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Abstract This article reviews research into the potential of vortex generators to mitigate shock-induced separation. Studies ranging from those conducted in the early post-war era to those performed recently are discussed. On the basis of the investigations described in this report, it is clear that vortex generators can alleviate shock-induced boundary layer separation. Yet, it will be shown, that their potential and efficiency varies considerably in practical applications. Much more success is reported in transonic test cases compared to separation induced in purely supersonic interactions.

Under a variety of flow conditions, the best performance is achieved with vortex generators with a height of roughly half the boundary layer thickness and a shape similar to a swept vane. Notwithstanding this, vortex generator performance is not as consistent as it is in low-speed applications. Further work is required before vortex generators can be implemented into the design process for eliminating shock-induced separation on transonic wings and in supersonic inlets.

Keywords Fluid Dynamics · Compressible Flow · Applied Aerodynamics

1 Introduction

The detrimental impact of shock waves on boundary layers is well known [1–4]. For one, shock waves introduce wave drag. Secondly, the high adverse pressure

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gradient imposed by a shock wave weakens the boundary layer, and in many instances leads to flow separation. These negative impacts must be confronted as shock wave / boundary layer interactions, SBLIs, are ubiquitous in high-speed aerodynamic systems.

Flow control is often employed to mitigate the detrimental impact of SBLI. Although, in principle, it is possible to address the wave drag and boundary layer separation issues simultaneously, in most instances flow control is only employed to address one of the negative consequences of SBLI. This is because the goals of reducing wave drag and reducing flow boundary layer separation are very often conflicting (preventing separation will more often than not increase wave drag and vice versa). As the title of this review suggests, the primary focus of this paper is the subject of boundary layer separation alleviation. There are many instances where boundary layer separation is the primary concern. These include: transonic wings approaching buffet and/or at high angle-of-attack where separation leads to instability and loss of manoeuvrability; and supersonic inlets where separation leads to reduced pressure recovery, increased flow distortion, and instabilities including inlet buzz and inlet unstart. Early examples of investigations which examined the detrimental consequences of shock-induced separation are given by Ferri [5], Davis et al. [6,7], and Todd [8].

A reduction in boundary layer separation can be achieved by either altering the inviscid part of the flow—the shock system—which in turn reduces the adverse pressure gradient, or by manipulating the boundary layer to make it more resilient to separation. Over the years, numerous options have been investigated to control the complex interaction between the inviscid pressure field and the viscous boundary layer, and to eradicate the detrimental impact of SBLI. One very simple flow control technology for direct manipulation of the boundary layer is the vortex generator, sometimes abbreviated to VG. The primary purpose of such a device is to introduce streamwise vorticity into the flow close to the surface. This leads to a bulk transfer of fluid and therefore momentum in the plane normal to that of the vortex. This transfer of momentum increases the wall shear stress and ‘fills out’ the boundary layer near the surface making it more resistant to adverse pressure gradients. Two vortex generator configurations and their corresponding momentum transfer process are shown schematically in Figure 1. Initially, vortex generators were employed to control boundary layers in subsonic flows. The first use of such a device to suppress a subsonic boundary layer separation is generally attributed to Taylor [9] of United Aircraft Corporation. However, it was not long before their potential use for mitigating shock-induced separation was noted, and one of, if not, the first studies with this in mind was undertaken by Donaldson [10] of the National Advisory Committee for Aeronautics (NACA). The promising results of this investigation led to a test flight the same year by Lina and Reed [11].

Since Taylor [9], vortex generators have been implemented in various aerodynamic systems. In the area of external aerodynamics, particularly aircraft wing design, vortex generators have been implemented on some designs, but they have not generally been employed for control of shock-induced separation. For a comprehensive review of their use on airfoils, see Lin [12]. In the field of internal aerodynamics, in particular inlet design, much work has been done to establish the benefits of vortex generators in the subsonic portion of the inlet, see references [13–17]. Nevertheless, their implementation in the supersonic portion of the inlet has been scarce. To the authors' knowledge, there exists only a single application of vortex generators in turbomachinery for SBLI control, by Gamerdinger [18].

In many applications where some form of boundary layer control is desired, partial or total removal of the boundary layer is been undertaken using boundary layer suction. This technique is often simply referred to as bleed. While highly effective, there are a number of disadvantages associated with the use of boundary layer suction, including additional weight, complexity, and cost. For a detailed account of the advantages and disadvantages of bleed the reader should refer to Seddon and Goldsmith [19].

Due to the limitations of boundary layer bleed there has always been interest in other boundary layer control technologies such as vortex generators, which are physically less complex and therefore potentially more efficient. For this reason, this is an active research area, and there has been a foray of investigations using vortex generators for mitigating shock-induced separation in recent year. With a plethora of recent articles and numerous older publications, now is an appropriate time to step back and summarize the work that has been conducted to date. It is hoped that this review can be used as a reference point for those entering this field and to help move this field forward by consolidating our knowledge and illuminating areas where our understanding is still lacking.

Hereafter, the salient publicly available investigations on vortex generators for control of shock-induced boundary layer separation are presented in approximately chronological order. A wide variety of investigations will be discussed. These investigations are summarized in Table 1. Investigations of both transonic and supersonic SBLIs will be considered in both fundamental (simpler, lab-scale boundary layer experiments) and applied settings (conditions at least somewhat representative of an application).

2 1945–1965

The first comprehensive study which employed vortex generators for shock-induced separation mitigation was conducted by Pearcey [20]. Although reported by the author as a 'preliminary study' this investigation is very thorough and detailed in scope. Pearcey [20] also includes data from a number of other authors in his report. For this reason, his study is a good starting point for a detailed discussion on the control of shock-induced separation using vortex generators.

Pearcey [20] varied a large number of parameters, and the reporting of these variables alone is, in itself, a prominent illustration of the range of possibilities when it comes to vortex generator design. A number of these variables are shown schematically in Figure 1. The degrees of freedom within the vortex generator design space include the choice of: vortex generator array, whether co-rotating (all vortices of similar rotation, Figure 1(a)), counter-rotating (vortices of alternate rotation, Figure 1(b)) or a combination of these two, one example being a bi-plane configuration (two vortices in one orientation followed by two in the opposite orientation); vortex generator height, h; vortex generator spanwise spacing, D (and d if in a counter-rotating configuration); streamwise position of vortex generators relative to the shock-induced separation, Δx; and vortex generator strength (streamwise circulation), K, where K is a function of vortex generator length, l, angle-of-attack, α, height, h, shape, and the characteristics of the incoming boundary layer.

The experimental investigation performed by Pearcey [20] was undertaken on a half-airfoil section mounted on the floor of a small wind tunnel. Using such a configuration, the upper surface of a supercritical airfoil can be simulated. A selection of the salient data obtained by Pearcey [20] is reproduced here in Figure 2. The experimental arrangement and the different vortex generator configurations Pearcey [20] examined are shown schematically in Figure 2(a). The vortex generator configurations tested are shown in the table presented in part (b) of Figure 2. The two-dimensional bump-type configuration in question generates a terminal shock on the leeward side of the bump, which, once positioned far enough aft, caused substantial shock-induced separation. Although the freestream velocity ahead of the bump is subsonic, the vortex generators are located in the supersonic portion of the flow as can be seen from the schlieren images of Figure 2(c). Schlieren photography and pressure measurements at the trailing edge of the bump were utilized as the main aids in determining the success of the vortex generators. A small number of detailed flow measurements were taken with a Pitot probe, but it will be shown that the lack of more of these detailed measurements is the weakness of this investigation. The relative merits of different vortex generator geometries and configurations were assessed on a trial and error basis, in conjunction with simple analytical methods. These analytical methods, originally derived by Jones [21], use potential flow theory and the image theorem to calculate the vortex trajectories of the vortex generator arrays.

On the basis of his trial and error approach, Pearcey [20] concluded that vortex generators with a height $h = 0.01c$ placed at $0.3c$, where c is the bump chord length, were about optimal for suppressing separation across a range of shock positions on the bump—though little evidence is presented in reference [20] to back-up this conclusion, unfortunately. It is data using this height and streamwise that Pearcey [20] bases the majority of his conclusions on. It is this height and streamwise positioning that are presented in Figure 2.

Looking at the data presented in Figure 2 in detail, what is apparent from the schlieren images of Figure 2(c) is the visible reduction in the boundary layer separation behind the shock-wave with the introduction of vortex generators. As anticipated, the introduction of streamwise vorticity clearly helps the boundary layer to remain attached. This improvement is shown quantitatively in Figure 2(d) which presents curves of trailing edge pressure, p_{te}, versus shock position, x_{SH}/c, for a number of vortex generator configurations. From this figure, it can be seen that all the vortex generator configurations increase the trailing-edge pressure when the shock is at or beyond 45% of chord ($x_{sh} > 0.45$). Unsurprisingly, this is also the location at which the shock becomes strong enough to instigate separation—evidenced by the sudden drop in the trailing-edge pressure in the uncontrolled instance (see Figure 2(d)).

Also evident in Figure 2(d) is the pronounced variation in the trailing-edge pressure between configurations. It can be seen that the bi-plane configuration B1—see Figure 2(a)—is particularly beneficial at reducing separation across the range of shock positions tested. When aligning vortex generators in an alternating arrangement—a counter-rotating arrangement—Pearcey [20] was also able to obtain good separation mitigation performance in some instances. For example, the counter-rotating configuration M2 performed with only slightly inferior separation mitigation performance when compared to bi-plane configuration B1.

While the counter-rotating configuration M2 performed favorably, Pearcey [20] reported that careful consideration of D, d, h, and K is required to delay vortex lift-off (wall-normal movement of the vortices such that they are no longer energizing the near wall flow) and hence maximize range (the streamwise distance over which the vortex generators are effective). In particular, Pearcey [20] suggests that careful consideration should be given to D/d, as this variable is key to determining the rate at which vortex lift-off occurs. This is because D/d establishes the relative vane spacing between vortices which induce a common upwash on each other ($\propto D$) and vortices which induce a common downwash on one another ($\propto d$). These regions of upwash and downwash are illustrated in Figure 1. The importance of D/d is exemplified in Figure 2(d), where configuration M1, $D/d = 2$, performs very unfavorably in comparison to counter-rotating configuration M2, $D/d = 4$. The spacing is the only factor different between the two configurations. As a result, it is almost certain—as Pearcey [20] suggests—that premature lift-off is the cause of the poor performance achieved with configuration M1.

The pressure distributions of Figure 2(d) also suggest that co-rotating configurations (all vortex generators producing a vortex of similar orientation) are not as good at reducing separation as the best bi-plane or counter-rotating configurations. While previous authors have generally found the co-rotating configuration to be largely unfavorable, for example Lina and Reed [11], Pearcey's [20] results suggest that as long as the spacing to height ratio (D/h) is greater than 3, and preferably in the range $D/h = 4$–8, some separation reduction is achieved. In addition, Pearcey [20] reports very long ranges over which co-rotating configurations are effective; up to $100h$. He suggests that such a large range of effectiveness is due to the favorable induced velocity field in the co-rotating case which leads to no wall-normal velocity components and hence no vortex lift-off. This is a reasonable conclusion, yet, the lack of detailed flow measurements downstream of the devices makes this hypothesis and the physics behind many of Pearcey's [20] observations impossible to confirm.

Around the same time as the work of Pearcey [20], interest was starting to build in using vortex generators for shock control in supersonic inlets. An early study to examine vortex generators on the compression surface separation on a simple axisymmetric two-shock inlet was conducted by Griggs [22]. The vortex generator configuration was of counter-rotating type based on previous unpublished work with $D/h = 3$ and $D/d = 2$ (constant spacing between both common flow up and common flow down pairs). Much like the majority of Pearcey's [20] work, no measurements were obtained near the vortex generators or the terminal shock. Griggs' [22] conclusions were, therefore, largely based on data obtained at the simulated engine face. Results from the schlieren imaging and engine face stagnation pressure measurements indicated that the vortex generators had little to no impact on the shock-induced separation. The schlieren and engine face pressure recovery measurements confirm noteworthy separation both with and without vortex generators. There was, nevertheless, a small increase in the stable mass flow range of the inlet when vortex generators were applied. Without measurements in the vicinity of the SBLI, Griggs [22] concedes that it is not possible to determine why the vortex generators were unsuccessful on this occasion. One possibility based on the observations of Pearcey [20] is that the vortex generator spacing $D/d = 2$ is too small and is causing early lift-off. Consequently, the vortex generators are ineffective once they reach the terminal SWBLI. It is also possible, however, that transitional effects are impacting the interaction due to the limited Reynolds number. Once again, the lack of detailed measurements makes it impossible to reach a firm conclusion.

