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# Scalar implicatures of embedded disjunction

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**Abstract** Sentences with disjunction in the scope of a universal quantifier,
*Every A is P or Q*, tend to give rise to distributive inferences that each of
the disjuncts holds of at least one individual in the domain of the quantifier,
*Some A is P & Some A is Q*. These inferences are standardly derived as an
entailment of the meaning of the sentence together with the scalar implica-
ture that it is not the case that either disjunct holds of every individual in
the domain of the quantifier, ¬*Every A is P & ¬Every A is Q* (plain negated
inferences). As we show, this derivation faces a challenge in that distributive
inferences may obtain in the absence of plain negated inferences. We address
this challenge by showing that on particular assumptions about alternatives, a
derivation of distributive inferences as scalar implicatures can be maintained
without in fact necessitating plain negated inferences. These assumptions ac-
cord naturally with the grammatical approach to scalar implicatures. We also
present experimental data that suggest that plain negated inferences are not
only unnecessary for deriving distributive inferences, but might in fact be un-
available.

## 1 Distributive inferences

Disjunction in the scope of a universal quantifier tends to give rise to existen-
tial inferences pertaining to each of the disjuncts, specifically, the inferences

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that each of the disjuncts holds of at least one individual in the domain of
the universal quantifier (we will henceforth refer to these as *distributive infer-
ences*). This observation can make sense of the fact that a sentence like (1)
is perceived as infelicitous in a context in which all of the speaker's brothers
are married to a woman (and none are known by the speaker to be married
to a man) – that is, in a context in which the distributive inferences of the
sentence, given in (2), are false.[1]

(1)     Every brother of mine is married to a woman or a man.


(2)     Distributive inferences:
        a.   Some brother of mine is married to a woman.
        b.   Some brother of mine is married to a man.

   Distributive inferences are standardly characterized and derived as scalar
implicatures (henceforth, SIs), not least because they bear the telltale sign of
SIs – they disappear in downward-entailing environments. For example, if we
embed the sentence in (1) under a downward-entailing operator, say, under the
predicate *doubt*, as in (3), its distributive inferences disappear: the sentence in
(3) entails that John doubts that every brother of mine is married, which is
a stronger meaning than would obtain if the distributive inferences were part
of the meaning of the embedded clause – that is, that John doubts that every
brother of mine is married and that some of them are married to a woman,
while others are married to a man.

(3)     John doubts that every brother of mine is married to a woman or a
        man.

   There are two types of approaches to SIs, both of which can accommo-
date distributive inferences: the pragmatic approach, an instance of which is
the neo-Gricean approach advocated by, for example, Sauerland (2004), and
the grammatical approach advocated by, for example, Chierchia et al. (2011).
Both types of approaches take the SIs of a sentence to be a product of an
exhaustification of the sentence relative to a constrained set of alternatives
induced by the sentence. The approaches agree that to properly understand
this computation, a general theory of alternatives is needed that assigns to
each expression an appropriate set of alternatives (see, e.g., Sauerland 2004,
Fox 2007, Katzir 2007). Moreover, they agree that these sets of alternatives
satisfy the following condition (in addition to the abovementioned authors, see

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[1] If in the context the speaker is taken to be opinionated about the alternatives induced
by a sentence that she utters and if the alternatives are taken to be relevant, it is generally
the case that an SI of the sentence based on the alternatives will be computed. Although a
subsequent cancellation of the SI may be possible, which may require a re-analysis of what
is relevant in the context (see Mayol and Castroviejo 2013 on conditions on SI cancellation),
the sentence is perceived to convey false information in the context if the SI is false (cf.
Gazdar 1979, Horn 1984, Levinson 2000, among others).

also Rooth 1992, Kratzer and Shimoyama 2002 for variants of this assumption and a more detailed discussion):

(4) *Standard assumption about alternatives*
A constituent, $\alpha = [\beta \ \gamma]$, has as its alternatives its subconstituents and the pointwise combinations of the alternatives to its subconstituents, $\text{ALT}(\alpha) = \{\alpha' \mid \exists \beta', \gamma' \colon \beta' \in \text{ALT}(\beta) \wedge \gamma' \in \text{ALT}(\gamma) \wedge (\alpha' = [\beta' \ \gamma'] \vee \alpha' = \beta \vee \alpha' = \gamma)\}$.

On the standard assumption about alternatives, a disjunctive constituent has as alternatives each of the disjuncts, a conjunctive alternative in which the disjunctive connective is replaced by the conjunctive one, and the alternatives induced by each of the disjuncts and their disjunctions and conjunctions (these latter alternatives are irrelevant for the purposes of this paper and will be ignored in the following; see Sauerland 2004, Fox 2007, Katzir 2007 for discussion). Formally:

(5) *Standard assumption about alternatives of disjunction*
A disjunctive constituent, $\alpha \ or \ \beta$, has as its alternatives the disjuncts as well as their conjunction, $\text{ALT}(\alpha \ or \ \beta) = \{\alpha \ or \ \beta, \ \alpha, \ \beta, \ \alpha \ and \ \beta\}$.

The two types of approaches to SIs differ with respect to the nature of exhaustification: on the pragmatic approaches the exhaustification involves pragmatic reasoning by conversational agents, while on the grammatical approach it takes place in grammar. Nonetheless, at first glance, distributive inferences emerge on both types of approaches in a similar way – through exhaustification of the matrix sentence, either by pragmatic reasoning or in grammar.

## 1.1 Distributive inferences on the neo-Gricean approach to SIs

On the neo-Gricean approach to SIs, SIs are derived by reasoning about speakers' mental states on the basis of two principles, a version of the Gricean Maxim of Quantity and the assumption of opinionatedness (see, e.g., Sauerland 2004 for a detailed exposition). Specifically, upon hearing an utterance of the sentence (1), the hearer is assumed to reason as follows: The sentence has the alternatives in (6) – the standard alternatives to the disjunctive constituent combined pointwise with the universal quantifier (we will henceforth refer to these as *plain alternatives*). (The sentence itself is an alternative as well, but we ignore it for reasons of brevity.)

(6) ALT(Every brother of mine is married to a woman or a man) =
{ Every brother of mine is married to a woman,
Every brother of mine is married to a man,
Every brother of mine is married to a woman and a man }

Since these alternatives are *ex hypothesi* relevant, stronger than the uttered sentence, and the speaker has not used them, we are licensed by the Maxim of Quantity to conclude that it is not the case that she believes any of them.

(7)    a.    Every brother of mine is married to a woman or a man.
       b.    ⇝ $\neg B_{\text{speaker}}$(Every brother of mine is married to a woman)
       c.    ⇝ $\neg B_{\text{speaker}}$(Every brother of mine is married to a man)

Furthermore, given that it holds according to the assumption of opinionatedness that for each of the alternatives in (6) the speaker either believes that it is true or that it is false, the hearer is licensed to conclude from (7) that the speaker believes that all the alternatives are false. This then yields the SIs of the sentence:

(8)    a.    Every brother of mine is married to a woman or a man.
       b.    ⇝ $B_{\text{speaker}}$(¬Every brother of mine is married to a woman)
       c.    ⇝ $B_{\text{speaker}}$(¬Every brother of mine is married to a man)

For ease of exposition, we will refer to these inferences – that is, inferences that correspond to the negation of the plain alternatives of a sentence – as *plain negated inferences*.

(9)    Plain negated inferences:
       a.    ¬Every brother of mine is married to a woman.
       b.    ¬Every brother of mine is married to a man.

Distributive inferences follow from the meaning of the sentence together with its plain negated inferences: if (I, the speaker, believe that) every brother of mine is married and not every one of them is married to a woman and not every one of them is married to a man, then (I, the speaker, believe that) some brother of mine is married to a woman and some brother of mine is married to a man.

## 1.2 Distributive inferences on the grammatical approach to SIs

On the grammatical approach to SIs, there is an exhaustification device, *exh*, in grammar that is akin to *only* and is responsible for generating SIs. Following much preceding work (e.g., Fox 2007, Chierchia et al. 2011), we represent *exh* as a clausal operator that takes two arguments: a set of relevant alternatives to the clause to which *exh* is adjoined (the domain of *exh*) and the meaning of the clause (the prejacent of *exh*). On this approach, the sentence in (1) may have a representation with a matrix exhaustification operator that operates on the set of plain alternatives described in (6).

(10)    a.    Every brother of mine is married to a woman or a man.
        b.    exh(C)(Every brother of mine is married to a woman or a man)
        c.    C = ALT(Every brother of mine is married to a woman or a man)

The import of the exhaustification operator is to convey that its prejacent is true but that the appropriately excludable relevant alternatives are false:

(11)     $\text{exh}(C)(p) = \lambda w.\ p(w) \wedge \forall q \in \text{Excl}(C,p)(\neg q(w))$

An alternative is thereby appropriately excludable, given a set of alternatives and the prejacent of *exh*, if it is in all the maximal sets of alternatives whose negation is jointly consistent with the prejacent, as per (12) (see Fox 2007).

(12)     *Definition of excludable alternatives* (as presented in Magri 2009)
   a.    $X = \{q_1, ..., q_n\} \subseteq C$ is a set of (jointly) negatable alternatives given C and p iff $p \wedge \neg q_1 \wedge ... \wedge \neg q_n \neq \bot$.
   b.    $X \subseteq C$ is a maximal set of (jointly) negatable alternatives given C and p iff there is no X' such that $X \subset X'$ and X' is a set of (jointly) negatable alternatives given C and p.
   c.    $\text{Excl}(C,p)$, the set of excludable alternatives given C and p, is the intersection of all maximal sets of (jointly) negatable alternatives given C and p.

In the case of (10), all the alternatives in the domain of the exhaustification operator are excludable since the conjunction of their negations with the prejacent is consistent. Accordingly, the output of the exhaustification is the conjunction of the prejacent and the plain negated inferences:

(13)     Every brother of mine is married to a woman or a man $\wedge$
         $\neg$Every brother of mine is married to a woman $\wedge$
         $\neg$Every brother of mine is married to a man

Overall the result is the same as in the neo-Gricean approach described above: distributive inferences are derived from the conjunction of the prejacent and the plain negated inferences.[2]


1.3 Summary

Distributive inferences can be derived in a closely related way in the pragmatic and the grammatical approaches to SIs – through exhaustification of the matrix sentence, either by abductive reasoning about speakers' mental states or by an application of a grammatical exhaustification device, respectively. On both types of derivations, if the alternatives to the matrix sentence are the plain alternatives – that is, the standard alternatives of disjunction combined pointwise with the universal quantifier, as exemplified in (6) – distributive inferences emerge as entailments of the sentence combined with plain negated inferences. This is summarized in (14).

