Massachusetts Institute of Technology
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ESD Working Paper Series

Optimal Selection of Sample Weeks for Approximating the Net Load in Generation Planning Problems

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ESD-WP-2013-03 January 2013 esd.mit.edu/wps

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***Abstract*— The increasing presence of variable energy resources (VER) in power systems –most notably wind and solar power– demands tools capable of evaluating the flexibility needs to compensate for the resulting variability in the system. Capacity expansion models are needed that embed unit commitment decisions and constraints to account for the interaction between hourly variability and realistic operating constraints. However, the dimensionality of this problem grows proportionally with the time horizon of the load profile used to characterize the system, requiring massive amounts of computing resources. One possible solution to overcome this computational problem is to select a small number of representative weeks, but there is no consistent criterion to select these weeks, or to assess the validity of the approximation. This paper proposes a methodology to optimally select a given number of representative weeks that jointly characterize demand and VER output for capacity planning models aimed at evaluating flexibility needs. It also presents different measures to assess the error between the approximation and the complete time series. Finally, it demonstrates that the proposed methodology yields a valid approximation for unit commitment constraints embedded in long-term planning models.**

***Index Terms*—Flexibility, Generation capacity expansion, Variable energy resources, Renewables integration, Power system modeling, Net load.**

NOMENCLATURE

CF_j^{SOLAR}	Solar PV power capacity factor level at hour *j*
CF_j^{WIND}	Wind power capacity factor level at hour *j*
CFE^X	Capacity factor error of technology *X*
D_j	Demand level at hour *j*
ECE^X	Energy contribution error of technology *X*
$j \in J$	Index for the set of hours
LDC	Load duration curve
$\widetilde{LDC}$	Load duration curve approximation
$NLDC$	Net load duration curve
$\widetilde{NLDC}$	Net load duration curve approximation
$RMSE^\nu$	Root-mean-square error
$NRMSE^\nu$	Normalized root-mean-square error
$\overline{S}$	Solar photovoltaic capacity installed
SUE^X	Start-up error of technology *X*
$\overline{W}$	Wind power capacity installed
κ	Net-load-based selection including peak hour week
λ	Load-based selection
ν	Net-load-based selection
ρ	Season-based selection
τ	Net-load-based selection including the peak hour day

I. INTRODUCTION

THE ongoing large-scale introduction of variable energy resources (VER) in electric power systems has substantially increased the variability and the uncertainty of the net load, the difference between the actual load and the variable energy output, also known as residual demand [1]. Under this new paradigm, resources that can respond quickly and balance the variability of the net load throughout a prolonged period of time need to be deployed in the system [2] [3] [4]. These resources provide what is generally referred to as *operational flexibility* or, simply, *flexibility* (although there is no common definition for it) [1] [5] [6]. Quantifying the operational flexibility needs in a power system requires using long-term investment models that account for the cost of deploying these resources, as well as including sufficient operational detail to account for the cycling costs and operational effects resulting from the additional variability introduced [7] [8] [9].

Capacity expansion problems are typically used to determine the optimal generation capacity mix and the level of reserves to supply electricity reliably to a given demand represented by its load duration curve (LDC). However, the additional fuel and O&M costs associated with a more intense cycling regime, induced by a greater variability in the system, is not captured in the classic version of the problem [10].

This work was supported by the National Science Foundation Grant No. 835414.

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Conversely, unit commitment (UC) problems represent the hour-by-hour dynamics of the system solving for commitment states, start-up and shutting-down of plants, but not for new investment decisions [11]. In the past, some authors have introduced methods that cope with some of the uncertainties present in the UC problem. For instance, the uncertain availability of plants has been considered through probabilistic reserve determination [12], and uncertainty in demand [13] and stochastic hydro-inflows in hydro-thermal systems [14] through stochastic programming. More recently, stochastic programming approaches [3] [15] [16] and stronger formulations of the problem [17] have been proposed to more effectively and efficiently model the uncertainty posed by VER, given an existing capacity mix.

Models that assess the need for flexibility for long-term planning in a system with VER need to simultaneously include capital investment decisions, operational decisions, and the uncertainty and variability from VER. Recently, formulations based on including capacity decisions within existing simplified UC problems have gained momentum, as a means to assess operational flexibility needs [7] [8] [9]. These formulations minimize the total system cost (the sum of the fixed and variable costs) over one year, by taking into account the annualized capital cost of the units and the total variable cost of a one-year unit commitment problem, including the main operational constraints (see Appendix).

These models are computationally very difficult to solve as each potential unit in the system has on the order of thousands of binary commitment variables associated. For instance, to make commitment and generation decisions from a pool of 300 generating units, one would need to solve a problem with over five million binary variables (counting each unit's commitment and start-up decisions over the 8,760 hours of a year). Yet, the high dimensionality of these models cannot be solved with a straightforward application of stochastic optimization techniques, as binary commitment variables are part of the subproblem in the classic Benders' decomposition formulation, which impedes closing the duality gap [18].

Morales et al. [3] point out that improving the computational tractability of models that assess the need for reserves in systems with VER might require an appropriate bundling of hourly wind-related values and a reduced but representative set of VER generation scenarios. Accordingly, simplified versions of such models have been proposed, all of which select a number of representative weeks to construct an approximate load profile of the system's yearly demand [7] [8] [9].

Kirschen et al. [9] based their selection of representative weeks on seasonality and well-known demand patterns. Conversely, Papavasiliou et al. [16] proposed a method that optimally determines the weights of a pre-selected number of representative wind power scenarios. However, in a system with a large penetration of VER, a sound method that selects a small number of representative weeks has to simultaneously provide a good account of the load duration curve while including potential correlations between the load and VER output.

This paper proposes a method that takes into account this correlation by optimally selecting a number of weeks from a one-year-horizon hourly load series, given the hourly availability of VERs over the same timeframe. First, we describe how the complexity of the problem posed by selecting weeks from a one-year data series increases with the number of weeks selected. We propose metrics for this application and we argue that a small number of weeks are sufficient to represent the net load duration curve, with negligible error. Second, we explore the trade-off between the added complexity of including the peak net load day in the problem and achieving a more accurate representation of the hours with non-served energy (NSE), which is critical for determining the economic feasibility of peaking units. Third, we demonstrate the improvement in accuracy from accounting for the correlation between demand and VERs, using a selection based on the net load duration curve, as opposed to a selection based on seasons or based on the system's load duration curve. Lastly, we validate the applicability of our methodology to power systems with a high penetration of VER by comparing the error values obtained in two different regions.

II. APPROXIMATING THE NET LOAD DURATION CURVE

A. Methodology

The methodology presented in this paper is based on selecting a fixed number of sample weeks from a one-year-horizon net load series. We focus on the net load because it represents the combined variability of demand and VERs that must be balanced with dispatchable generation. By jointly considering demand and VER generation, we account for the correlation between them.

The goal of this method is to develop useful approximations to be used within planning tools for systems with a very high penetration of VERs. To this end, we need to assume the hourly VER generation for a certain future penetration level as an input of the model. It is well-known that there is a smoothing effect on the variability of VER output with a larger geographical dispersion of the resources. However, for sufficiently large levels of dispersion and penetration, this smoothing effect reaches a saturation point above which there is minimal further reduction in variability [19]. In this paper, we assume that for the systems studied such a saturation point has been reached and that it is safe to upscale historical data to project hourly VER output for higher levels of penetration.

