INVESTIGATION OF THE CAUSES OF NEGATIVE DAMPING OF SYNCHRONOUS MACHINES

by

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Graduate House
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May 1, 1947

Professor Joseph S. Newell
Secretary of the Faculty
Massachusetts Institute of Technology

Dear Professor Newell:

In partial fulfillment of the requirements for the degree of Doctor of Science from the Massachusetts Institute of Technology, I hereby submit my thesis entitled, "Investigation of the Causes of Negative Damping of Synchronous Machines."

Respectfully yours,

Signature Redacted

Chung, Shih-Mu

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ABSTRACT

The investigation of the causes of negative damping of synchronous machines is taken in two parts — the mathematical analysis and the experimental verifications. For the mathematical analysis the following important assumptions are used:

(a) The machine is connected to a very large source.

(b) The machine constants are constant.

(c) The core losses are disregarded.

(d) The machine oscillates steadily with small amplitude and small frequency.

With these assumptions the differential equations based upon the Kirchhoff's laws are established for both the armature and the field windings. All the voltage and current quantities in them are expressed in terms of symmetrical components of instantaneous values. Thus we are dealing with complex quantities, and we have similar advantages to those obtained by using complex numbers in alternating-current circuits. In solving the differential equations, the method of superposition and the method of successive reflections are employed. The former method is used to handle the armature and field sources separately, while the latter one is used to avoid the difficulty of solving the equations simultaneously. For the purpose of simplification, the replacement of the armature-circuit resistance by an imaginary inductance has also been used

somewhere in the analysis. From the solutions of the currents the expressions of instantaneous electromagnetic torque are obtained. As the machine under investigation is in steady oscillation in addition to its synchronous speed, we have the angle displacement between the stator and rotor as

$$\theta = \theta_o + \omega t + \delta \sin bt \tag{1}$$

where

θ_o is the initial angle

ω is the synchronous velocity

δ is the amplitude of oscillation

b is equal to 2π times the frequency of oscillation.

Hence the velocity of the machine is

$$\frac{d\theta}{dt} = \omega + \delta\, b \cos bt \tag{2}$$

From the expression of the electromagnetic torque we may select all the terms containing the factor $\delta b \cos bt$ and collect them together as

$$T_d = B\, \delta\, b \cos bt \tag{3}$$

where T_d is the damping torque due to the electromagnetic action, and B is the corresponding damping coefficient. When B is positive, the damping torque T_d will act in such a way as will tend to increase the amplitude of oscillation. Therefore it contributes to the negative damping of the oscillation. On the contrary, if B is negative, it contributes to the positive damping. If B is zero, there will be no effect on the damping due to the electromagnetic action at all. Hence the object of the mathematical analysis is to determine the expressions of B for the different types of constructions of synchronous machines.

In the experimental test, due to its inherent character the synchronous machine can self-oscillate after any electrical or mechanical disturbance is introduced. If there is no net damping on the oscillation, the amplitude will remain constant and the frequency will be fixed by the moment of inertia of the moving system and the synchronizing torque of the synchronous machine. Thus the frequency of oscillation is usually very small in comparison with line frequency, and the amplitude of oscillation can be adjusted at will. When a machine oscillates, besides the damping due to the electromagnetic action there is always some extra damping due to the effects of load, windage, friction, hysteresis and eddy currents, etc. This extra damping is usually positive, i.e., it tends to diminish the amplitude of oscillation. Throughout the test the machine is adjusted for the condition of no net damping; then the damping due to the electromagnetic action is always negative and just enough to compensate the extra positive damping. In the extra damping, the part due to electromagnetic action of the coupled d-c machine can be expressed analytically as

$$B_{d.c.} = -\frac{k\,p\,\phi^2}{2R} \qquad (4)$$

where

$B_{d.c.}$ is the damping coefficient due to the electromagnetic action of the d-c machine k is equal to $\frac{550}{746}$ for the practical units,

ϕ is a constant and equals the ratio of $\frac{e}{\omega}$,

e is the induced emf of the d-c machine,

R is the resistance in the armature circuit of the d-c machine,

p is the number of poles of the synchronous machine under test.

This formula is derived without considering the effect of inductance in the armature circuit of the d-c machine.

If the d-c machine is operated as a motor (i.e., the synchronous machine is operated as generator), R is nearly equal to the armature resistance only; then the damping of the d-c machine will be too large to make the net damping of the system zero or negative. This explains why it was necessary to test the synchronous machines as synchronous motors. The coefficient of the extra damping, excluding the electromagnetic action of the d-c machine, should be practically constant in the tests. It is confirmed fairly well from the obtained results.

The condition of zero net damping of a machine in oscillation can be clearly indicated by the steady oscillation of the reading of a voltmeter which is connected as shown in Fig. 10. If a brush oscillograph is used instead of the voltmeter to record any variation of the amplitude of oscillation, we may also test the machine with either positive or negative resultant damping. The variation of the amplitude should follow the following formula:

$$\delta_2 = \delta_1 \varepsilon^{\frac{B'}{2J}(t_2 - t_1)} \tag{5}$$

where

δ_2, δ_1 are the amplitudes of the oscillation at time t_2 and t_1, respectively,

B' is the resultant damping coefficient,

J is the quotient of the moment of inertia of the moving system divided by the number of pairs of poles of the synchronous machine.

In order to have good verification, B' should not differ very much from zero, and any disturbance in the period between t_2 and t_1 must be avoided.

From the mathematical analysis, the damping coefficient due to the electromagnetic action of cylindrical-rotor synchronous machine without damper windings may be shown as

$$B = 2K \left\{ \frac{2E^2}{\omega\, x_m Z_s} \sin\alpha \cos 2\alpha + \frac{3x_m V\, k_b^2 \sin(\alpha - \delta_o)}{r_b Z_s^2 \omega (k_b^2 + b^2)} \left[2E \sin\alpha - V \sin(\alpha - \delta_o) \right] \right\} \tag{6}$$

If the field terminals are short-circuited instead of connected to the d-c source, E is zero, and the damping coefficient is

$$B = -\frac{6K\, x_m V^2 k_b^2 \sin^2(\alpha - \delta_o)}{r_b Z_s^2 \omega (k_b^2 + b^2)} \tag{7}$$

Hence B is always negative or the electromagnetic action always produces positive damping. Professor H. E. Edgerton explained, in the discussion of the paper, "Effect of Armature Resistance Upon Hunting of Synchronous Machines," by C. F. Wagner, in A.I.E.E., July 1930, that the cause of negative damping of the synchronous machines might be due to the single-phase secondary (field winding) induction-motor action. According to the formula 7, however, that action could not be the cause of negative damping under the assumed conditions.

Andreas Timasheff emphasized strongly in his paper, "Eine Erklärung der Schwingungsanfachung bei Synchronmaschinen," in Siemens Zeitschrift, vol. 15, No. 6, June 1935, that the phenomenon of negative damping of the synchronous machines may be explained by using the analogy of the relation between the speed and the induction-motor torque near standstill (considering short-circuited armature windings as secondary). He derived a formula for the damping coefficient of round-rotor synchronous machines with the notations used in this thesis as

$$B = \frac{4K\ E^2 \sin \alpha \ \cos 2\alpha}{\omega\ x_m Z_s} \tag{8}$$

By comparing the formulas 6 and 8, it is noticed that formula 8 is a part of formula 6. Hence, Timasheff's result explains only a part of the causes of negative damping of cylindrical-rotor synchronous machines without damper windings.

In the paper, "Stability of Synchronous Machines," by C. A. Nickle and C. A. Pierce, in A.I.E.E., vol. 49, January 1930, they analyzed the problem of damping of synchronous machines from the vector diagram and gave an analytic expression for the damping coefficient. If it is expressed with the notations of this thesis, we have the damping coefficient of a round-rotor synchronous machine without damper windings as

$$B = \frac{6K\, x_m V\, k_b^2 \sin(\alpha - \delta_o)}{r_b Z_s^2 \omega (k_b^2 + b^2)} \left[2E \sin\alpha - V \sin(\alpha - \delta\,)_o \right] \qquad (9)$$

This expression is entirely different from the formula 8. It is the other part of the formula 6. Therefore, C. A. Nickle's and C. A. Pierce's result also explains only a part of the causes of the negative damping of synchronous machines.

From the mathematical analysis of this thesis, the coefficient of damping due to the electromagnetic action of salient-pole synchronous machines without damper windings may be given as:

$$B = \frac{6K\, x_m V}{\omega\, r_b} \cdot \frac{k_b^2}{k_b^2 + b^2} \cdot \frac{Z_q^4}{(r_a^2 + x_d x_q)^3}$$

$$\times \left[\begin{array}{l} 2E \sin\alpha_1 - \dfrac{Z_d}{Z_q} V \sin(\alpha_2 - \delta_o) \\ \quad + \dfrac{x_d - x_q}{Z_q} V \sin\alpha_1 \cos(\alpha_1 - \delta_o) \end{array} \right] \sin(\alpha_1 - \delta_o)$$

$$+ \frac{4K\, E^2 r_a (x_d x_q - r_a^2)}{\omega\, x_m (r_a^2 + x_d x_q)^2}$$

$$+ \frac{4K\, E\, r_a (x_d - x_q)}{\omega\, x\, (r_a^2 + x_d x_q)^3} \left(\begin{array}{l} (E - V\cos\delta_o)(2r_a^2 - x_d x_q)x \\ \quad + (V \sin\delta_o)(2x_d x_q - r_a^2) r_a \end{array} \right) \qquad (10)$$

From the results in the paper by C. A. Nickle and C. A. Pierce, we have

$$B = \frac{6K\, x_m V}{\omega\, r_b} \cdot \frac{k_b^2}{k_b^2 + b^2} \cdot \frac{Z_q^4}{(r_a^2 + x_d x_q)^3}$$

$$\times \left[\begin{array}{l} 2E \sin \alpha_1 - \dfrac{Z_d}{Z_q} V \sin(\alpha_2 - \delta_o) \\ + \dfrac{x_d - x_q}{Z_q} V \sin \alpha_1 \cos(\alpha_1 - \delta_o) \end{array} \right] \sin(\alpha_1 - \delta_o) \qquad (11)$$

This expression is still only a part of the formula 10.

For the case of salient-pole synchronous machines with damper windings, the mathematical analysis is given in the Chapter V. The expression of the damping coefficient is very long, and it is not repeated here.

The analytic expression for the damping coefficient of a wound-rotor induction motor operated as a synchronous machine with the symmetrical field excitation may be given as:

$$B = \frac{2K\, E^2 \sin \alpha \cos 2\alpha}{\omega\, x_m Z_s}$$

$$+ \frac{2K\, x_m}{r_b Z_s^2 \omega} \left| \frac{k_b^2}{k_b^2 + b^2} \right| \left[\begin{array}{l} 2E\, V \sin \alpha \sin(\alpha + 2\beta - \lambda - \delta_o) \\ \quad - V^2 \cos(2\beta - \lambda) \end{array} \right]$$

$$\ldots\ldots \quad (12)$$

And Dr. L. Dreyfus gave the corresponding damping coefficient in his paper, "Einführung in die Theorie der selbsterregten Schwingungen synchroner Maschinen," in E.U.M., vol. 29, nos. 16 and 17, April 1911, as (with the notations used in this thesis):

$$B = \frac{2K\,E^2 \sin\alpha \cos 2\alpha}{\omega\, x_m Z_s} + \frac{2K\, x_s}{r_b Z_s^2 \omega}\left[2E\,V \sin\alpha \sin(\alpha - \delta_o) - V^2\right] \tag{13}$$

By comparing the formulas 12 and 13, we can see that if we let

$$b = 0 \tag{14}$$

then

$$\frac{k_b^2}{k_b^2 + b^2} = 1 \tag{15}$$

and

$$2\beta - \lambda = 0 \tag{16}$$

and the two expressions of B will be identical except that where it is x_s in the formula 13, it is x_m in the formula 12.

After investigating all the expressions of the damping coefficient B which have been obtained from the analysis, we can conclude that if there is no resistance in the armature circuit, any kind of synchronous machines will be inherently stable (i.e., with positive damping).

INVESTIGATION OF THE CAUSES OF NEGATIVE DAMPING OF SYNCHRONOUS MACHINES

CHAPTER I

INTRODUCTION

The existence of synchronizing torque and moment of inertia of synchronous machines can only provide the ability to keep the machines in oscillation about their equilibrium positions after some momentary disturbance has been introduced. In order to bring themselves back to their exact synchronous speed, it is necessary to have some positive damping torque to damp off the oscillations. The inherent windage, friction, and eddy currents, etc. will always supply some positive damping, but the electromagnetic action between the field and armature windings may produce either positive or negative damping, depending upon the load conditions, machine constants, and excitation. If the resultant damping effect is negative, the machine will then oscillate with larger and larger amplitude. Such cumulative oscillations will cause instability of the machine. For the treatment of any stability problem of synchronous machines, therefore, the condition of negative damping should be avoided. In the ordinary textbooks treating steady-state stability and transient-state stability of power systems, the synchronous machines in the system have already been assumed to provide enough positive damping to prevent the cumulative oscillations. In practice, fortunately, the amortisseur windings which may have been installed for other reasons usually furnish enough damping for stability except for abnormal conditions. In theory, however, the causes of the negative damping have never been completely examined and satisfactorily explained. This thesis attempts to accomplish this purpose.

As early as 1902, C. P. Steinmetz did first announce the possibility of negative damping of synchronous machines. In the following year, B. Hopkinson noticed the phenomenon of negative damping and pointed out that it was due to the presence of armature resistance. In 1911, Dr. Ludwig Dreyfus made an analysis on uniform air-gap synchronous machines with a damping winding in the quadrature axis having the same constants as that of the d-c field winding. He showed that the high excitation, large value of armature circuit resistance, and low line frequency are the favorable conditions for negative damping. In 1930, C. A. Nickle and C. A. Pierce made a more detailed analysis based on the vector diagram and gave the results that "the damping torque of any synchronous machine can become negative, giving instability, if the armature resistance is increased beyond a critical limiting value. ...... This value, for a salient-pole generator with normal excitation and no amortisseur winding, is

$$r = xq \tan \delta'$$

where r is armature-circuit resistance, xq is quadrature synchronous reactance, and δ' is the steady-state displacement angle. If r is less than this critical limiting value, the damping torque is positive; if greater, negative. ...... The damping of a generator increases in the positive direction with increase in load." In the same year, C. F. Wagner — with the energy point of view — also obtained the same limiting value for armature resistance to make a generator stable. In 1935, A. Timascheff in Germany gave an explanation for the phenomenon of negative damping of synchronous machines by the analogy of induction-motor torque at starting, and gave a very simple expression for the damping coefficient of round-rotor synchronous machines as

$$T_d = -\frac{E^2}{x_s}\,\frac{\sin 4\rho}{4}$$

where T_d is the damping coefficient (it is positive for positive damping), E is excitation emf, x_s is synchronous reactance, and ρ is the complement of the synchronous impedance angle. From this expression we can see that the damping coefficient does not depend on the load, and it is always negative except that the ratio of armature resistance to synchronous reactance is too large.

According to the different results shown above, we can see only one point common to them all, i.e., the armature-circuit resistance is to be blamed for the negative damping of synchronous machines. As to how it depends, the different authors claimed differently. Their disagreement seems to the present writer to be due to the fact that their methods of analysis are not rigorous enough. A more complete and rigorous solution is attempted by the writer in this thesis. He starts the mathematical analysis from the differential equations with self- and mutual inductances of the armature and field windings. The speed of the machine is assumed to vary sinusoidally of small amplitude about its synchronous speed. (The method of analysis can be extended to the cases of large amplitude of oscillations.) The voltages and currents are all expressed in terms of symmetrical components of instantaneous quantities (i.e., they are in complex quantities). The advantages of using such components are the same as the advantages in using complex numbers to solve steady-state problems in a-c circuits. By applying the method of superposition and the method of successive reflections, we are able to find the armature and field currents with each in terms of the first few terms of a Fourier series. In the analysis the stator resistance will be approximated by an imaginary inductance after a few reflections between the armature and field have been taken into account. Then, from the expressions of the currents, we can get the expression of the electromagnetic torque produced during the assigned oscillations. This expression of torque is also in terms of a Fourier series, and that term which is of the same frequency and is in phase with the variation of speed contributes to the damping action under investigation.

It is planned to begin the analysis on the simplest case first (the machine is symmetrically excited and has a uniform air gap). The analysis is then extended to the most complex case, in which the machine has salient poles with damper windings in both axes. By doing it in this way, the method of analysis is understood more easily and the causes of negative damping of synchronous machines more clearly defined. In the analysis, in order to normalize and simplify the problem, some assumptions and approximations have been introduced; to justify them, therefore, some experimental verifications are also included in this thesis. The machines to be analyzed are limited to polyphase machines only.

CHAPTER II

A WOUND-ROTOR INDUCTION MOTOR OPERATED AS A SYNCHRONOUS MACHINE IN OSCILLATION (WITH SYMMETRICAL FIELD EXCITATION)

A uniform air-gap synchronous machine with a damper winding in quadrature axis having the constants the same as those of a d-c field winding can be imitated by a wound-rotor induction motor with symmetrical excitation, so far as the theory is concerned. Then, the machine really provides polyphase windings on both stator and rotor. This facilitates the analysis to a great extent. We shall start to analyze this case first.

2.1 Assumptions for the Analysis

The machine used for the analysis can be shown with a connection diagram as in Fig. 1, where the power system is large and can be considered as balanced infinity bus of sinusoidal wave form.

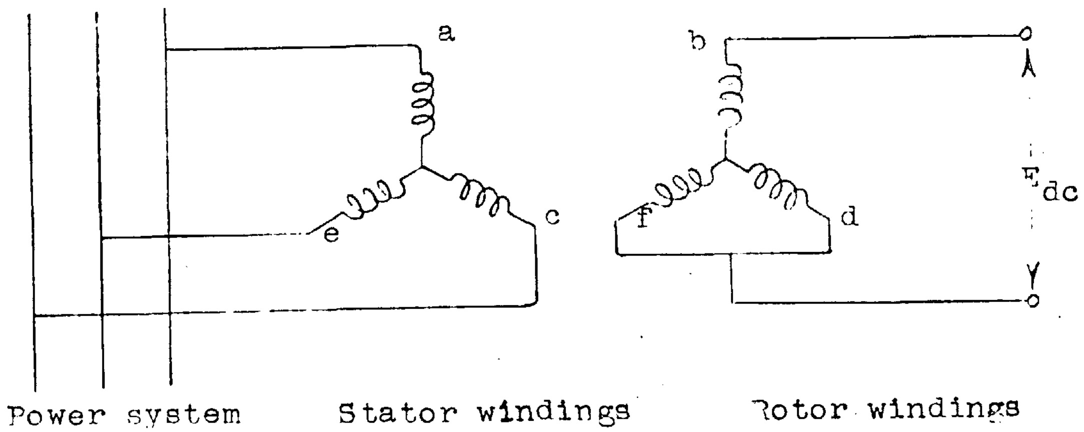


Figure 1, Connection Diagram.

The machine is oscillating sinusoidally with small amplitude and frequency about its equilibrium position in addition to its synchronous speed. By small amplitude it means that the sine of the amplitude does not differ very much from the amplitude. By small frequency it means small in comparison with the line frequency. The exciting source has a constant emf $E_{d.c.}$ which is independent of the machine oscillations. The flux distribution in the air gap is assumed sinusoidal, and the iron losses are neglected. As the machine is under operation of constant applied voltage throughout the investigation, we may consider that the magnetic saturation is nearly fixed at a certain value; then the machine constants used can be considered constant corresponding to the saturation. With these assumptions in mind, we can develop the fundamental equations for the analysis.

2.2 The Fundamental Equations for the Analysis

For three-phase induction machines we have the fundamental differential equations as follows:

$$v_{a1} = (r_a + L_a \frac{d}{dt})\, i_{a1} + \frac{3}{2} M \frac{d}{dt} (i_{b1} \varepsilon^{j\theta}) \tag{1}$$

$$v_{b1} = (r_b + L_b \frac{d}{dt})\, i_{b1} + \frac{3}{2} M \frac{d}{dt} (i_{a1} \varepsilon^{-j\theta}) \tag{2}$$

$$T = j\, K\, (i_{b1} i_{a2} \varepsilon^{j\theta} - i_{b2} i_{a1} \varepsilon^{-j\theta}) \tag{3}$$

where

v_{a1}, v_{b1} are the stator and rotor positive-sequence voltages in terms of instantaneous values referred to phases a and b , respectively.

i_{a1}, i_{b1} are the corresponding positive-sequence currents,

and $i_{a2} = \underset{\wedge}{i}_{a1}$, $i_{b2} = \underset{\wedge}{i}_{b1}$.

r_a , L_a are the stator resistance and stator self-inductance at line frequency.

r_b , L_b are the rotor resistance and rotor self-inductance under d-c conditions.

M is the maximum mutual inductance between one stator phase and one rotor phase.

θ is the position angle between the stator and rotor. It can be any function of time.

T is the electromagnetic torque produced.

j is equal to $\sqrt{-1}$.

K is a constant depending upon the units used. If the units of currents, inductance, and torque are in amp, henry, and lb. ft., respectively, we have

$$K = \frac{\text{no. of poles}}{2} \cdot \frac{550}{746} \cdot \frac{3}{2} M \cdot 3$$

The relative directions of θ and T can be shown as in Fig. 2.

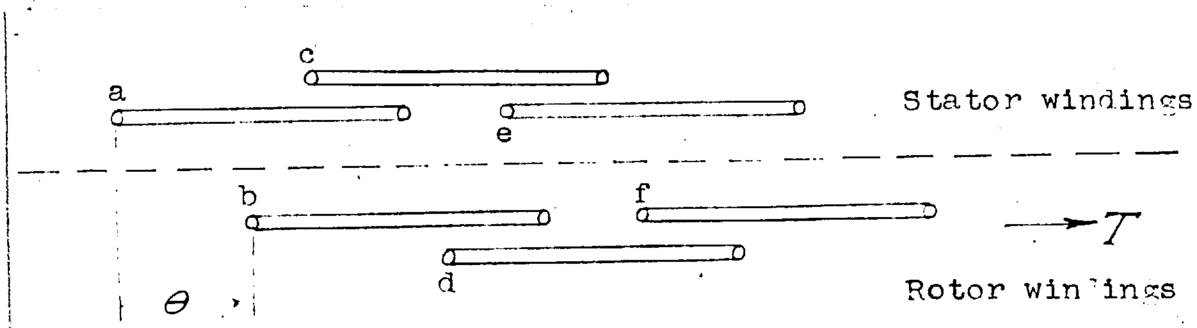


Figure 2, Winding Positions.

Although the eqs 1, 2, and 3 will hold good for θ being any function of time, yet, as we know, there are solutions of elementary functions only when θ is a linear function of time (i.e., the machine is operated at constant speed). For the problems which we are going to investigate, the speed of the machine is not constant but is pulsating. Based upon the assumption we have made,

$$\theta = \theta_0 + \omega t + \delta \sin bt \; \ldots\ldots \tag{4}$$

i.e.,

$$\frac{d\theta}{dt} = \omega + \delta b \cos bt \; \ldots\ldots\ldots \tag{4'}$$

where

θ_o is the initial angle.

ω is the synchronous angular velocity.

δ is the amplitude of oscillation. It is assumed less than 0.2 rad.

b is the angular velocity of oscillation. It is about 3 per cent of ω .

Since the applied voltage is balanced and sinusoidal, we have

$$v_{a1} = V \varepsilon^{j\omega t} \; \ldots\ldots \tag{5}$$

where V is a known complex constant. Its magnitude is equal to half of the maximum value of the applied phase voltage, and its argument is equal to the initial phase angle of the voltage applied to phase a . Due to the fact of symmetrical field excitation, we have

$$v_d = v_f$$

and

$$v_b - v_d = E_{d.c.}$$

Hence
$$v_{b1} = \frac{E_{d.c.}}{3} \tag{6}$$

Substituting the values of θ , v_{a1} , v_{b1} into eqs 1 and 2, we have

$$V\,\varepsilon^{j\omega t} = (r_a + L_a \frac{d}{dt})\, i_{a1} + \frac{3}{2} M \frac{d}{dt}\left[i_{b1}\varepsilon^{j(\theta_o + \omega t + \delta \sin nbt)}\right] \tag{7}$$

$$\frac{E_{d.c.}}{3} = (r_b + L_b \frac{d}{dt})\, i_{b1} + \frac{3}{2} M \frac{d}{dt}\left[i_{a1}\varepsilon^{-j(\theta_o + \omega t + \delta \sin nbt)}\right] \tag{8}$$

By solving the eqs 7 and 8 simultaneously for i_{a1} and i_{b1} and then substituting into eq 3, we can get an expression for the electromagnetic torque produced. From this expression we shall see what the damping depends upon and whether it is positive or negative.

2.3 Method of Successive Reflections for Coupled Circuits

Before solving eqs 7 and 8, a method which I call a method of successive reflections may be introduced. This method may be used to solve problems of coupled circuits when it is difficult to solve the differential equations simultaneously. It is a method of solving the differential equations one at a time for an infinite number of times. The solutions obtained are, then, in forms of infinite series. If the original equations have solutions of elementary functions, the series so obtained will converge to the same functions. Otherwise there is difficulty in summing up the series. Nevertheless, because of the characteristics of most coupled circuits the series will converge very rapidly. Then we may either sum up the first few terms of the series only, or approximate the sum of all the remaining terms in addition to the first few terms, to give the required solutions. The approximate value of the sum of all the remaining terms of the series may be found by solving the equations simultaneously after introducing some simplifying approximations (on the equations themselves after the corresponding reflections have been taken).

To illustrate this method, we may take, for example, a simple case of a static transformer with its secondary short circuited and its primary connected to a source as shown in Fig. 3, where

Z_a, Z_b are the self-impedances.

Z_m is the mutual impedance.

V is the applied voltage.

I_a, I_b are the currents to be solved.

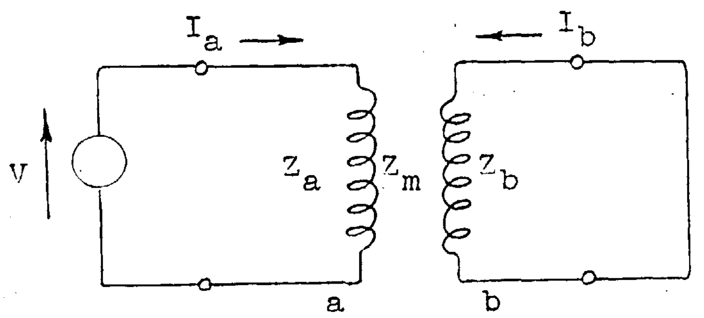


Figure 3, Short-circuited Transformer.

Applying the Kirchhoff's Laws to the two windings of the transformer, we have

$$V = I_aZ_a + I_bZ_m \quad \text{..........} \tag{9}$$

$$0 = I_aZ_m + I_bZ_b \quad \text{..........} \tag{10}$$

These two equations could be solved simultaneously and have exact solutions as

$$I_a = V \frac{Z_b}{Z_a Z_b - Z_m^2} \quad \cdots\cdots\cdots\cdots \quad (11)$$

$$I_b = V \frac{-Z_m}{Z_a Z_b - Z_m^2} \quad \cdots\cdots\cdots\cdots \quad (12)$$

If we want to solve them by the method of successive reflections, we can do it as follows:

First step: Assume the circuit b open and with applied voltage across circuit a, as shown in Fig. 4.

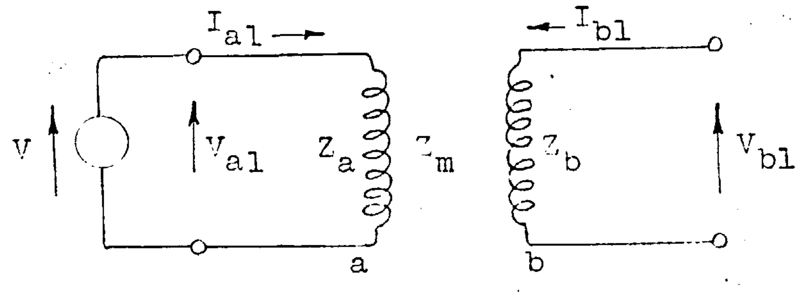


Figure 4, Transformer at No Load.

We have

$$V_{a1} = V$$

$$I_{a1} = \frac{V}{Z_a}$$

$$I_{b1} = 0$$

$$V_{b1} = I_a Z_m = \frac{V}{Z_a} Z_m$$

where the number following the letters in the subnotations indicates the component considered in each step.

Second step: Assume now circuit a open and a voltage of $(-V_{b1})$

is applied to the circuit b, as shown in Fig. 5.

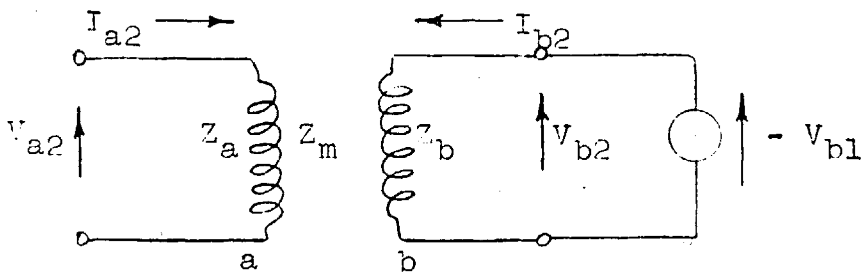


Figure 5, Transformer at No Load.

We have, then,

$$V_{b_2} = -V_{b_1} = -\frac{V}{Z_a} Z_m$$

$$I_{b_2} = \frac{V_{b2}}{Z_b} = -\frac{V}{Z_a} \frac{Z_m}{Z_b}$$

$$I_{a_2} = 0$$

$$V_{a_2} = I_{b_2} Z_m = -\frac{V}{Z_a} \frac{Z_m}{Z_b} Z_m$$

Third step: Assume now circuit b open again and a voltage of $(-V_{a_2})$ is applied to circuit a (i.e., the total voltage now acting on circuit a is $V_{a_1} + V_{a_2} - V_{a_2}$). As is shown in Fig. 6, we have

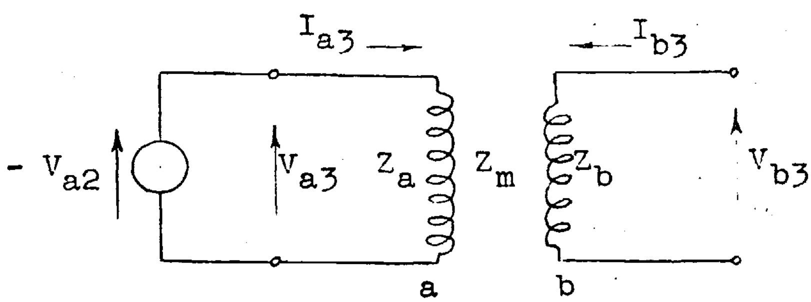


Figure 6, Transformer at No Load.

$$V_{a_3} = -V_{a_2} = \frac{V}{Z_a} \cdot \frac{Z_m}{Z_b} \cdot Z_m$$

$$I_{a_3} = \frac{V_{a_3}}{Z_a} = \frac{V}{Z_a} \cdot \frac{Z_m}{Z_b} \cdot \frac{Z_m}{Z_a}$$

$$I_{b_3} = 0$$

$$V_{b_3} = I_{a_3} Z_m = \frac{V}{Z_a} \cdot \frac{Z_m}{Z_b} \cdot \frac{Z_m}{Z_a} \cdot Z_m$$

Continue on the cycles again and again an infinite number of times. We shall then have the total values of currents and voltages as follows:

$$I_a = I_{a_1} + I_{a_2} + I_{a_3} + \ldots\ldots$$

$$= \frac{V}{Z_a} + 0 + \frac{V}{Z_a} \cdot \frac{Z_m^2}{Z_a Z_b} + 0 + \frac{V}{Z_a}\left(\frac{Z_m^2}{Z_a Z_b}\right)^2 + \ldots$$

$$= \frac{V}{Z_a}\left[1 + \frac{Z_m^2}{Z_a Z_b} + \left(\frac{Z_m^2}{Z_a Z_b}\right)^2 + \ldots\right]$$

$$= \frac{V}{Z_a} \cdot \frac{1}{1 - \frac{Z_m^2}{Z_a Z_b}} = V \frac{Z_b}{Z_a Z_b - Z_m^2}$$

$$I_b = I_{b_1} + I_{b_2} + I_{b_3} + \ldots$$

$$= 0 + \left(-\frac{V}{Z_a}\ \frac{Z_m}{Z_b}\right) + 0 + \left(-\frac{V}{Z_a}\ \frac{Z_m}{Z_b}\ \frac{Z_m^2}{Z_a Z_b}\right)$$

$$+ 0 + \left(-\frac{V}{Z_a}\ \frac{Z_m}{Z_b}\ \frac{Z_m^2}{Z_a Z_b} \cdot \frac{Z_m^2}{Z_a Z_b}\right) + \ldots\ldots$$

$$= -\frac{V}{Z_a}\cdot\frac{Z_m}{Z_b}\left[1 + \frac{Z_m^2}{Z_aZ_b} + \left(\frac{Z_m^2}{Z_aZ_b}\right)^2 + \ldots\right]$$

$$= -\frac{V}{Z_a}\cdot\frac{Z_m}{Z_b}\cdot\frac{1}{1-\frac{Z_m^2}{Z_aZ_b}} = V\,\frac{-Z_m}{Z_aZ_b - Z_m^2}$$

$$V_a = V_{a1} + V_{a2} + V_{a3} + \ldots$$

$$= V + V_{a2} - V_{a2} + V_{a4} - V_{a4} + \ldots = V$$

$$V_b = V_{b1} + V_{b2} + V_{b3} + \ldots$$

$$= V_{b1} - V_{b1} + V_{b3} - V_{b3} + \ldots = 0$$

We can see that the currents are the same as given by expressions 11, 12, and the voltages satisfy the given conditions of the original problem (i.e., with a voltage V applied to the circuit <u>a</u> and with the circuit <u>b</u> short-circuited). For a simple problem like this, of course, the method of successive reflections does not show any advantage at all, and gives an impression of tedious work instead. But for the problems which we are going to investigate, this method will serve as a powerful tool.

2.4 Solutions of the Stator and Rotor Currents when the Machine is in Steady Oscillation

To solve eqs 7 and 8 for i_{a1} and i_{b1}, the method of successive reflections will be used to obtain the components of currents of the first few reflections. After a few reflections have been taken, we can solve the equations simultaneously by approximating the stator resistance with an imaginary inductance to obtain the sums of the remaining components of the currents. That is, r_a is replaced by an imaginary inductance equal to $\frac{r_a}{j\omega}$. This is justified, as the machine is under steady oscillation of small frequency, and since under such conditions the components of stator currents will have frequencies very near to the line frequency. With line frequency, such replacement introduces no error at all. This method was first used by Professor W. V. Lyon to take into account core losses in electric machinery, in the paper "Transient Conditions of Electric Machinery," A.I.E.E. Transactions, 1923.

