Three Essays on Economic Growth

by

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Abstract

The first essay — Miracle and Poverty — develops a model of an integrated world economy in which productivity growth is sustained by exogenous technical progress and human capital accumulation. New products with better quality are being invented continually. Their production processes require workers with ever greater skill which provides incentive for human capital accumulation. I first show that there exists a stable distribution of relative skills within which low-skill workers remain relatively low-skill indefinitely. Then, I argue that differential increase in the labor force of developed and under-developed countries could help explaining the observed cross-country productivity growth patterns; particularly, the so-called economic miracle of some middle-income countries.

The second essay — Scientific Progress and Growth — models scientific progress in a way which distinguishes basic research from applied research. There are two contributions to endogenous growth literature. First, instead of assuming the law of motion $\dot{a}/a = f(R)$ as usual, I step back and derive two familiar-looking laws of motion, one for scientific knowledge and the other for productivity growth. Second, I introduce federally-funded research in order to reexamine empirical findings concerning the relationship between research and productivity growth, particularly, the findings that basic research has higher return than applied research and that privately-funded research has higher return than federally-funded research.

The third essay — Knowledge Spillover and Scale Effects — develops a general equilibrium model in which productivity growth is sustained by innovative activities: $\dot{a}/a = f(R)$. It combines the quality-ladder model with Krugman's model of product differentiation in a way that varieties of intermediate goods is endogenous. The production function for final good has Dixit-Stiglitz's preference. Under this setting, it is possible to demonstrate that there may not be scale effects: an increase in the size of the economy does not always lead to faster productivity growth. If knowledge spillover is *limited*, there is no scale effects. But, if knowledge of a firm spills over to every other firms, there is. Finally, this model suggests that the joint behavior of productivity growth slowdown and research intensity increase in OECD countries during 1960–1990 can be explained by an exogenous increase in the degree of sector-specificity of technology.

Thesis Supervisor: Robert M. Solow
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To my grandmother and father

whose departure leaves my world a darker place.

Acknowledgments

Mark Hopkins, a former president of Williams College, once said that "We are to regard the mind, not as a piece of iron to be laid upon the anvil and hammered into any shape, nor as a block of marble in which we are to find the statue by removing the rubbish, or as a receptacle into which knowledge may be poured; but as a flame that is to be fed..." My life has been fortunate to have so many wonderful teachers who kindle my love and passion for knowledge. It is to all my teachers that my debt of gratitude lies.

I thank Kru Mali Kansakul, my geometry teacher in the sixth grade who always tells me that my homework and proofs are clean, neat, and well-done, and always gives me my perfect ten. Her classes mark the beginning of my love for mathematics. At Williams College, I was a student of Professor Olga Beaver and Professor Cesar Silva, whose guidances kindle my passion for abstract measure theory and measurable dynamical systems. Mathematics is no longer a tedious manipulation of symbols; it becomes a sublime transformation of structures and properties. Professor Beaver almost persuaded me to become a mathematician. Professor Gordon Winston taught me an introductory economics and, three years later, a seminar on economic theories. Through him, economics is a wonderful and fun subject with so many possibilities for research.

I thank Professor Eckaus for his advice and comments. Every time I stop by his office, he would be kind to share with me some insights — insights that one can only gain with time and experiences. Professor Caballero always gives me extremely useful and important papers for my projects. Professor Kremer is always supportive of my works. His enthusiastic comments always give me more energy. I have learned many things from him. And it was he who first taught me theories of economic growth.

I especially thank Professor Blanchard for many of his insights and comments. They were invaluable in improving my works. He has been a wonderful advisor, extremely smart, kind and charming. His insistence that a project should not only be fun but also important will be long cherished by me. One day, I should be able to write such papers. Coincidentally, it was his charming-self and his charming letter that convinced me to come to MIT in the first place.

My greatest debt of gratitude is for Professor Solow. It was most fortunate to be introduced to him at the end of my first semester for research guidance, and to be his thesis advisee later on. His detailed comments — he reads every single thing — not only provide me interesting research directions to pursue, they also indicate what are lacking in my works. Looking back and comparing my very first work with my last paper, reveals how much I owe it to him. His enthusiasm and his kind encouragement always makes my days brighter. His guidances are invaluable. The mere thought that my work would delight my teacher makes research much more fun. It is more than fortunate and a joy to be his student.

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Introduction

This thesis consists of three chapters on economic growth. First chapter, Miracle and Poverty, looks at the broad picture of international economic inequalities. It asks why there are poor countries, why poor countries remain relatively poor overtime, and why some countries catch-up faster than others. This chapter contents that answering these questions properly requires that one first recognizes that countries are not alone in the world economy. So, one needs a theoretical model of an integrated world economy with countries being parts of the world trading. It will be shown that there is a tendency for countries to differentiate themselves as developed and under-developed countries. Products are being produced first in developed countries and then by less-developed countries later on, as observed in the product-cycle literature.

The model consists of three compact assumptions. First, human capital of different levels are not perfectly substitutable. That is, one cannot put two nurses together and perform a brain surgery. Appropriate training is required.[1] Consequently, existence of a distribution of skills and increasing wage profile as a function of human capital are to be expected. For that essential and primitive products such as rice must be produced by some workers. Given that it is not necessary for workers to accumulate much human capital to become farmers, there will be low-skill workers (producing rice) in this integrated world economy. Workers who accumulate human capital to produce advanced products such as microprocessors will be compensated in term of higher wages. It is this particular assumption that delivers several new theoretical results and distinguishes this model from others.

Second, human capital accumulation process is assumed to have constant returns, namely, $\dot{h}/h = s$ where s is the fraction of time a worker spent on education and h is the level of her

[1]This assumption is due to Stokey [1991].

human capital. This particular assumption concerning the cost of human capital accumulation allows low-skill workers to remain relatively low-skill indefinitely and delivers persistence economic inequalities among workers and countries. Third, I assumed that productivity growth is sustained by exogenous technical progress in term of new products invention. New products with better quality are being invented continually; their production processes require workers with ever greater skill which provides incentive for human capital accumulation. Thus, the engine of growth in this model is a combination of human capital accumulation and exogenous technical progress.

It will be shown that there will be division of labor — workers differentiate themselves into high- or low-skill workers. External benefits of human capital provides a tendency for (human capital) convergence within each country. Interaction between division of labor and this tendency leads countries to differentiate themselves as high- or low-skill countries. Finally, I use this model to argue that differential increase in the labor force of developed and under-developed countries could help explaining the observed cross-country productivity growth patterns; particularly, the so-called economic miracle of some middle-income countries. The main innovation there is that instead of assuming that there are two big countries — North and South — as done in many other models, I assume that all countries are small open economies. This approach allows some under-developed countries to catch-up faster than others.

Second chapter, Scientific progress and Growth, is based on empirical finding that there are distinctions between basic research expenditures and applied research expenditures. Specifically, Griliches [1986], Lichtenberg and Siegel [1991], and Mansfield [1980], find that investment in basic research has strong effect on productivity growth while other types of R&D has either a small impact or none at all. An estimated return for basic research is 133.8 percent; and that for applied research, statistically insignificant, 10.8 percent. This suggests that distinctions between various types of research are essential for understanding productivity growth. Particularly, it suggests that basic research is perhaps the engine of growth while applied research, though importance, is not.

Thus, in this chapter, I construct a model of growth with basic research as the engine of growth. Instead of assuming the law of motion — $\dot{a}/a = f(R)$ — as usual, I step back and *derive* two familiar looking laws of motion, one for scientific knowledge and the other for productivity growth:

$$\dot{\gamma} = \gamma f(R_b) \qquad \text{and} \qquad \dot{a} = \gamma f(R_d)$$

where γ is an index of scientific knowledge; a is productivity level; R_b and R_d are the number of researchers conducting basic research and applied research, respectively. Notice that basic research increases the effective efficiency of applied research but does not affect productivity level directly. It is applied research which increases the productivity level. Here, basic research is the engine of growth while applied research, though importance, is not.

Third chapter, Knowledge spillover and Scale effects, is written in response to a recent empirical evidence, Jones [1996], against research-based endogenous growth models. During 1965 and 1990 the number of scientists and engineers engaged in research and development in the United States has increased twofold from five hundred thousands to roughly one million; yet productivity growth rate has not been increasing as predicted by research-based endogenous growth models such as Romer [1991], Aghion and Howitt [1992], and Grossman and Helpman [1991]. This particular evidence leads Jones to reject this class of models and reject the law of motion as empirically untenable.

This chapter provides an example of a research-based endogenous growth model without scale effects. It argues that the law of motion $\dot{a}/a = f(R)$ exists at firms level, not the aggregate level. The critical feature of this model is that the number of firms is endogenous. Doubling the size of the economy, double the number of firms. Although the number of researchers do double, the number of researchers per firm as well as productivity growth rate remain unchanged. Put it differently, with free entry the aggregate number of researchers is not a good proxy of the number of researchers per firm. This is why it may be inappropriate to regress the aggregate number of researchers on productivity growth rates as Jones does in his paper. Consequently, I reinterpret Jones's empirical finding as a rejection of the assumptions (within those endogenous growth models) that the law of motion exists in the aggregate level or that there is fixed number of firms in the economy.

Chapter 1

Miracle and Poverty

"...why poor countries remain poor; why rich are rich; and why some catch-up faster than others."

This paper develops a model of an integrated world economy in which productivity growth is sustained by exogenous technical progress and human capital accumulation. New products with better quality are being invented continually. Their production processes require workers with ever greater skill, which provides incentive for human capital accumulation. I first show that there exists a stable distribution of relative skills within which low-skill workers remain relatively low-skill indefinitely. Then, I argue that differential increase in the labor force of developed and under-developed countries could help explaining the observed cross-country productivity growth patterns; particularly, the so-called economic miracle of some middle-income countries.

1.1 Introduction

Casual observation of cross-country productivity growth patterns during 1960 and 1985, reveals some definite and simple patterns. First, there exists a large income disparity between countries with the highest GDP per capita at 39 times value of the lowest. These economic inequalities increased steadily with the dispersion of relative output per worker increasing slightly.[1] Second, OECD countries with initially lower GDP per capita have experienced higher growth, providing evidence for income convergence among developed countries. Third, poor countries are not

[1] Barro and Sala-i-Martin [1995, p.2-3], Chari et. al. [1995, p.4]

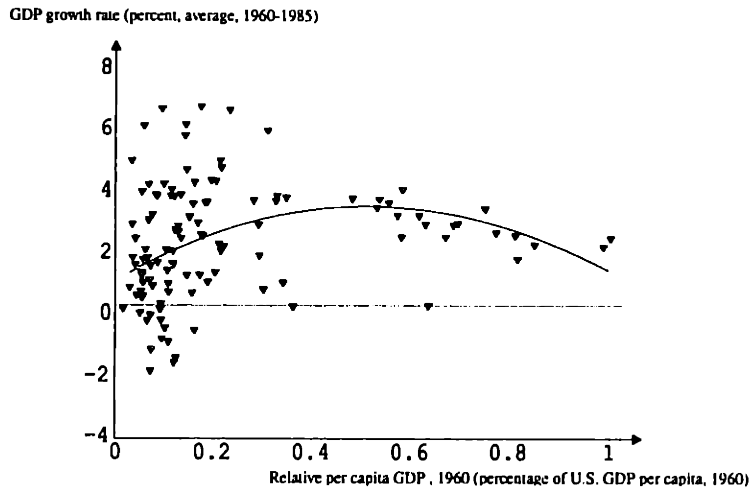

Figure 1-1: Average GDP growth rates, 1960-1985, and relative per capita GDP, 1960

catching-up on average. In general, the rates of growth of GDP per capita in low-income countries do not exceed those for developed countries. Fourth, there is a small subgroup of middle-income countries which perform exceptionally well. During 1965 and 1990, the average growth of GNP per capita for the so-called East Asia Miracle is around 5.5%, with the world average, around 2%.[2]

This cross-country variation in per capita income and national growth rates poses a challenge to existing models of economic growth. Originally, this observed large income disparity and the observed lack of correlation between GDP growth rates and initial per capita income level, are used by some economists as evidence for endogenous growth models. However, there are several problems with endogenous growth models with the major problem being that most of them "predict that the difference in the logarithm of per capita income between two nonidentical countries should increase without bound over time....Not only should we see the distribution of income spreading out, but we should see for any arbitrary subset of countries its distribution spreading out as well."[3] This prediction does not conform well with the observed income convergence among OECD countries nor with the observed roughly stable dispersion of relative output per worker during 1960 and 1985.

[2]Some have experienced negative growth during this period. For, Africa, 1965-1990 is known as the lost decades; the average growth of GNP per capita for Sub-Sahara Africa is around .3%.

[3]Parente and Prescott [1992, p.202] For example, an AK model with countries having different saving rates.

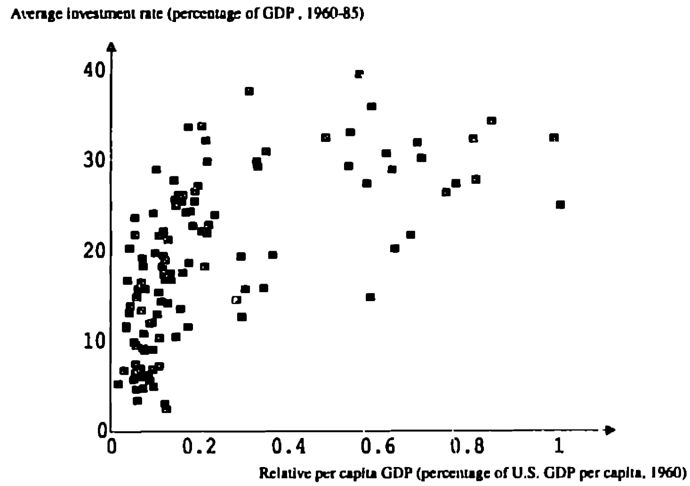

Figure 1-2: Average investment (percentage of GDP, 1960-1985) and relative per capita GDP

On the other hand, this observed large income disparity and lack of correlation between GDP growth rates and initial per capita income level seems, at first glance, to provide evidence against exogenous growth models; for that, exogenous growth model in its simple form with every country sharing an identical production function of a single good and identical saving behavior, predict income convergence. This is not so. Mankiw, Romer and Weil [1992] argue that by allowing *exogenous* saving and population growth rates to differ from country to country, an augmented Solow model which includes accumulation of human as well as physical capital provides an excellent description of this cross-country income disparity. It accounts for 80 percent of the cross-country variation in income. One also should not expect income convergence; rather, countries generally reach different steady states. Once differences in exogenous saving and population growth rates are accounted for, there is convergence in income per capita at roughly the rate that the model predicts.[4]

Grossman and Helpman [1994, p.27-30] provide discussion of the aforementioned work. They write that "In the estimation of the basic Solow model without schooling variable, the fraction of the variation in OECD country incomes 'explained' by population growth and the investment ratio is only 1 percent! Mankiw, Romer, and Weil get most of their mileage from the large differences in investment ratios and population growth rates between the rich and

[4]Mankiw, Romer, and Weil [1992, p.407-8]

poor countries."[5] As we can see in Figure 1-2, there exists large disparity in investment ratios among countries. Low-income countries invest much lower percentage of their GDP than those with initially high-income per capita with Madagascar invested only 1.4 percent of its GDP while the U.S., 24 percent.

My paper is a complement to Mankiw, Romer, and Weil [1992]. It asks two questions. First, this observed large disparity in investment ratios is critical for explaining international income disparity. Yet it is taken as exogenous by the authors. Is it possible to construct a model such that investment ratios are determined endogenously and that poor countries optimally invest less than rich countries?[6] Second, Figure 1-1 provides a fitted line, regressed by the World Bank.[7] This inverted U-shaped curve suggests that middle-income countries grow faster than low-income and high-income countries. This leads one to wonder if each country has its own steady state and the observed national growth rates merely reflect behavior out of steady states as being suggested by the authors, why middle-income countries are further below from its steady state than other countries. Is it possible to construct a model which systematically allows middle-income countries to catch-up faster than others, thereby delivering an inverted U-shaped curve?

These two questions provides impetus for constructing the following model of human capital accumulation and growth. It is an extension of Stokey [1991]. Productivity growth is sustained by exogenous technical progress *and* human capital accumulation. New products with better quality are being invented exogenously. Their production require workers with ever greater skill, providing reason for human capital accumulation. By introducing vertical product differentiation and by assuming that human capital of different levels are not perfectly substitutable, I construct an exogenous growth model which yields income disparity between workers. International division of labor gives rise to international economic inequalities. Persistent income disparity becomes possible in this model by importing with care, an aspect of

[5]For additional discussion, the reader is referred to the cited paper.

[6]Mankiw, Romer, and Weil [1992, p.409] write that "Overall, the findings reported in [thier] paper cast doubt on the recent trend among economists to dismiss the Solow growth model in favor of endogenous growth model that assume constant or increasing returns to scale in capital. One can explain much of the cross-country variation in income while maintaining the assumption of decreasing returns. This conclusion does not imply, however, that the Solow model is a complete theory of growth: one would like also to understand the determinants of saving, population growth and worldwide technological change, all of which the Solow model treats as exogenous."

[7]World Bank[1993, p.29]. The regression equation is: $GDPG = 0.013 + 0.062RGDP60 - 0.061RGDP60^2$.

endogenous growth models, formerly thought to be inconsistent with exogenous growth models, namely constant returns to human capital accumulation.[8] The resulted model yields insights into the working of an integrated world economy and provides a coherent explanation of the observed persistent income disparity and varieties of economic growth patterns.

Although roughly half of my model is inspired by Stokey [1991], there are several differences. In my model, productivity growth is sustained exogenously whereas in her model it is sustained endogenously. Second, my model is a variation of infinite horizon Ramsey models where workers continually accumulate their human capital; her model is a variation of overlapping generation models where workers first accumulate human capital and then work for the rest of their life. Workers in the same cohort have the same human capital level. Third, we also differ in the way we model the stock of knowledge. She models it simply as purely external effects where workers in the same cohort face with the same stock of knowledge. I do not. My approach has several advantages. Exogenous technical progress allows me to discuss the case when there are heterogeneity among workers. Infinite horizon Ramsey model allows me to discuss the case when workers differentiate themselves into high-skill and low-skill workers. And by allowing workers to have different stock of knowledge, I can discuss the variation in investment rates across countries.

The following section describes the model and its properties. I show that there exists a stable distribution of relative skills within which low-skill workers remain relatively low-skill indefinitely. The third section discusses two extensions. It allows for heterogeneity among countries so as to discuss a question asked at the beginning, why rich countries are rich. The other extension allows for external benefits of human capital in order to demonstrate that externality does not necessarily lead to income convergence among countries, and that there is a tendency for labor within each country to be homogenous. Poor countries optimally invest less than rich countries, as empirically observed. The fourth section suggests that differential increase in the labor force of developed and under-developed countries can help explaining the observed varieties of economic growth patterns. Specifically, this model provides a mechanism which allows some countries to catch-up faster than others while poor countries remains relatively poor. The fifth section concludes.

[8] This is an exogenous growth model with a twist.

1.2 A model of human capital accumulation and growth

This model is a multisector decentralized system with perfect foresight. Productivity growth is sustained by exogenous technical progress and human capital accumulation. New products with better quality are being invented exogenously; their production require workers of ever greater skill than before. Human capital of different levels are not perfectly substitutable. Better goods command higher prices; consequently, workers with greater skill command higher wages, providing incentive for human capital accumulation. International division of labor and constant returns to human capital accumulation sustains a stable distribution of relative skills and income.

I begin by describing my model, solve for the perfect foresight balanced growth path, and analyze its properties. My aim is to show that there exists a stable distribution of relative skills within which low-skill workers remain relatively low-skill indefinitely.

1.2.1 Model description

This is a model of an integrated world economy with one factor of production: labor of various skills. Each country is so small that it is negligible in the world economy; its population is *uniformly educated* and has the same human capital level.[9] L denotes the world total population. There is no cost of transportation and no national limitation to trade. All goods are traded freely. There are no financial assets. There are two types of goods: a final good and intermediate goods. The market for the final good is perfectly competitive. The final good is made of intermediate goods only and the production function is:

$$y = \left(\int_{-\infty}^{N} \left(e^i x_i \right)^\alpha di \right)^{\frac{1}{\alpha}} \tag{1.1}$$

where N is the best intermediate good available for production, x_i and $e^i = \exp(i)$ is the quantity and quality level of intermediate goods i, respectively. The number of products available

[9]This is a model of inequalities among countries. I will justify this simplifying assumption in the following section. It will be shown that external benefits of human capital within each country, provides tendency for its labor to have the same human capital level.

for production increases linearly at an exogenous rate g and quality rises exponentially.[10] It is assumed that $\alpha < 1$, the marginal product of each intermediate good at $x_i = 0$ goes to infinity, so that all available intermediate goods will be produced.

The market structure for each intermediate good is also perfectly competitive. The production function for intermediate good i is

$$x_i = l_i \tag{1.2}$$

where l_i is the number of workers employed in the production. Each worker is capable of producing 1 unit of any intermediate good providing that she has the required skill. Human capital of different levels are not perfectly substitutable; one must reach a certain level of skill in order to produce high quality products such as microprocessors and surgery.[11] Specifically, one must have human capital level at least e^i to produce an intermediate good i. It is this assumption that introduces comparative advantage between workers which provides reason for trade and, as we shall see, human capital accumulation.

Engine of growth

The engine of productivity growth in this model is the combination of human capital accumulation and exogenous introduction of better intermediate goods. Higher skill allows workers to produce better products. Better products command higher prices, providing incentive for human capital accumulation. For each worker, I assume as in Lucas [1988] that:

$$\dot{h} = \delta s h \tag{1.3}$$

where h is her current level of human capital; s, the fraction of time spent on education (or in school); δ measures educational efficiency level. I assume that there is constant returns to human capital accumulation.[12] External benefits of human capital will be discussed in the next

[10]These assumptions deliver constant (exponential) productivity growth. If the number of new products does not increase linearly, then productivity growth rate is either slowing down or increasing over time.

[11]For example, one cannot put two nurses together and perform a brain surgery. An appropriate training is required. This assumption is due to Stokey [1991].

[12]Constant returns is crucial for establishing a stable distribution of relative skills. Note that although workers at the frontier have constant returns to human capital accumulation in term of their human capital level, the returns in term of wages, as we shall see, is not.

section.

The relationship between constant returns to human capital accumulation and exogenous rate of new-product invention is the heart of this model. Each has its own role. The true engine of growth is exogenous technical progress: without new products, there is no incentive to accumulate additional human capital. Thus, exogenous technical progress imposes a constraint on human capital accumulation. On the other hand, without human capital accumulation there is no growth. My assumption of constant returns will deliver me a constant rate of human capital accumulation for each individual worker on the balanced growth path and is thus crucial for establishing a stable distribution of relative skills.[13]

Instantaneous production decision

Given perfect competition and constant returns to scale in the production function of the final and intermediate goods, there is no need to introduce firms. Each individual worker can set up a miniature factory on her own backyard. Let Y denotes the aggregate output of the final good. The market demand for intermediate good i, derived from equation (1.1), is given by:

$$X_i = \left(\frac{Pe^{\alpha i}}{p_i}\right)^{\frac{1}{1-\alpha}} Y \tag{1.4}$$

where p_i is the price of intermediate good i. The price for one unit of the final good is:

$$P = \left(\int_{-\infty}^{N} \left(e^i/p_i\right)^{\frac{\alpha}{1-\alpha}} di\right)^{\frac{\alpha-1}{\alpha}} \tag{1.5}$$

Given the production function and perfectly competitive labor market, a worker who produces one unit of intermediate good i, will earn her marginal revenue product: $w_i = p_i$. The relative

[13]One may wonder whether it is possible to construct a model of growth with only one of these assumptions. The answer is yes with some qualifications. First, Stokey [1991] constructs a model of endogenous growth with vertical product differentiation. Constant return to human capital sustains both productivity growth and income disparity. Goods are always available, the issue at hand is whether workers have the required skills. The technical difference between our models is the assumption concerning the marginal product of each intermediate good at $x_i = 0$. Stokey assumes that it is finite; I, infinite. (With infinite marginal product at $x_i = 0$, Stokey's model do not have a stable growth path, nor stable income distribution.) Second, is constant return to human capital accumulation neccesary for stable relative income disparity? I do not know the exact answer yet, but diminishing return and increasing return to human capital accumulation will not yield stable distribution of relative skills. On simplicity ground, constant return simplifies the model greatly; and it delivers persistent income disparity, something which is empirically observed.

employment of labor among various intermediate goods production is given by:

$$\frac{L_i}{L_j} = e^{(i-j)\frac{\alpha}{1-\alpha}} \left(\frac{w_i}{w_j}\right)^{\frac{1}{\alpha-1}} \tag{1.6}$$

where L_i is the aggregate number of workers producing intermediate good i. And w_i/w_j is the relative wages. This equation indicates that there is a direct connection between the wage profile and the allocation of labor among various intermediate goods production.

Perfect foresight variables: the wage profile

Let the price for one unit of the final good be the numeraire, $P = 1$, so that other prices will be given in term of the final good. Without any financial asset, there is only one perfect foresight variable: the wage profile $w(i,t)$ or the expected wage for producing one unit of intermediate good i at time t. This wage profile has a prominent role in this model as the sole incentive for investment in human capital. Each worker takes this profile as given when she makes her human capital accumulation decision.

Along a perfect foresight path, the expected wage profile coincides with the actual wage profile and nobody makes a mistake by over-accumulating human capital for her actual job. Workers with human capital level e^i always produce intermediate good i, the highest available job for their skill. This has three implications. First, at any point in time, the wage profile must be an increasing function of i, otherwise some workers could be better off by accumulating less human capital and yet earn as much wages as before. Consequently, better skills command higher wages. Second, this allows me to rewrite the wage profile as a function of human capital level, $w(h,t)$.[14] Third, according to equation (1.6), the shape of the wage profile determines the exact distribution of relative skills and vice versa.

1.2.2 The steady state

The steady state is characterized by a stable distribution of employment ($\ln h$) continuously shifting to the right at a constant rate, as illustrated in Figure 1-3. This, according to human

[14] I have already had $w(i,t)$. So, this function should have been written as $w(\ln h, t)$, to be exact. Reader should look at the parameter of these functions to determine which one of the two the author is referring to.