Therefore, for a certain penetration level of wind ($\overline{W}$) and solar PV ($\overline{S}$), the hourly variable output is calculated as the product of each variable technology's historic capacity factor (CF) and the total capacity deployed of that technology. Thus, the net load duration curve ($NLDC$) can be obtained by sorting the difference between the demand and the VER output in decreasing order:

$$\begin{aligned} NLDC_j &= D_j - CF_j^{WIND}\,\overline{W} - CF_j^{SOLAR}\,\overline{S} \\ \text{s.t.}: NLDC_{j-1} &- NLDC_j \geq 0 \quad \forall j \in J \end{aligned} \tag{1}$$

Similarly, the *approximate net load duration curve* ($\widetilde{NLDC}$) can be obtained in three consecutive steps: 1) sample a given number of weeks from a full year of demand and VER generation; 2) scale up the hours contained in the sample to one year; and 3) sort the series in decreasing order to form the approximate net load duration curve. The resulting $\widetilde{NLDC}$ is a discretized approximation of the *NLDC*.

To choose from among all possible approximations of a given number of weeks, we use least square error minimization between the *NLDC* and its approximation. The optimal approximation is given by the solution of the following optimization problem:

$$v^* \in \arg\min_{v} \left\| NLDC - \widetilde{NLDC}^{v} \right\|^2 \tag{2}$$

where $v \in \mathbb{Z}^n$ is the set of indices of the n weeks selected; $v^* \in \mathbb{Z}^n$ is the set of indices of the optimal week combination; and $NLDC, \widetilde{NLDC}^{v} \in \mathbb{R}^{8,736}$.

B. *Scaling the Sample*

We can assess the quality of the approximation by comparing each of the individual hours in the *NLDC* with its corresponding hour in the $\widetilde{NLDC}$. In order to establish this comparison, we need to have the same number of hours in the approximation as in the *NLDC*. To this end, each hour contained in the weeks sampled has to be expanded by an integer weight to complete the total number of hours in a year. For simplicity, although one year is composed of 52.14 weeks, we will assume that a year is formed by 52 weeks to preserve the integrality of the total number of weeks selected. For example, if a sample of four weeks is selected, the weight by which we have to scale each hour within the group of weeks selected to generate a one-year series is 52/4 = 13 (Fig. 1).

Notice that to apply equal weight to all weeks selected, the possible number of weeks is limited to the divisors of 52 (i.e., 1, 2, 4, 13, 26 and 52).

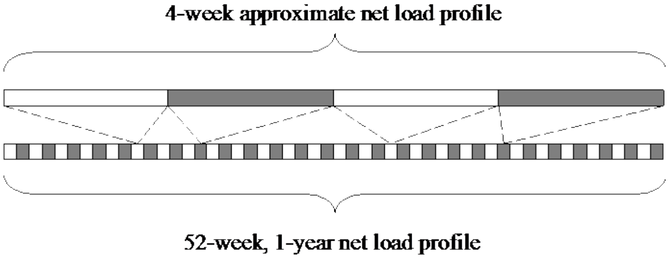
4-week approximate net load profile
52-week, 1-year net load profile

Fig. 1. Four-week approximate net load profile constructed with 4 weeks selected from the total of 52 weeks in one year. Each hour from the selected weeks is weighted by a factor of 13.

C. *Errory Metrics*

As mentioned earlier, the objective of the approximation is two-fold: first, to accurately approximate the yearly net energy demanded by the system; and second, to account for the operational detail of thermal units in the system. Thus, we need metrics that assess the approximation's performance from these two perspectives. The statistical metric for the error incurred with respect to the net energy is the root-mean-square error (*RMSE*) between the *NLDC* and its approximation:

$$RMSE^{v} = \sqrt{\frac{\sum_{j=1}^{8,736} \left(NLDC_j - \widetilde{NLDC}_j^{v} \right)^2}{8{,}736}} \tag{3}$$

This metric can be normalized relative to the range of net load values in the series with the normalized-root-mean-square error (*NRMSE*):

$$NRMSE^{v} = \frac{RMSE^{v}}{NLDC_{max} - NLDC_{min}} \cdot 100 \ \ [\%] \tag{4}$$

Although the sample of weeks is chosen to best represent the net load duration curve, we are primarily interested in how well the sample approximates the unit commitment decisions from a full year. To demonstrate the value of the approximation, we compare the results for energy contribution, capacity factor, and number of cycles performed per year from the approximation (scaled up to a year) with the results from running the UC for the full year. These three values give an indication of how thermal units are operated in the system. The difference in results between the two models is expressed in absolute values with their sign. We define error metrics for technology *X* as:

$$ECE^{X} = \frac{1}{|X|} \left(\sum_{i=1, i\in X}^{I} \widetilde{EC}_i - \sum_{i=1, i\in X}^{I} EC_i \right) \quad [p.u.] \tag{5}$$

$$CFE^{X} = \frac{1}{|X|} \left(\sum_{i=1, i\in X}^{I} \widetilde{CF}_i - \sum_{i=1, i\in X}^{I} CF_i \right) \quad [p.u.] \tag{6}$$

$$SUE^{X} = \frac{1}{|X|} \left(\sum_{i=1, i\in X}^{I} \widetilde{SU}_i - \sum_{i=1, i\in X}^{I} SU_i \right) \quad [\frac{startups}{year}] \tag{7}$$

where $X \subset I$ is the subset of indices corresponding to generating units of technology 'X'; and $|X|$ is the cardinality of the subset.

D. *How Many Weeks to Select?*

The selection algorithm consists of an exhaustive search throughout all the possible combinations of weeks to determine which combination yields the minimum error. This enumeration process is implemented by a number of nested loops equal to the number of weeks to be selected. As a result, although the computation time required to choose the optimal sample set grows proportionally with the number of possible combinations, the number of possible combinations grows as the factorial of the number of sample weeks (Table I).

TABLE I
NUMBER OF POSSIBLE COMPOSITE COMBINATIONS AND COMPUTING TIME AS A FUNCTION OF THE NUMBER OF WEEKS SELECTED

Number of weeks selected	Number of possible combinations	Computing time
1	$\binom{52}{1} = 52$	0.05 secs
2	$\binom{52}{2} = 1{,}326$	1.5 secs
4	$\binom{52}{4} = 270{,}725$	10 mins
5	$\binom{52}{5} = 2{,}598{,}960$	1h 40 mins
8	$\binom{52}{8} = 752{,}538{,}150$	~19 days
13	$\binom{52}{13} = 6.3501E + 11$	~46 years

Computing times from running a MATLAB algorithm having as many nested loops as weeks selected, on a commercial 2.4 GHz Intel Core 2 Duo machine with 4GB of memory

Incrementing the sample size by one week requires an additional nested loop in the sampling algorithm to search through all possible week combinations. On a standard desktop, the optimization procedure can be reasonably performed for sample sizes of up to 5 weeks (5 nested loops), but beyond that it becomes prohibitive.

E. Selecting a Large Number of Weeks

As an alternative to optimal selection for larger sample sizes, there are heuristic approximations that allow increasing the sample size without increasing the number of nested loops. For instance, we could sample subsets of some certain number of adjoining weeks instead of individual weeks (e.g., two-week intervals). With this heuristic we can easily expand the number of weeks selected, but with the limitation that optimality cannot be guaranteed anymore. Once these subsets have been selected, we can expand the sample to create a 52-week approximation by applying different weights to each subset of weeks. For instance, a 16-week sample might consist of four subsets of multiple weeks and scaled by different weights to approximate a full year (Fig. 2.).