In order to get the solutions clearer and easier, the method of superposition is also used to handle the stator and rotor sources separately. The difficulty of solving the eqs 7 and 8 arises from the factor $\varepsilon^{j\delta \sin bt}$. It is advisable to express this factor as a Fourier series with Bessel coefficients.

$$\varepsilon^{x} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \ldots$$

Then

$$\begin{aligned}
\varepsilon^{j\delta \sin bt} &= 1 + j\delta \sin bt - \frac{\delta^2}{2}\sin^2 bt \\
&\quad - j\frac{\delta^3}{6}\sin^3 bt + \frac{\delta^4}{24}\sin^4 bt + \ldots \\
&= 1 + j\delta \sin bt - \frac{\delta^2}{2}\left(\frac{1-\cos 2bt}{2}\right) \\
&\quad - j\frac{\delta^3}{6}\sin bt\left(\frac{1-\cos 2bt}{2}\right) \\
&\quad + \frac{\delta^4}{24}\left(\frac{1-\cos 2bt}{2}\right)^2 + \ldots \\
&= 1 + j\delta \sin bt - \frac{\delta^2}{4} + \frac{\delta^2}{4}\cos 2bt \\
&\quad - j\frac{\delta^3}{12}\sin bt + j\frac{\delta^3}{12}\sin bt \cos 2bt \\
&\quad + \frac{\delta^4}{96} - \frac{\delta^4}{48}\cos 2bt + \frac{\delta^4}{96}\cos^2 2bt + \ldots
\end{aligned}$$

$$= 1 + j\delta \sin bt - \frac{\delta^2}{4} + \frac{\delta^2}{4} \cos 2bt$$

$$- j\frac{\delta^3}{12} \sin bt + j\frac{\delta^3}{24} \sin 3bt - j\frac{\delta^3}{24} \sin bt$$

$$+ \frac{\delta^4}{96} - \frac{\delta^4}{48} \cos 2bt + \frac{\delta^4}{192} + \frac{\delta^4}{192} \cos 4bt + \ldots$$

$$= (1 - \frac{\delta^2}{4} + \frac{\delta^4}{96} + \frac{\delta^4}{192} + \ldots)$$

$$+ j\delta\ (1 - \frac{\delta^2}{12} - \frac{\delta^2}{24} + \ldots)\ \sin bt$$

$$+ \frac{\delta^2}{4}\ (1 - \frac{\delta^2}{12} + \ldots)\ \cos 2bt$$

$$+ j\frac{\delta^3}{24} \sin 3bt + \frac{\delta^4}{192} \cos 4bt + \ldots$$

$$= (1 - \frac{\delta^2}{4} + \frac{\delta^4}{64} + \ldots)$$

$$+ j\delta\ (1 - \frac{\delta^2}{8} + \ldots)\ \sin bt$$

$$+ \frac{\delta^2}{4}\ (1 - \frac{\delta^2}{12} + \ldots)\ \cos 2bt$$

$$+ j\frac{\delta^3}{24} \sin 3bt + \frac{\delta^4}{192} \cos 4bt + \ldots$$

$$= \left[(1 - \frac{\delta^2}{4} + \frac{\delta^4}{64} + \ldots) + \frac{\delta^2}{4}\ (1 - \frac{\delta^2}{12} + \ldots)\ \cos 2bt \right.$$

$$\left. + \frac{\delta^4}{192} \cos 4bt + \ldots \right]$$

$$+ j\left[\delta(1 - \frac{\delta^2}{8} + \ldots)\ \sin bt + \frac{\delta^3}{24} \sin 3bt + \ldots \right] \ldots \quad (13)$$

As we have assumed that δ is small, then, from expression 13, by neglecting high harmonics, we can have

$$e^{j\delta \sin nbt} = m + j\delta n \sin bt \ldots\ldots \qquad (14)$$

where

$$m = 1 - \frac{\delta^2}{4} + \frac{\delta^4}{64} + \dots = J_o(\delta) \quad (15)$$

$$n = 1 - \frac{\delta^2}{8} + \dots = \frac{2}{\delta} J_1(\delta) \quad (16)$$

For simplicity, we even may use

$$e^{j\delta \sin bt} = 1 + j\delta \sin bt \dots\dots \quad (17)$$

However, in order to show the principal effect of the value of δ on the damping under investigation, the relation 14 is advisable. If the amplitude δ is too large, we have to take more terms of higher harmonics; then the analysis becomes much more complex.

By applying the method of superposition, we can proceed with the analysis as follows:

A. Due to the field source alone

At present we may consider that there is field source only and the armature terminals are short-circuited. Then, from eqs 7 and 8 we have

$$0 = (r_a + L_a D)i_{a1} + \frac{3}{2} MD(i_{b1}\varepsilon^{j\theta}) \dots\dots \quad (18)$$

$$\frac{E_{d.c.}}{3} = (r_b + L_b D)i_{b1} + \frac{3}{2} MD(i_{a1}\varepsilon^{-j\theta}) \dots\dots \quad (19)$$

where

$$\theta = \theta_o + \omega t + \delta \sin bt$$

$$D \equiv \frac{d}{dt}$$

Then, applying the method of successive reflections, we have:

First Step. Assume at present the stator terminals open. (From now on, we shall use a second number in subnotations to indicate the components in each step, while the first number still denotes the sequence components.) Hence

$$i_{a11} = 0 \tag{20}$$

$$v_{b11} = \frac{E_{d.c.}}{3} = (r_b + L_b D) i_{b11}$$

i.e.,

$$i_{b11} = \frac{E_{d.c.}}{3r_b} \tag{21}$$

Also

$$v_{a11} = \frac{3}{2} MD(i_{b11} \varepsilon^{j\theta}) = \frac{ME_{d.c.}}{2r_b} D(\varepsilon^{j\theta}) \tag{22}$$

Second Step. Assume that the rotor terminals are open and a voltage of $(-v_{a11})$ is applied to the stator. We have

$$v_{a12} = -v_{a11} = -\frac{ME_{d.c.}}{2r_b} D(\varepsilon^{j\theta}) = (r_a + L_a D) i_{a12} \tag{23}$$

Let

$$k_a = \frac{r_a}{L_a} \tag{24}$$

We have the solution of equation 16,

$$i_{a12} = -\frac{ME_{d.c.}}{2r_b L_a} \varepsilon^{-k_a t} \int \varepsilon^{k_a t} \, d(\varepsilon^{j\theta}) \tag{25}$$

As

$$\begin{aligned} \varepsilon^{j\theta} &= \varepsilon^{j(\theta_o + \omega t + \delta \sin bt)} \\ &= \varepsilon^{j(\theta_o + \omega t)} \varepsilon^{j\delta \sin bt} \\ &= \varepsilon^{j(\theta_o + \omega t)} (m + j\delta n \sin bt) \end{aligned}$$

Then

$$d(\varepsilon^{j\theta}) = \begin{bmatrix} j\ \omega\ \varepsilon^{j(\theta_o+\omega t)}(m + j\ \delta\ n\ \sin bt) \\ + j\ \delta\ n\ b\ \varepsilon^{j(\theta_o+\omega t)} \cos bt \end{bmatrix} dt$$

$$= \varepsilon^{j(\theta_o+\omega t)}(j\ \omega\ m - \omega\ \delta\ n\ \sin bt + j\ \delta\ n\ b\ \cos bt)\ dt$$

and

$$\int \varepsilon^{k_a t}\ d(\varepsilon^{j\theta}) = j\ \omega\ m\ \varepsilon^{j\theta_o} \int \varepsilon^{(k_a+j\omega)t}\ dt$$

$$- \omega\ \delta\ n\ \varepsilon^{j\theta_o} \int \varepsilon^{(k_a+j\omega)t} \sin bt\ dt$$

$$+ j\ \delta\ n\ b\ \varepsilon^{j\theta_o} \int \varepsilon^{(k_a+j\omega)t} \cos bt\ dt$$

$$= \frac{j\ \omega}{k_a + j\omega}\ m\ \varepsilon^{(k_a+j\omega)t+j\theta_o}$$

$$- \frac{\omega\ \delta\ n\ \varepsilon^{(k_a+j\omega)t+j\theta_o}}{(k_a + j\omega)^2 + b^2} \left[(k_a + j\omega)\sin bt - b\cos bt \right]$$

$$+ \frac{j\ \delta\ n\ b\ \varepsilon^{(k_a+j\omega)t+j\theta}}{(k_a + j\omega)^2 + b^2} \left[(k_a + j\omega)\cos bt + b\sin bt \right]$$

$$= \frac{j\ \omega}{k_a + j\omega}\ m\ \varepsilon^{(k\ +j\omega)t+j\theta_o}$$

$$+ j\ \frac{\delta\ n\ \varepsilon^{(k_a+j\omega)t+j\theta_o}}{(k_a + j\omega)^2 + b^2} \left[j\omega(k_a + j\omega) + b^2 \right] \sin bt$$

$$+ j\ \frac{\delta\ n\ b\ k_a\ \varepsilon^{(k_a+j\omega)t+j\theta_o}}{(k_a + j\omega)^2 + b^2} \cos bt$$

Then, substituting into eq 25, we have

$$i_{a_{12}} = -\frac{ME_{d.c.}}{2r_bL_a}\left[\frac{j\,\omega}{k_a + j\omega}\;m\,\varepsilon^{j(\omega t+\theta_o)}\right.$$

$$+ j\,\frac{\delta\, n\left(j\omega(k_a + j\omega) + b^2\right)}{(k_a + j\omega)^2 + b^2}\;\varepsilon^{j(\omega t+\theta_o)}\sin bt$$

$$\left.+ j\,\frac{\delta\, n\, k_a b}{(k_a + j\omega)^2 + b^2}\;\varepsilon^{j(\omega t+\theta_o)}\cos bt\right] \tag{26}$$

and

$$i_{a_{12}}\varepsilon^{-j\theta} = i_{a_{12}}\varepsilon^{-j(\theta_o+\omega t+\delta\sin bt)}$$

$$= i_{a_{12}}\varepsilon^{-j(\theta_o+\omega t)}(m - j\,\delta\, n\sin bt)$$

$$= -\frac{ME_{d.c.}}{2r_bL_a}(m - j\,\delta\, n\sin bt)$$

$$X\left[\frac{j\,\omega\, m}{k_a + j\omega} + j\,\frac{\delta\, n\quad j\omega(k_a + j\omega) + b^2}{(k_a + j\omega)^2 + b^2}\sin bt\right.$$

$$\left.+ j\,\frac{\delta\, n\, k_a b}{(k_a + j\omega)^2 + b^2}\cos bt\right]$$

$$= -\frac{ME_{d.c.}}{2r_bL_a}\left\{\frac{j\,\omega\, m^2}{k_a + j\omega} + j\,\frac{\delta\, n\, m\quad j\omega(k_a + j\omega) + b^2}{(k\;\; + j\omega)^2 + b^2}\sin bt\right.$$

$$+ j\,\frac{\delta\, n\, m\, k_a b}{(k_a + j\omega)^2 + b^2}\cos bt + \frac{\delta\,\omega\, m\, n}{k_a + j\omega}\sin bt$$

$$+ \frac{\delta^2 n^2\left[j\omega(k_a + j\omega) + b^2\right]}{(k_a + j\omega)^2 + b^2}\sin^2 bt$$

$$\left.+ \frac{\delta^2 n^2 k_a b}{(k_a + j\omega)^2 + b^2}\cos bt\sin bt\right\}$$

Considering the fact that $b \ll \omega$, we have

$$i_{a_{12}} \varepsilon^{-j\theta} = -\frac{ME_{d.c.}}{2r_b L_a} \left[\frac{j\,\omega\,m^2}{k_a + j\omega} - \frac{\delta\,n\,m\,\omega}{k_a + j\omega} \sin bt \right.$$

$$+ j \frac{\delta\,n\,m\,k_a b}{(k\;\;+ j\omega)^2} \cos bt + \frac{\delta\,n\,m\,\omega}{k_a + j\omega} \sin bt$$

$$+ \frac{\delta^2 n^2 j\omega}{k_a + j\omega} \frac{(1 - \cos 2bt)}{2}$$

$$\left. + \frac{\delta^2 n^2 k_a b}{2(k_a + j\omega)^2} \sin 2bt \right]$$

$$= -\frac{ME_{d.c.}}{2r_b L_a} \left[\frac{j\,\omega}{k_a + j\omega} (m\;\; + \frac{\delta^2 n^2}{2}) + j \frac{\delta\,n\,m\,k_a b}{(k_a + j\omega)^2} \cos bt \right.$$

$$\left. - \frac{\delta^2 n^2 j\omega}{2(k_a + j\omega)} \cos 2bt + \frac{\delta^2 n^2 k_a b}{2(k_a + j\omega)^2} \sin 2bt \right] \qquad (27)$$

From relations 15 and 16 we have

$$(m^2 + \frac{\delta^2 n^2}{2}) = (1 - \frac{\delta^2}{4} + \frac{\delta^4}{64} + \ldots)^2 + \frac{\delta^2}{2} (1 - \frac{\delta^2}{8} + \ldots)^2$$

$$= (1 - \frac{\delta^2}{2} + \frac{\delta^4}{16} + \frac{\delta^4}{32} + \ldots)$$

$$+ \frac{\delta^2}{2} (1 - \frac{\delta^2}{4} + \ldots)$$

$$= (1 + \frac{3}{32} \delta^4 - \frac{1}{8} \delta^4 + \ldots)$$

$$= 1 - \frac{1}{32} \delta^4 + \ldots \doteq 1 \qquad (28)$$

Substituting this relation into eq 27, we get

$$i_{a_{12}}\varepsilon^{-j\theta} = -\frac{ME_{d.c.}}{2r_bL_a}\left\{\frac{j\,\omega}{k_a + j\omega} + j\,\frac{\delta\,n\,m\,k_a b}{(k\ \ + j\omega)^2}\ \cos bt\right.$$

$$\left. - j\,\frac{\delta^2 n^2\omega}{2(k_a + j\omega)}\ \cos 2bt + \frac{\delta^2 n^2 k_a b}{2(k_a + j\omega)^2}\sin 2bt\right\} \qquad (27')$$

Then the rotor voltage will be

$$v_{b_{12}} = \frac{3}{2}\,MD(i_{a_{12}}\varepsilon^{-j\theta})$$

$$= -\frac{3M^2E_{d.c.}}{4r_bL_a}\left\{- j\,\frac{\delta\,n\,m\,k_a b^2}{(k_a + j\omega)^2}\ \sin bt\right.$$

$$+ j\,\frac{\delta^2 n^2\omega\,b}{k_a + j\omega}\ \sin 2bt$$

$$\left. + \frac{\delta^2 n^2 k_a b^2}{(k_a + j\omega)^2}\cos 2bt\right\} \ldots. \qquad (29)$$

and

$$i_{b_{12}} = 0 \qquad (30)$$

<u>Third Step</u>: In order to obtain the sum of the remaining parts of i_{a_1} and i_{b_1}, respectively, we now consider that a voltage of $(-v_{b_{12}})$ is applied to the rotor while the stator terminals are short-circuited. Then we have

$$0 = (r_a + L_aD)\ i'_{a_1} + \frac{3}{2}\,MD(i'_{b_1}\ \varepsilon^{j\theta}) \qquad (31)$$

$$-v_{b_{12}} = (r_b + L_bD)i'_{b_1} + \frac{3}{2}\,MD(i'_{a_1}\ \varepsilon^{-j\theta}) \qquad (32)$$

where

$$i'_{a_1} = i_{a_1} - (i_{a_{11}} + i_{a_{12}})$$

$$i'_{b_1} = i_{b_1} - (i_{b_{11}} + i_{b_{12}})$$

and $v_{b_{12}}$ is given by expression 29.

It is actually as difficult to solve eqs 31 and 32 simultaneously as to solve the eqs 18 and 19 in the beginning. But in eqs 31, 32, the currents and voltages are all small quantities in comparison with the corresponding terms in eqs 18 and 19. In other words, the principal parts of i_{a_1}, i_{b_1} have already been obtained from the first two reflections; so we may apply some simplifying approximation in solving eqs 31 and 32 and introduce little error into i_{a_1} and i_{b_1} at all. By noticing the fact that <u>b</u> is usually about 3 per cent of ω, we may approximate the stator resistance r_a by an imaginary inductance $\frac{r_a}{j\omega}$. Hence we have, from eq 31,

$$0 = \left(\frac{r_a}{j\omega} + L_a\right) D\, i'_{a_1} + \frac{3}{2} MD(i'_b\, \varepsilon^{j\theta}) = 0$$

or

$$\left(\frac{r_a}{j\omega} + L\right) i'_{a_1} + \frac{3}{2} M\, i'_b\, \varepsilon^{j\theta} = 0 \tag{31}$$

Let

$$L_e = \frac{r_a}{j\omega} + L_a \tag{33}$$

We have

$$i'_{a_1} \varepsilon^{-j\theta} = -\frac{\frac{3}{2} M}{L_e}\, i'_{b_1} \tag{34}$$

Substituting eq 34 into eq 32, we get

$$-v_{b_{12}} = (r_b + L_b D) i'_{b_1} - \frac{\left(\frac{3}{2} M\right)^2}{L_e} D\, i'_{b_1}$$

or

$$-\frac{v_{b_{12}}}{L_b} = \left\{ \frac{r_b}{L_b} + \left[1 - \frac{\left(\frac{3}{2} M\right)^2}{L_b L_e} \right] D \right\} i'_{b_1} \tag{35}$$

Let

$$\sigma_b = 1 - \frac{\left(\frac{3}{2} M\right)^2}{L_b L_e} \tag{36}$$

$$k_b = \frac{r_b}{\sigma_b L_b} \tag{37}$$

Then, from eq 35, we have

$$-\frac{v_{b_{12}}}{\sigma_b L_b} = (k_b + D) i'_{b_1}$$

or

$$i'_{b_1} = \varepsilon^{-k_b t} \int \varepsilon^{k_b t} \left(- \frac{v_{b_{12}}}{\sigma_b L_b} \right) dt \tag{35'}$$

Then, from eqs 29 and 35', we get

$$\begin{aligned} i'_{b_1} = {} & j \frac{3M^2 E_{d.c.}}{4 r_b L_a \sigma_b L_b} \cdot \frac{\delta\ n\ m\ k_a b^2}{(k_a + j\omega)^2} \cdot \frac{(b \cos bt - k_b \sin bt)}{k_b^2 + b^2} \\ & - j \frac{3M^2 E_{d.c.}}{4 r_b L_a \sigma\ L_b} \cdot \frac{\delta^2 n^2 \omega b}{k_a + j\omega} \cdot \frac{(2b \cos 2bt - k_b \sin 2bt)}{k_b^2 + 4b^2} \\ & + \frac{3M^2 E_{d.c.}}{4 r_b L_a \sigma_b L_b} \cdot \frac{\delta^2 n^2 k_a b^2}{(k_a + j\omega)^2} \cdot \frac{(k_b \cos 2bt + 2b \sin 2bt)}{k_b^2 + 4b^2} \end{aligned} \tag{38}$$

Substituting eq 38 into eq 34, we get

$$i'_{a1}\varepsilon^{-j\theta} = j\,\frac{9M^3E_{d.c.}}{8r_bL_a\sigma_bL_bL_e}\cdot\frac{\delta\ n\ m\ k_a b^2}{(k_a + j\omega)^2}\cdot\frac{(k_b\ \sin bt - b\cos bt)}{k_b^2 + b^2}$$

$$+ j\,\frac{9M^3E_{d.c.}}{8r_bL_a\sigma_bL_bL_e}\cdot\frac{\delta^2n^2\omega b}{k_a + j\omega}\cdot\frac{(2b\cos 2bt - k_b\sin 2bt)}{k_b^2 + 4b^2}$$

$$- \frac{9M^3E_{d.c.}}{8r_bL_a\sigma_bL_bL_e}\cdot\frac{\delta^2n^2k_a b^2}{(k_a + j\omega)^2}\cdot\frac{(k_b\cos 2bt + 2b\sin 2bt)}{k_b^2 + 4b^2} \tag{39}$$

Therefore, due to field source alone, from eqs 21, 30, and 38 we have

$$i_{b1} = i_{b11} + i_{b12} + i'_{b1}$$

$$= \frac{E_{d.c.}}{3r_b} + j\,\frac{3M^2E_{d.c.}}{4r\ L\ \sigma\ L}\cdot\frac{\delta\ n\ m\ k_a b^2}{(k + j\omega)^2}\cdot\frac{(b\cos bt - k_b\sin bt)}{(k_b^2 + b^2)}$$

$$- j\,\frac{3M^2E_{d.c.}}{4r_bL_a\sigma_bL_b}\cdot\frac{\delta^2n^2\omega b}{k_a + j\omega}\cdot\frac{(2b\cos 2bt - k_b\sin 2bt}{(k_b^2 + 4b^2)}$$

$$+ \frac{3M^2E_{d.c.}}{4r_bL_a\sigma_bL_b}\cdot\frac{\delta^2n^2k_a b^2}{(k_a + j\omega)^2}\cdot\frac{k_b\cos 2bt + 2b\sin 2bt}{(k_b^2 + 4b^2)} \tag{40}$$

and from eqs 20, 27', and 39 we get

$$i_{a1}\varepsilon^{-j\theta} = i_{a11}\varepsilon^{-j\theta} + i_{a12}\varepsilon^{-j\theta} + i'_{a1}\varepsilon^{-j\theta}$$

$$= -\frac{ME_{d.c.}}{2r_bL_a}\cdot\frac{j\omega}{k_a + j\omega} - j\,\frac{\delta\ n\ m\ k_a b\ E_{d.c.}M}{(k_a + j\omega)^2\ 2r_bL_a}\cos bt$$

$$+ j\,\frac{ME_{d.c.}}{2r_bL_a}\cdot\frac{\delta^2n^2\omega}{2(k_a + j\omega)}\cos 2bt - \frac{ME_{d.c.}}{2r_bL_a}\cdot\frac{\delta^2n^2k_a b}{2(k_a + j\omega)^2}\sin 2bt$$

$$+ j\,\frac{9M^3E_{d.c.}}{8r\,L\,\sigma\,L\,L}\cdot\frac{\delta\,n\,m\,k_a b^2}{(k_a + j\omega)^2}\cdot\frac{(k_b \sin bt - b\cos bt)}{(k_b^2 + b^2)}$$

$$+ j\,\frac{9M^3E_{d.c.}}{8r_bL_a\sigma_bL_bL_e}\cdot\frac{\delta^2 n^2\omega b}{(k_a + j\omega)}\cdot\frac{(2b\cos 2bt - k_b \sin 2bt)}{(k_b^2 + 4b^2)}$$

$$- \frac{9M^3E_{d.c.}}{8r_bL_a\sigma_bL_bL_e}\cdot\frac{\delta^2 n^2 k_a b^2}{(k_a + j\omega)^2}\cdot\frac{(k_b\cos 2bt + 2b\sin 2bt)}{(k_b^2 + 4b^2)} \qquad (41)$$

Equations 40 and 41 may be represented in short by

$$i_{b1} = B_o + B_1 \sin(bt + \beta_1) + B_2 \sin(2bt + \beta_2) \qquad (40')$$

$$i_{a1}\varepsilon^{-j\theta} = A_o + A_1 \sin(bt + \alpha_1) + A_2 \sin(2bt + \alpha_2) \qquad (41')$$

where B_o , B_1 , B_2 , A_o , A_1 , A_2 are constants which can be determined easily from eqs 40 and 41.

Since the electromagnetic torque is

$$T = j\,K\,(i_{b1}i_{a2}\varepsilon^{-j\theta} - i_{b2}i_{a1}\varepsilon^{-j\theta}) \qquad (3)$$

then, if only that part of the torque which is varying with the same frequency as the speed is of interest, we may disregard the terms of second harmonics in the expressions 40' and 41' so long as

$$B_2 \ll B_o \qquad (42)$$

$$A_2 \ll A_o \qquad (43)$$

For small values of δ , from eqs 40 and 41, we know that the conditions 42 and 43 are satisfied. So we may express i_{b1} and

$i_{a1}\varepsilon^{-j\theta}$ as

$$i_{b1} = B_o + B_1 \sin(bt + \beta_1) \tag{40''}$$

$$i_{a1}\varepsilon^{-j\theta} = A_o + A_1 \sin(bt + \alpha_1) \tag{41''}$$

and introduce little error into the first harmonic torque to be investigated. Then we have

$$i_{b1} = \frac{E_{d.c.}}{3r_b} + j\,\frac{3M^2E_{d.c.}}{4r_bL_a\sigma_bL_b} \cdot \frac{\delta\ n\ m\ k_a b^2}{(k_a + j\omega)^2} \cdot \frac{(b\cos bt - k_b \sin bt)}{(k_b^2 + b^2)} \tag{44}$$

$$i_{a1}\varepsilon^{-j\theta} = -\frac{E_{d.c.}M}{2r_bL_e} - j\,\frac{E_{d.c.}M\,k_a\delta\ n\ m\ b}{(k_a + j\omega)^2\ 2r_bL_a}\cos bt$$

$$+ j\,\frac{9M^3E_{d.c.}}{8r_bL_a\sigma_bL_bL_e} \cdot \frac{\delta\ n\ m\ k\ b^2(k_b \sin bt - b\cos bt)}{(k_a + j\omega)^2\ (k_b^2 + b^2)} \tag{45}$$

B. Due to the armature source alone

Now we imagine that the armature source alone is applied and the rotor terminals are short-circuited. Then, from eqs 7 and 8,

$$V\,\varepsilon^{j\omega t} = (r_a + L_aD)i_{a1} + \frac{3}{2}MD(i_{b1}\varepsilon^{j\theta}) \tag{46}$$

$$0 = (r_b + L_bD)i_{b1} + \frac{3}{2}MD(i_{a1}\varepsilon^{-j\theta}) \tag{47}$$

In order to solve eqs 46 and 47, we may still use the method of successive reflections as in the case when the field source alone is applied. But for this case, as the frequency of the applied source is of line frequency, the replacement of stator resistance r_a by inductance $\frac{r_a}{j\omega}$ gives no error for the first reflection. Hence the results obtained will be the same by approximating the stator resistance with an imaginary inductance in the beginning and then solving them simultaneously as follows:

From eq 46 we have

$$V\,\varepsilon^{j\omega t} = L_e D\, i_{a1} + \frac{3}{2} MD(i_{b1}\varepsilon^{j\theta})$$

where

$$L_e = \frac{r_a}{j\omega} + L_a$$

i.e.,

$$\int V\,\varepsilon^{j\omega t} dt = \int \left[L_e D\, i_{a1} + \frac{3}{2} MD(i_{b1}\varepsilon^{j\theta}) \right] dt$$

or

$$\frac{V}{j\omega}\,\varepsilon^{j\omega t} = L_e i_{a1} + \frac{3}{2} M\, i_{b1}\varepsilon^{j\theta}$$

Then

$$i_{a1}\varepsilon^{-j\theta} = \frac{1}{L_e}\left(\frac{V}{j\omega}\,\varepsilon^{j(\omega t-\theta)} - \frac{3}{2} M\, i_{b1} \right)$$

$$= \frac{V}{j\omega L_e}\,\varepsilon^{j\phi} - \frac{\frac{3}{2}M}{L_e}\, i_{b1} \tag{48}$$

where

$$\phi = \omega t - \theta \tag{49}$$

Substituting eq 48 into eq 47, we get

$$(r_b + L_b D) i_{b1} + \frac{3}{2} MD \left[\frac{V}{j\omega L_e}\,\varepsilon^{j\phi} - \frac{\frac{3}{2}M}{L_e}\, i_{b1} \right] = 0$$

or

$$(k_b + D) i_{b1} = -\frac{3MV}{2j\omega L_e \sigma_b L_b} D\,\varepsilon^{j\phi}$$

Hence

$$i_{b1} = -\frac{3MV}{2j\omega\, L_e\sigma_b L_b}\ \varepsilon^{-k_b t}\int \varepsilon^{k_b t} d(\varepsilon^{j\phi}) \tag{50}$$

Since

$$\phi = \omega t - \theta = \omega t - (\theta_o + \omega t + \delta \sin bt) = -(\theta_o + \delta \sin bt)$$

Then

$$\varepsilon^{-k_b t}\int \varepsilon^{k_b t}\, d(\varepsilon^{j\theta}) = \frac{-j\,\delta\, n\, \varepsilon^{-j\theta_o}}{k_b^2 + b^2}(b^2 \sin bt + b\, k_b \cos bt) \tag{51}$$

Substituting eq 51 into eq 50, we have

$$i_{b1} = \frac{3MV\,\delta\, n\, \varepsilon^{-j\theta_o}}{2\omega\, L_e L_b \sigma_b (k_b^2 + b^2)}(b^2 \sin bt + bk_b \cos bt) \tag{52}$$

Substituting eq 52 into eq 48, we get

$$i_{a1}\varepsilon^{-j\theta} = \frac{V\,\varepsilon^{j\phi}}{j\,\omega\, L_e} - \frac{(\frac{3}{2}\,M)^2 V\,\delta\,\varepsilon^{-j\theta_o}}{\omega\, L_e^2 \sigma_b L_b (k_b^2 + b^2)}(b^2 \sin bt + b\, k_b \cos bt) \tag{53}$$

C. <u>Both the armature and field sources are applied</u>

By applying the principles of superposition, we know that the rotor and stator currents, when both armature and field sources are applied, are equal to the sums of the respective currents when the two sources are applied one at a time with the other short-circuited. Then, from eqs 44, 45, 52, and 53 we have

$$i_{b1} = \frac{E_{d.c.}}{3r_b} + j\,\frac{3M^2 E_{d.c.} k_a b^2 \delta\, n\, m(b \cos bt - k_b \sin bt)}{4r_b L\,\sigma_b L_b (k_a + j\omega)^2 (k_b^2 + b^2)}$$
$$+ \frac{3MV\,\delta\, n\, \varepsilon^{-j\theta_o}(b^2 \sin bt + b\, k_b \cos bt)}{2\omega\, L_e L_b \sigma_b (k_b^2 + b^2)} \tag{54}$$

$$i_{a1}\varepsilon^{-j\theta} = -\frac{E_{d.c.}M}{2r_bL_e} - j\,\frac{E_{d.c.}M\,k_a\delta\,n\,m\,b}{2r_bL\,(k\;+j\omega)^2}\,\cos bt$$

$$+ j\,\frac{9M^3E_{d.c.}k_ab^2\delta\,n\,m(k_b\sin bt - b\cos bt)}{8r_bL_a\sigma_bL_bL_e(k_a+j\omega)^2(k_b^2+b^2)}$$

$$+\frac{V\,\varepsilon^{j\phi}}{j\omega\,L_e} - \frac{9M^2V\,\varepsilon^{-j\theta_o}\delta\,n(b^2\sin bt + b\,k_b\cos bt)}{4\omega\,L_e\sigma_bL_b^2(k_b^2+b^2)} \tag{55}$$

Let δ_o be the power angle or displacement angle when the machine is in steady synchronous speed, and assume it to be positive for excitation emf leading applied voltage. We have

$$V = (v_{a1})_{t=0} = |V|\,\varepsilon^{j(\frac{\pi}{2}+\theta_o-\delta_o)}$$

$$= j\,|V|\,\varepsilon^{j(\theta_o-\delta_o)} \tag{56}$$

Let the exponential forms of the constants be

$$\frac{1}{L_e} = \frac{1}{\frac{r_a}{j\omega}+L_a} = \left|\frac{1}{L_e}\right|\,\varepsilon^{j\alpha} \tag{57}$$

$$\frac{1}{\sigma_b} = \frac{1}{1-\frac{(\frac{3}{2}M)^2}{L_eL_b}} = \left|\frac{1}{\sigma_b}\right|\,\varepsilon^{j\beta} \tag{58}$$

$$k_b = \frac{r_b}{\sigma_bL_b} = |k_b|\,\varepsilon^{j\beta} \tag{59}$$

$$k_b^2+b^2 = \left|k_b^2+b^2\right|\varepsilon^{j\lambda} \tag{60}$$

$$k_a + j\omega = \frac{r_a}{L_a} + j\omega = \frac{j\omega}{L_a}\left(\frac{r_a}{j\omega} + L_a\right)$$

$$= \frac{j\omega}{L_a} L_e = j \left|\frac{\omega L_e}{L_a}\right| \varepsilon^{-j\alpha} \tag{61}$$

$$\frac{1}{(k \;\; + j\omega)^2} = - \left|\frac{L_a}{\omega L_e}\right|^2 \varepsilon^{j2\alpha} \tag{62}$$

Then we can rewrite the equations 54 and 55 into exponential forms as follows:

$$i_{b1} = \frac{E_{d.c.}}{3r_b} - j \left|\frac{3M^2 E_{d.c.} k_a b^3 \delta \, m \, n}{4r_b L_a L_b \sigma_b (k_a + j\omega)^2 (k_b^2 + b^2)}\right| \varepsilon^{j(\beta+2\alpha-\lambda)} \cos bt$$

$$+ j \left|\frac{3M^2 E_{d.c.} k_a b^2 k_b \delta \, m \, n}{4r_b L_a L_b \sigma_b (k_a + j\omega)^2 (k_b^2 + b^2)}\right| \varepsilon^{j(2\beta+2\alpha-\lambda)} \sin bt$$

$$+ j \left|\frac{3MV \, b^2 \delta \, n}{2\omega \, L_e L_b \sigma_b (k_b^2 + b^2)}\right| \varepsilon^{j(\alpha+\beta-\lambda-\delta_o)} \sin bt$$

$$+ j \left|\frac{3MV \, b \, k_b \delta \, n}{2\omega \, L_e L_b \sigma \, (k_b^2 + b^2)}\right| \varepsilon^{j(\alpha+2\beta-\lambda-\delta_o)} \cos bt \tag{63}$$

$$i_{a1}\varepsilon^{-j\theta} = - \left|\frac{E_{d.c.} M}{2r_b L_e}\right| \varepsilon^{j\alpha} + j \left|\frac{E_{d.c.} M \, k_a b \, \delta \, n \, m}{2r_b L_a (k_a + j\omega)^2}\right| \varepsilon^{j2\alpha} \cos bt$$

$$- j \left|\frac{9M^3 E_{d.c.} k_a \delta \, b^2 k_b m \, n}{8r_b L_a L_b \sigma_b L_e (k_a + j\omega)^2 (k_b^2 + b^2)}\right| \varepsilon^{j(3\alpha+2\beta-\lambda)} \sin bt$$

$$+ j \left|\frac{9M^3 E_{d.c.} k_a \delta \, b^3 m \, n}{8r_b L_a L_b \sigma_b L_e (k_a + j\omega)^2 (k_b^2 + b^2)}\right| \varepsilon^{j(3\alpha+\beta-\lambda)} \cos bt$$

$$+ \left|\frac{V}{\omega L_e}\right| \varepsilon^{j(\alpha-\delta_o)}(m - j\,\delta\, n \sin bt)$$

$$- j \left|\frac{9M^2 V\,\delta\, b^2 n}{4\omega\, L_e^2 \sigma_b L_b (k_b^2 + b^2)}\right| \varepsilon^{j(2\alpha+\beta-\lambda-\delta_o)} \sin bt$$

$$- j \left|\frac{9M^2 V\,\delta\, b\, k_b n}{4\omega\, L_e^2 \sigma_b L\ (k_b^2 + b^2)}\right| \varepsilon^{j(2\alpha+2\beta-\lambda-\delta_o)} \cos bt \qquad (64)$$

From the above expressions of currents we can see that they do not depend upon θ_o , the initial position angle between the stator and rotor windings.

