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Quantitative empirical trends in technical performance

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Abstract

Technological improvement trends such as Moore's law and experience curves have been widely used to understand how technologies change over time and to forecast the future through extrapolation. Such studies can also potentially provide a deeper understanding of R&D management and strategic issues associated with technical change. However, this requires that methodological approaches for these analyses be addressed and compared to more effectively interpret results. Our analysis of methodological issues recommends less ambiguous approaches to: 1) the unit of analysis; 2) choice of the metrics within a unit of analysis; 3) the relationships among possible independent variables; and 4) qualitative and quantitative data quality considerations.

The paper then uses this methodology to analyze performance trends for 28 technological domains with the following findings:

1. Sahal's relationship is tested for several effort variables (for patents and revenue in addition to cumulative production where it was first developed).
2. The relationship is quite accurate when all three relationships, (a. an exponential between performance and time, b. an exponential of effort and time and c. a power law between performance and the effort variable) have good data fits (r^2 >0.7) .
3. The power law and effort exponents determined are dependent upon the choice of effort variable but the time dependence exponential is not.
4. In domains where the quantity of patents do not increase exponentially with time, Sahal's relationship gives poor estimates even though Moore's law is followed even for these domains.
5. Good data quality for any of the relationships depends upon adequate screening involving not only r^2 but also the confidence interval based upon two different statistical tests; by these measures, all 28 domains have high quality fits between the log of performance and time whereas less than ½ show this level of quality for power law fits with patents as the effort variable.

Overall, the results are interpreted as indicating that Moore's law is a better description of longer-term technological change when the performance data come from various designs whereas experience curves may be more relevant when a singular design in a given factory is considered.

Keywords: Technical Performance; Forecasting; quantitative performance trends; empirical quantitative trends

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1. Introduction

Since economic and social impacts of technologies are dependent upon the trend of performance, many theories of technological change (1-8) involve hypotheses about such trends over the life cycle of a technology. Nonetheless, there are major issues with the quantitative empirical analysis of such trends that make testing of such hypotheses problematic. One objective in this paper is to help make this aspect of technological change less ambiguous, more standardized and thus more useful to all researchers.

Trajtenberg opens his important 1990 paper (9) with the following statement: "The study of technological change has been hampered all along by the scarcity of appropriate data and, in particular, by the lack of good indicators of innovation having a wide coverage". His paper establishes the importance of citation counts in signaling patent significance thereby contributing to making the patent system a vital data source for people studying technical change; success is evidenced in the extensive ongoing literature using patents as an empirical means to understand technological progress (and to test theories about technological progress).

In his computed tomography (CT) scan case study at the heart of the 1990 paper, Trajtenberg also utilizes performance (including cost/price) measures for establishing the amount of technological progress in CT during the period from 1975- 1982 (10). Such performance and price trends are themselves a second potentially excellent way ("good indicator with wide coverage") to study technological change in addition to patents. Therefore it is not surprising that many researchers over the past decades have utilized price and performance trends as part of the empirical edifice for studying technological change (11-21). Despite the fact that some of the co-authors are among the researchers noted, we cannot strongly dispute Coccia's conclusion (22) that most studies of performance trends are (at least somewhat) "ad hoc".

This paper attempts to make technical performance trends a more reliable part of the empirical arsenal for those studying technological change by discussing and clarifying important methodological issues. Section 2 examines the ambiguities involved with describing *the quantitative technical performance trend of a technology.* Analysis of the issues involved is done in terms of four important factors that are discussed in more depth in the following section of the paper. Section 3 presents performance trend results for 28 "technologies" following the four factors identified and discussed in section 2 whereas section 4 interprets the results and discusses their implications in terms of the remaining ambiguity associated with reliably determining the quantitative technical performance trend of technologies.

2. Methodological Issues in analyzing performance trends

2.1 Important factors in reliable determination of quantitative technical performance trends

There are four important factors in understanding *the quantitative technical performance trend of a technology.* identifies the four most important factors in

achieving the goal. One is the unit of analysis; in other words, what do we mean by "a technology" when we measure *the quantitative technical performance trend of **a technology***? Second, what do we mean by "technical performance" when we measure *the quantitative **technical performance** trend of a technology*? This involves the complex subject of metrics and choice of the most appropriate metric as a dependent variable to characterize technical performance of a technology. Third, what independent variable is appropriate which depends upon the "trend" we are attempting to represent. Possible independent variables include time, production, sales revenue, profits, R&D spending, and numbers of patents. The fourth key factor in the framework proposed here is data quality.

2.2 Unit of analysis

Arriving at a non-ambiguous definition of a technology is very challenging and this affects all empirical and theoretical studies in the field of technological change [3]. There are a lot of different approaches to decomposition of technology to specific technologies but the broadest attempts by highly experienced and motivated experts is clearly the US (UPC) and International patent classes (IPC). The UPC has about 400 "top level" classes and about 135,000 subclasses (24) and the IPC (is structured with 628 4-digit classes and 71,437 subgroups at the most granular level of the hierarchy (25).

Most technological progress researchers find these "too detailed" for several reasons. First, since technologies evolve in unexpected directions, detailed decompositions such as the UPC and IPC constantly evolve over *time*. The second logical problem with decomposition of technology is the diversity of *inter-relationships* among different named technologies: technologies are related hierarchically, in complementary and in competitive manners. Indeed, some technologies are seen as general- purpose technologies that affect a great number of other technological fields by becoming embedded in them or serving to enable them by complementary relationships (26,27). The reality of this logical issue is supported by the fact that an average US patent is listed in 4.6 UPCs and 2.4 IPCs indicating impact on multiple streams of technology.

A second way to differentiate among technologies is using Dosi's notions (28) of "trajectories and paradigms" for technological progress. Dosi uses the idea of a paradigm as normal technology progress (analogous to Kuhn's interpretation of scientific progress) and trajectory as the economic focus on the technological problem solving process inherent in a paradigm. Much more recently, Martinelli (29) utilized Dosi's concepts in a study of the telecommunication switching industry and in doing so, developed the ideas further. Martinelli considers seven generations of switches (from 1870-2010) as described in the technical literature and identifies 4 overall paradigms (which she discusses based upon *knowledge* used by the engineers) containing the 7 generations that she equates with trajectories. The Dosi approach as evolved by

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[3] This is just as serious an issue for patents (where the sub-classes are too numerous to map to desired "technologies") as it is for technical performance trends (see 23).

Martinelli is more flexible than the patent classification system partly because it is less detailed. However, useful reductions in ambiguity without becoming too detailed are possible.

A third way to differentiate technologies starts with generic functional categories (17). One advantage of this approach is that a specific set of functional categories can be determined; an example is shown in figure 1. A second favorable aspect of the approach is the linking of technological direction to basic functional needs (transport, storage and transformation of basic technical quantities) which delineates a connection between economics and technology similar to Dosi's trajectories. A drawback of this approach is the breadth of the generic categories dictates that very different artifacts, very different scientific knowledge bases and also very distinct industries are contained in one category. From both governmental policy and industrial strategy viewpoints, a more in-depth decomposition is needed in order to distinguish among clearly different technologies. An approach that moves in this direction is to consider domains within functional categories: specifically, a technological domain is defined here as "artifacts[4] that fulfill a specific generic function utilizing a particular, recognizable body of scientific knowledge". This definition essentially decomposes generic functions along the lines of established bodies of scientific knowledge. This approach is in the same spirit as the Dosi/Martenelli framework [and with Arthur's later approach (30)] with the generic function connecting the domains to the economy and the body of scientific knowledge connecting the domain to science and other technical knowledge. Its advantage is that both generic function and domain (body of recognized scientific knowledge) are less ambiguous than the trajectory and paradigm concepts. This approach to the unit of analysis aspect is applied in Section 3 to 28 domains where sufficient data has been gathered to quantitatively describe the technical performance trend.

operations	Operands		
	Information	**Energy**	**Material**
Storage	Punch card Magnetic tape Hard disk	Super capacitor Li ion battery Lead acid battery	Silo, reservoir
Transfer	single cable coaxial cable	Belts, shafts Electrical power transmission lines	Aircraft (Boeing 747) Train, Sailing ship
Transformation	Sliderule Zuse's Z3 Harvard Mach II IBM Mainframe	Electrical Generator Hydropower turbine	Bridgeport Milling machine SLA machine (MakerBot Replicator)

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[4] Artifacts include systems, products, subsystems, processes, software and components

Figure 1: Generic functional categories with some selected artifacts shown in each functional category (the intersections of three fundamental technical "things" – operands- and three fundamental processes that add value -operations)

2.3 Dependent variable: performance metric selection

Once we have defined the technological domain, how should we measure performance to quantify improvements in performance? This depends on the purpose of the study. One *purpose* for studying trends of metrics is to indicate the significance of *usage* of a technology to the economy and society over time. The metrics used in such studies (and there are extensive such "diffusion" studies) include the amount consumed (31), fraction of potential users who become users (32), market penetration (33), and units produced (34). While this research is very important, it does not clarify trends in technical performance and the metrics used are not measures of technical performance.