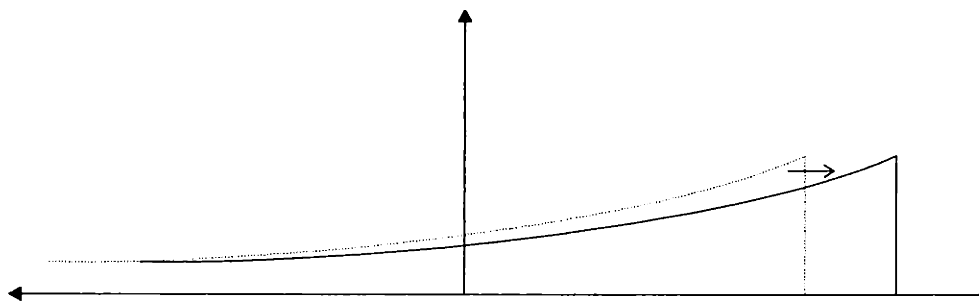

Figure 1-3: The evolution of the distribution of employment along a balanced growth path

capital accumulation equation, requires that each worker spends the same fraction of her time educating herself. The quality level of intermediate goods being produced by a given worker increases exponentially. The aggregate final good output increases exponentially, as a result.

In order to solve for a steady state, one must compute a wage profile which provides incentive for each worker — at all level of skills — to spend the same fraction of her time educating herself thereby sustaining a stable distribution of employment. This, as we shall see, requires that the elasticity of the wage profile with respect to human capital to be the same for all individual, independent of her human capital level. I thus consider the following candidate for the relative wages profile:

$$\frac{w(i,t)}{w(j,t)} = e^{(i-j)\beta} \tag{1.7}$$

where $\beta > 0$ so that the wage profile is an increasing function of product quality. β will be the elasticity of the wage profile with respect to human capital. I choose this functional form such that relative wages between any two intermediate goods does not depend on time, but only on the relative human capital required for their production.

Since workers are being paid their marginal revenue product, equation (1.7) implies that $p_i(t) = e^{(i-j)\beta} w(j,t)$. Substituting this expression into equation (1.5), I derive the implied wage profile:

$$w(j,t) = e^{j\beta} e^{n(t)(1-\beta)} \qquad \text{or} \qquad w(h,t) = h^{\beta} e^{n(t)(1-\beta)} \tag{1.8}$$

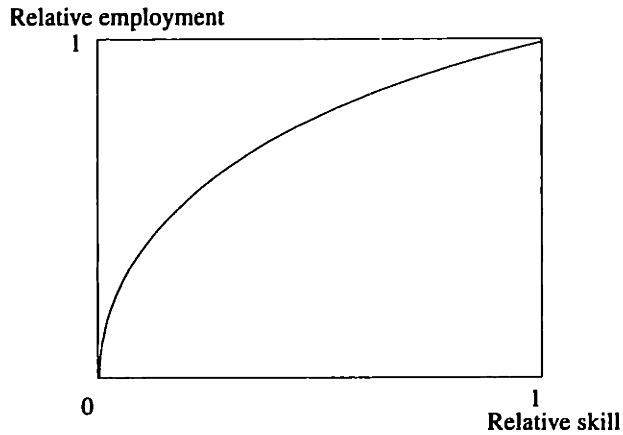

Figure 1-4: Relative employment as a function of relative skills

where $n(t)$ is the best intermediate good *being produced* at time t.[15] The wage profile will be represented by two variables: the expected elasticity of wage profile with respect to human capital, β, and the expected best intermediate good being produced at time t. This last equation also indicates that the wage being paid for producing a given intermediate good in term of the final good is increasing exponentially over time despite its declining relative wages with respect to the best intermediate good $n(t)$.

As mentioned before, equation (1.6) summarizes the relationship between the wage profile and the distribution of employment ($\ln h$) among various intermediate goods production at any point in time. My candidate for the wage profile implies that along a balanced growth path:

$$\frac{L_i}{L_j} = e^{(i-j)\frac{\alpha-\beta}{1-\alpha}} = \left(\frac{h_i}{h_j}\right)^{\frac{\alpha-\beta}{1-\alpha}} \tag{1.9}$$

The relative employment at any given intermediate good with respect to the number of workers employed at the currently best product, is declining over time. In other words, any given intermediate good will be produced less and less. Equation (1.9) also indicates that there exists a stable relationship between the relative employment with respect to that at the best intermediate good, $L_i/L_{n(t)}$, and the level of relative skills, $h_i/h_{n(t)}$. In other words, the

[15]Let us ignore, for three pages, the fact that the number of available intermediate goods $N(t)$ is given exogenously. The role of exogenous technical progress will be discussed there.

distribution of relative skills $h_i/h_{n(t)}$ will be stable in a steady state.

This relationship between relative employment and relative skills is illustrated in Figure 1-4. By construction, this curve passes through the origin and (1,1). Its curvature is determined by $(\alpha - \beta)/(1 - \alpha)$. When intermediate goods are more substitutable — larger α — more workers acquire higher skill; for that, the relative demand for high-quality goods increase. My model requires the endogenously determined elasticity of the wage profile β to be smaller than α in order for $\int L_i \, di$ to exist. Under this condition, the number of employment at a given intermediate good is an increasing function of product quality.[16]

Human capital accumulation

Let denote human capital level of individual ς by h_ς. The perfect foresight path requires that she does not over-accumulate her human capital for her job; thus, she always produce the best available intermediate good given her skill. She produces $i_\varsigma = \ln h_\varsigma$. Each worker faces with the following intertemporal maximization problem:

$$\max_{s} \int_0^\infty e^{-\rho t} \ln[(1-s)w(h_\varsigma, t)]dt$$

subject to

$$\dot{h}_\varsigma = \delta s h_\varsigma$$

where ρ is her discount rate. Each worker allocates her available time between working, at the wage rate $w(h_\varsigma, t)$, and educating herself in order to improve her human capital for higher paid job in the future. There are no financial assets in this economy. Each worker takes as given the two variables, β and $n(t)$, which represent the wage profile. Solving this problem is easy.[17] On a balanced growth path, it yields that

$$s = 1 - \frac{\rho}{\delta\beta} \qquad \text{and} \qquad \mu = \delta s = \delta - \frac{\rho}{\beta} \tag{1.10}$$

[16]This does not imply that most workers in the world have high-skill. Using equation (1.9), I derive the total measure of workers with skill lower than e^i to be $M_i = \int_{-\infty}^{i} L_i \, di = e^{(i-N)(\alpha-\beta)/(1-\alpha)}\mathcal{L}$ where $\mathcal{L}$ is the total labor engaging in production in the world economy. Since I can choose $(\alpha - \beta)/(1 - \alpha)$ to be as small as I like, the total measure of workers with skill less than e^i approaches $\mathcal{L}$ for any given i. In other words, it is possible for most workers to have low-skill in this model.

[17]I solve this maximization problem in detail in Appendix A.

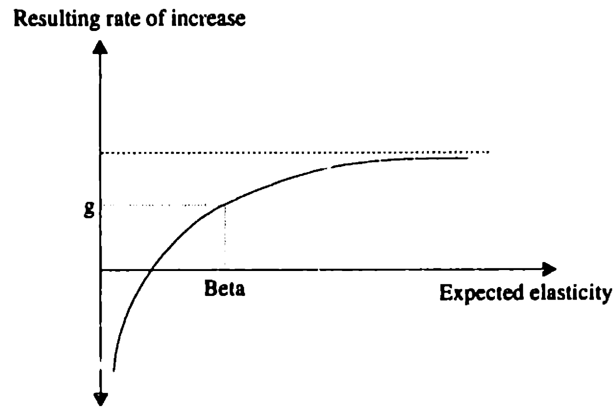

Figure 1-5: The resulting rate of increase μ as a function of the expected elasticity

where μ denotes the *resulting* rate of increase of new intermediate goods being produced. The fraction of time each individual spent on educating herself does not depend on her human capital level. Every individual increases her human capital and moves up the product ladder at the same rate μ. Consequently, the distribution of employment ($\ln h$) is stable and continuously shifting to the right at this constant rate.

The relationship between the resulting rate of increase of new intermediate goods being produced and the expected elasticity of the wage profile is illustrated in Figure 1-5. In this economy, the elasticity plays a prominent role in human capital accumulation process. With higher elasticity, wage increases are more responsive to human capital accumulation. There is greater incentive for each to spend more time educating herself. Each moves up the product ladder faster and, consequently, the resulting rate of increase of new intermediate goods being produced will be higher. The opposite happens with lower elasticity.

The case when $\beta = 0$ deserves additional discussion. This is the case when all workers earn the same wage rate as in the standard models. When the elasticity is expected to be zero, the wage profile is expected to be flat. Workers could have earned the same wage with lower human capital. There is no incentive for workers to keep up with the frontier. In fact, they will decumulate their human capital if possible. Thus, homogenous labor force, where every worker is identical and earns the same wage, is not to be expected in this model.

The perfect foresight path requires that the *expected* increases in the number of intermediate good being produced $\dot{n}(t)$ and the actual increases μ coincides. Since each worker can always form her expectation freely such that $\dot{n}(t)$ is equal to μ, it seems that any point on the curve in Figure 1-5 can be a steady state. This is not true, however. There is a unique steady state as I shall demonstrate.

Here is where exogenous technical progress plays a crucial role and where the assumption concerning the marginal product of intermediate good at $x_i = 0$ being infinity, is essential. Let us first consider the case when the rate of increase of new intermediate goods being *produced* μ is less than the rate of new products being *invented* g. There will be intermediate goods which have already been invented, but no workers have the skill to produce it. This cannot be an equilibrium, for workers will deviate given that others stick to their plans. Workers can do better by becoming a producer of this good. (Since the marginal product of each intermediate good is very large around $x_i = 0$, this worker can command high wages in return.) Therefore, $\mu < g$ cannot be an equilibrium. On the other hand, if the exogenous rate of new products being invented is smaller than the rate of human capital accumulation μ, then after some time someone will over-accumulate her human capital. These cannot be a perfect foresight path, either. Only when $g = \mu$, nobody has any incentive to deviate; the expected wage profile coincides with the actual wage profile; and the distribution of employment is stable and shifting to the right at the rate g. (These are the precise reasons for imposing exogenous technical progress onto this system despite constant returns to human capital accumulation.[18])

On the perfect foresight balanced growth path, the fraction of time spent on education and the elasticity of the wage profile are given by:

$$s = \frac{g}{\delta} \qquad \text{and} \qquad \beta = \frac{\rho}{\delta - g} \tag{1.11}$$

where I assume that $\delta > g$; that is, each individual is capable of catching up with the frontier of intermediate goods. This completes the basic model.

It is worthwhile to look back and give an analogy for the system constructed. Think of

[18] Of course, I do not strongly believe that the marginal product of each intermediate good at $x_i = 0$ is infinity, instead of being finite as Stokey [1991] assumes. Nor am I wishing to argue that it is the case in reality. There are two grounds for defending this model. First, this assumption of infinite derivative is not crucial. If I impose exogenous technical progress onto Stokey's model, with exogenous technical progress preceeding at a slower pace than what her system yields, I believe, I will get the same result. Second, and more importantly, this assumption allows me to introduce exogenous technical progress into this model; together they yield insights to several important empirical puzzles. Maybe the price is worth paying for.

traffic on a highway. If the highway is empty where one can drive as fast as one want, there is no stable structure to be observed. However, suppose I put a Model T in the middle and nobody can get pass her. Of course, people will get angry but some structure will eventually emerge. My assumption concerning exogenous technical progress provides that needed Model T. Stokey [1991] gets her Model T by assuming that the marginal product is finite. The emerging structures are the wage profile and the distribution of relative skills. Now, we are ready to study how these structures change under various conditions (when that Model T slows down, for instance).

1.2.3 Comparative static exercises

To understand the results of comparative static exercises in this subsection, one must make a distinction between a change in an exogenous parameter for a particular individual, and a change in an exogenous parameter for the whole system. In the former case, one uses equation (1.10) to determine its effects; in the latter case, equation (1.11). The difference between the two is that a change for a particular individual has no effect on the wage profile. Comparative static exercises for this individual have the usual signs: smaller discount rate and greater educational efficiency induce her to spend more time educating herself and move up the product ladder at a faster pace.

When a change in an exogenous parameter for the whole system occurs, the wage profile adjusts in response; the incentive structure that each individual worker faces, is altered. Some results of comparative static exercises — such as greater educational efficiency causes people to spend less time on education — might be puzzling at first. But, if one keeps in mind that the wage profile has been changed and these are comparative static exercises on the whole system, they will make sense.

I report results of comparative static exercises in Table 3.1. Each cell of the table indicates how a change in an exogenous parameter for **the whole system** affects the elasticity of the wage profile, the distribution of relative skills and the fraction of time spent on education. Below, I describe how the system adjusts when a change occur.

Effects of exogenous technical progress

Exogenous technical progress affects workers' decision and the system in a roundabout way. Inspection of equation (1.10) reveals that workers do not care about how fast new products are being introduced.[19] The fraction of time each individual spent on education only depends on her own discount rate, her educational efficiency, and the elasticity of the wage profile with respect to human capital. An increase in exogenous technical progress affects each individual only by altering the wage profile.

Comparative static exercises indicate that an increase in exogenous technical progress raises the elasticity of the wage profile. This is so because it is hard to be at the top, or keeping up with it. When the top is moving faster and requires one to spend more time keeping up and less time for fun, the system must provide enough incentive for this person to do so. Furthermore, there will be more relatively low-skill workers in the system. An interpretation of this is that the frontier is moving too fast, so more people fall further behind.

Effects of educational efficiency

An increase in educational efficiency has two effects: one on each individual and the other on the wage profile. With higher efficiency, each individual would spend more time educating herself providing that the elasticity of the wage profile remains the same. So, people catch up with the frontier at a faster rate, resulting in more relatively high-skill workers in this system. However, one unintended consequence of these actions is that with more people working closer to the frontier, the relative wages of advance products fall and thus the return from accumulating additional human capital falls as well. In the long run, with the elasticity of the wage profile keeps declining, the time each individual spent on eduction will decrease. This process keeps on going until there is no more incentive for catching up with the frontier. At that point, the fraction of time spent on education will be lower than before.[20] Notice that this should not be interpreted that it has a wrong sign; workers have now moved up the product ladder to a higher relative positions as a result of higher educational efficiency.

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[19] This statement implicitly assumes that there are no external benefits of human capital.

[20] Intuitively, with greater educational efficiency and constant g, the fraction of time spent on education will be lower on the balanced growth path.

Table 1.1: Comparative Static Results

Parameters	Elasticity of wage profile	Distribution of relative skills	Education
g	+	More low-skill	+
δ	–	More high-skill	–
ρ	+	More low-skill	No changes

This implication stands in the opposite direction of the prediction one would get from endogenous growth models. In those models, an increase in educational efficiency leads each individual to spend more time on education; consequently, productivity growth increases as well.

Effects of the discount rate

Similarly, an increase in the discount rate has two effects: one on each individual and the other on the wage profile. With higher discount rate, each individual cares more about current consumption and does not value future return from higher paid job as much as before. She spends less time educating herself. Therefore, workers fall further behind the frontier, resulting in the increase of more relatively low-skill workers. The relative wages for primitive products are now lower and thus the return from accumulating human capital is rising.

In the long run, with the elasticity of the wage profile keeps increasing, the fraction of time each individual spent on education rebounds from its drop. This process keeps on going until there is no more incentive to fall further behind the frontier; at that point, the fraction of time spent on eduction will be the same as before. (This is a direct consequence of equation (1.3).) Of course, one should not interpret this as a wrong sign, either. Workers now accumulate less human capital than they would have, if they had the old discount rate.

World population level

One implication that is not reported, is how an increase in the world population level affects the system. Inspecting the model reveals that the distribution of relative skills and the wage profile does not depend on the world population level. In the new equilibrium, there will be a proportional increase in employment of all skills. Put it differently, this model predicts that certain fractions of population will be surgeons, nurses, barbers, farmers...etc. There will be more surgeons in the world economy only if there are more people in the world. This implication will be important for explaining the observed varieties of economic growth patterns.

1.3 Two extensions

This section provides two extensions of the basic model. First, I introduce heterogeneity among countries so as to discuss, a question asked at the beginning, why rich countries are rich. Second, I introduce external benefits of human capital in order to demonstrate that (1) externality does not necessarily lead to income convergence among countries, (2) there is a tendency for labor within each country to be homogenous, which justifies my assumption at the beginning that workers in each country have the same human capital level, and (3) poor countries can optimally invest less than rich countries.

Additional remark concerning my definition of a country is in order. So far, one can think of this as a model of a closed economy with many individuals. Given that national boundaries have no effect whatsoever — any given worker will face the same incentives irrespective of her nationality — it makes no difference whether there exist national boundaries or not. Consequently, the distribution of relative skills within each country is indeterminate.[21]

One can still think of my first extension as a model of a closed economy with heterogenous workers; for that, there is no role for national boundaries in that extension, neither. This extension shows that workers with advantage in human capital accumulation or with greater patient eventually have higher human capital and higher income. It is only due to my assumption that each country consists of workers with the same characteristics that allows me to make a similar assertion in countries' level.

[21]The following mathematical construct should clarify this statement. Think of a unit rectangle, with x-axis denoting human capital level and y-axis, density of workers who have that human capital level. (For illustrative purpose, I assume that I have a uniform distribution.) One can slice or draw a curve through this rectangular in many ways. If one slices it horizontally, calling each horizontal line a country, then every country will have the same distribution of relative skills. If one slices it vertically, each country will be homogenous in term of skill. And there are many other ways to slice this rectangular as well, resulting in different distribution of skills within each country. Note that I can always do it in such a way that each country has measure zero, i.e. negligible. Since I can always do this *after* the world economy reach its equilibrium, the distribution of relative skills within each country is indeterminate.

Only in the second part of my second extension where there is external benefits of human capital within each country, that national boundaries do matter. Being in the same country with workers with high human capital raises one's effective educational efficiency. The transitional dynamic will be complex, due to forces acting at local and global levels. Consequently, I cannot discuss the distribution of relative skills within each country during a transitional path. However, it is possible to show that there exists a steady state where each country has homogenous labor force.[22] Then, I show that there exist a tendency for each individual to return to her country average human capital level after a shock to her human capital occurs.

Why rich countries are rich: heterogeneity among countries

One limitation of the basic model is that it does not provide a satisfactory answer to the question, why rich countries are rich. The position of a country within the world trading economy is determined purely by chance and history. In a more complete model, countries with advantages in human capital accumulation process or those with greater patient for future reward should have tendency to become richer than the rest of the world.

Toward this end, I extend my model to allow for heterogeneity in educational efficiency levels and subjective rates of discount. The world economy is populated with a distribution of people with different characteristic. The main task is to show that there exists a perfect foresight stable distribution of relative skills corresponding to that given distribution. I assume, as before, that population of any given country share the same characteristics.

First, suppose that there is heterogeneity in the subjective rates of discount. Denote the discount rate of individual ς by ρ_ς. A perfect foresight distribution of relative skills can be found by the following argument. In a steady state, the distribution of relative skills must be stable; the equilibrium wage profile must provide incentive for each individual to accumulate her human capital at the same given exogenous rate, thereby sustaining her relative skills with respect to the frontier. Since human capital accumulation decision remains unchanged,

[22] And this is the only stable steady state with respect to shock to each country average human capital level. The other obvious candidate for a steady state is when every country has the same distribution of relative skills. It is unstable, however. If a country experiences a positive shock, it will eventually accumulate more human capital than other countries.

according to equation (1.10), this requires that at any point in time:

$$g = \delta - \frac{\rho_\varsigma}{\beta_\varsigma} \tag{1.12}$$

where $\beta_\varsigma \equiv \beta(h_\varsigma, t)$ is the elasticity of the equilibrium wage profile with respect to human capital at h_ς. This condition imposes a relationship between each individual rate of discount and the elasticity of the wage profile at her position.

Given that, the relative skills of this individual with respect to the frontier remains unchanged in the steady state, this condition implies that the elasticity of her relative wages with respect to relative skills at her position must be constant and given by β_ς.[23] In other words, I will be able to determine the elasticity of the whole relative wages profile from my knowledge of each individual's relative skills.

Toward this end, consider a simple example of two types of individual with discount rate ρ_1 and ρ_2, where $\rho_1 > \rho_2$. In a steady state, the relative wages at any relative skills must have one of the two corresponding elasticity β_1 and β_2. Equation (3.6) implies that an individual with larger discount rate must face a higher elasticity. Although it may be possible to construct several wage profiles which yield stable distribution of relative skills, there is only one wage profile that is stable with respect to disturbance. The elasticity of relative wages must be a non-increasing function of relative skills as illustrated Figure 1-6. With this given profile, if a country with great impatience experiences a good fortune — its human capital level suddenly increases — it will fall into the region with smaller elasticity. Now, the loss from falling further behind the frontier does not cost as much given its taste for current consumption; therefore, its population spend less time educating themselves in comparison to others with the same human capital level. This country will fall further behind the frontier and back to where it comes from. Note that the breaking point f is determined by the relative mass of these two types of workers. With continuous distribution of discount rates, the elasticity of the wage profile will be continuous and declines with respect to human capital.[24] In sum, this extension implies

[23]For that, denote the relative wages by $z = w(h_\varsigma)/w(h_N)$, then $[dz/d(h_\varsigma/h_N)][h_\varsigma/zh_N] = w'(h_\varsigma)h_\varsigma/w(h_\varsigma)$. In other words, the elasticity of relative wages with respect to relative skills is determined by the elasticity of the wage profile with respect to human capital; and vice versa. I will use them interchangebly.

[24]An extension to a continuous distribution of discount rates can be done easily. One know that any continuous function can be approximated by a sequence of step functions with finite steps. For a given step function

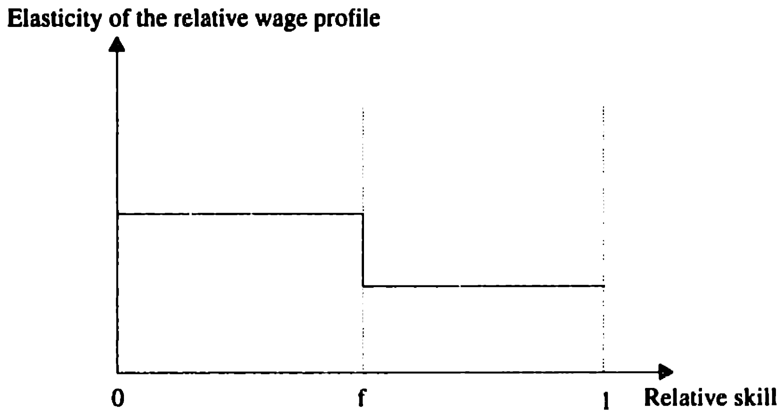

Figure 1-6: The elasticity of relative wages profile as a function of relative skills, given hetrogeneity in the discount rates.

that countries with greater patience will eventually have higher human capital level and higher income per capita, as expected.

Alternatively, one can introduce heterogeneity among countries' educational efficiency levels. Similar argument applies. In a steady state, human capital accumulation decision requires that at any point in time:

$$g = \delta_\varsigma - \frac{\rho}{\beta_\varsigma}$$

Countries with higher educational efficiency must face with lower elasticity of the wage profile with respect to human capital. As before, the elasticity of the wage profile must be non-increasing. Countries with higher efficiency in their human capital accumulation will eventually have higher human capital level and higher income per capita; and countries with lower efficiency, lower human capital level and lower income per capita, as expected. This provides an analytic interpretation to Landes' remark that "as time passes by, those most qualified make it; but those who do not make it lose ground and become less and less qualified."[25]

This extension has several implications. First, as in exogenous growth models, policies designed to encourage growth such as increasing efficiency of educational system, financial reform, or investment promotion cannot have long term effect on growth rates. These policies

approximating characteristic cumulative distribution, one get a corresponding step function of elaticity profile. Then, one takes the limiting function of the corresponding sequence.

[25] Landes [1990, p.12]

move the country's position in the world economy into the region where return to additional human capital accumulation is lower. This undermines the original policies; for that, each individual worker will adjust and spend less time educating herself. (This model however has a curious implication, retaining from its endogenous growth predecessors. If the government can non-optimally *force* everybody to spend more time educating herself, it may be able to raise income growth rate indefinitely until it reach the frontier.)

Second, some have suggested that there is positive relationship between stability and GDP growth. In its study of East Asian miracle, the World Bank [1993] finds that "macroeconomic stability and rapid export growth were two key elements in starting the virtuous circle of high rates of accumulation, efficient allocation, and strong productivity growth that form the basis for East Asia's success." Easterly and Levine [1995], in their study of Africa's growth tragedy, find political instability such as revolutions and assassinations, contributes negatively to growth. One could *stretch* this model and interpret these as changes in the subjective rates of discount for each individual in those countries. With less stability, the future return to human capital is "discounted more" Thus, countries with less stability move down the product ladder. But, there will be no long term effect on growth, however.

Third, there are some countries such as Burma[26], Laos, and Vietnam that have secluded themselves from the world economy for various reasons. It is interesting to ask, if a country had fallen further behind from the frontier by failing to accumulate human capital for whatever reasons, could this leave this country relative poor permanently? The answer is that it depends. In the context of this extension, if there is no change in educational efficiency level, this country will eventually return to its former relative position; if educational efficiency level is somehow lower as a result, this countries may remain relatively poor comparing to where it would have been; if the world is described by the basic model, then secluding from the world economy could be a serious policy mistake.[27]

Fourth, there are empirical findings and some theoretical models suggesting negative corre-

[26]Burma close itself from the world economy for 50 years. As a result, the economy is relatively backward comparing to the rest of the world. (My father told me that a house fire in Rangong could cause a serious damage because it will be fought with water in buckets and sand.)

[27]To discuss this satisfactorily, I need to provide a detail of how the world exogenous technical progress diffuse to this country. (One can expect that it is slow.) This is why I will not ask the standard question of which is better, autarky or free trade?

lations between population growth and educational efficiency. In my model context, countries with faster population growth — and consequently lower educational efficiency — are likely to move down the product ladder. This is why high population growth rate is not a good strategy for development.

External benefits of human capital

The extension above predicts that countries with lower educational efficiency (and thus lower GDP per capita) face higher return to human capital accumulation, thereby spending larger fraction of their time educating themselves. Empirical evidence as illustrated in Figure 1-2 suggests otherwise: poor countries have much lower investment rates than rich countries. This section demonstrates that by allowing for external benefits in human capital, the resulted extension delivers rising investment rate as a function of income, as empirically observed. Particularly, let us consider the following human capital accumulation process.

$$\dot{h}_\varsigma = \delta s_\varsigma h_\varsigma^{1-\sigma} W^\sigma \tag{1.13}$$

where $1 - \sigma$ is the relative importance of own human capital relative to the world aggregate human capital, W. This functional form captures the idea that with better environment, it is easier to increase one's human capital. Larger W raises the *effective* educational efficiency of each worker. Constant returns to human capital accumulation is preserved to sustain stable distribution of relative skills as before. The world aggregate human capital level is assumed to be a function of each individual human capital level and is homogenous of degree one.