Unfortunately, as mentioned above, the major drawback of early studies is the lack of detailed boundary layer / viscous flow field measurements in the vicinity of the vortex generators or the shock wave[1]. Consequently, conclusions on the separation suppression capabilities of the vortex generators are based entirely on implicit measurements of the net forces, wall pressure, and Pitot pressure readings. As a result, little is known about the details of the interaction itself. Yet, the studies discussed in this section provide a useful base on which to build. They also made it clear, however, that more detailed measurements were required.

3 1965–1985

Soon after the early work of Pearcey [20], flight tests were undertaken as part of a follow-on program to confirm the benefits Pearcey [20] had established in his laboratory experiments. Some of this flight-test data are summarized by Pearcey after-the-fact in reference [20], including flight-test data by Edwards [23] and previously unpublished flight-test data by Gould [24]. These data are reproduced once more here in Figure 3. Note that the naming con-

[1] LDV and PIV were not available at this time and other measurements such as Pitot probes and hot-wires are very difficult to employ in transonic flows

vention for the vortex generator configurations in Figure 3 is identical to that presented in Figure 2.

The preference obtained by Pearcey [20] in his laboratory experiments for bi-plane, counter-rotating and then co-rotating arrays, respectively, is not as transparent in the flight-test data. In the lift coefficient, C_{L}, test flight data obtained by Gould [24], presented in Figure 3(a)(i), there is little to differentiate the co- and counter-rotating arrangements. In fact, the co-rotating configuration performs better at low freestream Mach numbers, M_∞—compare the performance of configurations C1 and M4 in Figure 3(a)(i). No explanation is given for such a trend. The flight-test data of Gould [24] also illustrates the advantages of using triangular/swept vanes, configuration C5, over rectangular devices, configuration C1. The lift is clearly improved with configuration C5 over C1, as shown in Figure 3(a)(i). Correspondingly, the drag is much reduced as shown in Figure 3(a)(ii).

In the free flight tests of Edwards [23], shown in Figure 3(b), the roll rate performance presented in Figure 3(b)(i) indicates that the bi-plane configuration B3 and the counter-rotating configurations M2 give similar control authority on the flap they were employed on—with bi-plane configuration B3 slightly preferable. This is in agreement with Pearcey's [20] laboratory work. However, the corresponding drag coefficient data, presented in Figure 3(b)(ii), suggests that an appreciable drag penalty is incurred with the bi-planes. On the other hand, the counter-rotating configuration incurs a much smaller drag penalty. It is suggested that the drag penalty of configuration B3 far outweighs their additional control benefit. This is a reasonable conclusion in light of the available data.

As a consequence of all the data presented in his report, Pearcey [20] concludes that counter-rotating configurations are preferable as they provide the best compromise between shock-induced separation mitigation and increased parasitic drag. On the basis of his investigation he recommends $D/h = 10$, $D/d = 4$, $\alpha = 15$ and $l = 1.5h$. If some range can be sacrificed for the initial effectiveness then the number of pairs should be doubled to give $D/h = 5$. It is important to remember, however, that these recommended configurations are only a guideline and that the appropriate configuration will be dependent on the specifics of the application.

The next investigation worth noting is that of Mitchell and Davis [14] who unintentionally demonstrated a promising impact of vortex generators in a supersonic inlet study. This supersonic inlet investigation, like many of its forerunners, was concerned with studying vortex generators for boundary layer control in the subsonic portion of the inlet. Yet, in a handful of instances, Mitchell and Davis operated the inlet supercritically at a point where the terminal shock was located downstream of the vortex generators. The performance improvements obtained at this supercritical condition, compared to a similar case without vortex generators, was sufficiently large to be prominently noted by the authors. At one condition the inlet pressure recovery and the distortion were improved by more than 1% and 10%, respectively. The only plausible explanation for these performance improvements is that the vortex generators were mitigating a shock-induced separation occurring at these supercritical conditions. In the study of Mitchell and Davis [14], three vortex generator configurations were examined. While the configuration with the highest D/d of the three performed the best at the supercritical condition, the investigators established a slight preference for a different configuration over the entire operating range. While it is not possible to establish why such a variation in preference exists, this is clearly early evidence that the optimal vortex generator configuration may differ between subsonic and supersonic conditions.

On the basis of the promising results of Mitchell and Davis [14], Mitchell [25] undertook a more thorough study of vortex generators for shock control—using the same supersonic inlet. In these experiments, he explored the boundary layer behavior in the vicinity of the SBLI in much greater detail than any known previous investigation. The difficultly of obtaining such measurements and the effort employed by Mitchell [25] to overcome these difficulties is exemplified by Figure 4(a), which shows the complexity of the supersonic inlet utilized and the boundary layer rakes fabricated to achieve measurements within the vicinity of the SBLIs.

In his investigation, Mitchell [25] used a constant spacing between the converging and diverging vanes, $D/d = 2$, and the spacing between vane pairs was $D/h = 8.2$. Two vortex generator heights[2] and angles with respect to the oncoming flow were examined: $h/\delta = 1.2$ and 2.4 and $\alpha = 9°$ and 18°. Salient stagnation pressure data from Mitchell [25] with and without vortex generators are shown in Figure 4(b) and 4(c). In the cases presented in this Figure, the vortex generators were employed downstream of the reflected oblique shock wave but ahead of the terminal shock wave on the inlet centerbody.

Measurements taken with Pitot rakes—the centerbody throat exit rakes shown in Figure. 4(a)—downstream of the terminal shock, but still within the vicinity of the throat, are presented in Figure 4(b)(i) and 4(b)(ii). Each chart shows a different rake azimuthal position. In these charts the regions of upwash (behind converging pairs) and downwash (between diverging pairs) introduced by the vortex generators are unmistakable. In the downwash regions, there is a large net benefit to the stagnation pressure profile of the centerbody boundary layer. This benefit includes a substantial increase in the skin friction and hence resistance of the boundary layer to subsequent adverse pressure gradients. On the other hand, though, in the upwash regions the boundary layer exhibits a large area of low stagnation pressure and hence low velocity flow, as one would expect. However, on careful examination of the near-wall region, the shear stress in the upwash regions between controlled and uncontrolled cases is not dissimilar; there is no appreciable reduction in the shear stress. These measurements indicate that the vortex generators have resulted in an overall net increase in wall shear stress. This is solid evidence that the boundary layer

[2] Note that this is the first occasion where the boundary layer thickness at the vortex generators was evaluated; an accolade to the high quality of the measurements in this study

downstream of the vortex generators has a higher resistance to separation and that there will be a net benefit to inlet performance.

While the stagnation pressure profiles of Figure 4(b) demonstrate the impact of the vortex generators in the vicinity of the shock, they do not directly indicate how overall system performance is influenced. This can only be confirmed by measurements at the engine face. Some measurements of engine face pressure recovery obtained by Mitchell [25] are reproduced alongside those of the throat exit rakes in Figure 4(c). Charted in this figure is the variation of pressure recovery with terminal shock Mach number, which was varied by translating the cowl (Figure 4(c)(i)) and by varying the back pressure (Figure 4(c)(ii)). It is to be expected that the benefit of vortex generators will be restricted to conditions where shock-induced separation occurs—roughly at Mach numbers higher than 1.35, see Reference [4] for full details. Below this, there should be no benefit; in fact, the drag of the vortex generators should decrease the pressure recovery. The clarity by which this trend is observed in Figure 4(c) is striking—especially for the smaller vortex generators. With the smaller of the vanes at the higher angle of attack, a 2–3% improvement is recorded above the initiation of separation (above Mach 1.35). What is more, the inlet is also able to sustain increased supercritical operation before inlet unstart occurs (note data points further to the right with vortex generators present). These data demonstrate that vortex generators improve this inlet's performance both on- and off-design.

When comparing the results for the different vortex generator configurations examined by Mitchell [25] in Figure 4(c), it is evident that the smaller vortex generators, $h/\delta = 1.2$, are more beneficial. Furthermore, the flow is not sensitive to the vane angle—with only a marginal preference for the higher angle vanes. The fact that both increased angle and increased height will increase vortex generator strength, K, but that only angle improves performance, suggests that proximity of the vortices to the wall is important. It is evident that the vortices from the larger vanes, $h/\delta = 2.4$, are too far from the surface to effectively suppress the flow separation. Further investigation of vortex generators alongside bleed also indicated that a combined benefit could be achieved as long as the bleed was positioned downstream of the vortex generators. Even though Mitchell [25] determined a preference for smaller, higher angle of attack vanes, clearly, more vortex generator configurations need to be examined to obtain optimal performance. However, the restriction to a relatively small number of configurations is a result of the complexity and cost of such investigations.

Up until now, we have only discussed the implementation of vortex generators in transonic flows. However, at the same time as determining the potential of vortex generators for suppressing separation through the terminal shock, Mitchell [25] also employed vanes on the forward part of the inlet centerbody to control the oblique shock reflection initiated off the inlet cowl. These vortex generators can also be seen in Figure 4(a). Interestingly, unlike across the terminal shock, in this role, none of the vortex generator configurations achieved an improvement. In fact, a reduction in total pressure recovery between 0.5%

and 1.5% was observed, despite the vortex generator configurations being very similar to those used further downstream to control the terminal SBLI.

Not long after Mitchell [25], one of the earliest fundamental supersonic investigations involving vortex generators was completed by Gartling [26] in a purely supersonic SBLI. Gartling [26] examined the influence of vane-type vortex generators on a Mach 5, 35^o compression corner. While not a highly comprehensive investigation, it is only briefly mentioned here because it is another example of a case where no success was achieved when employing vortex generators for SBLI at purely supersonic speeds. We will return to this observation later.

4 1985–2005

After 1970 it appears that little was done to investigate vortex generators for shock-induced separation mitigation until McCormick [27]. His investigation builds on previous observations at subsonic speeds that much smaller vortex generators ($h < \delta$) can produce just as favorable, if not superior, performance to conventional vortex generators ($h \approx \delta$). The first report of such behavior is generally attributed to Rao and Kariya [28] who demonstrated that vortex generators as small as $h/\delta = 0.2$ could provide comparable levels of mixing and hence separation mitigation to those of more traditional devices, $h/\delta = 1$, but without such a large drag penalty. This preference for smaller vortex generators for separation suppression in subsonic flows has since been backed up by a number of research studies. A comprehensive summary of these is provided by Lin [12]. While Lin [12] refers to these smaller devices as “low-profile” vortex generators, they are also often referred to as “micro”-vortex generators or “sub-boundary layer” vortex generators.

McCormick’s investigation [27] was undertaken in a small-scale facility in which a slightly diverging channel is used to hold a normal shock steady and to allow the shock Mach number to be easily varied. The effects of low-profile vortex generators and a passive cavity on a terminal SBLI were then compared and contrasted. The configuration was axisymmetric and due to its simple nature was amenable to a variety of measurement techniques. The experimental setup utilized by McCormick and the vortex generators considered are shown here, schematically, in Figure 5(a).

Once again, favorable results are achieved in this transonic test case. This is the first instance in which surface-flow visualizations indicate *directly* that vortex generators can alleviate shock-induced separation. Unfortunately, the low quality of the images precludes their presentation here. Instead, quantitative pressure and boundary layer profile data is reproduced from McCormick [27] in Figure 5(b)–(f). Looking at the measurements of wall pressure recovery in Figure 5(b), it can be seen that with the introduction of vortex generators, there is no longer a distinct plateau that is indicative of boundary layer separation.

Unlike the majority of previous studies, vane-type vortex generators were not utilized by McCormick [27]. Instead, a modified wedge-type vortex generator named the Wheeler doublet—after Wheeler [29]—was employed. Due to the lack of a direct comparison with other shapes, the relative effectiveness of Wheeler doublets cannot be ascertained.

In addition to providing more extensive wall pressure measurements than previous authors, this is also the first instance in which a large number of Pitot profiles have been obtained in the vicinity of the shock—the Pitot probe can be seen in Figure 5(a). McCormick [27] undertook Pitot probe surveys at a variety of azimuthal positions. However, he reports only small azimuthal variations with differences of the boundary layer integral parameters no higher than the experimentally observed scatter of Figures 5(c)–(e). While there will certainly be span-wise variations near to the vortex generators, see for example Figure 4(b), much of the data obtained by McCormick is far downstream, between $20 - 60\delta$, and suggests that azimuthal variations have largely dissipated. As a result, the remaining data presented in [27], and reproduced here, only shows the span-wise average of the variables in question.

