness of trial by jury, as well as the sociological theorizing of Walras and Pareto, span two hundred years of attempts to create analogues of Newton’s laws for complex social systems. What is new? The computer. In

the last fifty years, the exponentially increasing ability of machines to process information has allowed the exploration of realms of complexity that were previously inaccessible.

Not that computers are so smart. It's just that human beings are relatively dumb, at least when it comes to performing mind-numbingly repetitive mathematical manipulations. In the past, to trace out the consequences of even the most over-simplified models for how proteins fold or how clouds form was virtually impossible. Now, however, the economist need not assume that agents are omniscient, that markets clear instantaneously, or that money is the only thing that matters. Though computerized models are still necessarily simplified, they can include much more detail than was previously possible.

The remainder of this paper will summarize a variety of endeavors in the sciences of complexity. Most of these endeavors focus on complex adaptive systems — systems that are composed of several or many subsystems and that change their behavior in an adaptive fashion in response to environmental stimuli. But there are many subfields of complexity, as will be seen. In each of these fields, the ability to perform detailed computer simulations has translated into significant advances in human understanding. Just how significant these advances are will be left to the individual reader to determine. The following reports represent only the author's opinions: they should be used as stepping stones, nothing more. The alphabetical list constructed here is by no means complete. Readers who wish to follow the sciences of complexity further can consult the various links listed at the end of this article.

### **Artificial life:**

Most of the basic concepts of the sciences of complexity have been around for as long as science itself. It is simply that these concepts have not been accessible to scientific investigation before the development of computers. Take for example the field of artificial life. This branch of the sciences of complexity studies artificially constructed living organisms, in particular, ones that live in a computer's memory. Artificial life is hardly a futuristic idea. Mary Shelley's *Frankenstein* and *The Golem* are late additions to the field. The first mention of artificial life in English that I have been able to discover occurs in the second sentence of Hobbes's *Leviathan* (1651): 'For seeing life is but a motion of limbs, the beginning whereof is some principal part within; why may we not say, that all *automata* (engines that move themselves by springs and wheels as doth a watch) have an artificial life?'

Despite the abiding interest in artificial life (*Frankenstein* and the *Golem*, it was not

until the twentieth century that the study of automata was pursued in earnest. In the 1950's, von Neumann analyzed the problem of self-reproducing organisms in the abstract, by investigating computerized organisms, or automata, that were capable of creating copies of themselves. He noted several features that a self-reproducing automaton must possess, all of which turned out to be features of living cells once DNA was identified as the genetic material. Contemporary offspring of von Neumann's idea exhibit a wide variety of 'biological' behavior, including parasitism, immunity, and malignancy. Anyone whose computer has been infected by a virus has had first hand experience of artificial disease and the difficulty of killing off an artificial life form.

The past twenty years have seen a flowering of the field of artificial life, including applications to computer science, internet communications, robotics, and notably animation. The series of conferences on artificial life instituted by Chris Langton at the Santa Fe Institute have spawned (if I may say so) a rapidly growing artificial biomass. A particularly elegant example of artificial life is Tom Ray's Tierra system. But many more examples of artificial life exist, for better or worse. (Who knows how Windows is using its strangely vast number of machine cycles?)

### **Biological complexity**

To speak of complex biological systems is redundant. Biology is the *ur* science of complexity. But it is also the science in which complex systems theory has potentially the most to contribute. I say 'potentially' because biology is already very successful without any explicit attempt to construct a formal theory of biological complexity. (Similarly, engineering has been very successful in designing, manufacturing, and operating complex systems without constructing a formal theory of engineering complexity.) Real life, as opposed to the artificial kind, is by definition the provenance of biologists, who have painstakingly elucidated many of the chemical processes that underlie life as we know it. But life as we know it evolved from earlier life forms we don't know as much about. When it comes to the chemical processes that resulted in the first life on earth more than three million years ago (discounting for the moment the speculative theory that life was 'seeded' from space), virtually nothing is known. Computer-based simulations of chemical reactions from which life might have arisen are currently being carried out by theoretical biologists, who are regarded with suspicion by some of their colleagues.