In this world, each worker takes the wage profile and the future path of $W(t)$ as given. Solving the corresponding human capital accumulation decision of each individual yields the following condition:

$$g = \delta \left(\frac{W}{h_\varsigma}\right)^\sigma - \frac{\rho + \sigma g}{\beta_\varsigma}$$

When $\sigma = 0$, I get back the by-now familiar condition. In a steady state with a stable distribution of relative skills, this equation requires that the elasticity of the wage profile is an increasing function of human capital level.[28] This is obvious. Since countries with relatively

[28] Notice that the elasticity can now be greater than α as long as the elasticity of the wage profile at the lower

lower skills have more knowledge to learn from, it is harder to be a forerunner. The wage profile must provide an appropriate incentive for them to keep their lead and not falling back. That is, the return from human capital accumulation for forerunners must be higher.

At this point, one might still be skeptical, feeling that knowledge spillover should be a stronger force and lead to income convergence as usual. The following thought experiment should help. The question is whether it is possible for every worker to have the same human capital level, thereby earning the same wage. The answer is no. Since the elasticity of the wage profile will be zero in this situation, falling back imposes no penalty. Workers can earn the same wage as others even with lower human capital level. Some will consume now and not keeping up with the frontier. Eventually, the price of the frontier product rises relatively; for that, now, not many workers have the skill to produce it. This keeps happening until there is enough incentive for the remaining workers not to fall behind: the elasticity of wage profile at this point must be large enough. By induction, one can work this logic, going down the product ladder until equilibrium wage profile and a stable distribution of relative skills emerge. In sum, introducing external benefits of human capital, though affecting the equilibrium wage profile, does not necessarily lead to income convergence among countries as Lucas [1993, p.255] suggested.

The fraction of time spent on education for each individual can be easily calculated from equation (1.13).

$$s_\varsigma = \frac{g}{\delta}\left(\frac{h_\varsigma}{W}\right)^\sigma$$

Under-developed countries with lower human capital spend less time on education. In other words, poor countries optimally invest less than rich countries, as empirically observed.

Recalling an assumption I made at the beginning, that labor within each country have the same human capital level. Here is an appropriate place to justify my assumption by demonstrating that although external benefits of human capital does not lead to income convergence among countries, it does lead to income convergence *within* each country. Consider the following

end around $h = 0$ is less than α. In addition, in order to maintain the concavity of the wage profile with respect to human capital, I need that $c(h) + \beta(h) - 1 < 0$ where $c(h,t)$ is the elasicity of $\beta(h,t)$. This condition will be easily satisfied with small σ and $W(t) \approx h_{N(t)}$.

human capital accumulation process.

$$\dot{h}_\varsigma = \delta s_\varsigma h_\varsigma^{1-\sigma} H_\varsigma^\gamma W^\eta$$

where $\gamma + \eta = \sigma$. The country average human capital level is denoted by H_ς, which captures external benefits of human capital within each country. Each worker takes the future paths of H_ς and W as given. Solving the corresponding human capital accumulation decision yields:

$$s_\varsigma = 1 - \frac{\rho + \sigma g}{\delta \beta_\varsigma} \left(\frac{h_\varsigma}{H_\varsigma} \right)^\gamma \left(\frac{h_\varsigma}{W} \right)^\eta \tag{1.14}$$

where β_ς is the elasticity of the wage profile at h_ς. Consider a world resting in a steady state with its equilibrium wage profile and stable distribution of relative skills. Suppose that individual ς somehow loses her human capital (or she's just born). I will show that she will converge back to her home country average human capital level H_ς. First, notice that equation (1.14) implies that she will spend a larger fraction of her time educating herself than another who has the same human capital level but residing in a country with average human capital, $H^* = h_\varsigma$. For that, she has higher effective educational efficiency due to external benefits of her countrymen. Thus, she accumulates her human capital faster than this other individual. Given that in a steady state where every worker, beside her, accumulating their human capital at the same rate, this particular worker who loses her human capital will converge back to her home country average human capital level, as claimed.

This difference between international external benefits of human capital and national one, is due to the fact that in the latter case, income convergence within a country has no effect on the wage profile.

1.4 Miracle and Poverty

This section provides a discussion of persistent income disparity and the observed varieties of cross-country economic performances during 1960-1985. It *suggests* that the observed varieties of growth patterns is a natural phenomena occurring as a response to differential increase in the labor force of developed and under-developed countries.

Why poor countries remain poor: persistent income disparity

Forty years ago, Professor Myrdal asked so passionately in his book *Development and Underdevelopment: a note on the mechanism of national and international economic inequality*[29] — "why and how have these international economic inequalities come to exist, why do they persist, and why do they tend to increase? What is the causal mechanism at work which produces these trends?" In order to evaluate my model, I revisit these questions.

My model suggests that observed inequalities is nothing but division of labor at work. Inequalities is natural outcome, given that there are many products in the world economy. Different products require workers of different skills, with advanced products such as microprocessors demanding more skill from their workers than primitive products such as rice. Since someone has to produce each of these products and it is not necessary to get a degree to become a farmer, existence of a distribution of skills is to be expected.

My extended model provides additional reason for the emergence of international inequalities. In the basic model, there is nothing preventing every country to be identical and has the same distribution of relative skills. It is shown that external benefits of human capital within a country, provides tendency for human capital of workers within a country to converge. Interaction between this tendency and division of labor, leads countries to differentiate themselves as high-skill or low-skill countries.[30] Since higher skills command higher wages, international inequalities come to exist.

An explanation of why international economic inequalities tend to increase is obvious in my model context. My integrated world economy yields a stable distribution of relative skills in which low-skill workers remain relatively low-skill indefinitely. Although, the distribution of relative income is constant over time, the distribution of wages in term of the final good is increasing.[31]

[29]This book is based on the three lectures which Professor Myrdal gave at that National Bank of Egypt in 1955. This book is later revised and published under the title *Economic Theory and Under-Developed Regions.*

[30]Note that my model requires that there are at least slight differences in the initial condition between countries. It then provides a mechanism which magnifies these differences, however slight, and allows countries to differntiate themselves into high-skill and low-skill countries. When all countries initial conditions are identical, one should not expect them to behave differently and international division of labor will not emerge. However, this situation is extremely unstable for that there is no incentive to keep up with others. Some countries will deviate and accumulate less human capital in order to enjoy higher current consumption. Of course, these countries pay the price later by becoming low-skill countries.

[31]Let X denote relative income; $Y = e^{gt}X$ is income in term of final good. Thus, $Var[Y] = e^{2gt}Var[X]$.

Persistence of economic inequalities is not answered satisfactorily, however. For the distribution of relative skills to be stable, there must be an appropriate incentive structure — consistent with costs of human capital accumulation — which keeps every worker happy with her relative position. This structure is given by the wage profile which provides incentive and disincentive for investment in human capital. However, the stability of this distribution is bought by assuming that there is a constant return to human capital accumulation. Since I assume this condition, I have not satisfactorily answer why economic inequalities persist nor why poverty will always be here. A question remains — why should there be constant return to human capital accumulation?

On a brighter side, this model provides an insight into the conventional wisdom that "lateness is an advantage; that the gap between what is and what can be is a tremendous opportunity; that the follower can benefit from the experience and knowledge of its predecessors and avoid their mistakes..., it will in fact grows faster than its forerunners."[32] Lateness in this model is not an advantage. Failing to accumulate human capital and falling further behind the frontier, "could" permanently leave the country in lower part of income distribution. As shown in the last section, knowledge spillover though providing benefits to followers does not necessarily lead to income convergence (due to its effects on the wage profile).

Why some catch-up faster than others: economic miracle.

In his third lecture, Professor Myrdal comments on the prospect of development that "The under-developed countries in their drive for economic development are in almost all respects up to very much greater difficulties than the now developed countries ever faced....[They] are late-comer. They have not the opportunity, as the now developed countries had, to advance as industrial islands in the surrounding world of backward nations which they could exploit as markets for manufactured goods and as sources of raw materials." Reflection on economic development during the forty years interim provides some brightening examples of economic successes for some middle-income countries. If Professor Myrdal is to be correct in suggesting that the now developed countries had developed by integrating backward countries as part of

Income disparity is increasing over time, given a stable distribution of relative skills.

[32]This quotation is taken out of context. Landes [1990, p.8] is refering to the doctrine of Gerchenkron, who wrote the famous "Economic Backwardness in Historical Perspective."

their production processes of primitive products, one can explain recent economic miracle by a force akin to his with differential increase in the labor force of developed and under-developed countries now providing "the surrounding world of backward nations" for these economic miracle.[33]

Empirical findings concerning cross-country economic performances during 1960 and 1985 is summarized by Figure 1-1. It indicates existence of varieties of cross-country economic growth patterns. Experience of a given country varies according to its position in the world distribution of income. First, poor countries do not grow faster than the rest of the world, on average. Second, there is evidence for income convergence among developed countries. OECD countries with initially lower GDP per capita have experienced higher growth than those with initially higher income. Third, some middle-income countries have achieved economic miracle.[34] These varieties of growth patterns have puzzled economists: is there any model that is capable of explaining these behavior *coherently*. Can one construct a model of growth in a way that these varieties of behavior are its inherent property or that these varieties are systematic response to changes in its underlining parameters? Specifically, is it possible to construct a model which systematically allows middle-income countries to catch-up faster than other, thereby delivering an inverted U-shaped curve as drawn in Figure 1-1?

My basic model predicts that each country has the same growth rate. Income disparity persists. Poor countries does not grow faster than the rest of the world. Varieties of economic performances is not its inherent property. Nonetheless, this model *suggests* that the observed varieties of economic performances could be a systematic response of an integrated world economy to differential increase in the labor force of developed and under-developed countries.

The logic runs as follow. Given exogenous technical progress and constant return to human capital accumulation, there is a unique steady state distribution of relative skills with its corresponding wage profile. Given that labor in each country tends to be homogenous — as I have shown, greater increase in the labor force of under-developed countries increases the world relative supply of unskilled labor. This changes the wage profile: the relative prices of advance products increase, increasing the return to human capital accumulation for low-skill

[33]It is this comment of Professor Myrdal that started this project.

[34]Of course, there are economic diaster as well: some countries experience negative average growth rates during the same period.

workers. Over time, the system adjusts itself so as to return to its steady state distribution of relative skills. For this to occur, developed countries catch up with the forerunner; and several middle-income countries experience spectacular economic performances in the process.[35]

According to 1980 World Development Report, during 1960-1970 the weighted average of average annual growth of labor force in low-, middle-, and high-income countries are 1.7%, 2.0%, and 1.2%, respectively. Those during 1970-1980 are 1.9%, 2.4% and 1.1%, respectively.[36] Notice that the largest increases occur in middle-income countries. Although changes in world labor force will has no effect on the long-run distribution of relative skills, relative skills with respect to the frontier of a given country changes during the adjustment process. After the system reaches its new steady state, one can then calculate the total gains in term of relative skills with respect to the frontier of a given country, if one is willing to make an additional assumption that the countries' ranking in term of skills does not change during the adjustment process.[37] This total gains is defined to be the ratio of relative skills after and before the adjustment process. It measures the additional growth in human capital with respect to that of the frontier, of a given country. In other words, it tells us how far a given country converges to the frontier after the adjustment process ended.

The total changes as a function of relative skills is depicted by Figure 1-7.[38] First, notice that developed countries are now closer to the frontier as a result of the increase in the labor force of under-developed countries. Developed countries with initially lower relative skills have made greater gain, indicating that income convergence among developed countries had occurred. Second, middle-income countries experience the largest convergence to the frontier. Third, lower-income countries achieve lesser gain than middle-income countries, as empirically observed. They can either experience convergence or divergence from the frontier depending

[35]Metaphorically, demographic differential creates pressure within the system. Think of water being gently drop on one side of a cup. Existing water will be disturbed and try to return to its steady state. It is this pressure that allows economic miracle and income convergence among developed countries to occur simultanously with persistent income disparity.

[36]The weighted averge of average annual growth of population in lower-, middle-, and high-income countries during 1960-1970 are 2.5%, 2.5%, and 1.0% respectively. Those during 1970-1978 are 2.2%, 2.4%, and 0.7% respectively.

[37]I make this assumption since it is difficult to calculate the transitional dynamic of this sytem. This assumption captures an idea that once an opportunity arise, a person who is closer to that opportunity will be in a better position to seize it.

[38]The detail derivation is given in Appendix B.

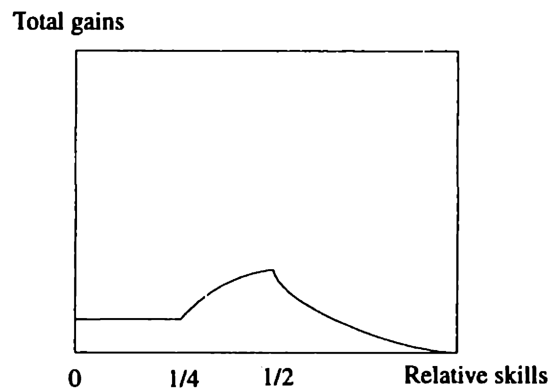

Figure 1-7: Total gains in relative positions, after the adjustment process ended, as a function of initial relative skills

on the relative increase in the labor force of lower- and middle-income countries.

The mechanism I mentioned at the beginning of this section is the principal reason behind convergence to the frontier. An increase in the labor force of countries with lower skill allows or forces countries with higher skill to move up the product ladder. (With more people in the world economy, there can now be more brain surgeons.) However, with relatively greater increase in the labor force of middle-income countries, the return from human capital accumulation for low-skilled labor is lower, resulting in slower human capital accumulation and lesser total gains for lower-income countries.

There are several implications. First, middle-income countries grow faster than the rest of the world, thereby providing an explanation for economic miracle and the inverted U-shaped curve. Second, high-income countries converge to the forerunner, confirming the evidence on income convergence among developed countries. Third, income disparity persists while income convergence among countries occur. The system always returns to its unique distribution of relative skills. On the other hand, there are some problems with this model. First, it predicts that on average poor countries may gain on the frontier (depending on the magnitude of the relative increase in the labor force of low- and middle-income countries). Here, I am willing to settle for the lesser prize of explaining the observed slower growth in poor countries in comparison to middle-income countries. Second, there are countries which had experienced the so-called economic disaster during 1960 and 1985. In the group of ten countries — mostly lower-income Sub-Sahara African countries — Chari et. al. [1995] find that their average growth of relative income to be around -3.1. Third, there are several middle-income countries that did not experience economic miracle. During 1960 and 1985, the average GDP growth rates of Argentina, Uruguay, and Venezuela are 0.7%, -0.1%, and -0.09%, respectively.

This inability of my model to predict experience of each country reflects the fact that there are several important variables and aspects of the real world left undiscussed by the model and that economic growth is **not** a simple subject. Negative growth experience, according to the first extension of my model, can be a result of instability within these countries. At any rate, the task of a theory is and always will be to explain the general trend and not each specific example.

1.5 Conclusions

Time and time again, economists ask: why poor countries remain poor; why rich countries are rich; and why some catch up faster than others. This paper attempts to provide answers for these questions.

The crucial step toward understanding these phenomena requires that one first constructs a model of persistent income disparity so that there will always be poor countries and rich countries; and that catching-up is a possibility. I combine exogenous growth models with an aspect of endogenous growth models, namely, constant returns to human capital accumulation. The central aspect of my model is the role of the wage profile in providing incentive for investment in human capital. With a special cost structure given by constant return to human capital accumulation, the wage profile brings about a stable international division of labor and persistent income disparity among countries.

This model suggests an explanation for the observed varieties of cross-country economic performances during 1960-1985. These varieties, it suggests, are systematic response of an integrated world economy. Differential increase in the labor force of developed and underdeveloped countries changes the wage profile, thereby altering incentive structure for investment in human capital. However, these changes are not uniform, resulting in varieties of responses from members of the world economy. Metaphorically, demographic differential creates pressure within the system which allows poor performance in low-income countries, economic miracle of some middle-income countries, and income convergence among developed countries to occur simultaneously with persistent income disparity.

Appendix A: Human capital accumulation

This appendix solves in detail the intertemporal human capital accumulation for each individual worker. Since there are three similar maximization problems — each with a different law of motion — in the main expositicn, I will solve the most general problem of the three:

$$\max_{s} \int_0^{\infty} e^{-\rho t} \ln[(1-s)w(h_\varsigma, t)]\, dt$$

subject to

$$\dot{h}_\varsigma = \delta s h_\varsigma^{1-\sigma} H_\varsigma^{\gamma} W^{\eta}$$

where $\gamma + \eta = \sigma$. H_ς denotes the country average human capital level and W denotes the world aggregate human capital level. Each worker allocates her time between working and educating herself for higher paid jobs in the future. Each takes the wage profile $w(h_\varsigma, t)$ and the future paths of $H(t)$ and $W(t)$, as given. The current value Hamiltonian is

$$\ln[(1-s)w(h_\varsigma, t)] + \lambda \delta s h_\varsigma^{1-\sigma} H_\varsigma^{\gamma} W^{\eta}$$

The first order conditions are

$$\begin{aligned} 1 &= (1-s)\lambda\delta h_\varsigma^{1-\sigma} H_\varsigma^{\gamma} W^{\eta} \\ \frac{\dot{\lambda}}{\lambda} &= \rho - \frac{w_1(h_\varsigma, t)}{\lambda w(h_\varsigma, t)} - (1-\sigma)\delta s h_\varsigma^{-\sigma} H_\varsigma^{\gamma} W^{\eta} \end{aligned}$$

On the perfect foresight balanced growth path, the fraction of time spent on eduction is a constant; thus, from the first order condition, I have $-\dot{\lambda}/\lambda = (1-\sigma)\dot{h}_\varsigma/h_\varsigma + \gamma \dot{H}_\varsigma/H_\varsigma + \eta \dot{W}/W$. By substituting $\gamma + \eta = \sigma$ and the perfect foresight growth rate $\dot{H}_\varsigma/H_\varsigma = \dot{W}/W = g$ and eliminating λ, I derive:

$$s = 1 - \frac{(\rho + \sigma g)}{\delta \beta_\varsigma} \left(\frac{h_\varsigma}{H_\varsigma}\right)^{\gamma} \left(\frac{h_\varsigma}{W}\right)^{\eta} \tag{1.15}$$

where $\beta_\varsigma = w_1(h_\varsigma, t)h_\varsigma/w(h_\varsigma, t)$ is the elasticity of the wage profile with respect to human capital at h_ς. When $\sigma = 0$ and $\gamma = \eta = 0$, there is no external benefits of human capital and I simply have $s = 1 - \rho/\delta\beta_\varsigma$.

Appendix B: Total gains in the relative positions

This appendix provides a detail derivation of Figure 1-7. With differential increase in the labor force of developed and under-developed countries, relative skills with respect to the frontier of a given country changes as a result. Its total gains after the system reaches a new steady state can be calculated if one is willing to assume that countries' ranking in term of skills does not change during the adjustment process. This total gains is defined to be:

$$G = \frac{(h^*_\varsigma / h^*_{\max})}{(h_\varsigma / h_{\max})}$$

where the asterisk indicates the corresponding values in the new steady state after the demographic change occurred. This index measures how far a country converges to the frontier as a response to the demographic change. Given that the distribution of relative skills is independent of the position of the frontier, I may assume that $h^*_{\max} = h_{\max}$. Then $\ln G = \ln h^*_\varsigma - \ln h_\varsigma = i^*_\varsigma - i_\varsigma$ serves as an alternative index for the total gains.

Below, I illustrate the procedure for calculating the total gains. For simplicity sake, choose α and β such that $(\alpha - \beta)/(1-\alpha) = 1$. Equation (1.9) becomes $L_i = e^{i-j} L_j$. By imposing the labor market clearing condition $\int_{-\infty}^{N} L_i \, di = (1-s)L \equiv \mathcal{L}$, I can calculate the density of workers who are producing the best intermediate good to be $L_N = \mathcal{L}$ and consequently $L_i = e^{i-N}\mathcal{L}$. From here, I can calculate the measure (mass) of workers with less human capital than a worker who produces intermediate good i. The cumulative measure of workers with human capital less than e^i is given by $M_i = \int_{-\infty}^{i} L_i \, di$. Substituting L_i with the last expression, I derive

$$M_i = e^{i-N}\mathcal{L} \tag{1.16}$$

Thus, I will know the ranking of each worker or how many workers have less skill than a given worker. Rewriting this equation, I have:

$$i - N = \ln M_i - \ln \mathcal{L} \tag{1.17}$$

This allows me to determine the relative position to the frontier of a particular worker once I know (1) the mass of total working labor force $\mathcal{L}$ and (2) the mass of workers with less skill than this particular worker M_i. Since this condition holds for all demographic condition, I also have a similar relationship $i^* - N = \ln M_i^* - \ln \mathcal{L}^*$ where $\mathcal{L}^*$ and M_i^* denote the *new* mass of total working labor force and the new mass of workers with skill less than e^i, respectively. The total gains is given by:

$$i^* - i = \ln(M_i^*/M_i) - \ln(\mathcal{L}^*/\mathcal{L}) \tag{1.18}$$

Calculating this total gains index is simple. What is left to figure out, is which product i^* a particular worker will be producing after the demographic change occurs and the adjustment process ended. Given that we know $\mathcal{L}$, M_i, and $\mathcal{L}^*$, we only need to calculate M_i^*. Put it differently, this last equation allows us to solve this problem by merely calculating the mass of workers with less human capital than the particular worker in the new steady state. To do this, one must employed the assumption, aforementioned.

For example, suppose that labor force of countries in the bottom half, who currently have relative skills less than 1/2 and produce $i < N + \ln 1/2$, increase proportionally at a rate z while the other half remains constant. Given that countries' ranking remains constant (due to my assumption) after the system reaches a new steady state the worker who is used to produced intermediate good i and have M_i workers with less skill than her, will now have

$$M^* = \begin{cases} zM_i & \text{if} \quad i < N + \ln 1/2 \\ (z-1)M_{(N+\ln 1/2)} + M_i & \text{if} \quad i > N + \ln 1/2 \end{cases}$$

workers with less skill than her. $\mathcal{L}^* = (z-1)M_{(N+\ln 1/2)} + \mathcal{L}$. Employing equation (1.18), I have

$$i^* - i = \begin{cases} \ln z - d & \text{if} \quad i < N + \ln 1/2 \\ \ln \frac{(z-1)M_{(N+\ln 1/2)} + M_i}{M_i} - d & \text{if} \quad i > N + \ln 1/2 \end{cases}$$

where $d = \ln(\mathcal{L}^*/\mathcal{L})$. When the initial relative skills is less than 1/2, $i < N + \ln 1/2$, the total gains is a positive constant; when the initial relative skills is greater than 1/2, $i > N + \ln 1/2$, the total gains is a declining function of initial relative skills.

An extension for a more general demographic changes is straight-forward *under* the sustaining assumption that countries's ranking does not change during the adjustment process. An interesting case is when the labor force of middle income country increases more than that of low- and high-income countries as empirically observed. I will have

$$i^* - i = \begin{cases} \ln z - d & \text{if} \quad i < N + \ln 1/4 \\ \ln \frac{(z-v)M_{(N+\ln 1/4)} + vM_i}{M_i} - d & \text{if} \quad N + \ln 1/4 < i < N + \ln 1/2 \\ \ln \frac{(z-v)M_{(N+\ln 1/4)} + (v-1)M_{(N+\ln 1/2)} + M_i}{M_i} - d & \text{if} \quad N + \ln 1/2 < i \end{cases}$$

where z and v are the rate of increase of low- and middle-income countries' labor forces. $d = \ln(\mathcal{L}^*/\mathcal{L})$ and $\mathcal{L}^* = (z-v)M_{(N+\ln 1/4)} + (v-1)M_{(N+\ln 1/2)} + \mathcal{L}$. Notice that when the initial relative skills is less than 1/4, $i < N + \ln 1/4$, the total gains is a constant function; when $N + \ln 1/4 < i < N + \ln 1/2$, it is an increasing function if $z < v$; and when the initial relative skills is greater than 1/2, $i > N + \ln 1/2$, it is a decreasing function. This function is drawn in Figure 1-7.

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Chapter 2

Scientific progress and Growth

This paper models scientific progress in a way which distinguishes basic research from applied research. There are two contributions to endogenous growth literature. First, instead of assuming the law of motion $\dot{a}/a = f(R)$ as usual, I step back and derive two familiar-looking laws of motion, one for scientific knowledge and the other for productivity growth. Second, I introduce federally-funded research in order to reexamine empirical findings concerning the relationship between research and productivity growth, particularly, the findings that basic research has higher return than applied research and that privately-funded research has higher return than federally-funded research.

2.1 Introduction

Many economists have long discussed the relationship between scientific progress and productivity growth. Kuznets wrote that "...a distinctive feature of modern industrial societies is their success in applying knowledge derived from scientific research to the economic sphere." Scientific knowledge is indispensable in innovative processes. It guides us in our search to improve quality of existing products, and in our search to create new inventions.

The role of fundamental knowledge in the creation of new inventions may be illustrated by history of the atomic bomb. "Chadwick in England discovered the neutron in 1932; and in 1934 Fermi in Italy bombarded all known elements with a beam of neutrons, and found that uranium yielded an isotope which he thought was a new element of atomic number 93 or

94....Hahn continued his experiment with Strassmann, and before Christmas, 1939, concluded, though he could hardly believe it himself, that the 'new' element was in fact an old one, barium, derived by splitting the uranium atom in half. Hahn immediately communicated his results to Meitner, then in Sweden, who discussed the results with his nephew and using Einstein's formula concluded that Hahn's result would implied that enormous energy must have been released in the fission process."[1] The basic lesson is clear despite too many names being mentioned: without Einstein's formula, nobody would have noticed that enormous energy has been released. It would have taken much longer time and much more resources to come up with the bomb. Einstein's formula — an advancement in scientific knowledge — facilitates the search for an atomic bomb.[2,3]

This paper is my first attempt at modelling the relationship between scientific progress and innovation and productivity growth. My model is built upon (1) my representation of a consumer product and (2) distinctions between basic research and applied research. Examining airplanes or computers, one finds that these products are complex, and become more complex and more productive over time. Airplanes carry more passengers to further destination; computers execute more instructions per second. This casual observation suggests that one can represent a consumer product by its degree of complexity or the number of parts of which it consists. Products with greater complexity are more productive.

In my model, productivity growth is a consequence of new parts being continually invented and added to existing products. Here, the relationship between scientific progress and innovation and productivity growth is crucial. Researchers employ existing scientific knowledge in their research to innovate new additional parts. With new parts added, productivity level increases. Sustaining constant productivity growth requires *in addition* that evermore complicated new parts are being innovated. It is here that scientific progress plays an essential role: scientific advances induce firms to optimally innovate more and more complicated new parts. Scientific progress thus sustains productivity growth.

[1] Schmookler [1966, p. 193]

[2] The distinction between purposeful discovery and accidental discovery is not crucial. Scientific knowledge increases the chance that researchers recognize their solution, when they see one.