The boundary layer integral parameters that are presented are highly informative. As shown in Figure 5(c)–(e) the integral parameters exhibit a healthy improvement with the vortex generators present: The growth of the displacement thickness, Figure 5(c), is significantly restrained when compared to the uncontrolled case, and although the momentum thickness, Figure 5(d), is initially increased above that of the uncontrolled case—due to the drag of the devices themselves—after 20δ, it falls below the level of the uncontrolled case and continues to flatten off at the end of the measurement region. As a consequence, the boundary layer shape factor, Figure 5(e), a standard measure of the boundary layer's resistance to separation remains well below that of the uncontrolled case throughout[3]. In light of this wealth of data, it is unfortunate that McCormick [27] does not present more than a single velocity profile in his paper.

While the data from surface-flow visualizations, static pressure measurements, and integral parameters indicate a beneficial influence of vortex generators, mass-averaged total pressure reproduced in Figure 5(f) exhibits a lower recovery with vortex generators compared to the baseline case. This strongly indicates that there is no overall utility of employing vortex generators in this case. This indicates that the reduction in the dissipation due to separation is outweighed by the increase in losses due to device drag. This suggests that the flow separation in this instance is not large enough to justify vortex generators. Alternatively, the parasitic drag of the Wheeler doublet may be too high to justify their use compared to alternate vortex generators.

The contrast in flow field performance presented between Figure 5(e) and Figure 5(f) exemplifies the difficulty of determining the overall aerodynamic benefit from small-scale fundamental experiments. This confusion substantiates the need for applied, yet simple, investigations where both detailed boundary layer performance and overall aerodynamic performance can be measured,

[3] Note that it would be preferable to use the incompressible shape factor to represent the boundary layer state – see Winter and Gaudet [30] for full details.

if vortex generator characteristics and potential are to be measured simultaneously.

Soon after McCormick [27], Barter and Dolling [31] conducted further supersonic compression corner experiments in the same facility as Gartling [26], but with a smaller ramp angle and hence weaker shock ($\theta = 28^o$). Following McCormick [27], Barter and Dolling [31] employed Wheeler doublets in preference to the traditional vanes used by Gartling [26]. In this instance, even with a proven vortex generator design, flow visualizations revealed that the separation remained largely intact. Once again, in purely supersonic conditions, vortex generators are not able to appreciably reduce shock-induced separation. However, beneficial effects to the interaction unsteadiness were observed. Near the compression corner, Barter and Dolling [31] measured a reduction in the maximum pressure loadings of around 20%, a reduction in the extent of the unsteady region, and a shift in the frequency spectra to higher frequencies. Such decreases in shock unsteadiness are thought to be beneficial for inlet stability—potentially delaying both inlet buzz and unstart. In this light, some benefit due to the vortex generators was achieved.

The next prominent study employing vortex generators for shock control was conducted by Wasserbauer et al. [32] using a complete inlet system much like Mitchell [25]. Unlike Mitchell [25], however, this inlet was not an axisymmetric design but had a more two-dimensional cross-section. This geometry led to a highly complex three-dimensional flow field as measured by Pitot rakes in the vicinity of the inlet throat. The introduction of vortex generators did show some benefit, particularly at supercritical conditions, where the terminal shock more than likely produced significant shock-induced separation. However, the complexity of the flow field, and the inconsistent outcomes of different vortex generator configurations, makes it very difficult to draw hard conclusions. What is immediately clear from the investigation of Wasserbauer et al. [32] is the need for further experimental investigations in simpler flow fields.

The next experimental study worthy of discussion has only been published within a much wider program on boundary layer control led by Ashill et al., see references [33–35]. The vast majority of this program was conducted subsonically with the primary data collected related to vortex production, trajectory, and decay, in order to identify optimum vortex generator geometries. Nevertheless, as part of this research program, the effect of low-profile vortex generators for shock control on transonic wings was investigated. At least some of this investigation is reported in both reference[35] and the review paper by Ashill et al. [33]. There are two novel aspects to this study when compared to previous investigations. For one, it is probably the first study in an applied setting to utilize smaller, low-profile vortex generators, $h/\delta^* = 1$, $(h/\delta \approx 0.2)$[4] for shock-induced separation mitigation. Secondly, this investigation was completed on a transonic airfoil in a relatively large facility (8 ft^2) with a large Reynolds number of 2×10^7 based on chord. The RAE 5243, airfoil section was

[4] estimate

specifically selected for this study because the position of the terminal shock only varies weakly with changes in Mach number and incidence in this design. This permits more straightforward streamwise placement of vortex generators. Such a choice of airfoil removes the need for the vortex generators to be effective over a wide range of streamwise distances. Ashill et al. [33] examined two low-profile vortex generator configurations: counter-rotating vanes and what are often referred to as ramps or wedges with $h/\delta^* \simeq 1$ and $D/h = 12$ (for examples of ramps/wedges, see Figures 8 and 9). For the vanes, $D/d = 1.4$ and for the wedges, $D/d = 1.26$.

Data which demonstrates the performance improvements afforded by vortex generators in this situation are reproduced in Figure 6. In the charts presented in Figure 6, the normal force, C_{N} and hence lift, L, were calculated using surface pressure measurements, and the drag, D, was measured using a rake downstream of the airfoil. From Figure 6(a), it can be seen that the normal force is improved by both the wedges and the vanes as the angle of attack is increased above 2°. These results signify that these low-profile vortex generators are able to curb the separation present at high angles of attack. Improved performance is also seen in the lift-to-drag comparison shown in Figure 6(b). This demonstrates that the device drag incurred by the vortex generators does not offset the gain in lift. The lift-to-drag measurements demonstrate near comparable performance between the wedges and vanes. Combining the results of Figures 6(a) and 6(b) indicates that vanes have preferable separation suppression capability, but also suffer from higher device drag. The separation reduction capabilities of both the vanes and wedges are seconded by the trailing-edge static pressure measurements presented in Figure 6: The drop-off in trailing-edge pressure, which is symptomatic of boundary layer separation, is delayed by the application of vortex generators, with vanes being more effective.

The investigation by Ashill et al. [35] is yet another example of a transonic test-case where vortex generators improve performance by reducing shock-induced separation. In this instance, the small size of the vortex generators is almost certainly helping to keep device drag low, leading to improved L/D, in other words, improved overall aerodynamic performance.

Like the applied investigations of Mitchell [25] and Wasserbauer et al. [32] though, the inability to obtain detailed measurements in the vicinity of the shock somewhat limits our understanding of the flow physics. For example, Ashill et al. [35] surmise that the beneficial hump in the L/D curve with the wedges at a $C_L = 0.4$, Figure 6(b), is due to the ramp shock from the wedges weakening the terminal shock and therefore reducing the wave drag (effectively multi-stage compression). Without further more detailed measurements, though, this is a mere hypothesis.

Further to the data of reference [35], Ashill et al. [36] present more experiments on vortex generator improvements on a swept wing. Similar levels of improvement were also achieved in this investigation. In this instance, a novel type of vortex generator, the wire-type vortex generator was employed, see Ashill et al. [36] for full details.

5 2005–present day

In the last ten years there have been many investigations on vortex generators in simple, fundamental flow fields. One of the first of the more recent studies was undertaken by Holden and Babinsky [37], as a follow-on to the work of Ashill et al. [35]. Holden and Babinsky [37] employed the same vortex generator configurations, namely wedge- and vane-type devices with $h/\delta = 0.20$. The primary objective of this study was to obtain more details than Ashill et al. [35] in the vicinity of the SBLI. This was achieved by utilizing a simpler flow field more amenable to measurements close to the SBLI. Another goal was to try and replicate the wedge-type vortex generator wave-drag benefit postulated by Ashill et al. [35] (as discussed in the preceding section).

In contrast to the overwhelming benefit of vortex generators reported by Ashill et al. [35], the benefits reported by Holden and Babinsky [37] are not as clear-cut. Importantly, both measured wall pressure distributions and Pitot pressure profiles exhibited near negligible improvements with vortex generators over the uncontrolled case. While no improvement in the pressure distributions was achieved, surface-flow visualizations did qualitatively suggest that vortex generators were able to mitigate the shock-induced separation, as when wedges and vanes were introduced, the accumulation of oil beneath the shock was reduced. Furthermore, when comparing the visualization with wedges to that with vanes, there is evidence that the vanes are better at reducing the extent of separation. On the basis of the surface flow visualizations, Holden and Babinsky [37] conclude that vortex generators alleviate the shock-induced separation. However, no definitive reason is given as to why there is no corresponding benefit in either wall pressure or Pitot pressure. As no detailed velocity measurements were achieved in the vicinity of the SBLI, it is difficult to suggest reasons for this apparent discrepancy. It is possible that the separation was too small such that its mitigation only has a minimal influence on the wall pressure and Pitot pressure results,with any improvement in stagnation pressure recovery counteracted by the drag of the devices themselves. Detailed velocity measurements would, however, be required to confirm these hypotheses.

Not long after the investigation of Holden and Babinsky [37], Bur et al. [38] studied a similar problem. A sample of their surface flow visualizations is shown in Figure 7. From these visualizations, it can be seen that the vortex generators have a marked impact on the near-wall topology; nevertheless, they are not able to significantly mitigate the separation. Instead, the separation is partitioned into a number of three-dimensional cells, separated by thin channels of attached flow. This effect is achieved across multiple vortex generator configurations. Bur et al. [38] examined vanes with heights of 0.5δ and 1δ, and both counter-rotating and co-rotating configurations (see [38] for full details). A similar partitioning of the separated region with thin channels of attachment has also been reported by Zare Shahneh and Motallebi [39].

In addition to the three-dimensional partitioning of the separated flow, the surface flow visualizations of Bur et al. [38] unveil a strong and complex interaction at the junction of the floor and sidewalls. Looking closely at these regions in Figure 7, it can be seen that there is separation in the corner region, which results in substantial cross-flow. Clearly, in a small facility such as the one utilized by Bur et al. [38], the flow in the corners has a strong influence on the overall flow. This is a potential issue, because many flow fields do not have corners, or at least not corners similar to those found in experimental facilities. If the influence of the corner regions were consistent between uncontrolled and vortex generator controlled flows this phenomenon would not be a concern. However, the flow visualizations of Bur et al. [38] demonstrate that the devices strongly alter the flow development in the corner regions: In the uncontrolled case, only minimal departures from two-dimensionality are observed as the sidewall is approached, while the vortex generator-controlled flow exhibits a complex, three-dimensional flow with notable separation near the junction of the floor and sidewall. Increased three-dimensionality accompanying the introduction of vortex generators was also observed by Holden and Babinsky [37].

Since Holden and Babinsky [37] observed enhanced three-dimensionality with vortex generators, a number of further studies have been conducted in the same facility to examine this phenomenon further. These studies include: Bruce et al. [40] [41], Titchener and Babinsky [42], Rybalko et al. [43], and Titchener and Babinsky [44] (see Table 1 for more details). In all of these studies, vortex generators were found to amplify three-dimensionality. Titchener and Babinsky [42] concluded that this amplification is caused by the relative lack of flow control in the corner regions compared to the center-span regions where the vortex generators are usually applied. This situation comes about because the reduction in separation downstream of the vortex generators subjects the corner regions to a higher adverse pressure gradient (due to the increase in effective area).

This process of loss redistribution is in many instances inevitable, because, not only is there no effective boundary layer control in the corners, but the corner flow is often as susceptible, if not more susceptible, to separation than the center-span regions (where separation is already having to be controlled). As a result, the uncontrolled regions often separate more extensively. In turn, these areas with increased loss often negate the benefit achieved elsewhere in the flow field. Three-dimensional effects can significantly complicate the flow, which might lead to erroneous conclusions about the effectiveness of vortex generators. These results demonstrate that care must be taken when evaluating vortex generators in small-scale facilities.

In a recent study by Titchener and Babinsky [44] the three-dimensionality was partially mitigated by introducing boundary layer bleed in the corners at the junctions of the floor and sidewalls. Flow visualizations from this investigation are presented here in Figure 10. With the addition of boundary layer bleed in the corners significant benefits were achieved in the presence of vortex generators across the entirety of the span, see Figure 10(b). Without flow control in the corners, Figure 10(a), the flow is largely separated. For further details, see References [44] and [45].