Particularly relevant to the origins of life is the notion of an 'autocatalytic set,' an idea suggested by Melvin Calvin and explored independently by Otto Rössler, Manfred Eigen,

Peter Schuster, Stuart Kauffman, and by many others. An autocatalytic set arises when a group of chemicals react with each other to produce other chemicals that in turn encourage or catalyze the original reactions. Starting at almost negligible concentrations in a given volume, such a set of chemicals and reactions can by mutual catalytic encouragement rapidly come to dominate the volume. The set effectively reproduces itself and can evolve if it discovers new reactions and creates new products. Eventually, the story goes, the evolving set hits upon the chemical reactions that make up life.

This is potentially a convincing story, and lacks only a detailed analysis of the chemical kinetics to be confirmed not just as good science but as superb science (Eigen received a Nobel Prize in part for his work on hypercycles, autocatalytic sets involving RNA). Dooyne Farmer, Norman Packard and Richard Bagley managed to program a Los Alamos computer with a simplified, artificial chemistry that exhibited autocatalytic sets; unfortunately, the actual chemical kinetics are too complicated to be analyzed even by the fastest computer available. If it is to be confirmed, the autocatalytic set hypothesis for the origins of life will have to await more powerful computers and more detailed chemical experiments.

There are many more biological applications of the sciences of complexity, not only at the level of RNA. Insect colonies, flocking and schooling behavior, Turing patterns in cell organization, the modeling of the immune system (pioneered by Alan Perelson among others), have all been analyzed by the process of detailed computer models. Frequently, these models, even when highly simplified, replicate *grosso modo* the features of the biological system studied. These correspondences between simplified, though somewhat complex models and the truly complex systems whose behavior they are designed to capture is a significant success of the sciences of complexity.

But this success is accompanied, if not by failure, but by a question. Do these correspondences between numerical experiments and the coarse-grained behavior of biological systems actually imply that the biological systems are operating in a way that corresponds to the structure of the computer model? Or might a variety of other models give similar behavior? Finding a general method to answer this nagging doubt remains an open problem in complex systems research.

### **Cellular automata**

Cellular automata are regular arrays of systems that are updated in parallel by local logical operations. They are capable of exhibiting a wide variety of complex behaviors, including computational universality (very simple systems can be computationally universal,

but the capacity for computational universality translates into the ability to exhibit arbitrarily complex behavior), fractal structures, adaptation, etc. Their homogeneous nature makes cellular automata attractive computational structures for investigating dynamics with spatial symmetries, such as the laws of physics.

### **Complexity, entropy, and the physics of information**

As noted above, information is a key quantity in the analysis of complex systems. Although all the formulae for information were developed by Maxwell, Boltzmann, and Gibbs in their development of statistical mechanics, concepts of information have had an uncertain role in physics until recently. The situation has now changed with the development of a formal theory of the physics of information processing, including quantum computing, quantum communication, etc. A good summary of this field can be found in Wojciech Zurek's book of the same name, and in the large existing literature in quantum computation.

### **Chaos theory**

Complexity is often confused with chaos. Murray Gell-Mann, author of *The Quark and the Jaguar*, a book that contains an engaging and penetrating discussion of the sciences of complexity, claims never to have given a talk on complex systems without someone coming up afterward to thank him for his talk on chaos theory. In fact, chaos is only one of the sciences of complexity. For all its ominous name and the hoopla surrounding its popularization, chaos is a relatively narrow mathematical discipline that concentrates on classical, deterministic, dynamical systems; and the scientific successes of chaos theory come from the intensity of its narrow focus. These successes are great, particularly in the case of low-dimensional chaos, where the elegant results of chaos theory have been confirmed in a wide variety of experiments. In high-dimensional (i.e., complex) chaotic systems, however, the applicability of chaos theory to observation is less clear-cut, owing to the 'curse of dimensionality' (the exponential explosion in the amount of data required to test the theory accurately).