[3] Combining ideas is not the only component of innovation, as Weitzman [1995] suggests; Scientific knowledge is the other essential component. Suppose that scientists are searching for Z. Combining A with B, they get C which is very far away from what they are looking for. It is scientific knowledge which links C and Z together that facilitates processes of innovation.

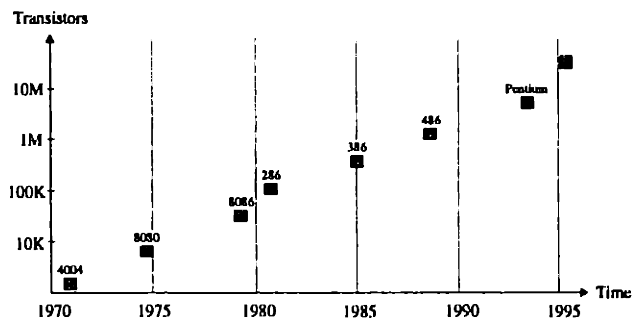

Figure 2-1: Intel microprocessors evolution

Microprocessors provide an example of what I have in mind. Figure 2-1 illustrates the number of transistors per cpu of some prominent Intel microprocessors. The number of transistors per cpu increases exponentially. So does the computing power of these microprocessors. Notice that not only new parts are being continually invented and added to existing microprocessors, but also these new parts are more and more complicated. Hidden in the background, of course, is scientific advancements in the fundamental knowledge of etching.

The second building block of my model is the distinction between basic research (knowledge) and applied research (knowledge). Basic research searches for *general* knowledge. Its outcome improves our understanding of nature. Although it does not provide useful production techniques by itself, it facilitates innovative processes by increasing the likelihood that new techniques will be discovered. Applied research, in contrast, aims at raising productivity level of existing products. It searches for such *specific* techniques that the resulting knowledge is useless to other researchers. Mathematically, it does not affect the probability that other researchers will discover another technique.[4]

This attempt to distinguish various types of R&D expenditures is motivated in part by existing empirical findings on differential return between basic research and applied research. Many studies — Mansfield [1980], Griliches [1986], Lichtenberg and Siegel [1991], for instance — have found that investment in basic research has a strong effect on productivity growth while investment in other types of R&D has either a small impact or none at all. An estimated return

[4] In search-theoretic framework, as in Evenson and Kiev [1976], applied research is represented by a draw from a given distribution. While scientific progress improves that distribution, a draw from a distribution does not change the distribution in any way, i.e. with replacement.

for basic research is 133.8 percent; and that for applied research, statistically insignificant 10.8 percent. This suggests that distinctions between various types of research are essential for understanding productivity growth.[5] Specifically, it suggests that basic research is perhaps the engine of growth while applied research, though importance, is not.

The following three sections describe in turn scientific progress, innovation, and productivity growth. Specifically, I derive two laws of motion — one for scientific knowledge and the other for productivity growth:

$$\dot{\gamma} = \gamma f(R_b) \qquad \text{and} \qquad \dot{a} = \gamma f(R_d)$$

where γ is an index of scientific knowledge; a is productivity level; R_b and R_d are the number of researchers conducting basic research and applied research, respectively. (The subscript d stands for development.) Scientific knowledge increases efficiency of applied research, but it does not affect productivity level directly. Fifth section incorporates these two laws of motion into a model of productivity growth. It describes the model and its properties. The sixth section use this model to discuss existing empirical findings concerning the relationship between research and productivity growth; particularly, the findings that basic research has higher return than applied research and that privately-funded research has higher return than federally-funded research. The seventh section concludes.

I employ the following approach. Certain equations are derived using probability theory. I, then, solve out my model based on certainty version of those equations. This particular approach is chosen since, I believe, I will not learn much more by being rigorous.

2.2 Scientific progress

"I want to know God's thoughts. The rest are details"[6]

I have the following picture in mind. God of nature and scientists are playing a game. God picks a number Ω from the unit interval; scientists have to find out what the number is exactly.

[5]Mansfield [1984, p.128] comments that "economists have devoted too little attention to [the composition of R&D expenditures]. For both analytical and policy purposes, the total R&D figures are hard to interpret because they include such a heterogenous mixture of activities. Basic research and applied research are mixed up with development. Long-term projects are mixed up with short term projects....To answer many important analytical and policy questions, it is essential to disaggregate R&D ."

[6]Einstein

Given that He could have chosen an irrational number, scientists may not find it. But they will know within which subinterval it lies and the subinterval becomes smaller over time.

I interpret this game as follow. Each number represents a model of nature based on certain collection of postulates. Finding Ω means that scientists uncover the true collection. Any number in its neighborhood represents a model of nature whose postulates are similar to those of the true model. By restricting the subinterval into a smaller subinterval, scientists reject a group of models whose assumptions are not possible *ex post.* As a consequence, *working models* become better approximation of the true model.

I give a mathematical representation of this process here. The unit interval can be represented by a diadic representation. Every number on the unit interval can be mapped to a sequence of 0 and 1 according to a function $f : [0,1] \to 2^N$ such that:

$$f(x) = \{y_i\} \quad \text{if and only if} \quad x = \sum_{i=1}^{\infty} \frac{y_i}{2^i}$$

where $\{y_i\} = \{y_1, y_2, y_3, ...\}$ and y_i is equal to zero or one. Each coordinate of the sequence represents a postulate concerning nature. In effect, nature is characterized by countably infinite number of possible postulates and every working model is represented by a certain sequence. If $y_i = 0$, the working model assumes that the i th postulate is false. Otherwise, it assumes that it is true. For example,

$$\begin{aligned} \text{Model } 0.625 &= \{10100...\} \\ \text{Model } 0.125 &= \{00100...\} \end{aligned}$$

Let the first coordinate represents the law of gravity. Model 0.625 assumes that nature obeys the law of gravity while model 0.125 assumes otherwise. And both models assume that nature obeys the third postulate.

Implicitly embedded in this construction is the assumption that postulates in earlier coordinates are more important. Getting the first coordinate wrong means that the working model is off from the true model by 0.5 whereas getting the 10th coordinate wrong means that it is off by roughly 0.001. Since a number closer to Ω represents a better working model then getting the 10th coordinate wrong does not hurt as much as getting the first coordinate wrong.

I assume a priori that the probability which any particular postulate is true is equal to 1/2. These probabilities are independent of each other. Consequently, the prior distribution of Ω is uniform over the unit interval. As scientists restrict the interval into a smaller subinterval, the *posteriori* distribution of Ω will be uniform over that subinterval as well.

The progress of sciences

My model identifies scientific progress with the length of the subinterval. Scientists test whether a given postulate is correct. Revelation of the first coordinate restricts the unit interval into either $[0, \frac{1}{2}]$ or $[\frac{1}{2}, 1]$. A further revelation of the second coordinate restricts the subinterval into $[0, \frac{1}{4}]$, $[\frac{1}{4}, \frac{1}{2}]$, $[\frac{1}{2}, \frac{3}{4}]$, or $[\frac{3}{4}, 1]$.[7]

Suppose that each test lasts exactly $1/h$ year.[8] h measures the efficiency of basic research. After t years of testing, the length of the subinterval will be $1/2^{ht}$. With P_b projects going on simultaneously, I assume that each postulate will be completely tested within $1/hP_b$ years. Consequently, the length of the subinterval will be $l(t) = 1/2^{hP_b t}$.

2.3 Innovation

Over time technologies become more and more productive. And they become more and more complex. Were a poor peasant from a remote province to visit Bangkok, he would be overwhelmed by technologies. Upon his return, he would tell his sons and daughters that things are much better and that things are much more complex. "Those metal carts are outrageous. I don't believe my eyes. They run much much faster than our buffalos. I can't even count how many pieces there are in those carts. You got to see them, kids!"

Technologies are techniques of making and doing things. Most of them are complex objects (processes) consisting of several parts (steps). Were we to design an airplane for ourselves, we

[7] In effect, scientists are testing which subinterval of the bisection, the true model occupies. This corresponds well with the way one would play a number guessing game. For instance, suppose that we have to guess for an integer between 0 and 100 and our opponent always signal whether our guess is too high or too low. The most efficient procedure is to guess the middle integer of the interval every time. On average, this minimizes the number of our guesses.

[8] I assume that each project is completed according to a poisson process with parameter $\lambda = h$. I choose $\lambda = h$ so that I can study how changes in the efficiency of basic research affect the system.

would begin by decomposing the plane into various parts such as wings, engines, flight control, body, etc. Then, we design each part and put them together. With little reflection, we realize that designing an airplane which flies is a difficult task. This, of course, this is not the lesson I draw. Most technologies can be viewed as a collection of parts; each part must be solved before the whole thing works. I represent technologies by the number of parts of which they consist.

As I mention earlier, technologies which are more productive tend to be more complex, although the converse is not necessarily true. One can still consider a collection of technologies with the highest productivity level for a given degree of complexity. Comparing these "best" technologies to one another, one finds that more complex technologies are more productive. Mathematically, I assume that technologies with N parts have productivity level equal to $a = N$; The productivity level rises linearly with the complexity level.[9]

Shoulders of giants

Newton once said "if I have seen further, it is by standing upon the shoulders of giants."[10] Most innovations are improvements of existing products. Looking at airplanes and computers one wonders how human being could imagine such a complex thing in the first place. Reading technical articles stirs similar impression. Sensible explanation can be advanced: human do not design airplane in one step. We did it step by step. We improve upon the existing product.

Within my framework, "standing on the shoulders of giants" can be formalized naturally. Suppose that the current state-of-the-art technology has productivity level a, i.e. it consists of a parts. Researchers can increase productivity level by adding newly innovated parts to that technology. The new productivity level is simply the sum of a and the number of new parts.

2.3.1 Designing new additional parts

Let us first consider a task of designing a single part. Applied researchers employ scientific knowledge in their innovative process, as in the atomic bomb example. With the correct model of nature, the probability that applied researchers will be able to construct a simple-and-working part, is one. Not knowing the true model, they make some assumptions and design

[9]This can be extended to $a = N^k$ where k is a positive real number.

[10]This statement is popularized by Robert Merton who traces its origin to earlier period.

their products accordingly. In effect, applied researchers choose a working model ω from the current subinterval. The probability that they will succeed with their wrong model is assumed to be:

$$p(\omega) = 1 - (\Omega - \omega)^2$$

As the distance between the true model and the working model gets smaller, the probability of success becomes higher.[11] Since Ω is distributed uniformly over a subinterval $[a, b]$, the expected probability of success with a working model ω is:

$$E[p(\omega)] = 1 - var(\Omega) - (E[\Omega] - \omega)^2$$

By optimally choosing the working model $\omega = E[\Omega] = (a + b)/2$, the maximum expected probability of success is given by $\wp = 1 - var(\Omega)$. Substituting $var(\Omega) = (a - b)^2/12$ into this expression, I have:

$$\wp = 1 - \frac{l(t)^2}{12}$$

where $l(t)$ is the length of the current subinterval.[12] $\wp$ serves as an index of scientific knowledge. Of course, for simple objects such as pencils, basic research has sharply diminishing returns. Yet, this does not imply that it is uneconomical to conduct basic research after some point in time. I will argue, on page 65, that the true return of basic research is in facilitating the discoveries of highly-complicated products.

Designing new additional parts Given the existing scientific knowledge, the probability of coming up with a working blueprint for a single part is $\wp$. If the new additional parts consists of n parts, then the probability of coming up with a working blueprint is $\wp^n$.

[11] This formula is adapted from Wilson(1976). It can be extended to $p(\omega) = 1 - |\Omega - \omega|^c$ where $c > 0$. It is implicitly defined over the set of those possible-to-be-constructed objects only. In other words, even if with the correct model of nature Ω, applied researchers cannot construct a perpetual motion machine.

[12] Alternatively, I can assume that applied researchers choose their working models randomly from the subinterval. Then, $\wp = 1 - l(t)^2/8$; The maximum expected probability of success is lower.

2.4 Productivity growth

Productivity growth is a consequence of new parts being continually invented and added to existing products. With extra parts, productivity level increases. To sustain constant productivity growth, evermore complicated new parts must be innovated. As it turns out, scientific advances induce firms to innovate more and more complicated new additional parts.

2.4.1 The rate of productivity improvement.

After applied researchers choose their optimal degree of complexity, n, for their projects, they begin their designing process. Each project is completed as if it were a Poisson process with parameter φ.[13] When a project is completed, researchers come up with a new blueprint. Whether this blueprint works or does not work, is uncertain and independently distributed. Specifically, the outcome for each single part has a Bernoulli distribution with:

$$z = \begin{cases} 1 & \text{with probability} \quad \wp \\ 0 & \text{with probability} \quad 1 - \wp \end{cases}$$

where z denotes the number of new working parts. If every single part of this new blueprint works, then the project is successful; these new parts will be added to the existing product; productivity level increases. Otherwise, nothing will change. Applied researchers start another project anew. With P_d applied projects going on simultaneously, new blueprints are completed according to a Poisson process with parameter φP_d; The speed of discovery increases.

My assumption that $a = N$ and "standing on the shoulders of giants" together imply that the expected productivity increases is simply the expected number of new additional parts. With a chosen (or given) optimal number of new additional parts, n, I have:

$$E[\dot{a}] = \varphi P_d n \wp^n \tag{2.1}$$

The expected rate of productivity increases is simply the speed of discovery multiplied by the expected number of new additional parts once one blueprint is discovered.

[13] φ is a constant and is independent of the complexity of the project. φ/h measures speed of discovery in applied research relative to basic research.

The optimal degree of complexity

Before applied researchers begin their designing process, they choose the degree of complexity for their projects so as to maximize the expected rate of productivity increases. The first order condition is given by:

$$n^* = \frac{1}{-ln\wp} \tag{2.2}$$

The optimal number of new additional parts depends on the stock of scientific knowledge. As sciences progress, the optimal complexity increases. The intuition is simple. With better understanding of nature, the chance of discovering complicated technologies improves. This induces applied researchers to be brave and opt for more and more complicated techniques.

Certainty version of the law of motion.

By substituting the optimal number of new additional parts into equation (2.1), I will get $E(\dot{a}) = -\varphi P_d/(\ln\wp)e$. Since solving the full model under stochastic framework is difficult without yielding much more interesting implications, I use the corresponding certainty version in my model. It is given by:

$$\dot{a} = \gamma\varphi P_d \tag{2.3}$$

where $\varphi \equiv \varphi_{old}/e$. And $\gamma \equiv -1/\ln\wp$ is the optimal number of new additional parts. It serves as *the index* of scientific knowledge for the rest of this paper. Without scientific progress, this law of motion cannot sustain productivity growth. The growth rate slows down and converges to zero despite new parts being continually added to existing products.[14]

Rewrite equation (2.3) such that $\dot{a}/a = (\gamma\varphi/a)P_d$. Comparing this with the well-known law of motion $\dot{a}/a = P_d$, reveals their only difference: the coefficient $(\gamma\varphi/a)$. This term measures the *effective efficiency* of applied research. Scientific advancements raise effective efficiency of applied research. On a balanced growth path, scientific knowledge and applied knowledge increases in such a way that this term remains constant; there is no longer any difference between these two equations.

[14] Thus, "standing on the shoulders of giants" cannot sustain productivity growth by itself. Note that this concept is not a crucial component of my model. It helps simplifying the law of motion for productivity improvement. It also fosters productivity changes since applied researchers do not have to start from scratch.

2.4.2 The rate of scientific progress

Here is the place to derive the law of motion for scientific knowledge. Recall that the maximum expected probability of success is $\wp = 1 - 2^{-2hP_b t}/12$ and that $\gamma = -1/\ln \wp$ is an index of scientific knowledge. As $\wp$ approaches one, $\gamma \approx 12 * 2^{2hP_b t}$. The law of motion for scientific knowledge is given by:

$$\dot{\gamma} = \gamma h P_b \tag{2.4}$$

where $h \equiv (2\ln 2)\, h_{old}$. With a constant number of basic research projects, scientific knowledge grows at a constant rate.

2.4.3 Summary of research sectors

I recapitulate my results here. Scientific progress increases efficiency of applied research but does not affect productivity level directly. It is applied research which increases productivity level. Mathematically, I derive the following laws of motion for scientific knowledge and applied knowledge:

$$\dot{\gamma} = \gamma h P_b \qquad \text{and} \qquad \dot{a} = \gamma \varphi P_d.$$

where P_b and P_d are the number of basic projects and applied projects, respectively. On the balanced growth path, the effective efficiency of applied research remains constant and $\dot{a}/a = \dot{\gamma}/\gamma$. Scientific progress thus sustains productivity growth.[15]

Three remarks are in order. First, for any given level of complexity, basic research has decreasing returns. For instance, consider a task of designing a simple object such as a pencil. After the probability of success reaches some level like 99%, marginal return to basic research is small. It seems to be uneconomical to do more basic research after some point. This is incorrect, however. Scientific progress increases the probability of discovering highly-complicated techniques. It is these yet-to-be-built highly-complicated techniques (objects) that is its real return. Scientific progress in my model reduces the expected cost of designing new parts systematically. The cost, in term of applied projects, of finding new parts with complexity n is $1/\wp^n$. For a

[15] In a way, my description of research sectors is similar to Aghion and Howitt [1992]. In their model, however, there is no scientific progress; A fixed number of new parts are being continually invented and added to existing products. They sustain growth by assuming directly that quality is an exponential function of parts: $a = e^N$.

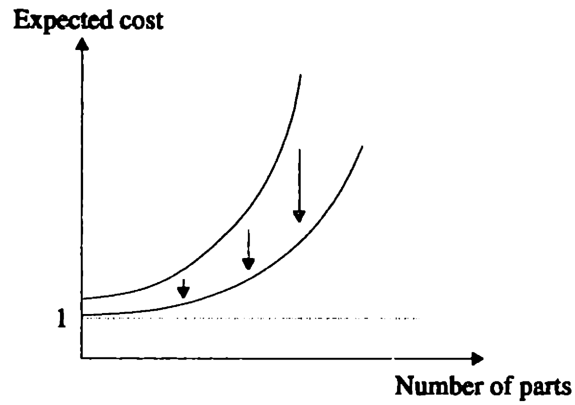

Figure 2-2: Expected cost, in term of applied projects, of finding new parts with complexity n.

given level of scientific knowledge, this is an exponential function. Scientific progress shifts the cost function down systematically, as in Figure 2-2. The cost of discovering complicated parts is reduced much more than the cost of discovering simple parts.[16,17]

Second, recall that $\wp = 1 - var(\Omega)$. Sustaining constant scientific progress requires that $var(\Omega)$ converges to zero exponentially. This requires two crucial conditions — that the length of the subinterval is halved every time a postulate is tested, and a constant rate of project completion. The first condition is satisfied by construction. I assume the second condition.[18]

Third, I explicitly assume that applied research does not contribute to scientific progress. A natural way to introduce this linkage is to let applied researchers infer the true model from their past experiences. This type of learning makes sure that $var(\Omega)$ converges to zero at the rate one over the number of applied projects to date. This is too slow and cannot sustain scientific progress by itself. This is why I assume this linkage away.

[16]Since $c(\cdot) = 1/\wp^n$, then $\dot{c}/c = -n(\dot{\wp}/\wp)$. The cost of discovering complicated parts is reduced more than the cost of discovering simple parts. This confirms my earlier statement that it is these yet-to-be-built highly complicated techniques that is the real return of basic research.

[17]Although my model bases on the distinction between numbers like 0.999 and 0.999001, this is just a mathematical way of summarizing how scientific progress affects the cost of discovering new techniques.

[18]By making an alternative assumption that it takes longer to test postulates in later coordinates, the law of motion for scientific knowledge will change. In particularly, I will be able to derive $\dot{\gamma} = \gamma^c h P_b$ where $c < 1$. Scientific progress no longer sustains productivity growth. One can then sustain constant productivity growth either by assuming constant population growth or by assuming that the number of basic and applied projects are functions of the final output. However, either approach will not fit well with the empirical finding that basic research has higher return than applied research. The other alternative — knowledge spillover — is discussed in section 2.6. Please also look at footnote 35.

The following section incorporates these two laws of motion into a model of productivity growth. My main purpose is to show that endogenous growth will be supported and to understand the basic implications of my research structure. In section 2.6, I use an extension of this model to reexamine empirical findings concerning the relationship between research and productivity growth.

2.5 A model of research and productivity growth

This model is a multisector decentralized system with perfect foresight. There are two types of goods: a final good and intermediate goods. There is no capital accumulation. The market structure for intermediate goods is monopolistic competition. There are two assumptions to be discussed. (1) The monopolist has a patent to its product forever. (2) Both basic research and applied research are not public good.

The first assumption is where I depart from the literature, the so-called quality ladder model. In such model, an intermediate good monopolist is replaced by an outside firm who has greater incentive to innovate. The monopolist does not engage in research activities; one has to calculate the expected length and value of the patent which can be very complicated. My first assumption simplifies much of the algebra and makes the model more transparent. The only effect that will be missing is the so-called "business stealing effect." Beside having a virtue of simplicity, this assumption corresponds well with the reality. There is an abundance of real world examples where existing firms conducting research: Boeing, IBM, Intel, for example. These firms stay in business for a long time. A simple explanation can be given — they have research cost advantage in innovative activities; they may have better information about their technology. (In quality ladder model, every firm has the same cost of research.) Since there are examples where existing firms are replaced by newcomers as well, my model is best viewed as a simpler model of endogenous growth for the other side of the spectrum.[19]

Research, I assume, are not public good. There are two justifications. First, casual observation reveals that firms usually engage in R&D in order to identify, assimilate, and exploit knowledge from the environment and incorporate those knowledge into their products. Das-

[19] An independent work by Per Krusell [1995] uses similar approach. Barro and Xala-i-Martin [1995] constructs a quality ladder model which monopolists have an advantage in innovative activities over outside firms.

gupta [1988, p.6] argue that "...even though the transmission cost of knowledge may be low, the cost of absorbing the information, of interpreting it and using it may well be very high. This is often the case with research output at the frontier of science and technology. Firms wishing to make use of the latest findings that are publicly available need to have scientists who can make it possible for them to do so....This provides one reason why private firms conduct research." Second, this assumption is crucial theoretically for the decentralization of this economy. If research are public goods, there will be a free-rider problem; with a large numbers of intermediate goods, there will be no or little research done in equilibrium. It is this assumption that provides an incentive for each firm to engage in basic and applied research.

Knowledge in a field of study often helps other fields in advancing their own knowledge. Chemistry is useful for biologists; mathematics is useful to many disciplines. Nonetheless, knowledge can be specific as well. If one would like to construct a rocket, one would not begin by buying economic textbooks; one must learn specific aspects of rockets building. Quick reflection reveals that knowledge must be somewhere between being specific and being applicable to other sectors.[20] This section considers the simplest case when both basic knowledge and applied knowledge are sector-specific. The following section discusses knowledge spillover.

2.5.1 Model description

The production structure is simple. There are two types of goods: intermediate goods and a final good. There are continuum of intermediate goods indexed by numbers between zero and one. The market structure for intermediate goods is monopolistic competition. The production function for the final good is:

$$y = \left(\int_0^1 (a_i x_i)^\alpha di \right)^{\frac{1}{\alpha}} \tag{2.5}$$

where a_i and x_i are the quality level and quantity of intermediate good i, respectively. Intermediate goods are assumed to be good substitute of each other, i.e. $0 < \alpha < 1$, so that the derived demand for each intermediate good is elastic.

[20] This must be one of the reason why there are many sub-disciplines with its own specific jargons and knowledge. The law of motion should have the following functional form: $\dot{\gamma}_i = \gamma_i^\sigma \Gamma^{1-\sigma} h R_{bi}^\theta$ where progress in a field relies on both its own specific knowledge and the external knowledge Γ which spills over from others. I discuss this particular law of motion in the next section.

Each intermediate good firm consists of three "departments" — intermediate good production, basic research, and applied research. Labor is the only input and can be used in any of the three activities. $\mathcal{L}$ denotes the total number of labor in the economy. L_i, R_{bi}, and R_{di} denote the number of production workers, basic researchers, and applied researchers in firm i, respectively. The production function for each intermediate good is:

$$X_i = L_i$$

The marginal cost is constant. This, together with the production function for the final good yield the constant markup result. The production functions for basic and applied research projects are:

$$P_{bi} = R_{bi}^{\theta} \qquad \text{and} \qquad P_{di} = R_{di}^{\theta}$$

where $0 < \theta < 1$. The marginal product of researchers approaches infinity as the number of researchers approaches zero. This assures that there will always be a positive number of both basic researchers and applied researchers.

Perfect foresight variables

There are three aggregate variables: the price of one unit of the final good P, the wage rate w, and the interest rate r. Consumers and producers solve their maximization problems taking the future path of these variables as given. Specifically, the wage rate and the interest rate are expected to be constant and P is expected to decline exponentially at the rate $\kappa > 0$. By choosing $w = 1$, the wage rate will serve as the numeraire.

Consumer's maximization problem

Let c denote each consumer's consumption level of the final good. (It also denotes consumption per capita since consumers are identical.) Each consumer has a logarithmic utility function. Each one takes w, r, P as given and solves the following intertemporal maximization problem.

$$\max_{c} \int_0^{\infty} e^{-\rho t} \ln c \, dt$$

subject to $\dot{b} = rb + w + d - Pc$. ρ is the discount rate, b is the amount of bond he purchases, and d is profit per capita. Consumers own equal shares of intermediate good firms so that profit will be distributed equally. Solving the intertemporal maximization problem, I get $\dot{c}/c = r + \kappa - \rho$.

Let κ^* denote the perfect foresight rate of decline in the price of one unit of the final good. On a balanced growth path, it turns out that $\dot{c}/c = \kappa^*$.[21] Consequently, the interest rate is equal to the discount rate:

$$r = \rho \tag{2.6}$$

Final good sector

The production function for the final good has constant returns to scale. By assuming that the final good market is perfectly competitive, there is no need to introduce firms. Each worker can set up a factory in her own backyard. Let Y denote the aggregate output of the final good and X_i, the total output of intermediate good i. The market demand curve for intermediate good i is given by:

$$X_i = \left(\frac{p_i}{P a_i^{\alpha}}\right)^{\frac{1}{\alpha-1}} Y \tag{2.7}$$

The price for one unit of the final good is $P = \left(\int_0^1 a_i^{\alpha/(1-\alpha)} p_i^{\alpha/(\alpha-1)} di\right)^{(\alpha-1)/\alpha}$ where p_i denotes price of the intermediate good i.