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[2] In the grammatical approach a parse without exhaustification would be implausible, since it would lead to the pragmatic inference that the speaker is not opinionated about the relevant alternatives (Fox 2007, 2013). See previous footnote.

(14)    *Exhaustification based on plain alternatives*
        For any sentence *Every A is P or Q*, if matrix exhaustification operates
        on its plain alternatives (*Every A is P, Every A is Q*), the distributive
        inferences (*Some A is P, Some A is Q*) are derived from the negation
        of the plain alternatives (¬*Every A is P*, ¬*Every A is Q*).


## 2 A puzzle about distributive inferences

In parallel to our discussion of the example in (1), we note that the slightly
modified variant of it in (15), which uses the present perfect, is also perceived
as infelicitous in a context in which all my brothers have been married to a
woman, but none of them have ever been married to a man.

(15)    [Every brother of mine has been married to a woman and none of
        them have been married to a man:]
        #Every brother of mine has been married to a woman or a man.

This is as expected in light of our above discussion: since the sentence in
(15) gives *ceteris paribus* rise to plain negated inferences and, consequently,
distributive inferences, a clash with the described context ensues – both con-
junction of the prejacent and plain negated inferences, given in (16), as well
as the distributive inferences, given in (17), are incompatible with the sup-
position that none of my brothers have ever been married to a man, which
explains the perceived infelicity of the sentence in the context. So far, so good.

(16)    Plain negated inferences of (15):
        a.    ¬Every brother of mine has been married to a woman.
        b.    ¬Every brother of mine has been married to a man.



(17)    Distributive inferences of (15):
        a.    Some brother of mine has been married to a woman.
        b.    Some brother of mine has been married to a man.

Strikingly, the felicity of sentence (15) improves markedly in a context in
which, say, the speaker has three brothers, Adam, Bob, and Carl, who got
married in college to Ann, Beth, and Christine, respectively; at some point
Adam and Ann got divorced and Adam married Arthur. That is, the felicity
of the sentence improves markedly in a context in which all of my brothers
have been married to a woman *and at least one of them has also been married
to a man*.[3]

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[3]   The minimal difference between the sentence in (1) and the sentence in (15)/(18) is that
in the former it is (contextually) impossible that both disjuncts hold of a brother of mine:
a brother of mine *being* married to a woman contextually entails him not being married to
a man (you can only be married to one individual at a given time). Accordingly, we get a
crisp judgment that the sentence is marked in any context in which, say, all of my brothers

(18)    [Every brother of mine has been married to a woman and some of
        them have been married to a man:]
        Every brother of mine has been married to a woman or a man.

The distributive inferences that the sentence in (18) gives rise to, spelled
out in (17), are compatible with the described context and they are in line with
the perceived felicity of the sentence. However, the inferences in (16), which
we have seen to be a necessary ingredient in the derivation of distributive
inferences on approaches that assume that matrix exhaustification operates
on plain alternatives, are incompatible with the described context, namely, all
of my brothers having been married to a woman. The distributional pattern
of distributive and plain negated inferences described in this section is thus
problematic for approaches that derive distributive inferences by relying on
matrix exhaustification that operates on plain alternatives – the contrast in
the felicity between (15) and (18) suggests that distributive inferences and
plain negated inferences can be dissociated.

(19)    *A puzzle about distributive inferences:*
        A disjunction in the scope of a universal quantifier may give rise to
        distributive inferences without giving rise to plain negated inferences.

In the following we show that the puzzle about distributive inferences can
be resolved on the grammatical approach to SIs without giving up the standard
assumption about alternatives.[4] The remainder of the paper has the following
structure: Section 3 resolves the puzzle about distributive inferences. Section
4 presents experimental data that suggest that distributive inferences are not
only possible in the absence of plain negated inferences, but are in fact prefer-
ably obtained in this way (or, perhaps, can be obtained only in this way) – a
state of affairs that we attempt to explain in Sect. 5. Section 6 concludes the
paper by pointing to several questions for future research.

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are married to a woman (i.e., the distributive inferences of (1) may not both be true in such
a context). This is not the case for the sentence used in (15)/(18): a brother of mine *having
been* married to a woman is compatible with him having been married to a man as well (i.e.,
the distributive inferences of (15)/(18) may both be true in contexts in which the negation
of one of the plain alternatives is false).

[4] Of course, the puzzle could be resolved by assuming different alternatives. For example,
as noted by a reviewer, one could assume that the set of relevant alternatives to (18) cor-
responds to {No brother of mine has been married to a man, No brother of mine has been
married to a woman}, in which case the exhaustification of (18) would yield distributive
inferences in the absence of plain negated inferences (that is, the negation of the alterna-
tives assumed here would correspond to the distributive inferences). We cannot explore all
the possibilities and their consequences for the theory of SIs here. (For illustration: if one
were to adopt *no brother* as an alternative of *every brother*, as assumed above, one would
incorrectly predict, say, that *Some boy read every book* should (be able to) convey that every
boy read some book, that is, ¬*Some boy read no book*.) Instead, we focus on one resolution
of the puzzle within the grammatical approach to SIs, a resolution that relies on standard,
uncontroversial assumptions about alternatives (again, see, e.g., Matsumoto 1995, Sauerland
2004, Fox 2007, Katzir 2007 for a detailed discussion of some constraints on alternatives).

## 3 A resolution of the puzzle

We have seen that the distribution of distributive inferences is not captured on the existing approaches to exhaustification if the sentence containing disjunction in the scope of a universal quantifier is taken to induce just plain alternatives – that is, alternatives in which the disjunction is either replaced by one of the disjuncts or by their conjunction:

(14)    *Exhaustification based on plain alternatives*
        For any sentence *Every A is P or Q*, if matrix exhaustification operates on its plain alternatives (*Every A is P, Every A is Q*), the distributive inferences (*Some A is P, Some A is Q*) are derived from the negation of the plain alternatives (¬*Every A is P*, ¬*Every A is Q*).

However, it is conceivable that adopting other alternatives for sentences of the form *Every A is P or Q* may lead to matrix exhaustification generating distributive inferences in the absence of plain negated inferences. More to the point, if instead of the plain alternative *Every A is P* one would have the alternative *Every A is only P* (more explicitly, *Every A is P but not Q*) and instead of the alternative *Every A is Q* one would have the alternative *Every A is only Q* (more explicitly, *Every A is Q but not P*), the sentence together with its SIs would entail distributive inferences but not plain negated inferences, as illustrated in (20) (in fact, the conjunction of the sentence with its SIs would in this case be equivalent to the conjunction of the sentence with its distributive inferences). We will refer to these new alternatives as *exhaustified alternatives*.

(20)    Every A is P or Q ∧
        ¬Every A is P but not Q ∧ ¬Every A is Q but not P
        a.    ⇒ Some A are P ∧ Some A are Q
        b.    ⇏ ¬Every A is P ∧ ¬Every A is Q

If one adopts exhaustified alternatives, distributive inferences can be derived as SIs even in the absence of plain negated inferences.

(21)    *Exhaustification based on exhaustified alternatives*
        For any sentence *Every A is P or Q*, if matrix exhaustification operates on its exhaustified alternatives (*Every A is only P, Every A is only Q*), the distributive inferences (*Some A is P, Some A is Q*) are derived without conveying the negation of the plain alternatives (¬*Every A is P*, ¬*Every A is Q*).

In light of the expedient prediction in (21), we have to explore whether the exhaustified alternatives are compatible with the standard assumption about alternatives, repeated below.

(4)    *Standard assumption about alternatives*
        A constituent, $\alpha = [\beta\ \gamma]$, has as its alternatives its subconstituents and the pointwise combinations of the alternatives to its subconstituents,

$$\text{ALT}(\alpha) = \{\alpha' \mid \exists \beta', \gamma' \colon \beta' \in \text{ALT}(\beta) \wedge \gamma' \in \text{ALT}(\gamma) \wedge (\alpha' = [\beta' \ \gamma'] \vee \alpha' = \beta \vee \alpha' = \gamma)\}.$$

We show in the following that exhaustified alternatives are compatible with the standard assumption about alternatives on the grammatical approach to SIs. We then present a derivation of distributive inferences in the absence of plain negated inferences. Subsequently, we point out that in order to avoid some outlandish predictions, the derivation must be coupled with a constraint on what counts as a legitimate domain of an exhaustification operator. We propose one such constraint.

### 3.1 Embedding *exh*

As reviewed in the introductory section, on the grammatical approach to SIs, SIs are generated by an exhaustification operator, *exh*, in grammar. Similar to other operators in grammar, *exh* may be embedded, in particular, it may be embedded in the scope of a universal quantifier (see, e.g., Chemla and Spector 2011, Chierchia et al. 2011, Magri 2011, Crnič 2013 for arguments in favor of embedded SIs). Furthermore, nothing prevents an occurrence of *exh* embedded under another occurrence of *exh* (Fox 2007). Accordingly, the sentence in (22a) may be parsed as having a structure with two occurrences of *exh*, namely (22b): one occurrence at the matrix level and one embedded immediately below the universal quantifier.[5]

(22)    a.    Every brother of mine has been married to a woman or a man.
        b.    exh($C_2$)(every brother$_x$ (exh($C_1$)(x has been married to a woman or a man)))

The meaning of the structure in (22b) depends on the resolution of the domains of the two occurrences of the exhaustification operator, $C_1$ and $C_2$. On this parse and on the standard assumption about alternatives, sentence (22a) may have as its alternatives the exhaustified alternatives − that is, alternatives that we have seen are required for generating distributive inferences in the absence of plain negated inferences. We focus on these alternatives and this derivation of the distributive inferences in the remainder of this section.

### 3.2 Exhaustified alternatives and pruning

The standard alternatives of the embedded disjunction in (22) are given in (23). The domain of the embedded exhaustification operator (the set referred to as $C_1$ in (22)) corresponds to a set that may contain (some of) them. We will say of the standard alternatives that do not end up in the domain of an exhaustification operator that they are 'pruned' from that domain.

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[5]  In fact, following Magri (2011), we will suggest in Sect. 5 that this is the only grammatical parse of the sentence – in other words, that the presence of both embedded and matrix *exh* is obligatory.

(23)    a.    x has been married to a woman
        b.    x has been married to a man
        c.    x has been married to a woman and a man

As noted above, pruning different alternatives from the domain of an exhaustification operator may lead to different meanings of the sentences in which the exhaustification operator occurs. Here we will focus on the consequences of pruning the conjunctive alternative in (23) from the domain of the embedded *exh* in (22) – that is, on the consequences of assuming that the domain of the embedded *exh* in (22) is the one provided in (24).