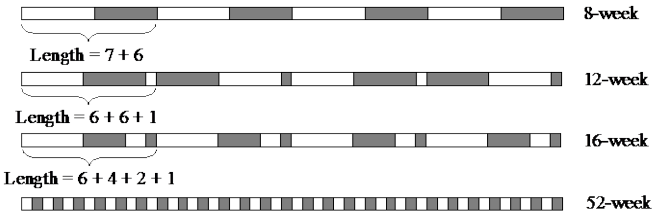

Fig. 2. Four-week approximate net load profile constructed with 4 weeks selected from the total of 52 weeks in one year. Each hour from the weeks picked is weighed by a factor of 13.

This approach allows us to increase the number of weeks sampled while avoiding a dramatic increase in computation time.

F. Number of Weeks and Net Energy Error

We test the proposed methodology using hourly demand and VER generation –solar and wind power– for the Electricity Reliability Council of Texas (ERCOT) [20] from 2009. In order to simulate a high penetration level of VERs, we upscale wind and solar outputs assuming an expected capacity of 30GW of wind power and 10 GW of solar PV [21].

We first describe how the errors in approximating the net load decrease with the number of weeks sampled. Fig. 3. shows graphically the results of selecting one, two and four weeks to characterize the NLDC.

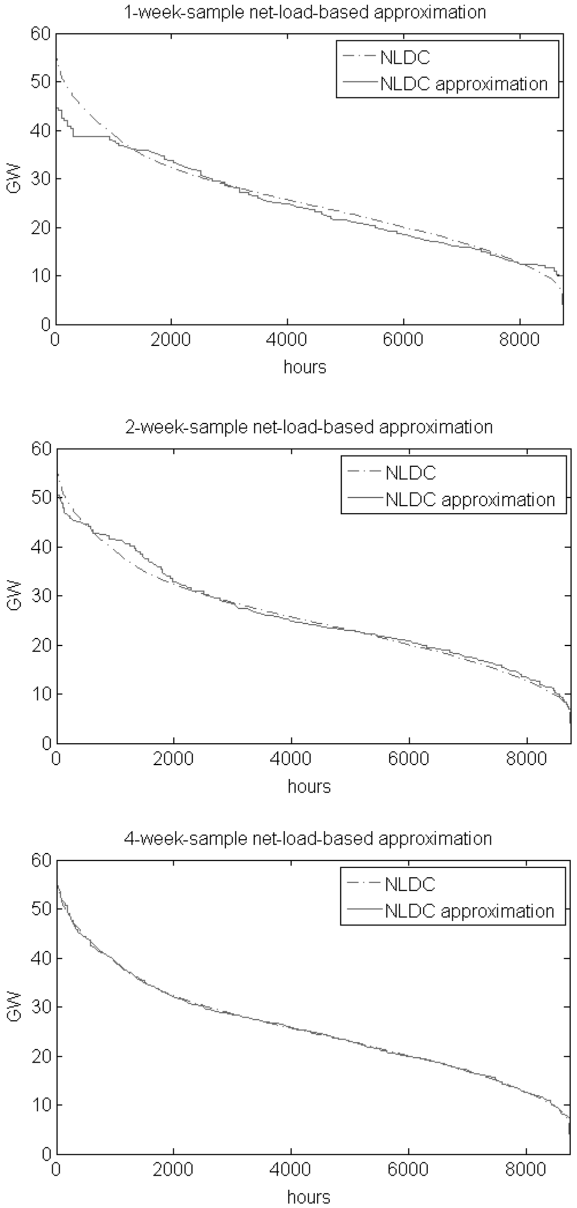

Fig. 3. Graphical representation of the system's LDC, NLDC and their respective approximations, built with one, two and four weeks, selected to fit the NLDC.

While the approximations based on one and two weeks produce an inaccurate representation of the *NLDC* with *RMSE* over 2%, the approximation using four weeks closely matches the shape of the original *NLDC*. For sample sizes larger than four weeks, errors are further reduced, although not monotonically (Table II).

TABLE II
NET ENERGY ERROR RELATIVE TO THE NUMBER OF WEEKS SELECTED

Number of weeks	Approximation v^*	$RMSE^v$	$NRMSE^v$ [%]	time [s]
1	{4}	2.3	4.2	0.066
2	{9,24}	1.233	2.3	1.52
4	{17,31,37,46}	0.299	0.5	688
8	{3,4,15,16,20,21, 31,32}	0.283	0.52	1,042
12	{12,13,14,34,35, 36,38,39,40,50,51, 52}	0.68	1.24	1,368
16	{3,4,5,6,14,15,16, 17,31,32,33,34,38, 39,40,41}	0.24	0.44	1,505

Summary of the error incurred by approximations constructed with an increasing number of weeks. The weeks are selected based on minimizing the error between the approximation and the NLDC. The column "Approximation" shows the weeks selected; "RMSE^v" is the root-mean-square error between the NLDC and its approximation; "NRMSE^v" is the normalized root-mean-square error between the NLDC and its approximation.

G. *Commitment Results and Errors*

Despite the small errors of the four-week approximation representing net load, its commitment performance must be tested. In order to do so, we run a capacity expansion model with UC constraints using the one-week, two-week, four-week and eight-week approximation, and compare the commitment results from this model with those from a UC model that uses the full one-year data series. The capacity expansion model with embedded unit commitment used in this analysis is fully described in the Appendix. This approach allows for determining capacity needs informed by the operational detail required to represent accurately the variability introduced by renewables.

The model was run for the ERCOT system, allowing power curtailment of both wind and solar PV, and using standard thermal units whose cost parameters and technical characteristics can also be found in the Appendix. We used CPLEX 12.4 under GAMS on a 64-bit dual-socket hexcore (i.e. 12 physical cores, 12 virtual cores total) Intel Nehalem (x5650) machine. The maximum duality gap tolerance was set to 0.01, producing a solution in approximately 4hrs. Stricter duality gap tolerances did not affect the main solution parameters but had a significant impact on the computation time. The computing time with commercial machines (2.4 GHz Intel Core 2 Duo machine with 4GB of memory) was in the order of 1.5 times greater than those experienced with the original 12-core machine. Capacity and commitment results obtained with the approximation are summarized in section A) of Tables III-VI.

To assess the error, we run a full one-year UC model for the generation units selected in the solution of the capacity expansion model with approximate UC. This commitment model uses the same parameters and applies the same constraints as those in the capacity expansion model, but uses hourly demand and VER data for the full year. A critical assumption is that the full year UC is performed for the capacity mix chosen by the capacity expansion model with approximate UC. Therefore, the corresponding full-year simulation for each approximation will have different results, since the capacity mixes vary. Results from the full-year run and the differences from the approximation results are presented in section B) of Tables III-IV.