2.5 Electromagnetic torque produced during the steady oscillation

For a machine of uniform air gap, the electromagnetic torque produced can be generally expressed in terms of symmetrical components as given by the eq 3 as

$$T = j\,K(i_{b1} i_{a2} \varepsilon^{j\theta} - i_{b2} i_{a1} \varepsilon^{-j\theta})$$

Let $i_{b1} i_{a2} \varepsilon^{j\theta} = x + j\,y$

where both x and y are real quantities. We have then

$$T = -2K\,y \qquad (65)$$

From the expressions 63 and 64, we get

$$y = \left(\frac{E_{d.c.}}{3r_b}\right)^2 \left|\frac{\frac{3}{2} M}{L_e}\right| \sin\alpha + \frac{E_{d.c.}}{3r_b} \left|\frac{V}{\omega L_e}\right| m \sin(\delta_o - \alpha)$$

$$+ \frac{E_{d.c.}}{3r_b} \left|\frac{V}{\omega L_e}\right| \delta\, n \cos(\delta_o - \alpha) \sin bt$$

$$- \left(\frac{E_{d.c.}}{3r_b}\right)^2 \left| \frac{\frac{3}{2} M\, k_a \delta\, b\, m\, n}{L_a (k_a + j\omega)^2} \right| \cos 2\alpha \cos bt$$

$$+ \frac{E_{d.c.}}{3r_b} \left| \frac{\frac{3}{2} M)^2 V\, b^2 \delta\, n}{\omega\, L_e^2 \sigma_b L_b (k_b^2 + b^2)} \right| \cos(2\alpha + \beta - \lambda - \delta_o) \sin bt$$

$$+ \frac{E_{d.c.}}{3r_b} \left| \frac{\frac{3}{2} M)^2 V\, b\, k_b \delta\, n}{\omega\, L_e^2 \sigma_b L_b (k_b^2 + b^2)} \right| \cos(2\alpha + 2\beta - \lambda - \delta_o) \cos bt$$

$$- \frac{E_{d.c.}}{3r_b} \left| \frac{\frac{3}{2} M)^2 V\, b^2 \delta\, n}{\omega\, L_e^2 \sigma_b L_b (k_b^2 + b^2)} \right| \cos\, (\beta - \lambda - \delta_o) \sin bt$$

$$- \frac{E_{d.c.}}{3r_b} \left| \frac{\frac{3}{2} M)^2 V\, b\, k_b \delta\, n}{\omega\, L_e^2 \sigma_b L_b (k_b^2 + b^2)} \right| \cos(2\beta - \lambda - \delta_o) \cos bt$$

$$+ \left| \frac{\frac{3}{2} M\, V^2 b^2 \delta\, m\, n}{\omega^2 L_e^2 \sigma_b L_b (k_b^2 + b^2)} \right| \cos(\beta - \lambda) \sin bt$$

$$+ \left| \frac{\frac{3}{2} M\, V^2 b\, k_b \delta\, m\, n}{\omega^2 L_e^2 \sigma_b L_b (k_b^2 + b^2)} \right| \cos(2\beta - \lambda) \cos bt$$

$$+ \left(\frac{E_{d.c.}}{3r_b}\right)^2 \left| \frac{\frac{3}{2} M)^3 k_a b^3 \delta\, m\, n}{L_e L_a \sigma_b L_b (k_a + j\omega)^2 (k_b^2 + b^2)} \right| \cos(\beta + \alpha - \lambda) \cos bt$$

$$- \left(\frac{E_{d.c.}}{3r_b}\right)^2 \left| \frac{\frac{3}{2} M)^3 k_a b^2 k_b \delta\, m\, n}{L_e L_a \sigma_b L_b (k_a + j\omega)^2 (k_b^2 + b^2)} \right| \cos(2\beta + \alpha - \lambda) \sin bt$$

$$+ \left(\frac{E_{d.c.}}{3r_b}\right)^2 \left| \frac{\frac{3}{2} M)^3 k_a b^2 k_b \delta\, m\, n}{L_e L_a L_b \sigma_b (k_a + j\omega)^2 (k_b^2 + b^2)} \right| \cos(3\alpha + 2\beta - \lambda) \sin bt$$

$$- \left(\frac{E_{d.c.}}{3r_b} \right)^2 \left| \frac{\frac{3}{2} M)^3 k_a b^3 \delta \ m \ n}{L_e L_a L_b \sigma_b (k_a + j\omega)^2 (k_b^2 + b^2)} \right| \cos(3\alpha + \beta - \lambda) \cos bt$$

$$- \frac{E_{d.c.}}{3r_b} \left| \frac{\frac{3}{2} M)^2 V \ k_a b^3 \delta \ m^2 n}{\omega \ L_e L_a L_b \sigma_b (k_a + j\omega)^2 (k_b^2 + b^2)} \right| \cos(\beta + \alpha - \lambda + \delta_o) \cos bt$$

$$+ \frac{E_{d.c.}}{3r_b} \left| \frac{\frac{3}{2} M)^2 V \ k_a b^2 k_b \delta \ m^2 n}{\omega \ L_e L_a L_b \sigma_b (k_a + j\omega)^2 (k_b^2 + b^2)} \right| \cos(2\beta + \alpha - \lambda + \delta_o) \sin bt$$

$$+ \ F \qquad (66)$$

where F is the sum of terms of small quantities having the factor δ^2 and can be neglected in our analysis.

Let Z_s be the magnitude of the synchronous impedance (including line impedance up to bus). We have

$$|\omega \ L_e| = Z_s \qquad (67)$$

$$\left| \frac{k_a}{L_a (k_a + j\omega)^2} \right| = \frac{\sin \alpha}{Z_s} \qquad (68)$$

Let

$$x_m = \omega \left(\frac{3}{2} M \right) \qquad (69)$$

and E be one-half of the maximum value of the excitation emf. We have

$$E = \omega \left(\frac{3}{2} M \right) \frac{E_{d.c.}}{3r_b} = x_m \left(\frac{E_{d.c.}}{3r_b} \right) \qquad (70)$$

Substituting these relations into eq 66 and rearranging, we have then, with V in real value,

$$y = \frac{E^2}{x_m Z_s} \sin \alpha + \frac{E\ V}{x_m Z_s}\ m \sin(\delta_o - \alpha)$$

$$+ \frac{E\ V}{x_m Z_s} \delta\ n \cos(\delta_o - \alpha) \sin bt$$

$$- \frac{E^2 \delta\ b}{\omega\ x_m Z_s} m\ n \sin \alpha \cos 2\alpha \cos bt$$

$$- 2 \frac{E\ V\ x\ \delta\ b}{r_b Z_s^2} \left| \frac{b\ k_b}{k_b^2 + b^2} \right| n \sin \alpha \sin(\alpha + \beta - \lambda - \delta_o) \sin bt$$

$$- 2 \frac{E\ V\ x_m \delta\ b}{r_b Z_s^2} \left| \frac{k_b^2}{k_b^2 + b^2} \right| n \sin \alpha \sin(\alpha + 2\beta - \lambda - \delta_o) \cos bt$$

$$+ \frac{V^2 x_m \delta\ b}{r_b Z_s^2 \omega} \left| \frac{b\ k_b}{k_b^2 + b^2} \right| m\ n \cos(\beta - \lambda) \sin bt$$

$$+ \frac{V^2 x_m \delta\ b}{r_b Z_s^2 \omega} \left| \frac{k_b^2}{k_b^2 + b^2} \right| m\ n \cos(2\beta - \lambda) \cos bt$$

$$+ 2 \frac{E^2 x_m \delta\ b^2}{r_b Z_s^2 \omega^2} \left| \frac{b\ k_b}{k_b^2 + b^2} \right| m\ n \sin^2\alpha \sin(2\alpha + \beta - \lambda) \cos bt$$

$$- 2 \frac{E^2 x_m \delta\ b^2}{r_b Z_s^2 \omega^2} \left| \frac{k_b^2}{k_b^2 + b^2} \right| m\ n \sin^2\alpha \sin(2\alpha + 2\beta - \lambda) \sin bt$$

$$- \frac{E\ V\ x_m \delta\ b^2}{r_b Z_s^2 \omega^2} \left| \frac{b\ k_b}{k^2 + b^2} \right| m^2 n \sin \alpha \cos(\beta + \alpha - \lambda + \delta_o) \cos bt$$

$$+ \frac{E\ V\ x_m \delta\ b^2}{r_b Z_s^2 \omega^2} \left| \frac{k_b^2}{k^2 + b^2} \right| m\ n \sin \alpha \cos(2\beta + \alpha - \lambda + \delta_o) \sin bt$$

$$\ldots\ldots \qquad (71)$$

For small values of δ, the values of m and n are each substantially as unity. In eq 71, then, we can see that the first two terms give the constant torque; the third term contributes the synchronizing action according to static characteristic; all the other terms having the factor $\sin bt$ will cause a modification of the synchronizing torque due to the machine oscillation; and all the terms having the factor $\cos bt$ give the damping action on the oscillation. In our analysis, therefore, only those terms having the factor $\cos bt$ are of interest.

2.6 Criterion for Negative Damping

As stated above, the damping torque of a synchronous machine comes from the terms having the factor $\cos bt$ in eq 71. Hence, if we let T_d be the damping torque due to electromagnetic action by neglecting eddy-current and hysteresis losses, and B be the corresponding damping coefficient, we have

$$
\begin{aligned}
T_d &= B\,\delta\, b \cos bt \\
&= -2K\,\delta\, b \cos bt \left[-\frac{E^2}{\omega\, x_m Z_s} \sin\alpha \cos 2\alpha \right. \\
&\quad - \frac{2EVx_m}{r_b Z_s^2 \omega} \left| \frac{k_b^2}{k_b^2 + b^2} \right| \sin\alpha \sin(\alpha + 2\beta - \lambda - \delta_o) \\
&\quad + \frac{V^2 x_m}{r_b Z_s^2 \omega} \left| \frac{k_b^2}{k_b^2 + b^2} \right| \cos(2\beta - \lambda) \\
&\quad + 2\,\frac{E^2 x_m b}{r_b Z_s^2 \omega^2} \left| \frac{b\, k_b}{k^2 + b^2} \right| \sin^2\alpha \sin(2\alpha + \beta - \lambda)
\end{aligned}
$$

$$- \frac{E\ V\ x_m b}{r_b Z_s^2 \omega^2} \left| \frac{b\ k_b}{k^2 + b^2} \right| \sin\alpha \cos(\beta + \alpha - \lambda + \delta_o) \Bigg]$$

$$= 2K\ \delta\ b \cos bt \left\{ \frac{E^2}{\omega\ x_m Z_s} \sin\alpha \cos 2\alpha \right.$$

$$+ \frac{x_m}{r_b Z_s^2 \omega} \left| \frac{k_b^2}{k_b^2 + b^2} \right| \Big[2E\ V \sin\alpha \sin(\alpha + 2\beta - \lambda - \delta_o) - V^2 \cos(2\beta - \lambda) \Big]$$

$$\left. + \frac{x_m b}{r_b Z_s^2 \omega^2} \left| \frac{b\ k_b}{k_b^2 + b^2} \right| \Big[E\ V \sin\alpha \cos(\beta + \alpha - \lambda + \delta_o) - 2E^2 \sin^2\alpha \sin(2\alpha + \beta - \lambda) \Big] \right\} \qquad (72)$$

and

$$B = 2K \left\{ \frac{E^2}{\omega\ x_m Z_s} \sin\alpha \cos 2\alpha \right.$$

$$+ \frac{x_m}{r_b Z_s^2 \omega} \left| \frac{k_b^2}{k_b^2 + b^2} \right| \Big[2E\ V \sin\alpha \sin(\alpha + 2\beta - \lambda - \delta_o) - V^2 \cos(2\beta - \lambda) \Big]$$

$$\left. + \frac{x_m b}{r_b Z_s^2 \omega^2} \left| \frac{k_b b}{k_b^2 + b^2} \right| \Big[E\ V \sin\alpha \cos(\beta + \alpha - \lambda + \delta_o) - 2E^2 \sin^2\alpha \sin(2\alpha + \beta - \lambda) \Big] \right\} \qquad (73)$$

When B is positive, T_d acts in the same direction as the increment of velocity; then it produces negative damping. If B is negative, T_d acts in the opposite direction as the increment of velocity; then it produces positive damping. And there will be zero damping due to the electromagnetic action only when $B = 0$. In other words, the criterion for negative damping of a symmetrically excited machine due to the action between the currents in the windings is

$$
\begin{aligned}
&\frac{E^2}{\omega\, x_m Z_s} \sin\alpha \cos 2\alpha \\
&+ \frac{x_m}{r_b Z_s^2 \omega} \left|\frac{k_b^2}{k_b^2 + b^2}\right| \Big[2E\,V \sin\alpha \sin(\alpha + 2\beta - \lambda - \delta_o) \\
&\qquad - V^2 \cos(2\beta - \lambda)\Big] \\
&+ \frac{x_m b}{r_b Z_s^2 \omega^2} \left|\frac{b\, k_b}{k_b^2 + b^2}\right| \Big[E\,V \sin\alpha \cos(\beta + \alpha - \lambda + \delta_o) \\
&\qquad - 2E^2 \sin^2\alpha \sin(2\alpha + \beta - \lambda)\Big] > 0 \qquad (74)
\end{aligned}
$$

2.7 Discussions and Conclusions

(a) If the armature-circuit resistance were zero, i.e., $\sin\alpha = 0$, there would be no possibility of having negative damping.

(b) If the excitation emf is zero, the damping is always positive.

(c) In expression 73, the last part is quite small in comparison with the other parts because of the fact $b \ll \omega$. Hence, for simplicity we may have

$$
\begin{aligned}
B = 2K \Bigg\{ &\frac{E^2}{\omega\, x_m Z_s} \sin\alpha \cos 2\alpha \\
&+ \frac{x_m}{r_b Z_s^2 \omega} \left|\frac{k_b^2}{k_b^2 + b^2}\right| \Big[2E\,V \sin\alpha \sin(\alpha + 2\beta - \lambda - \delta_o) \\
&\qquad - V^2 \cos(2\beta - \lambda)\Big] \Bigg\} \qquad (75)
\end{aligned}
$$

(d) If the armature terminals are short-circuited (i.e., $V = 0$), according to eq 75 the machine has always negative damping for non-excessive armature resistance. Since, with respect to the field flux, the torque-speed relation is similar to that of an induction motor at 100 per cent slip, at that speed the damping of an induction motor is negative if the resistance of secondary windings is not excessively high.

(e) In expression 75 the term proportional to the product EV will contribute to negative or positive damping as $\sin(\alpha + 2\beta - \lambda - \delta_o)$ is positive or negative. Physically it represents the interaction between the two sources when the machine is in oscillation.

(f) In expression 75 we can see that the term proportional to V^2 is much larger than the others except under abnormal conditions; then we can conclude that a symmetrically excited synchronous machine does not ordinarily have negative damping.

(g) From the expression 71 we can see that the modification of synchronizing action due to machine oscillation is principally due to the term

$$\frac{V^2 x_m \delta\, b}{r_b Z_s^2 \omega} \left| \frac{b\, k_b}{k_b^2 + b^2} \right| \cos(\beta - \lambda) \sin bt$$

It makes the synchronizing action stronger during oscillation than at constant speed.

CHAPTER III

A CYLINDRICAL ROTOR SYNCHRONOUS MACHINE WITHOUT DAMPER WINDINGS IN OSCILLATION

The method of analysis used for this case will be the same as that in the second chapter. But since there is a single-phase winding on the rotor instead of a polyphase winding, the differential equations for currents and voltages will be different and also the dampings under investigation.

3.1 The Differential Equations

If we express the voltages and currents in terms of symmetrical components of instantaneous quantities and imagine that there are two-phase windings on the rotor with one winding open (it is then actually a single-phase winding), we have, for a three-phase machine,

$$v_{a1} = (r_a + L_a D)\, i_{a1} + M\, D\, (i_{b1}\varepsilon^{j\theta}) \tag{1}$$

$$v_{b1} = (r_b + L_b D)\, i_{b1} + \frac{3}{2} M\, D\, (i_{a1}\varepsilon^{-j\theta}) \tag{2}$$

and

$$v_{a1} = V\, \varepsilon^{j\omega t} \tag{3}$$

$$v_b = v_{b1} + v_{b2} = E_{d.c.} \tag{4}$$

$$i_{b1} = i_{b2} = \frac{i_b}{2} \tag{5}$$

From the eqs 1 and 2 and the conjugate of eq 2, together with the relations 3, 4, and 5, we have

$$V \varepsilon^{j\omega t} = (r_a + L_a D)\, i_{a1} + \frac{M}{2}\, D(i_b \varepsilon^{j\theta}) \qquad (1')$$

$$E_{d.c.} = (r_b + L_b D)\, i_b + \frac{3}{2} M\, D\, (i_{a1}\varepsilon^{-j\theta} + i_{a2}\varepsilon^{j\theta}) \qquad (2')$$

and the electromagnetic torque produced is

$$T = j\, K\, (i_{a2}\varepsilon^{j\theta} - i_{a1}\varepsilon^{-j\theta})\, i_b \qquad \text{lb.ft.} \qquad (6)$$

where $K = \frac{\text{poles}}{2} \cdot \frac{550}{746} \cdot \frac{3M}{2}$ if the currents, inductance, and torque are in amperes, henries, and lb.ft., respectively.

The eqs 1', 2', and 6 are the fundamental equations of the analysis for a three-phase cylindrical-rotor synchronous machine without damper windings.

3.2 Solutions of Stator and Rotor Currents when the Machine is in Oscillation

The solutions of currents of eqs 1' and 2' can be obtained by applying the method of superposition to handle the armature and field sources separately.

A. Due to d-c source alone

The currents due to d-c source alone should satisfy the following two equations.

$$0 = (r_a + L_a D)\, i_{a1} + \frac{M}{2}\, D(i_b \varepsilon^{j\theta}) \qquad (7)$$

$$E_{d.c.} = (r_b + L_b D)\, i_b + \frac{3}{2} M\, D\, (i_{a1}\varepsilon^{-j\theta} + i_{a2}\varepsilon^{j\theta}) \qquad (8)$$

where

$$\theta = \theta_o + \omega t + \delta \sin bt$$

To solve eqs 7 and 8, we may again apply the method of successive reflections as follows:

First step: Assume the stator terminals open. (From now on, we shall use the same notations for stator current as in the second chapter. For the rotor current, since we shall deal with the total quantity instead of the symmetrical components, the numbers in the subnotation will indicate only the components in each step of the successive reflections.) We have

$$i_{a11} = 0 \tag{9}$$

and

$$E_{d.c.} = (r_b + L_b D) i_{b1}$$

or

$$i_{b1} = \frac{E_{d.c.}}{r_b} \tag{10}$$

where i_{b1} is the component of i_b in the first step; it is not the positive-sequence component of i_b.

$$\begin{aligned} v_{a11} &= \frac{M}{2} D(i_{b1} \varepsilon^{j\theta}) \\ &= \frac{M\, E_{d.c.}}{2r_b} D(\varepsilon^{j\theta}) \end{aligned} \tag{11}$$

Second step: Assume rotor terminals open.

$$i_{b2} = 0 \tag{12}$$

and

$$-\frac{M\, E_{d.c.}}{2r_b} D(\varepsilon^{j\theta}) = (r_a + L_a D) i_{a12} \tag{13}$$

By discarding the small terms of second harmonics, we have from the solution of eq 13,

$$i_{a12}\varepsilon^{-j\theta} = -\frac{M\,E_{d.c.}}{2r_bL_a}\left[\frac{j\omega}{k_a + j\omega} + j\,\frac{\delta\,n\,m\,k_a b}{(k_a + j\omega)^2}\ \cos bt\right] \tag{14}$$

$$v_{b2} = \frac{3}{2}\,M\,D\left[i_{a12}\varepsilon^{-j\theta} + i_{a22}\varepsilon^{j\theta}\right]$$

$$= \frac{3M^2E_{d.c.}}{2r_bL_a}\cdot\frac{k_a b^2\delta\,m\,n}{k_a^2 + \omega^2}\sin 2\alpha\,\sin bt \tag{15}$$

Third step: The remaining parts of the currents should satisfy the following equations:

$$0 = (r_a + L_aD)i'_{a1} + \frac{M}{2}D(i'_b\varepsilon^{j\theta}) \tag{16}$$

$$-v_{b2} = (r_b + L_bD)i'_b + \frac{3}{2}\,M\,D(i'_{a1}\varepsilon^{-j\theta} + i'_{a2}\varepsilon^{j\theta}) \tag{17}$$

By approximating the stator resistance r_a with an imaginary inductance $\frac{r_a}{j\omega}$, and letting

$$L_e = \frac{r_a}{j\omega} + L_a\ ,$$

eq 16 becomes

$$0 = L_eD\,i'_{a1} + \frac{M}{2}D(i'_b\varepsilon^{j\theta}) \tag{16 }$$

i.e.,

$$i'_{a1} = -\frac{M}{2L_e}\ i'_b\varepsilon^{j\theta}$$

or

$$i'_{a1}\varepsilon^{-j\theta} = -\frac{M}{2L_e}\,i'_b \tag{18}$$

Then

$$i'_{a1}\varepsilon^{-j\theta} + i'_{a2}\varepsilon^{j\theta} = -\frac{M\cos\alpha}{|L_e|}\, i_b \tag{19}$$

Substituting eq 19 into eq 17, we have

$$-v_{b2} = (r_b + L_b D)i'_b + \frac{3}{2} M\, D\left(-\frac{M\cos\alpha}{|L_e|}\, i'_b\right)$$

$$= \left[r_b + L_b\left(1 - \frac{\frac{3}{2}M^2\cos\alpha}{|L_e|\ L_b}\right)D\right] i'_b \tag{20}$$

Let

$$\sigma_b = 1 - \frac{\frac{3}{2}M^2\cos\alpha}{|L_e|\ L_b} \tag{21}$$

$$k_b = \frac{r_b}{\sigma_b L_b} \tag{22}$$

where σ_b , k_b are all real quantities. With eqs 21 and 22 we can rewrite eq 20 as

$$-\frac{v_{b2}}{\sigma_b L_b} = (k_b + D)i'_b \tag{20'}$$

Hence

$$i'_b = \varepsilon^{-k_b t}\int \varepsilon^{k_b t}\left(-\frac{v_{b2}}{\sigma_b L_b}\right) dt \tag{23}$$

Substituting eq 15 into eq 23, and then integrating, we get

$$i'_b = -\frac{3M^2 E_{d.c.}}{2r_b L_a \sigma_b L_b} \cdot \frac{k_a b^2 \delta\ m\ n \sin 2\alpha}{(k_a^2 + \omega^2)(k_b^2 + b^2)}\,(k_b \sin bt - b\cos bt) \tag{24}$$

Substitute eq 24 into eq 18 and get

$$i'_{a1}\varepsilon^{-j\theta} = \frac{3M^3 E_{d.c.}}{4r_b L_a L_e \sigma_b L_b} \cdot \frac{k_a b^2 \delta\ m\ n \sin 2\alpha}{(k_a^2 + \omega^2)(k_b^2 + b^2)}\,(k_b \sin bt - b\cos bt) \tag{25}$$

Therefore, from eqs 9, 10, 12, 14, 24, and 25, we have the currents due to the field source alone as

$$i_b = i_{b1} + i_{b2} + i_b'$$

$$= \frac{E_{d.c.}}{r_b} - \frac{3M^2 E_{d.c.}}{2r_b L_a \sigma_b L_b} \cdot \frac{k_a b^2 \delta \text{ m n } \sin 2\alpha}{(k_a^2 + \omega^2)(k_b^2 + b^2)} (k_b \sin bt - b \cos bt) \quad \text{.... (26)}$$

and

$$i_{a1}\varepsilon^{-j\theta} = i_{a11}\varepsilon^{-j\theta} + i_{a12}\varepsilon^{-j\theta} + i_{a1}'\varepsilon^{-j\theta}$$

$$= -\frac{M\ E_{d.c.}}{2r_b L_a} \left(\frac{j\omega}{k_a + j\omega} + j \frac{\delta \text{ n m } k_a b}{(k\ + j\omega)^2} \cos bt \right)$$

$$+ \frac{3M^3 E_{d.c.}}{4r_b L_a L_e \sigma_b L_b} \cdot \frac{k_a b\ \delta^2 \text{m n } \sin 2\alpha}{(k_a^2 + \omega^2)(k_b^2 + b^2)} (k_b \sin bt - b \cos bt) \quad \text{.... (27)}$$

B. <u>Due to the armature source alone</u>

The currents due to armature source alone with field terminals short-circuited should satisfy the following two equations:

$$V\ \varepsilon^{j\omega t} = (r_a + L_a D) i_{a1} + \frac{M}{2} D(i_b \varepsilon^{j\theta}) \quad (28)$$

$$0 = (r_b + L_b D) i_b + \frac{3M}{2} D(i_{a1}\varepsilon^{-j\theta} + i_{a2}\varepsilon^{j\theta}) \quad (29)$$

Since for the exact line frequency of current in stator winding the replacement of the stator resistance r_a by an imaginary inductance $\frac{r_a}{j\omega}$ introduces no error at all, then for this case we can obtain the same results by making this replacement in the beginning and then solving the equations simultaneously instead of applying the method of successive reflections.

From eq 28 we have

$$V\,\varepsilon^{j\omega t} = L_e D\, i_{a1} + \frac{M}{2} D(i_b \varepsilon^{j\theta})$$

or

$$i_{a1} = \frac{1}{L_e}\left[\frac{V\,\varepsilon^{j\omega t}}{j\omega} - \frac{M}{2}\, i_b \varepsilon^{j\theta}\right]$$

that is,

$$i_{a1}\varepsilon^{-j\theta} = \frac{1}{L_e}\left[\frac{V\,\varepsilon^{j\phi}}{j\omega} - \frac{M}{2}\, i_b\right] \tag{30}$$

where

$$\phi = \omega t - \theta = -\,\theta_o - \delta \sin bt \tag{31}$$

$$V = (v_{a1})_{t=0} = |V|\ \varepsilon^{j(90^o+\theta_o-\delta_o)}$$

$$= j\ |V|\ \varepsilon^{j(\theta_o-\delta_o)} \tag{32}$$

$$\frac{1}{L_e} = \frac{1}{|L_e|}\,\varepsilon^{j\alpha} \tag{33}$$

Substituting eqs 31, 32, and 33 into 30, we have

$$i_{a1}\varepsilon^{-jO} = \left|\frac{V}{\omega\, L_e}\right|\ \varepsilon^{j(\alpha-\delta_o-\delta \sin bt)} - \left|\frac{M}{2L_e}\right|\, i_b\varepsilon^{j\alpha}$$

$$= \left|\frac{V}{\omega\, L_e}\right|\ \varepsilon^{j(\alpha-\delta_o)}(m - j\ n\ \delta\ \sin\ bt) - \left|\frac{M}{2L_e}\right|\, i_b\varepsilon^{j\alpha} \tag{34}$$

Hence

$$\frac{3}{2}\, M\, D\, (i_{a1}\varepsilon^{-j\theta} + i_{a2}\varepsilon^{j\theta})$$

$$= 3M\ n\ \delta\ b\ \left|\frac{V}{\omega\, L_e}\right|\ \sin(\alpha - \delta_o)\cos\ bt - \frac{3M^2 \cos\,\alpha}{2\ L_e}\, D\ i_b \tag{35}$$

From eqs 29 and 35 we get

$$r_b + L_b\left(1 - \frac{3M^2 \cos\alpha}{2L_b\,|L_e|}\right) D \quad i_b$$

$$= \; - 3M\, n\, \delta\, b \left|\frac{V}{\omega\, L_e}\right| \sin(\alpha - \delta_o)\cos bt$$

or

$$(k_b + D)i_b = \; - \frac{3M\, n\, \delta\, b}{\sigma_b L_b} \left|\frac{V}{\omega\, L_e}\right| \sin(\alpha - \delta_o)\cos bt$$

i.e.,

$$i_b = \; - \frac{3M\, n\, \delta\, b}{\sigma_b L_b} \left|\frac{V}{\omega\, L_e}\right| \sin(\alpha - \delta_o)\; \varepsilon^{-k_b t}\int \varepsilon^{k_b t} \cos bt\; dt$$

$$= \; - \frac{3M\, n\, \delta\, b}{\sigma_b L_b} \left|\frac{V}{\omega\, L_e}\right| \sin(\alpha - \delta_o) \;.\; \frac{k_b \cos bt + b \sin bt}{(k_b^2 + b^2)} \qquad (36)$$

Substituting eq 36 into eq 34, we get

$$i_{a1}\varepsilon^{-j\theta} = \left|\frac{V}{\omega\, L_e}\right| \varepsilon^{j(\alpha-\delta_o)} \;(m - j\, n\, \delta \sin bt)$$

$$+ \frac{3M^2 n\, \delta\, b}{2\sigma_b L_b \omega} \left|\frac{V}{L_e^2}\right| \sin(\alpha - \delta_o)\varepsilon \quad . \frac{k_b \cos bt + b \sin bt}{(k_b^2 + b^2)}$$

.... (37)

C. <u>Both the armature and field sources are applied.</u>

From expressions 26, 27, 36, and 37, we have then the stator and rotor currents, when both the armature and field sources are applied, as follows:

$$i_b = \frac{E_{d.c.}}{r_b} - \frac{3M^2 E_{d.c.}}{2r_b L_a \sigma_b L_b} \cdot \frac{k_a b^2 \delta\ m\ n \sin 2\alpha}{(k^2 + \omega^2)(k^2 + b^2)} (k_b \sin bt - b \cos bt)$$

$$- \frac{3M\ n\ \delta\ b}{\sigma_b L_b} \left|\frac{V}{\omega\ L_e}\right| \sin(\alpha - \delta_o) \cdot \frac{k_b \cos bt + b \sin bt}{(k_b^2 + b^2)}$$

.... (38)

$$i_{a1}\varepsilon^{-j\theta} = -\frac{M\ E_{d.c.}}{2r_b L_e} - j \frac{M\ E_{d.c.} \delta\ n\ m\ k_a b}{2r_b L\ (k\ + j\omega)^2} \cos bt$$

$$\cdot \frac{3M^3 E_{d.c.} k_a b^2 \delta\ m\ n \sin 2\alpha}{4r_b L_a L_e \sigma_b L_b (k_a^2 + \omega^2)(k_b^2 + b^2)} (k_b \sin bt - b \cos bt)$$

$$+ \left|\frac{V}{\omega\ L_e}\right| \varepsilon^{j(\alpha - \delta_o)} (m - j\ n\ \delta \sin bt)$$

$$+ \frac{3M^2 n\ \delta\ b}{2\sigma_b L_b \omega} \left|\frac{V}{L_e^2}\right| \sin(\alpha - \delta\ )\ \varepsilon^{j\alpha} \cdot \frac{(k_b \cos bt + b \sin bt)}{(k_b^2 + b^2)}$$

.... (39)

3.3 Electromagnetic Torque Produced During the Steady Oscillations

Since the expression of electromagnetic torque is given by eq 6 as

$$T = j\ K(i_{a2}\varepsilon^{j\theta} - i_{a1}\varepsilon^{-j\theta})\ i_b$$

then, if we let

$$i_b i_{a1} \varepsilon^{-j\theta} = x + j\ y \qquad (40)$$

we have

$$T = 2K\,y \tag{41}$$

From eqs 38 and 39 we have

$$y = -\left(\frac{E_{d.c.}}{r_b}\right)^2 \left|\frac{M}{2L_e}\right| \sin\alpha + \frac{E_{d.c.}}{r_b}\left|\frac{V}{\omega L_e}\right| m \sin(\alpha - \delta_o)$$

$$- \frac{E_{d.c.}}{r_b}\left|\frac{V}{\omega L_e}\right| n\,\delta \cos(\alpha - \delta_o) \sin bt$$

$$+ \left(\frac{E_{d.c.}}{r_b}\right)^2 \frac{M\,k_a b\,\delta\,m\,n}{2L_a(k_a^2 + \omega^2)} \cos 2\alpha \cos bt$$

$$+ \frac{3M^2 E_{d.c.}\,n\,\delta\,b}{r_b\sigma_b L_b\omega}\left|\frac{V}{L_e^2}\right| \sin\alpha \sin(\alpha - \delta_o) \cdot \frac{(k_b\cos bt + b\sin bt)}{(k_b^2 + b^2)}$$

$$- \frac{3M\,m\,n\,\delta\,b}{\sigma_b L_b\omega^2}\left|\frac{V}{L_e}\right|^2 \sin^2(\alpha - \delta_o)\,\frac{(k_b\cos bt + b\sin bt)}{(k_b^2 + b^2)}$$

$$+ \frac{3M^3 E_{d.c.}^2 k_a b^2\delta\,m\,n \sin\alpha \sin 2\alpha}{2r_b^2 L_a\,|L_e|\,\sigma_b L_b(k_a^2 + \omega^2)(k_b^2 + b^2)}(k_b\sin bt - b\cos bt)$$

$$- \frac{3M^2 E_{d.c.} k_a b^2\delta\,m^2 n \sin 2\alpha \sin(\alpha - \delta_o)}{2r_b L_a\sigma_b L_b\omega(k_a^2 + \omega^2)(k_b^2 + b^2)}\left|\frac{V}{L_e}\right|(k_b\sin bt - b\cos bt)$$

$$+ F \tag{42}$$

where F is the sum of terms of small quantities having the factor δ^2 and can be discarded in our analysis.

Let V represent its magnitude and

$$Z_s = \left| \omega L_e \right|$$

$$x_m = \omega M$$

$$E = (\omega M) \frac{E_{d.c.}}{2r}$$

Then, from eq 42, with all in real values, we have

$$y = -\frac{2E^2}{x_m Z_s} \sin\alpha + 2\frac{E\,V}{x_m Z_s} m \sin(\alpha - \delta_o)$$

$$- \frac{2E\,V}{x_m Z_s} n\,\delta \cos(\alpha - \delta_o) \sin bt$$

$$+ \frac{2E^2 \delta\, b}{\omega\, x_m Z_s} m\,n \sin\alpha \cos 2\alpha \cos bt$$

$$+ \frac{6E\,V\,x_m k_b \delta\, b}{r_b Z_s \omega^2} n \sin\alpha \sin(\alpha - \delta_o) \cdot \frac{(k_b \cos bt + b \sin bt)}{(k_b^2 + b^2)}$$

$$- \frac{3V^2 x_m k_b \delta\, b}{r_b Z_s^2 \omega} m\,n \sin^2(\alpha - \delta_o) \cdot \frac{(k_b \cos bt + b \sin bt)}{(k_b^2 + b^2)}$$

$$+ \frac{6E^2 x\, k_b b^2 \delta}{r_b Z_s^2 \omega^2} m\,n \sin^2\alpha \sin 2\alpha \frac{(k_b \sin bt - b \cos bt)}{(k_b^2 + b^2)}$$

$$- \frac{3E\,V\,x_m k_b b^2 \delta}{r_b Z_s^2 \omega^2} m^2 n \sin\alpha \sin 2\alpha \sin(\alpha - \delta_o) \cdot \frac{(k_b \sin bt - b \cos bt)}{(k_b^2 + b^2)}$$

.... (43)

From eq 43 we can see that the first two terms contribute the steady-state constant torque; the third term causes the synchronizing action according to the static characteristic; all the other terms having the factor $\sin bt$ give the modification of synchronizing action due to the presence of oscillations; and all the terms having the factor $\cos bt$ produce torque varying in phase with the velocity and thus represent the damping action.

3.4 Criterion for Negative Damping

The angular velocity of the machine is

$$\frac{d\theta}{dt} = \omega + \delta\, b \cos bt$$

Then, if we let T_d and B be the damping torque and damping coefficient, respectively, we have

$$T_d = B\, \delta\, b \cos bt \tag{44}$$

From eqs 41 and 43, therefore, the damping coefficient due to the electromagnetic action is

$$B = 2K \left\{ \frac{2E^2 m\, n}{\omega\, x_m Z_s} \sin\alpha \cos 2\alpha \right.$$

$$+ \frac{3x_m V\, n}{r_b Z_s^2 \omega} \; \frac{k_b^2}{k_s^2 + b^2} \sin(\alpha - \delta_o) \left[2E \sin\alpha - m\, V \sin(\alpha - \delta_o) \right]$$

$$\left. + \frac{3x_m E\, m\, n\, b}{r_b Z_s^2 \omega^2} \; \frac{k_b b}{k_b^2 + b^2} \sin\alpha \sin 2\alpha \left[m\, V \sin(\alpha - \delta_o) - 2E \sin\alpha \right] \right\}$$

$$= 2K \left\{ \frac{2E^2 m\ n}{\omega\ x_m Z_s} \sin\alpha \cos 2\alpha \right.$$

$$+ \frac{3x_m V\ n\ k_b^2}{r_b Z_s^2 \omega (k_b^2 + b^2)} \left[\sin(\alpha - \delta\ ) - \frac{m\ E\ b^2}{V\ \omega\ k_b} \sin\alpha \sin 2\alpha \right]$$

$$\left. \cdot \left[2E \sin\alpha - m\ V \sin(\alpha - \delta_o) \right] \right\} \tag{45}$$

For small values of δ, both m and n are substantially equal to unity, and for an ordinary synchronous machine $b \ll \omega$. Hence we can simplify the expression 45 as

$$B = 2K \left\{ \frac{2E^2}{\omega\ x_m Z_s} \sin\alpha \cos 2\alpha \right.$$

$$\left. + \frac{3x_m V\ k_b^2}{r_b Z_s^2 \omega (k_b^2 + b^2)} \sin(\alpha - \delta_o) \left[2E \sin\alpha - V \sin(\alpha - \delta_o) \right] \right\} \tag{45'}$$

When B is positive, we have $T_d > 0$. That means the damping torque acts in the same direction as the change of the velocity of the machine. Hence, if the machine starts to oscillate, the electromagnetic action tends to enlarge the amplitude of oscillation. It is then negative damping. Therefore the criterion of negative damping due to electromagnetic action of a cylindrical-rotor synchronous machine without damper windings is $B > 0$, or

$$\left\{ \frac{2E^2}{\omega\ x_m Z_s} \sin\alpha \cos 2\alpha \right.$$

$$\left. + \frac{3x_m V\ k_b^2}{r_b Z_s^2 \omega (k_b^2 + b^2)} \sin(\alpha - \delta_o) \left[2E \sin\alpha - V \sin(\alpha - \delta_o) \right] \right\} > 0$$

$$\ldots\ (46)$$

3.5 Discussions and Conclusions

(a) If the resistance of the armature circuit were zero (i.e., $\alpha = 0$), there would be positive damping.