Intermediate good production

The labor market is perfectly competitive. Each intermediate good monopolist chooses her output such that her instantaneous profit is maximized. She takes her market demand curve and the wage rate as given. Her maximization problem is given by:

$$\max_{X_i} \; p_i X_i - w L_i$$

subject to $X_i = L_i$ and $w = 1$. Solving this maximization problem, I find that the instantaneous profit will be $\pi_i = (1-\alpha) X_i/\alpha$. Price is a constant markup over the wage rate, $p_i = 1/\alpha$,

[21] On the balanced growth path, $c = Y/\mathcal{L}$, $Y = AL$, and $A = 1/\alpha P$.

which simplifies the price of one unit of the final good to be:

$$P = \frac{1}{\alpha A} \tag{2.8}$$

where $A = (\int_0^1 a_i^{\alpha/(1-\alpha)} di)^{(1-\alpha)/\alpha}$ measures the aggregate productivity level. Using this and equations (2.5), (2.7), (2.8), I get:

$$Y = AL \tag{2.9}$$

where L denotes the total number of production workers in all sectors. Once this variable is determined, the aggregate output of the final good depends solely on the aggregate productivity level. Aggregate final output growth is sustained by aggregate productivity growth. The total output of intermediate good i is:

$$X_i = (a_i/A)^{\frac{\alpha}{1-\alpha}} L \tag{2.10}$$

The instantaneous output of each intermediate good depends solely on its productivity level relative to the aggregate productivity level. By raising the productivity of its product relative to the aggregate productivity level, each firm can increase its market demand, which is the reason why firms engage in research.

Firms' intertemporal research decision

Each firm hires labor to conduct basic research and applied research. It takes w, r, A, L as given and solve the following intertemporal maximization problem:

$$\max_{R_{bi}, R_{di}} \int_0^{\infty} e^{-rt} (\pi_i - R_{bi} - R_{di})\, dt$$

subject to

$$\begin{aligned} \pi_i &= \frac{(1-\alpha)}{\alpha} \left(\frac{a_i}{A}\right)^{\frac{\alpha}{1-\alpha}} L \\ \dot{\gamma}_i &= \gamma_i h R_{bi}^{\theta} \\ \dot{a}_i &= \gamma_i \varphi R_{di}^{\theta} \end{aligned}$$

where R_{bi} and R_{di} denote the number of basic researchers and applied researchers in firm i, respectively. When $0 < \alpha < 1/2$, the objective function is concave with respect to a_i as required for optimal control problems. I solve out this maximization problem in detail in Appendix A. Important results are summarized below.

2.5.2 The symmetric balanced growth path

Imposing symmetry between intermediate good firms, the subscript i can be ignored. L, R_b and R_d denote the aggregate number of production workers, basic researchers, and applied researchers, respectively. (Given that intermediate goods are indexed by the unit interval, these denote those corresponding variables at the aggregate level as well. So, I use them interchangeably.) Define $\Delta = (\gamma\varphi/ah)^{1/\theta}$. This variable is determined endogenously. It is very important. It directly relates to the effective efficiency of applied research; it measures the degree of exhaustion of potential pool of knowledge;[22] and it correlates positively with productivity growth rates since $\dot{a}/a = \rho\Delta$.[23]

On a symmetric balanced growth path, the number of production workers, basic researchers, and applied researchers are constant. Scientific progress sustains productivity growth. Consequently, γ/a and Δ are constant. The ratio of basic researchers to applied researchers is:

$$\frac{R_b}{R_d} = \Delta \tag{2.11}$$

Faster productivity growth is associated with larger fraction of researchers being devoted to basic research. The ratio of researchers to production workers is given by:

$$\frac{R}{L} = \theta\Delta \tag{2.12}$$

where $R = R_b + R_d$ denotes the aggregate number of researchers. Faster productivity growth is associated with larger fraction of labor force being devoted to research. Finally, the number

[22] Recall that γ measures the frontier of scientific knowledge and a measures the position of applied knowledge; Their ratio measures the relative position between applied knowledge and its potential. Low γ/a indicates that most of the potential has been exhausted.

[23] I show this in Appendix A.

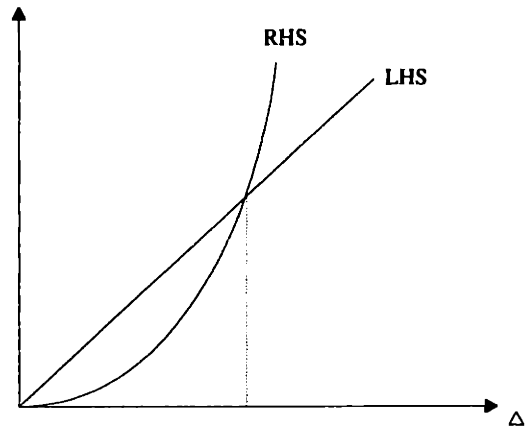

Figure 2-3: Determining the level of Δ.

of applied researchers per firm is given by:

$$R_d = \left(\frac{\rho}{h}\right)^{\frac{1}{\theta}} \Delta^{\frac{1-\theta}{\theta}} \tag{2.13}$$

Using equations (2.6), (2.11)–(2.13), and the labor market clearing condition, $\mathcal{L} = L + R_b + R_d$, I derive the following algebraic equation:

$$\mathcal{L}\left(\frac{h}{\rho}\right)^{\frac{1}{\theta}} \Delta = (\Delta + \frac{1}{\theta})(\Delta + 1)\Delta^{\frac{1-\theta}{\theta}} \tag{2.14}$$

Once Δ is determined, I can calculate exactly the number of labor employed in each activity, providing me the solution of this intertemporal maximization problem.[24,25]

2.5.3 Comparative static exercises.

Although it is not possible to solve this algebraic equation for an exact solution, I can still perform comparative static exercises using the diagram in Figure 2-3. Since there is always a positive number of basic researchers and applied researchers and thus $\Delta = R_b/R_d$ is always a

[24] Intermediate good monopolists earn positive profit on the symmetric balanced growth path when $(1-\alpha)/\alpha > \theta\Delta$. This condition can be satisfied by assuming that intermediate goods are not good substitutes, i.e. small α.

[25] Note that by solving a corresponding social planner's problem, one derive exactly the same algebraic equation. Thus, decentralized system yield identical outcomes as in the social planner problem. This result is not surprising given that there is no knowledge spillovers in this simple model.

positive number, I draw only the first quadrant. The left hand side of the equation is a linear line passing through the origin with slope $\mathcal{L}\left(h/\rho\right)^{1/\theta}$. The right hand side is a polynomial with degree greater than 2, passes through the origin as well. For simplicity sake, I assume that $0 < \theta < 1/2$ so that the slope of this polynomial approaches zero around the origin and rises monotonically. These two curves intersect exactly once in the first quadrant. The solution for Δ exists and is unique.

Results of comparative static exercises are reported in Table 2.1. The first three columns are the number of production workers, basic researchers, and applied researchers employed at each firm. The sign in each cell indicates how each variable responses to an increase in exogenous parameters. All signs are in the expected direction.

Productivity growth rate

On the symmetric balanced growth path, $\dot{a}/a = \rho\Delta = hR_b^{\theta}$. One can study how each exogenous parameter affects the productivity growth rate by analyzing its effects on either Δ or the number of basic researchers per firm.

The productivity growth rate is higher when the efficiency of basic research increase; when the discount rate decreases; and when population increase. With higher efficiency of basic research, slope of the linear curve in Figure 2-3 increases. Consequently, Δ and, thus, productivity growth increases. Intuitively, an increase in the efficiency of basic research lowers its research cost, inducing firms to hire more basic researchers.

When the discount rate or the interest rate decreases, slope of the linear curve decreases as well. Consequently, Δ decreases. According to equations (2.11) and (2.12), less labor will be devoted to research activities and smaller fraction of this smaller research sector engages in basic research. Thus, there will be fewer basic researchers per firm; productivity growth rate decreases.

With larger population, the available resources per firm increases (given that there is a fixed number of intermediate goods). The scale of production in each sector increases, reducing the cost of research per unit of output. This induces firms to hire more researchers, particularly basic researchers. In effect, productivity growth increases as the scale of the economy rises. So, my model predicts the so-called scale effects, a prediction which I would like to deemphasize. I

Table 2.1: Comparative static results

Parameters	Production workers	Basic researchers	Applied researchers	Growth rate
h	$-$	$+$	$+$	$+$
ρ	$+$	$-$	$-$	$-$
$\mathcal{L}$	$+$	$+$	$+$	$+$

shall demonstrate in Chapter 3 that this particular prediction comes from my assumption that there is a **fixed** number of intermediate goods.

Allocation of labor

The first three columns of Table 2.1 report how each parameter affects the number of production workers, basic researchers, and applied researchers employed at each firm. There are two comments. First, an increase in the efficiency of basic research, h, induces each firm to hires more basic researchers and more applied researchers. This indicates that basic research and applied research are *complementary* activities. Second, when the interest rate increases, firm cares less about the future. It hires more production workers and layoff researchers. This reduction comes at the expense of basic research more than applied research: the percentage of R&D expenditures devoted to basic research is lower.

Basic research intensity and growth

The ratio of basic researchers to applied researchers, on a balanced growth path, is simply Δ. Thus, my simple model predicts positive correlation between relative intensity of basic research with respect to applied research, and productivity growth. Manfields [1984, p.128] finds that "when a firm's total R&D expenditures were hold constant, its innovative output seems to be directly related to the percentage of its R&D expenditures devoted to basic research. The data on which this result is based pertain to the chemical and petroleum industries, areas where we have accumulated a considerable amount of data concerning the R&D and innovative activities of particular firms."

Research intensity and growth

My model also predicts positive relationship between research intensity, as measured by the fraction of labor force devoted to research, and productivity growth. Given the positive correlation between Δ and productivity growth, it suffices to demonstrate this prediction by showing another positive correlation between Δ and research intensity. Using equation (2.12) and the labor market clearing condition, I get:

$$\frac{R}{\mathcal{L}} = \frac{\theta\Delta}{\theta\Delta + 1}$$

Research intensity correlates positively with Δ and thus productivity growth, as claimed. This prediction is empirically rejected by the joint behavior of productivity growth slowdown and research intensity increase in OECD countries during 1960-1990. (Chapter 3 provides possible theoretical explanations which reconcile this model with the observed joint behavior.)

Productivity growth and productivity level

Basic research, in this model, determines productivity growth whereas applied research determines productivity level. On a balanced growth path, $\dot{a}/a = hR_b^{\theta}$; The number of basic researchers determines productivity growth rate. In addition, given the number of basic researchers, the entire path of γ will be known. Given that the productivity growth rate can be expressed as $g = (\gamma\varphi/a)R_d^{\theta}$, then:

$$a = \frac{\gamma\varphi}{g}R_d^{\theta}$$

The number of applied researchers will determine productivity level, as claimed

Empirically, many studies have found that investment in basic research has a strong effect on productivity growth rates while investment in other types of R&D (controlling for investment in basic research) has either a small impact or none at all. An estimated return on productivity growth rate for basic research is 133.8 percents; and that for applied research, statistically insignificant 10.8 percents. This finding suggests, as my model does, that basic research is perhaps the engine of growth while applied research, though importance, is not.

Efficiency of applied research

An interesting thing about productivity growth rate is that φ, the efficiency of applied research, does not show up as one of its determinants. This finding is a direct consequence of basic research being the engine of productivity growth.[26] Undoubtedly, this result depends on the way which one sustains productivity growth in one's model. For instance, if one uses a variation of AK model as the engine of growth, then efficiency of applied research will show up as one of the determinants. However, that particular model now predicts that after controlling for investment in basic research, applied research has strong and significant impact on productivity growth rates — a prediction which is not supported by existing empirical findings.[27]

Rewrite $\Delta = (\gamma\varphi/ah)^{\frac{1}{\theta}}$ such that $\gamma\varphi/a = h\Delta^{\theta}$. Since Δ is a function of exogenous parameters, $\gamma\varphi/a$ is a certain constant on the balanced growth path. Increasing efficiency of applied research reduces the ratio γ/a proportionally without affecting anything else in this model.

2.6 Implications

Here, I compare existing empirical findings concerning the relationship between research and productivity growth with my model's properties so as to develop additional implications to Table 2.1. It turns out to be fruitful — these findings suggest interesting extensions of my model and, at the same time, my model provides a framework for reexamining them. These findings are best summarized by Griliches [1986, p.151]:

> "There are three major findings: R&D contributed positively to productivity growth and seems to have earned a relatively high rate of return; basic research appears to be more important as a productivity determinant than other types of R&D; and privately financed R&D expenditures are more effective, at the firm level, than federally financed one."

[26] Similar results are found in Aghion and Howitt [1995], Mulligan and Xala-i-Martin [1993]. In the neoclassical growth model, there is a logic akin to this. By assuming that capital investment is not an engine of growth, the model predicts no correlation between saving rates and long-term growth rates. In addition, any efficiency coefficient of capital accumulation processdoes not show up as a determinant of long-term growth rates.

[27] Consider the following variation of an AK model: $Y = \sqrt{A}$, $\dot{A} = \gamma\varphi s_d Y$, and $\dot{\gamma} = hs_b Y$. Let s_b and s_d be constant. The productivity growth rate will be $g = \sqrt{2\varphi h s_d s_b}$.

Table 2.2: Regressions of DTFP on Various Measures of R & D

Type of R&D Intensity	R&D Expenditure/Sales		
	(a)	(b)	(c)
Total R&D	.132* (6.40)		
Basic R&D		1.338* (13.06)	
Applied R&D		.108 (1.14)	
Development		.014 (0.24)	
Company funded R&D			.353* (13.09)
Federally funded R&D			.026 (0.81)
R^2	.026	.051	.075

Source: Lichtenberg and Siegel [1991].

These findings are confirmed in a more recent study using a larger and more detail data set[28] by Lichtenberg and Siegel [1991].

I summarized their findings in Table 2.2. (It is a truncated table.) Research contributes positively to productivity growth with an estimated rate of return of 13.2 percent. Disaggregating research activities into basic research, applied research, and development reveals that investment in basic research has a strong impact on productivity growth while investment in other types of R&D apparently has either a small impact or none at all.[29] The estimated return for basic research is 133.8 percent; and that for applied research, statistically insignificant 10.8 percent. Disaggregation by sources of funds reveal that privately-funded research has a strong positive impact on productivity growth; federally-funded research does not.

At first glance, there seem to be a productivity "premium" on basic research, and a higher returns to privately-funded research, suggesting that (1) firms can gain by reallocating their resources toward basic research; and (2) there should be more privately-funded research and less federally-funded projects. Griliches [1986] and Lichtenberg and Siegel [1991] provide *econometric* reasons why such implications are not warranted. I provide complementary *theoretical* reasons against such implications.

[28]There are over 2,000 firms in this linked R&D-LRD data set. The sample period is between 1972 and 1986. Companies in this sample account for 84 percent of R&D performed by industrial firms in the US in 1976.

[29]Similar results can be founded in Mansfields [1980], Link [1981], Griliches et. al. [1984], and Griliches [1986].

Concerning the productivity premium on basic research, Griliches [1986] writes "Such findings are always subjected to a variety of econometric and substantive reservations. In this context the two major related issues are simultaneity and the question of how major divergences in private rates of return persist for such long periods....It is even more difficult to respond to the theoretical a priori argument that such results cannot be true since they implies widely differing rates of return to different activities under the control of the same firm." He, then, provides explanations which include that basic research is riskier than applied research and that higher returns to basic research represent a higher social but not higher private return.

My model suggests a complementary and simpler explanation. There is nothing puzzling about this finding. Basic research is the engine of growth while applied research is not. Basic research determines productivity growth rates; applied research, productivity level. Since the dependent variable in the regressions is productivity growth rates, applied research expenditures should not explain the variable once one controls for basic research expenditures. Nonetheless, applied research is important. Advances in scientific knowledge will not increase productivity level without applied research being conducted. Particularly, firms may already be at their optimal composition of research expenditures despite the observed "widely differing rates of return to their various activities". So, firms may not gain from reallocating their resources toward basic research, as being suggested.[30]

2.6.1 Federally-funded research

There are three related empirical findings concerning the effects of federal-funded research on the composition of private research expenditures and productivity growth rates which I would like to discuss. (I spend the rest of this paper discussing them.) They are:

i) Federally-funded research has negative effect on private expenditures on basic research.

ii) There is a complementarity between federally-funded basic research and industrial applied research. Additional National Institute of Health (NIH) spending on basic research increases the pharmaceutical industry's applied research expenditures later.

[30] Jovanovic and Nyarko [1995] use a different model to make an argument in the same direction.

iii) While privately-funded research is a significant determinant of productivity growth, federally-funded research does not appear to be a significant determinant.

In a cross-sectional study of 275 U.S. manufacturing firms, Link [1981] finds that federally-funded research reduces private expenditures on basic research. "A one dollar increase in federally-funded research increases company-funded research by 9.4 cents....This increases is primary directed away from basic research and toward development." Specifically, it reduces basic research by 8.1 cents. Disaggregating research activities into basic, applied, and development, reveals that this negative relationship between federally-funded research and private expenditure on basic research is primarily a result of federal basic research allocation. Link did not report the coefficient, however.

The second finding is based on Ward and Dranove [1995]. In their study of the vertical chain of research and development in the pharmaceutical industry, they find that federally-funded research does, several years later, generate privately-funded applied research. "Specifically, a 1 percent increase in research funding by the NIH leads to an estimated 2.5 percent increase in [applied] R&D expenditures by member of the Pharmaceutical Manufacturers of America, after a lag of seven years."

The last finding is reported in the last column of Table 2.2. Research contributes positively to productivity growth. Disaggregation by sources of funds reveals that while privately-funded research is a significant determinant of productivity growth, federally-funded research is not.[31] The estimated return for privately-funded research is 35.3 percent, that for privately-funded research is 2.6 percent. This is rather puzzling given that federally-funded research account for roughly 60 percent of the U.S. national expenditures for basic research and an estimated return to basic research, as we have seen, is 132 percent. Since this finding is not easy to explain, I will discuss it at the end of this section. I will argue in particular that the observed higher returns to privately-funded research does not necessarily imply that firms only benefit from research that is privately financed nor should there be less fed ral research projects.

[31] Lichtenberg and Seigel [1991, p. 213]. Similar results can be founded in Link [1981], Griliches [1986].

A simple extension

In order to discuss these findings, it is necessary to introduce federally-funded research into my model. I treat the level of federally-funded research as an exogenous variable and then analyze its effects on the composition of private research expenditures and productivity growth rates.

One can introduce federally-funded research into my decentralized system by making the following assumption. The government supports an across-the-board funding of research. It hires G_b and G_d researchers to conduct basic research and applied research in each firm at the going wage. (I ignore the government's intertemporal budget constraint.) Later, I will let $G_d = 0$. This assumption modifies the laws of motion to be:

$$\dot{\gamma}_i = \gamma_i h \tilde{R}_{bi}^{\theta} \qquad \text{and} \qquad \dot{a}_i = \gamma_i \varphi \tilde{R}_{di}^{\theta}$$

where $\tilde{R}_{bi} \equiv G_b + R_{bi}$ and $\tilde{R}_{di} \equiv G_d + R_{di}$ denote the *total* number of basic researchers and applied researchers — federally-funded researchers included — in sector i, respectively. This functional form assumes one and only one difference between federally-funded researchers and privately-funded researchers: their sources of funding. Federally-funded research is as effective as privately-funded research.[32] Solving the decentralized system taking the level of federally-funded research as exogenous parameters, I find the following results.

Federally-funded research crowds out the *corresponding* private research expenditures. At low level of funding, G_b and G_d being less than the outcome of my simple decentralized system, there is a complete crowding-out. Firms respond by cutting back their corresponding researchers one for one; for that, an increase in the number of federally-funded researchers reduces both the number of available labor and the marginal return of privately-employed researchers. The total number of basic researchers and applied researchers at each sector remain unchanged. Federally-funded research thus has no effect on the economy.

At *sufficiently* high level of funding, there is crowding-out but not complete. For that, firms cannot hire negative number of researchers. Consider, for instance, that the government funds only basic research. Firms respond by cutting back their own basic researchers one for one

[32] Recall that I would like to use my model to explain the third empirical finding: privately-funded research is more effective than federally-funded research. This assumption assures that I am not assuming the desired result.

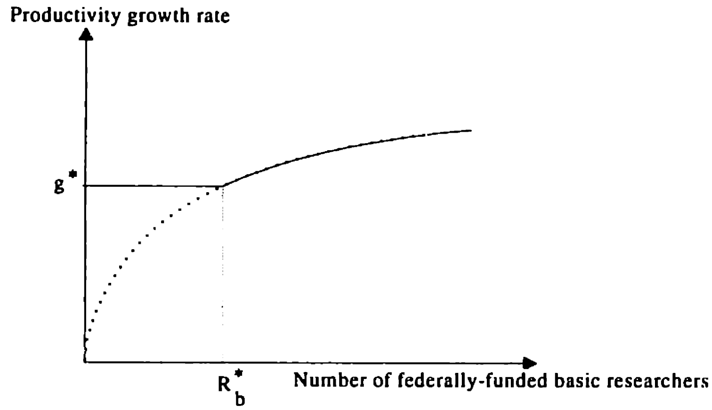

Figure 2-4: Productivity growth rate as a function of G_b.

until the number of federally-funded basic researchers reaches R_b^*, the outcome of my simple decentralized system. From there on, firms conduct no private basic research. Productivity growth is determined by the number of federally-funded basic researchers, as illustrated in Figure 2-4. Federally-funded research does increase productivity growth rates and can affect the allocation of labor.[33]

2.6.2 Knowledge spillover

My extended model predicts that federally-funded basic research has significant negative effect on private basic research as Link [1981] finds. Still, it cannot account for the complementarity between federally-funded basic research and privately-funded applied research as observed in the pharmaceutical industry. At low level, federal funding on basic research has no effect on private applied research expenditures. At sufficiently high level, it is likely to have a negative effect.[34] This suggests that I must modify my extended model. There are two obvious questions to ask: what is the difference between federally-funded research and privately-funded research? and what is missing in my model?

[33] The implications that federally-funded research completely crowd-out privately-funded research at low level of funding and that it has effects on the economy only when no firms conduct private research are too strong. They will be soften below.

[34] With higher growth rate of scientific knowledge, the marginal return of applied research increases. On the other hand, federally-funded research reduces the number of available labor and increases cost of research. This second effect dominates at a sufficiently high level of funding. (Appendix C.)

An answer to the first question is that private companies keep their findings secret whereas federally-funded research disclose their results. Yet, this distinction will not make any difference in my model since knowledge is specific to each sector. This brings us to the second question. What is missing in my model seems to be *knowledge spillover*. Knowledge in a field of study often helps other fields to advance their own knowledge. Chemistry is useful for biologists; mathematics is useful to many disciplines. In the following extension, I introduce into my model both knowledge spillover and the distinction between federally- and privately-funded research. The resulted model helps explaining the observed complementarity between federally-funded basic research and privately-funded applied research.

There are two possible places to introduce knowledge spillover. First, an across-the-board scientific progress facilitates *scientific* advancements in any particular field. Second, an across-the-board scientific progress facilitates advancements of *applied* knowledge in any particular field. I shall demonstrate that only the first type of knowledge spillover is important to our understanding of the long-run productivity growth.

The first type of spillover effect

Here, an across-the-board scientific progress facilitates *scientific* advancements in each particular field. I captures this effect by assuming the following functional form:[35,36]

$$\dot{\gamma}_i = \gamma_i^{\sigma}\Gamma^{1-\sigma}h\tilde{R}_{bi}^{\theta}$$

where $0 < \sigma < 1$ measures the importance of own specific knowledge relative to external knowledge, Γ. Scientific advancements in a particular field rely on both its specific knowledge

[35] From my information gathering structure, I could have derived this law of motion by assuming as in footnote 18 that it takes longer to test postulates in later coordinates, and thus $\dot{\gamma} = \gamma^{\sigma}hP_d$ where $\sigma < 1$. I sustain growth by assuming that improvements in general scientific knowledge increases efficiency of basic researchers in such a way that $P_d = \Gamma^c R_d^{\theta}$ where $c = 1 - \sigma$. Of course, there is no reason for the last equality to hold. It is assumed so that endogenous growth is possible.

[36] This change of assumption has the following effect. Due to spillover effect, the productivity growth rate of the decentralized system will be too low comparing to the optimum. Comparative static results will be the same, however. On the symmetric balanced growth path, this new assumption assumes that $\dot{\gamma}_i/\gamma_i = s^{1-\sigma}h\hat{R}_{1i}^{\theta}$ which is identical to my previous law of motion except the coefficient $s^{1-\sigma}$.

and the general scientific knowledge. I assume that:

$$\Gamma = s\left(\frac{G_b}{\tilde{R}_b}\right)T(\gamma_0, .., \gamma_i, .., \gamma_1)$$

where $T(\cdot)$ is homogenous of degree one and $T(1, .., 1, .., 1) = 1$. The endogenous coefficient $s(\cdot)$ measures the degree of knowledge disclosure. When firms have the same level of scientific knowledge, the general scientific knowledge will be given by $s(\cdot)\gamma$.

The degree of knowledge disclosure is assumed to be a monotonically increasing function of the *share* of federal funding within basic research category. With higher share, more information are disclosed and general scientific knowledge available to each firm will be greater. Given that empirically federally-funded basic research has a negative effect on privately-funded basic research, the elasticity of privately-funded basic researchers with respect to federally-funded basic researchers is assumed to be less than one. Consequently, the share of federal funding within basic research category is an increasing function of G_b. The degree of knowledge disclosure increases when federal funding increases.

On a symmetric balanced growth path, the law of motion can be rewritten as:

$$\dot{\gamma}_i = \gamma_i s^{1-\sigma} h \tilde{R}_{bi}^{\theta}$$

An increase in federal funding raises its share within basic research category, thereby increasing the degree of knowledge disclosure. The effective efficiency of basic research, $s^{1-\sigma}h$, increases. According to Table 2.1, there will be an increase in the number of applied researchers, an increase in the *total* number of basic researchers, and higher productivity growth. Notice that federal funding now has effects on the economy at all levels of funding.

Federally-funded basic research thus stimulates private expenditures on applied research, thereby explaining the observed complementarity in the pharmaceutical industry. The total number of basic researchers increases as the number of federally funded basic researchers increases. However, the effect on *private* basic research expenditures is likely to be negative, as observed by Link [1981]. (For that, privately-funded basic researchers is equal to the total minuses the number of federally-funded basic researchers.)

The second type of spillover effect.

Alternatively, one can assume that an across-the-board scientific progress facilitates advancements of *applied* knowledge in each particular field. This is captured by the following law of motion:

$$\dot{a}_i = \gamma_i^{\sigma}\Gamma^{1-\sigma}\varphi\tilde{R}_{di}^{\theta}$$

where Γ measures the general scientific knowledge and its definition is given above. Applied researchers rely on both their technology-specific knowledge and general knowledge in their innovative processes.[37] On a symmetric balanced growth path, I rewrite this law of motion as:

$$\dot{a}_i = \gamma_i s^{1-\sigma}\varphi\tilde{R}_{di}^{\theta}$$

As before, an increase in federal funding in basic research category raises the degree of knowledge disclosure. Consequently, effective efficiency of applied researchers, $s^{1-\sigma}\varphi$, increases. Yet, there will be no effect on equilibrium allocation of labor and productivity growth rates, as I have shown on page 77.[38] Specifically, this extension yields an outcome identical to my simple decentralized system when the funding is not too high. Of course, with greater effective efficiency of applied researchers, there will be greater exhaustion of the potential pool of knowledge, i.e. low γ/a.