TABLE III
A) ONE-YEAR UNIT COMMITMENT RESULTS BASED ON THE ONE-WEEK NET-LOAD-BASED APPROXIMATION

Technology	units installed	Capacity Installed [GW]	EC [p.u.]	CF [p.u.]	Startups/ year
Nuclear	8	8	0.245	1	0
Solar	10	10	0.043	0.141	–
Wind	30	30	0.190	0.206	–
Coal	13	6.5	0.188	0.943	0
CCGT	72	28.8	0.331	0.375	102
OCGT	18	5.4	0.003	0.017	145

B) ONE-YEAR UNIT COMMITMENT RESULTS BASED ON A FULL ONE-YEAR DATA SERIES AND COMMITMENT ERROR

Technology	EC [p.u.]	ECE [p.u.]	CF [p.u.]	CFE [p.u.]	Startups/ year	SUE [startups/year]
Nuclear	0.228	+ 0.017	1	+ 0.000	0	+ 0
Solar	0.056	- 0.013	0.198	- 0.054	–	–
Wind	0.199	- 0.009	0.233	- 0.027	–	–
Coal	0.176	+ 0.012	0.951	- 0.008	0	+ 0
CCGT	0.325	+ 0.006	0.395	- 0.020	64	+ 38
OCGT	0.007	- 0.004	0.048	- 0.031	148	- 3

Results summary from: A) a capacity expansion model with embedded unit commitment constraints based on a one-week sample; and B) a one-year full unit commitment model of the units installed, and commitment errors between the full run and the approximation. NSE = 7.75E-3 p.u.

TABLE IV
A) ONE-YEAR UNIT COMMITMENT RESULTS BASED ON THE TWO-WEEK NET-LOAD-BASED APPROXIMATION

Technology	units installed	Capacity Installed [GW]	EC [p.u.]	CF [p.u.]	Startups/ year
Nuclear	9	9	0.243	1	0
Solar	10	10	0.058	0.215	–
Wind	30	30	0.226	0.279	–
Coal	12	6	0.154	0.950	0
CCGT	87	34.8	0.317	0.338	74
OCGT	18	5.4	0.002	0.014	97

B) ONE-YEAR UNIT COMMITMENT RESULTS BASED ON A FULL ONE-YEAR DATA SERIES AND COMMITMENT ERROR

Technology	EC [p.u.]	ECE [p.u.]	CF [p.u.]	CFE [p.u.]	Startups/ year	SUE [startups/year]
Nuclear	0.257	- 0.014	1	+ 0.000	0	+ 0
Solar	0.057	+ 0.001	0.198	+ 0.017	–	–
Wind	0.196	+ 0.030	0.229	+ 0.050	–	–
Coal	0.162	- 0.008	0.950	+ 0.000	0	+ 0
CCGT	0.325	- 0.008	0.327	+ 0.011	57	+ 17
OCGT	0.003	- 0.001	0.019	- 0.005	97	+ 0

Results summary from: A) a capacity expansion model with embedded unit commitment constraints based on a two-week sample; and B) a one-year full unit commitment model of the units installed, and commitment errors between the full run and the approximation. NSE = 1.01E-3 p.u.

TABLE V
A) ONE-YEAR UNIT COMMITMENT RESULTS BASED ON THE FOUR-WEEK NET-LOAD-BASED APPROXIMATION

Technology	units installed	Capacity Installed [GW]	EC [p.u.]	CF [p.u.]	Startups/ year
Nuclear	8	8	0.226	1	0
Solar	10	10	0.063	0.223	–
Wind	30	30	0.198	0.233	–
Coal	15	7.5	0.199	0.939	0
CCGT	87	34.8	0.31	0.315	54
OCGT	30	9	0.003	0.012	61

B) ONE-YEAR UNIT COMMITMENT RESULTS BASED ON A FULL ONE-YEAR DATA SERIES AND COMMITMENT ERROR

Technology	EC [p.u.]	ECE [p.u.]	CF [p.u.]	CFE [p.u.]	Startups/ year	SUE [startups/year]
Nuclear	0.228	- 0.002	1	+ 0.000	0	+ 0
Solar	0.057	+ 0.007	0.199	+ 0.024	–	–
Wind	0.199	- 0.001	0.233	+ 0.000	–	–
Coal	0.199	+ 0.000	0.933	+ 0.006	0	+ 0
CCGT	0.313	- 0.003	0.316	- 0.001	59	- 5
OCGT	0.003	+ 0.000	0.012	+ 0.000	65	- 4

Results summary from: A) a capacity expansion model with embedded unit commitment constraints based on a four-week sample; and B) a one-year full unit commitment model of the units installed, and commitment errors between the full run and the approximation. NSE = 1.17E-4 p.u.

TABLE VI
A) ONE-YEAR UNIT COMMITMENT RESULTS BASED ON THE EIGHT-WEEK NET-LOAD-BASED APPROXIMATION

Technology	units installed	Capacity Installed [GW]	EC [p.u.]	CF [p.u.]	Startups/ year
Nuclear	10	10	0.284	1	0
Solar	10	10	0.063	0.220	–
Wind	30	30	0.186	0.219	–
Coal	11	5.5	0.148	0.950	0
CCGT	89	35.6	0.316	0.312	66
OCGT	26	7.8	0.003	0.012	71

B) ONE-YEAR UNIT COMMITMENT RESULTS BASED ON A FULL ONE-YEAR DATA SERIES AND COMMITMENT ERROR

Technology	EC [p.u.]	ECE [p.u.]	CF [p.u.]	CFE [p.u.]	Startups/ year	SUE [startups/year]
Nuclear	0.285	- 0.001	1	+ 0.000	0	+ 0
Solar	0.056	+ 0.007	0.197	+ 0.023	–	–
Wind	0.192	- 0.006	0.225	- 0.006	–	–
Coal	0.149	- 0.001	0.950	+ 0.000	0	+ 0
CCGT	0.315	+ 0.001	0.310	+ 0.002	57	+ 9
OCGT	0.002	+ 0.001	0.011	+ 0.001	67	+ 4

Results summary from: A) a capacity expansion model with embedded unit commitment constraints based on an eight-week sample; and B) a one-year full unit commitment model of the units installed, and commitment errors between the full run and the approximation. NSE = 1.29E-4 p.u.

Comparing the errors from the different sample sizes, it can be observed that the errors corresponding to the one-week and two-week sample runs are greater than those found in the four-week and eight-week sample runs. Both energy contribution errors and capacity factor errors are at least twice as large in the first two cases as in the last two. Also, start-up errors in the first two cases are close to or well over 20 cycles per year, indicating that they represent poorly the cycling behavior of the thermal units, a main concern of flexibility studies. On the other hand, the average start-ups per year in the four-week and eight-week sample are very similar to those found in the full-year UC simulation.

Additionally, the values of NSE in the four-week and eight-week cases are one order of magnitude below those produced by the one-week and the two-week sample. This difference shows how the last two samples produce a more adequate capacity mix than the first two as net load is better represented. Errors are not appreciably reduced by using a sample size of eight weeks.

Based on these results, an approximation of the net load from four-weeks of data using this approach appears to represent well the full one-year data series. The optimal four-week approximation of net load will be used for the remainder of this paper.

III. REPRESENTING THE PEAK NET LOAD

One of the critical parameters for determining adequacy in capacity expansion and flexibility assessment studies is the value-of-lost-load (VOLL) [22]. An adequate system typically has less than one day with non-served energy (NSE) every 10 years (or less than 2.4 hours every year), although it varies from one system to another. In the model used, the VOLL is assumed to be fixed, and the amount of NSE is derived endogenously from the cost minimization model (see Appendix). Nevertheless, the amount of NSE does not depend exclusively on the VOLL. It is also determined by the system's peak load or, in situations with a high penetration of VER, the peak net load. Hence, it is important to examine how the inclusion of the peak net load in the sample affects the final solution of the problem.