Alongside the numerous transonic studies, there have been multiple efforts to investigate the use of vortex generators in purely supersonic interactions. Verma et al. [46] examined the influence of vortex generators at a supersonic compression corner much like Gartling [26] and Barter and Dolling [31]. Verma et al. [46] employed wedge-type devices upstream of a Mach 2, 24^o compression corner. Once again, like Gartling [26] and Barter and Dolling [31], only localized variations in the separation extent were observed, with no reduction in the overall separation footprint. In contrast to various transonic investigations, no investigation using vortex generators to control a supersonic compression corner has reported attached flow channels or the corresponding breakup of separation into discrete cells or blocks.

Barring the recent compression corner study of Verma et al. [46], all recent studies in purely supersonic interactions employing vortex generators have considered the reflected oblique SBLI. One prominent investigation on the reflected oblique SBLI is that reported by Babinsky et al. [47]. Surface flow visualizations from this work are reproduced here in Figure 8(a) and (b) alongside corresponding computational fluid dynamics (CFD) results by Ghosh et al. [48] in Figure 8(c) and (d). These investigations were accomplished at Mach 2.5 with an impinging shock wave generated by a 7° shock-generating wedge. From the visualizations in Figure 8 it is clear that the vortex generators, in this case micro-ramps (a ramp-type device designed by Anderson et al. [49]), are not able to eliminate the separation. There is, however, some deformation of the separation, and perhaps even attached channels through the separated region. This partitioning is not dissimilar to that reported by Bur et al. [38] and shown in Figure 7. A partial removal of the separation has also been noted by Lee et al. [50] who undertook both experiments and CFD to investigate the performance of arrays of micro-ramps downstream of an oblique shock reflection at Mach 3.

From the surface flow visualizations of Babinsky et al. [47] and Bur et al. [38] one tentatively concludes, as these authors do, that the vortex generators have been successful at reducing the extent of shock-induced separation. On visual inspection the separation footprint is reduced when vortex generators are introduced (see Figures 7 and 8). The partitioning of a separated region has also been reported in the investigation of Blinde et al. [51]. In this case, micro-ramps were employed upstream of a Mach 1.8 oblique shock reflection with 10° of initial flow turning. There is, however, evidence in the particle image velocimetry (PIV) results of Blinde et al. [51] that there may not be a reduction, but instead a redistribution of the separated regions. A sample of the PIV results of Blinde et al. [51], with and without devices, is reproduced here in Figure 9. Each image in this figure is a wall-normal slice of the velocity field. Close to the wall, at $y/\delta = 0.1$, shown here in Figure 9(a), it is evident from the PIV data that the separation is broken up into a number of cells, but not eradicated across the span. This is in agreement with Babinsky et al. [47] and Bur et al. [38]. However, Blinde et al. [51] also present a PIV slice further away from the wall at $y/\delta = 0.4$. From this PIV plane, reproduced here in Figure 9(b), it is evident that in the areas where separation remains in the vortex generator-controlled case that the separated regions are taller as the low velocity regions extend much higher above the surface. Consequently, it is not clear whether the total volume of separation has actually been reduced by the application of vortex generators. It is clear that we need to be careful when basing conclusions on limited surface flow visualizations. Unfortunately, though, this is often one of the few flow diagnostic techniques available in such flow fields.

In addition to surface flow visualization, Babinsky et al. [47], and also Herges et al. [52], undertook boundary layer profile measurements as well as surface flow visualizations. In both instances, improvements in the boundary layer shape parameter, H, were achieved with devices. While the results of Babinsky et al. [47] and Herges et al. [52] do suggest the boundary layer is now less susceptible to separation further downstream, to determine definitively whether there is an overall improvement, either the stagnation pressure recovery or energy thickness needs to be calculated far downstream of the separation at the plane of interest. In contrast, Herges et al. [53] did take overall measurements using stagnation pressure probes at the simulated engine face. However, in this instance no measurements were obtainable in the vicinity of the SBLI. In terms of performance, no real benefit was achieved by Herges et al. [53] using micro-ramps in an axisymmetric external compression inlet. Unfortunately, it is not possible to explain this result with engine face data alone. It has become clear that both measurements in the vicinity of the SBLI and at the plane of real interest are necessary.

The results presented above on supersonic SBLI investigations lead us to conclude that the utility of vortex generators in supersonic SBLIs remains unclear, as no investigation has demonstrated a clear aerodynamic benefit of employing vortex generators. This is in stark contrast with the results obtained in the transonic case, where a number of studies have clearly demonstrated the aerodynamic benefits of vortex generators, including Ashill et al. [35] and Titchener and Babinsky [44].

While good performance has been achieved in some transonic cases, the disappointing results of Holden and Babinsky [37] and Bur et al. [38] are reminders that we have not conquered this problem completely. It is postulated that the separation in these two cases was just so small that no measurable gains could be achieved in these instances. It is also possible, however, that further losses due to three-dimensional effects were introduced in these instances.

The need for a sizeable region of separation postulated above could explain why such minimal success has been achieved in supersonic interactions, as supersonic SBLIs often produce small separations in comparison to transonic interactions (compare, for example, the size of the separated regions in Figures 8 and 10). The details of why this tends to be the case is beyond the scope of this article. The reader should refer to Reference [4] for details on the flow mechanisms in SBLIs.

6 Parametric studies

As there is such a wide variety of factors that affect vortex generator shock-induced separation mitigation performance, it is very difficult to make general statements in regard to preferential vortex generator designs. The optimal vortex generator configuration for a particular application will, of course, depend on many factors not considered herein. Nevertheless, this section attempts to establish some general design guidelines.

The separation mitigation capability is determined by factors that fall into one of two categories: Firstly, the vortex generator configuration, for example vortex generator height, shape, and positioning. These factors are relatively straightforward to vary in a fundamental study without altering the uncontrolled shock-induced separation scenario—the baseline for any comparison. The influence of these factors is therefore relatively easy to establish. The second group of factors encompasses flow characteristics such as the state of the boundary layer, Mach number, and Reynolds number preceding the vortex generators. These factors are directly related to the flow field geometry. These flow characteristics influence the vorticity production from the devices and its development downstream. Importantly, these variables also alter the nature of the shock-induced separation *ab initio*—with or without vortex generators. As a result, it is more difficult to establish the effect of these variables on vortex generator control effectiveness. As a consequence, this section only aims to establish overarching trends within the first group of variables based on the investigations listed in Table 1. Care should be taken when interpreting any trends, however, as the influence of the flow characteristics will inevitably be present.

Data on the vortex generator device variables height, shape, and streamwise positioning are presented in Figures 11, 12, and 13 respectively. In each figure, the investigations of Table 1 are shown on the abscissa and the variable of interest on the ordinate. The order in which the investigations are placed on the abscissa is based on what the authors perceive was the overall separation effectiveness of each study. In instances where little success was achieved the investigation is placed more to the left, while more successful investigations are placed further to the right. Success has been gauged using the available data, much of which has already been presented in the preceding sections. The static pressure rise has been used as the preferred performance metric, as changes in the wall pressure distribution due to vortex generator control are the best surrogate for reduced flow blockage, i.e., reduced flow separation. In the absence of wall pressure data, flow visualizations have been used, followed by pressure recovery, and finally downstream boundary layer profiles. For investigations in which an optimization was attempted by the authors, multiple data-points are included, with a filled-in marker to indicate the preferred height/shape/position. Note this is a largely subjective analysis and that blanks in Figures 11, 12, and 13 indicate instances where data was unavailable. It is also important to point out that our definition of success is based solely on separation mitigation and does not take into account device drag (as device drag is not often measured).

In Figure 11, non-dimensional vortex generator height is charted for the majority of investigations listed in Table 1. In a few cases where the boundary layer thickness was not documented it has been estimated using straightforward flat-plate boundary layer analysis. From Figure 11 it can be seen that a wide variety of heights have been utilized—anywhere from 0.2δ to 2.5δ. Although there is considerable scatter in the data, the more successful investigations, those of McCormick [27], Titchener and Babinsky [44], and Ashill et al. [35], tended to utilize relatively small vortex generators in the range 0.2δ to 0.5δ. Notwithstanding this, Mitchell [25] demonstrated that large vortex generator heights, $h/\delta = 1.2$, can also be successful. Nevertheless, the trend towards smaller vortex generators is consistent with the subsonic findings of Rao and Kariya [28] who concluded that smaller vortex generators can produce similar mixing levels without the high drag penalty of larger vortex generators. With the addition of wave-drag in transonic and supersonic flows one would expect there to be additional benefits to going to smaller vortex generators. However, there must be a limit below which the drop in vorticity production outweighs the gains of reduced drag and proximity to the wall. This limit remains unknown. This is partly because in the majority of the successful studies there was little variation in device height. In addition to this lower limit on device height the sensitivity to device height remains unknown. A gap remains for investigations which determine the optimum device height and sensitivity for vortex generator separation control.

The types of vortex generator employed by the numerous studies in Table 1 are plotted versus success at mitigating separation in Figure 12. In this instance, the ordinate spans shapes from the simplest vane-type to the complex Wheeler doublet, with hybrid shapes spanning the region in-between. Note that the naming convention for vortex generator shapes used here is taken from Ashill et al. [33]. Again, there is considerable scatter, and the judgement of control success remains somewhat subjective. However, a number of the more successful studies expressed a preference for geometries close (at least functionally close) to a traditional vane shape. For example, Pearcey [20], Titchener and Babinsky [44], Rybalko et al. [43], Holden and Babinsky [37], and Ashill et al. [35] all found either a swept or modified vane geometry gave the most favorable performance.

Figure 12 also illustrates that vane-type vortex generators have not been considered for the overwhelming majority of supersonic SBLIs. Instead, for many of these investigations, a ramp- or wedge-type vortex generator was utilized. This tendency to use ramp-type vortex generators under supersonic conditions is largely based on the computational work conducted by Anderson et al. [49] who established a liking for such a design—including its advantage in terms of robustness. In contrast, many investigations performed in the transonic regime have utilized both vane- and ramp-type vortex generators. Importantly, in instances where both vane- and ramp-type devices were examined—for example Holden and Babinsky [37] and Ashill et al. [35]—an aerodynamic preference for vane-type devices has been established. As a consequence, a comparison of vane-type and ramp-type vortex generators in a supersonic SBLI would be worthwhile to determine whether the lack of success at separation mitigation is due to the choice of ramp-type devices or whether it is a result of the differing flow physics, as previously discussed. The investigation of Gartling [26] suggests the lack of success is not due to the type of vortex generator employed, but further investigation is required to confirm this conclusion. The general preference for vane-type vortex generators in transonic flows is in contrast to the extensive subsonic vortex generator study by Ashill et al. [33]. In this investigation ramp-type vortex generator were found to be more effective than the vane-type in terms of vorticity production.

In general there is a preference in the investigations listed in Table 1 for counter-rotating vortex generator configurations. In such a configuration, it has been shown that the relative placement of common-flow-up and common-flow-down devices, D/d, is an important vortex generator parameter. This variable has not really been investigated in recent studies. In the authors' opinion, this variable requires further investigation. For example, it is possible that the inherently low D/d of ramp-type devices, which will result in premature lift-off of the devices' vortices, is responsible for their poor performance. Further data is required for confirmation. At this time, the best data on D/d available for transonic and supersonic flows is still that of Pearcey [20]. In addition to counter-rotating configurations, the novel bi-plane configuration examined only by Pearcey [20] also performed favorably. This type of configuration should also be investigated further.

The streamwise positioning of vortex generators with respect to the separation line for a number of the investigations in Table 1 is shown in Figure 13. The first observation that can be made based on this figure is the very wide range of streamwise positions over which vortex generators have been tested. Distances range from only a couple of vortex generator heights, $h/\delta = 2$, to more than one hundred device heights, $h/\delta = 100$. Examining Figure 13 it can be seen that success has been achieved with devices both close (Titchener and Babinsky [44]) and far from the separated region (Ashill et al. [35]). Again, due to a lack of parametric variation in streamwise positioning in the successful studies the precise effect of this parameter remains uncertain. In instances where optimisation of the streamwise position was attempted, a preference for distances non-dimensionalized on vortex generator height in the region $\Delta x/h = 15$–30 was established. Once again, further investigation is required to better establish preferred streamwise distance and the sensitivity to this variable.

In the above discussion there has been no real mention of device drag. This is because this quantity is not often measured. When implementing vortex generator control, however, this loss must be accounted for. It is felt that fundamental studies on vortex generator device drag would be highly beneficial to this area.

7 Conclusions

This article collated a wide range of investigations describing vortex generator control of shock-induced separation. On the basis of this discussion the following conclusions can be made.