(b) From eq 46 we can see that the damping coefficient consists of three parts, namely, (1) a part proportional to the square of the excitation emf, (2) a part proportional to the product of the applied voltage and excitation emf, and (3) a part proportional to the square of the applied voltage. The first part always contributes to the negative damping except when the armature resistance is greater than the armature reactance. It exists even when the armature terminals are short-circuited. Hence it represents the electromagnetic action which is similar to that of an induction motor at about 100 per cent slip.

The second part represents the damping due to interaction of the two applied sources during the oscillation of the machine. It is zero when either the armature or the field terminals are short-circuited. It is positive or negative, depending on α being greater or less than δ_o.

The third part represents the electromagnetic action of an induction motor with single-phase secondary near zero slip. It always causes positive damping.

(c) If the machine has positive damping at small loads, it will have positive damping at larger loads also.

(d) High excitation promotes the negative damping when the machine runs as a motor.

(e) From (c) and (d) we can see why the damping of an over-excited synchronous condenser without damper windings may be negative.

(f) From eqs 41 and 43, with m and n both equal to unity, the additional synchronizing torque due to oscillations of the machine is

$$- 2K \left\{ \frac{3x_m V\, k_b^2 \delta\, b}{r_b Z_s^2 \omega (k_b^2 + b^2)} \left[2E \sin\alpha - V \sin(\alpha - \delta_o) \right] X \right.$$

$$\left. \left[\frac{b}{k_b} \sin(\alpha - \delta_o) + \frac{E\, b}{V\, \omega} \sin\alpha \sin 2\alpha \right] \right\} \sin bt \qquad (47)$$

(g) From eq 46, if we could disregard the first term, we would have the conclusion that a cylindrical-rotor synchronous generator without damper winding will not have negative damping if the armature resistance does not exceed a certain limiting value, which is

$$r_a = Z_s \sin \delta_o \qquad (48)$$

or

$$\sin\alpha = \sin\delta_o$$

i.e.,

$$\alpha = \delta_o \qquad (48')$$

And if the machine is operated as a motor, the value of excitation emf to insure positive damping due to electromagnetic action is

$$E < \frac{V \sin(\alpha - \delta_o)}{2 \sin\alpha} \qquad (49)$$

CHAPTER IV

A SALIENT-POLE SYNCHRONOUS MACHINE WITHOUT DAMPER WINDINGS IN OSCILLATION

The saliency of the pole structure of salient-pole synchronous machines introduces much difficulty in the mathematical analysis. Then, in order to normalize the saliency effect on the analysis, we shall use some simplifying assumptions as follows:

(a) The flux produced by the field current alone is distributed sinusoidally along the air gap; or, in other words, the mutual inductance between any armature winding and the field winding varies as the cosine of the position angle between them.

(b) The self-inductance of any armature phase and the mutual inductance between any two armature phases have each a constant value and a second harmonic variation with respect to the position of the poles.

With the above assumptions and the others used for cylindrical-rotor machines, we can develop the fundamental differential equations for our analysis.

4.1 The Fundamental Differential Equations

For three-phase salient-pole synchronous machines without damper windings, the fundamental differential equations for voltages and currents in terms of symmetrical components of instantaneous quantities are as follows:

$$v_{a1} = (r_a + L_a D)i_{a1} + L'_a D(i_{a2}\varepsilon^{j2\theta}) + M\,D(i_{b1}\varepsilon^{j\theta}) \tag{1}$$

$$v_{a2} = (r_a + L_a D)i_{a2} + L_a' D(i_{a1}\varepsilon^{-j2\theta}) + M\, D(i_{b2}\varepsilon^{-j\theta}) \tag{2}$$

$$v_{b1} = (r_b + L_b D)i_{b1} + \frac{3}{2} M\, D(i_{a1}\varepsilon^{-j\theta}) \tag{3}$$

$$v_{b2} = (r_b + L_b D)i_{b2} + \frac{3}{2} M\, D(i_{a2}\varepsilon^{j\theta}) \tag{4}$$

where L_a and L_a' may be expressed in terms of constants in two-reaction theory as

$$L_a = \frac{x_d + x_q}{2\omega} \tag{5}$$

$$L_a' = \frac{x_d - x_q}{2\omega} \tag{6}$$

while x_d and x_q are synchronous reactances referred to d-axis and q-axis, respectively. Now

$$v_{a1} = V_1\varepsilon^{j\omega t} \tag{7}$$

$$v_{a2} = V_2\varepsilon^{-j\omega t} \tag{8}$$

$$i_{b1} = i_{b2} = \frac{i_b}{2} \tag{9}$$

$$v_{b1} + v_{b2} = E_{d.c.} \tag{10}$$

where V_1 is a constant complex quantity having a magnitude equal to one-half of the maximum value of the applied voltage, and V_2 is the conjugate of V_1 .

From eqs 1, 2, 3, 4, 7, 8, 9, and 10 we have, then,

$$V_1\varepsilon^{j\omega t} = (r_a + L_aD)i_{a1} + L_a'D(i_{a2}\varepsilon^{j2\theta}) + \frac{M}{2}D(i_b\varepsilon^{j\theta}) \tag{11}$$

$$V_2\varepsilon^{-j\omega t} = (r_a + L_aD)i_{a2} + L_a'D(i_{a1}\varepsilon^{-j2\theta}) + \frac{M}{2}D(i_b\varepsilon^{-j\theta}) \tag{12}$$

$$E_{d.c.} = (r_b + L_bD)i_b + \frac{3}{2}M\,D(i_{a1}\varepsilon^{-j\theta} + i_{a2}\varepsilon^{j\theta}) \tag{13}$$

Although these equations hold good for θ being any function of time or, in other words, for any kind of motion of the machine, it is not always easy to get solutions.

The electromagnetic torque produced by a salient-pole synchronous machine without damper windings can be expressed in terms of symmetrical components of instantaneous quantities of currents as follows:

$$T = j\,K\left\{ i_b(i_{a2}\varepsilon^{j\theta} - i_{a1}\varepsilon^{-j\theta}) + \frac{2L_a'}{M}\left[(i_{a2}\varepsilon^{j\theta})^2 - (i_{a1}\varepsilon^{-j\theta})^2\right]\right\} \tag{14}$$

where $K = \frac{\text{poles}}{2} \cdot \frac{550}{746} \cdot \frac{3M}{2}$ if torque T is in lb.ft. and currents, inductance in amperes, henries, respectively.

4.2 Solutions of the Stator and Rotor Currents When the Machine is in Steady Oscillation

The solutions of eqs 11, 12, and 13 can be obtained by applying the method of superposition to handle the armature and field sources separately, and the method of successive reflections to avoid the difficulty in solving the equations simultaneously.

A. Due to the field source alone

The currents due to the field source alone (i.e., armature terminals short-circuited) should satisfy the following equations:

$$0 = (r_a + L_a D)i_{a1} + L_a' D(i_{a2}\varepsilon^{j2\theta}) + \frac{M}{2} D(i_b \varepsilon^{j\theta}) \tag{15}$$

$$0 = (r_a + L_a D)i_{a2} + L_a' D(i_{a1}\varepsilon^{-j2\theta}) + \frac{M}{2} D(i_b \varepsilon^{-j\theta}) \tag{16}$$

$$E_{d.c.} = (r_b + L_b D)i_b + \frac{3}{2} M\, D(i_{a1}\varepsilon^{-j\theta} + i_{a2}\varepsilon^{j\theta}) \tag{17}$$

In order to solve eqs 15, 16, and 17, we then apply the method of successive reflections as follows:

First step: Assume the stator terminals open, we get

$$i_{a11} = 0 \tag{18}$$

$$i_{b1} = \frac{E_{d.c.}}{r_b} \tag{19}$$

where i_{b1} is the component of i_b in the first step of the reflection. It does not represent the positive-sequence component of i_b now and later.

Then, with m=1, n=1 , we have

$$v_{a11} = \frac{M}{2} D(i_{b1}\varepsilon^{j\theta}) = \frac{M\ E_{d.c.}}{2r_b}\ D(\varepsilon^{j\theta})$$

$$= \frac{M\ E_{d.c.}}{2r_b} D\ \ \varepsilon^{j(\theta_o+\omega t+\delta \sin bt)}$$

$$= \frac{M\ E_{d.c.}}{2r_b} D\ \ \varepsilon^{j(\theta_o+\omega t)}(1 + j\ \delta\ \sin\ bt)$$

$$= \frac{M\ E_{d.c.}}{2r_b}\quad j\omega\ \varepsilon^{j(\theta_o+\omega t)} + j\ \frac{(\omega + b)\delta}{2}\ \varepsilon^{j(\theta_o+\omega t+bt)}$$

$$- j\ \frac{(\omega - b)\delta}{2}\ \varepsilon^{j(\theta_o+\omega t-bt)} \tag{20}$$

Second step: Assume field terminals are open and a voltage of $(-v_{a11})$ is applied to the armature. Then we get

$$i_{b2} = 0 \tag{21}$$

where i_{b2} is the component of i_b in the second step of the reflection. The component of i_{a1} should satisfy the following two equations.

$$- v_{a11} = (r_a + L_a D)i_{a12} + L_a' D(i_{a22}\varepsilon^{j2\theta}) \tag{22}$$

$$- v_{a21} = (r_a + L_a D)i_{a22} + L_a' D(i_{a12}\varepsilon^{-j2\theta}) \tag{23}$$

where v_{a21} and i_{a22} are the conjugates of v_{a11} and i_{a12}, respectively, and v_{a11} is given by the expression 20.

In order to solve eqs 22 and 23, it is advisable to put v_{a11} into three parts as

$$v_{a11} = L_{\omega} + L_{\omega+b} + L_{\omega-b} \tag{20'}$$

with

$$L_{\omega} = j\omega \frac{M\,E_{d.c.}}{2r_b} \varepsilon^{j(\theta_o+\omega t)} \tag{24}$$

$$L_{\omega+b} = j \frac{(\omega + b)\delta}{2} \cdot \frac{M\,E_{d.c.}}{2r_b} \varepsilon^{j(\theta_o+\omega t+bt)} \tag{25}$$

$$L_{\omega-b} = - j \frac{(\omega - b)\delta}{2} \cdot \frac{M\,E_{d.c.}}{2r_b} \varepsilon^{j(\theta_o+\omega t-bt)} \tag{26}$$

Then, applying the principle of superposition again, we may handle L_{ω} , $L_{\omega+b}$, and $L_{\omega-b}$, separately.

By substituting L_{ω} alone for v_{a11} and the conjugate of L_{ω} for v_{a21} into eqs 22 and 23, we have

$$- j\omega \frac{M\,E_{d.c.}}{2r_b} \varepsilon^{j(\theta_o+\omega t)} = (r_a + L_a' D)i_{a12} + L_a' D(i_{a22}\varepsilon^{j2\theta}) \tag{27}$$

$$j\omega \frac{M\,E_{d.c.}}{2r_b} \varepsilon^{-j(\theta_o+\omega t)} = (r_a + L_a D)i_{a22} + L_a' D(i_{a12}\varepsilon^{-j2\theta}) \tag{28}$$

Now we can replace D by $j\omega$ in eq 27 and by $-j\omega$ in eq 28 with the introduction of a very small error. The error will be absent when δ , b , L_a' , or r_a is zero. Thus we get

$$- j\omega \frac{M\ E_{d.c.}}{2r_b} \varepsilon^{j(\theta + \omega t)} = (r_a + j\omega\ L_a)i_{a12} + j\omega\ L'_a i_{a22}\varepsilon^{j2\theta}$$

..... (27')

$$j\omega \frac{M\ E_{d.c.}}{2r_b} \varepsilon^{-j(\theta_o + \omega t)} = (r_a - j\omega\ L_a)i_{a22} - j\omega\ L'_a i_{a12}\varepsilon^{-j2\theta}$$

..... (28')

Multiplying eq 27' by $\varepsilon^{-j\theta}$ and eq 28' by $\varepsilon^{j\theta}$ and then solving them simultaneously, we have

$$i_{a22}\varepsilon^{j\theta} = \frac{-\frac{M\ E_{d.c.}}{2r_b}(L_e\varepsilon^{j\delta \sin bt} - L'_a\varepsilon^{-j\delta \sin bt})}{L_e^{\ 2} - (L'_a)^2}$$

$$= -\frac{M\ E_{d.c.}}{2r_b}\ \frac{(L_e - L'_a) + j(L_e + L'_a)\delta \sin bt}{L_e^{\ 2} - (L'_a)^2} \qquad (29)$$

where

$$L_e = \frac{r_a}{j\omega} + L_a \qquad (30)$$

With the substitution of $L_{\omega+b}$ alone for v_{a11} and the conjugate of $L_{\omega+b}$ for v_{a21} into eqs 22 and 23, we have

$$- j\frac{(\omega + b)\delta}{2} \cdot \frac{M\ E_{d.c.}}{2r_b} \varepsilon^{j(\theta + \omega t + bt)} = (r_a + L_a D)i_{a12} + L'_a D(i_{a22}\varepsilon^{j2\theta})$$

..... (31)

$$j\frac{(\omega + b)\delta}{2} \cdot \frac{M\ E_{d.c.}}{2r_b} \varepsilon^{-j(\theta_o + \omega t + bt)} = (r_a + L_a D)i_{a22} + L'_a D(i_{a12}\varepsilon^{-j2\theta})$$

..... (32)

Replacing D by $j(\omega+b)$ in eq 31, and then multiplying the equation by $\dfrac{\varepsilon^{-j\theta}}{j(\omega + b)}$, we have

$$-\frac{\delta}{2} \cdot \frac{M\,E_{d.c.}}{2r_b}\,\varepsilon^{j(bt-\delta \sin nbt)} = \left[\frac{r_a}{j(\omega + b)} + L_a\right] i_{a12}\varepsilon^{-j\theta} + L_a i_{a22}\varepsilon^{j\theta} \qquad \text{..... (33)}$$

From eq 33, disregarding the small term having the factor δ^2, we get

$$-\frac{\delta}{2} \cdot \frac{M\,E_{d.c.}}{2r_b}\,\varepsilon^{jbt} = \left[\frac{r_a}{j(\omega + b)} + L_a\right] i_{a12}\varepsilon^{-j\theta} + L_a' i_{a22}\varepsilon^{j\theta} \qquad \text{..... (33')}$$

Correspondingly, we have its conjugate as

$$-\frac{\delta}{2} \cdot \frac{M\,E_{d.c.}}{2r_b}\,\varepsilon^{-jbt} = \left[\frac{r_a}{-j(\omega + b)} + L_a\right] i_{a22}\varepsilon^{j\theta} + L_a' i_{a12}\varepsilon^{-j\theta} \qquad \text{..... (34)}$$

Solving eqs 33' and 34 simultaneously, we get

$$i_{a22}\varepsilon = \frac{-\dfrac{\delta}{2} \cdot \dfrac{M\,E_{d.c.}}{2r_b}\left\{\left[\dfrac{r_a}{j(\omega + b)} + L_a\right]\varepsilon^{-jbt} - L_a'\varepsilon^{jbt}\right\}}{\left|\dfrac{r_a}{j(\omega + b)} + L_a\right|^2 - (L_a')^2} \qquad \text{..... (35)}$$

Similarly, with the substitution of $L_{\omega-b}$ alone for v_{a11} and the conjugate of $L_{\omega-b}$ for v_{a21} into eqs 22 and 23, we shall have the solution as

$$i_{a22}\varepsilon^{j\theta} = \frac{\frac{\delta}{2} \cdot \frac{M\,E_{d.c.}}{2r_b}\left\{\left[\frac{r_a}{j(\omega - b)} + L_a\right]\varepsilon^{jbt} - L_a'\varepsilon^{-jbt}\right\}}{\left|\frac{r_a}{j(\omega - b)} + L_a\right|^2 - (L_a')^2} \qquad \ldots\ldots \quad (36)$$

Therefore, with the substitution of $(L_\omega + L_{\omega+b} + L_{\omega-b})$ for v_{a11} and the conjugate of $(L_\omega + L_{\omega+b} + L_{\omega-b})$ for v_{a21} into eqs 22 and 23, the solution of $i_{a22}\varepsilon^{j\theta}$ should be the sum of the expressions 29, 35, and 36. That is,

$$\begin{aligned} i_{a22}\varepsilon^{j\theta} = &- \frac{M\,E_{d.c.}}{2r_b} \; \frac{(L_e - L_a') + j(L_e + L_a)\delta \sin bt}{|L_e|^2 - (L_a')^2} \\ &- \frac{\frac{\delta}{2} \cdot \frac{M\,E_{d.c.}}{2r_b}\left[\frac{r_a}{j(\omega + b)} + L_a - L_a'\right]\cos bt}{\left|\frac{r_a}{j(\omega + b)} + L_a\right|^2 - (L_a')^2} \\ &+ j\,\frac{\frac{\delta}{2} \; \frac{M\,E_{d.c.}}{2r_b}\left(\frac{r_a}{j(\omega + b)} + L_a + L_a'\right)\sin bt}{\left|\frac{r_a}{j(\omega + b)} + L_a\right|^2 - (L_a')^2} \\ &+ \frac{\frac{\delta}{2} \cdot \frac{M\,E_{d.c.}}{2r_b}\left(\frac{r_a}{j(\omega - b)} + L_a - L_a'\right)\cos bt}{\left|\frac{r_a}{j(\omega - b)} + L_a\right|^2 - (L_a')^2} \\ &+ j\,\frac{\frac{\delta}{2} \cdot \frac{M\,E_{d.c.}}{2r_b}\left(\frac{r_a}{j(\omega - b)} + L_a + L_a'\right)\sin bt}{\left|\frac{r_a}{j(\omega - b)} + L_a\right|^2 - (L_a')^2} \end{aligned}$$

$$= -\frac{M\,E_{d.c.}}{2r_b}\cdot\frac{L_e - L_a'}{|L_e|^2 - (L_a')^2}$$

$$- j\,\frac{M\,E_{d.c.}}{2r_b}\,C_1\delta\,\sin bt$$

$$- \frac{M\,E_{d.c.}}{2r_b}\,C_2\delta\,\cos bt \qquad (37)$$

where

$$C_1 = \frac{L_e + L_a'}{|L_e|^2 - (L_a')^2}\quad \frac{1}{2}\left[\frac{\dfrac{r_a}{j(\omega+b)} + L_a + L_a'}{\left|\dfrac{r_a}{j(\omega+b)} + L_a\right|^2 - (L_a')^2} + \frac{\dfrac{r_a}{j(\omega-b)} + L_a + L_a'}{\left|\dfrac{r_a}{j(\omega-b)} + L_a\right|^2 - (L_a')^2}\right]$$

and

$$C_2 = \frac{1}{2}\left(\frac{\dfrac{r_a}{j(\omega+b)} + L_a - L_a'}{\left|\dfrac{r_a}{j(\omega+b)} + L_a\right|^2 - (L_a')^2} - \frac{\dfrac{r_a}{j(\omega-b)} + L_a - L_a'}{\left|\dfrac{r_a}{j(\omega-b)} + L_a\right|^2 - (L_a')^2}\right)$$

By considering the fact $b \ll \omega$, we may simplify the above expressions with

$$\frac{\dfrac{r_a}{j(\omega+b)} + L_a + L_a'}{\left|\dfrac{r_a}{j(\omega+b)} + L_a\right|^2 - (L_a')^2} = \frac{\dfrac{r_a}{j(\omega+b)} + L_a + L_a'}{\left(\dfrac{r_a}{\omega+b}\right)^2 + L_a^2 - (L_a')^2}$$

$$= b\,\frac{\partial}{\partial\omega}\left[\frac{\dfrac{r_a}{j\omega} + L_a + L_a'}{\left(\dfrac{r_a}{\omega}\right)^2 + L_a^2 - (L_a')^2}\right] + \frac{\dfrac{r_a}{j\omega} + L_a + L_a'}{\left(\dfrac{r_a}{\omega}\right)^2 + L_a^2 - (L_a')^2}$$

$$= b \frac{\partial}{\partial \omega} \left[\frac{L_e + L_a'}{|L_e|^2 - (L_a')^2} \right] + \frac{L_e + L_a'}{|L_e|^2 - (L_a')^2} \qquad \text{(A)}$$

$$\frac{\dfrac{r_a}{j(\omega - b)} + L_a + L_a'}{\left|\dfrac{r_a}{j(\omega - b)} + L_a\right|^2 - (L\ )^2} = - b \frac{\partial}{\partial \omega} \left[\frac{L_e + L_a'}{|L_e|^2 - (L_a')^2} \right]$$

$$+ \frac{L_e + L_a'}{|L_e|^2 - (L_a')^2} \quad \cdots\cdots \qquad \text{(B)}$$

$$\frac{\dfrac{r_a}{j(\omega + b)} + L_a - L_a'}{\left|\dfrac{r_a}{j(\omega + b)} + L_a\right|^2 - (L_a')^2} = b \frac{\partial}{\partial \omega} \left[\frac{L_e - L_a'}{|L_e|^2 - (L_a')^2} \right]$$

$$+ \frac{L_e - L_a'}{|L_e|^2 - (L_a')^2} \quad \cdots\cdots \qquad \text{(C)}$$

and

$$\frac{\dfrac{r_a}{j(\omega - b)} + L_a - L_a'}{\left|\dfrac{r_a}{j(\omega - b)} + L_a\right|^2 - (L_a')^2} = - b \frac{\partial}{\partial \omega} \left[\frac{L_e - L_a'}{|L_e|^2 - (L_a')^2} \right]$$

$$+ \frac{L_e - L_a'}{|L_e|^2 - (L_a')^2} \quad \cdots\cdots \qquad \text{(D)}$$

Hence

$$C_1 = \frac{L_e + L_a'}{|L_e|^2 - (L_a')^2} - \frac{1}{2} \left\{ b \frac{\partial}{\partial \omega} \left[\frac{L_e + L_a'}{|L_e|^2 - (L_a')^2} \right] + \frac{L_e + L_a'}{|L_e|^2 - (L_a')^2} \right.$$

$$\left. - b \frac{\partial}{\partial \omega} \left[\frac{L_e + L_a'}{|L_e|^2 - (L_a')^2} \right] + \frac{L_e + L_a'}{|L_e|^2 - (L_a')^2} \right\}$$

$$= \frac{L_e + L_a'}{|L_e|^2 - (L_a')^2} - \frac{L_e + L_a'}{|L_e|^2 - (L_a')^2} = 0 \qquad \text{(E)}$$

and

$$C_2 = \frac{1}{2}\left\{ b \frac{\partial}{\partial\omega}\left[\frac{L_e - L_a'}{|L_e|^2 - (L_a')^2}\right] + \frac{L_e - L_a'}{|L_e|^2 - (L_a')^2} \right.$$

$$\left. + \, b \frac{\partial}{\partial\omega}\left[\frac{L_e - L_a'}{|L_e|^2 - (L_a')^2}\right] - \frac{L_e - L_a'}{|L_e|^2 - (L_a')^2} \right\}$$

$$= b \frac{\partial}{\partial\omega}\left[\frac{L_e - L_a'}{|L_e|^2 - (L_a')^2}\right] = b \frac{\partial}{\partial\omega}\left[\frac{\frac{r_a}{j\omega} + L_a - L_a'}{\left(\frac{r_a}{\omega}\right)^2 + L_a^2 - (L_a')^2}\right]$$

$$= b \, \frac{-\frac{r_a}{j\omega^2}\left[\left(\frac{r_a}{\omega}\right)^2 + L_a^2 - (L_a')^2\right] + \frac{2r_a^2}{\omega^3}\left(\frac{r_a}{j\omega} + L_a - L_a'\right)}{\left[\left(\frac{r_a}{\omega}\right)^2 + L_a^2 - (L_a')^2\right]^2}$$

$$= jb \frac{r_a}{\omega^2} \cdot \frac{\frac{r_a}{j\omega}^{2} + 2 \frac{r_a}{j\omega}(L_a - L_a') + L_a - (L_a')^2}{\left[|L_e|^2 - (L_a')^2\right]^2}$$

$$= jb \frac{r_a}{\omega^2} \cdot \frac{L_e - (L_a')^2 - 2 \frac{r_a}{j\omega} L_a'}{\left[|L_e|^2 - (L_a')^2\right]^2} \qquad \text{(F)}$$

Substituting expressions E and F into eq 37, we have

$$i_{a22}\,\varepsilon^{j\theta} = -\frac{M\,E_{d.c.}}{2r_b}\cdot\frac{L_e - L_a'}{|L_e|^2 - (L_a')^2}$$

$$-\,j\,\frac{M\,E_{d.c.}}{2r_b}\cdot\frac{r_a b\,\delta}{\omega^2}\cdot\frac{L_e^2 - (L_a')^2 - 2\,\frac{r_a}{j\omega}\,L_a'}{\left[\,|L_e|^2 - (L_a')^2\right]^2}\cos bt$$

$$\ldots\ldots \quad (38)$$

From the results of the preceding chapter we can see that the effects due to the components of currents of the further reflections are small and can be neglected for the sake of simplicity. Hence, due to the field source alone, we have

$$i_b = \frac{E_{d.c.}}{r_b} \qquad (39)$$

$$i_{a2}\,\varepsilon^{j\theta} = -\frac{M\,E_{d.c.}}{2r_b}\cdot\frac{L_e - L_a'}{|L_e|^2 - (L_a')^2}$$

$$-\,j\,\frac{M\,E_{d.c.}}{2r_b}\cdot\frac{r_a b\,\delta}{\omega^2}\cdot\frac{L_e^2 - (L_a')^2 - 2\,\frac{r_a}{j\omega}\,L_a'}{\left[\,|L_e|^2 - (L_a')^2\right]^2}\cos bt$$

$$\ldots\ldots \quad (40)$$

B. <u>Due to the armature source alone</u>.

The currents due to the armature source alone, with the field terminals short-circuited, should satisfy the following equations:

$$V_1\varepsilon^{j\omega t} = (r_a + L_a D)i_{a1} + L_a D(i_{a2}\varepsilon^{j2\theta}) + \frac{M}{2}\,D(i_b\varepsilon^{j\theta}) \qquad (41)$$

$$V_2\varepsilon^{-j\omega t} = (r_a + L_aD)i_{a2} + L_a'D(i_{a1}\varepsilon^{-j2\theta}) + \frac{M}{2}D(i_b\varepsilon^{-j\theta}) \tag{42}$$

$$0 = (r_b + L_bD)i_b + \frac{3}{2}MD(i_{a1}\varepsilon^{-j\theta} + i_{a2}\varepsilon^{j\theta}) \tag{43}$$

With the same reasoning as in the preceding chapters, we can solve eqs 41, 42, and 43 simultaneously by replacing r_a with an imaginary inductance $\pm\frac{r_a}{j\omega}$. Thus, from eqs 41 and 42 we have

$$i_{a2}\varepsilon^{j\theta} = \frac{L_e\dfrac{V_2\varepsilon^{-j\phi}}{-j\omega} - L_a'\dfrac{V_1\varepsilon^{j\phi}}{j\omega} - (L_e - L_a')\dfrac{M}{2}i_b}{|L_e|^2 - (L_a')^2} \tag{44}$$

where

$$L_e = \frac{r_a}{j\omega} + L_a \tag{45}$$

$$\phi = -\theta_o - \delta\sin bt \tag{46}$$

and

$$V_1 = |V_1|\,\varepsilon^{j(90^\circ+\theta_o-\delta_o)}$$

$$= jV\,\varepsilon^{j(\theta_o-\delta_o)} \tag{47}$$

Hence

$$i_{a1}\varepsilon^{-j\theta} + i_{a2}\varepsilon^{j\theta} = \frac{1}{|L_e|^2 - (L_a')^2}\Bigg[(L_e - L_a')\frac{V_2\varepsilon^{-j\phi}}{-j\omega}$$

$$+ (\hat{L}_e - L_a')\frac{V_1\varepsilon^{j\phi}}{j\omega} - \left(\frac{L_e + \hat{L}_e}{2} - L_a'\right)M\,i_b\Bigg]$$

$$\ldots\ldots \tag{48}$$

Let

$$\frac{L_e - L_a'}{|L_e|^2 - (L_a')^2} = \frac{1}{L}\,\varepsilon^{-j\alpha_1} \tag{49}$$

where both L and α_1 are real values. Then

$$\frac{\hat{L}_e - L_a'}{|L_e|^2 - (L_a')^2} = \frac{1}{L}\,\varepsilon^{j\alpha_1} \tag{50}$$

$$\frac{\frac{L_e + \hat{L}_e}{2} - L_a'}{|L_e|^2 - (L_a')^2} = \frac{\cos\alpha_1}{L} \tag{51}$$

And the eq 48 can be written as

$$i_{a1}\varepsilon^{-j0} + i_{a2}\varepsilon^{j0} = \frac{2V}{L\omega}\cos(\alpha_1 - \delta_o - \delta\sin bt) - \frac{M\cos\alpha}{L}\,i_b \quad \cdots\cdots\cdots\cdots \tag{52}$$

From eqs 43 and 52 we get

$$0 = (r_b + L_b D)i_b + \frac{3MV}{L\,\omega}\,D\left[\cos(\alpha_1 - \delta_o - \delta\sin bt)\right] - \frac{3M^2\cos\alpha_1}{2L}\,D\,i_b$$

i.e.,

$$\left[r_b + L_b\left(1 - \frac{3}{2}\cdot\frac{M^2\cos\alpha_1}{L_b L}\right)D\right] i_b = -\frac{3MV}{L\,\omega}\,D\left[\cos(\alpha_1 - \delta_o - \delta\sin bt)\right]$$

or

$$(k_b + D)i_b = -\frac{3MV}{L\,\omega\,\sigma\,L_b}\,D\left[\cos(\alpha_1 - \delta_o - \delta\sin bt)\right] \tag{53}$$

where

$$\sigma = \left(1 - \frac{3}{2}\cdot\frac{M^2\cos\alpha_1}{L_b L}\right) \tag{54}$$

$$k_b = \frac{r_b}{L_b\,\sigma} \tag{55}$$

The solution of eq 53 is

$$i_b = -\frac{3MV}{L\,\omega\,\sigma\,L_b}\,\varepsilon^{-k_b t}\int \varepsilon^{k_b t}\, d\left[\cos\,(\alpha_1 - \delta_o - \delta\,\sin\,bt)\right]$$

$$= -\frac{3MV}{L\,\omega\,\sigma\,L_b}\cdot\frac{\delta\,\sin(\alpha_1 - \delta_o)}{k_b^2 + b^2}\,(b^2\sin\,bt + b\,k_b\cos\,bt) \qquad \ldots\ldots \quad (56)$$

Substituting eq 56 into eq 44, we get

$$i_{a2}\varepsilon^{j\theta} = \frac{V}{L\,\omega}\,\varepsilon^{-j(\alpha_1-\delta_o-\delta\sin bt)} + j\,\frac{2L_a'V}{\omega\left[\,|L_e|^2 - (L_a')^2\right]}\,\sin(\delta_o + \delta\,\sin\,bt)$$

$$+ \frac{3M^2V}{2L^2\omega\,\sigma\,L_b}\,\varepsilon^{-j\alpha_1}\cdot\frac{\delta\,\sin(\alpha_1 - \delta_o)}{k_b^2 + b^2}\,(b^2\sin\,bt + b\,k_b\cos\,bt) \qquad \ldots\ldots \quad (57)$$

C. <u>When both the armature and field sources are applied</u>.

By the method of superposition we know that the stator and rotor currents, when both the armature and field sources are applied, are respectively equal to the sum of the corresponding currents when the sources are applied one at a time. Therefore, from expressions 39, 40, 56, and 57, we have

$$i_b = \frac{E_{d.c.}}{r_b} - \frac{3MV}{L\,\omega\,\sigma\,L_b}\cdot\frac{\delta\,\sin(\alpha_1 - \delta_o)}{k_b^2 + b^2}\,(b^2\sin\,bt + b\,k_b\cos\,bt) \qquad \ldots\ldots \quad (58)$$

$$i_{a2}\varepsilon^{j\theta} = -\frac{M}{2L}\,\frac{E_{d.c.}}{r_b}\,\varepsilon^{-j\alpha_1}$$

$$-\,j\,\frac{M\,E_{d.c.}}{2r_b}\cdot\frac{r_a b\,\delta}{\omega^2}\cdot\frac{L_e^2-(L_a')^2-2\,\frac{r_a}{j\omega}\,L_a'}{\left[\,|L_e|^2-(L_a')^2\right]^2}\cos bt$$

$$+\,\frac{V}{L\,\omega}\,\varepsilon^{-j(\alpha_1-\delta_o-\delta\sin bt)}+j\,\frac{2L_a'V}{\omega\left[\,|L_e|^2-(L_a')^2\right]}\sin(\delta_o+\delta\sin bt)$$

$$+\,\frac{3M^2V}{2L^2\omega\,\sigma\,L_b}\,\varepsilon^{-j\alpha_1}\,\frac{\delta\,\sin(\alpha_1-\delta_o)}{k_b^2+b^2}\,(b^2\sin bt+b\,k_b\cos bt)$$

..... (59)

4.3 Electromagnetic Torque Produced During the Steady Oscillation

When a salient-pole synchronous machine without damper windings is in steady oscillation of small amplitude, the currents i_b and $i_{a2}\varepsilon^{j\theta}$ are given by the expressions 58 and 59. By substituting them into expression 14, we can get the electromagnetic torque in terms of the applied voltages, machine constants, and displacement angle, etc. Before making the substitutions, it is advisable to express $i_b i_{a2}\varepsilon^{j\theta}$ and $(i_{a2}\varepsilon^{j\theta})^2$ in rectangular co-ordinates as follows:

$$i_b i_{a2}\varepsilon^{j\theta} = x_1 + j\,y_1 \qquad (60)$$

and

$$(i_{a2}\varepsilon^{j\theta})^2 = x_2 + j\,y_2 \qquad (61)$$

where x_1 , x_2 , y_1 , and y_2 are all real quantities. Then the expression 14 can be written as

$$T = -2K(y_1 + \frac{2L_a'}{M} y_2) \tag{62}$$

From expressions 58 and 59, by neglecting the small terms having the factor δ^2, we get

$$
\begin{aligned}
y_1 = {} & \left(\frac{E_{d.c.}}{r_b}\right)^2 \cdot \frac{M}{2L} \sin \alpha_1 - \left(\frac{E_{d.c.}}{r_b}\right)^2 \cdot \frac{M\, r_a b\, \delta}{2\omega^2} \cdot \frac{-\left(\frac{r_a}{\omega}\right)^2 + L_a^2 - (L_a')^2}{\left[|L_e|^2 - (L_a')^2\right]^2} \cos bt \\
& - \frac{E_{d.c.}}{r_b} \cdot \frac{V}{L\,\omega} \sin(\alpha_1 - \delta_o - \delta \sin bt) \\
& + \frac{E_{d.c.}}{r_b} \cdot \frac{2L_a' V}{\omega\left[|L_e|^2 - (L_a')^2\right]} \sin(\delta_o + \delta \sin bt) \\
& - \frac{E_{d.c.}}{r_b} \cdot \frac{3M^2 V \sin \alpha_1}{L^2 \omega\, \sigma\, L_b} \cdot \frac{\delta \sin(\alpha_1 - \delta_o)}{k_b^2 + b^2} \cdot (b^2 \sin bt + b\, k_b \cos bt) \\
& + \frac{V}{L\,\omega}^2 \cdot \frac{3M}{\sigma\, L_b} \cdot \frac{\delta \sin^2(\alpha_1 - \delta_o)}{k_b^2 + b^2} \cdot (b^2 \sin bt + b\, k_b \cos bt) \\
& - \frac{V^2}{\omega^2\left[|L_e|^2 - (L_a')^2\right]} \cdot \frac{6M\, L_a'}{L\, \sigma\, L_b} \cdot \frac{\delta \sin \delta_o \sin(\alpha_1 - \delta_o)}{k_b^2 + b^2} (b^2 \sin bt + b k_b \cos bt)
\end{aligned}
$$

$$\cdots\cdots \quad (63)$$

$$y_2 = -\left(\frac{M\ E_{d.c.}}{2L\ r_b}\right)^2 \sin 2\alpha_1 - \left(\frac{V}{\omega\ L}\right)^2 \sin 2(\alpha - \delta_o - \delta \sin bt)$$

$$+ \frac{M\ V\ E_{d.c.}}{\omega^2 L^2 r_b} \sin(2\alpha_1 - \delta_o - \delta \sin bt)$$

$$- \frac{2M\ L_a' V\ E_{d.c.}}{\omega\ L\ r_b \left[\ |L_e|^2 - (L_a')^2\right]} \cos \alpha_1 \sin(\delta_o + \delta \sin bt)$$

$$+ \frac{3M^3 E_{d.c.} V}{2r_b L^3 \omega\ \sigma\ L_b} \cdot \frac{\delta \sin 2\alpha_1 \sin(\alpha_1 - \delta_o)}{k_b^2 + b^2} \cdot (b^2 \sin bt + b\ k_b \cos bt)$$

$$+ \frac{4L_a' V^2}{\omega^2 L \left[\ |L_e|^2 - (L_a')^2\right]} \cos(\alpha_1 - \delta_o - \delta \sin bt) \sin(\delta_o + \delta \sin bt)$$

$$- \frac{3M^2 V^2}{\omega^2 L^3 \sigma\ L_b} \cdot \frac{\delta \sin(\alpha_1 - \delta_o) \sin(2\alpha_1 - \delta_o)}{k_b^2 + b^2} (b^2 \sin bt + b\ k_b \cos bt)$$

$$+ \frac{6M^2 V^2 L_a' \cos \alpha_1}{\omega^2 L^2 \sigma\ L \left[\ |L_e|^2 - (L_a')^2\right]} \cdot \frac{\delta \sin(\alpha_1 - \delta_o) \sin \delta_o}{k_b^2 + b^2} (b^2 \sin bt + bk_b \cos bt)$$

$$+ \quad \frac{E_{d.c.}}{r_b}^{\ 2} \frac{M^2 r_a b\ \delta \cos \alpha_1}{2\omega^2 L} \cdot \frac{-\left(\frac{r_a}{\omega}\right)^2 + L_a^2 - (L_a')^2}{\left[\ |L_e|^2 - (L_a)^2\right]^2} \cos bt$$

$$- \left(\frac{E_{d.c.}}{r_b}\right)^2 \frac{M^2 r_a b\ \delta \sin \alpha_1}{2\omega^2 L} \cdot \frac{\frac{2r_a}{\omega}(L_a - L_a')}{\left[|L_e|^2 - (L_a')^2\right]^2} \cos bt$$

$$- \frac{M\ E_{d.c.} V}{\omega\ L\ r_b} \cdot \frac{r_a b\ \delta \cos(\alpha_1 - \delta_o)}{\omega^2} \cdot \frac{-\left(\frac{r_a}{\omega}\right)^2 + L_a^2 - (L_a')^2}{\left[|L_e|^2 - (L_a')^2\right]^2} \cos bt$$

$$+ \frac{M\ E_{d.c.} V}{\omega\ L\ r_b} \cdot \frac{r_a b\ \delta \sin(\alpha_1 - \delta_o)}{\omega^2} \cdot \frac{\frac{2r_a}{\omega}(L_a - L_a')}{\left[|L_e|^2 - (L_a')^2\right]^2} \cos bt$$

$$- \frac{2M\ L_a' E_{d.c.} V}{r_b \omega} \cdot \frac{r_a b\ \delta \sin \delta_o}{\omega^2} \cdot \frac{\frac{2r_a}{\omega}(L_a - L_a')}{\left[|L_e|^2 - (L_a')^2\right]^3} \cos bt$$

...... (64)

From expressions 62, 63, and 64 we can see that the electromagnetic torque produced during the steady oscillation of small amplitude can be grouped into three parts, namely,

(a) A part with each term having a factor δ b cos bt,

(b) A part with each term having a factor $\delta\ b^2 \sin bt$,

and (c) A part including all the other terms.