A second purpose is to help one visualize future engineering problems or future design directions. The metrics used in such work (they are numerous but usually as part of more comprehensive design trend studies) include pressure ratio (13), Temperature achieved in an artifact (13), energy efficiency (35), mass balance (35) and others. Again, although such technical metrics are unquestionably important, effective analysis of technical change and its social and cultural impact requires that technical performance metrics must go beyond these technical metrics. Indeed, an appropriate definition for this purpose is that *technical performance metrics are the properties of artifacts that are coupled to economic usage but are independent of amount of usage, number of possible users, competitive offerings, or the scarcity or depletion of resources that are used in building the artifacts*.

The "ideal" metric for assessing technical capability is one that would assess the economic value of an artifact independently of purely economic variables such as scarcity and strength of demand. An ideal metric would combine (in the "correct" weight) all performance factors that have a role in a *purchase/use* decision. Thus, these "techno-economic" metrics would measure the performance of an artifact as viewed by a user and not design variables as viewed by an engineer (the technical metrics) and also not the number of users or depletion effects as present in more marketing or economically focused metrics. The desire for such ideal metrics has also been discussed as part of hedonic pricing research (10, 36,37).

The prior work has shown that developing a metric that correctly combines all performance factors that define the value of an artifact for users (to establish trends) is vey problematic not least because different customers/users place different weights on different attributes. Nonetheless, reductions in ambiguity relative to current practice are possible (sometimes technical, economic and techno-economic metrics- in our terminology- are mixed together in ways that make all statements about trends or

findings extremely ambiguous). As a summary of this subsection, six principles can be used in choosing metrics to describe quantitatively the *technical performance* of technologies:

1. The proposed metric should be user focused- not producer or designer focused (what makes this artifact class more valuable to a user? is the relevant question to ask).
2. The proposed metric should not assess the number of users or revenue generated by the artifacts under study; the intent is to measure technological performance, not adoption.
3. The proposed metric should not incorporate depletion effects such as often happens with study of natural resource extraction technologies. Managi et al (16) clearly show that depletion effects tend to decrease output that is improving due to increases in technical capability; oscillation of improvement and deterioration of output is shown by them to be the result in one case (oil extraction in the Gulf of Mexico measured by yield of oil per depth of well) and similar oscillations are seen in results from other researchers for similar metrics for other cases [(14)-coal extraction in West Virginia; (38, 39)- fossil fuels more generally]
4. Increase in the metric should make the artifact more appealing (thus indicators that reduce appeal -such as cost- should be in the denominator);
5. The proposed metric should not be extensive; for example, building height, kiln size, batch size, windmill power, computer computation magnitude, etc. are not good measures of technical performance. These metrics depend upon technical performance (as do those in item 2) but also reflect investment and usage factors that can render the underlying technical performance trend ambiguous.
6. For an intensive metric, the *minimum* number of performance indicators in the metric is two (desired output per some constraint) but incorporation of more indicators helps to minimize the "omitted variable problem".

2.4 Multiplicity of Independent Variables

Trends in technical performance are usually determined as a function of either time or effort. Intuitively, many feel more comfortable with identifiable activities for improvement of technology as opposed to the "passive" passage of time as an implied driver of technological change[5]. As an example is this quotation from MacDonald and Schrattenholzer (40)

> *"For most products and services, however, it is not the passage of time that leads to cost reductions, but the accumulation of experience. Unlike a fine wine, a technology design that is left on the shelf does not become better the longer it sits unused."*

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[5] This thought indicates a potentially unjustified confluence of causation with the functional relationship whereas without additional work only correlation is known.

Counterbalancing this apparent drawback of using time is the fact that measurement of effort introduces more needed data searching. More importantly, measurement of time is unambiguous whereas effort is ambiguous and can be assessed according to several distinct concepts. The original research by Wright (41) and further extensions (42-45) use cumulative production as the independent variable (the equation used will be discussed below). Although Wright treated learning as within a single plant (and for specific airplane designs), the same independent variable is now sometimes used more widely raising further unit of analysis issues. In particular, many researchers now treat cumulative production of an entire (usually global) industry as the independent variable. However, this requires more careful definition of "industry" than is sometimes done. In addition, this broad approach almost always introduces ambiguity about the initial values of output needed for cumulative production and thus also introduces data manipulation issues. To put it simply, determining when unrecorded early units were produced is very problematic.

Another issue involves defining effort since R&D and new designs –not just production- are important. The quotation above (40) is also interesting in that the implied "unit of analysis" is a "technology design" and one could counter that technological change does not proceed simply by continuing to accumulate experience on existing designs but through invention and creation of new designs. Recognizing this, some who take the broader view argue that cumulative production is not then "learning by doing" but instead an indirect –more or less total- measure of relevant *effort*. More direct measures of such broader relevant effort include number of patents, R&D spending, and sales revenue: all of these as well as cumulative production have issues in initial values and are more difficult to obtain. For these as well as historical reasons, much of the practice for independent variables for effort remains cumulative production [despite identification of significant issues in interpreting such studies (46)].

In addition to its passive nature, time as the independent variable conceptually seems to assume technology development is fully exogenous to what is happening in the economy. Since the consensus is that there are strong endogenous aspects of technology development, a fully exogenous assumption is counter-intuitive to those thinking primarily about causes. However, time indirectly contains the endogenous drivers as well as any exogenous drivers. For example if the production rate of an artifact is constant, then cumulative production and time are proportional (with the proportionality constant the rate of production) so learning by doing for factory workers is also implicitly contained within the time variable. Similar arguments apply to R&D spending, revenue and numbers of patents with a direct relationship realized if the rates of each are constant over time. The obvious weakness of these indirect entailments for time is that the effort-independent- variable (patent production, revenue or R&D spending) is not necessarily constant over time; a similar weakness for cumulative production occurs because profits/patents etc. are not directly proportional to cumulative production since cost or revenue per unit is in fact the dependency usually studied and R&D spending and patents are dependent on revenue-not units.

A preliminary conclusion could be that time casts "too wide a net" to give adequate emphasis to the endogenous affects in technological progress but that any specific effort variable "casts too narrow a net" to adequately capture all the endogenous efforts and captures none of the broader effects including "spillover" from efforts outside the implicit unit of analysis.

Perhaps surprisingly given this qualitative story of differences in the approaches, in a very important way the two approaches are equivalent and measure the same things. Important steps in showing this equivalence have been taken by Sahal, Nordhaus and Nagy et al (47-49). The mathematical relationships (and the inter-relationship among them) is given to specify this equivalence. A generalization of Moore's Law[6] that includes only performance q is

$$q = q_0 \exp\{k(t-t_0)\} \tag{1}$$

where $q_0 = q$ at $t = t_0$ and k is a constant; generalizing to an equation that includes cost as well as performance gives

$$q/c = q_0/c_0 \exp\{k(t-t_0)\} \tag{2}$$

where c is price/cost and $c = c_0$ at $t = t_0$
Wright's equation is usually formulated as describing only cost and relates it to cumulative production (p) as a power law:

$$c = B\,p^{-w} \tag{3}$$

where B is the cost for the first unit of production and w is a constant
A generalization of Wright's Law consistent with Equation 2 is

$$q/c = (q/c)_0\,p^{w} \tag{4}$$

$(q/c)_0$ is the value of q/c at 1 unit of production.
Sahal showed that that if the cumulative production, p, (or production) also follows an exponential relationship with time, namely

$$p = p_0 \exp g\{(t-t_0)\} \tag{5}$$

where g is a constant and $p = p_0$ at $t = t_0$, then eliminating time between equations 2 and 5 yields equation 4 with

$$k = wg \tag{6}$$

Thus, Sahal showed that the Wright and Moore formulations were equivalent when production follows an exponential (Equation 5) and the key parameters are simply related (Equation 6). An important issue is why one might expect equation (5) to hold. Nordhaus pointed out (48) that as user-based performance increases or cost decreases according to equation 2, basic economics (demand elasticity) would result in demand (hence production) increases. Since Equation 2 is exponential, demand and hence production would "automatically" (if demand elasticity is constant) follow the exponential relationship in equation (5). Thus, if either the Moore or Wright equation holds, Nordhaus' research indicates the other will be followed as well.