Discussion

The introduction into my model of both the distinction between federally- and privately-funded research and knowledge spillover turns out to be fruitful. The modified model fits empirical findings better. It predicts complementarity between federal funding within basic research category and private expenditures on applied research. This relationship exists until the level of funding is too high. It also predicts that an increase federal funding on basic researcher has negative effect on private expenditures on basic research.

The real return for this exercise is elsewhere, however. The observed complementarity between federal-funded basic research and industrial applied research suggests the existence

[37] Please look at Chapter 3, footnote 5.

[38] Of course, this result is not robust, depending on what is the engine of growth. However, emprirical evidence strongly suggests that basic research is an engine of growth, as assumed in my model.

of a particular type of knowledge spillover among sectors. This knowledge spillover must be between the production of scientific knowledge themselves. It in effect suggests that federally-funded research generates more knowledge spillover than privately-funded research, thereby providing a theoretical reason why there should **not** be less federally-funded projects.[39]

Of course, this should not suggest that the government should fund all basic research in the economy. Ward and Dranove [1995] find that the National Institute of Health allocated research funding differently than the industries. "NIH increase funding in those disease categories experiencing increases in mortality, whereas pharmaceutical firms devote additional funding to diseases with increasing prevalence." Thus, there may be other inefficiency associated with the allocation of federal funding among various research projects that is not included in my model.

Here is an appropriate place to reexamine the third finding that privately-funded research is a significant determinant of productivity growth, while federally-funded research is not. Lichtenberg and Siegel [1991] write that "At first glance, this result implies that firms only benefit from research that is privately financed. Perhaps private companies, rather than the federal government, are best able to judge the potential returns to industrial R&D. It must be emphasized, however, that there are two alternative explanations for this finding, each associated with the difficulties of measuring the benefits resulting from government-funded R&D projects....At the present time, it is difficult to know whether the standard econometric framework underestimates the impact of federal R&D on economic growth."

A complementary theoretical suggestion can be provided. This particular regression estimates the return to federally-funded research holding private research expenditures constant and it finds higher return to privately-funded research. So far, I illustrate that (1) it is useful to distinguish various type of R&D with basic research perhaps as an engine of growth, (2) federally-funded research crowd out the corresponding privately-funded research, and (3) there is a complementarity between federal funding on basic research and privately-funded applied research. These together suggest that it might be fruitful to reestimate the return to federally-funded *basic* research holding privately-funded *basic* research constant (instead of estimating the return to *total* federal funding holding *total* private research constant as usual). Until such

[39] Although this implication is already a consequence of Ward and Dranove's finding, our justification for federally-funded research differs. They argue that federally-funded research stimulates private expenditure on applied research. My justification adds that federally-funded research creates knowledge spillover as well.

estimation is made, it is difficult to evaluate the implication of the finding that privately-funded research has higher returns.

2.7 Conclusions

I constructed a basic framework for analyzing the relationship between scientific progress and productivity growth. There are two steps. The first step opens the black box of innovation. Instead of assuming the law of motion $\dot{a}/a = f(R)$ as usual, I step back and describe how basic knowledge and applied knowledge is accumulated over time. By introducing facilitating assumptions, I derive two familiar-looking laws of motion, one for scientific knowledge and the other for productivity growth. The second step incorporates these laws of motion into an endogenous growth model. I show that the perfect foresight balanced growth path exists with the expected comparative static results.

Using this model and its extensions, I reexamine empirical findings concerning the relationship between research and productivity growth. There are three insights. First, the observed productivity "premium" on basic research, does not imply that firms should reallocate more resources toward basic research. The premium perhaps reflects different natures and aims among various types of research expenditures. Second, the observed complementarity between federally- funded basic research and industrial applied research provides a circumstantial evidence that federally-funded research generates more knowledge spillover. This knowledge spillover must be between the production of scientific knowledge themselves. Finally, to evaluate the important finding whether there are higher returns –- in term of productivity growth –- to privately-funded research, it is essential to disaggregate R&D. Since basic research determines long-run productivity growth and federal funding changes the composition of private research expenditures, it may be fruitful to estimate the return to federally-funded basic research, holding private basic research constant (instead of estimating the return to total federal funding holding total private research constant as usual).

Appendix A: Decentralized system

In this appendix, I solve the intertemporal research decision for firm i, in detail. I relax the assumption that each consumer has a logarithmic utility function to be any isoelastic utility function, c^{β}/β where $\beta < 1$. The maximization problem is given by:

$$\max_{R_{bi},R_{di}} \int_0^{\infty} e^{-rt} \left(\pi_i - R_{bi} - R_{di}\right) dt$$

subject to

$$\begin{aligned} \pi_i &= \frac{(1-\alpha)}{\alpha} \left(\frac{a_i}{A}\right)^{\frac{\alpha}{1-\alpha}} L \\ \dot{\gamma}_i &= \gamma_i h R_{bi}^{\theta} \\ \dot{a}_i &= \gamma_i \varphi R_{di}^{\theta} \end{aligned}$$

where R_{bi} and R_{di} denote the number of basic researchers and applied researchers in firm i, respectively. The current value Hamiltonian is:

$$H(t) = \pi_i - R_b - R_d + \lambda_1 \gamma h R_b^{\theta} + \lambda_2 \gamma \varphi R_d^{\theta}$$

The first order conditions are:

$$\begin{aligned} 1 &= \lambda_1 \gamma h \theta R_b^{\theta-1} \\ 1 &= \lambda_2 \gamma \varphi \theta R_d^{\theta-1} \\ \frac{\dot{\lambda}_1}{\lambda_1} &= r - \frac{h(R_b + R_d)}{R_b^{1-\theta}} \\ \frac{\dot{\lambda}_2}{\lambda_2} &= r - \frac{\gamma\varphi}{a} \left(\frac{a_i}{A}\right)^{\frac{\alpha}{1-\alpha}} \frac{\theta L}{R_d^{1-\theta}} \end{aligned}$$

The subscript i is omitted whenever it does not create confusion. L denotes the total number of production workers in the economy. On a balanced growth path, the rate of growth of various variables are linked by the following expression:

$$\frac{\dot{\gamma}}{\gamma} = \frac{\dot{a}}{a} = \frac{\dot{Y}}{Y} = r\Delta \tag{2.15}$$

The number of basic researchers, applied researchers, and production workers are constant. Scientific progress sustains productivity growth. Since $\dot{\gamma}/\gamma = \dot{a}/a$, I have:

$$\frac{R_b}{R_d} = \Delta \tag{2.16}$$

where $\Delta = (\gamma\varphi/ah)^{1/\theta}$. From the first order conditions, $\lambda_1/\lambda_2 = \varphi/h\,(R_d/R_b)^{\theta-1}$; on the balanced growth path, $\dot{\lambda}_1/\lambda_1 = \dot{\lambda}_2/\lambda_2$. Eliminating λ_1 and λ_2 and substituting $a_i = A$, I get:

$$\frac{R}{L} = \theta\Delta \tag{2.17}$$

where $R = R_b + R_d$. Since $1 = \lambda_1\gamma h\theta R_b^{\theta-1}$, on the balanced growth path $\dot{\lambda}_1/\lambda_1 = -\dot{\gamma}/\gamma$. Eliminating λ_1 and γ, I get:

$$R_d = \left(\frac{r}{h}\right)^{\frac{1}{\theta}} \Delta^{\frac{1-\theta}{\theta}} \tag{2.18}$$

Equations (2.16) and (2.18) imply that $R_b = (r\Delta/h)^{\frac{1}{\theta}}$. Substituting this expression into the law of motion for scientific knowledge, I have $\dot{\gamma}/\gamma = r\Delta$, as claimed above. Productivity growth positively correlates with Δ. Finally, using equations (2.16), (2.17), (2.18), and the labor market clearing condition, $\mathcal{L} = L + R_b + R_d$, I derive an algebraic equation:

$$\mathcal{L}\left(\frac{h}{r}\right)^{\frac{1}{\theta}} \Delta = (\Delta + \frac{1}{\theta})(\Delta + 1)\Delta^{\frac{1-\theta}{\theta}} \tag{2.19}$$

Consumer's maximization problem

Each consumer has an isoelastic utility function c^β/β where $\beta < 1$. The intertemporal consumption problem is given by:

$$\max \int_0^\infty e^{-\rho t}\frac{c^\beta}{\beta}\,dt$$

subject to $\dot{b} = rb + w + d - Pc$. Solving this intertemporal maximization problem, I get:

$$\frac{\dot{c}}{c} = \frac{1}{1-\beta}\left(r + \kappa - \rho\right) \tag{2.20}$$

The perfect foresight equilibrium

When $\beta \neq 0$, the interest rate is no longer equal to the discount rate. I have to express the interest rate as a function of ρ. On a perfect foresight balanced growth path, $\dot{c}/c = \kappa^*$ and $\dot{c}/c = r\Delta$. Therefore, $\kappa^* = r\Delta$. Substituting this into equation (2.20), I get:

$$r = \frac{\rho}{1+\beta\Delta}$$

Interest rate is negatively related to Δ. Replacing r in equation (2.19) with this expression, I have:

$$\mathcal{L}\left(\frac{h}{\rho}\right)^{\frac{1}{\theta}} (1+\beta\Delta)^{\frac{1}{\theta}} \Delta = (\Delta + \frac{1}{\theta})(\Delta+1)\Delta^{\frac{1-\theta}{\theta}}$$

If one solves the corresponding social planner's problem, one will derive an identical algebraic equation: outcomes of my decentralized system are the same as outcomes of the social planner's problem despite its monopolisticly competitive market structure. This is not surprising since there is no externality in this simple model.

Transitional dynamic

This subsection provides a *partial* insight to the transitional dynamic of Δ. I assume symmetric research decision among firms. $x = R_b/R_d$ denotes the ratio of basic researchers to applied researchers. I will construct a phase diagram for x and Δ. Since $\Delta = (\gamma\varphi/ah)^{1/\theta}$, then $\theta\dot{\Delta}/\Delta = \dot{\gamma}/\gamma - \dot{a}/a = hR_b^\theta - \gamma\varphi R_d^\theta/a$. Therefore, $\dot{\Delta}/\Delta = 0$ when:

$$x = \Delta \tag{2.21}$$

Next, using the first order conditions, $\lambda_1/\lambda_2 = (\varphi/h)\, x^{1-\theta}$. Thus, $(1-\theta)\dot{x}/x = \dot{\lambda}_1/\lambda_1 - \dot{\lambda}_2/\lambda_2$. Consequently, $\dot{x}/x = 0$ when:

$$\Delta^\theta x^{1-\theta} = \frac{1}{\theta}\frac{R}{L} \tag{2.22}$$

The phase diagram is illustrated in Figure 2-5. I have a saddle-path dynamic with the ratio of basic researchers to applied researchers being a jump variable. The stable arm has negative slope. With greater stock of scientific knowledge relative to productivity level, firms allocates

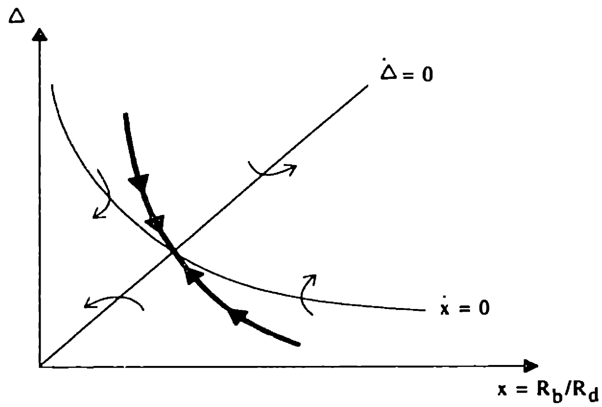

Figure 2-5: Transitional dynamic of Δ.

relatively more researchers toward applied research in order to exhaust the potential pool of knowledge. Productivity level grows faster than scientific knowledge; Δ declines and approaches its equilibrium level.

This diagram is *incorrect*, however. It assumes that the ratio of researchers to production workers, $y = R/L$, is a constant. This ratio changes along the transition path. The law of motion for this ratio in term of Δ and x is given by

$$\frac{1-\theta}{1+y}\frac{\dot{y}}{y} = r - \Delta^{\theta}\frac{\theta h}{y}{\frac{yx\mathcal{L}}{(1+y)(1+x)}}^{\theta}$$

Of course, the correct diagram should have three dimensions. This, I do not know how to draw. This is why this section provides only a partial insight.

Appendix B: Social planner's problem

The social planner allocates available resources — labor — between three competing activities. There are two decisions for him: the division of labor between production and research, and the division of labor between basic research and applied research. By allocating more labor toward production, the social planner increases current consumption while giving up future productivity increases. The second margin is between basic research and applied research. When the social planner allocates researchers among the two activities, he is trading off productivity growth with productivity level. Basic research determines productivity growth whereas applied research, productivity level. The maximization problem is:

$$\max_{R_b, R_d} \int_0^\infty e^{-\rho t} \frac{c^\beta}{\beta} \, dt$$

subject to

$$\begin{aligned} c &= \frac{aL}{\mathcal{L}} \\ \dot{\gamma} &= \gamma h R_b^\theta \\ \dot{a} &= \gamma \varphi R_d^\theta \\ \mathcal{L} &= L + R_b + R_d \end{aligned}$$

where ρ is the discount rate and $\beta < 1$. The current value Hamiltonian is

$$H(t) = \frac{c^\beta}{\beta} + \lambda_1 \gamma h R_b^\theta + \lambda_2 \gamma \varphi R_d^\theta$$

The first order conditions are

$$\begin{aligned} c^\beta &= \lambda_1 \gamma h \theta R_b^{\theta-1} L \\ c^\beta &= \lambda_2 \gamma \varphi \theta R_d^{\theta-1} L \\ \frac{\dot{\lambda}_1}{\lambda_1} &= \rho - \frac{h(R_b + R_d)}{R_b^{1-\theta}} \\ \frac{\dot{\lambda}_2}{\lambda_2} &= \rho - \frac{\gamma\varphi}{a} \frac{\theta L}{R_d^{1-\theta}} \end{aligned}$$

On a symmetric balanced growth path, the rate of growth of various variables are linked by the following expression:

$$\frac{\dot{\gamma}}{\gamma} = \frac{\dot{a}}{a} = \frac{\dot{c}}{c} \tag{2.23}$$

The number of production workers, basic researchers, and applied researchers are constant. Scientific progress sustains productivity growth. Since $\dot{\gamma}/\gamma = \dot{a}/a$, I have:

$$\frac{R_b}{R_d} = \Delta \tag{2.24}$$

where $\Delta = (\gamma\varphi/ah)^{1/\theta}$. From the first order conditions, $\lambda_1/\lambda_2 = \varphi/h\,(R_d/R_b)^{\theta-1}$; on a balanced growth path, $\dot{\lambda}_1/\lambda_1 = \dot{\lambda}_2/\lambda_2$. Eliminating λ_1 and λ_2, I get:

$$\frac{R}{L} = \theta\Delta \tag{2.25}$$

where $R = R_b + R_d$. Since $c^\beta = \lambda_1\gamma h\theta R_b^{\theta-1}L$, on a balanced growth path, $\beta\,(\dot{c}/c) = \dot{\lambda}_1/\lambda_1 + \dot{\gamma}/\gamma$. Eliminating c, λ_1, γ, and using equation (2.24), I derive:

$$R_d = \left(\frac{\rho}{h}\right)^{\frac{1}{\theta}} \Delta^{\frac{1-\theta}{\theta}} (1+\beta\Delta)^{-\frac{1}{\theta}} \tag{2.26}$$

Finally, using equations (2.24), (2.25), (2.26), and the full employment condition, $\mathcal{L} = L + R_b + R_d$, I derive the following algebraic equation:

$$\mathcal{L}\left(\frac{h}{\rho}\right)^{\frac{1}{\theta}} (1+\beta\Delta)^{\frac{1}{\theta}}\,\Delta = (\Delta + \frac{1}{\theta})(\Delta+1)\Delta^{\frac{1-\theta}{\theta}} \tag{2.27}$$

Once Δ is determined, I can compute the number of production workers, basic researchers, and applied researchers. This provides the solution to my social planner's problem.

Appendix C: Federal funding

This section solves my extended model in detail. It focuses on the case where the level of federally-funded basic research G_b is **larger** than the outcome of my decentralized system. Firms stop conducting their own basic research. For simplicity sake, I assume that there is no federally-funded applied research. The intertemporal research decision is given by:

$$\max_{R_{di}} \int_0^\infty e^{-rt}(\pi_i - R_{di})\,dt$$

subject to

$$\begin{aligned} \pi_i &= \frac{(1-\alpha)}{\alpha}\left(\frac{a_i}{A}\right)^{\frac{\alpha}{1-\alpha}} L \\ \dot{a}_i &= \gamma_i \varphi R_{di}^\theta \\ \dot{\gamma}_i &= \gamma_i h G_b^\theta \end{aligned}$$

As usual, I ignore the subscript i. The current valued Hamiltonian is:

$$H(t) = \pi_i - R_d + \lambda_2 \gamma \varphi R_d^\theta$$

The first order conditions are:

$$\begin{aligned} 1 &= \lambda_2 \gamma \varphi \theta R_d^{\theta-1} \\ \frac{\dot{\lambda}_2}{\lambda_2} &= r - \frac{\gamma\varphi}{a}\left(\frac{a_i}{A}\right)^{\frac{\alpha}{1-\alpha}} \frac{\theta L}{R_d^{1-\theta}} \end{aligned}$$

On a balanced growth path, scientific progress sustains productivity growth: $\dot{\gamma}/\gamma = \dot{a}/a$. The number of applied researchers and production workers are constant. Using the two laws of motion, I get:

$$R_d = \frac{G_b}{\Delta} \tag{2.28}$$

where $\Delta = (\gamma\varphi/ah)^{1/\theta}$ and G_b is the number of federally-funded basic researchers. Since $1 = \lambda_2\gamma\varphi\theta R_d^{\theta-1}$, then $\dot{\lambda}_2/\lambda_2 = -\dot{\gamma}/\gamma$. Substituting $r = \rho$ and eliminating λ_2 and γ, I have:

$$L = \frac{G_b}{\theta\Delta}\left(\frac{\rho}{h}G_b^{-\theta} + 1\right) \tag{2.29}$$

Employing equations (2.28), (2.29), and the labor market clearing condition, I derive the exact solution of this model:

$$\Delta = \frac{(1+\frac{1}{\theta})G_b + \frac{\rho}{\theta h}G_b^{1-\theta}}{\mathcal{L} - G_b} \tag{2.30}$$

Comparative static results

I report the results of comparative static exercises in Table 2.3. Each cell reports how an increase in each parameter affect the number of applied research and the number of production workers in each firm. All signs are in the expected directions.

Note that the effect of federal funding on the number of applied researchers in each firm is ambiguous. On one hand, an increase in federally-funded basic research increases each firm's efficiency of applied research. On the other, it reduces the number of available labor in the market. Since $R_d = G_b/\Delta$, implicitly differentiating equation (2.30) yields the direction of the

Table 2.3: Comparative static results

Parameters	Applied researchers	Production workers
h	+	−
ρ	−	+
$\mathcal{L}$	+	+
G_b	Ambiguous	−

effect of federal funding on applied research.

$$sign \ \frac{d(G_b/\Delta)}{dG_b} = sign \ -1 + \frac{\rho}{h}\frac{1}{G_b^{\theta}\Delta}$$

The term -1 indicates that an increase in the number of federally-funded researchers reduces the number of available labor in the market. Since ρ/h is a constant, the magnitude of this second term only depends on the magnitude of $G_b^{\theta}\Delta$. With sufficiently high level of G_b, an additional increase in federally-funded basic researchers reduces the number of applied researchers.

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Chapter 3

Knowledge spillover and scale effects

This paper develops a general equilibrium model in which productivity growth is sustained by innovative activities: $\dot{a}/a = f(R)$. It combines the quality-ladder model with Krugman's model of product differentiation in a way that varieties of intermediate goods is endogenous. The production function for final good has Dixit-Stiglitz's preference. Under this setting, it is possible to demonstrate that there may not be scale effects: an increase in the size of the economy does not always lead to faster productivity growth. If knowledge spillover is *limited*, there is no scale effects. But, if knowledge of a firm spills over to every other firms, there is. Finally, this model suggests that the joint behavior of productivity growth slowdown and research intensity increase in OECD countries during 1960-1990 can be explained by an exogenous increase in the degree of sector-specificity of technology.

3.1 Introduction

During the last decade, there has been a revival of interest in modelling productivity growth. The most interesting approach, exemplified by Romer [1990], Aghion and Howitt [1992], Grossman and Helpman [1991], is to construct a model such that purposive, profit-seeking investment in knowledge plays a critical role in the long run growth. Their essential assumption is that the law of motion for productivity increases is given by $\dot{a}/a = f(R)$ where R is the number of researchers and a can be either the quality level or the number of product varieties.

It is well-known that this class of models predicts faster productivity growth when the size

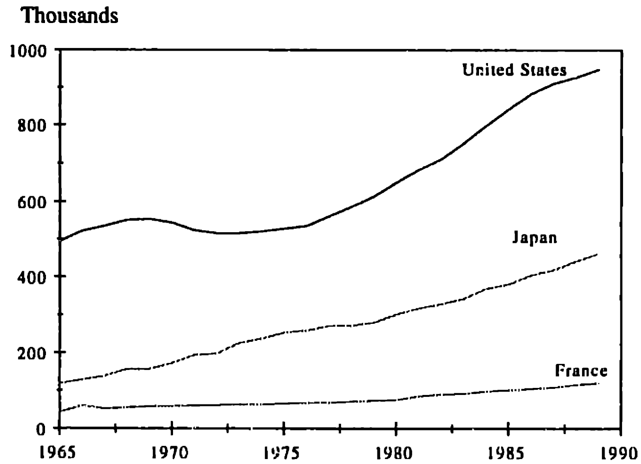

Figure 3-1: Scientists and engineers engaged in research and development (in thousands).

of the economy increases, the so-called *scale effects.* It suggests that India with its population of 800 millions should grow faster than the United States, and that the United States with its expanding economy should enjoy faster productivity growth now than thirty years ago. Unfortunately, these predictions are not confirmed in the data. Between 1965 and 1990, the number of scientists and engineers engaged in research and development in the United States has increased twofold from five hundred thousands to roughly one million. Yet, productivity growth rate has not been increasing as predicted. (Similar trends are found in other OECD countries.) Based on this particular evidence, Jones [1995a] rejects this class of models. Particularly, he rejects the law of motion as empirically untenable.

In a following paper, Jones [1995b] argues that this evidence could be explained should he modify the law of motion to be $\dot{a} = a^{\phi} f(R)$ where $\phi < 1$. Innovation increases productivity level, a, but not proportionally. Sustaining productivity growth requires that more and more resources are allocated to innovative activities. Specifically, productivity growth rate remains constant despite exponentially increasing number of researchers. The scale effects is eliminated. Unfortunately, his model predicts that countries with faster population growth grow faster. Several cross-countries studies, De Long and Summers [1991] for instance, have found that population growth and per capita output growth are either uncorrelated or negatively correlated. In addition, his model can not explain the observed joint behavior of productivity growth slowdown and rising research intensity, as measured by the number of researchers per labor force population.

This paper takes a different approach. It argues that the law of motion $\dot{a}/a = f(R)$ exists at firms level, not the aggregate level. It constructs a theoretical model whose critical feature is that the number of firms is endogenous. Doubling the size of the economy, double the number of firms. Although the aggregate number of researchers double, the number of researchers per firm as well as productivity growth rates remain unchanged. Put it differently, with free entry the aggregate number of researchers is not a good proxy of the number of researchers per firm. This is why it may be inappropriate to regress the aggregate number of researchers on productivity growth rates as Jones does in his paper. I then reinterpret Jones' empirical finding as a rejection of the assumption — within those endogenous growth models — that there is a *fixed* number of firms in the economy.[1] My model illustrates in addition that the prediction of scale effects is not a necessary consequence of $\phi = 1$.

My model of productivity growth without scale effects turns out to be special. Young [1995] independently develops a model with similar feature. He shows that although there is no scale effects with Dixit-Stiglitz's preference, there can be either positive or negative scale effects with a more general production structure such as Salop model of spatial competition. My paper complements his finding: the elimination of scale effects depends as well on the assumption concerning knowledge spillover. If knowledge spillover is limited to firms within certain technological distance, there is no scale effects. But, if knowledge of a firm spills over to every other firms and raises their research efficiency, there will be positive scale effects. The magnitude of scale effects depends on the strength of knowledge spillover.

The following section describes the model and its properties. The third section discusses existing empirical findings, particularly the observed productivity growth slowdown and research intensity increase in OECD countries during 1960–1990. It turns out to be difficult to reconcile these two phenomena given that most endogenous growth models, including Jones [1995b], predict a positive correlation between productivity growth rates and research intensity. Two explanation are examined: an exogenous decrease in the degree of substitutability between

[1] Romer [1990] assumes that there is only one research sector in the economy.

intermediate goods[2], and an exogenous increase in the degree of sector-specificity of technology. The latter fits existing empirical findings better.

3.2 A model of research and productivity growth

This model is a multisectors decentralized system with perfect foresight. It combines the quality-ladder model with Krugman's model of product differentiation in a way that varieties of intermediate goods is endogenous. Productivity growth is sustained by innovations at the firm level. There are no capital accumulation nor population growth.

I begin by describing my model. I solve for the symmetric balanced growth path and analyzes its properties, particularly the relationship between knowledge spillover and scale effects.

3.2.1 Model description

There are two types of goods: a final good and intermediate goods. The market for final good is perfectly competitive. The final good is made of intermediate goods and the production function is:

$$y = \left(\int_0^m (a_i x_i)^\alpha di\right)^{\frac{1}{\alpha}} \tag{3.1}$$

where m denotes the endogenous varieties of intermediate goods; a_i and x_i, the quality and quantity of intermediate good i, respectively. Intermediate goods are assumed to be good substitutes, i.e. $0 < \alpha < 1$, so that the derived demand for each intermediate good is elastic.

The market structure for intermediate goods is monopolistic competition and there is free entry. Each firm engages in two activities — intermediate good production and research. Labor is the only input. $\mathcal{L}$ denotes the labor force population; L_i and R_i, the number of production workers and researchers employed by firm i, respectively. The production function for each intermediate good is:

$$X_i = L_i - F \tag{3.2}$$

where X_i is the total output of intermediate good i. F is the fixed cost of production which assures that there is finite varieties of intermediate goods in the equilibrium. One can think of

[2] This explanation is my interpretation of Young [1995].

it as the amount of labor devoted to tasks such as administration. Marginal cost is constant and average cost is declining.