The third part represents the sum of synchronous torque and synchronizing torque according to the static characteristic. The second part causes the modification of synchronizing torque due to the presence of the oscillations. The first part varies in phase with the variation of the machine speed and causes either positive or negative damping action. Hence, in this analysis, only the first part is of interest, and it will be denoted by a symbol T_d.

Thus we have

$$T_d = -2K\,\delta\,b\left\{-\left(\frac{E_{d.c.}}{r_b}\right)^2 \cdot \frac{M\,r_a}{2\omega^2} \cdot \frac{-\left(\frac{r_a}{\omega}\right)^2 + L_a^2 - (L_a')^2}{\left[\,|L_e|^2 - (L_a')^2\right]^2}\right.$$

$$-\frac{E_{d.c.}}{r_b} \cdot \frac{3M^2V\sin\alpha_1}{L^2\omega\,\sigma\,L_b} \cdot \frac{k_b}{k_b^2 + b^2} \cdot \sin(\alpha_1 - \delta_o)$$

$$+\left(\frac{V}{L\,\omega}\right)^2 \cdot \frac{3M}{\sigma\,L_b} \cdot \frac{k_b}{k_b^2 + b^2} \cdot \sin^2(\alpha_1 - \delta_o)$$

$$-\frac{V^2}{\omega^2\left[\,|L_e|^2 - (L_a')^2\right]} \cdot \frac{6M\,L_a'}{L\,\sigma\,L_b} \cdot \frac{k_b}{k_b^2 + b^2}\sin\delta_o\sin(\alpha_1 - \delta_o)$$

$$+\frac{3M^2L_a'E_{d.c.}V}{r_bL^3\omega\,\sigma\,L_b} \cdot \frac{k_b}{k_b^2 + b^2} \cdot \sin 2\alpha_1\sin(\alpha_1 - \delta_o)$$

$$-\frac{6M\,L_a'V^2}{\omega^2L^3\sigma\,L_b} \cdot \frac{k_b}{k_b^2 + b^2} \cdot \sin(\alpha_1 - \delta_o)\sin(2\alpha_1 - \delta_o)$$

$$+\frac{12M(L_a')^2V^2\cos\alpha_1}{\omega^2L^2\sigma\,L_b\left[\,|L_e|^2 - (L_a')^2\right]} \cdot \frac{k_b}{k_b^2 + b^2} \cdot \sin(\alpha_1 - \delta_o)\sin\delta_o$$

$$+\left(\frac{E_{d.c.}}{r_b}\right)^2 \cdot \frac{M\,L_a'r_a\cos\alpha_1}{\omega^2L} \cdot \frac{-\left(\frac{r_a}{\omega}\right)^2 + L_a^2 - (L_a')^2}{\left[\,|L_e|^2 - (L_a')^2\right]^2}$$

$$- \left(\frac{E_{d.c.}}{r_b}\right)^2 \frac{M\, L_a' r_a \sin \alpha_1}{\omega^2 L} \cdot \frac{\frac{2r_a}{\omega}(L_a - L_a')}{\left[\,|L_e|^2 - (L_a')^2\right]^2}$$

$$- \frac{2L_a' E_{d.c.} V}{\omega\, L\, r_b} \cdot \frac{r_a}{\omega^2} \cdot \frac{-\left(\frac{r_a}{\omega}\right)^2 + L_a^2 - (L_a')^2}{\left[\,|L_e|^2 - (L_a')^2\right]^2} \cos(\alpha_1 - \delta_o)$$

$$+ \frac{2L_a' E_{d.c.} V}{\omega\, L\, r_b} \cdot \frac{r_a}{\omega^2} \cdot \frac{\frac{2r_a}{\omega}(L_a - L_a')}{\left[\,|L_e|^2 - (L_a')^2\right]^2} \sin(\alpha_1 - \delta_o)$$

$$\left. - \frac{4(L_a')^2 E_{d.c.} V}{r_b \omega} \cdot \frac{r_a}{\omega^2} \cdot \frac{\frac{2r_a}{\omega}(L_a - L_a')}{\left[\,|L_e|^2 - (L_a')^2\right]^3} \sin \delta_o \right\} \cos bt \qquad \ldots\ldots \quad (65)$$

Since

$$\omega\, L_a = \frac{x_d + x_q}{2}\,, \qquad \omega\, L_a' = \frac{x_d - x_q}{2}\,,$$

we have

$$|L_e|^2 = \left(\frac{r_a}{\omega}\right)^2 + L_a^2 = \frac{1}{\omega^2}\left[r_a^2 + \left(\frac{x_d + x_q}{2}\right)^2\right]$$

$$|L_e|^2 - (L_a')^2 = \frac{1}{\omega^2}(r_a^2 + x_d x_q)$$

$$-\left(\frac{r_a}{\omega}\right)^2 + L_a^2 - (L_a')^2 = \frac{1}{\omega^2}(x_d x_q - r_a^2)$$

$$\frac{1}{L} = \frac{|L_e - L_a'|}{|L_e|^2 - (L_a')^2} = \frac{\omega\sqrt{r_a^2 + x_q^2}}{r_a^2 + x_d x_q} = \frac{\omega\, Z_q}{r_a^2 + x_d x_q}$$

$$\alpha_1 = \tan^{-1}\left[\frac{r_a}{\omega(L_a - L_a')}\right] = \tan^{-1}\left(\frac{r_a}{x_q}\right)$$

Let

$$x_m = M\,\omega \tag{66}$$

$$E = \frac{E_{d.c.}}{r_b}\,\frac{M\,\omega}{2} \tag{67}$$

Substituting all these relations together with $k_b = \dfrac{r_b}{\sigma\, L_b}$ into expression 65, and rearranging, we have

$$T_d = 6K\,\frac{x_m}{r_b}\cdot\frac{Z_q^2}{(r_a^2 + x_d x_q)^2}\cdot\frac{k_b^2}{k_b^2 + b^2}\cdot\frac{\delta\, b}{\omega}\cdot\sin(\alpha_1 - \delta_o)$$

$$\times\left[2E\,V\,\sin\alpha_1 - V^2\sin(\alpha_1 - \delta_o) + \frac{V^2(x_d - x_q)}{Z_q}\,\sin\delta_o\right.$$

$$- \frac{E\,V\,Z_q(x_d - x_q}{r_a^2 + x_d x_q}\,\sin 2\alpha_1 + \frac{V^2 Z_q(x_d - x_q)}{r_a^2 + x_d x_q}\,\sin(2\alpha_1 - \delta_o)$$

$$\left.- \frac{V^2 x_q(x_d - x_q)^2}{Z_q(r_a^2 + x_d x_q)}\,\sin\delta_o\right]\cos bt$$

$$+ 4K\,\frac{\delta\, b}{\omega}\left\{\frac{E^2 r_a(x_d x_q - r_a^2)}{x_m(r_a^2 + x_d x_q)^2}\right.$$

$$+ \frac{r_a(x_d - x_q)}{x_m(r_a^2 + x_d x_q)^3}\left[E^2 x_q(2r_a^2 - x_d x_q)\right.$$

$$+ E\ V\ Z_q(x_d x_q - r_a^2)\cos(\alpha_1 - \delta_o)$$

$$- E\ V\ Z_q r_a x_q \sin(\alpha_1 - \delta_o)$$

$$\left. \left. + E\ V\ r_a x_q(x_d - x_q)\sin\ \delta_o \right] \right\} \cos\ bt \qquad (65')$$

Let

$$Z_d = \sqrt{x_d^2 + r_a^2} \qquad (68)$$

and

$$\alpha_2 = \tan^{-1}\ \frac{r_a}{x_d} \qquad (69)$$

Then we can rearrange eq 65' with

$$\left[2E\ V\ \sin\ \alpha_1 - \frac{E\ V\ Z_q(x_d - x_q)}{r_a^2 + x_d x_q}\ \sin\ 2\alpha_1 \right]$$

$$= 2E\ V\ \sin\ \alpha_1\ \left(1 - \frac{x_d x_q - x_q^2}{r_a^2 + x_d x_q}\right)$$

$$= \frac{2E\ V\ Z_q^2}{r_a^2 + x_d x_q}\ \sin\ \alpha_1 \qquad (70)$$

$$\left[-\sin(\alpha_1 - \delta_o) + \frac{x_d - x_q}{Z_q}\ \sin\ \delta_o \right]$$

$$= \frac{x_d}{Z_q}\sin\ \delta_o - \frac{r_a}{Z_q}\cos\ \delta_o \qquad (71)$$

$$\sin(2\alpha_1 - \delta_o) = \frac{2x_q r_a}{Z_q^2}\cos\delta_o - \frac{x_q^2 - r_a^2}{Z_q^2}\sin\delta_o \tag{72}$$

$$\left[\sin(2\alpha_1 - \delta_o) - \frac{x_q(x_d - x_q)}{Z_q^2}\sin\delta_o\right]$$

$$= \frac{2x_q r_a}{Z_q^2}\cos\delta_o - \frac{x_d x_q - r_a^2}{Z_q^2}\sin\delta_o \tag{73}$$

$$\left[- V^2\sin(\alpha_1 - \delta_o) + V^2\frac{(x_d - x_q)}{Z_q}\sin\delta_o + \frac{V^2 Z_q(x_d - x_q)}{r_a^2 + x_d x_q}\sin(2\alpha - \delta_o)\right.$$

$$\left.- \frac{V^2 x_q(x_d - x_q)^2}{Z_q(r_a^2 + x_d x_q)}\sin\delta_o\right]$$

$$= \frac{V^2 Z_q}{(r_a^2 + x_d x_q)}\left[\frac{x_d(r_a^2 + x_d x_q)}{Z_q^2}\sin\delta_o - \frac{r_a(r_a^2 + x\ x\ )}{Z_q^2}\cos\delta_o\right.$$

$$\left.+ \frac{2x_q r_a(x_d - x_q)}{Z_q^2}\cos\delta_o - \frac{(x_d x_q - r_a^2)(x_d - x_q)}{Z_q^2}\sin\delta_o\right]$$

$$= \frac{V^2 Z_q}{(r_a^2 + x_d x_q)}\left\{\left[\frac{x_d(r_a^2 + x_q^2)}{Z_q^2} + \frac{r_a^2(x_d - x_q)}{Z_q^2}\right]\sin\delta_o\right.$$

$$\left.- \left[\frac{r_a(r_a^2 + x_q^2)}{Z_q^2} - \frac{x_q r_a(x_d - x_q)}{Z_q^2}\right]\cos\delta_o\right\}$$

$$= \frac{V^2 Z_q}{(r_a^2 + x_d x_q)} \left[(x_d \sin \delta_o - r_a \cos \delta_o) \right.$$

$$\left. + \frac{r_a(x_d - x_q)}{Z_q^2} (r_a \sin \delta_o + x_q \cos \delta_o) \right]$$

$$= \frac{V^2 Z_q}{(r_a^2 + x_d x_q)} \left[Z_d \sin(\delta_o - \alpha_2) \right.$$

$$\left. + (x_d - x_q) \sin \alpha_1 \cos(\alpha_1 - \delta_o) \right]$$

$$= \frac{V^2 Z_q^2}{(r_a^2 + x_d x_q)} \left[\frac{Z_d}{Z_q} \sin(\delta_o - \alpha_2) + \frac{x_d - x_q}{Z_q} \sin \alpha_1 \cos(\alpha_1 - \delta_o) \right] \quad \ldots\ldots \quad (74)$$

$$Z_q(x_d x_q - r_a^2)\cos(\alpha_1 - \delta_o) - Z_q r_a x_q \sin(\alpha_1 - \delta_o)$$

$$+ r_a x_q (x_d - x_q) \sin \delta_o$$

$$= (x_d x_q - r_a^2)(x_q \cos \delta_o + r_a \sin \delta_o)$$

$$- r_a x_q (r_a \cos \delta_o - x_q \sin \delta_o) + r_a x_q (x_d - x_q) \sin \delta_o$$

$$= (x_d x_q - 2r_a^2) x_q \cos \delta_o + (2 x_d x_q - r_a^2) r_a \sin \delta_o \quad \ldots\ldots \quad (75)$$

Substituting eqs 70, 74, and 75 into 65', we have

$$T_d = 6K \frac{x_m}{r_b} \cdot \frac{Z_q^4}{(r^2 + x x)^3} \cdot \frac{k_b^2}{k_b^2 + b^2} \cdot \frac{\delta b}{\omega} \cdot \sin(\alpha_1 - \delta_o)$$

$$\times \left[2EV \sin \alpha_1 - V^2 \frac{Z_d}{Z_q} \sin(\alpha_2 - \delta_o) + V^2 \frac{x_d - x_q}{Z_q} \sin \alpha_1 \cos(\alpha_1 - \delta_o) \right] \cos bt$$

$$+ 4K \cdot \frac{\delta b}{\omega} \left\{ \frac{E^2 r_a (x_d x_q - r_a^2)}{x_m (r^2 + x_d x_q)^2} \right.$$

$$+ \frac{E\, r_a (x_d - x_q)}{x_m (r_a^2 + x_d x_q)^3} \left[(E - V \cos \delta_o)(2r_a^2 - x_d x_q) x_q \right.$$

$$\left. \left. + (2x_d x_q - r_a^2) r\, V \sin \delta_o \right] \right\} \cos bt \qquad (76)$$

4.4 Criterion for Negative Damping

If the damping torque is expressed in a product of a factor B multiplying the change of the machine speed as

$$T_d = B\left(\frac{d\theta}{dt} - \omega\right) \qquad (77)$$

then, in our case, we have

$$T_d = B\, \delta b \cos bt \qquad (78)$$

When B is positive, the torque T_d tends to increase or decrease the speed of the machine according to whether the speed is above or below its average value. Then, if the machine once starts to oscillate, the amplitude will tend to become larger and larger. This condition is called negative damping. On the contrary, if B is negative, the amplitude of the oscillation will tend to become smaller and smaller. The condition is then called positive damping. If B is zero, the machine tends to oscillate with constant amplitude, with zero damping.

Comparing expressions 76 and 78, we have

$$B = \frac{6K}{\omega} \cdot \frac{x_m}{r_b} \cdot \frac{Z_q^4}{(r_a^2 + x_d x_q)^3} \cdot \frac{k_b^2}{k_b^2 + b^2} \cdot \sin(\alpha_1 - \delta_o)$$

$$\times \left[2EV \sin \alpha_1 - V^2 \frac{Z_d}{Z_q} \sin(\alpha_2 - \delta_o) + V^2 \frac{x_d - x_q}{Z_q} \sin \alpha_1 \cos(\alpha_1 - \delta_o) \right]$$

$$+ \frac{4K}{\omega} \cdot \frac{E^2 r_a (x_d x_q - r_a^2}{x_m (r_a^2 + x_d x_q)^2}$$

$$+ \frac{4K}{\omega} \quad \frac{E\, r_a (x_d - x_q)}{x\, (r^2 + x_d x_q)^3} \left[(E - V \cos \delta_o)(2r_a - x_d x_q) x_q \right.$$

$$\left. + (2x_d x_q - r_a) r_a V \sin \delta_o \right] \qquad (79)$$

Therefore the criterion for the negative damping of salient-pole synchronous machines without damper windings is for expression 79 to be greater than zero.

4.5 Discussions and Conclusions

(a) The assumptions in our mathematical derivations disregarded all the core losses. Then the expression obtained for either total torque or damping torque does not include the effects due to the hysteresis and eddy currents in both the armature and the pole structure.

(b) If the armature resistance of a salient-pole synchronous machine without damper windings were zero, we should have

$$B = -\frac{6K}{\omega} \quad \frac{x_m}{r_b} \cdot \left(\frac{V}{x_d}\right)^2 \cdot \frac{k_b^2}{k_b^2 + b^2} \sin^2\delta_o \tag{80}$$

The value of B would be always less than zero for any value of δ_o , and the machine would always provide positive damping.

(c) If the machine runs with the field source short-circuited, we have

$$B = \frac{6K}{\omega} \cdot \frac{x_m}{r_b} \cdot \frac{Z_q^3}{(r_a^2 + x_d x_q)^3} \cdot \frac{k_b^2}{k_b^2 + b^2} \cdot \sin(\alpha_1 - \delta_o)$$

$$\times \left[V^2(x_d - x_q)\sin \alpha_1 \cos(\alpha_1 - \delta_o) - V^2 Z_d \sin(\alpha_2 - \delta_o) \right] \tag{81}$$

There will be negative or positive damping as the factor

$$\left[(x_d - x_q)\sin \alpha_1 \cos(\alpha_1 - \delta_o) - Z_d \sin(\alpha_2 - \delta_o) \right] \sin(\alpha_1 - \delta_o)$$

is greater or less than zero. Therefore, for negative values of δ_o (i.e., motor action) the damping is always positive.

(d) In expression 79, the last part is small under ordinary operations. So B may be expressed essentially as follows:

$$B = \frac{6K}{\omega} \cdot \frac{x_m}{r_b} \cdot \frac{Z_q^4}{(r_a^2 + x_d x_q)^3} \cdot \frac{k_b^2}{-k_b^2 + b^2} \cdot \sin(\alpha_1 - \delta_o)$$

$$\times \left[2EV \sin \alpha_1 - V^2 \frac{Z_d}{Z_q} \sin(\alpha_2 - \delta_o) + V^2 \frac{x_d - x_q}{Z_q} \sin \alpha_1 \cos(\alpha_1 - \delta_o)\right]$$

$$+ \frac{4K}{\omega} \cdot \frac{E^2 r_a (x_d x_q - r_a^2)}{x_m (r_a^2 + x_d x_q)^2} \tag{82}$$

where the term proportional to E^2 is always positive unless $r_a^2 > x_d x_q$; and for $E=V$ and δ_o being positive, the sum of the other terms is always positive when α_1 is greater than δ_o or

$$r_a > x_q \tan \delta_o \tag{83}$$

In other words, an ordinary salient-pole synchronous generator with normal excitation and no amortisseur winding will have negative damping due to the electromagnetic action between the armature and field windings when the relation between the armature resistance and the q-axis synchronous reactance satisfies the expression 83.

(e) The favorable conditions for the negative damping of a synchronous motor without amortisseur winding are high excitation, large armature resistance, ~~large ratio of~~ $\cancel{x_d}$ ~~to~~ $\cancel{x_q}$, and small power angle.

(f) From expressions 63 and 64, we can conclude that the additional synchronizing torque to that calculated from the static characteristic is

$$\Delta T_s = 6K \frac{x_m}{r_b} \cdot \frac{Z_q^4}{(r_a^2 + x_d x_q)^3} \cdot \frac{k_b b}{k_b^2 + b^2} \cdot \frac{\delta b}{\omega} \sin(\alpha_1 - \delta_o) \sin bt$$

$$\left[2EV \sin \alpha_1 - V^2 \frac{Z_d}{Z_q} \sin(\alpha_2 - \delta_o) + V^2 \frac{x_d - x_q}{Z_q} \sin \alpha_1 \cos(\alpha_1 - \delta_o) \right] \qquad \ldots\ldots \quad (84)$$

CHAPTER V

OSCILLATION OF SALIENT POLE SYNCHRONOUS MACHINE WITH FIELD WINDINGS IN BOTH AXES

The relative positions of the windings of a three-phase synchronous machine with auxiliary field winding in q-axis can be shown as follows:

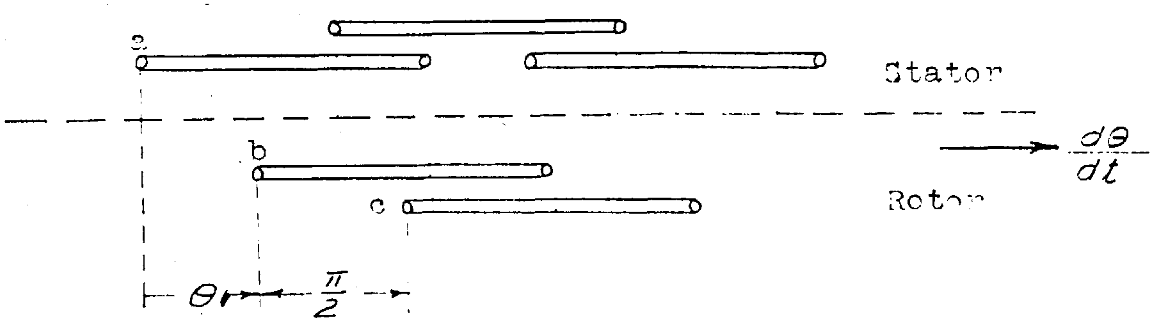


Figure 7. Winding Positions.

where a is one phase of the armature windings,
b is the main field winding,
c is the auxiliary field winding in q-axis.

Windings b and c are in space quadrature, and they have different circuit constants. Hence they do not form a balanced two-phase system together. In our analysis, then, we shall consider that each rotor winding is a phase of a system of two different but balanced two-phase systems with the other phase open-circuited. Thus the methods used before can be applied to this case also.

5.1 The Fundamental Equations

The fundamental differential equations in terms of symmetrical components of a three-phase salient-pole synchronous machine are:

$$v_{a1} = (r_a + L_a D)i_{a1} + L_a' D(i_{a2}\varepsilon^{j2\theta})$$

$$+ M_d D(i_{b1}\varepsilon^{j\theta}) + j\, M_q D(i_{c1}\varepsilon^{j\theta}) \qquad (1)$$

$$v_{b1} = (r_b + L_b D)i_{b1} + \frac{3}{2} M_d D(i_{a1}\varepsilon^{-j\theta}) \qquad (2)$$

$$v_{c1} = (r_c + L_c D)i_{c1} - j\,\frac{3}{2} M_q D(i_{a1}\varepsilon^{-j\theta}) \qquad (3)$$

where

M_d is the maximum mutual inductance between the main field winding and any phase on the armature.

M_q is the maximum mutual inductance between the auxiliary winding in the q-axis and any phase on the armature. (The mutual inductances between any rotor winding and any phase on the armature are assumed to vary as the cosine of the position angle between them.)

r_c is the resistance of the ~~damper~~ auxiliary winding.

L_c is the self-inductance of the auxiliary winding.

v_{c1} and i_{c1} are the positive-sequence voltage and current of the auxiliary winding in terms of instantaneous quantities.

All the other notations represent the same quantities as before.

Since the armature terminals are connected to a balanced three-phase source, the field winding is connected to a d-c source, and the auxiliary winding is short-circuited, we have

$$v_{a1} = V_1 \, \varepsilon^{j\omega t} \tag{4}$$

$$v_{b1} + v_{b2} = E_{d.c.} \tag{5}$$

$$v_{c1} + v_{c2} = 0 \tag{6}$$

Since the rotor windings belong to the different balanced two-phase systems with one phase of each system open-circuited, we have

$$i_{b1} = i_{b2} = \frac{i_b}{2} \tag{7}$$

$$i_{c1} = i_{c2} = \frac{i_c}{2} \tag{8}$$

where i_c is the current in the ~~damper~~ auxiliary winding, and i_{c2} is its negative-sequence component. Therefore, from eqs 1, 2, 3, and their conjugates, together with the eqs 5 to 8, we get

$$\begin{aligned} V_1\varepsilon^{j\omega t} = {} & (r_a + L_a D)i_{a1} + L_a' D(i_{a2}\varepsilon^{j2\theta}) \\ & + \frac{M_d}{2} D(i_b\varepsilon^{j\theta}) + j\,\frac{M_q}{2} D(i_c\varepsilon^{j\theta}) \end{aligned} \tag{9}$$

$$\begin{aligned} V_2\varepsilon^{-j\omega t} = {} & (r_a + L_a D)i_{a2} + L_a' D(i_{a1}\varepsilon^{-j2\theta}) \\ & + \frac{M_d}{2} D(i_b\varepsilon^{-j\theta}) - j\,\frac{M_q}{2} D(i_c\varepsilon^{-j\theta}) \end{aligned} \tag{10}$$

$$E_{d.c.} = (r_b + L_b D)i_b + \frac{3}{2} M_d D \;\; i_{a1}\varepsilon^{-j\theta} + i_{a2}\varepsilon^{j\theta} \tag{11}$$

$$0 = (r_c + L_c D)i_c - j\,\frac{3}{2} M_q D \;\; i_{a1}\varepsilon^{-j\theta} - i_{a2}\varepsilon^{j\theta} \tag{12}$$

These four equations are sufficient for solving the currents in the different windings. After the currents are determined, we can get the electromagnetic torque by the following expression:

$$T = j\,K\left\{ i_b(i_{a2}\varepsilon^{j\theta} - i_{a1}\varepsilon^{-j\theta}) + j\,\frac{M_q}{M_d}\left[i_c(i_{a2}\varepsilon^{j\theta} + i_{a1}\varepsilon^{-j\theta})\right] + \frac{2L_a'}{M_d}\left[(i_{a2}\varepsilon^{j\theta})^2 - (i_{a1}\varepsilon^{-j\theta})^2\right]\right\} \tag{13}$$

where $K = \frac{\text{poles}}{2} \cdot \frac{550}{746} \cdot \frac{3}{2} M_d$; if T is in lb.ft. and currents and inductances are in amperes and henries, respectively.

Let

$$i_b i_{a2}\varepsilon^{j\theta} = x_1 + j\,y_1 \tag{14}$$

$$(i_{a2}\varepsilon^{j\theta})^2 = x_2 + j\,y_2 \tag{15}$$

$$i_c i_{a2}\varepsilon^{j\theta} = x_3 + j\,y_3 \tag{16}$$

Then we have

$$T = -\,2K\left(y_1 + \frac{M_q}{M_d}\,x_3 + \frac{2L_a'}{M_d}\,y_2\right) \tag{17}$$

5.2 Solutions of the Stator and Rotor Currents When the Machine is in Oscillation

The method of superposition and the method of successive reflections will still be used for solving the currents from the equations 9 to 12. However, due to the presence of the additional winding on the rotor, the process will be much more involved.

A. Due to the field source alone

By applying the method of superposition, we are now considering the field source to be applied alone with the armature terminals short-circuited. The currents, then, should satisfy the following equations:

$$0 = (r_a + L_a D)i_{a1} + L_a' D(i_{a2}\varepsilon^{j2\theta}) + \frac{M_d}{2} D(i_b \varepsilon^{j\theta}) + j\frac{M_q}{2} D(i_c \varepsilon^{j\theta}) \tag{18}$$

$$0 = (r_a + L_a D)i_{a2} + L_a' D(i_{a1}\varepsilon^{-j2\theta}) + \frac{M_d}{2} D(i_b \varepsilon^{-j\theta}) - j\frac{M_q}{2} D(i_c \varepsilon^{-j\theta}) \tag{19}$$

$$E_{d.c.} = (r_b + L_b D)i_b + \frac{3}{2} M_d D\left(i_{a2}\varepsilon^{j\theta} + i_{a1}\varepsilon^{-j\theta}\right) \tag{20}$$

$$0 = (r_c + L_c D)i_c - j\frac{3}{2} M_q D\left(i_{a2}\varepsilon^{j\theta} - i_{a1}\varepsilon^{-j\theta}\right) \tag{21}$$

It is difficult to solve them simultaneously. However, the method of successive reflections can be used to obtain the principal parts of their solutions. We proceed as follows:

First step: We assume the armature terminals are open. Hence we have

$$i_{b1} = \frac{E_{d.c.}}{r_b} \tag{22}$$

$$i_{a11} = 0 \tag{23}$$

$$i_{a21} = i_{a11} = 0 \tag{24}$$

$$i_{c1} = 0 \tag{25}$$

$$v_{a11} = \frac{M_d}{2} D(i_{b1}\varepsilon^{j\theta}) = \frac{M_d E_{d.c.}}{2r_b} D\left[\varepsilon^{j(\theta_o+\omega t+\delta\sin bt)}\right]$$

$$= j\frac{M_d E_{d.c.}}{2r_b}\left[\omega\,\varepsilon^{j(\theta_o+\omega t)} + \frac{\delta}{2}(\omega+b)\varepsilon^{j(\theta_o+\omega t+bt)} - \frac{\delta}{2}(\omega-b)\varepsilon^{j(\theta_o+\omega t-bt)}\right]$$

$$= \underset{\wedge}{v}_{a21} \tag{26}$$

where i_{b1} and i_{c1} represent the components of i_b and i_c, respectively, in the first step of the method of successive reflections. They do not and will not represent the positive-sequence components now and later.

Second step: Assume now both the main field winding and the auxiliary winding are open and a voltage of $(-v_{a11})$ as its positive-sequence component is applied to the armature. Then we have

$$i_{b2} = 0 \tag{27}$$

$$i_{c2} = 0 \tag{28}$$

and

$$- v_{a11} = (r_a + L_a D) i_{a12} + L_a' D (i_{a22} \varepsilon^{j2\theta}) \tag{29}$$

$$- v_{a21} = (r_a + L_a D) i_{a22} + L_a' D (i_{a12} \varepsilon^{-j2\theta}) \tag{30}$$

From eqs 26, 29, and 30, by the same method as that used in the preceding chapter, we get

$$i_{a22}\varepsilon^{j\theta} = - \frac{M_d E_{d.c.}}{2L\, r_b} \varepsilon^{-j\alpha_1}$$

$$- j \frac{M_d E_{d.c.}}{2r_b} \cdot \frac{r_a b\, \delta}{\omega^2} \cdot \frac{L_e^2 - (L_a')^2 - 2 \frac{r_a}{j\omega} L_a'}{\left[|L_e|^2 - (L_a')^2 \right]^2} \cos bt$$

$$\ldots\ldots \quad (31)$$

$$v_{b2} = \frac{3}{2} M_d D \left(i_{a22}\varepsilon^{j\theta} + i_{a12}\varepsilon^{-j\theta} \right)$$

$$= \frac{3M_d^2 E_{d.c.}}{2r_b} \cdot \frac{r_a b^2 \delta}{\omega^2} \cdot \frac{2 \frac{r_a}{\omega} (L_a - L_a')}{\left[|L_e|^2 - (L_a')^2 \right]^2} \sin bt \tag{32}$$

$$v_{c2} = - j \frac{3}{2} M_q D \left(i_{a22}\varepsilon^{j\theta} - i_{a12}\varepsilon^{-j\theta} \right)$$

$$= \frac{3M_q M_d E_{d.c.}}{2r_b} \cdot \frac{r_a b^2 \delta}{\omega^2} \cdot \frac{-\left(\frac{r_a}{\omega}\right)^2 + L_a^2 - (L_a')^2}{\left[|L_e|^2 - (L_a')^2 \right]^2} \sin bt$$

$$\ldots\ldots \quad (33)$$

where v_{b2} , v_{c2} , i_{b2} , and i_{c2} represent the components v_b , v_c , i_b , and i_c in the second step. They are not the negative-sequence components.

Third step: Now we can solve the remaining parts of the currents by considering that voltages of $(-v_{b2})$ and $(-v_{c2})$ are applied to the main and auxiliary field windings, respectively, with the armature terminals short-circuited and replacing the armature resistance by imaginary inductances. Thus, if we let i_a' , i_b' , and i_c' be the remaining parts of the respective currents, they will satisfy the following equations:

$$0 = L_e D\, i_{a1}' + L_a' D(i_{a2}' \varepsilon^{j2\theta})$$

$$+ \frac{M_d}{2} D(i_b' \varepsilon^{j\theta}) + j \frac{M_q}{2} D(i_c' \varepsilon^{j\theta}) \tag{A}$$

$$0 = L_e D\, i_{a2}' + L_a' D(i_{a1}' \varepsilon^{-j2\theta})$$

$$+ \frac{M_d}{2} D(i_b' \varepsilon^{-j\theta}) - j \frac{M_q}{2} D(i_c' \varepsilon^{-j\theta}) \tag{B}$$

$$-v_{b2} = (r_b + L_b D) i_b' + \frac{3}{2} M_d D\left(i_{a2}' \varepsilon^{j\theta} + i_{a1}' \varepsilon^{-j\theta} \right) \tag{C}$$

$$-v_{c2} = (r_c + L_c D) i_c' - j\, M_q D\left(i_{a2}' \varepsilon^{j\theta} - i_{a1}' \varepsilon^{-j\theta} \right) \tag{D}$$

Then we have

$$i_b = \frac{E_{d.c.}}{r_b} + i_b' \tag{34}$$

$$i_c = i_c' \tag{35}$$

$$i_{a2}\varepsilon^{j\theta} = i'_{a2}\varepsilon^{j\theta} - \frac{M_d E_{d.c.}}{2L\ r_b}\,\varepsilon^{-j\alpha_1}$$

$$- j\,\frac{M_d E_{d.c.}}{2r_b}\cdot\frac{r_a b\ \delta}{\omega^2}\cdot\frac{L_e^2 - (L_a')^2 - 2\,\frac{r_a}{j\omega}\,L_a'}{\left[\,|L_e|^2 - (L_a')^2\right]^2}\cos bt$$

$$\ldots\ldots \tag{36}$$

As b is small in comparison with ω, the effective values of v_{b2} and v_{c2} as shown by expressions 32 and 33 will be small in comparison with v_d and v_q appearing later in eqs 57' and 58'. Therefore i_b', i_c', and $(i'_{a2}\varepsilon^{j\theta})$ may be disregarded so far as their effects on the torque are concerned.