(24)    $C_1$ = {x has been married to a woman, x has been married to a man}

In this case, *exh* is locally vacuous and does not affect the (assignment-dependent) meaning of the sister of the universal quantifier, given in (25).

(25)    $\text{exh}(C_1)$(x has been married to a woman or a man) =
        $\lambda w$. x has been married to a woman or a man in w

Neither of the two alternatives in $C_1$ is excludable given the prejacent and $C_1$ (see the definition of excludable alternatives in (12)): both alternatives form their own maximal set of negatable alternatives, and so neither alternative is in the intersection of such sets, that is, in the set of excludable alternatives. And since there are no excludable alternatives, the import of embedded exhaustification is vacuous. However, as we will see shortly, embedded exhaustification, though locally vacuous, turns out to affect the alternatives for the matrix *exh* and, thereby, the overall meaning of the sentence.

    The alternatives to the sister of the matrix exhaustification operator that may enter into the computation of the exhaustified meaning are given in (26). They are built on the two disjuncts and on the conjunctive connective; that is, they are derived from the structure in (22) in line with the standard assumption about alternatives.[6]

(26)    a.    every brother$_x$ $(\text{exh}(C_1)$(x has been married to a woman))
        b.    every brother$_x$ $(\text{exh}(C_1)$(x has been married to a man))
        c.    every brother$_x$ $(\text{exh}(C_1)$(x has been married to a woman and a man))

The meaning of the prejacent of the matrix exhaustification operator in (22) is that every brother has been married to a woman or a man; as noted with respect to (25), the embedded *exh* does not affect the meaning of the scope of the universal quantifier. The alternatives based on the two disjuncts correspond to every brother of mine having been married to a woman but not to

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[6] On the standard assumption about alternatives, further alternatives can be derived from the structure in (22) – for example, alternatives without the embedded *exh*. We assume here that these alternatives are pruned from the domain of the matrix *exh* (if they were not pruned, matrix exhaustification would yield plain negated inferences, as discussed in Sect. 1). But see Sect. 5.2 (especially footnote 19), where we propose that, in fact, alternatives without the embedded *exh* cannot be generated.

a man, given in (27a), and to every brother of mine having been married to a man but not to a woman, given in (27b); the conjunctive alternative corresponds to every brother of mine having been married both to a man and to a woman, given in (27c).

(27)  a.  every brother$_x$ (exh($C_1$)(x has been married to a woman)) =
          λw. every brother of mine has been married to a woman but not to a man in w
     b.  every brother$_x$ (exh($C_1$)(x has been married to a man)) =
          λw. every brother of mine has been married to a man but not to a woman in w
     c.  every brother$_x$ (exh($C_1$)(x has been married to a man and a woman)) = λw. every brother of mine has been married to a man and a woman in w

Note that the alternatives based on the two disjuncts correspond to the exhaustified alternatives needed to derive distributive inferences in the absence of plain negated inferences, as summarized in (21). If the domain of the matrix exhaustification operator in (22), $C_2$, consists of the alternatives in (26), the sentence conveys that they are all false (since they are all excludable):

(28)  exh($C_2$)(every brother$_x$ (exh($C_1$)(x has been married to a woman or a man))) = λw. every brother of mine has been married to a woman or a man in w ∧ ¬every brother of mine has been married to a woman but not to a man in w ∧ ¬every brother of mine has been married to a man but not to a woman in w ∧ ¬every brother of mine has been married to a man and a woman in w

The meaning in (28) satisfies our main desiderata: it entails the distributive inferences but not the plain negated inferences. First: the distributive inferences are entailed since if it holds that every one of my brothers has been married to a man or a woman and that not every one of my brothers has been married to a woman but not to a man, then some brother of mine must have been married to a man, and vice versa. Second: the plain negated inferences are not entailed since it may well be the case that every brother of mine has been married to a woman as long as at least one (but not all) of them has also been married to a man, as well as that every brother of mine has been married to a man as long as at least one (but not all) of them has also been married to a woman.

(29)  a.  (28) ⇒ Some brother of mine has been married to a woman
                ∧ Some brother of mine has been married to a man
     b.  (28) ⇏ ¬Every brother of mine has been married to a woman
                ∧ ¬Every brother of mine has been married to a man

Moreover, it is worth noting that we obtain distributive inferences in the absence of plain negated inferences also if we prune the conjunctive alternative

from the domain of the matrix exhaustification operator — that is, if we assume that the domain of the matrix exhaustification operator is the following:

(30)     $C_2$' = {every brother$_x$ (exh($C_1$)(x has been married to a woman)),
                every brother$_x$ (exh($C_1$)(x has been married to a man))}

Unsurprisingly, the meaning that we get on such resolution is logically weaker than the one we get if we do not prune the conjunctive alternative. In fact, the meaning that we get on this resolution is equivalent to the conjunction of the prejacent and the distributive inferences. (Again, a different pruning of alternatives may lead to a different interpretation of the sentence. We address other available prunings in Sect. 5.2.)

(31)     exh($C_2$')(every brother$_x$(exh($C_1$)(x has been married to a woman or a man))) = $\lambda$w. every brother of mine has been married to a woman or a man in w $\wedge$ ¬every brother of mine has been married to a woman but not to a man in w $\wedge$ ¬every brother of mine has been married to a man but not to a woman in w

To summarize, we have shown that if a sentence containing a disjunction in the scope of a universal quantifier, like (32a) below, has a parse on which both the matrix and the embedded clause are exhaustified, as in (32b), which is a possible parse on the grammatical approach to SIs, then the standard assumption about alternatives allows the sentence to have as its alternatives the exhaustified alternatives necessary to derive distributive inferences in the absence of plain negated inferences, as summarized in (21).

(32)     a.    Every brother of mine has been married to a woman or a man.
         b.    exh($C_2$)(every brother$_x$ (exh($C_1$)(x has been married to a woman or a man)))

More to the point, if the conjunctive alternative is pruned from the domain $C_1$ of embedded *exh* and no alternative (or just the conjunctive alternative) is pruned from the domain of matrix *exh*, the sentence entails distributive inferences without entailing plain negated inferences.

This resolves the puzzle about distributive inferences. The solution adheres to the standard assumption about alternatives and to those of the grammatical approach to SIs. However, the assumption that conjunctive alternatives can sometimes be pruned is not without consequences. Our next task is to ensure that it does not lead to wrong predictions elsewhere.

### 3.3 A constraint on pruning

On the grammatical approach to SIs, if one were allowed to freely prune conjunctive alternatives, one would predict that plain disjunction may have a conjunctive meaning, contrary to fact (see Chierchia 2010, Fox and Katzir 2011, Meyer 2012, Ivlieva 2013, and, in particular, Katzir 2013). We illustrate this

in the following: if sentence in (33a) had a recursively exhaustified structure, as given in (33b), where the domains of exhaustification contained alternatives built on just one of the disjuncts but did not contain conjunctive alternatives, the sentence would entail that both disjuncts are true.

(33)    a.   John ate cake or ice cream.
        b.   $\text{exh}(C_2)((\text{exh}(C_1)(\text{John ate cake or ice cream})))$
        c.   $C_1 = \{\text{John ate cake, John ate ice cream}\}$
        d.   $C_2 = \{\text{exh}(C_1)(\text{John ate cake}), \text{exh}(C_1)(\text{John ate ice cream})\}$

The meaning of the sentence would then be that it is true that John ate cake or ice cream, but false that he ate only cake and false that he ate only ice cream, as in (34) (note that both alternatives in $C_2$ are excludable given the prejacent and $C_2$; that is, the prejacent conjoined with the negated alternatives is consistent). This exhaustified meaning, in turn, is equivalent to John eating both cake and ice cream (Singh et al. 2013).

(34)    $\text{exh}(C_2)((\text{exh}(C_1)(\text{John ate cake or ice cream}))) = \lambda w.$ John ate cake or ice cream in w $\wedge \neg$John ate cake but not ice cream in w $\wedge \neg$John ate ice cream but not cake in w

(35)    $(34) \Leftrightarrow \lambda w.$ John ate cake in w $\wedge$ John ate ice cream in w

The problem is obviously that the sentence in (33) never conveys such a conjunctive meaning. The explanation of the puzzle that we provided in the preceding subsection thus leads us to expect that, all else being equal, disjunction may have readings that we in fact fail to observe.

(36)    Prediction:
        On the grammatical approach to SIs and the assumption of unconstrained pruning of alternatives, plain disjunction may convey a conjunctive meaning.

The need to constrain the pruning of alternatives in order to avoid undesirable results has been independently acknowledged and tackled by Fox and Katzir (2011). However, their constraint, though successful in blocking conjunctive meaning for simple disjunctive sentences, would block the pruning that we rely on in this paper. We thus propose a different constraint: the pruning of alternatives needs to result in structures that are asymmetrically

entailed by those among their counterparts in which at least some of those alternatives have not been pruned.[7],[8]

(37)     *Constraint on pruning*
         $\text{exh}(C)(S)$ is licensed for $C \subseteq \text{ALT}(S)$ only if for any C', $C \subset C' \subseteq \text{ALT}(S)$, $\text{exh}(C')(S)$ asymmetrically entails $\text{exh}(C)(S)$.[9]

*Conjunctive meaning of plain disjunction* The constraint in (37) correctly rules out the parse of a plain disjunctive sentence entertained in (33): although pruning of the conjunctive alternative from the domain of the embedded *exh* complies with (37), subsequent pruning of the conjunctive alternative from the domain of the matrix *exh* does not. The reasoning goes as follows. Pruning of the conjunctive alternative from the domain of embedded *exh* is legitimate because the meaning that we obtain – that John ate cake or ice cream, that is, the exhaustification is vacuous – is entailed by the meaning of the structure in which the conjunctive alternative is not pruned – that John ate cake or ice cream but not both. However, subsequent pruning of the conjunctive alternative from the domain of the matrix *exh* leads to a meaning – that John ate cake and ice cream – that entails the meaning of the structure in which the conjunctive alternative is not pruned – that John ate cake or ice cream.