Comparing the four-week approximation ν^* with the *NLDC* (Fig. 4.), the approximation underestimates the peak hour by 4.5%. If embedded within a capacity expansion model, this underestimate would lead to insufficient investment in generation capacity.

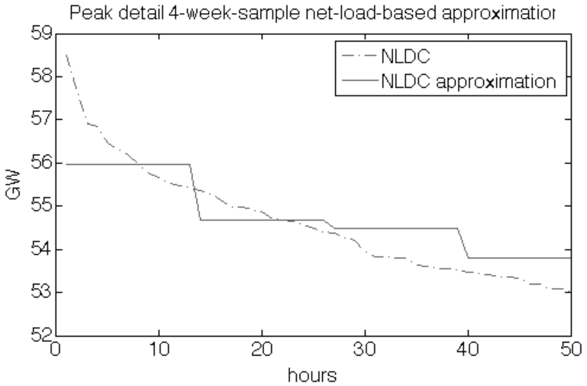

Fig. 4. Detail of the system's NLDC peak and its respective four-week-based approximation. The peak value in both series is 58.5GW and 56 GW respectively. Note that since the number of weeks selected is four, the step-size of the $\widetilde{NLDC}$ is 13.

One possible solution to this issue is to require the inclusion of the week containing the peak net load as one of the weeks in the sample. We denote this approximation by κ^*. The κ^*approximation will be constructed with the week containing the peak net load plus an optimal selection of three weeks, conditional on having the peak net load week in the approximation.

Alternatively, we can revisit the initial assumption that the year is composed of 52 weeks (in reality it is composed of 52 weeks and one day), and add the peak net load day to one four-week approximation, to make a total of 8,760 hours in one year. This approximation will be denoted by τ^*. Fig. 5. shows the detail of the peak net load as achieved by these two alternative approximations that contain the peak net load day.

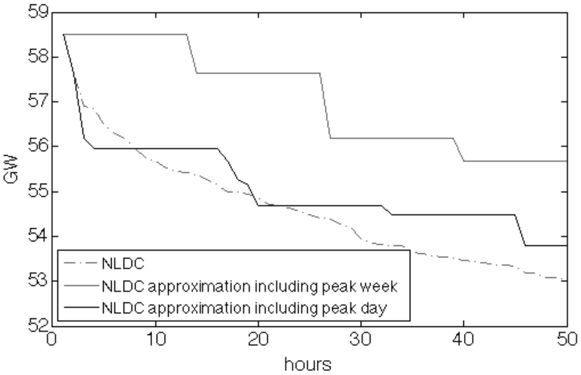

Fig. 5. Detail of the system's NLDC peak and its κ^* and τ^*four-week-based approximations. The peak value in both series is 58.5GW and 56 GW respectively. Note that since the number of weeks selected is four, the step-size of the $\widetilde{NLDC}$ is 13.

The accuracy of each approximation can be assessed by the *NRMSE* metric and by the results obtained from running the capacity expansion model (see Appendix) using the three different approximations:

TABLE VII
STANDARD ERROR RELATIVE TO THE NUMBER OF WEEKS SELECTED

Approximation	No Peak (ν^*)	Peak Week (κ^*)	Peak Day (τ^*)
Weeks selected	{17,31,37, 46}	{29,41, 46,48}	{17,21,31,45} + peak net load day (day #197)
NRMSE [%]	0.546	0.821	0.537
Total Capacity Installed [GW]	99.3	102.3	99.4
NSE [p.u.]	3.79E-4	9.74E-5	1.24E-4
System cost [%]	–	-0.54	-1.86

Summary of the error sum of squares incurred by three approximations: 1) neglecting the peak net load; 2) including the week containing the peak net load hour (52 weeks); and 3) including the day containing the peak net load hour (52 weeks + 1 day). The peak net load hour is in day #197, in week #29

The results show that approximations τ^* and κ^*, with greater peak capacities and lower NSE, represent the peak net load more precisely than the ν^* approximation. However, the NRMSE is greater for the approximation κ^* as it is forcing a suboptimal week in the sample. This error is reduced with approximation τ^* as the optimal selection is done conditional on having introduced the peak net load day. As a result of better resolution in the peak of the *NLDC*, approximations that consider the peak net load yield a higher system capacity and lower NSE. Although the impact on the overall approximation of the *NLDC* is small, including the peak net load day in the approximation increases the accuracy in the peak hours.

IV. COMPARISON TO OTHER APPROACHES

Here we demonstrate the advantages of our approach by comparing our results to those from using other sample week selection methods found in the literature.

A. *Results from Using a Season-based Selection*

The most common approach to selecting a sample of weeks to approximate a year is to identify seasonal demand patterns, and sampling one week from each of the seasons [9]. For example, the maximum, average and minimum demand from each week can be plotted (Fig. 6.) to identify the four seasons. One week from each season is then chosen randomly to make up a sample of four weeks.

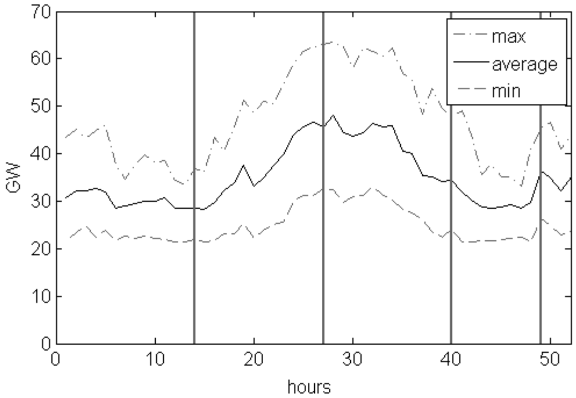

Fig. 6. Graphical representation of the maximum, average and minimum weekly demand for the ERCOT system throughout 2008.

Here we demonstrate on an example approximation. We sample one week from each season randomly, and scale up the data to a full year to construct a season-based approximation ρ^*. For this example, the sample $\rho^* = \{14,27,40,49\}$ has a *NRMSE* between the *NLDC* and its approximation of 2.78, as compared with 0.5 from our four-week approximation.

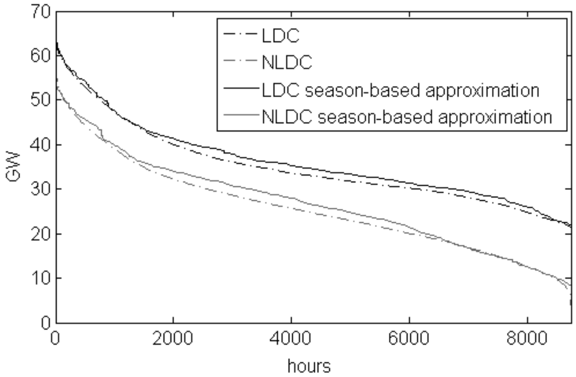

Fig. 7. Graphical representation of the system's LDC, NLDC and their respective four-week approximations selected through seasonality criteria.

Table VIII shows the capacity and commitment results using the approximation, and commitment results with their errors.