B. Due to the armature source alone

We can now consider the field source short-circuited. Then the currents should satisfy the following equations:

$$V_1\varepsilon^{j\omega t} = (r_a + L_a D)i_{a1} + L_a' D(i_{a2}\varepsilon^{j2\theta})$$

$$+ \frac{M_d}{2}\,D(i_b\varepsilon^{j\theta}) + j\,\frac{M_q}{2}\,D(i_c\varepsilon^{j\theta}) \tag{37}$$

$$V_2\varepsilon^{-j\omega t} = (r_a + L_aD)i_{a2} + L_a'D(i_{a1}\varepsilon^{-j2\theta})$$

$$+ \frac{M_d}{2} D(i_b\varepsilon^{-j\theta}) - j\frac{M_q}{2} D(i_c\varepsilon^{-j\theta}) \tag{38}$$

$$0 = (r_b + L_bD)i_b + \frac{3}{2} M_d(i_{a1}\varepsilon^{-j\theta} + i_{a2}\varepsilon^{j\theta}) \tag{39}$$

$$0 = (r_c + L_cD)i_c - j\frac{3}{2} M_q(i_{a1}\varepsilon^{-j\theta} - i_{a2}\varepsilon^{j\theta}) \tag{40}$$

By the same reasoning as in the preceding chapter, we may use the simplifying approximations of replacing the stator resistance r_a to the positive-sequence current by an imaginary inductance $\frac{r_a}{j\omega}$, and to the negative-sequence current by $\frac{r_a}{-j\omega}$. Then we have

$$(r_a + L_aD)i_{a1} = \left(\frac{r_a}{j\omega} + L_a\right)D\, i_{a1} \tag{41}$$

$$(r_a + L_aD)i_{a2} = \left(\frac{r_a}{-j\omega} + L_a\right)D\, i_{a2} \tag{42}$$

Substituting eqs 41 and 42 into the eqs 37 and 38, and integrating with respect to time t, we get

$$\frac{V_1\varepsilon^{j\omega t}}{j\omega} = L_e i_{a1} + L_a' i_{a2}\varepsilon^{j2\theta} + \frac{M_d}{2} i_b\varepsilon^{j\theta} + j\frac{M_q}{2} i_c\varepsilon^{j\theta}$$

or

$$\frac{V_1\varepsilon^{j\phi}}{j\omega} = L_e i_{a1}\varepsilon^{-j\theta} + L_a' i_{a2}\varepsilon^{j\theta} + \frac{M_d}{2} i_b + j\frac{M_q}{2} i_c \tag{43}$$

and

$$\frac{V_2\varepsilon^{-j\phi}}{-j\omega} = L_e i_{a2}\varepsilon^{j\theta} + L_a' i_{a1}\varepsilon^{-j\theta} + \frac{M_d}{2} i_b - j\frac{M_q}{2} i_c \qquad (44)$$

where

$$L_e = \frac{r_a}{j\omega} + L_a'$$

and

$$\phi = \omega t - \theta = -\theta_o - \delta \sin bt$$

From eqs 43 and 44 we get

$$i_{a2}\varepsilon^{j\theta} = \frac{L_e \dfrac{V_2\varepsilon^{-j\phi}}{-j\omega} - L_a' \dfrac{V_1\varepsilon^{j\phi}}{j\omega} - (L_e - L_a')\dfrac{M_d}{2} i_b + j(L_e + L_a')\dfrac{M_q}{2} i_c}{|L_e|^2 - (L_a')^2} \quad \cdots\cdots \qquad (45)$$

From eq 45 and its conjugate we have

$$i_{a1}\varepsilon^{-j\theta} + i_{a2}\varepsilon^{j\theta} = \frac{1}{|L_e|^2 - (L_a')^2}\left[(L_e - L_a')\frac{V_1\varepsilon^{j\phi}}{j\omega} + (L_e - L_a')\frac{V_2\varepsilon^{-j\phi}}{-j\omega} - (L_a - L_a')M_d i_b + \frac{r_a}{\omega} M_q i_c\right] \qquad (46)$$

and

$$i_{a1}\varepsilon^{-j\theta} - i_{a2}\varepsilon^{j\theta} = \frac{1}{|L_e|^2 - (L_a')^2}\left[(L_e + L_a')\frac{V_1\varepsilon^{j\phi}}{j\omega} - (L_e + L_a')\frac{V_2\varepsilon^{-j\phi}}{-j\omega} + \frac{r_a}{j\omega} M_d i_b - j(L_a + L_a')M_q i_c\right] \qquad (47)$$

Let

$$V_1 = V\,\varepsilon^{j\alpha_o} \tag{48}$$

$$\frac{L_e - L_a'}{|L_e|^2 - (L_a')^2} = \frac{1}{L_1}\,\varepsilon^{-j\alpha_1} \tag{49}$$

i.e.,

$$L = L_1 \tag{49'}$$

$$\frac{L_e + L_a'}{|L_e|^2 - (L_a')^2} = \frac{1}{L_2}\,\varepsilon^{-j\alpha_2} \tag{50}$$

Then

$$V_2 = V\,\varepsilon^{-j\alpha_o} \tag{51}$$

$$\tan\alpha_1 = \frac{r_a}{x_q} \tag{52}$$

$$\tan\alpha_2 = \frac{r_a}{x_d} \tag{53}$$

$$\frac{\frac{r_a}{\omega}}{|L_e|^2 - (L_a')^2} = \frac{\sin\alpha_1}{L_1} \tag{54}$$

$$\frac{L_a - L_a'}{|L_e|^2 - (L_a')^2} = \frac{\cos\alpha_1}{L_1} \tag{55}$$

$$\frac{L_a + L_a'}{|L_e|^2 - (L_a')^2} = \frac{\cos\alpha_2}{L_2} \tag{56}$$

and eqs 46 and 47 can be shown as

$$i_{a1}\varepsilon^{-j\theta} + i_{a2}\varepsilon^{j\theta} = \frac{2V}{\omega L_1}\sin(\phi + \alpha_o + \alpha_1) - \frac{M_d \cos\alpha_1}{L_1} i_b + \frac{M_q \sin\alpha_1}{L_1} i_c \qquad \ldots\ldots \quad (46')$$

$$i_{a1}\varepsilon^{-j\theta} - i_{a2}\varepsilon^{j\theta} = \frac{2V}{j\omega L_2}\cos(\phi + \alpha_o + \alpha_2) + \frac{M_d \sin\alpha_1}{j L_1} i_b + \frac{M_q \cos\alpha_2}{j L_2} i_c \qquad \ldots\ldots \quad (47')$$

Substituting eqs 46', 47', into eqs 39 and 40, and rearranging, we have

$$(k_b + D)i_b + \frac{3}{2} \cdot \frac{M_d M_q \sin\alpha_1}{\sigma_b L_b L_1} D\, i_c$$

$$= -\frac{3V M_d}{\sigma_b L_b L_1 \omega} D\left[\sin(\phi + \alpha_o + \alpha_1)\right] \qquad (57)$$

and

$$(k_c + D)i_c - \frac{3}{2} \quad \frac{M_d M_q \sin\alpha_1}{\sigma_c L_c L_1} D\, i_b$$

$$= \frac{3V M_q}{\sigma_c L_c L_2 \omega} D\left[\cos(\phi + \alpha_o + \alpha_2)\right] \qquad (58)$$

where

$$\sigma_b = 1 - \frac{\frac{3}{2} M_d^2 \cos\alpha_1}{L_b L_1}, \qquad k_b = \frac{r_b}{\sigma_b L_b}$$

$$\sigma_c = 1 - \frac{\frac{3}{2} M_q^2 \cos\alpha_2}{L_b L_2}, \qquad k_c = \frac{r_c}{\sigma_c L_c}$$

Since

$$\phi = \omega t - \theta = \omega t - (\theta_o + \omega t + \delta \sin bt)$$

$$= -\theta_o - \delta \sin bt$$

$$= -(\alpha_o + \delta_o - \frac{\pi}{2}) - \delta \sin bt$$

$$= \frac{\pi}{2} - \alpha_o - \delta_o - \delta \sin bt$$

then

$$\phi + \alpha_o + \alpha_1 = \frac{\pi}{2} - \delta_o + \alpha_1 - \delta \sin bt$$

$$\sin(\phi + \alpha_o + \alpha_1) = \cos(\delta_o - \alpha_1 + \delta \sin bt)$$

$$= \cos(\delta_o - \alpha_1) - \delta \sin(\delta_o - \alpha_1)\sin bt$$

$$D\left[\sin(\phi + \alpha_o + \alpha_1)\right] = -\delta\, b \sin(\delta_o - \alpha_1)\cos bt \qquad (59)$$

Similarly,

$$D\left[\cos(\phi + \alpha_o + \alpha_2)\right] = \delta\, b \cos(\delta_o - \alpha_2)\cos bt \qquad (60)$$

Substituting eqs 59 and 60 into eqs 57 and 58, we have

$$(k_b + D)i_b + \frac{3}{2} \cdot \frac{M_d M_q \sin \alpha_1}{\sigma_b L_b L_1} D\, i_c$$

$$= \frac{3V\, M_d \delta\, b}{\sigma_b L_b L_1 \omega} \sin(\delta_o - \alpha_1)\cos bt$$

$$= \frac{1}{\sigma_b L_b} \cdot v_d \qquad (57')$$

$$(k_c + D)i_c - \frac{3}{2} \cdot \frac{M_d M_q \sin \alpha_1}{\sigma_c L_c L_1} D\, i_b$$

$$= \frac{3V\, M_q \delta\, b}{\sigma_c L_c L_2 \omega} \cos(\delta_o - \alpha_2) \cos bt$$

$$= \frac{1}{\sigma_c L_c} \cdot v_q \tag{58'}$$

where

$$v_d = \frac{3V\, M_d \delta\, b}{L_1 \omega} \sin(\delta_o - \alpha_1) \cos bt$$

$$v_q = \frac{3V\, M_q \delta\, b}{L_2 \omega} \cos(\delta_o - \alpha_1) \cos bt$$

Equations 57' and 58' can be easily solved by using the ordinary method of complex numbers in alternating-current circuits. Thus we can get

$$i_b = I_b \cos(bt + \beta_b) \tag{61}$$

$$i_c = I_c \cos(bt + \beta_c) \tag{62}$$

where the constants I_b , I_c , β_b , and β_c are given by the following expressions:

$$I_b \varepsilon^{j\beta_b} = \frac{M_d L_2 (r_c + jb\, \sigma_c L_c) \sin(\delta_o - \alpha_1) - jb\, M\, M_q L_1 \cos(\delta_o - \alpha_2)}{(r_b + jb\, \sigma_b L_b)(r_c + jb\, \sigma_c L_c) - b^2 M^2} \cdot \frac{3V\, \delta\, b}{L_1 L_2 \omega}$$

$$\cdots\cdots \tag{63}$$

$$I_c \varepsilon^{j\beta_c} = \frac{M_q L_1 (r_b + jb\, \sigma_b L_b) \cos(\delta_o - \alpha_2) + jb\, M\, M_d L_2 \sin(\delta_o - \alpha_1)}{(r_b + jb\, \sigma_b L_b)(r_c + jb\, \sigma_c L_c) - b^2 M^2} \cdot \frac{3V\, \delta\, b}{L\, L\, \omega}$$

$$\cdots\cdots \tag{64}$$

$$\text{with} \quad M = \frac{3M_d M_q \sin \alpha_1}{2L_1} \tag{65}$$

Then, from eqs 45, 61, and 62 we have

$$i_{a2}\varepsilon^{j\theta} = \frac{L_e V_2 \varepsilon^{-j\phi} + L_a' V_1 \varepsilon^{j\phi}}{-j\omega\left[|L_e|^2 - (L_a')^2\right]} - \frac{M_d I_b}{2L_1}\varepsilon^{-j\alpha_1}\cos(bt + \beta_b)$$

$$+ j\,\frac{M_q I_c}{2L_2}\varepsilon^{-j\alpha_2}\cos(bt + \beta_c) \tag{66}$$

C. <u>When both the armature and field sources are applied</u>

The solutions of the rotor and stator currents, when both the armature and field sources are applied, will be the sums of the respective solutions when each source is applied alone. Thus, from expressions 34, 35, 36, 61, 62, and 66, we have

$$i_b = \frac{E_{d.c.}}{r_b} + + I_b \cos(bt + \beta_b) \tag{67}$$

$$i_c = I_c \cos(bt + \beta_c) \tag{68}$$

$$i_{a2}\varepsilon^{j\theta} = -\frac{E_{d.c.}M_d}{2L_1 r_b}\varepsilon^{-j\alpha_1}$$

$$- j\,\frac{M_d E_{d.c.}}{2r_b} \cdot \frac{r_a b\delta}{\omega^2} \cdot \frac{L_e^2 - (L_a')^2 - 2\frac{r_a}{j\omega}L_a'}{\left[|L_e|^2 - (L_a')^2\right]^2}\cos bt$$

$$+ \frac{L_e V_2 \varepsilon^{-j\phi} + L_a' V_1 \varepsilon^{j\phi}}{-j\omega\left[|L_e|^2 - (L_a')^2\right]}$$

$$- \frac{M_d I_b}{2L_1} \varepsilon^{-j\alpha_1} \cos(bt + \beta_b)$$

$$+ j \frac{M_q I_c}{2L_2} \varepsilon^{-j\alpha_2} \cos(bt + \beta_c) \qquad (69)$$

5.3 Electromagnetic Torque Produced During the Steady Oscillations

The electromagnetic torque of a salient-pole synchronous machine with an auxiliary field winding in the q-axis has been expressed in terms of stator and rotor currents by expression 13 or 17. The currents when the machine oscillates steadily are given by expressions 67, 68, and 69. Therefore the electromagnetic torque produced during the steady oscillations can be determined as follows.

By neglecting the small terms having the factor δ^2 , we get

$$y_1 = \left(\frac{E_{d.c.}}{r_b}\right)^2 \cdot \frac{M_d}{2L_1} \sin \alpha_1 - \left(\frac{E_{d.c.}}{r_b}\right)^2 \frac{M_d r_a b\, \delta}{2\omega^2} \cdot \frac{-\left(\frac{r_a}{\omega}\right)^2 + L_a^2 - (L_a')^2}{\left[\,|L_e|^2 - (L_a')^2\right]^2} \cos bt$$

$$- \frac{E_{d.c.}}{r_b} \cdot \frac{V}{L_1\omega} \sin(\alpha_1 - \delta_o - \delta \sin bt)$$

$$+ \frac{E_{d.c.}}{r_b} \cdot \frac{2L_a'V}{\omega\left[\,|L_e|^2 - (L_a')^2\right]} \sin(\delta_o + \delta \sin bt)$$

$$+ \frac{E_{d.c.} M_d}{r_b L_1} I_b \sin \alpha_1 \cos(bt + \beta_b)$$

$$+ \frac{E_{d.c.} M_q}{2 r_b L_2} I_c \cos \alpha_2 \cos(bt + \beta_c)$$

$$+ \frac{V}{L_1\omega} I_b \sin(\delta_o - \alpha_1)\cos(bt + \beta_b)$$

$$+ \frac{2L_a'V}{\omega\left[\,|L_e|^2 - (L_a')^2\right]} I_b \sin \delta_o \cos(bt + \beta_b) \qquad (70)$$

$$x_3 = -\frac{E_{d.c.}M_d}{2r_bL_1} I_c \cos \alpha_1 \cos(bt + \beta_c)$$

$$+ \frac{V}{L_1\omega} I_c \cos(\delta_o - \alpha_1)\cos(bt + \beta_c) \qquad (71)$$

$$y_2 = -\left(\frac{M\,E_{d.c.}}{2L_1r_b}\right)^2 \sin 2\alpha_1 - \left(\frac{V}{\omega\,L_1}\right)^2 \sin 2(\alpha_1 - \delta_o - \delta \sin bt)$$

$$+ \frac{M_dV\,E_{d.c.}}{\omega\,L_1^2r_b} \sin(2\alpha_1 - \delta_o - \delta \sin bt)$$

$$- \frac{2M_dL_a'V\,E_{d.c.}}{\omega\,L_1r_b\left[\,|L_e|^2 - (L_a')^2\right]} \cos \alpha_1 \sin(\delta_o + \delta \sin bt)$$

$$+ \frac{4L_a'V^2}{\omega^2L_1\left[\,|L_e|^2 - (L_a')^2\right]} \cos(\alpha_1 - \delta_o - \delta \sin bt)\sin(\delta_o + \delta \sin bt)$$

$$+ \frac{E_{d.c.}}{r_b}^2 \cdot \frac{M_d^2r_ab\,\delta \cos \alpha_1}{2\omega^2L_1} \cdot \frac{-\left(\frac{r_a}{\omega}\right)^2 + L_a^2 - (L_a')^2}{\left[\,|L_e|^2 - (L_a')^2\right]^2} \cos bt$$

$$- \left(\frac{E_{d.c.}}{r_b} \right)^2 \cdot \frac{M_d^2 r_a b \, \delta \sin \alpha_1}{2\omega^2 L_1} \cdot \frac{\frac{2r_a}{\omega} (L_a - L_a')}{\left[|L_e|^2 - (L_a')^2 \right]^2} \cos bt$$

$$- \frac{M_d E_{d.c.} V}{\omega L_1 r_b} \cdot \frac{r_a b \, \delta \cos(\alpha_1 - \delta_o)}{\omega^2} \cdot \frac{-\left(\frac{r_a}{\omega} \right)^2 + L_a - (L_a')^2}{\left[|L_e|^2 - (L_a')^2 \right]^2} \cos bt$$

$$+ \frac{M_d E_{d.c.} V}{\omega L_1 r_b} \cdot \frac{r_a b \, \delta \sin(\alpha_1 - \delta_o)}{\omega^2} \cdot \frac{\frac{2r_a}{\omega} (L_a - L_a')}{\left[|L_e|^2 - (L_a')^2 \right]^2} \cos bt$$

$$- \frac{2M_d L_a' E_{d.c.} V}{r_b \omega} \cdot \frac{r_a b \, \delta \sin \delta_o}{\omega^2} \cdot \frac{\frac{2r_a}{\omega} (L_a - L_a')}{\left[|L_e|^2 - (L_a')^2 \right]^3} \cos bt$$

$$- \frac{M_d^2 E_{d.c.}}{2L_1^2 r_b} \cdot I_b \sin 2\alpha_1 \cos(bt + \beta_b)$$

$$- \frac{M_d M_q E_{d.c.}}{2L_1 L_2 r_b} \cdot I_c \cos(\alpha_1 + \alpha_2) \cos(bt + \beta_c)$$

$$+ \frac{M_d V}{\omega L_1^2} \cdot I_b \sin(2\alpha_1 - \delta_o) \cos(bt + \beta_b)$$

$$+ \frac{M_q V}{\omega L_1 L_2} \cdot I_c \cos(\alpha_1 + \alpha_2 - \delta_o) \cos(bt + \beta_c)$$

$$- \frac{2M_d V \, L_a' \sin \delta_o \cos \alpha_1}{\omega L_1 \left[|L_e|^2 - (L_a')^2 \right]} I_b \cos(bt + \beta_b)$$

$$+ \frac{2M_q V \, L_a' \sin \delta_o \sin \alpha_2}{\omega L_2 \left[|L_e|^2 - (L_a')^2 \right]} I_c \cos(bt + \beta_c) \qquad (72)$$

Then, as shown by the formula 17, the electromagnetic torque produced is

$$T = -2K\left(y_1 + \frac{M_q}{M_d}x_3 + \frac{2L_a'}{M_d}y\right) \tag{73}$$

with y_1 , x_3 , y_2 given by the above expressions. If we expand the factors $\cos(bt+\beta_b)$ and $\cos(bt+\beta_c)$, we can see that the torque still consists of the three parts, namely:

(a) a part proportional to $\delta b \cos bt$, causing the damping action,

(b) a part proportional to $\delta b^2 \sin bt$, causing the modification of synchronizing action,

(c) a part including all the other terms to represent the sum of the synchronous torque and synchronizing torque according to static characteristics.

For the purpose of our investigation, only the part causing damping action is of special interest. If we let

$$\frac{-\left(\frac{r_a}{\omega}\right)^2 + L_a^2 - (L_a')^2}{\left[\,|L_e|^2 - (L_a')^2\right]^2} = \frac{1}{L_3^2} \tag{74}$$

$$\frac{\frac{2r_a}{\omega}(L_a - L_a')}{\left[\,|L_e|^2 - (L_a)^2\right]^2} = \frac{1}{L_4^2} \tag{75}$$

$$\frac{2L_a'}{|L_e|^2 - (L_a')^2} = \frac{1}{L_5} \tag{76}$$

and

$$\frac{E_{d.c.}}{2r_b}\,\omega\, M_d = E \tag{77}$$

then we have the part of the torque causing damping action, or the damping torque, as

$$T_d = 2K \left\{ \frac{2E^2 r_a \delta\, b}{\omega^4 M_d L_3^2} - \frac{2E}{\omega\, L_1} I_b \cos\beta_b \sin\alpha_1 \right.$$

$$- \frac{E\, M_q}{\omega\, L_2 M_d} I_c \cos\beta_c \cos\alpha_2 - \frac{V}{\omega\, L_1} I_b \cos\beta_b \sin(\delta_o - \alpha_1)$$

$$- \frac{V}{\omega\, L_5} I_b \cos\beta_b \sin\delta_o + \frac{E\, M_q}{\omega\, L_1 M_d} I_c \cos\beta_c \cos\alpha_1$$

$$- \frac{V\, M_q}{\omega\, L_1 M_d} I_c \cos\beta_c \cos(\delta_o - \alpha_1) - \frac{4E^2 L_a' r_a b\, \delta \cos\alpha_1}{\omega^4 L_1 M_d L_3^2}$$

$$+ \frac{4E^2 L_a' r_a b\, \delta \sin\alpha_1}{\omega^4 L_1 M_d L_4^2} + \frac{4E\, V\, L_a' r_a b\, \delta \cos(\alpha_1 - \delta_o)}{\omega^4 L_1 M_d L_3^2}$$

$$- \frac{4E\, V\, L_a' r_a b\, \delta \sin(\alpha_1 - \delta_o)}{\omega^4 L_1 M_d L_4^2} + \frac{4E\, V\, L_a' r_a b\, \delta \sin\delta_o}{\omega^4 M_d L_5 L_4^2}$$

$$+ \frac{2E\, L_a'}{\omega\, L_1^2} I_b \cos\beta_b \sin 2\alpha_1 + \frac{2E\, L_a' M_q}{\omega L_1 L_2 M_d} I_c \cos\beta_c \cos(\alpha_1 + \alpha_2)$$

$$- \frac{2V\, L_a'}{\omega\, L_1^2} I_b \cos\beta_b \sin(2\alpha_1 - \delta_o) - \frac{2V\, L_a' M_q}{\omega\, L_1 L_2 M_d} I_c \cos\beta_c \cos(\alpha_1 + \alpha_2 - \delta_o)$$

$$+ \frac{2V\, L_a'}{\omega\, L_1 L_5} I_b \cos\beta_b \cos\alpha_1 \sin\delta_o$$

$$- \frac{2V\, L_a' M_q}{\omega\, L_2 L_5 M_d} I_c \cos\beta_c \sin\alpha_2 \sin\delta_o \Bigg\} \cos bt$$

$$= 2K \Bigg\{ \frac{2E^2 r_a \delta\, b}{\omega^4 M_d L_3^2} - \frac{2E}{\omega\, L_1} I_b \cos\beta_b \sin\alpha_1 - \frac{E\, M_q}{\omega\, L_5 M_d} I_c \cos\beta_c$$

$$- \frac{V}{\omega\, L_1} \left[I_b \cos\beta_b \sin(\delta_o - \alpha_1) + \frac{M_q}{M_d} I_c \cos\beta_c \cos(\delta_o - \alpha_1) \right]$$

$$- \frac{4E\, L_a' r\, b\, \delta}{\omega^4 L_1 M_d L_3^2} \left[E\cos\alpha_1 - V\cos(\alpha_1 - \delta_o) \right]$$

$$+ \frac{4E\, L_a' r_a b\, \delta}{\omega^4 L_1 M_d L_4^2} \left[E\sin\alpha_1 - V\sin(\alpha_1 - \delta_o) \right]$$

$$+ \frac{2L_a' I_b \cos\beta_b}{\omega\, L_1^2} \left[E\sin 2\alpha_1 - V\sin(2\alpha_1 - \delta_o) \right]$$

$$+ \frac{2L_a' M_q I_c \cos\beta_c}{\omega\, L_1 L_2 M_d} \left[E\cos(\alpha_1 + \alpha_2) - V\cos(\alpha_1 + \alpha_2 - \delta_o) \right]$$

$$- \frac{V}{\omega\, L_5} I_b \cos\beta \sin\delta_o + \frac{4E\, V\, L_a' r_a \delta\, b \sin\delta_o}{\omega^4 M_d L_5 L_4^2}$$

$$+ \frac{2V\, L_a'}{\omega\, L_1 L_5} I_b \cos\beta_b \cos\alpha_1 \sin\delta_o$$

$$- \frac{2V\, L_a' M_q}{\omega\, L_2 L_5 M_d} I_c \cos\beta_c \sin\alpha_2 \sin\delta_o \Bigg\} \cos bt \qquad (78)$$

5.4 Criterion for the Negative Damping

If we put the expression for the damping torque in the following form,

$$T_d = B\ \delta\ b \cos bt \quad , \tag{79}$$

then, by the same reasoning as before, we can conclude that the machine will have negative damping due to the electromagnetic action only when $B > 0$. In other words, the criterion for the negative damping of a synchronous machine is

$$\Bigg\{ \frac{2E^2 r_a}{\omega^4 M_d L_3^2} - \frac{2E}{\omega L_1} \cdot \frac{I_b \cos \beta_b}{\delta\ b} \sin \alpha_1 - \frac{E\ M_q}{\omega L_5 M_d} \cdot \frac{I_c \cos \beta_c}{\delta\ b}$$

$$- \frac{V}{\omega L_1} \left[\frac{I_b \cos \beta_b}{\delta\ b} \sin(\delta_o - \alpha_1) + \frac{M_q I_c \cos \beta_c}{M_d \delta\ b} \cos(\delta_o - \alpha_1) \right]$$

$$- \frac{4E\ L_a' r_a}{\omega^4 L_1 M_d L_3^2} \left[E \cos \alpha_1 - V \cos(\alpha_1 - \delta_o) \right]$$

$$+ \frac{4E\ L_a' r_a}{\omega^4 L_1 M_d L_4^2} \left[E \sin \alpha_1 - V \sin(\alpha_1 - \delta_o) \right]$$

$$+ \frac{2L_a' I_b \cos \beta_b}{\omega L_1^2 b\ \delta} \left[E \sin 2\alpha_1 - V \sin(2\alpha_1 - \delta_o) \right]$$

$$+ \frac{2L_a' M_q I_c \cos \beta_c}{\omega L_1 L_2 M_d b\ \delta} \left[E \cos(\alpha_1 + \alpha_2) - V \cos(\alpha_1 + \alpha_2 - \delta_o) \right]$$

$$- \frac{V}{\omega L_5} \cdot \frac{I_b \cos \beta_b}{b \delta} \sin \delta_o + \frac{4E\ V\ L_a' r_a}{\omega^4 M_d L_5 L_4^2} \sin \delta_o$$

$$+ \frac{2V\ L_a'}{\omega L_1 L_5} \cdot \frac{I_b \cos \beta_b}{b \delta} \cos \alpha_1 \sin \delta_o$$

$$- \frac{2V\ L_a' M_q}{\omega L_2 L_5 M_d} \cdot \frac{I_c \cos \beta_c}{b \delta} \sin \alpha_2 \sin \delta_o \Bigg\} > 0 \qquad (80)$$

5.5 Discussions and Conclusions

(a) Since the assumptions in the analysis disregarded the core losses in both the armature and the rotor, the damping action due to the hysteresis and eddy currents is not included in the results which we have obtained.

(b) If the machine has two or more rotor windings in each axis, the formulas 78 and 80, etc. still hold, provided that the constants of the main and auxiliary field windings are replaced by their respective equivalent constants.

(c) If the armature resistance is zero, the formula 78 will be changed to

$$T_d = 2K \Bigg\{ - \frac{E\ M_q}{\omega L_5 M_d} I_c \cos \beta_c - \frac{V}{\omega L_1} \left(I_b \cos \beta_b \sin \delta_o + \frac{M_q}{M_d} I_c \cos \beta_c \cos \delta_o \right)$$

$$+ \frac{2L_a' I_b \cos \beta_b}{\omega L_1^2} V \sin \delta_o + \frac{2L_a' M_q I_c \cos \beta_c}{\omega L_1 L_2 M_d} \left(E - V \cos \delta_o \right)$$

$$- \frac{V}{\omega L_5} I_b \cos \beta_b \sin \delta_o + \frac{2V\ L_a'}{\omega L_1 L_5} I_b \cos \beta_b \sin \delta_o \Bigg\} \cos bt$$

$$= 2K \left\{ \frac{E\ M_q}{\omega\ M_d} I_c \cos\beta_c \left(\frac{2L_a'}{L_1 L_2} - \frac{1}{L_5} \right) \right.$$

$$- \frac{M_q}{M_d} \cdot \frac{V}{\omega\ L_1} I_c \cos\beta_c \cos\delta_o \left(1 + \frac{2L_a'}{L_2}\right)$$

$$\left. + \frac{V\ I_b \cos\beta_b \sin\delta_o}{\omega\ L_1} \left(\frac{2L_a'}{L_1} - 1 - \frac{L_1}{L_5} + \frac{2L_a'}{L_5} \right) \right\} \cos bt \qquad (81)$$

And, from formulas 49, 50, 63, 64, 65, and 76, with $r_a = 0$, we have

$$\left(\frac{2L_a}{L_1 L_2} - \frac{1}{L_5} \right) = \frac{2L_a'(L_a - L_a')(L_a + L_a')}{\left[L_a^2 - (L_a')^2\right]^2} - \frac{2L_a'}{L_a^2 - (L_a')^2}$$

$$= \frac{2L_a'}{L_a^2 - (L_a')^2} \quad \frac{2L_a'}{L_a^2 - (L_a')^2} = 0$$

$$\left(1 + \frac{2L_a'}{L_a}\right) = \frac{L_a^2 - (L_a')^2 + 2L_a'(L_a + L_a')}{L_a^2 - (L_a')^2} = \frac{L_a + L_a'}{L_a - L_a'}$$

$$\left(\frac{2L_a'}{L_1} - 1 - \frac{L_1}{L_5} + \frac{2L_a'}{L_5} \right) = \left(\frac{2L_a'}{L_1} - 1\right)\left(1 + \frac{L_1}{L_5}\right)$$

$$= \left[\frac{2L_a'(L_a - L_a')}{L_a^2 - (L_a')^2} - 1 \right] \times \left[1 + \frac{2L_a'}{L_a - L_a'} \right]$$

$$= -\frac{L_a - L_a'}{L_a + L_a'} \cdot \frac{L_a + L_a'}{L_a - L_a'} = -1$$

$$I_b \cos \beta_b = \frac{3V\ \delta\ b\ M_d}{L_1\omega} \cdot \frac{r_b \sin \delta_o}{r_b^2 + (b\ \sigma_b L_b)^2}$$

$$I_c \cos \beta_c = \frac{3V\ \delta\ b\ M_q}{L_2\omega} \cdot \frac{r_c \cos \delta_o}{r_c^2 + (b\ \sigma_c L_c)^2}$$

Hence the formula 81 can be rewritten as

$$T_d = -\ 2K \left\{ \frac{3M_q^2 V\ \delta\ b}{M_d \omega^2 L_1 L_2} \cdot \frac{L_a + L_a'}{L_a - L_a'} \cdot \frac{r_c \cos^2 \delta_o}{r_c^2 + (b\ \sigma_c L_c)^2} \right.$$

$$\left. + \frac{3M_d V^2 \delta\ b}{\omega^2 L_1^2} \cdot \frac{r_b \sin^2 \delta_o}{r_b^2 + (b\ \sigma_b L_b)^2} \right\} \cos bt$$

$$= -\ \frac{6K\ M_d V^2 \delta\ b}{\omega^2 L_1^2} \left[\left(\frac{M_q}{M_d} \cdot \frac{L_a + L_a'}{L_a - L_a'} \right)^2 \cdot \frac{r_c \cos^2 \delta_o}{r_c^2 + (b\ \sigma_c L_c)^2} + \frac{r_b \sin^2 \delta_o}{r_b^2 + (b\ \sigma_b L_b)^2} \right] \cos bt$$

..... (81')

i.e.,

$$B = -\ \frac{6K\ M_d V^2}{\omega^2 L_1^2} \left[\left(\frac{M_q}{M_d} \cdot \frac{L_a + L_a'}{L_a - L_a'} \right)^2 \cdot \frac{r_c \cos^2 \delta_o}{r_c^2 + (b\ \sigma_c L_c)^2} + \frac{r_b \sin^2 \delta_o}{r_b^2 + (b\ \sigma_b L_b)^2} \right]$$

$$= -\ \frac{6K\ V^2}{M_d} \left[\left(\frac{M_q}{x_q} \right)^2 \cdot \frac{k_c \cos^2 \delta_o}{k_c^2 + b^2} + \left(\frac{M_d}{x_d} \right)^2 \cdot \frac{k_b \sin^2 \delta_o}{k_b^2 + b^2} \right] \qquad (82)$$

The value of B is always negative, no matter what will be the value of δ_o. Therefore the machine always provides a positive damping due to the electromagnetic action.

(d) If the pole structure of the machine is nonsalient, we have

$$L_a' = 0 \quad , \qquad \frac{1}{L_5} = \frac{2L_a'}{L_a^2 - (L_a')^2} = 0 \quad ,$$

and $\frac{1}{L_1} = \frac{1}{L_2} = \frac{\omega}{Z_s} \quad , \qquad \alpha_1 = \alpha_2$

where Z_s is the magnitude of synchronous impedance. Then the damping torque is

$$T_d = 2K \left\{ \frac{2E^2 r_a \delta\, b}{\omega^4 M_d L_3^2} - \frac{2E}{\omega\, L_1} I_b \cos\beta_b \sin\alpha_1 \right.$$

$$\left. - \frac{V}{\omega\, L_1} \left[I_b \cos\beta_b \sin(\delta_o - \alpha_1) + \frac{M_q}{M_d} I_c \cos\beta_c \cos(\delta_o - \alpha_1) \right] \right\} \cos bt$$

$$\ldots\ldots \quad (83)$$

Thus we can see that the presence of the short-circuited windings in the q-axis increases the positive damping action.