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[7] A reviewer inquires about an explanatory motivation for the constraint in (37). While we abstain from extensive speculation on the issue here, let us nonetheless hint at a possible proposal. To set the stage, let us assume that the context is structured: it includes a partition of the context set, which is induced by the question under discussion and relative to which certain possible worlds are equivalent (see, e.g., Groenendijk and Stokhof 1984, Roberts 2012). The guiding intuition behind the proposal, then, is that pruning of alternatives, which corresponds to a shift to a new, more coarse-grained context (that is, a context in which any two possible worlds that were equivalent prior to pruning remain equivalent but not vice versa), should result in the information conveyed by the speaker to be more coarse-grained as well (that is, the cell or the union of cells picked out in the new context by an exhaustified sentence should be a superset of the one(s) it picked out prior to pruning). More concisely, a shift in the coarse-grainedness of the context should be matched by an appropriate shift in the coarse-grainedness of the information provided by the speaker or, equivalently, switching to a question under discussion that seeks less information than the prior question calls for an answer that provides less information than the prior answer did. A more serious investigation of this proposal and of its potential extension to other types of domain restriction is beyond the scope of this paper.

[8] Katzir (2013) has also proposed a new constraint on pruning. His constraint makes the same predictions as the one put forward in (37), at least with respect to the data discussed in this paper. The main difference between the two constraints is that the one in (37) may be more readily generalized to other alternative-sensitive operators. We must leave a thorough investigation and comparison of the different constraints on pruning, and of their extensions to alternative-sensitive operators more generally, to another occasion.

[9] The constraint on pruning could be weakened so that it relies on plain entailment instead of asymmetric entailment. We chose the stronger formulation primarily in order to simplify the discussion in Sect. 5.2. If the weaker formulation were chosen, this would not affect the discussion in the current section, but we would need to adopt a further preference/principle that would rule out 'redundant pruning' in Sect. 5.2. Since, as it stands, we lack direct empirical support for either of the two formulations, we allow presentational simplicity to guide us. As stated in footnote 8, a detailed investigation of different constraints on pruning must be left to another occasion.

More to the point, if the domain of the matrix exhaustification operator contains all the alternatives, as represented in (38c), none of the alternatives are excludable with respect to it and the prejacent.

(38)   a.   $exh(C_2')(exh(C_1)(\text{John ate cake or ice cream}))$
       b.   $C_1 = \{\text{John ate cake, John ate ice cream}\}$
       c.   $C_2' = \{exh(C_1)(\text{John ate cake}), exh(C_1)(\text{John ate ice cream}),$
             $exh(C_1)(\text{John ate cake and ice cream})\}$

For example, the prejacent conjoined with the negations of the two non-conjunctive alternatives is consistent and entails the conjunctive alternative, showing that the conjunctive alternative is not in every maximal set of jointly negatable alternatives and thus that it is not excludable (see (12) for the characterization of excludable alternatives). Accordingly, the structure fails to trigger any SI and fails to entail that John ate cake and ice cream, which is the meaning of the parse of the sentence in which the conjunctive alternative is pruned, computed in (34).

(39)   $exh(C_2')(exh(C_1)(\text{John ate cake or ice cream})) \not\Rightarrow$
       $exh(C_2)(exh(C_1)(\text{John ate cake or ice cream}))$

This means that the parse on which the conjunctive alternative is pruned from the domain of matrix exhaustification – a parse that yields the unwanted conjunctive interpretation of plain disjunction – is ruled out by the constraint on pruning in (37) that requires pruning to lead to weaker meanings.[10]

(40)   Consequence of the constraint on pruning for sentence (33):
       For all $C_2'$, $C_2 \subset C_2' \subseteq ALT(exh(C_1)(\text{John ate cake or ice cream}))$,
       $exh(C_2')(exh(C_1)(\text{J. ate cake or ice cream})) \not\supseteqq$
       $exh(C_2)(exh(C_1)(\text{J. ate cake or ice cream}))$

*Distributive inferences without plain negated inferences* In contrast, the proposed resolutions of domains of the two exhaustification operators in (22) that yield distributive inferences in the absence of plain negated inferences comply with the constraint on pruning. Consider the parse on which the conjunctive alternatives are pruned from the domain of both *exh*:

(41)   a.   Every brother of mine has been married to a woman or a man.
       b.   $exh(C_2)(\text{every brother}_x (exh(C_1)(\text{x has been married to a woman}$
             $\text{or a man})))$
       c.   $C_1 = \{\text{x has been married to a woman,}$
             $\text{x has been married to a man}\}$

---

[10] Building on preceding observations by Braine and Rumain (1981), Singh et al. (2013) show that there is a population of children that interpret plain disjunction conjunctively. To account for this behavior, Singh et al. propose that that population of children lacks conjunctive alternatives altogether (more generally, they lack substitution alternatives – that is, alternatives that are derived by substituting lexical items in the prejacent). Their proposal is thus compatible with our constraint on pruning in (37).

      d.    $C_2$ = {every brother$_x$ (exh($C_1$)(x has been married to a woman)),
                every brother$_x$ (exh($C_1$)(x has been married to a man))}

We have already seen that the pruning of the conjunctive alternative from the domain of the embedded *exh* in (41) satisfies the constraint on pruning: we obtain an inclusive disjunctive meaning, which is weaker than the exclusive meaning that we obtain if the conjunctive alternative is not pruned.

The pruning of the conjunctive alternative from the domain of the matrix *exh* in (41) results in the meaning given in (42).

(42)     exh($C_2$)(every brother$_x$ (exh($C_1$)(x has been married to a woman or a man))) = $\lambda$w. every brother of mine has been married to a woman or a man in w $\wedge$ ¬every brother of mine has been married to a woman but not to a man in w $\wedge$ ¬every brother of mine has been married to a man but not to a woman in w

To check whether the constraint on pruning is satisfied, we need to check whether the meaning in (42) is entailed by the structure in which the domain of the matrix *exh* contains not only the alternatives in (41d) but also the conjunctive alternative, given in (43d).

(43)     a.    Every brother of mine has been married to a woman or a man.
         b.    exh($C_2$')(every brother$_x$ (exh($C_1$)(x has been married to a woman or a man)))
         c.    $C_1$ = {x has been married to a woman,
                x has been married to a man}
         d.    $C_2$' = {every brother$_x$ (exh($C_1$)(x has been married to a woman)),
                every brother$_x$ (exh($C_1$)(x has been married to a man)),
                every brother$_x$ (exh($C_1$)(x has been married to a woman and a man))}

Now, all the alternatives in $C_2$' are excludable with respect to the prejacent and $C_2$' since the conjunction of the prejacent and the negated alternatives is consistent:

(44)     exh($C_2$')(every brother$_x$ (exh($C_1$)(x has been married to a woman or a man))) = $\lambda$w. every brother of mine has been married to a woman or a man in w $\wedge$ ¬every brother of mine has been married to a woman but not to a man in w $\wedge$ ¬every brother of mine has been married to a man but not to a woman in w $\wedge$ ¬every brother of mine has been married to a man and to a woman in w

Since this meaning asymmetrically entails the meaning that we obtain by pruning the conjunctive alternative (note that (44) differs from (42) only in the former having an additional conjunct), the representation in which the conjunctive alternative is pruned satisfies the constraint on pruning in (37).

## 3.4 Summary

In this section we have provided an account of the puzzle about distributive inferences – that is, the fact that disjunction in the scope of a universal quantifier may give rise to distributive inferences in the absence of plain negated inferences. Our starting point was the observation, restated below, that such inferences can be derived on the assumption of exhaustified alternatives.

(21) *Exhaustification based on exhaustified alternatives*
For any sentence *Every A is P or Q*, if matrix exhaustification operates on its exhaustified alternatives (*Every A is only P, Every A is only Q*), the distributive inferences (*Some A is P, Some A is Q*) are derived without conveying the negation of the plain alternatives (¬*Every A is P, ¬Every A is Q*).

While on approaches to SIs that do not allow for embedded exhaustification, exhaustified alternatives are unavailable on the standard assumption about alternatives, on the grammatical approach to SIs they are available if the respective sentences are exhaustified at the embedded level.

(45)  a.  Every A is P or Q.
      b.  $\text{exh}(C_2)(\text{every } A_x (\text{exh}(C_1)(x \text{ is P or Q})))$

If the conjunctive alternative is pruned from the domain of embedded *exh* (and no other alternatives are pruned), we obtain distributive inferences in the absence of plain negated inferences.

(46)  $\text{exh}(C_2)(\text{every } A_x (\text{exh}(C_1)(x \text{ is P or Q})))$
      $\Rightarrow$  Every A is P or Q $\land$ ¬Every A is only P $\land$ ¬Every A is only Q
      $\Rightarrow$  Some A is P $\land$ Some A is Q
      $\nRightarrow$  ¬Every A is P $\land$ ¬Every A is Q

To avoid the overgeneration that unmitigated pruning would bring about, we proposed to constrain pruning by requiring it to result in meanings that are logically weaker than the meanings that would be obtained without pruning or by pruning fewer alternatives.

(37)  *Constraint on pruning*
$\text{exh}(C)(S)$ is licensed for $C \subseteq \text{ALT}(S)$ only if for any C', $C \subset C' \subseteq \text{ALT}(S)$, $\text{exh}(C')(S)$ asymmetrically entails $\text{exh}(C)(S)$.
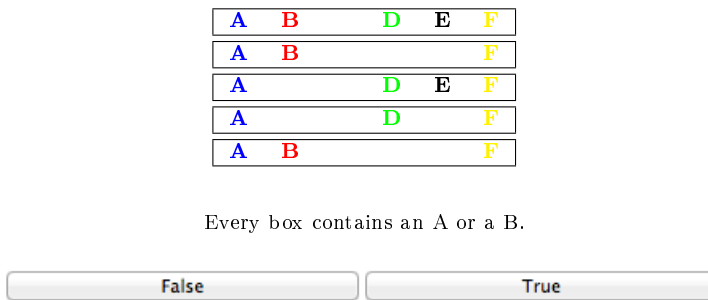
## 4 Experiment

In the preceding section, we have shown how to derive distributive inferences in the absence of plain negated inferences. We now present the results of a sentence verification experiment that suggest that such a derivation of distributive inferences is not only possible but, in our task, preferred to a derivation with

plain negated inferences. More precisely, the experiment provides evidence for the computation of distributive inferences, but not of plain negated inferences.

## 4.1 Experimental items and predictions

A sentence verification experiment was devised in which participants were presented with a picture accompanied by a sentence and asked to evaluate whether the sentence provided a true or false description of the picture. The pictures had the form exemplified by the experimental item in Fig. 1: there were five oblong boxes, with each box containing some letters from A to F, positioned into six columns.



Every box contains an A or a B.