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[6] Q in the original or actual Moore's Law is the number of transistors on a wafer.

Beyond these theoretical considerations, Nagy et al (49) carried out an important and relatively extensive empirical investigation of these relationships. For 62 cases (but where only price of the artifacts was considered), they found for most cases that production followed exponentials with time and that Equation 6 showed minimal deviation *in all 62 cases*. The research by Sahal, Nordhaus and Nagy et al. shows that attempting to use fits to equations 1 through 4 to distinguish among the intuitively different interpretations of technological progress is not easily done.

In this work, we consider other effort variables as a further test of what has been done with production. In particular, we pursue invention as a driver of technical change and utilize the number of patents in a domain as an effort-variable in 28 different domains. In one domain, we consider production, revenue and the number of patents as effort variables and compare the results. As background for those results, we demonstrate in this section that Sahal's relationship is again expected if any effort- variable follows an exponential relationship with time. In particular, if we simply decompose the derivative of ln. performance vs. time defining E as any effort variable, we obtain:

$$\text{dln } Q/dt = \text{dlnQ/dlnE} \times \text{dlnE/dt} \qquad\qquad (7)$$

The left hand side of Equation (7) is the familiar slope of a ln performance vs. time plot which is k (= equation 2 exponent). The first term on the right hand side of (7) is the power law exponent (w in equations 3 and 4) and the second term is the slope of the exponential fit of the effort-variable with time, g in equation 5. Thus, for *any* effort variable, Sahal's relationship (equation 6) holds, k = w g. where g is now the exponent of equation 5 for any effort variable and w is the exponent of a power law (equation 4) or the slope of a log performance vs. log effort plot. Of course, for the relationship to hold the effort variable must be the same for both terms on the right hand side of Equation 7. As we will see in the results, each of these quantities can depend upon the effort variable selected. Note that equation 7 holds for cumulative or annual versions of the effort variables as long as equation 6 holds.

2.5 Data Quality
An important reason for greater reliance on patent than performance data by scholars is that the quality of patent data is perceived to be higher than that of performance data. For patent data, there is a highly motivated group of experts (the patent examiners and other patent attorneys as well as courts of law) who maintain the definitional quality of a patent and the meaningfulness of various metadata associated with each patent. To promote the use of performance data by researchers, similar quality levels should be the goal. This section describes some important quality issues for technical performance trend data and suggests a standard set of quantitative statistical tests.

One can often obtain technical performance data from a variety of sources combining them into metrics that vary with time and other independent variables. Although the

entire data set thus obtained can be of interest: for determining the trend, only *non-dominated observations* are typically used. Non-dominated observations are those for which the metric is not surpassed in magnitude by the value achieved by the metric at lower values (earlier time, smaller number of patents, smaller cumulative production, etc.) –they are "record setters". Although this reduces the amount of data available for analysis, it is the usual preferred practice because of concern that dominated points may be exceedingly high on a missing variable introducing noise.

Numerous other data quality issues are worth considering. These include:
- Range of independent variable over which data is collected (more range is better)
- Number of observations (more is better);
- Source of observed data (private communication is weaker than publicly available data but archival Journals are better yet with multiple journal articles the best to be expected;
- Three or more *user*-oriented attributes in metric is best, two or more user-oriented attributes in metric is next best, use of non-user-oriented attribute in metric or use of an extensive metric is the poorest;

In addition to these considerations, quality analysis continues with exponential fits (equation 1 or 2) for the log-linear plots and with power law fits (equation 3 or 4) for the log-log graphs. In addition to determining best estimates of parameters such as k and w as discussed in section 3.4, the regression analysis gives r^2 values for each regression which are useful estimates of the explanatory power of the estimated equation. We also find two other statistical estimates of value in analyzing the regression results. Both give estimates of the reliability of the parameters (k and w) derived by the regression analysis. The first is a standard F test analysis and the second is to analyze the distribution of k (or w) estimates by a point-removal method. In the point-removal method (50), regression analyses are performed with each separate observation eliminated from the data set. For n observations, this results in n regressions with n different estimates of k (or w) and the standard deviation of the set of estimates gives information about how sensitive the overall k estimate is to singular observations.

3. Results

3.1 Overview of results

This section of the paper will first present some key results that are relevant to the four issues just discussed from analysis of data sets from 28 domains. We will first consider the unit of analysis results specifying the domains investigated in Section 3.2. Section 3.3 first reports results for one domain (integrated circuits processing) concerning different independent variables and then focuses on the use of patents as an independent variable because patents are the most direct measure of inventive effort in a domain. We then report the dependent variable (performance metric) results in

section 3.4 and the results section closes (as does our discussion of issues in section 2) with results relative to data quality in section 3.5.

The Supplementary Information file (see section 7 for overview and a link) contains the key data for the 28 domains and for 71 different performance metrics within those domains. The SI also contains annual patent counts from 1976 to 2013 for each domain; the quantity of patents is used as an effort variable for each domain to compare with time dependence. We obtained these highly relevant patent sets by use of a classification overlap method (51) developed earlier; the application of the method for the 28 domains is described in (50).

3.2 Unit of Analysis: Technological domains

The 28 domains for which we have sufficiently high quality data sets are shown in Figure 2 demonstrating their position in the generic functions as displayed in Figure 1. The figure also shows for each of these domains the generic functions and branches of scientific knowledge that define each domain. Domains do define a narrower unit of analysis than simply using generic functions; one example is that we show three separate domains for information storage where magnetism, optics and electronic structure of solids are the relevant scientific fields of knowledge. Indeed, all generic functions in figure 2 with more than one domain represent a narrower unit of analysis than chosen by Koh and Magee (17). However, we must also note that our differentiation based upon distinct branches of knowledge is not simply determined. In fact, the differences sometimes hinge as well on differentiation among artifacts. The wireless telecommunication domain is different from electrical (or wired) telecommunication as much by artifact classes than by differences in branches of science since both rely essentially on electromagnetism as first developed by Faraday.

#	Information	Energy	Material
Storage	**Semiconductor Information storage** *(Solid-state physics, chemistry)*	**Electrochemical batteries** *(Electro-chemistry)*	
	Magnetic information storage *(magnetic materials)*	**Capacitors** *(Electrostatics)*	

	Optical information storage *(Optical materials)*	**Flywheel** *(Mechanics, materials)*	
Transportation	**Electrical telecommunication** *(Electromagnetism)*	**Electrical power transmission** *(electromagnetics)*	**Aircraft transport** *(Aerodynamics, mechanics)*
	Optical telecommunication *(photonics, optics)*	**Superconductivity** *(solid state physics)*	
	Wireless telecommunication *(Electromagnetism))*		
Transformation	**IC Processors** *(Solid-state physics, chemistry)*	**Combustion engines** *(Thermodynamics, mechanics)*	**Milling Machines** *(Mechanics, dynamics)*
	Electronic computation *(Solid-state physics, computation)*	**Electrical motors** *(Electromagnetism)*	**3D printing** *(Materials, computation)*
	Camera Sensitivity *(Photonics)*	**Solar PV power** *(Solid-state physics)*	**Photolithography** *(Chemistry, optics)*
	MRI *(Nuclear physics)*	**Wind turbines** *(Aerodynamics, mechanics)*	
	CT scan *(Atomic physics, computation)*	**Fuel cells** *(Physical Chemistry)*	
	Genome sequencing *(biology, genomics)*	**Incandescent Lighting** *(materials)*	
		LED lighting *(Solid-state physics)*	

Figure 2: The 28 technological domains defined for this study shown in the generic functional format shown in Figure 1. The *italicized phrase* is the scientific knowledge base for the domain.