(e) From the expression of the total electromagnetic torque we can also find that the additional amount to the static synchronizing torque may be expressed in a similar form as expression 78 by simply changing $I_b \cos\beta_b$, $I_c \cos\beta_c$, $\cos bt$ into $-I_b \sin\beta_b$, $-I_c \sin\beta_c$, $\sin bt$, respectively, and omitting all the other terms. It is, then,

$$\Delta T_s = 2K \left\{ \frac{2E}{\omega L_1} I_b \sin \beta_b \sin \alpha_1 + \frac{E M_q}{\omega L_5 M_d} I_c \sin \beta_c \right.$$

$$+ \frac{V}{\omega L_1} \left[I_b \sin \beta_b \sin(\delta_o - \alpha_1) + \frac{M_q}{M_d} I_c \sin \beta_c \cos(\delta_o - \alpha_1) \right]$$

$$- \frac{2L_a' I_b \sin \beta_b}{\omega L_1^2} \left[E \sin 2\alpha_1 - V \sin(2\alpha_1 - \delta_o) \right]$$

$$- \frac{2L_a M_q I_c \sin \beta_c}{\omega L_1 L_2 M_d} \left[E \cos(\alpha_1 + \alpha_2) - V \cos(\alpha_1 + \alpha_2 - \delta_o) \right]$$

$$+ \frac{V}{\omega L_5} I_b \sin \beta_b \sin \delta_o \left(1 - \frac{2L_a'}{L_1} \cos \alpha_1\right)$$

$$\left. + \frac{2V L_a' M_q}{\omega L_2 L_5 M_d} I_c \sin \beta_c \sin \alpha_2 \sin \delta_o \right\} \sin bt \qquad (84)$$

(f) The formula 80 can be expressed directly in terms of voltages, resistances, reactances, and power angle, etc., as we have done correspondingly in the preceding cases. It remains as it is only for the sake of simplicity.

CHAPTER VI

LABORATORY EXPERIMENTS

The criteria for the negative damping due to the electromagnetic action of synchronous machines have been derived analytically in the preceding chapters. It is advisable to have some experimental verifications with the machines in the laboratory. As the machines under test will be in oscillatory motion, any flexible mechanical coupling between shafts must be avoided. Otherwise, uncontrollable or unpredictable extra-damping would be introduced. Hence, for our test, the M.I.T. Alternator No. 95 is used. It is rated at 220 volts, 3 phases, 60 cycles, 118 amperes, 1200 rpm, and 49 kilovoltamperes. It has salient-pole structure without damper windings. Rigidly coupled to its shaft there are a d-c machine and a pilot 3-phase generator. The d-c machine may serve either as a driving motor or as a load generator. The pilot 3-phase generator is used for power-angle measurement. In order to reduce the damping due to the electromagnetic action of the d-c machine, it is always advisable to operate it as a generator (i.e., to test the synchronous machine as a synchronous motor).

6.1 Machine Constants

The synchronous impedances of the M.I.T. Alternator No. 95 have been accurately determined by Professor Charles Kingsley Jr. by different methods. They are:

$$x_d = 0.985 \text{ ohm}$$
$$x_q = 0.53 \text{ ohm}$$
$$x_e = 0.21 \text{ ohm}$$
$$r_a = 0.043 \text{ ohm}$$

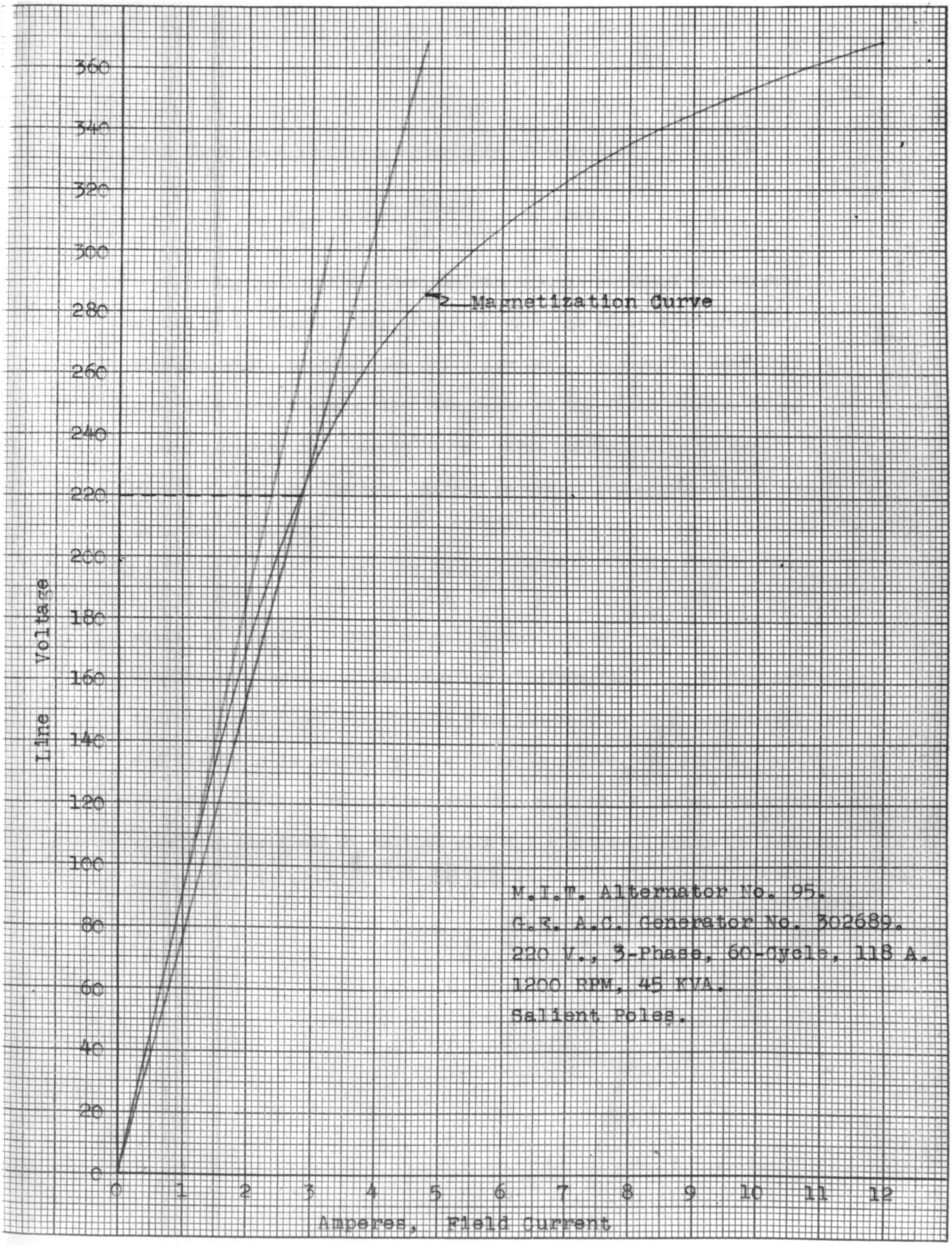
where x_d, x_q are unsaturated reactances. The magnetization curve or open-circuit characteristic is given on p. 117. As in the analysis, the machine is assumed under almost constant saturation. The saturation factor may be determined by assuming the generated emf of 220 volts. Hence, from the open-circuit characteristic we have the saturation factor as

$$k = \frac{260}{220} = 1.18$$

Then, for simplicity, we may have the saturated reactances as

$$x_d = 0.21 + \frac{0.985 - 0.21}{1.18}$$
$$= 0.21 + \frac{0.775}{1.18}$$
$$= 0.21 + 0.657 = 0.867 \text{ ohm}$$

$$x_q = 0.21 + \frac{0.53 - 0.21}{1.18}$$
$$= 0.21 + \frac{0.32}{1.18}$$
$$= 0.21 + 0.271 = 0.481 \text{ ohm}$$



360
340
320
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
0
1
2
3
4
5
6
7
8
9
10
11
12
Line Voltage
Amperes, Field Current
Magnetization Curve
M.I.T. Alternator No. 95.
G.E. A.C. Generator No. 302689.
220 V., 3-Phase, 60-Cycle, 118 A.
1200 RPM, 45 KVA.
Salient Poles.

From the open-circuit characteristic we have also

$$x_m = \omega M = \frac{220\sqrt{2}}{\sqrt{3} \times 2.85} = 63 \text{ ohms}$$

In the mathematical derivations we have excluded the effects of the core losses. They, together with the effects of friction, windage, etc., produce some extra damping. In order to meet this condition more closely, the inductance of the field winding to be used in computations should be the value corresponding to the direct-current excitation. It can be determined appropriately with the formula

$$L_b = L_b' \times \frac{M}{M'} \tag{1}$$

where M' is the maximum mutual inductance between the field winding and any one of the armature windings, and L_b' is the self-inductance of the field winding, when the machine is at standstill and the field is excited with a 60-cycle source. M and L_b are the corresponding inductances when the machine is run at synchronous speed and the field is excited with a d-c source. Since the leakage coefficient is given as

$$\sigma = 1 - \frac{3}{2} \cdot \frac{M^2 \cos \alpha_1}{L_b L}$$

$$= 1 - \frac{3}{2} \cdot \frac{M}{L_b} \cdot \frac{x_m x_q}{r_a^2 + x_q x_d} \qquad ,$$

from formula 1, and by neglecting the stator resistance, we have

$$\sigma = 1 - \frac{3}{2} \cdot \frac{M'}{L_b'} \cdot \frac{x_m}{x_d} \qquad (2)$$

Thus, by measuring L_b', M', and with the values of x_d and x_m, we get

$$L_b = 28.6 \text{ h}$$

$$\sigma = 0.258$$

and

$$\omega \sigma L_b = 377 \times 0.258 \times 28.6 = 2780 \text{ ohms}$$

6.2 Connection Diagrams

Any electrical or mechanical disturbance could cause a synchronous machine to oscillate about the equilibrium position with respect to its rotating field. The amplitude of oscillation will be constant, increasing or decreasing when the total damping of the system is zero, negative, or positive, respectively. In order to test the damping, then, the synchronous machine may be connected as for normal operation except that the armature terminals are connected to the bus through rheostats for adjusting the damping due to the electromagnetic effect. Hence we can show the connection diagram for our test as Fig. 8. Since the synchronous machine is tested as a synchronous motor, the coupled d-c machine is operated as a separately excited generator and delivers its load to a rheostat (Fig. 9). The coupled pilot a-c generator is used for power-angle measurements. It is connected to a two-pole phase shifter as shown in Fig. 10.

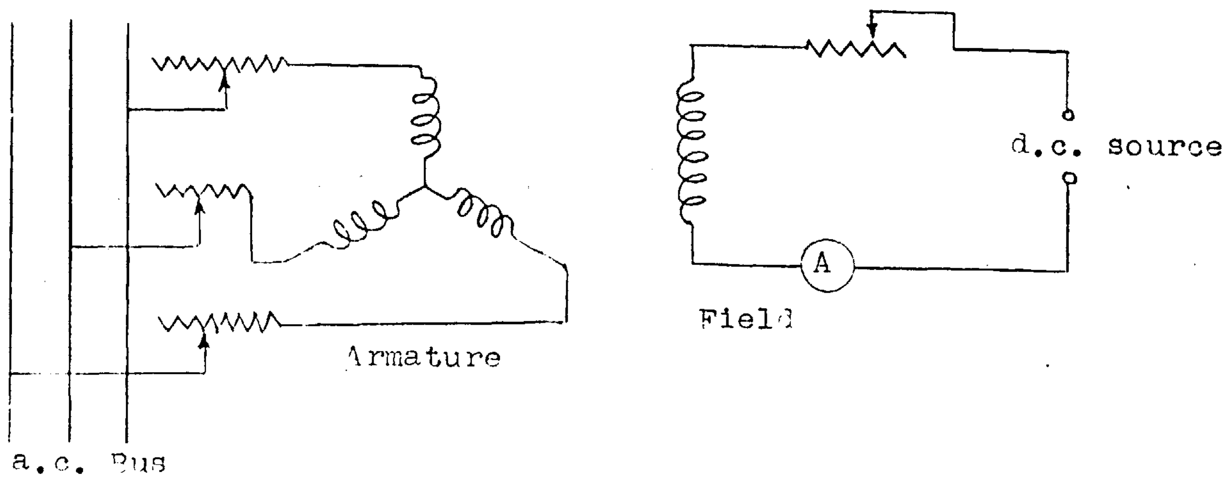


Figure 8, Synchronous Machine.

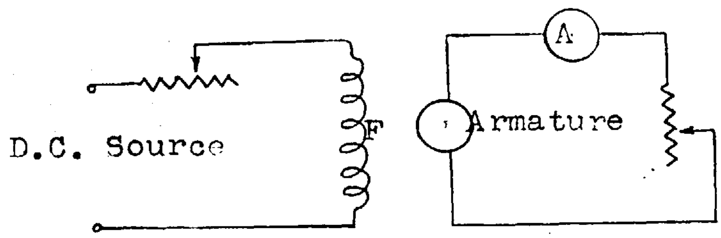


Figure 9, D. C. Machine.

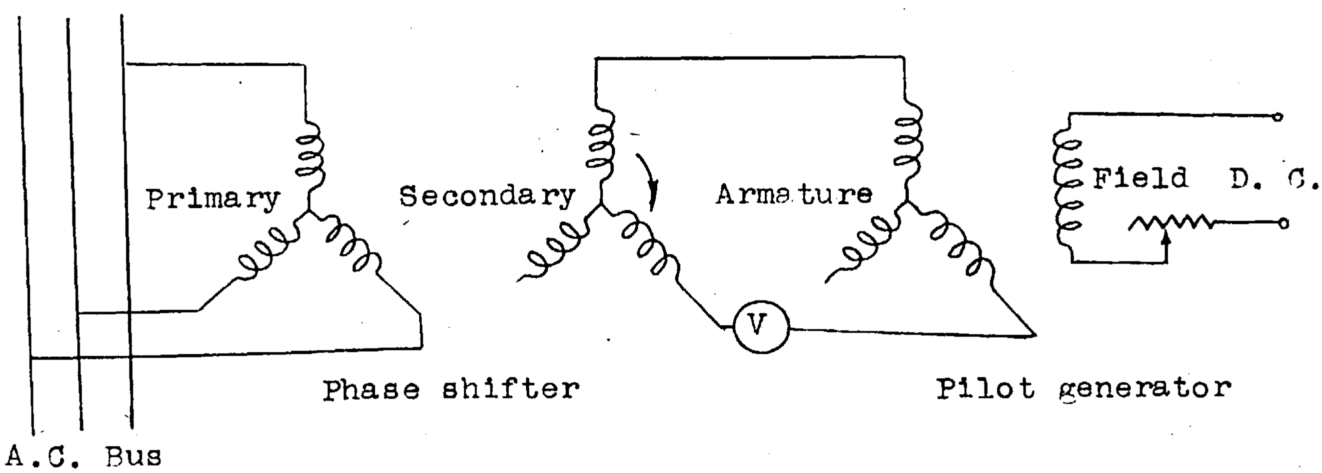


Figure 10.

6.3 Power-Angle Measurements

The power angle of a synchronous machine is usually measured with a pilot generator connected as shown in Fig. 11. The pilot generator is rigidly coupled to the main synchronous machine with their field axes in exact alignment. Thus their excitation emfs are in phase with each other. If the pilot generator is excited to a voltage equal to the applied emf, the voltmeter shown in the figure will give a reading V_δ such that

$$V_\delta = 2V \sin \frac{\delta_o}{2} \qquad (3)$$

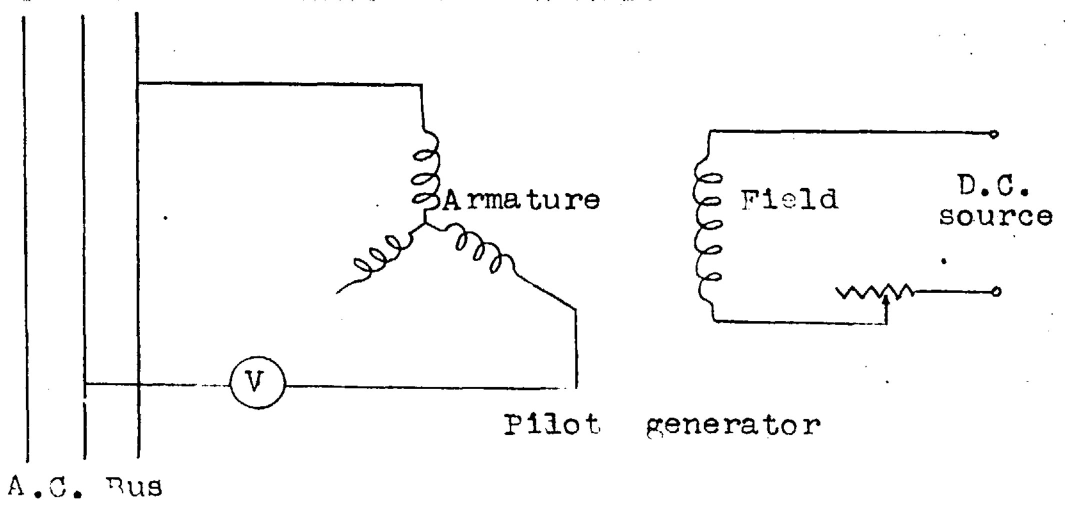


Figure 11.

Hence the value of δ_o can be calculated from the voltage measurement only. This method for measuring the power angle is very simple but has the following disadvantages:

(a) The pilot generator and the main synchronous machine cannot be easily coupled together with their field axes in exact alignment.

(b) The wave form of the emf of the pilot generator will affect the result very much, especially for small values of δ_o.

(c) The voltage across the voltmeter shown in the figure is uncontrollable. It is not suitable when the machine is oscillating and the variation of the amplitude of oscillation is of special interest.

In order to avoid these disadvantages, in addition to the pilot generator a phase shifter is also used. The connection diagram is shown as Fig. 10. The phase shifter has its secondary windings mounted on a rotor which can be turned manually. We can read the angle of rotation from a graduated disk with its smallest division equivalent to one electrical degree. The pilot generator is now excited to a voltage equal to the output voltage of the phase shifter, and it may be coupled to the main synchronous machine with an arbitrary angle between their field axes. Before we excite the a-c bus, the synchronous machine is run at no load as a synchronous generator with its excitation emf equal to the voltage to be applied to the a-c bus. We may, then, turn the rotor of the phase shifter to a position so that the shown voltmeter gives a reading suitable for our purpose (as we might connect a brush oscillograph across the voltmeter to record the variation of the power angle when the machine is in oscillation). Then, with the a-c bus excited and the synchronous machine loaded, the power angle will be equal to the angle through which we have to turn the rotor of the phase shifter to give the indicated voltmeter the same reading as before. It is, then, not affected by the wave form of the emf of the pilot generator so long as the machine is not oscillating. When the conditions of the synchronous machine are adjusted to give zero resultant damping, the machine could oscillate with constant amplitude and a frequency fixed by its synchronizing torque and the moment of inertia of the system. As the frequency is usually a few cycles per second, the needle of the voltmeter will be forced to oscillate correspondingly with constant amplitude about its average position. In other words, the constant-amplitude oscillation of the reading of the voltmeter can serve as an indication of zero resultant damping. Similarly, if the damping of the machine is positive or negative, the reading of the voltmeter will oscillate with decreasing or increasing amplitude, respectively. But if the increment of the amplitude of the machine oscillation is of interest, it is advisable to replace the voltmeter by a brush oscillograph to record the corresponding variation of voltage. (The wave form of the pilot generator's emf does affect the magnitude of this variation.)

One record is shown in Fig. 12 to illustrate the phenomena of negative, positive, and zero damping of the synchronous machine by simply changing its excitation. When we are taking the oscillograph, any electrical or mechanical disturbance should be avoided. Otherwise, the record will contain irregularities.

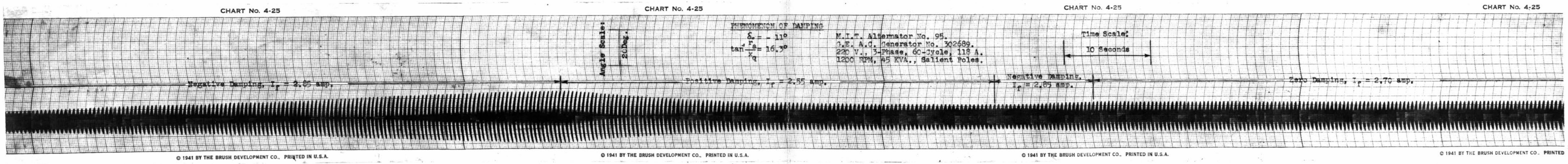


CHART No. 4-25
CHART No. 4-25
CHART No. 4-25
CHART No. 4-25
PHENOMENON OF DAMPING
$\delta_o = -11^\circ$
$\tan^{-1}\frac{r_a}{x_q} = 16.3^\circ$
M.I.T. Alternator No. 95.
G.E. A.C. Generator No. 302689.
220 V., 3-Phase, 60-Cycle, 118 A.
1200 RPM, 45 KVA., Salient Poles.
Angle Scale:
20 Deg.
Time Scale:
10 Seconds
Negative Damping, I_f = 2.85 amp.
Positive Damping, I_f = 2.55 amp.
Negative Damping, I_f = 2.85 amp.
Zero Damping, I_f = 2.70 amp.
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6.4 Numerical Computations and Comparison of Results

From the mathematical analysis we have the expression of the damping coefficient for synchronous machines of salient-pole structure without damper windings as follows.

$$B = \frac{3K\, x_m V}{\omega\, r_b} \cdot \frac{k_b^2}{k_b^2 + b^2} \cdot \frac{Z_q^4}{(r_a^2 + x_d x_q)^3}$$

$$\times \left\{ 2E \sin \alpha_1 - \frac{Z_d}{Z_q} V \sin(\alpha_2 - \delta_o) + \frac{x_d - x_q}{Z_q} V \sin \alpha_1 \cos(\alpha_1 - \delta_o) \right\} \sin(\alpha_1 - \delta_o)$$

$$+ \frac{K}{\omega} \cdot \frac{2E^2 r_a (x_d x_q - r_a^2)}{x_m (r_a^2 + x_d x_q)^2}$$

$$+ \frac{K}{\omega} \cdot \frac{2E\, r_a (x_d - x_q)}{x_m (r_a^2 + x_d x_q)^3} \left\{ (E - V \cos \delta_o)(2r_a^2 - x_d x_q) x_q + (V \sin \delta_o)(2x_d x_q - r_a^2) r_a \right\}$$

When B is positive, it means negative damping due to the electromagnetic action of the synchronous machine. Where

$$\alpha_1 = \tan^{-1} \frac{r_a}{x_q}$$

$$\alpha_2 = \tan^{-1} \frac{r_a}{x_d}$$

$$k_b = \frac{r_b}{\sigma\, L_b} = \frac{r_b}{\omega\, \sigma\, L_b}\, \omega$$

$b = 2\pi$ times the frequency of oscillation

E = excitation emf between lines in effective value

V = applied voltage between lines in effective value

$$K = \frac{550}{746} \cdot \frac{\text{No. of poles}}{2} \cdot \frac{x_m}{2\omega} = 0.184$$

B is in lb.ft. per elec. rad. per sec.

The units of voltage, current, impedance, and angular velocity are volt, ampere, ohm, and radians per second, respectively. If we let

$$A_1 = \frac{3x_m V}{r_b} \cdot \frac{k_b^2}{k^2 + b^2} \cdot \frac{Z_q^4}{(r_a^2 + x_d x_q)^3} \tag{4}$$

$$A_2 = \left[2E \sin \alpha_1 - \frac{Z_d}{Z_q} V \sin(\alpha_2 - \delta_o) + \frac{x_d - x_q}{Z_q} V \sin \alpha_1 \cos(\alpha_1 - \delta_o) \right] \sin(\alpha_1 - \delta_o) \tag{5}$$

$$A_3 = \frac{2E^2 r_a (x_d x_q - r_a^2)}{x_m (r_a^2 + x_d x_q)^2} \tag{6}$$

$$A_4 = \frac{2E\, r_a (x_d - x_q)}{x_m (r_a^2 + x_d x_q)^3} \left[(E - V \cos \delta_o)(2r_a^2 - x_d x_q) x_q + V \sin \delta_o (2x_d x_q - r_a^2) r_a \right] \tag{7}$$

Then we have

$$B = \frac{K}{\omega} (A_1 A_2 + A_3 + A_4) \tag{8}$$

For the purpose of verifying this formula, the following two tests are made.

<u>Test 1</u>. The object of this test is to show the relations of the damping coefficient B' and the excitation emf E with respect to the armature-circuit resistance r_a when the machine is operated as a synchronous motor at constant power angle and apparent zero damping.

The machine is always operated at $V = 226$ volts, $r_b = 81.5$ ohms, and $\delta_o = -11^o$. When it is adjusted to oscillate with constant amplitude (i.e., zero damping), we have the following data:

Armature-circuit Resistance r_a	Field Current I_f	Excitation emf E	Angular Velocity of Oscillation b
0.125	2.95	228	$0.0389\ \omega$
0.141	2.70	208	$0.0367\ \omega$
0.155	2.50	194	$0.0356\ \omega$
0.174	2.28	176	$0.0333\ \omega$
0.185	2.10	163	$0.0322\ \omega$
0.228	1.90	148	$0.0311\ \omega$
0.281	1.70	132	$0.0289\ \omega$

Where the values of b are obtained from the measured periods of oscillations.

(a) For $r_a = 0.125$ ohm

and $E = 228$ volts.

We have

$$\alpha_1 = \tan^{-1} \frac{0.125}{0.481} = \tan^{-1} 0.259 = 14.5^o$$

$$\sin \alpha_1 = \sin 14.5^\circ = 0.251$$

$$\alpha_2 = \tan^{-1} \frac{0.125}{0.867} = \tan^{-1} 0.144 = 8.2^\circ$$

$$Z_q^2 = 0.125^2 + 0.481^2 = 0.447^2 = 0.247$$

$$Z_d^2 = 0.125^2 + 0.867^2 = 0.876^2 = 0.768$$

$$r_a^2 + x_d x_q = 0.125^2 + 0.867 \times 0.481$$
$$= 0.0156 + 0.417 = 0.433$$

$$x_d - x_q = 0.867 - 0.481 = 0.386$$

$$2r_a^2 - x_d x_q = 0.031 - 0.417 = -0.386$$

$$2x_d x_q - r_a^2 = 0.834 - 0.0156 = 0.818$$

$$x_d x_q - r_a^2 = 0.417 - 0.0156 = 0.401$$

$$k_b = \frac{81.5}{2780}\,\omega = 0.0293\,\omega$$

$$A_1 = \frac{3 \times 63 \times 226}{81.5} \cdot \frac{0.0293^2}{0.0293^2 + 0.0389^2} \cdot \frac{0.247^2}{0.433^2}$$

$$= 524 \times \frac{86}{237} \times \frac{610}{815} = 142$$

$$A_2 = (2E \times 0.25 - \frac{0.376}{0.497} \times 226 \sin 19.2^\circ$$

$$+ \frac{0.386}{0.497} \times 226 \times 0.251 \cos 25.5^\circ) \sin 25.5^\circ$$

$$= (0.502E - 131 + 39.8) \times 0.431$$

$$= (114.5 - 131 + 39.8) \times 0.431$$

$$= 23.3 \times 0.431 = 10.0$$

$$A_3 = \frac{2E^2 \times 0.125 \times 0.401}{63 \times 0.433^2} = 0.00846E^2$$

$$= 0.00846 \times 228^2 = 441$$

$$A_4 = \frac{2E \times 0.125 \times 0.386}{63 \times 0.433^3} \left\{ \begin{array}{l} (E - 222)(-0.386) \times 0.481 \\ \quad - 43.1 \times 0.818 \times 0.125 \end{array} \right\}$$

$$= -\frac{2 \times 228 \times 0.125 \times 0.386}{63 \times 0.433^3} (1.11 + 4.4) = -23.8$$

Therefore

$$B = \frac{K}{\omega}(142 \times 10.0 + 441 - 23.8)$$

$$= \frac{K}{\omega}(1420 + 441 - 23.8) = 1837 \frac{K}{\omega} = 0.895$$

(b) For $r_a = 0.141$ ohm

and $E = 208$ volts.

We have

$$\alpha_1 = \tan^{-1} \frac{0.141}{0.481} = \tan^{-1} 0.293 = 16.3^\circ$$

$$\sin \alpha_1 = \sin 16.3^\circ = 0.281$$

$$\alpha_2 = \tan^{-1} \frac{0.141}{0.867} = \tan^{-1} 0.163 = 9.25^\circ$$

$$Z_q^2 = 0.141^2 + 0.481^2 = 0.501^2 = 0.251$$

$$Z_d^2 = 0.141^2 + 0.867^2 = 0.877^2 = 0.768$$

$$r_a^2 + x_d x_q = 0.141^2 + 0.867 \times 0.481 = 0.437$$

$$x_d - x_q = 0.386$$

$$2r_a^2 - x_d x_q = 0.0398 - 0.417 = -0.377$$

$$2x_d x_q - r_a^2 = 0.834 - 0.0199 = 0.814$$

$$x_d x_q - r_a^2 = 0.417 - 0.0199 = 0.397$$

$$k_b = 0.0293\ \omega$$

$$A_1 = 524 \times \frac{0.0293^2}{0.0293^2 + 0.0367^2} \times \frac{0.251^2}{0.437^2}$$

$$= 524 \times \frac{86}{220} \times \frac{630}{835} = 154$$

$$A_2 = (2E \times 0.281 - \frac{0.877}{0.501} \times 226 \sin 20.35^\circ$$

$$+ \frac{0.386}{0.501} \times 226 \times 0.281 \cos 27.3^\circ) \sin 27.3^\circ$$

$$= (0.562E - 137 + 433) \times 0.458$$

$$= (117 - 137 + 43.3) \times 0.458$$

$$= 23.3 \times 0.458 = 10.7$$

$$A_3 = \frac{2E^2 \times 0.141 \times 0.397}{63 \times 0.437^2} = 0.00930E^2$$

$$= 0.0093 \times 208^2 = 402$$

$$A_4 = \frac{2E \times 0.141 \times 0.386}{63 \times 0.437^3} \begin{bmatrix} (E - 222)(-0.377) \times 0.481 \\ -43.1 \times 0.814 \times 0.141 \end{bmatrix}$$

$$= \frac{2 \times 208 \times 0.141 \times 0.386}{63 \times 0.0835} (2.54 - 4.94) = -10.3$$

Therefore

$$B = \frac{K}{\omega} (154 \times 10.7 + 402 - 10.3)$$

$$= \frac{K}{\omega} (1645 + 402 - 10.3) = 2037 \frac{K}{\omega} = 0.994$$

(c) For $r_a = 0.155$ ohm

$E = 194$ volts

We have

$$\alpha_1 = \tan^{-1} \frac{0.155}{0.481} = \tan^{-1} 0.322 = 17.85^\circ$$

$$\sin \alpha_1 = \sin 17.85^\circ = 0.307$$

$$\alpha_2 = \tan^{-1} \frac{0.155}{0.867} = \tan^{-1} 0.1785 = 10.1^\circ$$

$$z_q^2 = 0.155^2 + 0.481^2 = 0.506^2 = 0.255$$

$$z_d^2 = 0.155^2 + 0.867^2 = 0.882^2 = 0.778$$

$$r_a^2 + x_d x_q = 0.155^2 + 0.481 \times 0.867 = 0.441$$

$$x_d - x_q = 0.386$$

$$2r_a^2 - x_d x_q = 0.048 - 0.417 = -0.369$$

$$2x_d x_q - r_a^2 = 0.834 - 0.024 = 0.810$$

$$x_d x_q - r_a^2 = 0.417 - 0.024 = 0.393$$

$$k_b = 0.0293\ \omega$$

$$A_1 = 524 \times \frac{0.0293^2}{0.0293^2 + 0.0356^2} \times \frac{0.255^2}{0.441^3}$$

$$= 524 \times \frac{86}{212} \times \frac{650}{857} = 161$$

$$A_2 = (2E \times 0.307 - \frac{0.882}{0.506} \times 226 \sin 21.1^\circ$$

$$+ \frac{0.386}{0.506} \times 226 \times 0.307 \cos 28.85^\circ) \sin 28.85^\circ$$

$$= (0.614E - 142 + 46.5) \times 0.484$$

$$= (0.614 \times 194 - 142 + 46.5) \times 0.484$$

$$= 23.5 \times 0.484 = 11.4$$

$$A_3 = \frac{2E^2 \times 0.155 \times 0.393}{63 \times 0.441^2} = 0.00992E^2$$

$$= 0.00992 \times 194^2 = 374$$

$$A_4 = \frac{2E \times 0.155 \times 0.386}{63 \times 0.441^3} \begin{bmatrix} (E - 222)(-0.369) \times 0.481 \\ - 43.1 \times 0.810 \times 0.155 \end{bmatrix}$$

$$= \frac{2 \times 194 \times 0.155 \times 0.386}{63 \times 0.0857} (4.96 - 5.42) = -1.97$$

Therefore

$$B = \frac{K}{\omega} (161 \times 11.4 + 374 - 1.97)$$

$$= \frac{K}{\omega} (1840 + 374 - 1.97) = 2212 \frac{K}{\omega} = 1.08$$

(d) For $r_a = 0.174$ ohm

and $E = 176$ volts

We have

$$\alpha_1 = \tan^{-1} \frac{0.174}{0.481} = \tan^{-1} 0.361 = 19.85^\circ$$

$$\sin \alpha_1 = \sin 19.85^\circ = 0.340$$

$$\alpha_2 = \tan^{-1} \frac{0.174}{0.867} = \tan^{-1} 0.201 = 11.35^\circ$$

$$Z_q^2 = 0.174^2 + 0.481^2 = 0.512^2 = 0.262$$

$$Z_d^2 = 0.174^2 + 0.867^2 = 0.884^2 = 0.780$$

$$r_a^2 + x_d x_q = 0.174^2 + 0.417 = 0.447$$

$$x_d - x_q = 0.386$$

$$2r_a^2 - x_d x_q = 0.0606 - 0.417 = -0.356$$

$$2x_d x_q - r_a^2 = 0.834 - 0.0303 = 0.804$$

$$x_d x_q - r_a^2 = 0.417 - 0.0303 = 0.387$$

$$k_b = 0.0293\ \omega$$

$$A_1 = 524 \times \frac{0.0293^2}{0.0293^2 + 0.0333^2} \times \frac{0.262^2}{0.447^3}$$

$$= 524 \times \frac{86}{197} \times \frac{688}{900} = 175$$

$$A_2 = (2E \times 0.340 - \frac{0.884}{0.512} \times 226 \sin 22.35^\circ$$

$$+ \frac{0.386}{0.512} \times 226 \times 0.34 \cos 30.85^\circ)\sin 30.85^\circ$$

$$= (0.68E - 148.2 + 49.8) \times 0.512$$

$$= (0.68 \times 176 - 148.2 + 49.8) \times 0.512$$

$$= 21.2 \times 0.512 = 10.8$$

$$A_3 = \frac{2E^2 \times 0.387 \times 0.174}{63 \times 0.447^2} = 0.01055E^2$$

$$= 0.01055 \times 176^2 = 326$$

From the results shown in the preceding cases, A_4 is very small and can be neglected. Therefore

$$B = \frac{K}{\omega} (175 \times 10.8 + 326)$$

$$= \frac{K}{\omega} (1890 + 326) = 2216 \frac{K}{\omega} = 1.08$$

(e) For $r_a = 0.185$ ohm

and $E = 163$ volts

We have

$$\alpha_1 = \tan^{-1} \frac{0.185}{0.481} = \tan^{-1} 0.385 = 21^\circ$$

$$\sin \alpha_1 = \sin 21^\circ = 0.359$$

$$\alpha_2 = \tan^{-1} \frac{0.185}{0.867} = \tan^{-1} 0.213 = 12^\circ$$

$$Z_q^2 = 0.185^2 + 0.481^2 = 0.516^2 = 0.266$$

$$Z_d^2 = 0.185^2 + 0.867^2 = 0.889^2 = 0.788$$

$$r_a^2 + x_d x_q = 0.185^2 + 0.417 = 0.451$$

$$x_d - x_q = 0.386$$

$$x_d x_q - r_a^2 = 0.417 - 0.0342 = 0.383$$

$$k_b = 0.0293\ \omega$$

$$A_1 = 524 \times \frac{0.0293^2}{0.0293^2 + 0.0322^2} \times \frac{0.266^2}{0.451^3}$$

$$= 524 \times \frac{86}{190} \times \frac{708}{920} = 183$$

$$A_2 = (2E \times 0.359 - \frac{0.889}{0.516} \times 226 \sin 23^\circ$$

$$+ \frac{0.386}{0.516} \times 226 \times 0.359 \cos 33^\circ) \sin 33^\circ$$

$$= (0.718E - 152 + 51) \times 0.545$$

$$= (0.718 \times 163 - 152 + 51) \times 0.545$$

$$= 16 \times 0.545 = 8.72$$

$$A_3 = \frac{2E^2 \times 0.185 \times 0.383}{63 \times 0.451^2} = 0.0111\ E^2$$

$$= 0.0111 \times 163^2 = 295$$

A_4 is neglected.