**Fig. 1** An example of an experimental item used in the experiment

The position of the letters in the respective columns stayed fixed throughout the experiment, following alphabetical order from left to right. What varied was which letters were shown in a given box: for example, the letter A did not appear in every box, but if it did, it would always appear in the first position. The distribution of letters satisfied certain constraints across all trials, such that the right choice of letters would allow us to create any configuration of interest (see below). Specifically, on each trial there were two letters that were present in every box (e.g., A and F in Fig. 1); there was one letter that was missing from every box (e.g., C in Fig. 1); there was always a pair of letters such that every box contained a member of the pair, but no single member of the pair was in every box (e.g., B and D in Fig. 1); and there was always a pair of letters such that each of the two letters was in some box, but some box contained neither letter (e.g., B and E, D and E in Fig. 1). The pertinent experimental sentences had the form exemplified in (47), where the two letters varied across trials and ranged from A to F. After the picture and the paired sentence were displayed, the subject had to evaluate whether the sentence was a true description of the picture.

(47)    Every box contains an A or a B.

The primary goal of the experiment was to establish whether there is a relation between the computation of distributive inferences and the computation of plain negated inferences.

(48)    a.    Experimental sentence:
               Every box contains an A or a B.
        b.    Distributive inferences:
               Some box contains an A ∧ Some box contains a B
        c.    Plain negated inferences:
               ¬Every box contains an A ∧ ¬Every box contains a B

Recall that we have described two derivations of distributive inferences. Section 1 presented a derivation of distributive inferences that relies on the exhaustification of the matrix sentence on the basis of plain alternatives. This derivation crucially requires generating plain negated inferences. The operative principle was stated in (14), repeated here.

(14)    *Exhaustification based on plain alternatives*
        For any sentence *Every A is P or Q*, if matrix exhaustification operates on its plain alternatives (*Every A is P, Every A is Q*), the distributive inferences (*Some A is P, Some A is Q*) are derived from the negation of the plain alternatives (¬*Every A is P*, ¬*Every A is Q*).

Section 3 presented a derivation of distributive inferences that relies on exhaustified alternatives, a derivation that is available on the standard assumption about alternatives on the grammatical approach to SIs, but is not available on pragmatic approaches. This derivation does not generate plain negated inferences. The operative principle in this case was (21), repeated here.

(21)    *Exhaustification based on exhaustified alternatives*
        For any sentence *Every A is P or Q*, if matrix exhaustification operates on its exhaustified alternatives (*Every A is only P, Every A is only Q*), the distributive inferences (*Some A is P, Some A is Q*) are derived without conveying the negation of the plain alternatives (¬*Every A is P*, ¬*Every A is Q*).

We now describe the different conditions of the experiment and the responses predicted by the two derivations of distributive inferences. On the one hand, in certain cases the appropriate responses to picture-sentence pairs based on the two derivations match. They match, first, for picture-sentence pairs where every box in the picture contains one of the two letters and where the plain negated inferences of the sentence are true (both exhaustifications are true, M1); second, for picture-sentence pairs where not every box in the picture contains one of the two letters (both exhaustifications are false, M2 and M3); and third, for picture sentence pairs where every box contains one of the two letters and no box contains the other letter – that is, for picture-sentence

pairs for which the distributive inferences are false (both exhaustifications are false, M4). (See Fig. 2 for a visual presentation of these conditions.)

(49)    Matching conditions:
        M1: Prejacent is true, plain negated (distributive) inferences are true
        M2: Prejacent is false, both letters in the disjunction are in some box
        M3: Prejacent is false, only one letter in the disjunction is in some box
        M4: Prejacent is true, distributive (plain negated) inferences are false

(50)    Predicted responses to matching conditions (both derivations):
        M1: True (prejacent is true, exhaustified inferences are true)
        M2: False (prejacent is false)
        M3: False (prejacent is false)
        M4: False (prejacent is true, exhaustified inferences are false)

On the other hand, the predicted appropriate responses come apart for picture-sentence pairs for which the distributive inferences of the sentence are true but the plain negated inferences are false.

(51)    Distinguishing conditions:
        D1: Prejacent is true, distributive inferences are true, but plain negated
            inferences are false; only one of the letters in the sentence is in
            all of the boxes
        D2: Prejacent is true, distributive inferences are true, but plain negated
            inferences are false; both letters in the sentence are in all of the
            boxes

    On the exhaustification described in Sect. 1 these sentences should, to the extent that SIs are computed, be judged as false, while on the exhaustifications discussed in Sect. 3 some of these sentences should be judged as true (see qualification in footnote 11). More specifically, we will be interested in the following prediction. The rejection rate of M4 provides an evaluation of the base derivation rate of plain negated inferences. If (a) distributive inferences are tied to negated plain inferences, D1 and D2 should be just like M4, but if (b) distributive inferences can be derived independently of plain negated inferences, we expect that the rejection rate could be lower for D1 and D2.

(52)    Predicted responses to distingushing conditions:
        D1, D2: False (exhaustification with plain alternatives)
        D1: True (exhaustification with exhaustified alternatives)
        D2: True/False (exhaustification with exhaustified alternatives)[11]

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[11]  If the conjunctive alternative is pruned from the domain of the embedded but not the matrix *exh*, represented in (28), the predicted response is True for D1 and False for D2. If the conjunctive alternative is pruned from the domains of both the embedded and the matrix *exh*, represented in (31), the predicted response is True for both D1 and D2. In any case,

The conditions and the predicted responses for the two types of exhaustification are summarized in the table in Fig. 2; the conditions on which the two types of exhaustification make distinguishable predictions are highlighted.

| Conditions | Predicted responses | |
|---|---|---|
| | Plain alternatives | Exhaustified alternatives |
| M1 *(B or D)* | T | T |
| M2 *(B or E)* | F | F |
| M3 *(B or C)* | F | F |
| M4 *(A or C)* | F | F |
| D1 *(A or B)* | F | T |
| D2 *(A or F)* | F | T |



M1: Every box contains a B or a D    M4: Every box contains an A or a C
M2: Every box contains a B or an E    D1: Every box contains an A or a B
M3: Every box contains a B or a C    D2: Every box contains an A or an F

**Fig. 2** Summary of the conditions

## 4.2 Participants and procedure

Fifty-three native English-speaking participants performed the experiment on Amazon Turk, for a payment each of $1.05. Each of the conditions appeared eight times, with the exception of the first control condition, M1, on which both distributive and plain negated inferences are true; condition M1 appeared sixteen times to counterbalance negative responses. There were also sixty filler items that did not contain a disjunction. Two participants were excluded from the analysis due to poor performance on the first three matching conditions (less than 75% correct responses).[12]

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the predictions of exhaustification based on plain and exhaustified alternatives are distinct. See Sects. 4.3 and 5.3 for discussion of further intricacies.

[12] We obtain practically indistinguishable results when we do not exclude the data for these participants. The means and standard errors of the conditions without screening for poor performance ($n = 53$) are as follows. M1: mean 93, std. error 1; M2: mean 5, std. error 1.1; M3: mean 1, std. error 0.5; M4: mean 75.2, std. error 2.1; D1: mean 93.3, std. error 1.2; D2: mean 92.2, std. error 1.3. Furthermore, a by-participants Wilcoxon signed-rank test reveals that the differences between the distinguishing conditions D1 and D2 and the matching condition M4 are significant (M4 vs. D1: W = 424, Z = -4.72; M4 vs. D2: W = 397.5, Z = -4.47, $ps < 0.001$ after correction for multiple comparisons). Moreover, the difference between the matching condition M4 and the matching condition M1 is significant as well (M1 vs. M4: W = 372, Z = -4.3, $p < 0.001$ after correction for multiple comparisons). See footnote 23 for some further discussion of the excluded participants.

## 4.3 Results

We analyzed the proportions of participants' responses to the experimental sentences. Figure 3 presents the main result. We see that participants responded as expected in the first three matching conditions: the mean of True responses to the sentences in condition M1 was 97% (std. error 1%), while the mean of True responses to the sentences in conditions M2 and M3 was about 5% (std. error 1.4%) and 1% (std. error 0.4%), respectively.
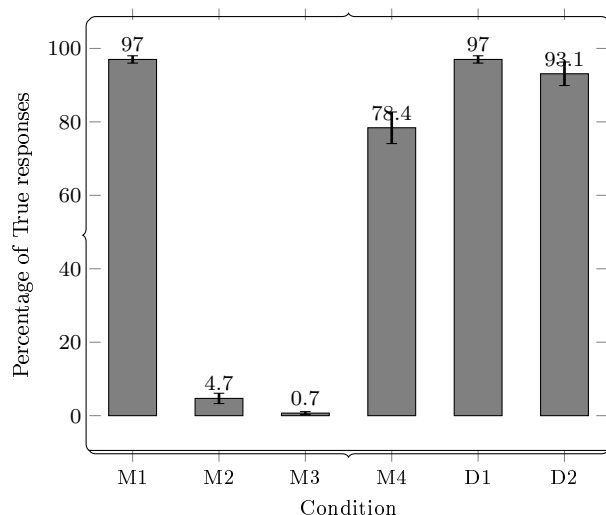


**Fig. 3** Percentage of True responses with error bars representing standard error

There is a contrast between the responses of participants in the remaining three conditions: on the one hand, the mean of True responses in conditions D1 and D2 was about 97% (std. error 1%) and 93% (std. error 3.2%), respectively; recall that these are the conditions in which distributive inferences are true but plain negated inferences are false. On the other hand, the mean of True responses was only 78% (std. error 4.3%) in condition M4, that is, in a condition in which distributive inferences are false. A by-participants ($n = 51$) Wilcoxon signed-rank tests reveal that the differences between the distinguishing conditions in which distributive inferences are true but plain negated inferences are false, D1 and D2, and the matching condition on which distributive inferences are false, M4, is significant (M4 vs. D1: W = 369, Z = -4.53; M4 vs. D2: W = 324.5, Z = -4.24, $p$s < 0.005 after correction for multiple comparisons). Moreover, the difference between the matching condition M4, on which distributive inferences are false, and the matching condition M1, on which both distributive and plain negated inferences are true, is significant as well (M1 vs. M4: W = 345, Z = -4.2, $p$ < 0.001 after correction for multiple

comparisons). No other relevant pairwise comparison provided a significant difference (in all comparisons W $> 40$, $p > 0.2$),[13] except for the difference between the matching conditions M2 and M3.[14] Thus, the acceptability ratings of the sentences in the distinguishing conditions D1 and D2 are at the level of those in the matching condition M1, on which both distributive and plain negated inferences are true; the acceptability ratings drop significantly in the matching condition M4, on which the prejacent is true but distributive inferences are false.