We therefore find that even narrower units of analysis are possible than a domain. The case of magnetic storage is of interest in that data were gathered for two distinctly different artifacts in the domain- magnetic storage disks and magnetic tape. Figure 3 shows exponential plots for these two different artifacts contained within the magnetic information storage domain. The visual signal from this graph (before 1990) and the quantitative information in Table 1 indicates that these two classes of artifact are reasonably distinguishable and can be treated as separate unit of analyses for trend data. Moreover, Table 1 shows that the statistically significant difference between the rate of advance of these classes of artifacts is not very large so one can choose to combine or not these artifacts as one's research goals dictate. A similar finding

determined that the combustion engine domain remain singular rather than devolving into the separate artifact-based sub-domains of turbines, internal combustion engines and diesel engines.
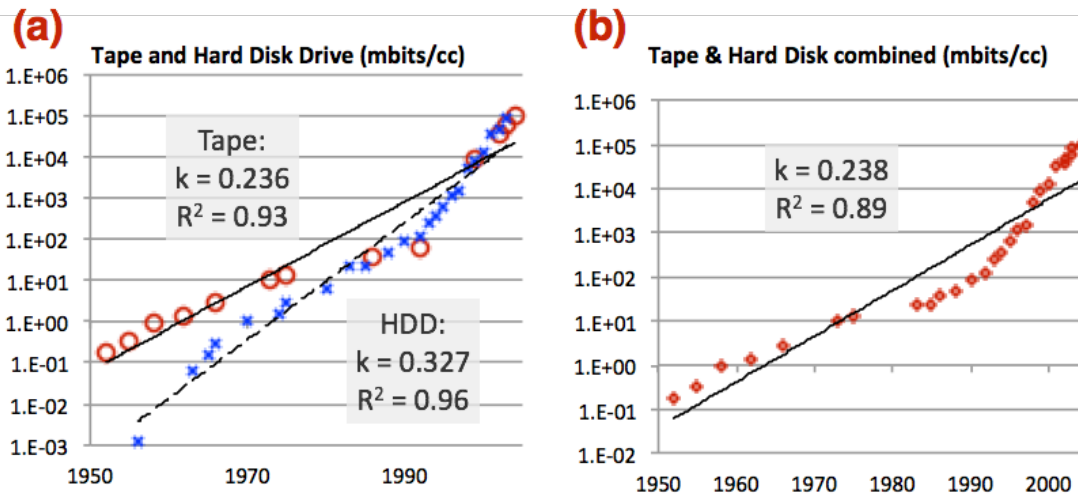

Figure 3: Performance of magnetic tape and Hard Disk Drive
Figure 3a-separate fits (circles for HDD and crosses tape); Figure 3b- combined data fit

	K	R^2
Magnetic tape	0.236	0.93
Hard Disk drive	0.327	0.96
Combined - Tape and Hard Disk drive	0.238	0.88

Table 1: combined and individual k and R^2 for tape and disk drive artifacts in the magnetic information storage domain

3.3 Independent variable results

3.3.1 Performance vs. time and three effort-variables for Integrated Circuits
Before analyzing the results we have considering patents as an effort-variable, it is useful to recall that Section 2.4 showed that Sahal's relationship is generalizable to any effort -variable. An empirical confirmation of this is possible by examination of one of our 28 domains- namely integrated circuits. In particular, we obtained detailed production and revenue data for the IC domain from (52). Table 2 shows empirical estimates of g and w for ICs for all three effort-variables along with r^2 for each estimate. g describes the exponential between the effort-variable and time (Equation 5) whereas w is the power law fit for performance vs. the effort variable (Equation 4). These results

indicate acceptable power-law fit quality for the IC domain for Moore's performance variable (transistors/ die) vs. each of the three effort-variables- production/demand, revenue and patents for this domain/industry. The results in Table 2 also indicate acceptable fit to the exponential with time for each of these effort-variables.

Independent – effort-variable	g from equation 5 and (r^2)	w from Equation 4 and (r^2)	Estimated k from Equation 6; {empirically det.}
Production/demand	0.59 (.97)	0.6 (.99)	0.35; {0.36}
Revenue	0.095 (.91)	3.4 (.88)	0.32; {0.36}
Number of Patents	0.114 (.76)	3.0 (.86)	0.34; {0.36}

Table 2: Empirical values of g and w for IC processors with the dependent variable (Q) = transistors/die. The empirical value of k for this dependent variable is 0.36 in good agreement with the values estimated from Sahal's relationship (k = g w)

It is first interesting to note that the estimates for w and g are dependent upon the particular effort-variable; g is much higher and w much lower for production as opposed to revenue. This striking result is a natural outcome of the fact that this domain has improved rapidly so that the revenue per transistor has greatly diminished over time. As a result, the exponential increase with time (g) is much lower for revenue than for production. Similarly, the log of increase in performance with a logarithmic increase in effort (w) is understandably much larger for revenue than for production again because of the much more rapid increase in production compared to revenue with the same performance increase. In a given domain, the amount of R&D spending is approximately proportional to revenue and the number of patents is approximately proportional to R&D spending (53); thus, g and w for revenue and patents are expected to be similar. Table 2 confirms empirically that g and w are much more similar for patents and revenue than for production/demand but also shows that patents have increased slightly more rapidly (11.4% per year) compared to revenue (9.5% per year) due to increases in the R&D/revenue ratio in this domain over time (54).

Despite the systemic change in w and g for the three effort-variables, Table 2 shows good agreement between direct determination of k and the value of k calculated from Sahal's relationship. The estimates are definitely within the confidence interval for k for the IC processors. The strong agreement with all three effort-variables for IC processors is a strong confirmation of the usefulness of Sahal's relationship and that patents can potentially be used as an effort variable. In fact, some (3) have argued that use of an invention-oriented effort- variable is superior to time or production. In fact, for the number of transistors per die to increase over time obviously takes more invention than continued production of a given design and thus patents are arguably superior for this case relative to production. However, since much helpful invention occurs outside the domain of interest, time may still be better. We now turn to results for using patents as an effort-variable for all 28 of our domains. We first show some plots to calibrate the reader to different levels of fit found in the data for equations 2, 4 and 5.

3.3.1 Performance vs. time and patent output for other technological domains

Figure 4a shows log-linear plots of performance with time (equation 2) for four domains (optical telecom, LEDs, batteries and 3D printing) and four relevant performance metrics (we call each of these domains with a specific metric a domain-metric-pair). Figure 4b shows the log of performance vs. log of patents[7] (Equation 4) for the same four-domain-metric-pairs and Figure 4c plots log patents vs. time (equation 5) for the same four pairs. These four domain-metric-pairs are chosen because they represent the full range of quality of fits in our larger data set. In particular, the LED and optical Telecom plots show good r^2 values and subjectively good fits for all three plots. However, 3D printing and batteries show poorer subjective fits and r^2 values in Figures 4b and 4c. It is important to note that Sahal's relationship is not accurate in cases with such poor fit since the parameters on the right hand side of Equation 7 are not constant. In fact, the k estimated from equation 6 for 3D printing and batteries are off from the directly determined value by factors greater than 1.5 (much greater for 3D printing) but are within a factor of 1.2 for optical telecom and LEDs. These results indicate that Sahal's relationship is accurate for cases where good fits (r^2 >0.75) exist for k, w and g. An apparent reduction in accuracy of the relationship occurs as the fits deteriorate. This finding does not depend upon the nature of the effort-variable but instead upon whether an exponential describes well the relationship between the effort-variable and time.