TABLE VIII

A) ONE-YEAR UNIT COMMITMENT RESULTS BASED ON THE FOUR-WEEK SEASON-BASED APPROXIMATION

Technology	units installed	Capacity Installed [GW]	EC [p.u.]	CF [p.u.]	Startups/ year
Nuclear	11	11	0.294	1	0
Solar	10	10	0.046	0.171	–
Wind	30	30	0.209	0.260	–
Coal	14	7	0.173	0.924	0
CCGT	88	35.2	0.276	0.293	76
OCGT	29	8.7	0.003	0.014	80

B) ONE-YEAR UNIT COMMITMENT RESULTS BASED ON A FULL ONE-YEAR DATA SERIES AND COMMITMENT ERROR

Technology	EC [p.u.]	ECE [p.u.]	CF [p.u.]	CFE [p.u.]	Startups/ year	SUE [startups/year]
Nuclear	0.314	- 0.020	1	+ 0.000	0	+ 0
Solar	0.056	- 0.010	0.196	- 0.025	–	–
Wind	0.189	+ 0.020	0.221	+ 0.039	–	–
Coal	0.180	- 0.007	0.900	+ 0.024	0	+ 0
CCGT	0.260	+ 0.017	0.259	+ 0.034	68	+ 8
OCGT	0.002	+ 0.001	0.006	+ 0.008	53	+ 27

Results summary from: A) a capacity expansion model with embedded unit commitment constraints based on a four-week sample; and B) a one-year full unit commitment model of the units installed, and commitment errors between the full run and the approximation. NSE = 3.65E-4 p.u.

B. Results of Ignoring the Correlation between Load and VERs

The season-based approach above has two sources of error. First, it randomly chooses a week from each season rather than minimizing error. Second, this approach usually focuses on load rather than net load, which ignores the correlation between demand and VER.

To differentiate between these two sources of error, we demonstrate the errors from using an optimal selection procedure, but aimed at approximating the *LDC*, which ignores the correlation between the load and VERs. Analogous to the objective in Section II, the objective now is to find the combination that yields the least square error between the *LDC* and its approximation, $\widetilde{LDC}$:

$$\lambda^* \in \arg\min_{\lambda} \left\| LDC - \widetilde{LDC}^{\lambda} \right\|^2 \qquad (8)$$

where $\lambda \in \mathbb{Z}^n$ is the set of indices of the *n* weeks selected; $\lambda^* \in \mathbb{Z}^n$ is the set of indices of the optimal week combination; and $LDC, \widetilde{LDC}^{\lambda} \in \mathbb{R}^{8,736}$.

We repeat the enumeration process required for optimal selection, by constructing all possible approximations for a certain number of weeks, selecting now the combination that best represents the *LDC*. Accordingly, the $\widetilde{NLDC}^{\lambda}$ is built with the net load associated with the weeks selected through this process. Fig 8. shows the approximations to both the *LDC* and the *NLDC* obtained for the 4-week case.

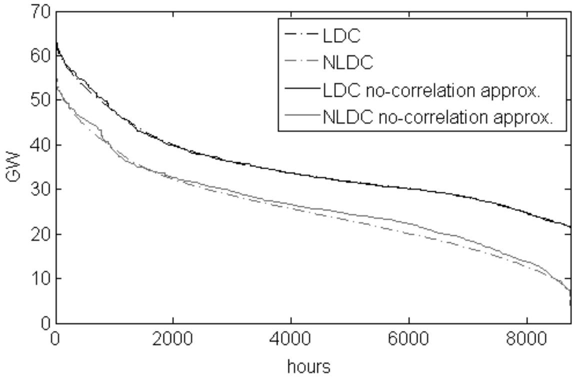

Fig. 8. Graphical representation of the system's LDC, NLDC and their respective four-week approximations, selected to fit the LDC.

For all the cases explored (one, two, four, eight, twelve and sixteen week samples), the resulting error metrics are summarized in Table IX.

TABLE IX

STANDARD ERROR RELATIVE TO THE NUMBER OF WEEKS SELECTED

Number of weeks	Approximation λ^*	$RMSE^{\lambda}$	$NRMSE^{\lambda}$ [%]	time [s]
1	{38}	3.953	7.22	0.06
2	{10,23}	2.311	4.22	1.61
4	{11,16,27,40}	1.271	2.32	694
8	{5,6,16,17,24,25,39,40}	1.242	2.27	992
12	{4,5,6,24,25,26,39,40,41,42,43,44}	0.627	1.14	1,419
16	{3,4,5,6,15,16,17,18,24,25,26,27,39,40,41,42}	0.583	1.07	1,542

Summary of the error incurred by approximations constructed with an increasing number of weeks. The weeks are selected based on minimizing the error between the approximation and the *LDC*. The column "Approximation" shows the weeks selected; "$RMSE^{\lambda}$" is the root-mean-square error between the *NLDC* and its approximation; "NRMSE$^{\lambda}$" is the normalized root-mean-square error between the *NLDC* and its approximation.

The errors corresponding to the different approximations are significantly greater when the selection is based on the *LDC* (λ^*) than when the selection is based on approximating the *NLDC* (ν^*). By fitting the *LDC*, we do not necessarily select the weeks that characterize best the *NLDC* or, in other words, a selection based on the *LDC* is suboptimal relative to how well it characterizes the *NLDC*.

More importantly, the errors in the approximate capacity, commitment, and startup results obtained with λ^* are larger when the sample is based on the *LDC* (Table X). As the level of penetration of VER increases, so does the importance of accounting for the correlation between load and VERs.

TABLE X

A) One-Year Unit Commitment Results Based on the Four-Week Load-Based Approximation

Technology	units installed	Capacity Installed [GW]	EC [p.u.]	CF [p.u.]	Startups/ year
Nuclear	10	10	0.285	1	0
Solar	10	10	0.062	0.218	–
Wind	30	30	0.162	0.190	–
Coal	14	7	0.185	0.928	0
CCGT	86	34.4	0.303	0.309	55
OCGT	21	6.3	0.002	0.012	69

B) One-Year Unit Commitment Results Based on a Full One-Year Data Series and Commitment Error

Technology	EC [p.u.]	ECE [p.u.]	CF [p.u.]	CFE [p.u.]	Startups/ year	SUE [startups/year]
Nuclear	0.285	+0.000	1	+ 0.000	0	+ 0
Solar	0.056	+ 0.006	0.197	+ 0.021	–	–
Wind	0.193	- 0.031	0.225	- 0.035	–	–
Coal	0.183	+ 0.002	0.915	+ 0.013	0	+ 0
CCGT	0.281	- 0.022	0.286	+ 0.023	65	- 10
OCGT	0.002	+ 0.000	0.012	+ 0.000	81	- 12

Results summary from: A) a capacity expansion model with embedded unit commitment constraints based on a four-week sample; and B) a one-year full unit commitment model of the units installed, and commitment errors between the full run and the approximation. NSE = 3.98E-4 p.u.

V. Validity Assessment

In this section we demonstrate the application of the method to another example system. We use load, wind CF and solar CF data corresponding to Germany in 2008 and repeat the four-week sampling method described in Section II. The sample is then used as an approximation of the net load in a capacity expansion model, and the resulting generating capacity serves as an input to a full-year UC model. Results are shown in Table XI using the same format as in previous simulations.