Therefore $$B = \frac{K}{\omega}(183 \times 8.72 + 295)$$

$$= (1592 + 295)\frac{K}{\omega} = 1890\frac{K}{\omega} = 0.921$$

(f) For $r_a = 0.228$ ohm

and $E = 148$ volts

We have

$$\alpha_1 = \tan^{-1} \frac{0.228}{0.481} = \tan^{-1} 0.474 = 25.4^\circ$$

$$\sin \alpha_1 = \sin 25.4^\circ = 0.429$$

$$\alpha_2 = \tan^{-1} \frac{0.228}{0.867} = \tan^{-1} 0.263 \doteq 14.7^\circ$$

$$Z_q^2 = 0.228^2 + 0.481^2 = 0.531^2 = 0.282$$

$$Z_d^2 = 0.228^2 + 0.867^2 = 0.899^2 = 0.805$$

$$r_a^2 + x_d x_q = 0.228^2 + 0.417 = 0.469$$

$$x_d - x_q = 0.386$$

$$x_d x_q - r_a^2 = 0.417 - 0.052 = 0.365$$

$$k_b = 0.0293\ \omega$$

$$A_1 = 524 \times \frac{0.0293^2}{0.0293^2 + 0.0311^2} \times \frac{0.282^2}{0.469^3}$$

$$= 524 \times \frac{86}{183} \times \frac{795}{1030} = 190$$

$$A_2 = (2E \times 0.429 - \frac{0.899}{0.531} \times 226 \times \sin 25.7^\circ$$

$$+ \frac{0.386}{0.531} \times 226 \times 0.429 \cos 36.4^\circ) \sin 36.4^\circ$$

$$= (0.858\ E - 165.5 + 56.5) \times 0.593$$

$$= (0.858 \times 148 - 165.5 + 56.5) \times 0.593$$

$$= 18 \times 0.593 = 10.65$$

$$A_3 = \frac{2E^2 \times \ 0.228 \times 0.365}{63 \times 0.469^2} = 0.01205\ E^2$$

$$= 0.01205 \times 148^2 = 262$$

A_4 is neglected.

Therefore $$B = \frac{K}{\omega}\,(190 \times 10.65 + 262)$$

$$= \frac{K}{\omega}\,(2030 + 262) = 2292\,\frac{K}{\omega} = 1.12$$

(g) For $r_a = 0.281$ ohm
and $E = 132$ volts

We have

$$\alpha_1 = \tan^{-1} \frac{0.281}{0.481} = \tan^{-1} 0.583 = 30.3^\circ$$

$$\sin \alpha_1 = \sin 30.3^\circ = 0.505$$

$$\alpha_2 = \tan^{-1} \frac{0.281}{0.867} = \tan^{-1} 0.324 = 17.95^\circ$$

$$Z_q^2 = 0.281^2 + 0.481^2 = 0.557^2 = 0.310$$

$$Z_d^2 = 0.281^2 + 0.867^2 = 0.910^2 = 0.825$$

$$r_a^2 + x_d x_q = 0.281^2 + 0.417 = 0.496$$

$$x_d - x_q = 0.386$$

$$x_d x_q - r_a^2 = 0.417 - 0.079 = 0.338$$

$$k_b = 0.0293\ \omega$$

$$A_1 = 524 \times \frac{0.0293^2}{0.0293^2 + 0.0289^2} \times \frac{0.310^2}{0.496^3}$$

$$= 524 \times \frac{86}{170} \times \frac{96}{122} = 209$$

$$A_2 = (2E \times 0.505 - \frac{0.910}{0.557} \times 226 \sin 28.95^\circ$$

$$+ \frac{0.386}{0.557} \times 226 \times 0.505 \cos 41.3^\circ) \sin 41.3^\circ$$

$$= (1.01\ E - 179 + 59.5) \times 0.660$$

$$= (1.01 \times 132 - 179 + 59.5) \times 0.66$$

$$= 13.5 \times 0.66 = 8.91$$

$$A_3 = \frac{2E^2 \times 0.281 \times 0.338}{63 \times 0.496^2} = 0.01225\ E^2$$

$$= 0.01225 \times 132^2 = 214$$

A_4 is neglected.

Therefore $$B = \frac{K}{\omega} (209 \times 8.91 + 214)$$

$$= \frac{K}{\omega} (1860 + 214) = 2074 \frac{K}{\omega} = 1.01$$

Therefore, when the synchronous machine is operated at a fixed power angle of (-11°) and then it is adjusted to oscillate without net damping, the damping coefficient due to its electromagnetic action can be tabulated as follows:

r_a	I_f	B
0.125	2.95	0.895
0.141	2.70	0.994
0.155	2.50	1.08
0.174	2.28	1.08
0.185	2.10	0.921
0.228	1.90	1.12
0.281	1.70	1.01

If there were no extra damping due to the load, windage, friction, hysteresis and eddy currents, etc., the value of B should be zero when the machine shows no damping on its oscillations. As for reference, we may calculate the values of E which would make B zero for the corresponding armature-circuit resistances. From the above calculations, with A_4 disregarded, we have

(a) For $r_a = 0.125$ ohm, and

$$B = \left[142(0.502\ E - 131 + 39.8) \times 0.431 + 0.00846\ E\right] \frac{K}{\omega}$$

$$= (0.00846\ E^2 + 30.7\ E - 5520) \frac{K}{\omega} = 0\ ,$$

$E = 170$ volts or $I_f = 2.21$ amp

(b) For $r_a = 0.141$ ohm, and

$$B = \left[154(0.562\ E - 137 + 43.3) \times 0.458 + 0.0093\ E^2\right] \frac{K}{\omega}$$

$$= (0.0093\ E^2 + 39.5\ E - 6600) \frac{K}{\omega} = 0\ ,$$

$E = 161$ volts or $I_f = 2.09$ ohm

(c) For $r_a = 0.155$ ohm, and

$$B = \left[161(0.614\ E - 142 + 46.5) \times 0.484 + 0.00992\ E^2\right] \frac{K}{\omega}$$

$$= (0.00992\ E^2 + 47.8\ E - 7450) \frac{K}{\omega} = 0\ ,$$

$E = 151$ volts or $I_f = 1.96$ amp

(d) For $r_a = 0.174$ ohm, and

$$B = \left[175(0.68\ E - 148.2 + 49.8) \times 0.512 + 0.01055\ E^2\right] \frac{K}{\omega}$$

$$= (0.01055\ E^2 + 60.9\ E - 8800) \frac{K}{\omega} = 0\ ,$$

$E = 142$ volts or $I_f = 1.85$ amp

(e) For $r_a = 0.185$ ohm, and

$$B = \left[183(0.718\ E - 152 + 51) \times 0.545 + 0.0111\ E^2\right] \frac{K}{\omega}$$

$$= (0.0111\ E^2 + 71.6\ E - 10080) \frac{K}{\omega} = 0\ ,$$

$E = 137$ volts or $I_f = 1.77$ amp

(f) For $r_a = 0.228$ ohm, and

$$B = \left[190(0.858\ E - 165.5 + 56.5) \times 0.593 + 0.01205\ E^2\right] \frac{K}{\omega}$$

$$= (0.01205\ E^2 + 96.5\ E - 12250) \frac{K}{\omega} = 0\ ,$$

$E = 125$ volts or $I_f = 1.63$ amp

(g) For $r_a = 0.281$ ohm, and

$$B = \left[209(1.01\ E - 179 + 59.5) \times 0.66 + 0.01225\ E^2\right] \frac{K}{\omega}$$

$$= (0.01225\ E^2 + 139\ E - 16500) \frac{K}{\omega} = 0\ ,$$

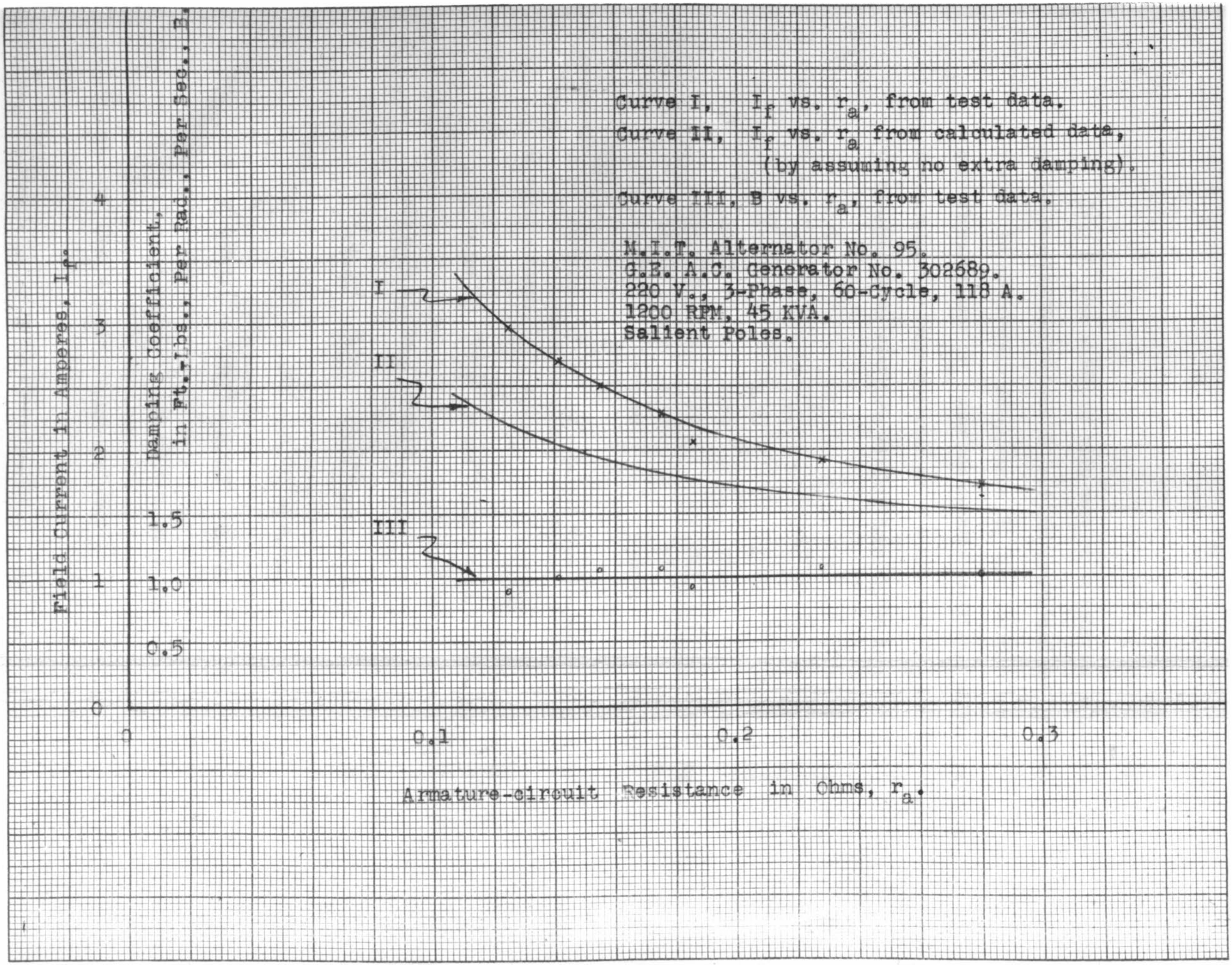
$E = 117$ volts or $I_f = 1.52$ amp

From the results which have been obtained, three curves can be plotted as shown on p. 142. The curve I shows the relation between the field current and the armature-circuit resistance with the data obtained from test. The machine is adjusted to oscillate without damping. The curve II shows the same relation as curve I but with the data calculated for the condition $B = 0$. The curve III indicates the values of B for the different values of r_a when the machine oscillates without damping.

Since the machine always possesses the extra positive damping due to the effects of load, windage, friction, hysteresis and eddy currents, etc., the curves I and II are not expected to coincide with each other. The curve II has to be below the curve I as they are shown.

As the machine is operated at constant average power angle δ_o , its average load is practically constant throughout the test and so the damping due to the load. Therefore the total extra damping is practically constant. When the machine oscillates without damping, the damping due to the main electromagnetic action should be equal and opposite to the total extra damping. Hence the calculated damping coefficient B should be nearly constant. It checks essentially the curve III.

In the region above the curve I, the machine is unstable, while in the region below the curve I it is stable.


Curve I, I_f vs. r_a, from test data.
Curve II, I_f vs. r_a from calculated data, (by assuming no extra damping).
Curve III, B vs. r_a, from test data.
M.I.T. Alternator No. 95.
G.E. A.C. Generator No. 302689.
220 V., 3-Phase, 60-Cycle, 118 A.
1200 RPM, 45 KVA.
Salient Poles.
I
II
III
Field Current in Amperes, I_f.
Damping Coefficient, in Ft.-Lbs., Per Rad., Per Sec., B.
0
1
2
3
4
0.5
1.0
1.5
0
0.1
0.2
0.3
Armature-circuit Resistance in Ohms, r_a.

Test 2. The object of this test is to show the relations of the damping coefficient B and the excitation emf E with respect to the average power angle δ_o for constant armature-circuit resistance r_a when the machine oscillates without damping. The machine is always operated at $V = 226$ and $r_a = 0.141$. When it is adjusted to oscillate without damping, we have:

δ_o	I_f	E	b
-6.0°	1.85	144	$0.0311\,\omega$
-9.0°	2.30	178	$0.0344\,\omega$
-11.0°	2.70	208	$0.0367\,\omega$
-13.5°	3.10	240	$0.0389\,\omega$
-15.5°	3.50	270	$0.0400\,\omega$
-17.0°	3.90	300	$0.0422\,\omega$

where the values of b are obtained from the measured periods of oscillations.

$$\alpha_1 = \tan^{-1} \frac{0.141}{0.481} = \tan^{-1} 0.293 = 16.3^\circ$$

$$\sin \alpha_1 = \sin 16.3^\circ = 0.281$$

$$\alpha_2 = \tan^{-1} \frac{0.141}{0.867} = \tan^{-1} 0.163 = 9.25^\circ$$

$$Z_q^2 = 0.141^2 + 0.481^2 = 0.501^2 = 0.251$$

$$Z_d^2 = 0.141^2 + 0.867^2 = 0.877^2 = 0.768$$

$$r_a^2 + x_d x_q = 0.141^2 + 0.867 \times 0.481 = 0.437$$

$$x_d - x_q = 0.867 - 0.481 = 0.386$$

$$2r_a^2 - x_d x_q = 0.0398 - 0.417 = -\ 0.377$$

$$2x_d x_q - r_a^2 = 0.834 - 0.0199 = 0.814$$

$$x_d x_q - r_a^2 = 0.417 - 0.0199 = 0.394$$

$$A_1 = \frac{3 \times 63 \times 226 \times 0.251^2}{0.437^3} \times \frac{k_b^2}{r_b(k_b^2 + b^2)}$$

$$= 32200 \times \frac{k_b^2}{r_b(k_b^2 + b^2)}$$

$$A_2 = \Big[\ 2 \times 0.281\ E - \frac{0.877}{0.501} \times 226 \sin(9.25^\circ - \delta_o)$$

$$+ \frac{0.386}{0.501} \times 226 \times 0.281 \cos(16.3^\circ - \delta_o)\Big]\ \sin(16.3^\circ - \delta_o)$$

$$= \Big[\ 0.562\ E - 395 \sin(9.25^\circ - \delta_o)$$

$$+ 48.9 \cos(16.3^\circ - \delta_o)\Big]\ \sin(16.3^\circ - \delta_o)$$

$$A_3 = \frac{2E^2 \times 0.141 \times 0.397}{63 \times 0.437^2} = 0.00930\ E^2$$

$$A_4 = \frac{2E \times 0.141 \times 0.386}{63 \times 0.437^3}\ \Big[(E - 226 \cos\delta_o) \times 0.481 \times (-0.377)$$

$$+ (226 \sin\delta_o) \times 0.141 \times 0.814\Big]$$

$$= 0.00375\ E(226 \cos\delta_o - E) + 0.538\ E \sin\delta_o$$

(a) For $\delta_o = -6.0^\circ$, $E = 144$, we get

$$r_b = \frac{220}{1.85} = 119$$

$$k_b = \frac{119}{2780}\,\omega = 0.0428\,\omega$$

$$\frac{k_b^2}{r_b(k_b^2 + b^2)} = \frac{0.0428^2}{119(0.0428^2 + 0.0311^2)} = 0.00550$$

$$A_1 = 32200 \times 0.0055 = 177$$

$$A_2 = \left[0.562 \times 144 - 395 \sin 15.25^\circ + 48.8 \cos 22.3^\circ \right] \sin 22.3^\circ$$
$$= (80.7 - 104 + 45.1) \times 0.380$$
$$= 21.8 \times 0.380 = 8.28$$

$$A_3 = 0.0093 \times 144^2 = 193$$

$$A_4 = 0.00375 \times 144 \times 81 + 0.538 \times 144(-0.1045)$$
$$= 43.7 - 8.1 = 35.6$$

Therefore $B = (177 \times 8.28 + 193 + 35.6) \frac{K}{\omega}$

$$= (1460 + 193 + 35.6) \frac{K}{\omega} = 1689 \frac{K}{\omega} = 0.828$$

(b) For $\delta_o = -9.0^\circ$, $E = 178$, we get

$$r_b = \frac{220}{2.30} = 95.5$$

$$k_b = \frac{95.5}{2780} \omega = 0.0344\ \omega$$

$$\frac{k_b^2}{r_b(k_b^2 + b^2)} = \frac{0.0344^2}{95.5(0.0344^2 + 0.0344^2)} = 0.00525$$

$$A_1 = 32200 \times 0.00525 = 169$$

$$A_2 = \left[0.562 \times 178 - 395 \sin 18.25^2 + 48.8 \cos 25.3^\circ \right] \sin 25.3^\circ$$
$$= (100 - 124 + 44.2) \times 0.427$$
$$= 20.2 \times 0.427 = 8.63$$

$$A_3 = 0.0093 \times 178^2 = 295$$

$$A_4 = 0.00375 \times 178 \times 45 - 0.538 \times 178 \times 0.1565$$
$$= 30.0 - 14.95 = 15$$

Therefore $B = (169 \times 8.63 + 295 + 15)\ \frac{K}{\omega}$

$$= (1460 + 295 + 15)\ \frac{K}{\omega} = 1770\ \frac{K}{\omega} = 0.865$$

(c) For $\delta_o = -11.0^\circ$ and $E = 208$, we have

$$r_b = \frac{220}{2.70} = 81.5$$

$$k_b = \frac{81.5}{2780}\ \omega = 0.0294\ \omega$$

$$\frac{k_b^2}{r_b(k_b^2 + b^2)} = \frac{0.0294^2}{81.5(0.0294^2 + 0.0367^2)} = 0.00480$$

$$A_1 = 32200 \times 0.0048 = 154$$

$$A_2 = (0.562 \times 208 - 395 \sin 20.25^\circ + 48.8 \cos 27.3^\circ) \sin 27.3^\circ$$
$$= (117 - 137 + 43.8) \quad 0.458$$
$$= 23.3 \times 0.458 = 10.7$$

$$A_3 = 0.0093 \times 208^2 = 402$$

$$A_4 = 0.00374 \times 208 \times 14 - 0.538 \times 208 \times 0.19$$
$$= (10.9 - 21.4) = -10.5$$

Therefore $B = (154 \times 10.7 + 402 - 10.5)\ \frac{K}{\omega}$

$$= (1650 + 402 - 10.5)\ \frac{K}{\omega} = 2041\ \frac{K}{\omega} = 1.005$$

(d) For $\delta_o = -13.5^\circ$ and $E = 240$, we have

$$r_b = \frac{220}{3.10} = 71.0$$

$$\frac{k_b^2}{r_b(k_b^2 + b^2)} = \frac{0.0256^2}{71.0(0.0256^2 + 0.0389^2)} = 0.00424$$

$$A_1 = 32200 \times 0.00424 = 136$$

$$\begin{aligned} A_2 &= (0.562 \times 240 - 395 \sin 22.75^\circ + 48.8 \cos 29.8^\circ)\sin 29.8^\circ \\ &= (134.5 - 153 + 42.4) \times 0.497 \\ &= 23.9 \times 0.497 = 11.9 \end{aligned}$$

$$A_3 = 0.0093 \times 240^2 = 535$$

$$\begin{aligned} A_4 &= 0.00374 \times 240(-21) - 0.538 \times 240 \times 0.234 \\ &= -18.8 - 30.2 = -49 \end{aligned}$$

Therefore
$$\begin{aligned} B &= (136 \times 11.9 + 535 - 49)\,\frac{K}{\omega} \\ &= (1620 + 535 - 49)\,\frac{K}{\omega} = 2106\,\frac{K}{\omega} = 1.035 \end{aligned}$$

(e) For $\delta_o = -15.5^\circ$ and $E = 270$, we have

$$r_b = \frac{220}{3.50} = 63.0$$

$$k_b = \frac{63}{2780}\,\omega = 0.0227\,\omega$$

$$\frac{k_b^2}{r_b(k_b^2 + b^2)} = \frac{0.0227^2}{63(0.0227^2 + 0.0400^2)} = 0.00390$$

$$A_1 = 32200 \times 0.0039 = 125$$

$$A_2 = (0.562 \times 270 - 395 \sin 24.75^\circ + 48.8 \cos 31.8^\circ)\sin 31.8^\circ$$
$$= (151.5 - 165.5 + 41.5) \times 0.527$$
$$= 27.5 \times 0.527 = 14.5$$

$$A_3 = 0.0093 \times 270^2 = 677$$

$$A_4 = -0.00374 \times 270 \times 52 - 0.538 \times 270 \times 0.267$$
$$= -52.5 - 38.8 = -91.3$$

Therefore $B = (125 \times 14.5 + 677 - 91.3) \frac{K}{\omega}$

$$= (1815 + 677 - 91.3) \frac{K}{\omega} = 2401 \frac{K}{\omega} = 1.16$$

(f) For $\delta_o = -17.0^\circ$ and $E = 300$, we have

$$r_b = \frac{220}{3.90} = 56.5$$

$$k_b = \frac{56.5}{2780}\,\omega = 0.0203\,\omega$$

$$\frac{k_b^2}{r_b(k_b^2 + b^2)} = \frac{0.0203^2}{56.5(0.0203^2 + 0.0422^2)} = 0.00331$$

$$A_1 = 32200 \times 0.00331 = 106.5$$

$$A_2 = (0.562 \times 300 - 395 \sin 26.25^\circ + 48.8 \cos 33.3^\circ)\sin 33.3^\circ$$
$$= (168.5 - 175 + 40.8) \times 0.548$$
$$= 34.3 \times 0.548 = 18.8$$

$$A_3 = 0.0093 \times 300^2 = 837$$

$$A_4 = -0.00374 \times 300 \times 84 - 0.538 \times 300 \times 0.293$$
$$= -94.3 - 47.3 = -141.6$$

Therefore $B = (106.5 \times 18.8 + 837 - 141.6)\,\frac{K}{\omega}$

$$= (2000 + 837 - 141.6)\,\frac{K}{\omega} = 2695\,\frac{K}{\omega} = 1.325$$

The values of B calculated above may be tabulated as follows:

δ_o	I_f	B
-6.0°	1.85	0.828
-9.0°	2.30	0.865
-11.0°	2.70	1.005
-13.5°	3.10	1.035
-15.5°	3.50	1.160
-17.0°	3.90	1.325

For reference we may also calculate the values of E or I_f which will make B zero for the corresponding power angles. From the above calculations, with the small A_4 neglected, we have:

(a) If $\delta_o = -6.0^\circ$, and

$$B = \frac{K}{\omega}\left[177(0.526\,E - 104 + 45.1) \times 0.38 + 0.0093\,E^2\right]$$
$$= \frac{K}{\omega}(37.8\,E - 3960 + 0.0093\,E^2) = 0\,,$$

$E = 102$ or $I_f = 1.30$

(b) If $\delta_o = -9.0^o$, and

$$B = \frac{K}{\omega}\left[169(0.562\ E - 124 + 44.2) \times 0.427 + 0.0093\ E^2 \right]$$

$$= \frac{K}{\omega}(40.6\ E - 5760 + 0.0093\ E^2) = 0 ,$$

$E = 137$ or $I_f = 1.75$

(c) If $\delta_o = -11.0^o$, and

$$B = \frac{K}{\omega}\left[154(0.562\ E - 137 + 43.3) \times 0.458 + 0.0093\ E^2 \right]$$

$$= \frac{K}{\omega}(39.7\ E - 6620 + 0.0093\ E^2) = 0 ,$$

$E = 160$ or $I_f = 2.05$

(d) If $\delta_o = -13.5^o$, and

$$B = \frac{K}{\omega}\left[136(0.562\ E - 153 + 42.4) \times 0.497 + 0.0093\ E^2 \right]$$

$$= \frac{K}{\omega}(38.0\ E - 7450 + 0.0093\ E^2) = 0 ,$$

$E = 188$ or $I_f = 2.45$

(e) If $\delta_o = -15.5^o$, and

$$B = \frac{K}{\omega}\left[125(0.562\ E - 165.5 + 41.5) \times 0.527 + 0.0093\ E^2 \right]$$

$$= \frac{K}{\omega}(37.1\ E - 8160 + 0.0093\ E^2) = 0 ,$$

$E = 209$ or $I_f = 2.70$

(f) If $\delta_o = -17.0^o$, and

$$B = \frac{K}{\omega}\left[106.5(0.562\ E - 175 + 40.8) \times 0.548 + 0.0093\ E^2 \right]$$

$$= \frac{K}{\omega}(32.8\ E - 7830 + 0.0093\ E^2) = 0 ,$$

$E = 225$ or $I_f = 2.90$

Thus, from the results which have been obtained, three curves can be plotted as shown on p. 152. The curve (a) shows the relation between the field current I_f and the power angle δ_o with the data from test (the machine oscillates without net damping). The curve (b) shows the same relation as the curve (a), but with the data calculated for the condition $B = 0$. The curve (c) indicates the values of B for the different values of δ_o when the machine oscillates without net damping.

On the curve (a) the resultant damping is zero, while on the curve (b) the damping due to the main electromagnetic action alone is zero. These two curves should coincide with each other if there was no extra damping due to the effects of load, windage, friction, hysteresis and eddy currents, etc. As the extra damping is always positive, the main electromagnetic action alone for the condition of the curve (a) should always produce negative damping (i.e., positive values of B) to compensate it. Thus the curve (a) should be above the curve (b), and curve (c) should be above the abscissa as they are shown.

Since the d-c generator is separately excited and its excitation is kept constant throughout the test, we have

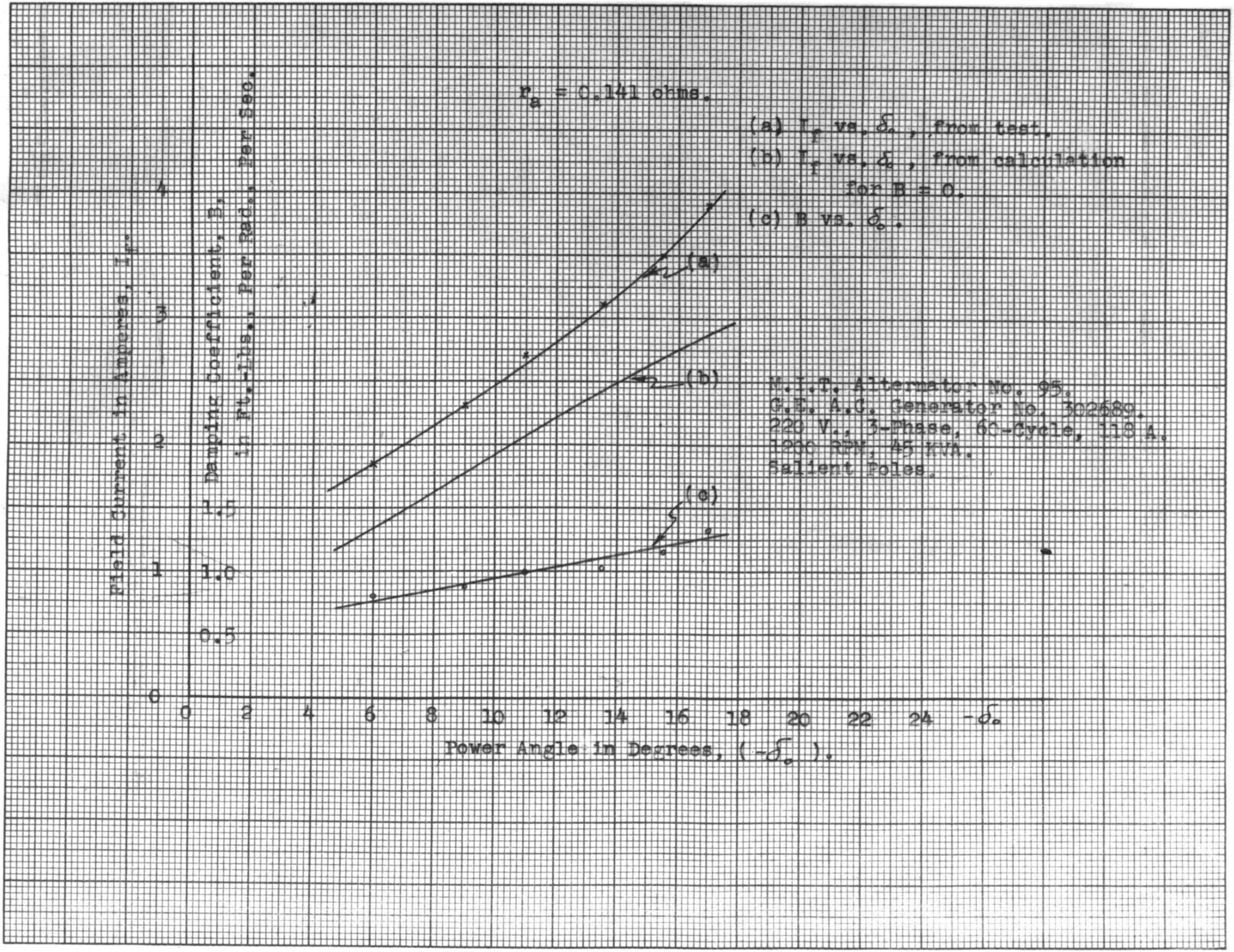
$$e = \phi\,\omega \tag{9}$$

$$i = \frac{e}{R} = \frac{\phi\,\omega}{R} \tag{10}$$

where

e is the induced emf of the d-c generator.

i is the armature current of the d-c generator.



r_a = 0.141 ohms.
(a) I_f vs. δ_o, from test.
(b) I_f vs. δ_o, from calculation for B = 0.
(c) B vs. δ_o.
(a)
(b)
(c)
M.I.T. Alternator No. 95.
G.E. A.C. Generator No. 302689.
220 V., 3-Phase, 60-Cycle, 118 A.
1200 RPM, 45 KVA.
Salient Poles.
Field Current in Amperes, I_f.
Damping Coefficient, B, in Ft.-Lbs., Per Rad., Per Sec.
0
1
2
3
4
0.5
1.0
1.5
0
2
4
6
8
10
12
14
16
18
20
22
24
$-\delta_o$
Power Angle in Degrees, ($-\delta_o$).

R is the total resistance in the armature circuit of the d-c generator.

ϕ is a constant (the effect of the armature reaction is disregarded).

ω is the angular velocity of the machine.

If the effect of the inductance in its armature circuit is neglected, the formulas 9 and 10 hold good even when ω varies. Hence the electromagnetic torque (it is considered positive in the direction of the speed) of the d-c generator is

$$T_{d.c.} = -k\frac{p\,e\,i}{2\omega} = -\frac{k\,p\,\phi^2\omega}{2R} \tag{11}$$

and the damping coefficient due to $T_{d.c.}$ is

$$B_{d.c.} = \frac{d\,T_{d.c.}}{d\,\omega} = -\frac{k\,p\,\phi^2}{2R} \tag{12}$$

where p is the number of poles of the synchronous machine under test, and k is equal to $\frac{550}{746}$ for the practical units. As ϕ is constant, the magnitude of $B_{d.c.}$ is inversely proportional to R or directly proportional to the average power of the d-c generator. In the testing range the average power increases with the average power angle of the synchronous machine. Therefore the total extra damping of the system increases with the increase of the average power angle. As the machine oscillates with constant amplitude, the damping due to the main electromagnetic action must be equal and opposite to the total extra damping. Hence B should increase with the increase of δ_o. It is fairly confirmed with the curve (c).

In the region above the curve (a) the machine is unstable, while below the curve (a) it is stable. For the same power angle it is more stable with smaller field current; and for the same field current it is more stable with a larger power angle.

6.5 Discussions

The experimental verifications for the analytic formula of the damping coefficient B will be much better if there are some simple methods to determine directly the extra damping due to the effects of load, windage, friction, hysteresis and eddy currents, etc. The damping coefficient due to the load of the d-c machine (i.e., due to its main electromagnetic action) has been given by formula 12 as

$$B_{d.c.} = -\frac{k\,p\,\phi^2}{2R}, \quad \text{with} \quad \phi = \frac{e}{\omega}$$

This is theoretically correct if the effect of the inductance in its armature circuit can be disregarded. It is derived for the condition that the d-c machine is operated as a generator delivering its load to a rheostat. However, it still holds good when the machine is connected back to the d-c source either as a generator or as a motor, so long as the voltage of the d-c source is constant. With such an arrangement the value of R is nearly equal to the armature resistance instead of the load resistance plus armature resistance, and hence the magnitude of $B_{d.c.}$ is much larger. The moving system is not able to have negative or zero net damping. This is the reason why it is advisable to test the synchronous machine as a synchronous motor instead of synchronous generator. The coefficient of the extra damping, excluding that due to the load of the d-c machine, can hardly be expressed analytically or determined experimentally. However, in the investigation, as the applied voltage is constant it should be practically constant.

Throughout the tests the synchronous machine is adjusted for zero net damping. It oscillates with constant amplitude and a voltmeter, as shown in Fig. 10, is used to indicate this required condition. If we replace the voltmeter by a brush oscillograph which is able to record the variations of the amplitude of oscillation, we may also adjust the machine to oscillate with either positive or negative damping, and then check the damping with the value calculated from the derived formula. The amplitude should vary exponentially with respect to the time as

$$\delta_2 = \delta_1 \varepsilon^{B'/2J(t_2-t_1)} \tag{13}$$

where

δ_1 and δ_2 are the amplitudes of oscillations at the times t_1 and t_2, respectively.

B' is the resultant damping coefficient, and it is positive for a negative damping.

J is the quotient of the moment of inertia of the moving system divided by the number of pairs of poles of the synchronous machine under investigation.

In the mathematical analysis we assume that the machine does oscillate with constant amplitude. Hence, in the test the resultant damping should not be too far from zero in order to have good verification.

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BIOGRAPHICAL SKETCH

The writer, born in Pukiang, Chekiang, China, on the eighth of July, 1911, was awarded the degree of Bachelor of Science in Electrical Engineering, Chiao Tung University, Shanghai, in July 1936. Taught in the Department of Electrical Engineering, Tsing Hua University, Pieping (moved to Kunming during the war), from 1936 to 1943. Awarded the degree of Master of Science in Electrical Engineering, Massachusetts Institute of Technology, in February 1945. Elected an Associate Member of the Society of the Sigma Xi in June 1945. Presented to the Faculty of the Massachusetts Institute of Technology in May 1947, for the degree of Doctor of Science in Electrical Engineering, a thesis entitled, "Investigation of the Causes of Negative Damping of Synchronous Machines."