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[7] We also tested cumulative patents and got similar results. An additional issue in use of cumulative patents (as with any cumulative variable) is that one does not have actual data for many years (in our patents, we cannot apply COM before 1976) and estimation techniques do not actually add any information. Since equation 7 works for cumulative or annual effort variables and the exponents g and w) are the same for annual or cumulative variables, we use the actual data rather than an arbitrary reworking of it.
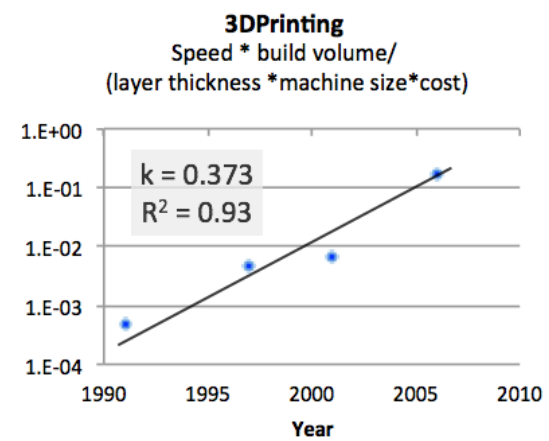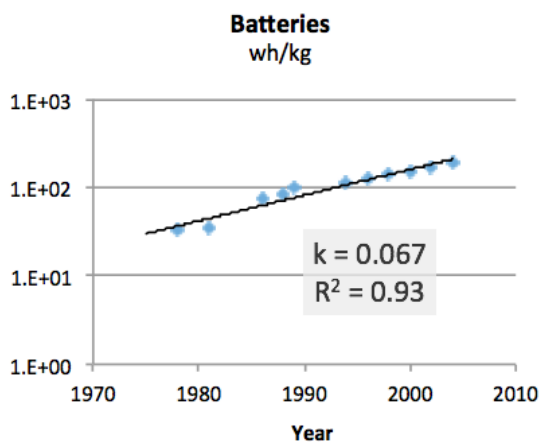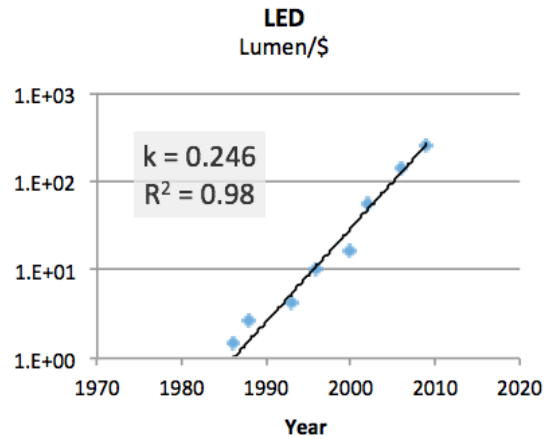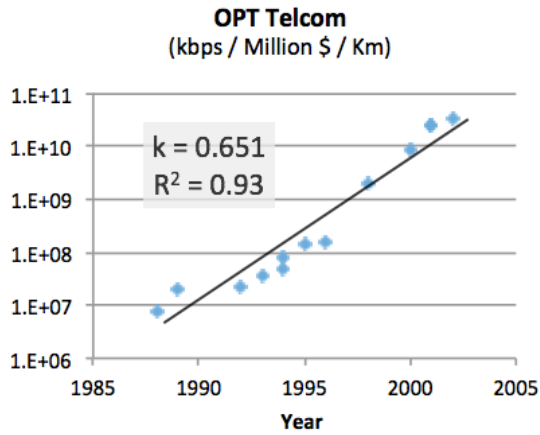
Figure 4A: Technological performance (log) against time for four domains (optical telecommunications, LED lighting, electro-chemical batteries, and 3D printing)
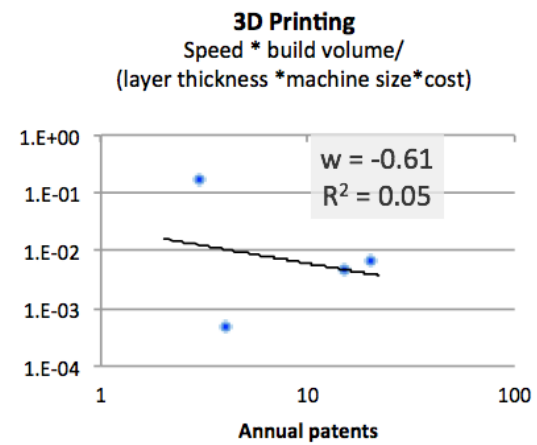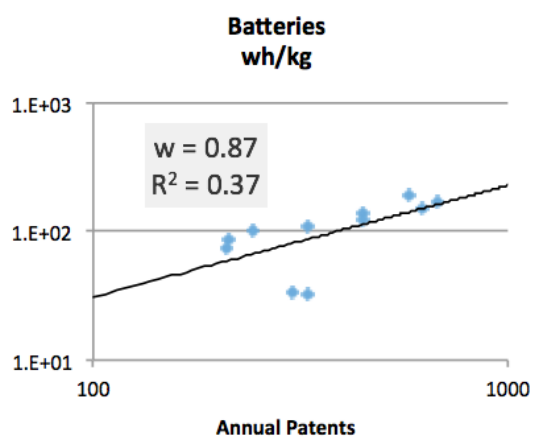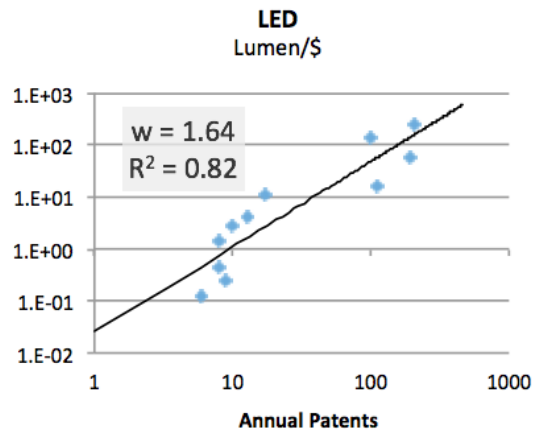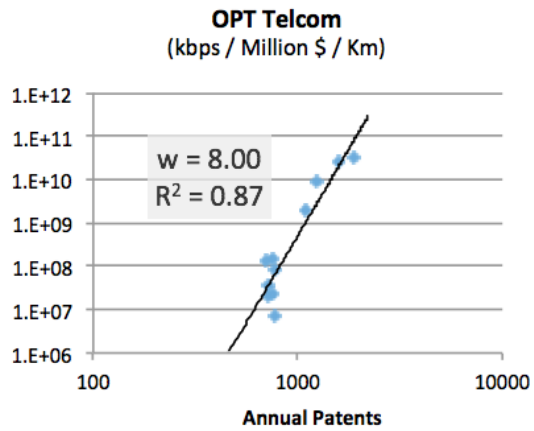
Figure 4B: Power law fit for four domains (optical telecommunications, LED lighting, electrochemical batteries and 3D SLA printing). The metric for each is above the graph.
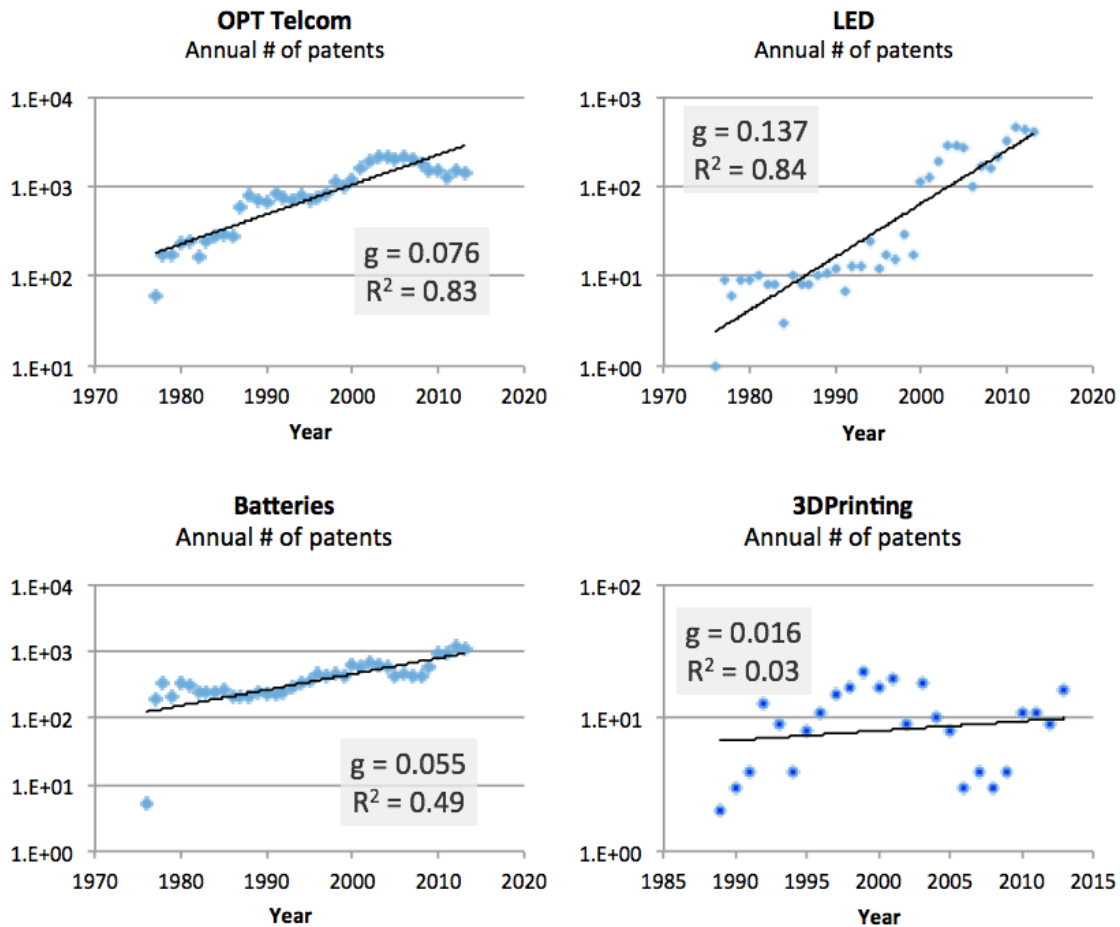
Figure 4C: Annual patents against time for four domains (optical telecommunications, LED lighting, electro-chemical batteries, and 3D printing)