First and foremost, in a number of instances, the implementation of vortex generators has led to a reduction in boundary layer separation. This fact has been observed explicitly in fundamental experiments using surface flow visualizations. What is more, the separation suppression capabilities of vortex generators has also been demonstrated implicitly in applied settings. Noteworthy improvements in L/D have been achieved on a two-dimensional transonic wing and increased pressure recovery has been attained in a viable supersonic inlet. These results demonstrate that a net reduction in loss can be achieved, i.e., the reduction in dissipation due to flow separation outweighs the introduced device drag.

The effectiveness of vortex generators is especially evident at transonic conditions. In a number of transonic test cases shock-induced separation has been markedly reduced. The results in supersonic flow fields are in stark contrast to those achieved at transonic conditions. In the purely supersonic case, there has not been a single investigation, neither applied nor fundamental, that has demonstrated a resounding beneficial influence with vortex generators. It has been suggested in this review that this inability to achieve a marked benefit is in large part due to a lack of separation to mitigate initially. Many uncontrolled supersonic baseline flow fields tend to feature only small, and thus not very detrimental, separation. This minimizes the potential benefit of any control strategy. In both transonic and supersonic instances a partitioning of the separated region into more three-dimensional cells is sometimes observed. While the overall footprint of the separation is smaller, their extent may not be significantly different. On the basis of this review, some rough guidelines have been established on preferable vortex generator configurations in terms of size, shape, and positioning. The investigations presented within this review suggest that: counter-rotating configurations work particularly well as long as the device spacing $D/d \geq 4$; vortex generators with heights 0.2δ to 0.5δ are most effective; superior performance is achieved using vane-type vortex generator shapes compared to ramp/wedge-type vortex generators; and no substantiated preference has been ascertained for the streamwise positioning, but somewhere between 15–30 $\Delta x/h$ may be a good initial value.

More recent investigations have illustrated two main points. Firstly, the importance of three-dimensional effects in small-scale facilities and the inherent difficulty of reproducing conditions applicable to our desired applications has become apparent. Considerably more work is required, both experimental and computational, before these three-dimensional flow fields can be fully understood. In the meantime, it is important to recognize their existence and probable effects. Secondly, the need for flow separation over a large enough length to justify employing vortex generators has become obvious. It is natural to then ask what extent of flow separation is required to justify the application of vortex generators. This question cannot be definitively answered here, however, the authors suggest, based on the data herein, that a flow field with a separation extending $\geq 5 - 10\delta$ will more than likely profit from the introduction of vortex generators.

In short, the above results suggest that vortex generators are able to alleviate shock-induced separation, and in instances provide an overall aerodynamic benefit. In the authors' opinion the potential benefit of vortex generators is of great enough value that there is significant merit in continuing research in this area. The overall benefit of introducing vortex generators must, however, be calculated as part of a system-wide optimization.

Table 1 Studies utilizing VGs for shock separation control

Authors	Year	Type of Investigation	Wind tunnel	SBLI	M_∞	Re (m^{-1})
Lina and Reed [11]	1950	flight test	-	terminal shock on wing	≃1–1.4	7.8×10^6
Griggs [22]	1958	inlet study; canonical 2-shock axi. inlet	RAE no. 4 supersonic wind tunnels	terminal and ds. diffuser	2-3	12×10^6
Pearcey [20]	1961	fund. SBLI study	NPL supersonic facility	terminal shock on two-dimensional bump	1–1.4	-
Edwards [23]	1966	free-flight test	-	terminal shock on airfoil	1.35	31×10^6
Gartling [26]	1970	fund. SBLI study	University of Texas, Austin, Mach 5 facility	compression corner (35 ^{o})	4.67	47×10^6
Mitchell [25]	1971	inlet study; NASA 60/40 inlet	NASA John H. Glenn 10 ft. × 10 ft.	oblique reflection, terminal and ds diffuser	2.5	6.2×10^6
McCormick [27]	1993	fund. SBLI study	UTRC supersonic facility	normal with ds. divergence	1.56-1.65	15×10^6
Barter and Dolling [31]	1995	fund. SBLI study	University of Texas, Austin, Mach 5 facility	compression corner (28 ^{o})	5	52×10^6
Wasserbauer et al. [32]	1996	inlet study; two-dimensional bifurcated mixed compression	NASA John H. Glenn 10 ft. × 10 ft.	terminal and ds. diffuser	2.0–2.8	7.2×10^6
Ashill et al. [35]	2001	airfoil study	DERA 8 ft. high-speed facility	terminal shock on airfoil	0.67–0.71	30×10^6
Holden and Babinsky [37]	2007	fund. SBLI study	CUED SST	normal straight-channel	1.5	28×10^6
Babinsky et al. [47]	2009	fund. SBLI study	CUED SST	oblique reflection (7 ^{o})	2.5	40×10^6
Blinde et al.[51]	2009	fund. SBLI study	Delft University TST-27 trans-supersonic facility	oblique reflection (10 ^{o})	1.84	37×10^6
Bur et al. [38]	2009	fund. SBLI study	ONERA Meudon Center S8Ch	terminal shock on two-dimensional bump	1.45	14×10^6
Zare Shahneh and Motallebi [39]	2009	fund. SBLI study	Queen Mary high-speed facility	normal with ds. divergence	1.4	16×10^6
Herges et al.[52]	2010	fund. SBLI study	University of Illinois supersonic wind tunnel	normal with ds. divergence	1.4	30×10^6

Authors	Year	Type of Investigation	Wind tunnel	SBLI	M_∞	Re (m^{-1})
Lee et al.[50]	2010	fund. SBLI study	TGF Wright-Patt.	oblique reflection (8 ^{o})	2.98	1.8×10^6
Titchener and Babinsky [42]	2011	fund. SBLI study	CUED SST	terminal shock and ds. diffuser	1.4	31×10^6
Rybalko et al. [43]	2012	fund. SBLI study	CUED SST	normal shock followed by ds. diffuser	1.4	26×10^6
Verma et al. [46]	2012	fund. SBLI study	NAL trisonic blow-down facility	compression corner (24 ^{o})	2	25.3×10^6
Herges et al. [53]	2012	inlet study; Gulfstream inlet	NASA John H. Glenn 8 ft. × 6 ft.	terminal with ds. diffuser	1.7-1.8	15.7×10^6
Titchener and Babinsky [44]	2013	fund. SBLI study	CUED SST	terminal shock and ds. diffuser	1.4	31×10^6

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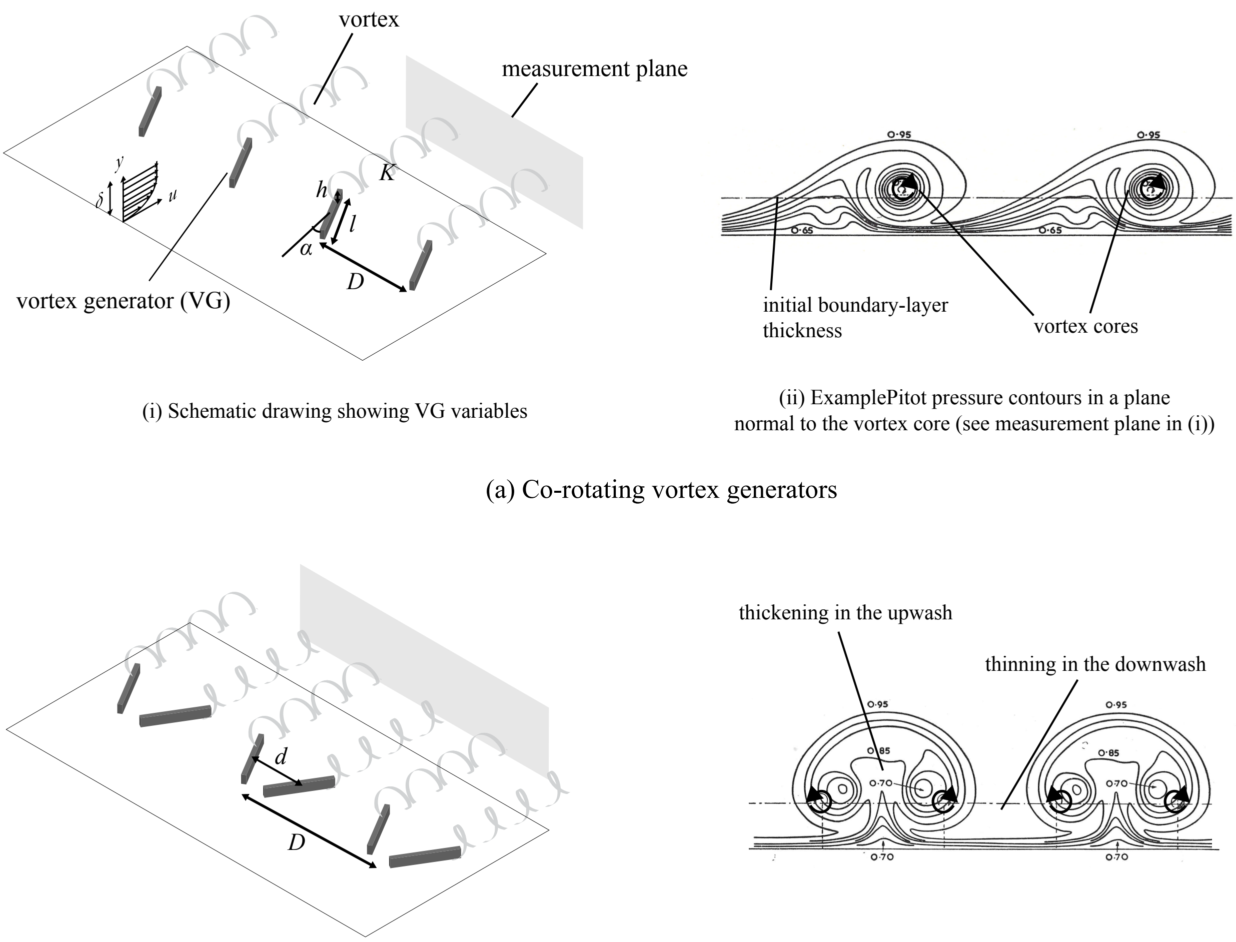


(i) Schematic drawing showing VG variables

(ii) ExamplePitot pressure contours in a plane normal to the vortex core (see measurement plane in (i))

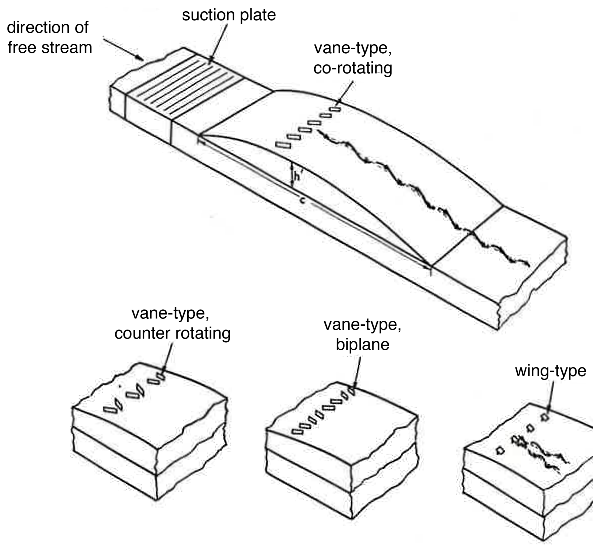
(a) Co-rotating vortex generators

(i) Schematic drawing showing VG variables

(ii) Example Pitot pressure contours in a plane normal to the vortex core (see measurement plane in (i))

(b) Counter-rotating vortex generators

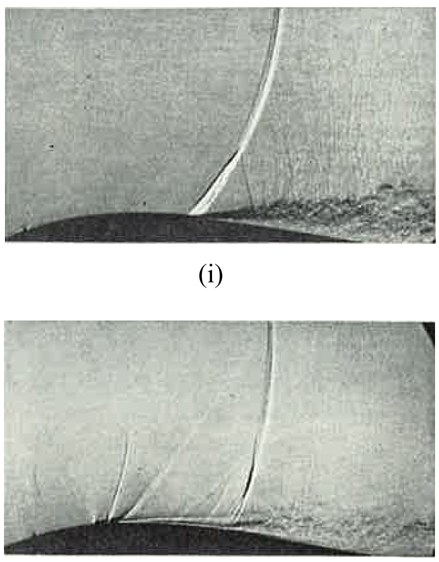
Fig. 1 Diagramatic representation of simple vane-type vortex generators. The variables available in the co-rotating vortex generators design space and shown in (a)(i) with the additional variables available in the counter-rotating setup shown in (b)(i). Example Pitot pressure contours for co-rotating and counter-rotating systems, reproduced from pages 1282 and 1287 of Pearcey, Boundary Layer and Flow Control: Its Principles and Applications, Vol. 2, part IV, 1961 [20] are shown in (a)(ii) and (b)(ii). Further details of this experimental investigation are given in Table 1