An exploration of individual participants' responses reveals that the population is not homogeneous – specifically, groups of participants appear to employ different response strategies on certain conditions. We focus on two conditions in the following: condition M4, where participants could be divided into different populations, and condition D1, where we observe homogeneous behavior. We return to other conditions in the next section, where we elaborate on possible sources for the differences in participants' response patterns.

First: We have seen that the responses on condition M4, on which the distributive inferences are false, are significantly different from the responses on all other conditions. The question is whether this difference results from a homogenous population that on average tends to reject the sentence more often on this condition than on others, or whether there are distinct subpopulations that each behave in a more uniform fashion. Individual participants' behavior can be gleaned from the breakdown of the responses by number of times a participant responded with True, represented on the left side of Fig. 4.[15]

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[13] There is no significant difference between the distinguishing conditions D1 and D2 (D1 vs. D2: W $= 41$, Z $= -0.16$, $p > 0.5$), nor between the distinguishing conditions D1 and D2 and the matching condition M1 (M1 vs. D1: W $= 44$, Z $= -0.55$, $p > 0.5$; M1 vs. D2: W $= 70.5$, Z $= -1.22$, $p > 0.2$).

[14] A Wilcoxon signed-rank test shows that the difference between the matching condition M2, on which the prejacent is false and the picture contains both of the letters mentioned in the experimental sentence, and the matching condition M3, on which the prejacent is false and the picture contains just one of the letters mentioned in the experimental sentence, is significant as well (M2 vs. M3: W $= 82$, Z $= -2.55$, $p < 0.05$). We defer pursuit of the reasons for this difference to another occasion.

[15] Each of the fifty-one participants was presented with eight items of condition M4. Among the fifty-one participants, three participants responded with True to none of the eight items; three participants responded with True to one item; three participants responded with True to four items; three participants responded with True to five items; seven participants responded with True to six items; seven participants responded with True to seven items; twenty-five participants responded with True to eight items.
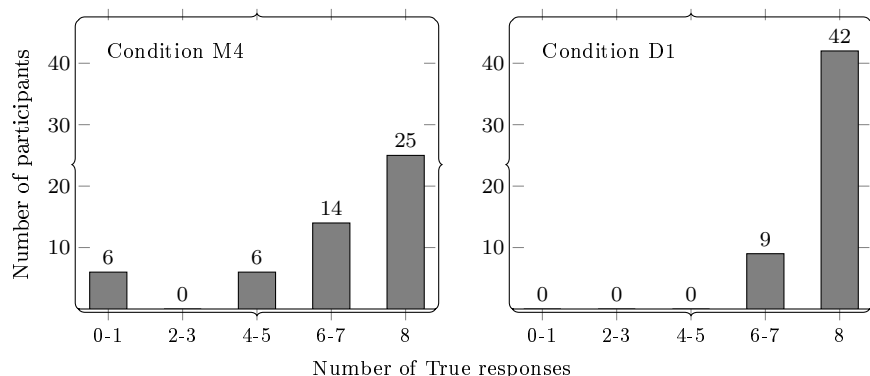
**Fig. 4** Responses on condition M4, on which distributive inferences are false, and on condition D1, on which distributive inferences are true but plain negated inferences are false. (All fifty-one participants evaluated eight items of each condition.)

The population is not homogeneous on this condition. More to the point, the data suggest that there are at least two populations of participants that differ in their response strategies – that is, participants that compute SIs and participants that do not (see the next section for further elaboration). This distribution of response strategies is in line with previous experimental studies on SIs, which have observed that there tends to be a substantial subpopulation of participants that appear not to compute SIs (suggestively dubbed 'logicians' by Noveck 2000, Bott and Noveck 2004).

Second: Unlike in the case of condition M4, participants' behavior appears to be homogeneous on condition D1, on which distributive inferences are true but plain negated inferences are not. More to the point, the data suggest that participants do not to compute plain negated inferences. The breakdown of the responses is represented on the right side of Fig. 4.[16]

All in all, the results presented in this section are concordant with distributive inferences being generated in the absence of plain negated inferences. This conforms to our observation that distributive and plain negated inferences can be dissociated. However, we found no evidence for other readings that should in principle be available – specifically, no evidence for plain negated inferences (as we will see in detail shortly). We thus need to refer to certain additional principles that would disfavor (or block) the unattested readings.

## 5 Comprehensive set of predicted readings

We have seen that although distributive inferences in the absence of plain negated inferences cannot be derived on approaches that rely on exhaustification based on plain alternatives, they can be derived on approaches that rely

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[16]   Each of the fifty-one participants was presented with eight items of condition D1. Among the fifty-one participants, forty-two responded with True to eight of the eight items; six responded with True to seven items; three responded with True to six items.

on exhaustification based on exhaustified alternatives. These alternatives are available on the grammatical approach to SIs and the standard assumption about alternatives.

In the case of experimental sentences from the preceding section, repeated below in (53), distributive inferences are derived without plain negated inferences from parses of the form provided in (54). The parses contain two occurrences of *exh* and, crucially, the conjunctive alternative is pruned from the domain of the embedded *exh* (whether the conjunctive alternative is also pruned from the domain of the matrix *exh* does not affect the derivation of distributive inferences in the absence of plain negated inferences, as discussed in Sect. 3; see also below).

(53)  a.  Experimental sentence:
           Every box contains an A or a B.
      b.  Distributive inferences:
           Some box contains an A $\wedge$ Some box contains a B
      c.  Plain negated inferences:
           $\neg$Every box contains an A $\wedge$ $\neg$Every box contains a B


(54)  a.  exh($C_2$)(every box$_x$ (exh($C_1$)(x contains an A or a B)))
      b.  $C_1$ = {x contains an A, x contains a B}
      c.  $C_2$ = {every box$_x$ (exh($C_1$)(x contains an A)),
               every box$_x$ (exh($C_1$)(x contains a B))}
      d.  $\lambda$w. every box contains an A or a B in w $\wedge$ some box contains an A in w $\wedge$ some box contains a B in w


5.1 Ambiguity in exhaustification

5.1.1 Predicted possible readings

In addition to the parse in (54), the grammatical approach to SIs allows for several other parses of the sentence in (53a) and, accordingly, for several other readings of the sentence.

*(i)* The grammatical approach allows for a reading with no distributive inferences. The reading is derived, say, from a parse with no exhaustification operators.


*(ii)* The grammatical approach allows for a reading that entails plain negated inferences, as discussed in Sect. 1. The reading is derived, say, from a parse without embedded *exh*, where the domain of the matrix *exh* contains disjunct alternatives:

(55)  a.  exh(C)(every box contains an A or a B)))
      b.  C = {every box contains an A, every box contains a B}

    c.    $\lambda$w. every box contains an A or a B in w $\wedge$ $\neg$every box contains
an A in w $\wedge$ $\neg$every box contains a B in w

(56)    Plain negated inferences:
    a.    $\neg$Every box contains an A
    b.    $\neg$Every box contains a B

*(iii)* The grammatical approach allows for readings that entail the negation of
the conjunctive alternative to the matrix sentence. These readings are derived,
say, by the matrix *exh* having the conjunctive alternative in its domain. One
such parse is provided in (57), in which the conjunctive alternative has been
pruned from the domain of the embedded *exh* but not of the matrix *exh*.

(57)    a.    $\mathrm{exh}(C_2)$(every box$_x$ $\mathrm{exh}(C_1)$(x contains an A or a B))
    b.    $C_1 = \{$x contains A, x contains a B$\}$
    c.    $C_2 = \{$every box$_x$ $(\mathrm{exh}(C_1)$(x contains an A)),
           every box$_x$ $(\mathrm{exh}(C_1)$(x contains a B)),
           every box$_x$ $(\mathrm{exh}(C_1)$(x contains an A and B))$\}$
    d.    $\lambda$w. every box contains an A or a B in w $\wedge$ some box contains an
A in w $\wedge$ some box contains a B in w $\wedge$ $\neg$every box contains an
A and a B in w

(58)    Matrix negation of conjunctive alternative:
    $\neg$Every box contains an A and a B

*(iv)* Finally, the grammatical approach allows for readings with embedded
strengthening of disjunction, which can be derived by the embedded *exh* having
the conjunctive alternative in its domain.

(59)    a.    every box$_x$ $(\mathrm{exh}(C)$(x contains an A or a B))
    b.    $C = \{$x contains an A, x contains a B, x contains an A and B$\}$
    c.    $\lambda$w. every box contains an A or a B but not both A and B in w
        (= $\lambda$w. every box contains just one of A or B in w)

(60)    Embedded negation of conjunctive alternative:
    Every box contains an A or a B but not both A and B
    (= Every box contains just one of A or B)

### 5.1.2 Readings supported by the experiment

The results of the experiment described in Sect. 4, however, provide support
for only some of these readings of disjunction under a universal quantifier:
(a) readings with distributive inferences but no plain negated inferences, and

(b) a reading with no distributive inferences. In other words, we found no evidence for the existence of any of the other readings that can in theory be generated on the grammatical approach to SIs – in particular, readings that entail plain negated inferences. While on the condition on which distributive inferences were false, M4, the mean of responses was significantly lower than on other pertinent conditions and there was a population that systematically judged the test sentences as false, this was not the case on conditions on which plain negated inferences were false (see the discussion of the contrast between conditions M4 and D1 in the preceding section, esp. Figs. 3-4).

### 5.1.3 The puzzle

On the face of it, the results of the experiment described in Sect. 4 present a puzzle for the grammatical approach to SIs: namely, the approach admits representations and thus readings for which we lack evidence.[17] Accordingly, an account is needed on which either (a) grammatical means rein in the representations admitted by the grammatical approach, or (b) there is a selection mechanism that picks out the preferred representations from those admitted by the approach, or (c) on which both (a) and (b) hold and together yield limitations on attested readings.

The remaining goal of this paper is to provide a tentative account for the puzzle by relying on the strategy described under (c). The account is tentative insofar as it would be easy to devise a variety of alternative accounts which might lead to distinct and perhaps empirically more adequate predictions in other domains of SI computation. We leave the development and proper comparison of such competing accounts to another occasion (see Sect. 6 for some further discussion).