Figure 5 is a distribution of r^2 for all 28 domains for the three key fit variables (k, g and w). We note that only 2 of the 28 r^2 values are less than 0.8 for k but that the majority are less than 0.8 for w and for g. This demonstrates that Moore's Law is followed even when a relevant effort-variable does not increase exponentially with time. This is an important result because some might interpret Sahal's result to mean that one needs to have exponential increases with time for effort -variables to get exponentials of performance with time. The results in Figure 5 show that such a conclusion is clearly not true. Moreover, many cases of low r^2 for w shows that Wright's Law is also not followed well for this effort-variable when patent numbers do not increase exponentially with time. These results lead us to restrict ourselves to time as the independent variable in the remainder of the results section.
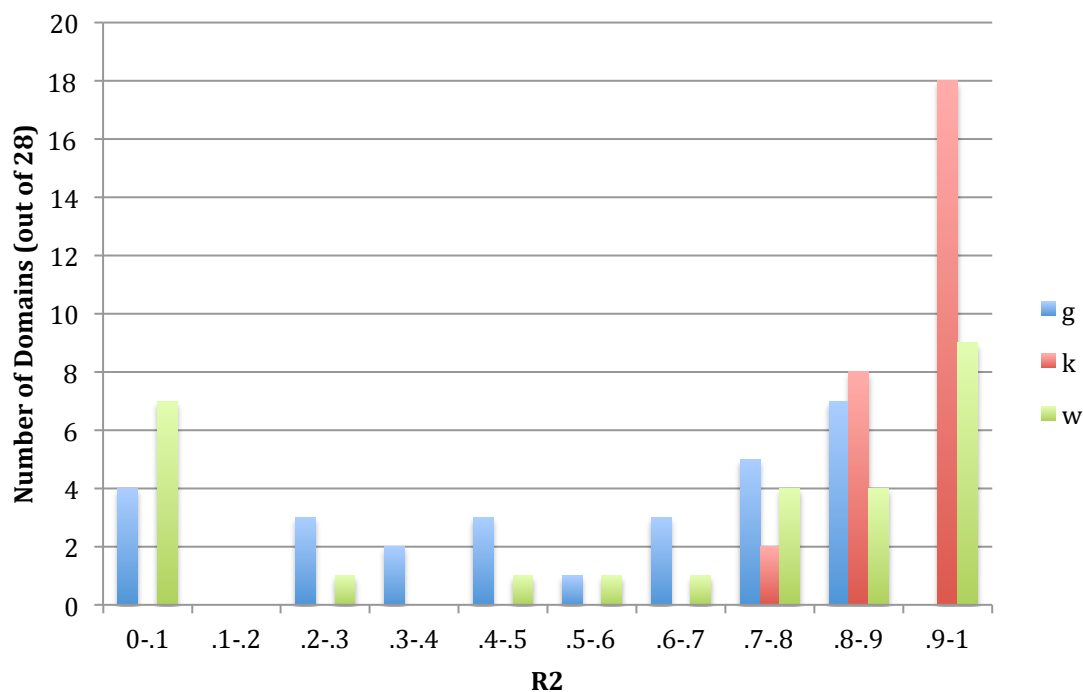
Figure 5: Distribution of r^2 for all 28 domains for k, g and w. (Note that w could not be calculated for two of the domains)

3.4 Dependent variable (performance metrics) results

For 12 of our domains as specified in section 3.2, we only found data for a single metric; however, for the remainder we had multiple metrics and overall we studied progress results for 71 distinct domain- metric pairs. Table 3 compares the statistical information for metrics that contain cost (usually estimated as equal to price) as part of the metric with those that do not contain cost. Our rationale for doing this was the reality that cost introduces sources of noise (depletion, subsidies, profit variation with time) that should make the relationship less strong. Although the average values for the standard deviations do show some deterioration for metrics containing cost, the r^2 values are quite similar for both kinds of metrics. Thus, including cost in a metric does cause some statistical downgrading but the effect is small enough that the important cost variable is certainly important to provide in all cases where it is available.

	R2	STDEV	PRM STDEV
WITH PRICE	0.869	4.4%	2.8%
WITHOUT PRICE	0.868	1.6%	0.8%

Table 3: Comparison of statistical criteria for domain-metric-pairs containing and not containing cost.

Another important issue is to consider the magnitude of the effect different metrics have on estimating quantitative technological trends in a given domain. Figure 6 shows the 16 domains where multiple metrics were studied (59 distinct domain-metric-pairs

for the sixteen domains where we had multiple metrics).   For each domain, the figure shows the mean k and the variation in k (calculated from the multiple metric results). Four of these domains (electric power transmission, MRI, photolithography and wireless information transmission) demonstrate significant variation where the standard deviation is > 0.5 times the mean. For the other 12 domains, the variation is small enough that a similar quantitative estimate of technological progress would arise with use of any single metric. However, the four exceptions show that assuming this always holds is not a reliable assumption.
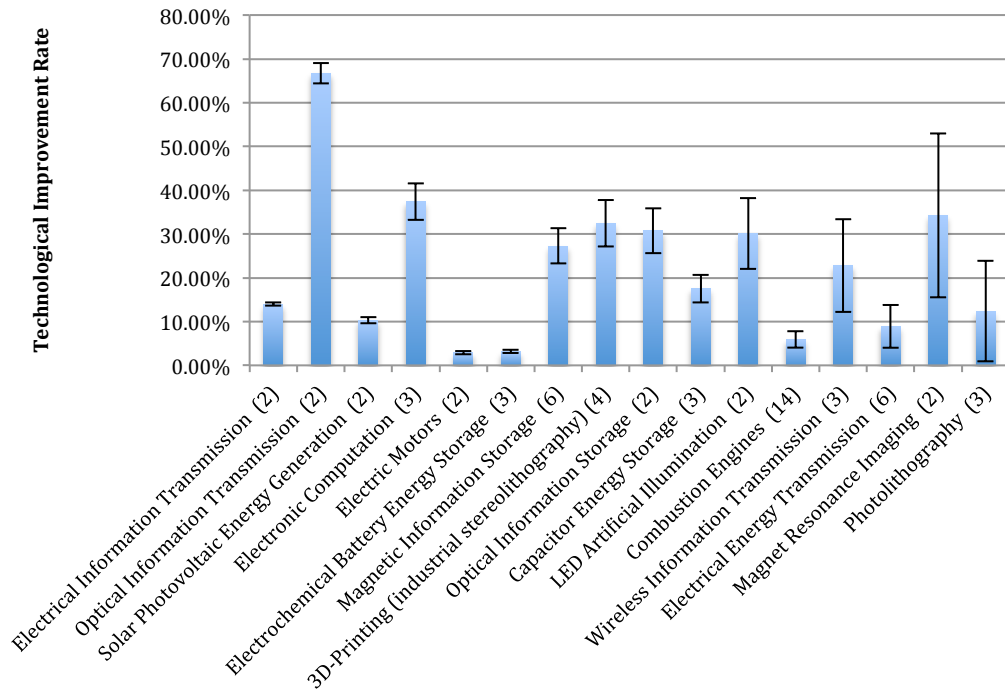

Figure 6: The k values for the 16 domains with multiple metrics showing the number of performance metrics for each technology in parentheses and the standard deviation for the estimates of the multiple metrics in the error bars