Knowledge and research efficiency

Each intermediate good firm owns a patent to her product forever and improves its quality by engaging in research. In particular, she has research cost advantage to outside firms.[3] (She may has better information about her technology.) Consequently, she will never be replaced and plans a sequential R&D program.

As a by-product, research yields better understanding of nature which provides guidance for future research. "Simply understanding the second law of thermodynamics rules out some proposed designs; knowledge of the relationship between electricity and magnetism further narrows the range of plausible power systems; recent research on superconductivity provides intriguing clues as to promising design avenues to explore."[4] In this sense, knowledge generated by past research allows researchers to screen their candidates and focus their search, thereby improving research efficiency.

It is crucial to distinguish between two sources of knowledge: *internal* knowledge and *external* knowledge. Internal knowledge is knowledge which each firm accumulates as a by-product of its research and which is specific to its technology. For the sake of simplicity, a_i not only serves as the quality level but also as an index for the stock of internal knowledge. Under this assumption, firms with better product have greater stock of knowledge. External knowledge, on the other hand, is knowledge which spills over from other firms' research. Both types increase research efficiency. I assume that

$$\dot{a}_i = a_i^{1-\sigma}\Gamma_i^{\sigma} h R_i \tag{3.3}$$

where $0 \leq \sigma \leq 1$. Quality improvement is a function of the number of researchers, R_i; the research productivity level, h; the stock of internal knowledge, a_i; and knowledge spillover from other firms, Γ_i. Notice that $1-\sigma$ measures the extent to which internal knowledge is important *relative* to external knowledge. An increase in this parameter not only increases the potency of

[3]This assumption simplifies my model significantly. And it is here that I depart from the quality-ladder model. Barro and Sala-i-Martin [1995] provides the condition under which current monopolist conducts all R&D within the standard quality-ladder model.

[4]Nelson [1982]

internal knowledge, but also decreases the potency of knowledge which spills over from other firms. This parameter also measures the degree of sector-specificity of technology.[5]

The special case when $\sigma = 1$ deserves additional discussion. The law of motion becomes $\dot{a}_i = \Gamma_i h R_i$; firms rely solely on external knowledge for guidance in their research projects. Under this setting, knowledge is freely transmitted and publicly available. Nonetheless, R&D spending does *not* increase the quality of other firms' products, but contributes to the stock of knowledge thereby increasing research efficiency of other firms.[6] In other words, knowledge is a public good, but R&D spending is not. Firms engage in R&D in order to identify, assimilate, and exploit knowledge from the environment and incorporate those knowledge into their products. Dasgupta [1988, p.6] discusses this very idea: he argues that "...even though the transmission cost of knowledge may be low, the cost of absorbing the information, of interpreting it and using it fruitfully may well be very high. This is often the case with research output at the frontiers of science and technology. Firms wishing to make use of the latest findings that are publicly available need to have scientists who can make it possible for them to do so....This provides one reason why private firms conduct research."

Let us return to the relationship between external knowledge and firms' stock of internal knowledge. I assume that knowledge spillover from other firms is given by:

$$\Gamma_i = \int_0^m e^{-\frac{2}{\beta} d(i,j)} a_j \, dj$$

[5]The parameter $1-\sigma$ measures the degree of sector-specificity of technology for the following reason. Suppose that quality level is characterized by the number of parts of which the product consists, and each firm "stands on the shoulders of giants" as in Chapter 1. Before each project, firms optimally choose their number of additional parts; $1-\sigma$ fraction of which is built based on its own technology-specific internal knowledge and σ fraction of which, based on external knowledge. Suppose that the probability of success in designing each corresponding part is given by $\wp_i = 1 - 1/a_i$ and $\wp' = 1 - 1/\Gamma_i$. The expected quality improvement will be

$$E(\dot{a}) = \left(\frac{(1-\sigma)}{a_i} + \frac{\sigma}{\Gamma_i} \right)^{-1} h R_i$$

This equation is similar to equation (2.3); the only difference is the elasticity of substitution between internal knowledge and external knowledge. Replacing equation (2.3) with this law of motion, the resulted model behaves similarly (which is why I interpret $1-\sigma$ as a measure of sector-specificity).

[6]My model differs from Spence [1984] in a crucial way. In his model, quality improvement is given by $\dot{a}_i = R_i + \beta \sum R_j$: result of own-research improves both the quality of own product and the quality of other firms' products; R&D spendings are substitutes.

My model could be viewed as an extension of Cohen and Levinthal [1989] to a continuous time model. They assume that $\dot{a} = R_i + \beta(R_i) \sum R_j$ where $\beta(R_i)$ is an endogenous degree of absorption. In thier model, firms need to conduct R&D to incorporate lastest findings into their products; R&D spendings are complements.

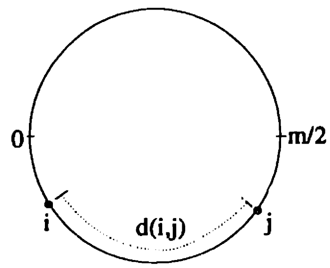

Figure 3-2: A representation of technological distance between firm i and firm j.

where $d(i,j)$ measures the technological distance between firm i and firm j which captures the concept that how useful another idea is, depends on how close the two technologies are from each other. $\beta > 0$ measures the extent of knowledge spillover. When it increases, there is more knowledge spillover from other firms. This specific functional form is not crucial but helps simplifying my notation.

There are several ways to represent the technological distance. I could have postulated that firms locate along a straight line. As new firms enter, the length of that line increases. Technological distance is simply the length of the interval between firms' location. However, the stock of external knowledge is not symmetric among firms; thus, it is not possible to solve for the symmetric balanced growth path. To get around this problem, I postulate that firms locate along the circumference of a circle with length m as in Figure 3-2. When new firms enter, the circumference expands accordingly. Technological distance is given by the shortest distance between firms' location along the circumference.[7] Under this representation, knowledge spillover from other firms on the symmetric balanced growth path is:

$$\Gamma(m) = \beta a[1 - e^{-\frac{m}{\beta}}] \tag{3.4}$$

The stock of external knowledge increases when the number of firms increases.

If there are a lot of firms in the economy, or if knowledge spillover is limited to firms within a certain technological distance, the stock of external knowledge is in constant proportion to

[7] There is a small problem with this representation. When new firms enter, the technological distance between some existing firms changes. The only defense which I can offer is that this representation greatly simplifies the algebra and one will not gain additional insight by assuming the first representation.

the stock of internal knowledge[8] which simplifies the productivity growth rate on the symmetric balanced growth path to be:

$$\frac{\dot{a}_i}{a_i} = \beta^{\sigma} h R_i \tag{3.5}$$

Productivity growth is an increasing function of the extent of knowledge spillover, β; the relative importance of external knowledge[9], σ; the research productivity level, h; and the number of researchers per firm. Note that $\beta^{\sigma} h$ measures research efficiency level.

Below, I solve my model assuming that the ratio Γ/a is constant and independent of the number of firms in the economy. Knowledge spillover from new firms does not increase research efficiency of existing firms. I then use these results to discuss what will happen in the other case when the stock of external knowledge does depend on the number of firms in the economy.

Perfect foresight variables

There are three aggregate variables: the price of the final good P, the wage rate w, and the interest rate r. Consumers and producers solve their maximization problems taking as given the future path of these variables. Specifically, the wage rate and the interest rate are expected to be constant and P is expected to decline exponentially at the rate $\kappa > 0$. In effect, the real wage rate rises at the same rate. By choosing $w = 1$, the nominal wage rate serves as the numeraire.

Consumer's maximization problem

Let c denote consumption of the final good per capita. Each consumer has a logarithmic utility function. Each one takes w, r, and P as given and solves the following intertemporal maximization problem.

$$\max_{c} \int_0^{\infty} e^{-\rho t} \ln c \, dt$$

subject to her budget constraint, $\dot{b} = rb + w + d - Pc$. ρ is the discount rate; b is the amount of bond she purchases; and d is the expected profit per capita (which already takes into account the endogenous number of firms). Consumers own equal shares of all firms so that profit is

[8] In the first case, I get $\Gamma/a = \beta$. In the other case, $\Gamma/a = \beta[1 - e^{-2d^{*}/\beta}] = \beta'$ where d^{*} is the maximum range of knowledge spillover.

[9] When $\beta > 1$.

distributed equally. Solving the intertemporal maximization problem, I get the first order condition, $\dot{c}/c = r + \kappa - \rho$.

Let κ^* denote the perfect foresight rate of decline in the price of the final good. On the balanced growth path, it turns out that $\dot{c}/c = \kappa^*$.[10] Consequently, the interest rate is equal to the discount rate:

$$r = \rho \tag{3.6}$$

Final good sector

The production function for the final good — equation (3.1) — has constant return to scale. By assuming that the final good market is perfectly competitive, there is no need to introduce firms. Each worker can set up a factory in his own backyard. Let Y denote the aggregate output of the final good. The market demand curve for intermediate good i is given by:

$$X_i = \left(\frac{p_i}{P a_i^{\alpha}}\right)^{\frac{1}{\alpha-1}} Y \tag{3.7}$$

The price for one unit of the final good is $P = \left(\int_0^m a_i^{\alpha/(1-\alpha)} p_i^{\alpha/(\alpha-1)} di\right)^{(\alpha-1)/\alpha}$ where p_i denotes the price of intermediate good i.[11]

Intermediate good production and instantaneous profit

The labor market is perfectly competitive. Each intermediate good monopolist chooses her output such that her profit is maximized. She takes her market demand curve — equation (3.7) — and the wage rate as given. Her maximization problem is:

$$\max_{X_i} \; p_i X_i - wL_i$$

subject to $X_i = L_i - F$. Solving this problem, I find that the instantaneous profit will be $\pi_i = (1-\alpha) w X_i/\alpha - wF$. Price is a constant markup over the wage rate, $p_i = w/\alpha$, which

[10] On the balanced growth path, $c = Y/\mathcal{L}$, $Y = A(L - mF)$, and $A = w/\alpha P$.

[11] When $a_i = 1$ and $p_i = 1$, the price for one unit of the final good is $P = m^{(\alpha-1)/\alpha} \neq 1$. This is so since it requires only $m^{-1/\alpha}$ unit of each intermediate good to produce one unit of the final good.

simplifies the price for one unit of final good to be:

$$P = \frac{w}{\alpha A} \tag{3.8}$$

where $A = \left(\int_0^m a_i^{\alpha/(1-\alpha)} di\right)^{(1-\alpha)/\alpha}$ measures the aggregate productivity level. Using equations (3.7), (3.8), and the definition of aggregate productivity level, I derive:

$$Y = A(L - mF) \tag{3.9}$$

where L denotes the total number of production workers in all firms. When this variable and the endogenous number of firms are determined, the aggregate output only depends on the aggregate productivity level. Aggregate productivity growth sustains aggregate output growth. The total output of intermediate good i is given by:

$$X_i = \left(\frac{a_i}{A}\right)^{\frac{\alpha}{1-\alpha}} (L - mF) \tag{3.10}$$

When the total number of production workers and the endogenous number of firms are determined, the instantaneous production level of intermediate good i depends only on its quality level *relative* to the aggregate productivity level. In other words, by raising the quality of its product relative to the aggregate productivity level, each firm can increase its market demand, which is the reason why firms engage in research. The elasticity of demand with respect to product quality is given by $\alpha/(1-\alpha)$.

Intertemporal research decision

Each firm improves the quality of her product by engaging in research. With higher quality, the demand for her product increases. She plans a sequential research program taking into consideration the dynamic return to her research, especially the dynamic return to her stock of internal knowledge. She takes the future path of w, r, m, A, L, and Γ_i as given and tries to maximize the value of her discounted stream of future profit. Her intertemporal maximization problem is given by:

$$\max_{R_i} \int_0^\infty e^{-rt} (\pi_i - wR_i)\, dt$$

subject to

$$
\begin{aligned}
\dot{a}_i &= a_i^{1-\sigma}\Gamma_i^{\sigma} h R_i \\
\pi_i &= \frac{(1-\alpha)w}{\alpha} X_i - wF \\
X_i &= \left(\frac{a_i}{A}\right)^{\frac{\alpha}{1-\alpha}} (L - mF)
\end{aligned}
$$

When $0 < \alpha < 1/2$, the objective function is concave with respect to a_i as required by optimal control problems. In addition, in order to properly ignore the instantaneous production decision including shutting-down decision from this intertemporal maximization problem, I must assume that each firm has a long-term agreement with its administrative workers F such that these workers will always be paid, regardless of other decisions by the firm.[12] Under this assumption, firms will never shut down and its expected maximum instantaneous profit at a point in time is $\pi_i = (1-\alpha)wX_i/\alpha - wF$ where $X_i = \left(\frac{a_i}{A}\right)^{\frac{\alpha}{1-\alpha}} (L - mF)$. Profit is simply a function of the firm's productivity level a_i. On the balanced growth path each firm, as we shall see, always earns positive instantaneous profit, $\pi_i > 0$.

I solve out this maximization problem in detail in Appendix A. The corresponding social planner's problem is solved in Appendix B. Important results are summarized below.

3.2.2 The symmetric balanced growth path

Imposing symmetry between intermediate-good firms, the subscript i can be ignored. As mentioned before, I assume either that there are a lot of firms or that knowledge spillover is limited, so that on the symmetric balanced growth path, $\Gamma/a = \beta$. Consequently, the number of researchers per firm is:

$$R_i = \frac{1}{\sigma}\left(\frac{L}{m} - F - \frac{r}{\beta^{\sigma} h}\right) \tag{3.11}$$

Firms hire less researchers when the interest rate increases; more when research efficiency $\beta^{\sigma} h$ is higher; and more when the relative potency of internal knowledge increases (holding research efficiency constant). Finally, $L/m - F$ denotes the expected total output per firm, i.e. the expected scale of production. When it increases, firms hire more researchers since the cost of

[12]This is a technical condition. I can alternatively assume that each firm incurs a fixed cost C at the beginning. The resulted model retains all of my basic implications.

research per unit output decreases.[13]

Entry decision

To simplify my model, I assume that all entries occur at time zero. There is no transitional dynamic given that there is only one state variable, a_i; the economy is always on the balanced growth path. The number of researchers per firm and the number of varieties are constant.

The value of discounted stream of profit is driven down to zero by entry. Given that there is no fixed cost of starting a new firm, entry occurs until the instantaneous profit from production is equal to research outlay: $\pi_i = wR_i \geq 0$. Substituting the instantaneous profit with equation (3.10) and the labor market clearing condition, I find that on the symmetric balanced growth path:

$$m = \frac{(1-\alpha)\mathcal{L}}{R_i + F} \tag{3.12}$$

Notice the similarity between this condition and Krugman's condition: $m = (1-\alpha)\mathcal{L}/F$. By introducing the possibility of research, Krugman's condition is modified slightly. The number of researchers per firm is now treated as if they were additional fixed cost of production.

3.2.3 Comparative static exercises

By imposing the labor market clearing condition: $\mathcal{L} = L + mR_i$, equation (3.6), and equation (3.12), I eliminate three endogenous variables — L, m, and r — from equation (3.11). I get:

$$\beta^\sigma h R_i = \frac{\rho R_i}{\varepsilon F - (\sigma - \varepsilon) R_i} \tag{3.13}$$

where $\varepsilon = \alpha/(1-\alpha)$ is the elasticity of demand with respect to product quality. It is instructive to express this condition in this particular form before solving for the productivity growth rate and the number of researchers per firm. The left hand side and the right hand side of this condition are drawn in Figure 3-3. The LHS is the relationship between productivity growth rate and the number of researchers per firm on the symmetric balanced growth path. It is a straight line passing through the origin with slope $\beta^\sigma h$. The RHS is drawn under the

[13] Wilson [1976]

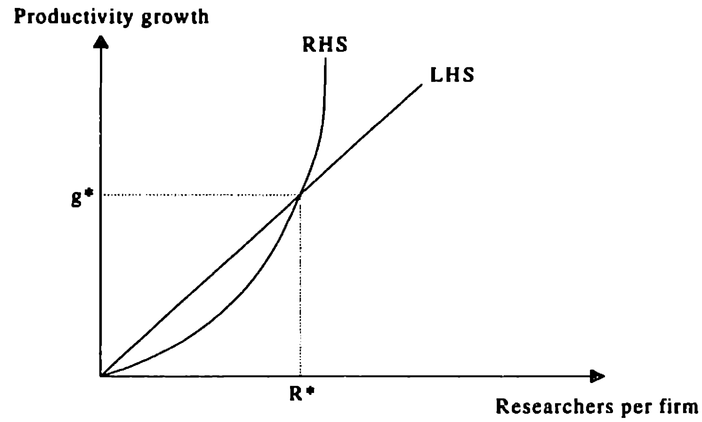

Figure 3-3: Determination of productivity growth rate and the number of researchers per firm.

assumption that $\sigma > \varepsilon$. (I discuss this condition below.) It passes through the origin with slope $\rho/\varepsilon F$. Its slope increases to infinity as the number of researchers per firm approaches a certain constant.

When the LHS has higher slope at the origin than the RHS, i.e. $\beta^\sigma h > \rho/\varepsilon F$, the two curves intersects exactly once and this intersection determines productivity growth rate. On the symmetric balanced growth path, the productivity growth rate and the number of researchers per firm are given by:

$$g = \frac{(\beta^\sigma h\varepsilon F - \rho)}{(\sigma - \varepsilon)} \qquad \text{and} \qquad R_i = \frac{1}{\beta^\sigma h}\frac{(\beta^\sigma h\varepsilon F - \rho)}{(\sigma - \varepsilon)} \tag{3.14}$$

Intuitively, an increase in the discount rate reduces productivity growth rate; for that, it lowers the return to research thereby reducing the number of researchers per firm. This requires that $\beta^\sigma h\varepsilon F > \rho$ and $\sigma > \varepsilon$: the discount rate cannot be too high and the relative potency of external knowledge is large enough. (When the discount rate is too high, firms do not engage in research.) Both conditions can be relaxed as shown in Appendix A.[14]

I report results of comparative static exercises in Table 2.1. Each cell of the table indicates how each variable responses to an increase in each exogenous parameter. The variable in the last column is the research intensity as measured by the number of researchers per labor force

[14]Specifically, both conditions can be relaxed by introducing diminishing return to the production function of projects: $\dot{a}/a = \beta^\sigma h R_i^\theta$ where $0 < \theta < 1$.

population. All cells have the expected sign.

Allocation of labor

Labor are allocated between two productive activities: intermediate good production and research. In addition, out of these production workers, some are spent as fixed cost, i.e. devoted to tasks such as administration. According to equation (3.12), the total number of researchers and workers spent as fixed cost is $m(R_i + F) = (1 - \alpha)\mathcal{L}$. That is, $1 - \alpha$ fraction of the labor force engages in either research or fixed cost activities; the rest, α fraction of the labor force, in the production of intermediate goods.

Employing equation (3.12) and the labor market clearing condition, I get

$$L_i = \frac{\alpha R_i}{1-\alpha} + \frac{F}{1-\alpha}$$

When firms hire more researchers, they hire more production workers and their production scale, $L_i - F$, rises. In other words, this model predicts a positive correlation between firm size and its research department size.

Productivity growth rate

The productivity growth rate is higher when the research productivity level increases; when the fixed cost increases; and when the discount rate decreases. With higher research productivity level, firms hire more researchers. The productivity growth rate is higher. Similarly, with higher fixed cost, the degree of increasing return at the firm level increases. Each firm hires more production workers leading to an expansion of its production scale. The cost of research per unit output declines. Firms consequently hire more researchers; the productivity growth rate is higher. Finally, with lower discount rate, the interest rate decreases. The future return to research is discounted less. Firms hire more researchers; the productivity growth rate increases.

In Appendix B, I show that the social planner allocates more researchers per firm than the decentralized system. Productivity growth rate is higher under the social planner which is not surprising given that the social return to research is higher than the private return.

Table 3.1: Comparative Static Results

Parameters	Growth rate	# of Firms	Researchers/Firm	Research Intensity
h	+	−	+	+
β	+	−	+	+
F	+	−	+	+
ρ	−	+	−	−
$\mathcal{L}$	No changes	+	No changes	No changes
α	+	−	+	+

Knowledge spillover

This model has an interesting implication that when the extent of knowledge spillover increases, productivity growth rate increases as well. There are two possible assumptions concerning R&D spending: that they are substitutes or complements. The former case is exemplified by Spence [1984] who assumes that $\dot{a} = R_i + \beta \sum R_j$. In his model, knowledge spillover reduces incentive for research. Unfortunately, this implication is at odd with some industries such as electronics which are characterized both by high degree of knowledge spillover and high R&D spending per firm.[15] On the other hand, if R&D spending are complements, as suggested by Cohen and Levinthal [1989] and my model, an increase in the extent of knowledge spillover increases the stock of external knowledge and consequently research efficiency. Firms hire more researchers to assimilate and absorb knowledge spillover from other firms. R&D spending per firm increases, which provides an explanation for the observation concerning electronic industry.

In addition, a study by Cockburn and Henderson [1993] of ten major pharmaceutical firms finds that "their results are consistent with the idea that there are significant spillover of knowledge across firms. Important patents per discovery dollars are likely to be higher if competitors have recently obtained a number of important patents in the area, and far from leading to a 'mining out' of opportunities, competitors' research appears to be a *complementary*

[15] Spence discusses this example on p.115 and admits that "such industries are something of a puzzle" in the context of his model. He then argues that "firms might imperfectly anticipate or even ignore the effects of their own R&D invetments on the costs of other firms."

activity to own R&D." It also finds that competitors' investment has a positive and significant impact on own research productivity, which is consistent with my model.

Scale effects

Inspecting equation (3.14), one finds that productivity growth rate does not depend on the size of the economy. A population increase leads to a proportional expansion of intermediate good varieties. As a result, available resources per firm remains constant. Firms hire the same number of researchers; productivity growth rate remains unchanged. The scale effects is eliminated. (However, there is a one-time jump in consumption level. Double the size of the economy doubles the number of intermediate goods. Due to the love-for-varieties assumption in Dixit-Stiglitz's production function, consumption level increases.)

Young [1995] independently constructs a model with similar feature: allowing entry to eliminate scale effects. He shows that although there is no scale effects with Dixit-Stiglitz's preference, there can be either positive or negative scale effects with a more general production structure such as Salop model of spatial competition. My paper complements his finding: the elimination of scale effects depends on the assumption concerning knowledge spillover as well.

On page 107, I assume either that there are a lot of firms in the economy or that knowledge spillover is limited to firms within a certain technological distance. In effect, R&D spending of new firms does not improve research efficiency of existing firms. Consequently, there is no scale effects in the resulted model. Now, let me discuss what will happen in the other case when the ratio Γ/a is an increasing function of the number of firms in the economy. In this case, R&D spending of new firms raises the stock of external knowledge and improves research efficiency of existing firms. Graphically, the LHS of equation (3.13) shifts upward. Firm hire more researchers and productivity growth rate increases. There will be positive scale effects. Its magnitude depends on the extent of improvement in research efficiency due to knowledge spillover from new firms. In my model, $\Gamma = \beta a[1 - e^{-\frac{m}{\beta}}]$ — the stock of external knowledge approaches βa as the number of firms increases. The magnitude of scale effects diminishes as the number of firms increases.

This model suggests that when economic integration increases research efficiency of firms in both economies, it is likely that it will increase productivity growth rates as well. Otherwise, economic integration might have only a level effect.

Population growth

My model cannot handle population growth. As in Romer [1990], Aghion and Howitt [1992], and Grossman and Helpman [1991], population growth leads to an ever-increasing productivity growth rate; The economy explodes in a finite time.

In the current literature, there exists models which are capable of handling population growth, namely Jones [1995b] and Kremer [1993]. In Jones' model, he assumes that $\dot{a} = a^{\phi} f(R)$ where $0 < \phi < 1$. Due to diminishing return in his research sector, there is no scale effects. A constant fraction of population engages in research. The productivity growth rate depends positively on population growth rate, which is not supported by the data.

3.3 Discussion

My model explains the lack of scale effects in OECD countries during 1960–1990 in term of increasing intermediate good varieties and limited knowledge spillover. However, there is another serious problem. During the same period, research intensity as measured by the number of scientists and engineers engaged in research per 10,000 labor force population has risen in these countries. Yet, their productivity growth rates have been declining at the same time.

Explaining productivity growth slowdown during 1970–1990 is not an easy task. Many economists have written on this topic and several explanations have been advanced: for example, mismeasurement problem, a decline in R&D spending, a decline in scientific opportunity, or the oil price shocks. Explaining the behavior of productivity growth slowdown and research intensity increase together is an even more difficult task. Most explanations for productivity growth slowdown predict that research intensity should decline, which is in the opposite direction of what is found in the data. For example, if a decline in scientific opportunities or "mining out" of opportunities causes the slowdown, firms should reallocate their labor toward a more productive activity, production. Research intensity of the economy decreases.

Moreover, most R&D-based endogenous growth models predict a positive correlation between productivity growth rates and research intensity. In Table 2.1, a shock to an exogenous

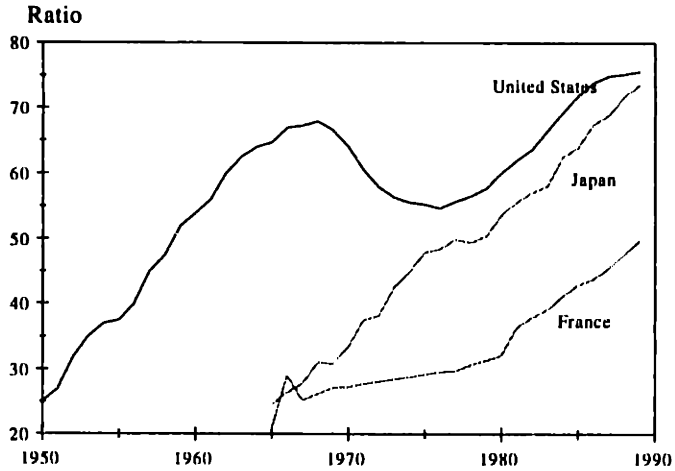

Figure 3-4: Scientists and engineerings engaged in R&D per 10,000 labor force population.

parameter affects productivity growth rate and research intensity level in the same direction. Similarly, Jones [1995b] admits that "the rise in R&D intensity is itself somewhat of puzzle in the context of his model. Along a balanced growth path, nothing in the model delivers a rising R&D intensity. One possibility is that these increases occur for reason outside the model, e.g. as a reflection of changing tax policies or a strengthening of property rights. Another possibility is that this rises reflects the relabelling of individuals who previously were not officially counted as 'scientists and engineers engaged in R&D'."