### 5.2 Unavailable readings

### 5.2.1 The principles

We submit that the empirically observed limitation of available readings–to (a) readings with distributive inferences but no plain negated inferences, and to (b) a reading with no distributive inferences–emerges from an interaction of two grammatical principles. One of these we already introduced above, namely, the constraint on pruning:

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[17] In general, compared to the pragmatic approach, the grammatical approach provides for many more readings of sentences in which scalar items are embedded under other operators; this is so because, all else being equal, the grammatical approach allows for recursive exhaustification and exhaustification in embedded scope positions. Some authors have construed this distinction as an argument for the grammatical approach to SIs (see, e.g., Fox and Hackl 2006, Chierchia et al. 2011, Chemla and Spector 2011, Magri 2011, Crnič 2013, among others; but see Russell 2006, Geurts and Pouscoulous 2009 for a differing view). In any event, it is clear that the grammatical approach would need to constrain the distribution of embedded exhaustification (e.g., Fox and Spector 2009, Chierchia et al. 2011).

(37)    *Constraint on pruning*
        $\text{exh}(C)(S)$ is licensed for $C \subseteq \text{ALT}(S)$ only if for any C', $C \subset C' \subseteq \text{ALT}(S)$, $\text{exh}(C')(S)$ asymmetrically entails $\text{exh}(C)(S)$.

The other grammatical principle relates to obligatoriness of exhaustification: we assume that an exhaustification operator is generated at every scope position—an assumption that has been extensively discussed and argued for by Magri (2011).[18],[19]

(61)    *Obligatoriness of exhaustification*
        Every phrase of type t either is a sister of an exhaustification operator or has an exhaustification operator as one of its daughters.

5.2.2 Deriving the absence of plain negated inferences

An immediate consequence of these two principles is that parses that give rise to plain negated inferences are ruled out unless disjunction is locally strengthened to convey exclusive meaning.[20] Recall that we generated such readings in Sect. 1 by relying on matrix exhaustification based on plain alternatives. On the assumption of obligatoriness of exhaustification at every scope position, this would correspond to a parse on which all the alternatives of the embedded *exh* are pruned:

(62)    a.    $\text{exh}(C_2)$(every $\text{box}_x$ $(\text{exh}(C_1)$(x contains an A or a B)))
        b.    $C_1 = \emptyset$
        c.    $C_2 = \{$every $\text{box}_x$ $(\text{exh}(C_1)$(x contains an A)),
                    every $\text{box}_x$ $(\text{exh}(C_1)$(x contains a B))$\}$

However, this parse is ruled out by the constraint on pruning; specifically, the constraint is violated by embedded exhaustification:

(63)    a.    $\text{exh}(C_1)$(x contains an A or a B))
        b.    $C_1 = \emptyset$

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[18] Under this approach a sentence will lack an SI if the domain of the obligatory exhaustification operator lacks excludable alternatives.

[19] If we take the alternatives to an expression to be other well-formed expressions in the language that are derived from the expression by certain manipulations (say, by replacement of scalar items with other scalar items), as is commonly assumed (e.g., Sauerland 2004, Katzir 2007), the grammatical principle in (61) has the consequence that all embedded clauses in alternatives will contain an exhaustification operator. This consequence is pertinent for our discussion in the main text because it greatly constrains the number of possible parses we need to consider. See Magri (2011) for various questions raised by the principle of obligatoriness of exhaustification in (61), e.g., questions pertaining to economy conditions on the distribution of the exhaustification operator.

[20] The pertinent parse with the exclusive construal of disjunction is one on which no alternative is pruned from the domain of the embedded *exh* and on which the matrix *exh* contains at least the disjunct alternatives. The sentence entails on this parse that it is not the case that every box contains just A, that it is not the case that every box contains just B, and that it is not the case that every box contains just one of A or B. Together with the prejacent, these inferences entail plain negated inferences.

      c.    $\lambda$w. x contains an A or a B in w

There is a proper superset $C_1$' of the set $C_1$, given in (64b) below, relative to which embedded exhaustification yields a meaning that does not asymmetrically entail the meaning in (63) but is, rather, equivalent to it. This is due to the fact that neither alternative in $C_1$' is excludable and, accordingly, neither alternative is negated by exhaustification. It follows that (63) and, as a consequence, (62) violate the constraint on pruning.

(64)     a.    exh($C_1$')(x contains an A or a B))
          b.    $C_1$' = {x contains an A, x contains a B}
          c.    $\lambda$w. x contains an A or a B in w

(65)     Entailment relation between (63) and (64):
          exh($C_1$)(x contains an A or a B))
          $\Leftrightarrow$ exh($C_1$')(x contains an A or a B))

(66)     Consequence of the constraint on pruning:
          For all $C_1$', $C_1 \subset C_1' \subseteq$ ALT(x contains an A or a B),
          exh($C_1$')(x contains an A or a B))
          $\not\sqsupseteq$ exh($C_1$)(x contains an A or a B))

The parse that yields plain negated inferences without embedded strenthening of disjunction is thus correctly ruled out by the two grammatical principles. This explains why no participants exhibited a tendency of responding with False to any of the conditions on which plain negated inferences are false (in particular, to either of conditions M4 and D1).[21] The idealized response profile accompanying the precluded parse is represented in Fig. 5.

| Parse | Idealized response profile | Inferences | | | |
|---|---|---|---|---|---|
| | | Dist. | Mat. neg. conj. | Emb. neg. conj. | Plain neg. |
| (62) | [M4: 0], [D1: 0], [D2: 0] | ✓ | ✓ | ✗ | ✓ |

**Fig. 5** Idealized response profile corresponding to the precluded parse (62), with a specification of what inferences are entailed by the parse.

By accounting for the unavailability of plain negated inferences, we have achieved the main goal of this section. In the remainder of the section, we explore some more fine-grained predictions of our proposal (pertaining to the parses of the experimental sentence that are admitted on our proposal), discuss

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[21] Recall that all participants responded with True to at least 6 of the eight condition D1 items (see Fig. 4 and the accompanying discussion).

how they square with our experimental results, and point to some issues this
raises for future research.

## 5.3 Available readings

The two principles in (37) and (61) do not affect the availability of other parses
and thus other readings of the sentence. In particular, they allow for (a) parses
that yield distributive inferences in the absence of plain negated inferences, as
discussed in Sect. 3, and (b) parses that yield no distributive inferences.

### 5.3.1 Readings with distributive inferences

There are three parses of the experimental sentence that comply with the
principles introduced above and entail distributive inferences.

*First parse:* If the conjunctive alternatives are pruned from the domains of
both embedded and matrix *exh*, we obtain a parse that entails distributive
inferences in the absence of the matrix negation of the conjunctive alternative
and in the absence of plain negated inferences. On this parse, repeated below,
the experimental sentence should be judged as false in condition M4, in which
distributive inferences are false, while it should be judged as true in condi-
tions D1 and D2, in which distributive inferences are true but plain negated
inferences are false. (See the first row of the summary in Fig. 6.)

(54)    a.    $\text{exh}(C_2)$(every box$_x$ (exh($C_1$)(x contains an A or a B)))
        b.    $C_1$ = {x contains an A, x contains a B}
        c.    $C_2$ = {every box$_x$ (exh($C_1$)(x contains an A)),
                    every box$_x$ (exh($C_1$)(x contains a B))}
        d.    $\lambda$w. every box contains an A or a B in w $\wedge$ some box contains an
              A in w $\wedge$ some box contains a B in w

*Second parse:* If the conjunctive alternative is pruned from the domain of
the embedded *exh* but not the matrix *exh*, the sentence entails distributive
inferences and the matrix negation of the conjunctive alternative, but not plain
negated inferences. On this parse, repeated below, the experimental sentence
should be judged as false in condition M4, in which distributive inferences are
false, and in condition D2, in which the matrix negation of the conjunctive
alternative is false; the experimental sentence should be judged as true in
condition D1, in which distributive inferences and the matrix negation of the
conjunctive alternative are true. (See the second row of the summary in Fig.
6.)

(57)    a.    $\text{exh}(C_2)$(every box$_x$ exh($C_1$)(x contains an A or a B))
        b.    $C_1$ = {x contains A, x contains a B}
        c.    $C_2$ = {every box$_x$ (exh($C_1$)(x contains an A)),
                    every box$_x$ (exh($C_1$)(x contains a B)),

<div align="center">every box$_x$ (exh(C$_1$)(x contains an A and B))}</div>

d.   $\lambda$w. every box contains an A or a B in w $\wedge$ some box contains an A in w $\wedge$ some box contains a B in w $\wedge$ $\neg$every box contains an A and a B in w

*Third parse:* If the conjunctive alternative is not pruned from the domain of the embedded *exh*, the sentence entails distributive inferences and embedded strengthening of disjunction (and plain negated inferences, if disjunct alternatives are not pruned from the domain of the matrix *exh*). On this parse, represented in (67), the experimental sentence should be judged as false in all conditions in which distributive inferences or embedded strengthening of disjunction are false (which in our experiment means all conditions). (See the third row of the summary in Fig. 6.)

(67)   a.   exh(C$_2$)(every box$_x$ (exh(C$_1$)(x contains an A or a B)))
     b.   C$_1$ = {x contains an A, x contains a B, x contains an A and a B}
     c.   C$_2$ = {every box$_x$ (exh(C$_1$)(x contains an A)),
              every box$_x$ (exh(C$_1$)(x contains a B))}
     d.   $\lambda$w. every box contains an A or a B but not both in w $\wedge$ some box contains an A in w $\wedge$ some box contains a B in w

| Parse | Idealized response profile | Inferences | | | |
|---|---|---|---|---|---|
| | | Dist. | Mat. neg. conj. | Emb. neg. conj. | Plain neg. |
| (54) | [M4: 0], [D1: 8], [D2: 8] | ✓ | ✗ | ✗ | ✗ |
| (57) | [M4: 0], [D1: 8], [D2: 0] | ✓ | ✓ | ✗ | ✗ |
| (67) | [M4: 0], [D1: 0], [D2: 0] | ✓ | ✓ | ✓ | ✓ |

**Fig. 6** Idealized response profiles corresponding to the admitted parses (54), (57), (67), with a specification of what inferences are entailed (esp. distributive inferences).

*Experiment results:* In our sample, we find participants that can be classified as consistently disambiguating the experimental sentences in favor of the parse represented in (54), on which the sentence induces distributive inferences but no other inferences, and participants that can be classified as disambiguating the experimental sentence in favor of the parse represented in (57), on which the sentence induces distributive inferences and matrix negated conjunction inference.[22] There appear to be no participants that computed embedded

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[22]   The response profiles of the participants whose behavior suggests that they disambiguate the experimental sentence in favor of the parse (54) are in (i) and of those whose behavior suggests that they disambiguate the experimental sentence in favor of the parse (57) are in (ii) (the participant identification number is followed by their response profile).