(a) Experimental setup showing the various vortex generator configurations employed

	h/c	D/h	D/d	l/h	α (deg)
Co-rotating					
C1	0.01	4	-	4	20
C2	0.01	8	-	4	20
C3	0.02	4	-	4	25
C4	0.012	5	-	4	25
C5	0.01	4	-	triangular	12
Counter-rotating					
M1	0.01	5	2	2.5	15
M2	0.01	10	4	2.5	15
M3	0.01	5	4	1.25	15
M4	0.01	6.3	2.7	3.6	15
M5	0.004	6.3	2.7	3.6	15
Bi-plane					
B1	0.01	11.25	4.5	2.5	15 , 20
B2	0.01	10	4	2.5	15
B3	0.01	5	4	1.6	15
Tandem row					
T1 row 1	0.02	8	4	wing type (square)	15
T1 row 2	0.01	16	4	4	20

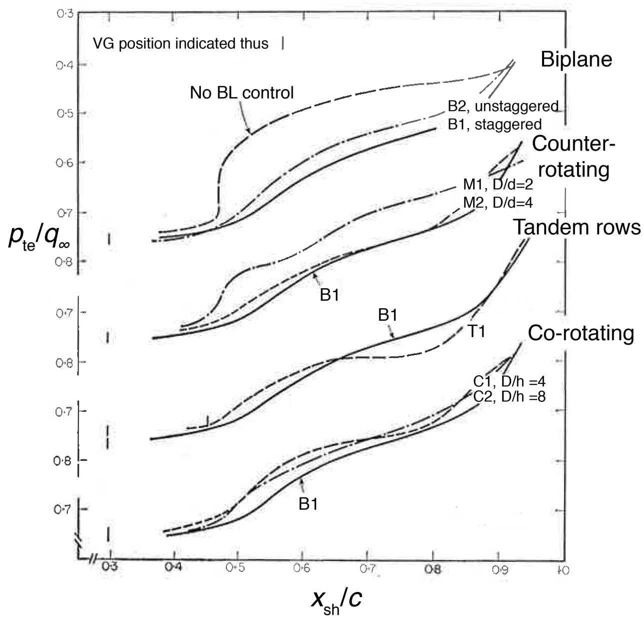
(b) Vortex Generator configurations



(i)

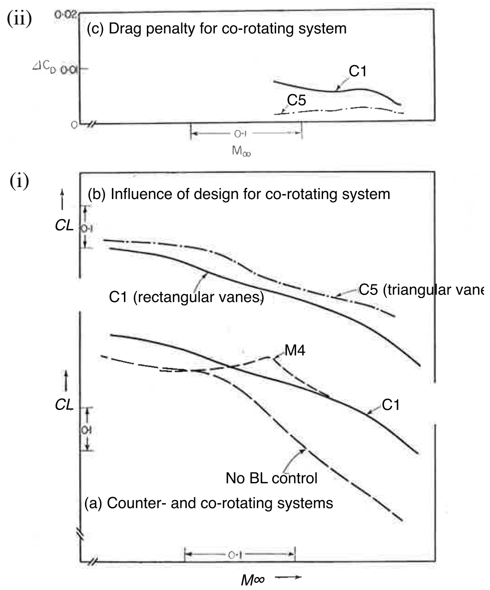
(ii)

(c) Schlieren images (i) without and (ii) with vortex generators

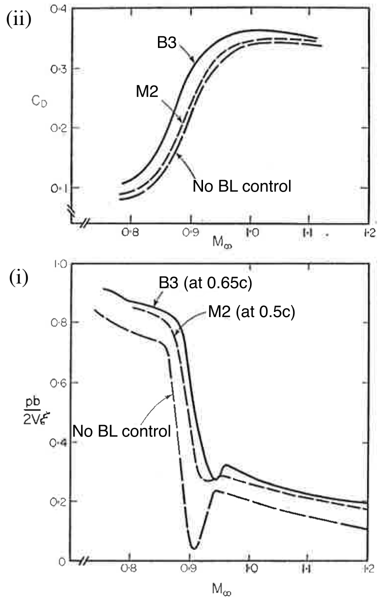


(d) Trailing-edge pressure versus shock position for various vortex generator configurations

Fig. 2 A summary of the salient results of Pearcey [20]. Data reproduced from pages 1272, 1303, 1275, and 1304 respectively of Pearcey, Boundary Layer and Flow Control: Its Principles and Applications, Vol. 2, part IV, 1961 [20]. Further details of this experimental investigation are given in Table 1



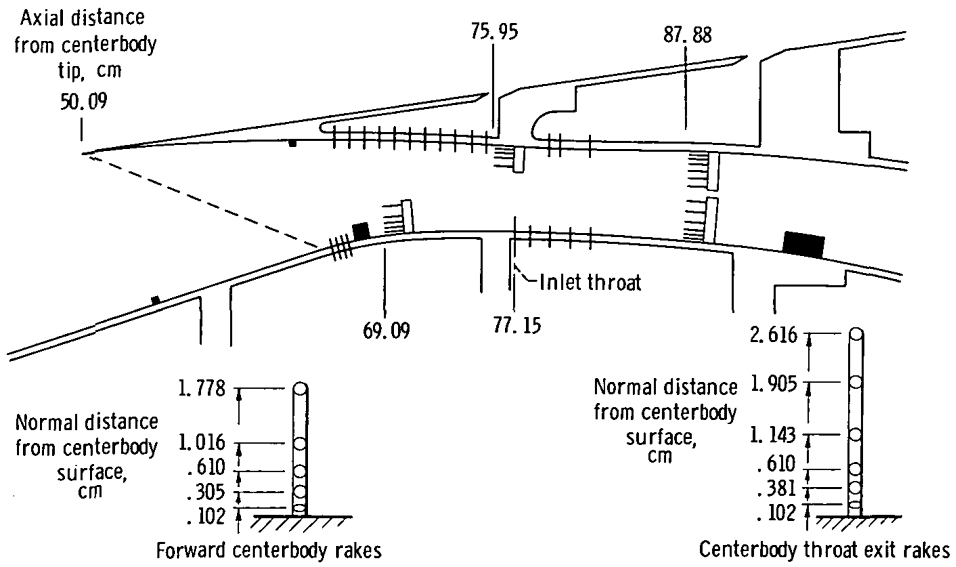
(a) Effect of vortex generators on a flight-determined buffet boundary and drag from the investigation of Gould [24]



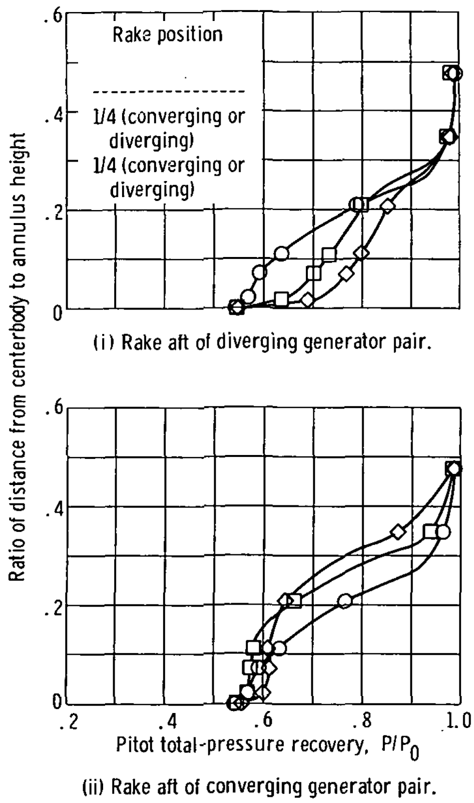
(b) Free-flight tests on drag and roll performance from the investigation of Edwards [23]

(i)

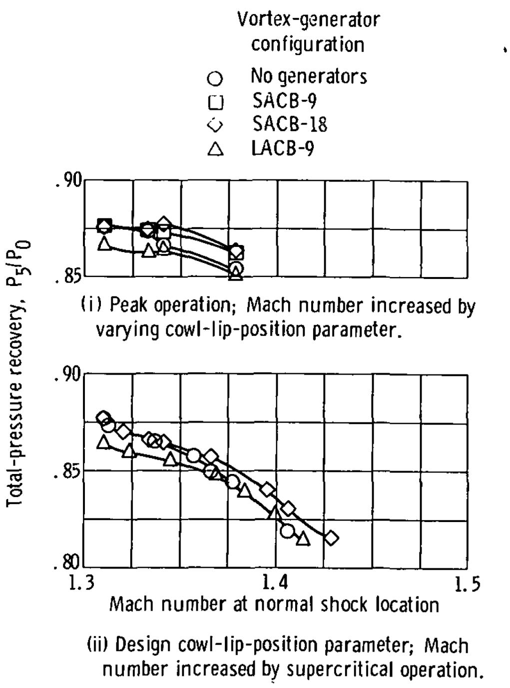
Fig. 3 Flight-test (a) and free-flight (b) data with and without vortex generators. Data reproduced from pages 1308 and 1309 respectively, from Pearcey, Boundary Layer and Flow Control: Its Principles and Applications, Vol. 2, part IV, 1961 [20]. Further details of this experimental investigation are given in Table 1



(a) Schematic representation of the inlet and rake instrumentation

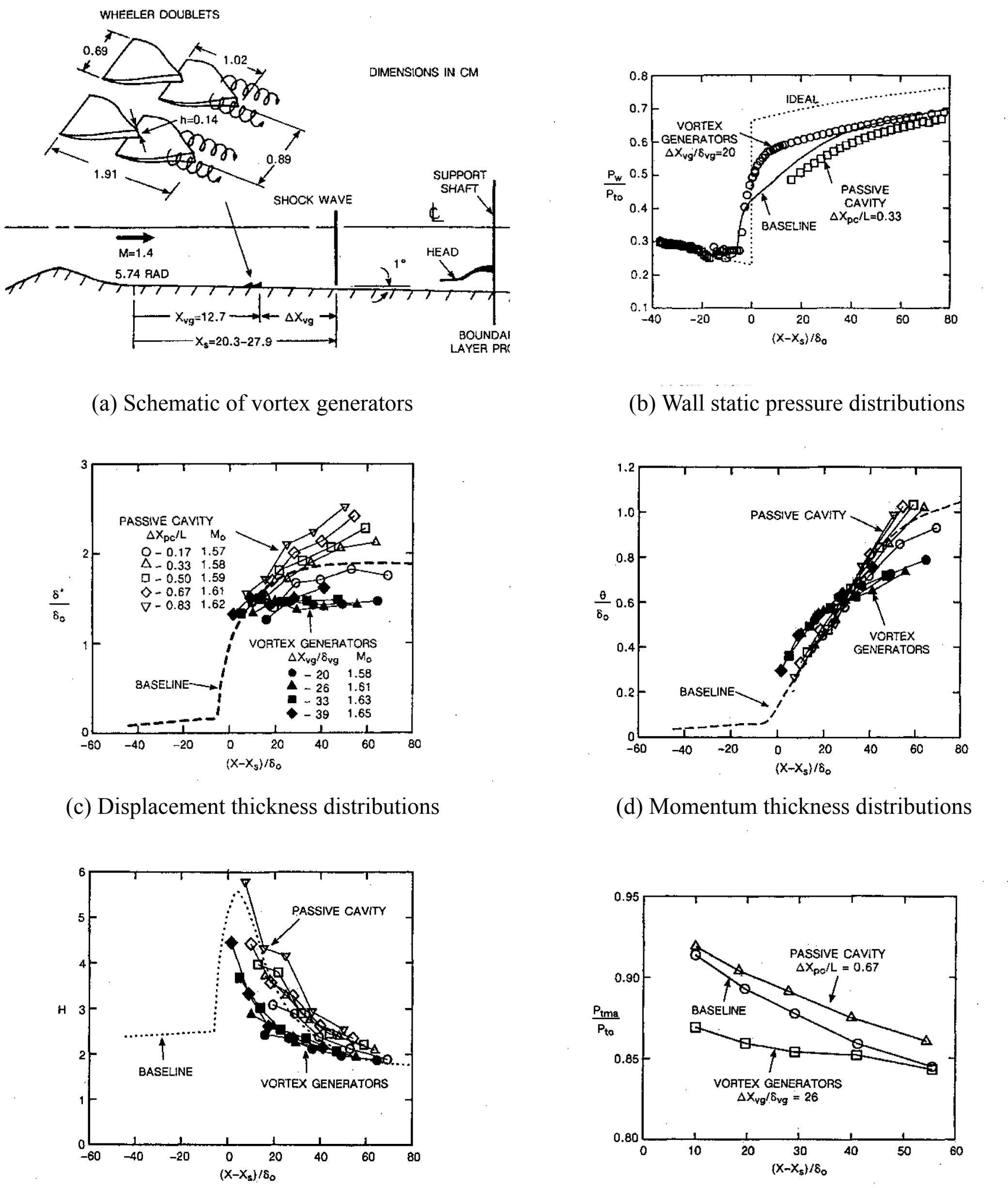


(b) Performance of aft centerbody VG configurations at peak recovery conditions, as measured by throat exit rakes