### **Control theory**

Control theory is *ur* complex engineering discipline. Engineers have been trying to control complex systems ever since they started building them. The successes of modern control theory in characterizing the controllability and observability of complex engineered systems are manifold: I will not catalog them here.

## **The economy as an evolving complex system**

A signal success of the sciences of complexity is their application to economics. As any student of economics knows, one of the primary frustrations of classical economics is its reliance on patently wrong assumptions, such as perfect rationality, perfect markets, etc. (Note that similar frustrations accrue to the student of physics, with its frictionless surfaces, ideal gases, etc.) The ability to perform detailed computer modeling allows the researcher to relax these unrealistic assumptions, constructing models where economic agents act in ways more congruent with actual agents. A particularly appealing example of the application of the sciences of complexity to economics is the artificial stock exchange, created by John Holland with economist Brian Arthur and physicist Richard Palmer along lines suggested in discussions with the Nobel laureates Kenneth Arrow (economics) and Phil Anderson (physics). In this electronic arena, mindless but greedy automata bid against each other's strategies, producing speculative bubbles and crashes and other real-life phenomena that classical economics with its perfect markets has difficulty reproducing.

More recently, the sciences of complexity have developed a new branch called *econophysics*. By attempting to solve problems loosely borrowed from economics with techniques from physics, game theory, and applied mathematics, econophysicists have constructed a new and exciting field (albeit one held in suspicion by both economists and physicists). A particularly impressive example is the Prediction Company (founded by Doyne Farmer and Norman Packard) which has used a complex adaptive 'ecosystem' of models to predict and make money on markets. In this ecosystem, models compete to predict financial time series and replicate variants in proportion to their success. The ones that are actually used to make bets on future behavior are those that have survived eNature, red in tooth and claw.

## **Genetic algorithms and evolutionary programming**

A wide variety of evolutionary, adaptive programming techniques are now commonplace. If you buy a car these days, its computer will start to adapt its engine's response to your own personal driving style, whether you like it or not. One of the original and most successful examples of evolutionary programming is John Holland's genetic algorithm—a computerized analog of the processes of mutation and recombination that underlie biological evolution. Genetic algorithms show how computers can learn to cope with complexity by imitating how living creatures cope with their complex environments. Genetic algorithms are widely used in industry, as are a plethora of adaptive programming techniques.

The trick, of course, with adaptive programming techniques, is to be able to guarantee their convergence and robustness in a variety of settings. Finding such guarantees represents a significant open question of complex system research.

### **Fractals**

Fractals are scale-invariant patterns that frequently arise in the context of chaotic systems, cellular automata, phase transitions, etc. Intricate in appearance, and fascinating because of their self-similar nature, they are nonetheless typically rather simply generated patterns.

### **Lattice gases**

Lattice gases are cellular automata whose behavior is isomorphic to actual physical gases and fluids. They are a useful application of complex system techniques to solve hard computational problems of fluid dynamics.

### **Neural networks**

Neural networks are nonlinear computation systems whose dynamics are loosely modeled on the behavior of actual networks of neurons in the brain. Neural networks are useful for modeling a variety of nonlinear behaviors and patterns. They have been used successfully in many complex engineered systems, particularly those that are designed to adapt to their environment. Neural networks sometimes suffer from problems of inefficiency of representation, lack of robustness, and instability.

### **Non-linear dynamics**

Almost all physical systems are nonlinear at some scale or another (with the possible exception of quantum mechanics, in which the Schroedinger equation has been tested to be linear to a high degree of accuracy). Non-linear dynamics is a large field with many contributions to the study of complex systems (see, e.g., the comments on chaos above).