This section discusses two theoretical explanations for this joint behavior of productivity growth slowdown and research intensity increase, namely an exogenous decrease in the degree of substitutability between intermediate goods and an exogenous increase in the degree of sector-specificity of technology. The latter explanation fits empirical findings better.

A decrease in the degree of substitutability between intermediate goods

Young [1995] demonstrates that there can be positive, zero, or negative scale effects depending on whether the elasticity of demand with respect to product quality is increasing, constant, or decreasing in the number of varieties.[16] He argues that his model with *negative scale effects* "provides a nice match with the OECD experience, where increases in scale have been associated

[16] Young [1995, p.28]

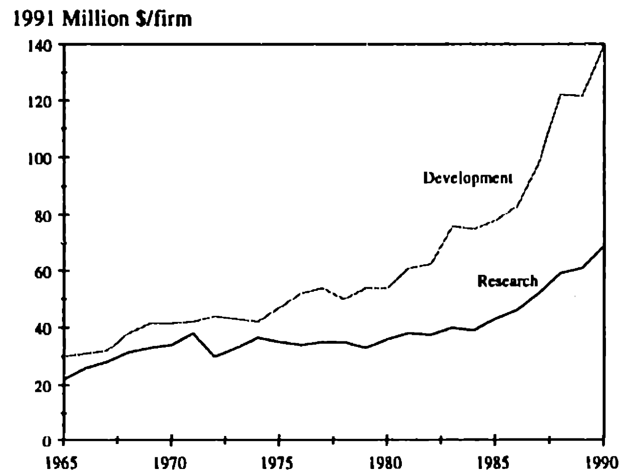

Figure 3-5: Mean R&D spending per firm in 1991 dollars

with declining growth rates and rising share of R&D research in the total labor force." Given that there is no knowledge spillover between firms in his model, research efficiency is independent of the size of the economy. With negative scale effects, a population increase leads to a more than proportional increase in the number of firms. Each firm hire *less* researchers resulting in slower productivity growth.

By introducing into my model, an ad hoc linkage between the number of varieties and the degree of substitutability between intermediate goods, the resulted model behaves like Young's model. Specifically, suppose that an increase in the number of varieties "somehow" leads to a reduction in the degree of substitutability between intermediate goods. The modified model predict that the elasticity of demand with respect to product quality, $\alpha/(1-\alpha)$, is now decreasing in the number of varieties. When the size of the economy increases, the number of firms increases more than proportionally. Each firm hires less researchers resulting in slower productivity growth. So, these predictions are in congruence with those of Young's model.

However, my model predicts a decrease in research intensity level. This difference is due to the fact that Young includes fixed cost as part of R&D spending, but I do not. If I also count "fixed cost" workers as if they were researchers, then a reduction in the degree of substitutability between intermediate goods will lead to a larger share of R&D research in the total labor force. For that, on the balanced growth path, $1-\alpha$ fraction of the labor force engages in either research or fixed cost activities.

Young's model and my modified model predict that firms have become smaller during 1960–1990. Unfortunately, this implication does not fit empirical findings well. Cockburn and Henderson [1993] provide an evidence from pharmaceutical industry which indicates that mean R&D spending per firm has risen between 1960 and 1989. In addition, Bound et. al. [1982] find that firms with larger R&D spending have lower patents per million R&D dollars as illustrated in Figure 3-6.[17] If firms were to become smaller as predicted, then the ratio of patents received per R&D dollar spent, according to this figure, should have risen. But to the contrary, it is well-known that the patent-R&D ratio has been declining steadily during this period.[18]

These are discriminating evidence against explaining the joint behavior of productivity growth slowdown and research intensity increase in term of an exogenous decrease in the degree of substitutability between intermediate goods. So, one need to look for an alternative theoretical explanation, which brings us to the next subsection. (Note that, this evidence concerning R&D spending per firm must be interpreted with care given that real wage is rising exponentially in my model. The variable to look at is the number of researchers per firm.)

An increase in the degree of sector-specificity of technology

So far, data suggests that the period between 1960 and 1990 is characterized by productivity growth slowdown, rising research intensity, increasing R&D spending per firm, and declining patents-R&D ratio. At the first glance, obvious candidates — given that several empirical studies at the firm level find positive relationship between the number of researchers and productivity growth rates[19] — are a reduction in research productivity level; a reduction in the extent of knowledge spillover; or a slowdown of exogenous scientific progress.[20] Unfortunately these hypotheses, as mentioned before, are capable of explaining productivity growth slowdown,

[17]Since this is a cross-sectional finding, one can replace R&D spending with the number of researchers per firm. (R&D spending is equal to wR_i in my model.) So, one can derive from this diagram that firms with larger research department or more researchers should have lower patents per researcher.

[18]Griliches [1990, p.1674]. Kortum [1993] finds that the ratio of patents per one million 1982-dollars of R&D has declined steadily from 3.5 to 1 during 1957–1990. He finds no evidence that the patent-R&D ratio has been driven down by rising real wages of research. As a result, the patents per researchers ratio has declined as well.

[19]Griliches [1986], Lichtenberg and Siegel [1991], Hall and Mairesse [1995].

[20]I can modify my model so that productivity growth is sustained by exogenous scientific progress. Specifically, the quality improvement is given by $\dot{a} = \Gamma h R_i$; but Γ now measures the stock of exogenous scientific knowledge. A slowdown of exogenous scientific progress is represented by a slower growth rate for Γ.

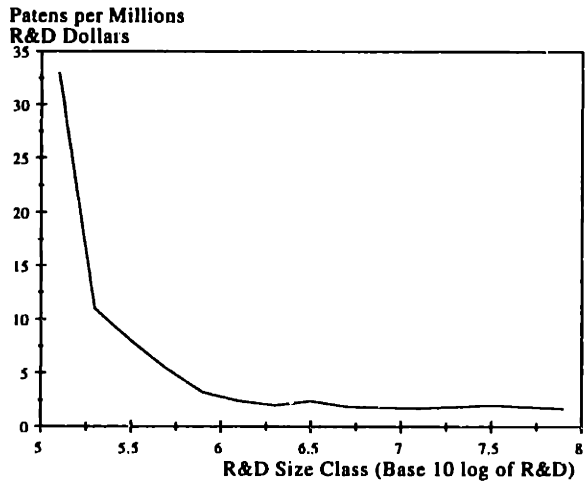

Figure 3-6: Patents per million R&D dollars by R&D size class for firms with both R&D and patents.

but not the rising research intensity nor the increasing R&D spending per firm.

My model suggests only one theoretical explanation which is capable of reconciling these empirical findings simultaneously, namely an increase in the degree of sector-specificity of technology. Intuitively, when the degree of sector-specificity increases, firms rely more on their internal knowledge. So, they hire more researchers. Their R&D spending increases and their patent-R&D ratio decreases. With the stock of external knowledge larger than the stock of internal knowledge, $\beta > 1$, the increase in the relative importance of internal knowledge reduces research efficiency of researchers.[21] If the reduction in research efficiency outweighs the additional researchers each firm hires, then it is possible for productivity growth rate of each firm to decrease.

Graphically, an exogenous increase in the degree of sector-specificity shifts both the LHS and the RHS of equation (3.13) down as illustrated in Figure 3-7. When the new intersection lies within the shaded area, firms hire more researchers and yet the productivity growth rate decreases. But, this is not always the case. On the balanced growth path, equation (3.14) determines the productivity growth rate and the number of researchers per firm. Implicit

[21]This outcome depends on several assumptions, particularly that the stock of external knowledge is larger than the stock of internal knowledge $\beta > 1$. Since it is not easy to convince the reader of this last assumption, I am willing to accept that this particular theoretical explanation is *suggestive* but not yet convincing.

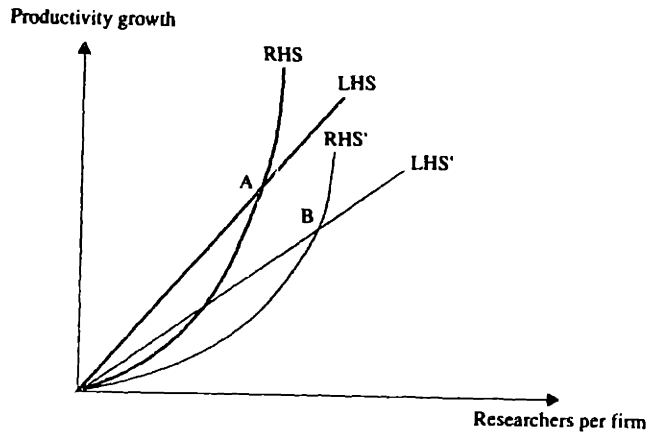

Figure 3-7: The effects of an exogenous increase in the degree of sector-specificity of technology.

differentiation yields the following conditions:

$$\frac{\partial R_i}{\partial \sigma} < 0 \quad \text{if and only if} \quad 1 + (\sigma - \varepsilon)\ln\beta < \frac{\beta^{\sigma} h\varepsilon F}{\rho} \qquad \text{(Condition I)}$$

$$\frac{\partial g}{\partial \sigma} > 0 \quad \text{if and only if} \quad 1 - (\sigma - \varepsilon)\ln\beta < \frac{\rho}{\beta^{\sigma} h\varepsilon F} \qquad \text{(Condition II)}$$

It is easy to show by induction that an increase in the degree of sector-specificity always raises the number of researchers per firm. Condition I holds for all value of $\sigma \geq \varepsilon$. First, since the existence of the symmetric balanced growth path requires that $\beta^{\sigma} h\varepsilon F > \rho$, this condition holds when $\sigma = \varepsilon$. Next, the slope of the LHS of this condition with respect to σ is $\ln\beta$ and that of the RHS is $(\beta^{\sigma} h\varepsilon F/\rho)\ln\beta$. The former is always smaller. The value of the LHS increases at a slower rate than that of the RHS. By induction, once Condition I holds at $\sigma = \varepsilon$, it holds for all value of $\sigma > \varepsilon$.

On the other hand, an increase in the degree of sector-specificity does not always reduce productivity growth rate. Condition II does not hold for all value of $\sigma \geq \varepsilon$. First, since the existence of the symmetric balanced growth path requires that $\beta^{\sigma} h\varepsilon F > \rho$, this condition fails at $\sigma = \varepsilon$. Next, the slope of LHS of this condition with respect to σ is $-\ln\beta$ and that of the RHS is $-\ln\beta(\rho/\beta^{\sigma} h\varepsilon F)$. The former is larger in absolute term. The value of the LHS decreases at a faster rate than the value of the RHS. Given that the LHS is larger than the RHS at $\sigma = \varepsilon$, this condition either fails for all value of σ or it holds after some value of σ.

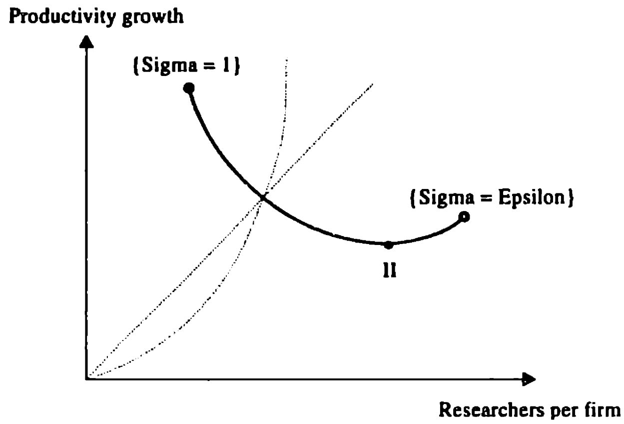

Figure 3-8: The symmetric balanced growth path as a function of σ.

Figure 3-8 illustrates the symmetric balanced growth path as a function of the degree of sector-specificity, where σ is between ε and 1. When $\sigma = \varepsilon$, Condition I holds but Condition II does not. Both productivity growth rate and the number of researchers per firm decline as the degree of sector-specificity decreases. Point II is where Condition II becomes binding (if it ever does). Beyond this point, an exogenous increase in the degree of sector-specificity leads to an increase in the number of researchers per firm but a decline in productivity growth rate. Note that, research intensity level increases as well. For that, research intensity level is given by $mR_i/\mathcal{L} = (1-\alpha) - mF/\mathcal{L}$ and an increase in the number of researches per firm reduces the number of firms. In addition, with more researchers per firm, the patents per researcher ratio falls.

In sum, my model is capable of explaining the joint behavior of productivity growth slowdown and rising research intensity in OECD countries during 1960–1990. It is shown that an exogenous increase in the degree of sector-specificity of technology leads to productivity growth slowdown; research intensity increase; rising R&D spending per firm; and declining patent-R&D ratio. This explanation fits empirical findings better than Young's explanation that there has been a decrease in the degree of substitutability between intermediate goods as the size of the economy increases. An additional evidence is given by Caballero and Jaffee [1993]. They study the patents statistics and argue that there is a documented narrowing of research. "In the nutshell, the citations data show that recent cohorts of patent are less cited than the older ones (controlling for obsolescence), suggesting that they are less potent in generating spillover....With shorter shoulders to stand on, current inventors have to spend more on telescopes in order to see as far as their predecessors did....An interpretation is that research is steadily becoming 'narrower' and, hence, generates fewer spillover because each new idea is relevant to a smaller and smaller set of technological concerns." This provides an additional evidence that there has been a reduction in the relative potency of external knowledge.

3.4 Conclusions

This paper combines the quality-ladder model with Krugman's model of product differentiation in a way that varieties of intermediate goods is endogenous. I use this model to illustrate that it may be inappropriate to regress the aggregate number of researchers on productivity growth rates and that the more appropriate variable is the number of researchers per firm. Intuitively, with free entry, the relationship between the aggregate number of researchers and the number of researchers per firm may not exist.

A model of endogenous growth without scale effects is constructed. Whether there is scale effects, depends on my assumption concerning knowledge spillover. When knowledge spillover is limited to firms within a certain technological distance, there is no scale effects. When knowledge of a firm spills over to every other firms in the economy and increases their research efficiency, there will be positive scale effect. The strength of scale effects depends on the strength of knowledge spillover. Finally, my model *suggests* that the observed productivity slowdown and research intensity increase in OECD countries during 1960–1990 can be explained in term of an increase in the degree of sector-specificity of technology. This explanation fits existing empirical facts well since it also implies that R&D spending per firm increases and the patent-R&D ratio decreases.

Appendix A: Decentralized system

This appendix solves the inteı temporal research decision for each intermediate good monopolist. I relax one of my assumption in order to demonstrate that the two conditions imposed on page 109 — $\beta^{\sigma} h\varepsilon F > \rho$ and $\sigma > \varepsilon$ — arc its consequences. Specifically, instead of assuming that productivity growth is a linear function in the number of researchers, I assume that $\dot{a}_i = a_i^{1-\sigma}\Gamma_i^{\sigma} h R_i^{\theta}$ where $0 < \theta < 1$. This functional form delivers variation to the marginal return of researchers: as the number of researchers approaches zero, the marginal return goes to infinity. The discount rate no longer has to be "small enough" to encourage research activities. Firms always engage in research and development on the symmetric balanced growth path.

Firm plans a sequential research program in order to maximize its discounted stream of future profit. Its intertemporal maximization problem is

$$\max_{R_i} \int_0^{\infty} e^{-rt} \left(\pi_i - wR_i\right) dt$$

subject to

$$\begin{aligned} \dot{a}_i &= a_i^{1-\sigma}\Gamma_i^{\sigma} h R_i^{\theta} \\ \pi_i &= \frac{(1-\alpha)\, w}{\alpha} X_i - wF \\ X_i &= \left(\frac{a_i}{A}\right)^{\frac{\alpha}{1-\alpha}} (L - mF) \end{aligned}$$

The current value Hamiltonian is

$$H = \pi_i - wR_i + \lambda_i a_i^{1-\sigma}\Gamma_i^{\sigma} h R_i^{\theta}$$

The first order conditions are

$$w = \lambda_i a_i^{1-\sigma}\Gamma_i^{\sigma} h \theta R_i^{\theta-1} \tag{3.15}$$

$$\frac{\dot{\lambda}_i}{\lambda_i} = r - (1-\sigma)\left(\frac{\Gamma_i}{a_i}\right)^{1-\sigma} h R_i^{\theta} - \frac{w}{\lambda_i a_i}\left(\frac{a_i}{A}\right)^{\frac{\alpha}{1-\alpha}} (L - mF) \tag{3.16}$$

The symmetric balanced growth path

The subscribe i is omitted whenever it does not create confusion. I assume either that there are a lot of firms or that knowledge spillover is limited so that on the symmetric balanced growth path, $\Gamma/a = \beta$. On the symmetric balanced growth path, the aggregate productivity level is $A = m^{\frac{1-\alpha}{\alpha}} a$ and the number of researchers per firm is a constant.

Equation (3.15) implies that on the symmetric balanced growth path $\dot{\lambda}/\lambda = -\dot{a}/a$. Eliminating λ from equation (3.16), I get

$$R_i^\theta = \frac{1}{\sigma}\left((\frac{L}{m} - F)(\theta R_i^{\theta-1}) - \frac{r}{\beta^\sigma h} \right) \tag{3.17}$$

By imposing the labor market clearing condition, $\mathcal{L} = L + mR_i$; the free entry condition, $\mathcal{L}/m = (R_i + F)/(1-\alpha)$; and the condition from consumer's maximization problem, $r = \rho$; I eliminate three endogenous variables — r, L, and m — from equation (3.17). I get

$$\beta^\sigma h R_i^\theta = \frac{\rho R_i}{\theta\varepsilon F - (\sigma - \theta\varepsilon)R_i} \tag{3.18}$$

where $\varepsilon = \alpha/(1-\alpha)$. It is an algebraic equation in one variable, R_i. Solving this equation provides solutions for the symmetric balanced growth path. Although a closed-form solution for this equation exists only when $\theta = 1$ or $\theta = 1/2$, I can still perform comparative static exercises for other values of θ using the diagram in Figure 3-9.

The left hand side of equation (3.18) is the relationship between productivity growth rate and the number of researchers per firm on the symmetric balanced growth path. It passes through the origin and its slope at that point is infinity. On the other hand, there are two cases for the right hand side of equation (3.18), depending on the sign of $\sigma - \theta\varepsilon$. The left diagram illustrates the case when the relative potency of external knowledge is "sufficiently" large: $\sigma > \theta\varepsilon$. The RHS passes through the origin with slope $\rho/\theta\varepsilon F$. Its slope increases to infinity as the number of researchers per firm approaches $\theta\varepsilon F/(\sigma - \theta\varepsilon)$. The two curves intersect exactly once and this intersection determines the productivity growth rate on the balanced growth path.

The right diagram illustrates the case when the relative potency of external knowledge is "sufficiently" small: $\sigma < \theta\varepsilon$. The RHS passes through the origin with the same slope as in the other case. But, as the number of researchers per firm increases, its slope now decreases to

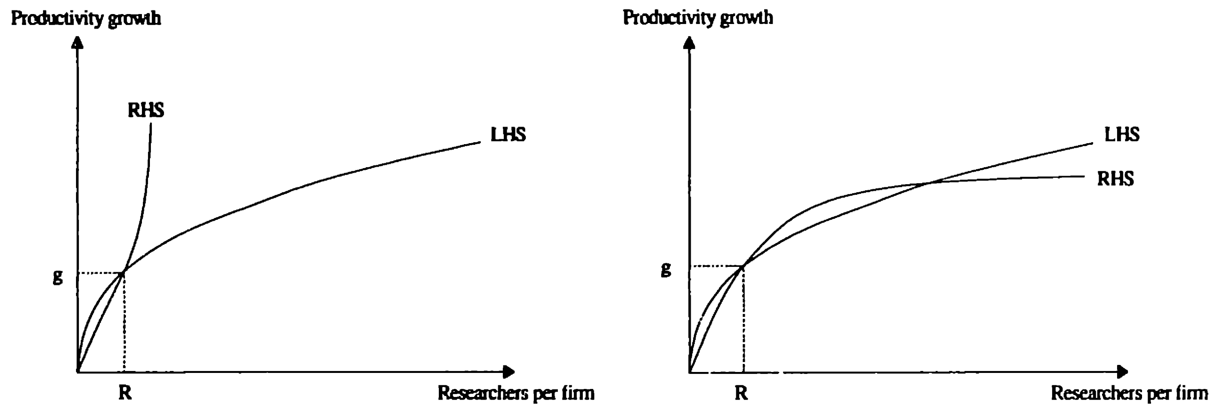

Figure 3-9: Determining the symmetric balanced growth path.

zero and its value approaches $\rho/(\theta\varepsilon - \sigma)$ from below. The two curves intersect twice or not at all.[22] When they do, the lower intersection provides the solution for the symmetric balanced growth path. The higher intersection cannot be a solution since it predicts incorrectly that firms conduct more research when the interest rate increases. Intuition suggests otherwise: since such increase reduces the return to research, firms should conduct less research, not more.

Appendix B: Social planner's problem

This appendix solves the corresponding social planner's problem. The social planner chooses the number of intermediate good varieties and he allocates labor between intermediate good production and research. At these two margins, the social planner trades off current consumption level with consumption growth. Increasing the varieties raises current consumption but reduces consumption growth. Similarly, increasing production workers raises current consumption but reduces consumption growth.

As in the decentralized system, I focus on the symmetric balanced growth path and assume either that there are a lot of intermediate good varieties or that knowledge spillover is limited so that on the symmetric balanced growth path, $\Gamma/a = \beta$. The social planner's problem is

[22] The two curves intersect when the discount rate is sufficiently high.

given by:

$$\max_{m,R_i} \int_o^{\infty} e^{-\rho t} \ln(c)\, dt$$

subject to

$$\begin{aligned} c &= \frac{m^{\frac{1}{\alpha}} a}{\mathcal{L}} (\frac{\mathcal{L}}{m} - R_i - F) \\ \dot{a} &= a\beta^{\sigma} h R_i^{\theta} \end{aligned}$$

There are two sources of increasing return in this model: at the aggregate level and at the firm level. Both are crucial.[23] The aggregate-level increasing return comes from the love-for-varieties assumption embedded in the Dixit-Stiglitz's production function. The firm-level increasing return is due to the fixed cost of production. As the social planner chooses the number of intermediate-good varieties, he trades off increasing return at the two levels.

The current value Hamiltonian is

$$H = \ln c + \lambda a \beta^{\sigma} h R_i^{\theta}$$

The first order conditions are

$$m = \frac{(1-\alpha)\mathcal{L}}{R_i + F} \tag{3.19}$$

$$1 = \lambda a \beta^{\sigma} h \theta R_i^{\theta - 1} (\frac{\mathcal{L}}{m} - R_i - F) \tag{3.20}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho - \beta^{\sigma} h R_i^{\theta} - \frac{1}{\lambda a} \tag{3.21}$$

Current consumption is maximized when the number of varieties is $m^* = (1-\alpha)\mathcal{L}/F$ and when there is no research being done. Around m^*, reducing varieties has second-order effect on current consumption but first-order effect on consumption growth (should the social planner allocate the extra production workers toward research activities in the remaining varieties.) Therefore, on the symmetric balanced growth path, the number of varieties will be less than

[23] One cannot relax either of them. For instance, if $F = 0$,then the social planner will choose infinite number of varieties. Alternatively, if there is no aggregate increasing return, he will choose the smallest possible number of varieties.

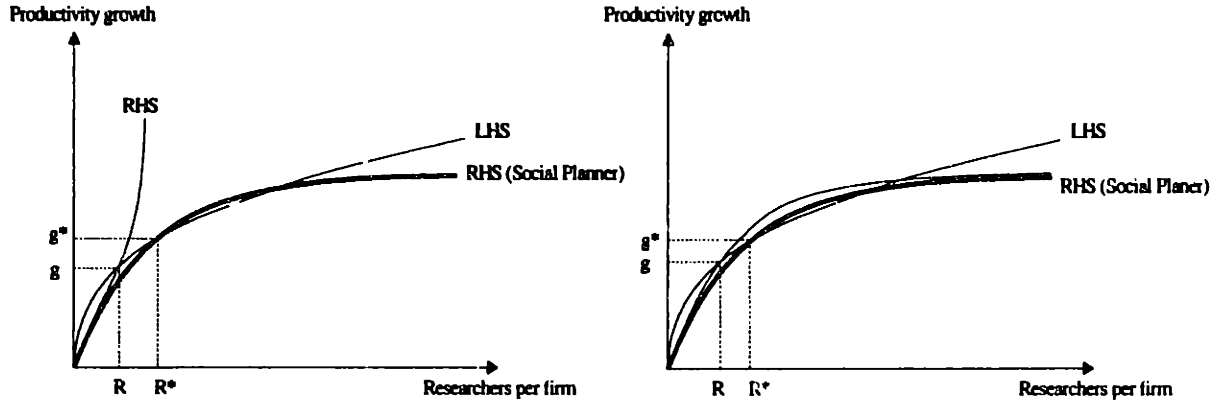

Figure 3-10: Comparison between the social planner's problem and the decentralized system.

m^* and a positive fraction of the labor force engages in research which is confirmed by equation (3.19).

Comparing equation (3.19) with $m^* = (1-\alpha)\mathcal{L}/F$, one notices a slight difference. The number of researchers per sector is now treated as if they were additional fixed cost of production. Once the social planner chooses his productivity growth rate and the number of researchers per sector, he then chooses the number of varieties such that current consumption is maximized given the generalized fixed cost.

The symmetric balanced growth path

On the symmetric balanced growth path, the number of researchers per sector is a constant. Consequently, according to equation (3.19), the number of varieties is a constant as well. Equation (3.20) implies that on the symmetric balanced growth path, $\dot{\lambda}/\lambda = -\dot{a}/a$. Eliminating λ and m from equation (3.21), I get

$$\beta^\sigma h R_i^\theta = \frac{\rho R_i}{\theta\varepsilon(R_i + F)} \tag{3.22}$$

Comparing this equation with equation (3.18) in Appendix A, one finds that the social planner sets σ in the denominator of the RHS of that equation equal to zero. Knowledge spillover has been internalized in the social planner's problem.

The RHS of equation (3.22) passes through the origin with slope $\rho/\theta\varepsilon F$. Its slope decreases to zero and its value approaches $\rho/\theta\varepsilon$ as the number of researchers per sector increases. When the LHS and the RHS intersect, they intersect twice.[24] As in Appendix A, the lower intersection provides the solution for the symmetric balanced growth path.

Since the RHS of is always smaller than that of equation (3.18), then the social planner always allocates more researchers per sector than the decentralized system as illustrated in Figure 3-10. The productivity growth rate is higher under the social planner, which is not surprising since the social return to research is higher than the private return.

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[24] The symmetric balanced growth path exists when the discount rate is sufficiently large, $\rho > \beta^{\sigma} h\varepsilon(1/\theta - F)^{\theta-1}$, and the diminishing return to researchers is strong enough, $1/\theta > F$. In addition, $\theta < 1$.

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