

14.121 Microeconomic Theory I: Waiver Exam

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Name:

Instructions:

- This is a closed-book examination. Put away your books, handouts, notes, calculators, laptops, cellular phones, etc.
- **Print your name clearly on the front cover of the answer booklet.** Write your answers clearly. We won't grade unreadable answers.
- The examination is for 90 minutes. If you run out of time, do not panic. Answer the questions you are able to get to as thoroughly as possible.
- There are five questions and a total of 100 points.
- In the exam, you are asked to "prove" certain statements. In doing so, you may rely on any results from mathematics you wish, but you should clearly state the steps of your argument and the theorems you reference, if appropriate. Results from economics should be proven unless noted. **When in doubt, state all assumptions and proceed.** Partial credit will be given for clear logic and careful reasoning.

1) [20 points] Consumer theory

The n -good Cobb-Douglas utility function is:

$$u(\mathbf{x}) = A \prod_{i=1}^n x_i^{\alpha_i},$$

where $A > 0$ and $\sum_{i=1}^n \alpha_i = 1$.

- a) Derive the Marshallian demand functions.
- b) Derive the indirect utility function.
- c) Compute the expenditure function.
- d) Compute the Hicksian demands.

2) [15 points] Consumer theory

- a) Suppose that $u : \mathbb{R}^n \times \mathbb{R}^k$ is a continuously differentiable function, and consider the problem:

$$\max_{z \in \mathbb{R}^n} u(z, \alpha)$$

where $\alpha \in \mathbb{R}^k$. Think of \mathbb{R}^k as the space of parameters and \mathbb{R}^n as the space of choice variables. State and prove the envelope theorem. Give an intuition for the theorem.

- b) Now suppose we are given $g : \mathbb{R}^n \rightarrow \mathbb{R}^m$ and suppose that our problem is

$$\begin{aligned} \max_{z \in \mathbb{R}^n} u(z, \alpha) \\ \text{subject to } g(z) = 0. \end{aligned}$$

State the envelope theorem for this constrained maximization problem. (Note: you do not need to prove it)

- c) State and prove Roy's identity.

3) [25 points] Producer theory

Consider the profit maximization problem:

$$\max_{k, l} pF(k, l) - rk - wl$$

where $F(k, l)$ is the production function, r is the rental rate of capital, w is the wage rate of labor.

- a) Suppose that F is not differentiable. When are k and l substitutes in production? Provide an example of a production function with this property (note: example can be differentiable).
- b) Demonstrate that if k and l are substitutes, then the optimal capital choice as a function of exogenous parameters, $k^*(r, w)$, is weakly increasing in w and likewise $l^*(r, w)$ is weakly increasing in r . Show this without assuming F is differentiable if you can.

- c) Suppose that the initial wage is w_0 . When wages change to w , write the optimization problems that the short-run demand for labor solves and the long-run demand for labor solves.
- d) How the short-run demand compare to the long-run demand when k and l are substitutes? Provide a proof and give economic intuition.
- e) Explain how the results of part d) changes if k and l are complements? What if k and l are complements when both are small, and substitutes when both are large?

4) [25 points] General equilibrium

Suppose there are I consumers, and x_i denotes the demand of consumer i at prices p and wealth w_i (obtained by utility maximization). Let $\bar{x} = \sum_{i \in I} x_i$ be the aggregate demand. All consumers face the same prices p , although their wealth and preferences could differ.

A benevolent planner has the power to re-allocate \bar{x} among the consumers but does not have additional resources. That is, he can change the original allocation, $(x_i)_{i \in I}$, to some re-allocation $(x'_i)_{i \in I}$ provided that $\sum_i x'_i = \bar{x}$. His aim is to find a feasible re-allocation (x'_i) such that $\sum_{i \in I} x'_i = \bar{x}$ and $u_i(x'_i) > u_i(x_i)$ for all $i \in I$, a **weak** Pareto improvement.

- a) Prove that there does not exist a weak Pareto improvement. Please highlight clearly what assumptions on preferences you use in the argument.

A feasible re-allocation, (x'_i) with $\sum_i x'_i = \bar{x}$, is said to be a Pareto improvement if $u_i(x'_i) \geq u_i(x_i)$ for all $i \in I$ and $u_j(x'_j) > u_j(x_j)$ for at least one $j \in I$.

- b) Prove that there does not exist a Pareto improvement over $(x_i)_{i \in I}$. Please highlight what assumptions on preferences you use in the argument.
- c) Do there exist preferences where there exists a Pareto improvement, but there does not exist a weak Pareto improvement? If so, provide an example. If not, explain your reasoning.

5) [15 points] Externalities

Assume the cost of commuting to work for an individual with wage w is $wf(n)$ by car and $wx + t$ by subway where n is the number of cars on the road, and x and t are constants. Let there be a total of N commuters. Assume that $f(0) = 0$ and $f', f'' > 0$.

- a) Assume everyone makes the same wage. What will be the equilibrium number of drivers? How will the equilibrium number change with the wage?
- b) What is the socially efficient number of drivers? How does this number change with wage?
- c) What are three ways to enforce the social optimum? Make a case for the most plausible one.

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