The distribution of the rate of improvement (k) for all 71 domain metric pairs studied is given in Figure 7; this demonstrates that the inter-domain variation is greater than the intra domain variation even allowing for the 4 exceptions just discussed.
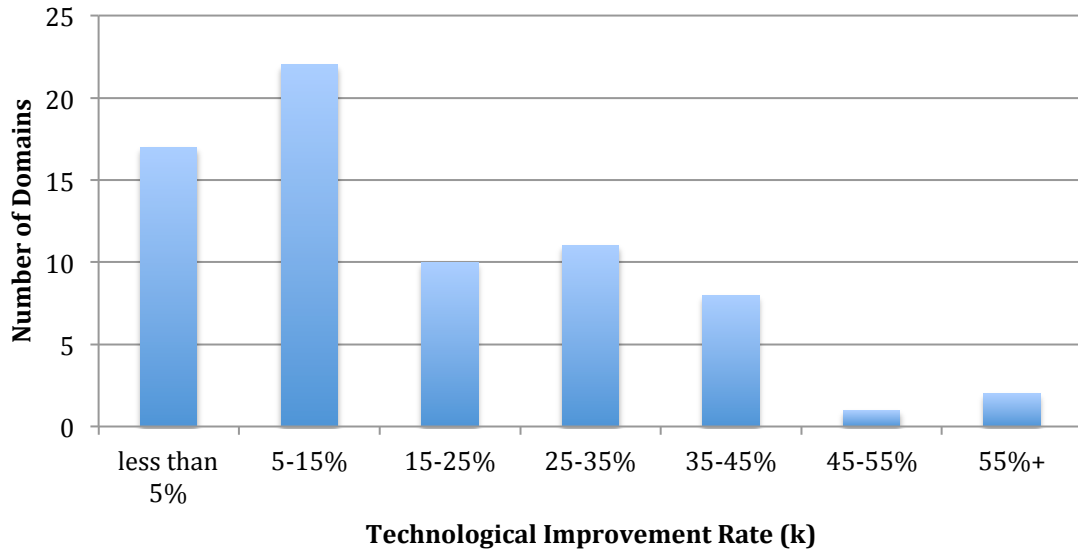
Figure 7: Distribution of k values for all 71 domain metric pairs

3.5 Data Quality

Figure 8 shows the distribution of r^2 values for all 71 cases of the fits of log of the performance metrics with time. The worst fits still have r^2 greater than 0.5 but the subset in Figure 6 are even stronger. Figure 9 shows the variance calculated by the methods discussed in section 2.5 for these 71 cases. These results demonstrate that – even for metrics with minimal data- exponential fits are an excellent model for technological improvement with time. We found it possible for all 28 domains to find metrics that have r^2 values greater than 0.7 and variations less than 0.07 by both variation calculations.
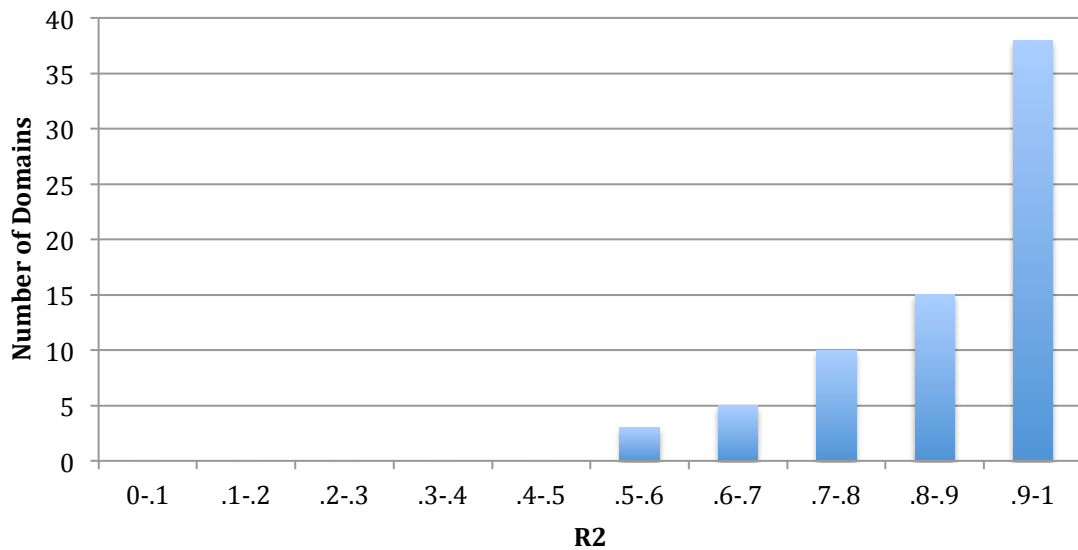

Figure 8; distribution of r^2 values for fits of Equation 2 for 71 domain-metric pairs
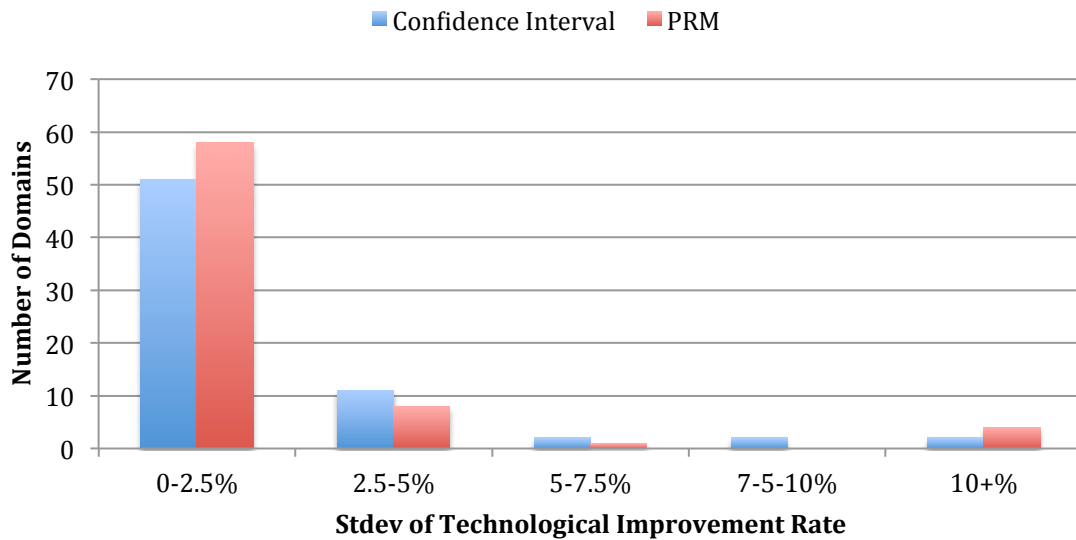
Figure 9: standard deviations for all 71 domain metric pairs for both confidence interval and PRM methods

4.Discussion

This section first discusses the research reported here in regard to our desire to develop a less ambiguous approach for describing *the quantitative technical performance trend of a technology*. After this examination of the research process, we briefly explore the implications of the research results on quantitative technical performance trends in 28 technological domains to general understanding of technological change. We conclude the discussion by consideration of open questions for further research using quantitative technical performance trends to more deeply understand technological change.

A summary of possible practices for reducing ambiguity regarding the four identified factors is given in Table 4. Examination of the research process addresses three questions: 1) How do these practices reduce ambiguity? 2) What evidence from section 3 supports our assertion that these practices reduce ambiguity? 3) What open issues remain?

One contribution this paper makes to reduction of ambiguity in this complex area is the explicit framework evident in the paper and Table 4. By delineating four distinct factors needing sufficient definition, we have attempted to make clear how this complex problem can be decomposed to issues that can be separately addressed. For each factor, the specific practices were followed in section 3. Our application of the framework in section 3 identified no important issues that show this framework is too narrow or is missing significant elements.

In regard to the unit of analysis, the recommended practices resulted in the 28 domains given in Figure 2. This result is a definite reduction in ambiguity from approaches that simply use "off-the-shelf" names for technology. It is also more specific than the generic categories given in Figure 1. However, we must note that some ambiguity remains. For example, we cannot say for sure that Figure 1 is complete and we do not have a specific list of recognized bodies of scientific knowledge. Moreover, very different artifacts are still contained in a single domain; for example, the domain of combustion engines (recognized body of knowledge thermodynamics, generic function of transforming energy) includes not only Diesels and Otto combustion engines but also airplane turbines and electric power turbines. We mentioned in section 3.2 that all of these artifacts have improved at roughly the same (low) rate; nonetheless, interpretations of trends in more depth may require more specific units of analysis.

Three further points can be made relative to the unit of analysis factor. First, some of our domains can be considered as contained within other domains (for example, photolithography within IC processors). Second, some domains are general purpose technologies (IC processors with large effects on numerous other domains such as camera sensitivity, solid state memory, solar photovoltaics, etc.) Third, analyses at more aggregated units of analysis are of interest in describing the *quantitative technical performance trends of **associated groups of technologies***- a closely related objective to what we have labeled the primary purpose. Overall, a precise and unchanging definition of "a technology" remains elusive despite the modest progress described in this paper.

The realities of data availability and time led to the 71 domain metric pairs covered in this paper. The principals guiding choice of metrics (the dependent variables whose trend we examine) are briefly summarized in Table 4. The principals receive implicit support from the acceptable quality found in all 28 cases. Use of single (intensive) metrics to characterize the overall quantitative technological trend is supported somewhat since 12 of the 16 domains where multiple metrics were examined showed that progress in a domain was quite similar for diverse metrics. However, the results also demonstrate that this is not true for the other four domains. An important future goal of research is to explain these domain and metric attribute differences for the domains and metrics studied here and for more domains and metrics. Similarly, the results support the principals relative to the superiority of using multiple attributes to arrive at more complete metrics but in one of our domains (superconductivity) a singular value metric (critical Temperature) gave an adequate estimate for the progress rate.