(c) Effect of aft centerbody VG configurations on compressor-face total-pressure recovery

Fig. 4 A summary of the salient pressure data obtained by Mitchell [25]. Figure reprinted from Mitchell, Experimental Investigation of the Performance of Vortex Generators Mounted in the Supersonic Portion of a Mixed-Compression Inlet, NASA TM X-2405, 1971 [25]. Courtesy of NASA. Further details of this experimental investigation are given in Table 1



(a) Schematic of vortex generators

(b) Wall static pressure distributions

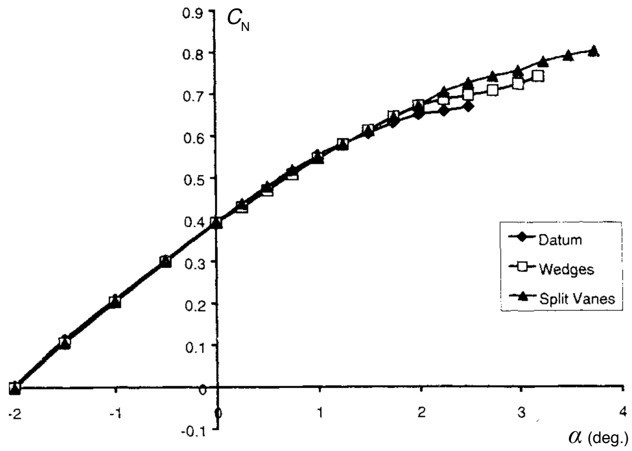
(c) Displacement thickness distributions

(d) Momentum thickness distributions

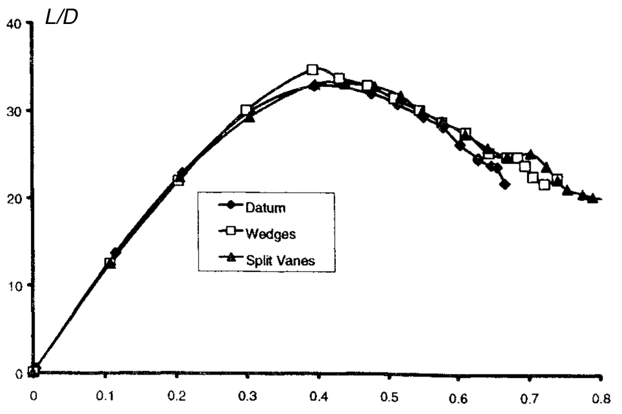
(e) Shape factor distributions

(f) Mass-averaged total pressure distributions

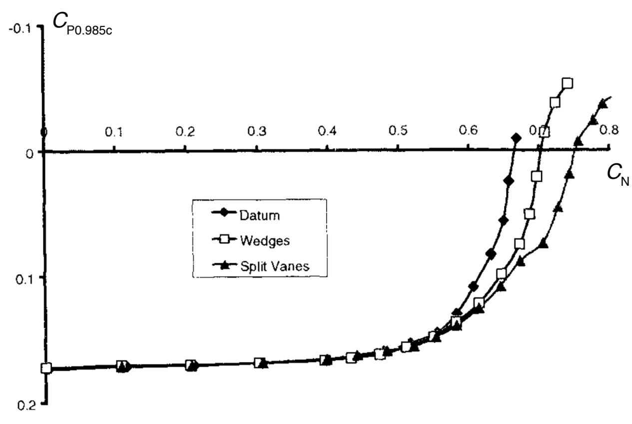
Fig. 5 Measurements of the influence of vortex generators through a terminal SBLI by McCormick [27]. Figures reprinted from McCormick, Shock/boundary layer Interaction Control with Vortex Generators and Passive Cavity AIAA Journal, Vol. 31, no. 1, 91–96, 1992 [27]. Reprinted with the permission of United Technologies Corporation. Further details of this experimental investigation are given in Table 1



(a) Normal force versus incidence

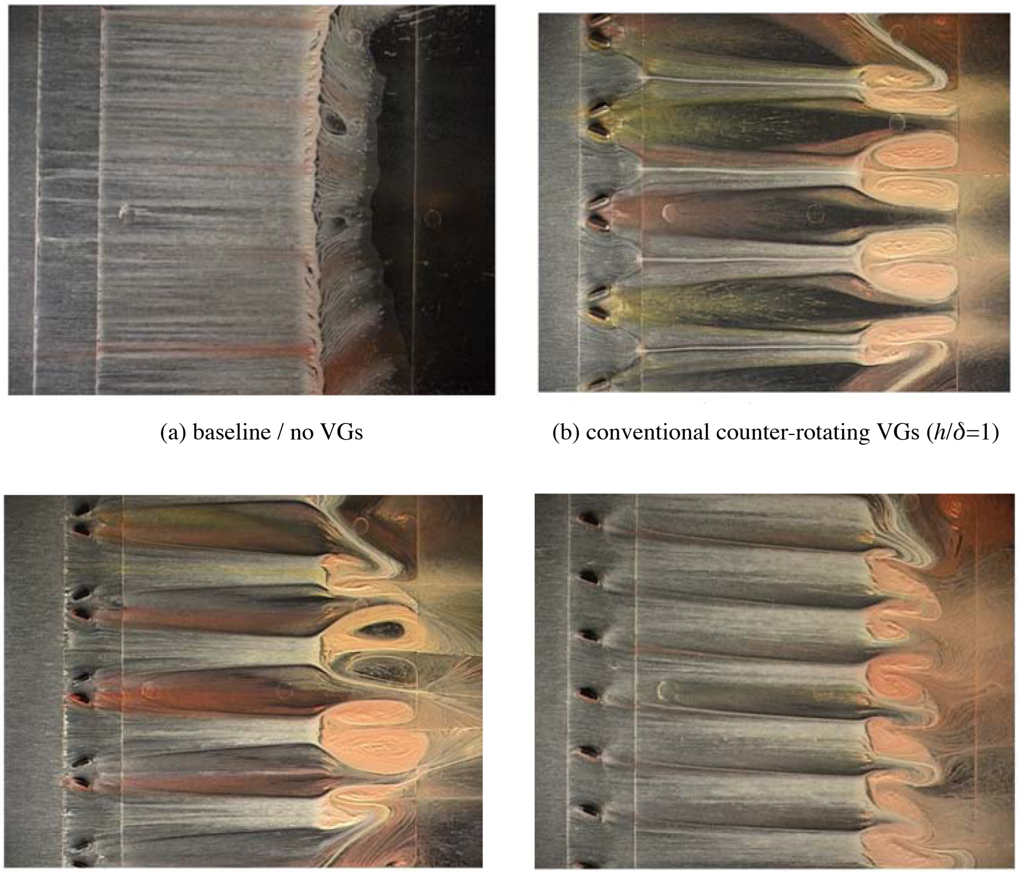


(b) Lift-to-drag ratio versus lift coefficient



(c) Pressure coefficient near the trailing edge versus normal force

Fig. 6 Pressure measurements on a RAE 5243 transonic wing by Ashill et al. [35] with and without vortex generators. Figure reproduced from Ashill et al., Research at DERA on Sub Boundary Layer Vortex Generators (SBVGs), AIAA 2001-0887, 2001 [35]. Reprinted with permission of Her Majesty's Stationary Office (HMSO). Further details of this experimental investigation are given in Table 1



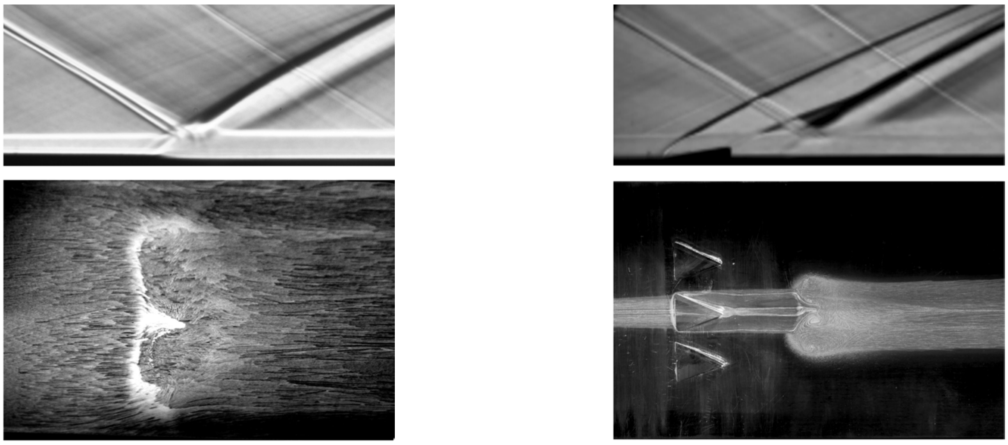
(a) baseline / no VGs

(b) conventional counter-rotating VGs (h/δ=1)

(c) counter-rotating VGs (h/δ=0.5)

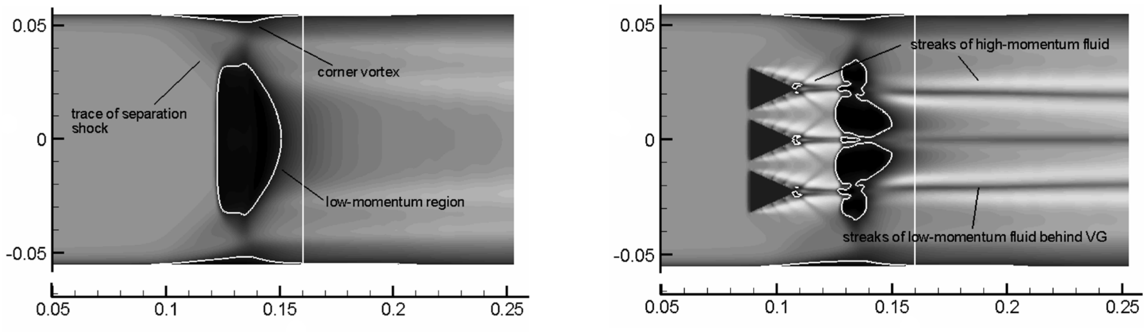
(d) co-rotating VGs (h/δ=0.5)

Fig. 7 Oil-flow visualizations from the terminal SBLI investigation of Bur et al. [38]. Reprinted with permission from Bur et al., Separation control by vortex generator devices in a transonic channel flow, Shock Waves, Vol. 19, 521–530, 2009 [38]. Further details of this experimental investigation are given in Table 1