TABLE XI

A) One-Year Unit Commitment Results Based on the Four-Week Net-Load-Based Approximation

Technology	units installed	Capacity Installed [GW]	EC [p.u.]	CF [p.u.]	Startups/ year
Nuclear	30	30	0.530	1	0
Solar	10	10	0.019	0.106	–
Wind	30	30	0.105	0.198	–
Coal	16	8	0.130	0.924	0
CCGT	78	31.2	0.215	0.390	68
OCGT	17	5.1	0.001	0.015	97

B) One-Year Unit Commitment Results Based on a Full One-Year Data Series and Commitment Error

Technology	EC [p.u.]	ECE [p.u.]	CF [p.u.]	CFE [p.u.]	Startups/ year	SUE [startups/year]
Nuclear	0.532	- 0.002	1	+ 0.000	0	+ 0
Solar	0.017	+ 0.002	0.096	+ 0.010	–	–
Wind	0.102	+ 0.003	0.192	+ 0.006	–	–
Coal	0.131	- 0.001	0.926	- 0.002	0	+ 0
CCGT	0.216	- 0.001	0.391	- 0.001	76	- 8
OCGT	0.001	+ 0.000	0.013	+ 0.002	99	- 2

Results summary for Germany from: A) a capacity expansion model with embedded unit commitment constraints based on a four-week sample; and B) a one-year full unit commitment model of the units installed, and commitment errors between the full run and the approximation. NSE = 3.92E-5 p.u.

The magnitude of the commitment errors is the same as that in the errors obtained with the ERCOT 2009 system. This result gives additional confidence in the applicability of the approximation method to power systems with a high penetration of VER. Although not a formal proof, the results presented here taken together suggest that our approximation method will exhibit improved performance for a range of systems.

VI. Conclusion

This paper introduced a methodology to sample weeks based on historical demand and VER generation that can be used in capacity expansion models with unit commitment constraints.

We have explored how the error of the approximation depends on the number of weeks and conclude that the error incurred with a four-week approximation is small with respect to both net energy and commitment error metrics.

We argue that in order to best reflect the possible correlation between demand and VER, the selection should be based on the net load instead of attending to other aspects such as demand's seasonal patterns. We demonstrate that errors are smaller with the proposed method than with other methods found in the literature. We also apply the method to another power system to demonstrate the consistency of the results.

Our method provides an effective approach for reducing the computation time for capacity expansion with embedded unit commitment constraints, while still capturing the relevant characteristics of a system with VER. Given the increased concern about flexibility in future generation planning, this approach provides a means to evaluate the flexibility of alternative future generation mixes.

Appendix

Formulation of the Capacity Expansion Problem with Unit Commitment Constraints

We formulate a model to represent the capacity expansion problem with embedded unit commitment constraints where investment, unit commitment and energy dispatch decisions are made jointly by solving a unique problem.

A. Indices and Variables

Two indices will be used in this model: $i \in I$, where I denotes the set of generating units that can be potentially built; and, $j \in J$, where J denotes the set of hours in a year (or, alternatively, the total number of hours contained in the weeks sampled). In addition, $W \subset I$, is the subset of wind units; $S \subset I$, is the subset of solar units, where $s \in S$; $T \subset I$, is the subset of thermal power units (nuclear, coal, CCGTs and OCGTs); and $G \subset I$, is the subset of gas-fired power plants – combined cycle gas turbines and combustion turbines (CCGTs and OCGTs)–.

Building decisions are modeled using binary variables $y_i \in \{0,1\}$; commitment states are $u_{ij} \in \{0,1\}$; start-up decisions are $z_{ij} \in \{0,1\}$; shut-down decisions are $v_{ij} \in \mathbb{R}_+$; and power output decisions are represented by $x_{ij} \in \mathbb{R}_+$. An extra variable $w_{ij} \in \mathbb{R}_+$ (where $w_{ij} = x_{ij} - u_{ij}\,\underline{P_i}$), has been introduced to separate the total output of each generator between its minimum stable level and the remainder output level to facilitate the formulation of ramping rate constraints.

B. Objective Function

The objective function of our model minimizes the total cost of the system, which is the sum of fixed costs (C_i^{FIX}), variable costs (C_i^{VAR}) and start-up costs (C_i^{STUP}). It is formulated as a two-stage decision problem, in which the first stage problem is to choose the investment (capacity) in individual power plants; and the second stage solves for operation (start-up, commitment and energy dispatch) decisions:

$$\min_{u,v,x,y,z} \sum_i \left(C_i^{FIX} y_i + \sum_j \left(C_i^{VAR} x_{i,j} + C_i^{STUP} z_{i,j} \right) \right) \qquad (9)$$

C. Operational Constraints

The model is subject to the classic constraints included in a unit commitment model: demand constraints, commitment state constraints, minimum and maximum output constraints (introducing the new variable $w_{ij} = x_{ij} - u_{ij}\underline{P_i}$), ramping rates constraints and minimum up and down time constraints. These constraints are explained in further detail below.

The demand constraints guarantee that the demand level (D_j) is met by the aggregate power output of all generators that are built in the system, including both thermal plants and renewables:

$$\sum_i x_{i,j} \geq D_j \qquad \forall j \in J \qquad (10)$$

State constraints link commitment states with start-up and shut-down decisions. Note that even if v_{ij} has been defined in the positive real domain, it will only adopt binary values as the commitment states and start-up decisions are all binary.

$$u_{i,j} - u_{i,j-1} = z_{ij} - v_{ij} \qquad \forall i \in T, \forall j \in J \qquad (11)$$

Ramping limit constraints account for the physical limitations imposed by power plants' thermal and mechanical inertias. These equations are constructed using the $w_{i,j}$ auxiliary variable to avoid the constraint becoming active when off-line power plants start-up and jump from zero to the minimum output.

$$w_{i,j} - w_{i,j-1} \leq \overline{R}_i^U \qquad \forall i \in T, \forall j \in J \qquad (12)$$

$$w_{i,j-1} - w_{i,j} \leq \overline{R}_i^D \qquad \forall i \in T, \forall j \in J \qquad (13)$$

Unit minimum and maximum output constraints are defined in terms of the interval between each unit's minimum and maximum output levels:

$$w_{i,j} \leq u_{i,j}\left(\overline{P}_i - \underline{P_i}\right) \qquad \forall i \in T, \forall j \in J \qquad (14)$$

Minimum up and down times are implemented following the formulation in [23]. In this formulation $\overline{M}_i^U$ and $\overline{M}_i^D$ represent the minimum time that a power plant has to remain on or off after a start-up or shut-down respectively, and jj is an index for the hours in the time series.

$$u_{t,j} \geq \sum_{jj>j-\overline{M}_i^U}^{j} z_{t,jj} \qquad \forall i \in T, \forall j \in J \qquad (15)$$

$$1 - u_{t,j} \geq \sum_{jj>j-\overline{M}_i^D}^{j} v_{t,jj} \qquad \forall i \in T, \forall j \in J \qquad (16)$$

Lastly, our model includes a coupling constraint that links the two decision stages –building and operating – to ensure that only units that have been built generate:

$$x_{i,j} \leq \overline{P}_i y_i \qquad \forall i \in I, \forall j \in J \qquad (17)$$

D. Treatment of Renewables

Wind and solar PV energy outputs are treated in the model as functions of each technology's capacity factor (CF). Capacity factors reflect the availability of wind or solar resources for a specific hour at a certain location. Hence, for a particular hour of the year, the output of the total wind or solar power in our system will be determined by the product of each technology's total capacity installed and its respective CF.

$$x_{i,j} = \overline{P}_i \, CF_j^{WIND} \qquad \forall i \in W, \forall j \in J \qquad (18)$$

$$x_{i,j} = \overline{P}_i \, CF_j^{SOLAR} \qquad \forall i \in S, \forall j \in J \qquad (19)$$

where $\overline{P}_w$ and $\overline{P}_s$ are the maximum capacities of a unit wind farm and a unit solar farm respectively (in this model, for instance, 1GW). Note that in this model the unit size of renewable power plants –wind and solar PV– does not affect the outcomes of the model, as long as the sum of the capacity of individual wind and solar plants in the system adds up separately to the total wind and solar capacity in place for some certain scenario.