Perhaps the most important contribution of this paper to reduction of ambiguity in the quantitative description of technical performance trends is our broader consideration of possible independent variables. In particular, we identify production –the most popular choice in the literature- but also revenue, R&D spending and quantity of patents issued as potentially useful "effort-variables". Our results demonstrate that Sahal's relationship is valid for integrated circuits using either revenue, or patents or production

as the effort- variable. The results also show that patents do not always follow an exponential increase with time and in such domains, one usually also does not find good fits for power laws between performance and patents despite having a good fit for an exponential between performance and time. For study of quantitative trends in performance, always reporting time is a firm conclusion. Time is always available and requires no more work. Fitting the exponential with time in addition to the power law with an effort- variable (if sufficient data exists) also appears sensible especially since this formulation of technical trends is found here to be the most accurate over a wide range of technological domains. We also recommend explicit discussion of the specific algorithm for estimating missing data when using cumulative effort-variables. This is unfortunately rarely done now and seriously limits the utility of such work since the importance of unknown assumptions cannot be checked. It is preferable to simply use the effort variable (annual or some other fixed period) directly rather than cumulative versions that are undocumented as to how non-existing effort variables are estimated.

Table 4

Factor	Recommended Practices
Unit of Analysis- definition of "a technology"	1. Choose domains – artifacts that serve a specific generic function from figure 1 while utilizing knowledge from a specific branch of scientific knowledge 2. Gather as much reliable data as possible concerning dependent and corresponding independent variables as described in the next two topics; 3. Examine trends of any multiple potential sub-domain artifact classes (such as magnetic tape and disk drives in the magnetic memory domain) and use this unit of analysis to the extent possible.
Dependent Metric for technical capability	1. The metric should be user-focused 2. The metric should not involve the number of users, market share or sales revenue of the artifacts 3. The metric should not incorporate resource depletion 4. Increase in the metric should correspond to increasing appeal of the artifacts being studied 5. The proposed metric should be intensive-not extensive 6. The incorporation of more attributes is positive with the minimum acceptable number being two.
Independent variables and fits	1. Always report time with any dependent metric observation 2. Report all available effort-independent-variables (patent numbers, R&D spending, sales revenue and cumulative

	production) along with each dependent metric observation. 3. Record the unit of analysis for each effort independent variable and performance observation 4. Test Sahal's relationship when possible
Quality	1. Report r^2 and two measures of variation in r^2 for each domain-metric pair and for each independent variable for which data is gathered, 2. A high quality standard is that r^2 must be greater than 0.7 and that variance of both types must be <0. 07
Analysis	1. Visual inspection of linear graphs of each dependent variable against each independent variable is the suggested first analysis step. 2. Visual inspection of a) log-linear plots of each dependent variable with time and b) log-log plots of each dependent variable with each "effort" independent variable is the second analysis step 3. The major analysis step is regression analysis including calculation of r^2 and variance for parameter estimates for each of the graphs in items 1 and 2 in this listing. This step is essential for the testing of Sahal's relationship as recommended above.

One practical implication of the results reported in this paper is for technological forecasting. Technical performance and cost are important to understand and future values of them are critical to such issues as potential diffusion and firm profitability. The high reliability of the fits to equation 2 (generalized Moore's law) is not evidence for imprudent forecasting but it does support the back-casting results (49) in indicating the value of such an approach in technological forecasting.

Achieving reliable knowledge about quantitative technical performance trends has been demonstrated in this paper. Having high-reliability knowledge concerning quantitative technical performance trends for a wide variety of technological domains opens up a number of research questions of significance to understanding technological change and thus improving our foundation for technological forecasting. A major one is causal explanation for the great variation in rates of advance over the full range of technologies (our 28 domain "sample" shows variation with time that varies from 3.1 % per year (electric motors) to 65.1% per year (optical telecommunication). Testing causal explanations for quantitative technical capability trends as outlined in (56) is possible with this wide range of variation for 28 domains.

Another important research issue is developing a broader domain-based technological map. One key question in this regard: how many domains exist in which one might measure a technical capability trend? Further work will be needed to even arrive at

approximate answers to this question but a deeper understanding of the relationship among domains may be a necessary first step in developing a meaningful domain-based technological map. To what extent domains overlap and how they interact (containment, competition for a need, enabling are three identified possibilities but others might exist) are important parts of this research topic.

Two other topics involve hypotheses about the trends and we have not fully identified meaningful analytical procedures to address them. S curves are hypothesized to be the usual trend for technical performance when plotted linearly against time or effort. Visual inspection of linear plots were done for all 71 domain metric pairs and none unequivocally appeared to be S curves; however, a desire for a more clearly objective way of determining the reality of S curves is needed. Unfortunately, statistical tools are limited by the fact that logistic (and other equation forms giving S curves) contain additional variables: there are cases (57) when these curves have been fit to data predicting emerging S curves that have not (even 30 years later) appeared. A second hypothesis about the technical capability trends is that they show major discontinuities (6). Testing this hypothesis is not straightforward because increases in technical performance must in reality be discontinuous since advances are typically made by introduction of new designs (inventions and products). However, the level of discontinuity is dependent upon the time between new products and it is not known how many new product observations are missed. One might want to only note discontinuities that in fact are breaks from an existing exponential or power law fit. Objective means for deciding what constitutes a major technological break is also needed to address these questions. Overall, the results reported here give no support to S curves, discontinuities or life cycle hypotheses in regard to technical change.

Our final important topic for future research (that may well greatly extend work on dependent variable metrics) is the linking of technical performance change with productivity changes with time. Although it is widely agreed that technological change is a major source of economic growth (58, 43, 59), there are not approaches that start with quantitative trends in technological performance and proceed to economic growth. This is at least partly due to the difficulty of the problem but the lack of attempts is disappointing. A simpler beginning issue in this regard might be linking technical performance trends with innovation and diffusion. It is widely intuitively understood that the metrics attempt to measure what is better and that what is better is generally what diffuses (33,32) but formal treatment has not been attempted. In fact, most diffusion models implicitly consider the relative performance and cost of a diffusing artifact to be constant so a doable first step might be to eliminate this assumption: using real data would be best as a test of any model so again the quality and availability of such data –one output of this paper- is an essential part of the research program.

5. Conclusions

Twenty-eight technologies (technological domains) are studied in this paper exploring their improvement as a function of time and effort where effort is measured by the annual number of patents published for the technology. A total of 71 different performance metrics were studied for these 28 domains. The paper analyzes issues in empirically determining quantitative technical performance trends in terms of four factors (unit of analysis, dependent variables, independent variables, and data quality) and based upon the results makes recommendations for all four factors relative to future work of this kind.

A major finding is that the results indicate that Moore's Law appears to be more fundamental than Wright's power law for these 28 domains (where performance data are record breakers from numerous designs and different factories). This conclusion is supported by:

1. The performance metrics in all 28 technological domains have strong exponential correlations with time (Moore's law generalization).
2. In contrast, most of these same performance metrics in the 28 domains, have much weaker log-log correlations with patents (Wright's law generalization).
3. Wright's law is followed only in those domains where published patents in the domain show a strong exponential correlation with time. For these domains, Sahal's relationship is followed: $k = w\,g$, where k is the Moore's Law exponent, w the Wright power law exponent and g the patent growth exponent.

We also find that of the sixteen domains for which we have multiple metrics, the intra-domain variation is much smaller than the inter-domain variation. Indeed, in only four of these domains are the variations of performance improvement with time significant when comparing different metrics.

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7. Supplementary Information

In addition to the information presented in this paper, we have compiled key data used into a Microsoft Excel file that can be easily obtained by copying the following link into a web browser where it can be viewed and/or downloaded:

http://bit.ly/mageeetalSIMay2014

This document contains three worksheets which are accessible by clicking the tabs at the bottom of the excel window.

- 28 Domains with k, g and w – this worksheet contains the k, g, and w values along with the r^2 values for each of the regressions for the 28 technological domains.
- 71 domain-metric-pairs with Statistical information – this worksheet includes the 71 domain-metric-pairs for the 28 technological domains (sixteen have more than one metric for which trends are determined) along with the relevant statistical information

- Domain Annual Patenting Rates – this worksheet shows the annual number of patents for each of the 28 domains.