(a) Schlieren (above) and oil-flow visualization (below) without VGs

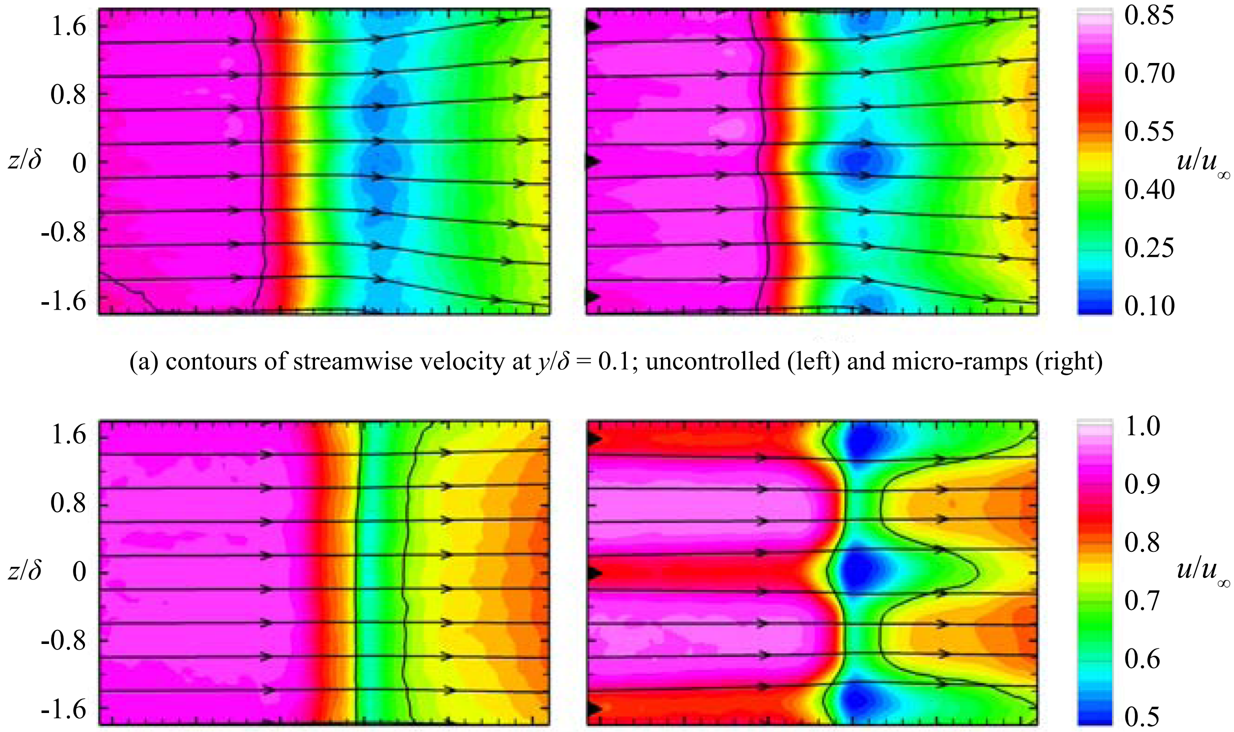
(b) Schlieren (above) and oil-flow visualization (below) with VGs



(c) Computed near-surface velocity contours without VGs (range: -5 ms^{-1} (dark) to 35 ms^{-1} (light))

(d) Computed near-surface velocity contours with VGs (range: -5 ms^{-1} (dark) to 35 ms^{-1} (light))

Fig. 8 Flow-visualizations from the reflected oblique SBLI investigation of Babinsky et al. [47] plus corresponding computations of Ghosh et al. [48]. Figures reproduced from Babinksy et al., Microramp Control of Supersonic Oblique Shock-Wave/boundary layer Interactions, AIAA Journal, Vol. 47, no. 3, 668–675, 2009 [47] ((a) and (b)) and Ghosh et al., Numerical Simulations of Effects of Micro Vortex Generators Using Immersed-Boundary Methods, AIAA Journal, Vol. 48, no. 1, 92–103, 2010 [48] ((c) and (d)) with permission from AIAA and Jack Edwards respectively. Further details of this experimental investigation are given in Table 1



(a) contours of streamwise velocity at $y/\delta = 0.1$; uncontrolled (left) and micro-ramps (right)

(b) contours of streamwise velocity at $y/\delta = 0.6$; uncontrolled (left) and micro-ramps (right)

Fig. 9 Streamwise Particle Image Velocimetry (PIV) measurements of a reflected oblique SBLI by Blinde et al. [51]. Figure reproduced with permission from Blinde et al., Effects of micro-ramps on a shock wave/turbulent boundary layer interaction, Shock Waves, Vol. 19, 507–520, 2009 [51]. Further details of this experimental investigation are given in Table 1

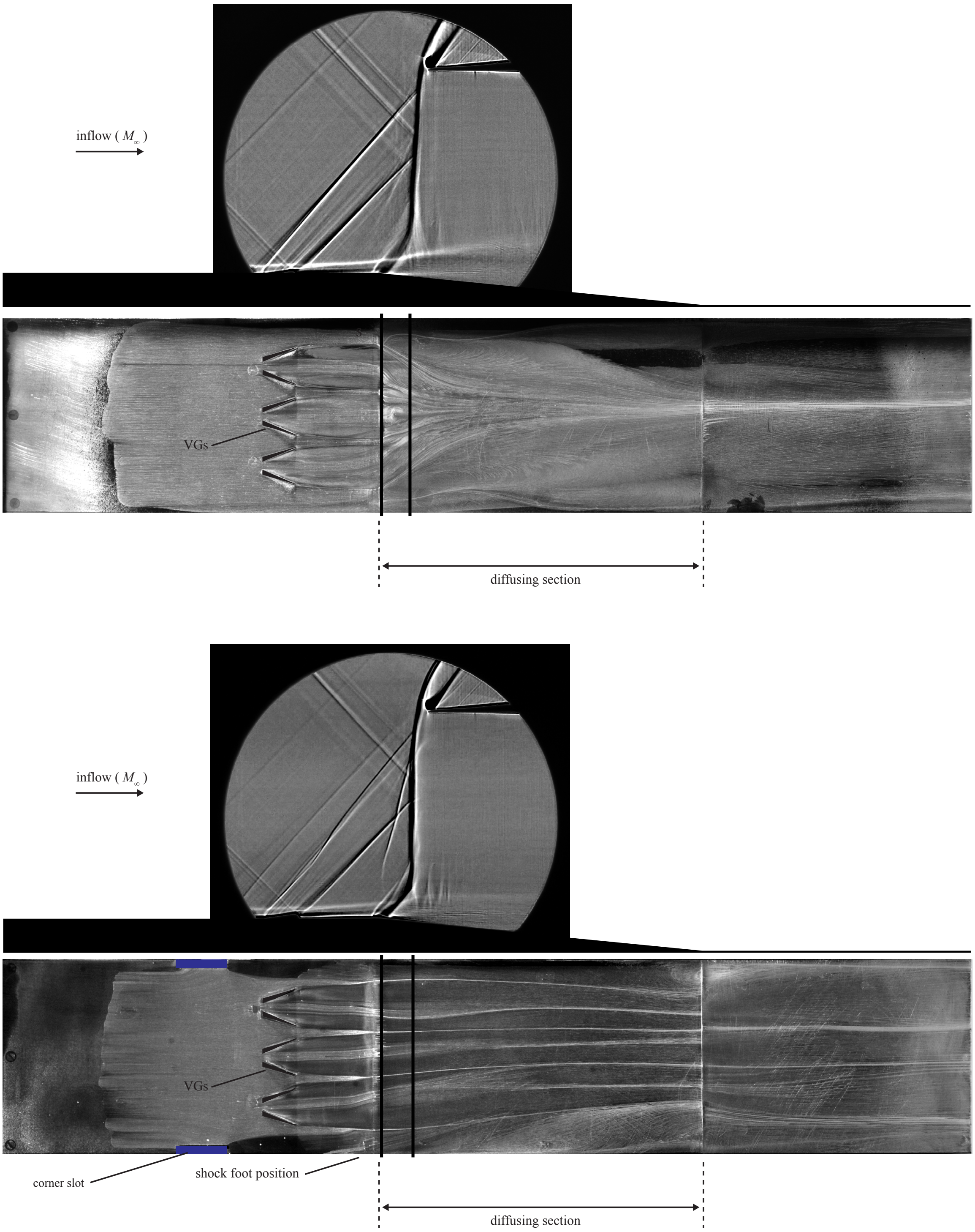


Fig. 10 Schlieren and oil-flow visualizations with vortex generators by Titchener and Babinsky [45]. Figure adapted from Titchener and Babinsky, Shock Wave/boundary layer Interaction Control Using a Combination of Vortex Generators and Bleed, AIAA Journal, Vol. 51, No. 5 (2013), pp. 1221-1233 [44]. Figure also available in Titchener, N. A., PhD thesis, 2013, University of Cambridge [45]. Further details of this experimental investigation are given in Table 1

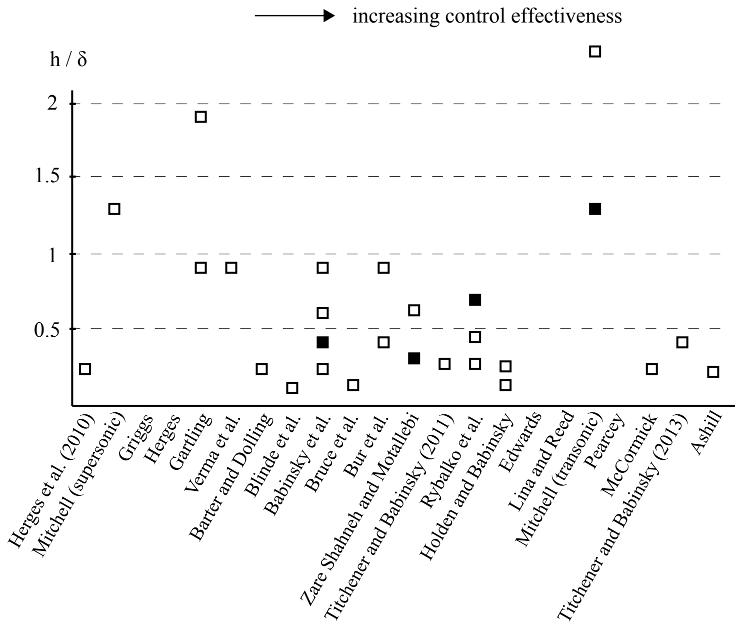


Fig. 11 Vortex generator heights relative to boundary layer thickness examined by the authors in Table 1 (filled-in symbols indicate instances where optimization was attempted)

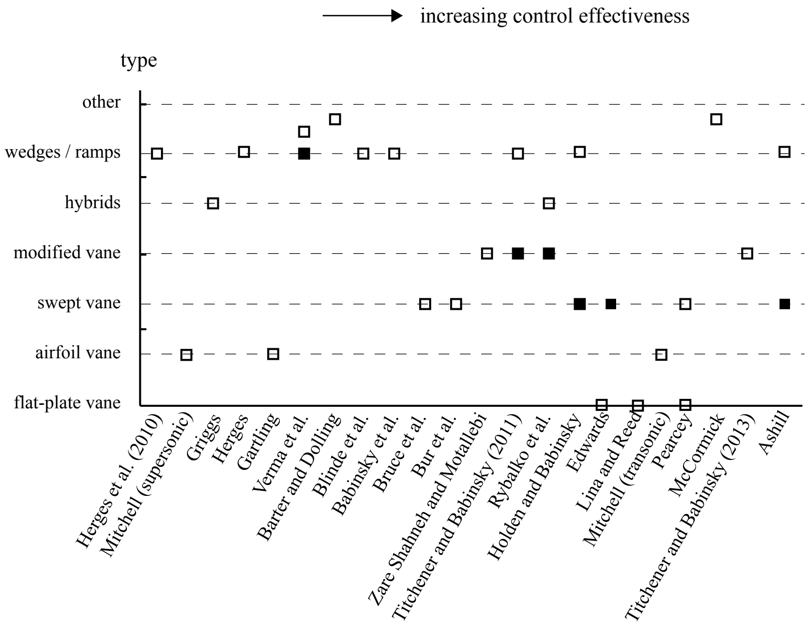


Fig. 12 Vortex generator types examined by the authors in Table 1 (filled-in symbols indicate instances where optimization was attempted)

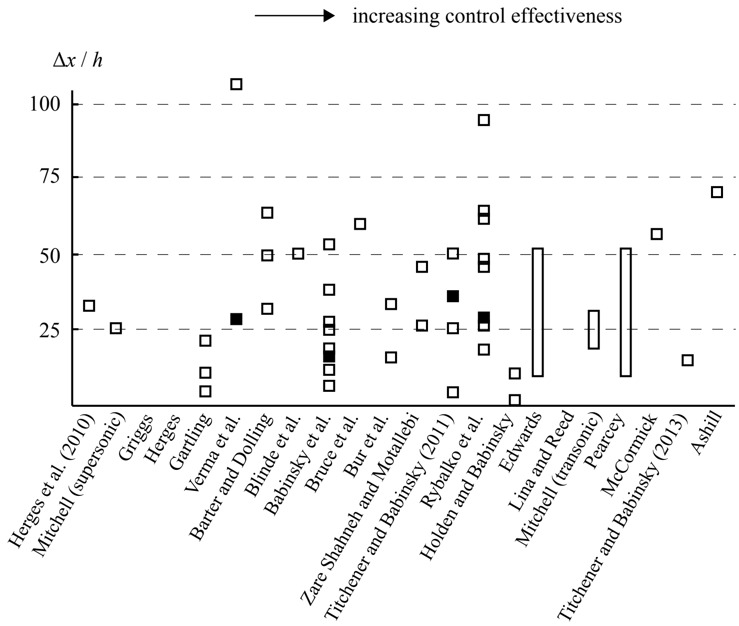


Fig. 13 Vortex generator positioning in terms of number of heights upstream of separation examined by the authors in Table 1 (filled-in symbols indicate instances where optimization was attempted)