The model also offers the possibility of introducing curtailment as an extra degree of freedom to ensure that hourly demand is met by generation. For wind and solar power, this feature is implemented through substituting the equality constraints by inequality constraints:

$$x_{i,j} \leq \overline{P}_i \, CF_j^{WIND} \qquad \forall i \in W, \forall j \in J \qquad (20)$$

$$x_{i,j} \leq \overline{P}_i \, CF_j^{SOLAR} \qquad \forall i \in S, \forall j \in J \qquad (21)$$

Allowing VER curtailment also has a direct impact on the computational performance of the model. Substituting for the equality with an inequality increases the number of feasible solutions, reducing the time taken by the solver to find feasible integer solutions from which to start the optimization algorithm. Moreover, it also reflects the actual behavior of some ISOs for whom curtailment is an additional resource at their disposal to help secure the safe operation of the system.

E. Operating Reserves

Power systems need to secure a certain amount of spinning and non-spinning capacity to guarantee that generation constantly meets demand and that the frequency of the system lies within the reference values. These frequency boundaries are set to protect the system from going unstable. Reserves are typically categorized according to their response time and the duration of the event they are required to mitigate. However, these categories differ from one power system to the next. Milligan et al. [2] surveys how reserves are used in a number of power systems and proposes a general classification of reserve types. In this paper we use the same classification, but using the European naming convention (primary, secondary and tertiary reserves).

Operating reserves are modeled by requiring the system to have a certain amount of spinning and non-spinning reserves ready to be deployed at every hour. These reserves will cope with random and non-random fluctuations as well as with contingencies that may occur in the system.

In addition to the conventional deployment of reserves, power systems with a significant amount of renewables require an extra reserve capacity to compensate for the difference between the renewable generation forecast and the actual power generated. Also, similarly to reserves designated to meet demand when thermal power plants fail, reserves require also to contemplate the possibility of VER failing. This type of reserve is included by requiring the system to have a total positive and negative regulation capacity proportional to the amount of wind and solar power installed. Accordingly, we can define the operating reserves included in the model as follows:

- **Primary reserves** ($r_j^{PRI\_U}$ and $r_j^{PRI\_D}$): up and down spinning reserves that can respond in seconds to frequency disturbances originated by contingency events. These events can be due to unpredicted demand shifts or the failure of a thermal or renewable power plant.

$$r_j^{PRI\_U} \geq \alpha D_j + \max_i\left(\overline{P}_i\right) \qquad \forall j \in J \tag{22}$$

$$r_j^{PRI\_D} \geq \beta D_j \qquad \forall j \in J \tag{23}$$

- **Secondary reserves** ($r_j^{SEC\_U}$ and $r_j^{SEC\_D}$): spinning reserves used mainly to reduce the area control error due to random oscillations like those coming from VER forecast errors. Hence, in this model secondary reserves are proportional to the forecasted VER output. These reserves can be up and down reserves.

$$\begin{aligned} r_j^{SEC_U} &\geq \gamma \left[CF_j^{WIND} \sum_{i \in W} y_i \, \overline{P}_i \right] \\ &+ \delta \left[CF_j^{SOLAR} \sum_{i \in S} y_i \cdot \overline{P}_i \right] \qquad \forall j \in J \end{aligned} \tag{24}$$

$$\begin{aligned} r_j^{SEC_D} &\geq \varepsilon \left[CF_j^{WIND} \sum_{i \in W} y_i \, \overline{P}_i \right] \\ &+ \theta \left[CF_j^{SOLAR} \sum_{i \in S} y_i \, \overline{P}_i \right] \qquad \forall j \in J \end{aligned} \tag{25}$$

- **Tertiary reserves** (r_j^{TER}): non-spinning reserves that maintain area control error and frequency due to contingencies. These reserves have a slower time response than other reserves and they are typically used for load following purposes and to substitute faster reserves that have to be restored to pre-event levels.

$$\begin{aligned} r_j^{TER} &\geq \zeta \sum_{i \in W} y_i \, \overline{P}_i \\ &+ \eta \sum_{i \in S} y_i \, \overline{P}_i \qquad \forall j \in J \end{aligned} \tag{26}$$

According to these definitions, positive (27) and negative (28) spinning reserve constraints are formulated as follows:

$$\sum_i \left(u_{i,j} \, \overline{P}_i - x_{i,j} \right) \geq r_j^{PRI\_U} + r_j^{SEC\_U} \qquad \forall j \in J \tag{27}$$

$$\sum_i \left(x_{i,j} - u_{i,j} \, \underline{P_i} \right) \geq r_j^{PRI\_D} + r_j^{SEC\_D} \qquad \forall j \in J \tag{28}$$

Likewise, non-spinning reserves will be provided by the available off-line capacity:

$$\sum_i \left(y_i - u_{i,j} \right) \overline{P}_i \geq r_j^{TER} \qquad \forall j \in J \tag{29}$$

The reserves coefficients α, β, γ, δ, ε, θ, ζ and η are all positive real numbers that take values within the interval [0,1]. The values of these coefficients are specified by each system's ISO attending to the system's reserves needs, which are dependent on the interconnection capacity with other systems and the accuracy of VER forecasts among other factors.

F. Model data

Cost parameters and technical characteristics of the thermal power plants used in the analysis are shown in Tables XII – XIV:

TABLE XII

FIXED COSTS OF THERMAL POWER PLANTS

Technology	Capital cost [k$/MW]	Life [years]	WACC [%]	Fixed O&M [$/MW-year]	Annualized capital cost [k$/MW year]
Nuclear[2]	5,335	40	7	88,750	489
Coal[1]	3,167	40	10	35,970	360
CCGT[1]	978	20	10	14,390	129
OCGT[1]	974	40	10	6,980	106

[1] [24]

[2] Fixed costs of Nuclear: [25]

TABLE XIII

VARIABLE COSTS OF THERMAL POWER PLANTS

Technology	Variable O&M [$/MWh]	Heat rate [MBTU/MWh]	Fuel price [$/MBTU]	Variable cost [$/MWh]
Nuclear[1]	2.04	10.49	0.43	6.5
Coal[1]	4.25	8.8	2.22	23.8
CCGT[1]	3.43	7.05	7.81	58.5
OCGT[1]	14.7	10.85	7.81	99.4

[1] [24]

TABLE XIV

TECHNICAL PARAMETERS OF THERMAL POWER PLANTS

Technology	$\overline{P}_i$[GW]	$\underline{P}_i$[GW]	Maximum ramp [GW/h][1]	Start-up costs [M$/start-up]	Minimum down time [hrs]	Minimum up time [hrs]
Nuclear	1	0.9	0.03	1	20	20
Coal	0.5	0.1	0.21	0.05	6	6
CCGT	0.4	0.1	0.32	0.02	3	3
OCGT	0.3	0.15	0.36	0.004	0	0

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