### **Protein folding**

Protein folding is a difficult problem: it can be modeled in analogy to simulated annealing. For a protein to fold to its ground state is similar to a spin-glass finding its ground state, a problem that is known to be NP-hard. Yet the proteins found in our cells fold all the time, with little difficulty. Presumably our cells evolved to produce proteins that

fold easily. Figuring out just how proteins fold is a key step in unravelling the mechanisms by which cells do the complex set of tasks that they do.

### **Self-organized criticality/edge of chaos**

In critical phenomena such as phase transitions, physical systems exhibit large fluctuations (as in the phenomenon of critical opalescence). Rather than falling off in a Gaussian fashion, the distribution in the size of fluctuations is a power law. In self-organized criticality, a variety of systems, most notably simulated sand piles, move naturally to a domain in which fluctuations are large and exhibit a power-law distribution. This fact may be related to some of the power laws found in nature, such as the Richter scale. Then again, it may not: the jury is still out.

The edge of chaos refers to an apparently related phenomenon. If a dynamical system has both regular and chaotic regimes, and one tunes some parameter to move the system's dynamics from the regular to the chaotic regime, the system often exhibits apparently complex behavior at the edge of chaos. For example, regular systems are not good at mixing; chaotic systems mix exponentially well; at the edge of chaos, the mixing is sometimes polynomial. Similarly, regular systems produce information at a logarithmic rate; chaotic systems produce information at a linear rate; systems at the edge of chaos can produce information at a log-polynomial rate. Once again, the jury is still out as to the significance of these observations.

### **Spin glasses**

Spin glasses are collections of spins, ordered irregularly as in a glass, with couplings that are to some degree random. Spin glasses are known to exhibit a variety of apparently complex behavior. In particular, finding the ground state of some spin glasses is an NP-hard problem. That is, spin glasses are simple physical systems that exhibit computationally sophisticated abilities. Spin glasses represent a uniquely physics-based paradigm for the study of complex systems.

Even as they expand exponentially, the sciences of complexity continue to provoke debate. Many concepts in complex systems seem to be poorly defined ('emergent behavior,' 'edge of chaos,' 'self-organized criticality,' etc.). Many scientists regard the sciences of complexity with suspicion, some of it surely justified. But while the debate on their legitimacy continues, the sciences of complexity have quietly pervaded everyday science and engineering. Almost a decade ago, having just received my Ph.D., I attended the

first Santa Fe Institute summer school for the study of complex systems. There I learned about many of the techniques described in these books, such as genetic algorithms, cellular automata, and simulated annealing. At the time, those techniques seemed to me far out, abstract, and not necessarily practical. Now I am a professor in the department of Mechanical Engineering at MIT. While I write this, graduate students are applying genetic algorithms to find the least wasteful way to stamp parts out of sheet metal, programming cellular automata to analyze air conditioning, and using simulated annealing to optimize designs for engines. Ideas from the theory of information and computation are woven together in a method called Axiomatic Design, and put to work making better freezers and injection molds. A good working definition of a complex system is one that has to get and process large amounts of information in order to function. Cells, brains and ecosystems are not alone in their complexity: increasingly, they are being joined by buildings, cars, and washing machines. The systematic study of complex systems is here to stay. And engineers, just as always, are at its forefront.

References: Probably the best way to sample so eclectic a field is through the web. Useful websites include that of the

Santa Fe Institute, [www.santafe.edu](http://www.santafe.edu)

New England Complex Systems Institute, [www.necsi.org](http://www.necsi.org)

Complexity On-Line, [complex.csu.edu.au/complex/](http://complex.csu.edu.au/complex/)

Complexity International, [www.csu.edu.au/complex\\_systems/complex.html](http://www.csu.edu.au/complex_systems/complex.html)

Michigan Center for the Study of Complex Systems, [www.pscs.umich.edu/](http://www.pscs.umich.edu/)

Duke Center for Nonlinear and Complex Systems, [www.phy.duke.edu/research/cncs/](http://www.phy.duke.edu/research/cncs/).

As with the web in general and with complex systems in particular, caveat emptor.