(i)    S.125 $\langle$[M4: 1], [D1: 6], [D2: 8]$\rangle$, S.143 $\langle$[M4: 0], [D1: 8], [D2: 8]$\rangle$, S.152 $\langle$[M4: 1], [D1: 6], [D2: 8]$\rangle$

(ii)   S.120 $\langle$[M4: 0], [D1: 7], [D2: 3]$\rangle$, S.123 $\langle$[M4: 0], [D1: 6], [D2: 2]$\rangle$, S.229 $\langle$[M4: 1], [D1: 8], [D2: 0]$\rangle$

strengthening of disjunction − that is, participants that can be classified as consistently disambiguating the experimental sentence in favor of the parse represented in (67).[23] We return to missing embedded strengthing of disjunction in Sect. 5.4.

| Parse | Idealized response profile | Number of participants |
|---|---|---|
| (54) | [M4: 0], [D1: 8], [D2: 8] | 3 |
| (57) | [M4: 0], [D1: 8], [D2: 0] | 3 |
| (67) | [M4: 0], [D1: 0], [D2: 0] | 0 |

**Fig. 7** Number of participants exhibiting a preference to disambiguate the sentence in favor of the respective parse.

### 5.3.2 Readings without distributive inferences

There are three parses of the experimental sentence that comply with the conditions introduced above and that do not entail distributive inferences.

*First parse:* If all the alternatives are pruned from the domain of the matrix *exh* and the conjunctive alternative is pruned from the domain of the embedded *exh*, the sentences entail neither distributive inferences nor matrix or embedded negation of the conjunctive alternative. On this parse, represented in (68), the experimental sentence should be judged as true in all pertinent conditions − in particular, in condition M4, in which distributive inferences are false. (See the first row of the summary in Fig. 8.)

(68)  a.  $\text{exh}(C_2)(\text{every box}_x \ (\text{exh}(C_1)(x \text{ contains an A or a B})))$
      b.  $C_1 = \{x \text{ contains an A}, x \text{ contains a B}\}$
      c.  $C_2 = \emptyset$
      d.  $\lambda w.$ every box contains an A or a B in w

*Second parse:* If the conjunctive alternative is pruned from the embedded *exh* and disjunct alternatives are pruned from the matrix *exh*, the sentence entails no distributive inferences but it does entail the matrix negation of the conjunctive alternative. On this parse, represented in (69), the experimental sentence should be judged as true in conditions M4 and D1, in which the matrix negation of the conjunctive alternative is true, and as false in condition D2, in which the matrix negation of the conjunctive alternative is false. (See the second row of the summary in Fig. 8.)

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[23] This statement should perhaps be qualified. The behavior of the two participants that were precluded from the analysis is consistent with them computing an embedded strengthening of disjunction. In particular, on conditions in the experiment not pertaining to disjunction (filler sentences of the form *Some boxes contain an A*), they behave similarly to other participants (specifically, they respond with True to sentences that are true descriptions of the picture on their strengthened meaning).

(69)     a.   $\mathrm{exh}(C_2)$(every box$_x$ $(\mathrm{exh}(C_1)$(x contains an A or a B)))
     b.   $C_1 = \{$x contains an A, x contains a B$\}$
     c.   $C_2 = \{$every box$_x$ $(\mathrm{exh}(C_1)$(x contains an A and a B))$\}$
     d.   $\lambda$w. every box contains an A or a B in w $\wedge$ ¬every box contains an A and a B in w

*Third parse:* If the conjunctive alternative is not pruned from the domain of the embedded *exh* and disjunct alternatives are pruned from the matrix *exh*, the sentence entails embedded strengthening of disjunction. On this parse, represented in (70), the experimental sentence should be judged as true in condition M4, in which distributive inferences are false, and as false in conditions D1 and D2, in which plain negated inferences are false.[24] (See the third row of the summary in Fig. 8.)

(70)     a.   $\mathrm{exh}(C_2)$(every box$_x$ $(\mathrm{exh}(C_1)$(x contains an A or a B)))
     b.   $C_1 = \{$x contains an A, x contains a B, x contains A and B$\}$
     c.   $C_2 = \{$every box$_x$ $(\mathrm{exh}(C_1)$(x contains an A and a B))$\}$
     d.   $\lambda$w. every box contains an A or a B but not both A and B in w ($= \lambda$w. every box contains just one of A or B in w)

| Parse | Idealized response profile | Inferences | | | |
|---|---|---|---|---|---|
| | | Dist. | Mat. neg. conj. | Emb. neg. conj. | Plain neg. |
| (68) | [M4: 8], [D1: 8], [D2: 8] | ✗ | ✗ | ✗ | ✗ |
| (69) | [M4: 8], [D1: 8], [D2: 0] | ✗ | ✓ | ✗ | ✗ |
| (70) | [M4: 8], [D1: 0], [D2: 0] | ✗ | ✓ | ✓ | ✗ |

**Fig. 8** Idealized response profiles corresponding to the admitted parses (68)–(70), with a specification of what inferences are entailed (not distributive inferences).

*Experiment results:* As discussed in Sect. 4, the majority of participants can be classified as consistently disambiguating the experimental sentence in favor of the parse represented in (68), on which no SIs are computed.[25] Furthermore, there are also participants that can be classified as consistently disambiguating the experimental sentence in favor of the parse represented in (69), on which the sentence entails the negation of the conjunctive alternative to the matrix sentence.[26] Again, there appear to be no participants that computed embedded

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[24]  The sentence should also be judged as false in condition M1, in which both distributive and plain negated inferences are true, since the embedded strengthening of disjunction is false in this condition as well (there is at least one box that contains both letters mentioned in the experimental sentence; see the description of the items in the preceding section).

[25]  Each of the fifty-one participants was presented with eight items of condition D1. Among the fifty-one participants, thirty-two participants responded with True to at least seven of the eight items in condition M4. All of these participants also responded with True to at least seven of the eight items in both conditions D1 and D2. See footnote 15 for further details.

[26]  The response profile of the participant whose behavior suggests that they disambiguate the experimental sentence in favor of the parse (69) is provided in (i).

negated conjunction inferences – that is, participants that might be classified as consistently disambiguating the experimental sentence in favor of the parse represented in (70) (though see footnote 23).

| Parse | Idealized response profile | Number of participants |
|-------|---------------------------|------------------------|
| (68) | [M4: 8], [D1: 8], [D2: 8] | 32 |
| (69) | [M4: 8], [D1: 8], [D2: 0] | 1 |
| (70) | [M4: 8], [D1: 0], [D2: 0] | 0 |

**Fig. 9** Number of participants exhibiting a preference to disambiguate the test sentence in favor of a certain parse

### 5.4 Disambiguation strategies

All six of the parses of the experimental sentence that are admitted on our account should, all else being equal, be equally available to all participants; the readings they give rise to are summarized in (71). However, certain preferences appear to emerge – that is, participants' response behavior tentatively suggests that certain groups of participants systematically disambiguate the sentence in favor of certain parses. This finding is only tentative, since both the number of participants apparently exhibiting a preference for a specific disambiguation and the number of items on which this conclusion is based are low. Additional experiments are needed to determine the distribution of disambiguation strategies among participants.

(71)     *Summary of predicted readings on our proposal*
Parse: $\text{exh}(C_2)(\text{every box}_x \ (\text{exh}(C_1)(x \text{ contains an A or a B})))$
*Admitted readings:*
$C_1$ includes $\{x \text{ contains an A, } x \text{ contains a B}\}$

- No additional inferences
- Matrix negated conjunction inference, no other inferences
- Embededded negated conjunction inference, no other inferences
- Distributive inferences, no other inferences
- Distributive inferences, matrix negated conjunction inference
- Distributive inferences, embededded negated conjunction inference

*Precluded readings:*
$C_1 = \emptyset$

- Plain negated inferences, no embedded negated conjunction inference

There are further factors besides the grammatical ones discussed above that might influence the selection of a particular parse for a sentence and thus yield limitations on the attested readings, such as the lack of embedded strengthening of disjunction in our experiment. An important factor is, arguably, whether

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(i)     S.235 $\langle$[M4: 8], [D1: 6], [D2: 1]$\rangle$

the respective reading is relevant in the context (see, e.g., Gualmini et al. 2008, Singh et al. 2013). We speculate that our failure to find participants who compute embedded strengthening of disjunction might be due to the respective reading not being made relevant enough in our sentence-picture pairs. That is to say, as discussed by Chemla and Spector (2011), embedded SIs based on substitution alternatives are difficult to detect in sentence-picture matching tasks unless special care is taken in the construction of pertinent experimental items. A detailed investigation of this hypothesis would, however, require more space than we can allot to it here.

To summarize, Sect. 4 presented experimental data that suggest that disjunction under universal quantifiers gives rise either (a) to distributive inferences in the absence of plain negated inferences or (b) to no distributive inferences at all. We have shown that this pattern can be construed as following from an interaction of the constraint on pruning and a principle of exhaustification at every scope position (Magri 2011) – both of which have been independently motivated. Although the two conditions correctly rule out certain parses, ambiguity in exhaustification is still permitted and, to some extent, reflected in participants' behavior. We have suggested that further factors may be involved in what disambiguations are chosen by the parser, such as relevance given a question under discussion. We must leave the development of a more comprehensive theory of parsing of exhaustified structures for another occasion.

## 6 Conclusion and outlook

Disjunction in the scope of a universal quantifier, *Every A is P or Q*, tends to give rise to distributive inferences, *Some A is P & Some A is Q*, which are inferences that bear telltale signs of SIs. We have shown in Sect. 2 that these inferences are not necessarily accompanied by plain negated inferences, *¬Every A is P & ¬Every A is Q*, which constitutes the puzzle about distributive inferences. We have seen that although this puzzle is problematic for approaches to distributive inferences that take them to be generated by matrix exhaustification based on plain alternatives, as described in (14), it can be resolved on approaches that provide for matrix exhaustification based on exhaustified alternatives, as per (21). We have shown that exhaustified alternatives are naturally available on the grammatical approach to SIs, combined with the standard assumption about alternatives, not least because the grammatical approach to SIs provides for embedded exhaustification. Approaches to SIs that do not provide for embedded exhaustification might be able to resolve the puzzle by not adopting the standard assumption about alternatives. The remainder of the paper discussed an apparent tendency among, at least, participants in our experiment to compute distributive inferences in the absence of plain negated inferences – a tendency that we proposed springs from the constraint on pruning and exhaustification being obligatory at every scope position.

The behavior of disjunction in the scope of universal quantifiers as well as our analysis of it raise several questions that we hope to pursue in the future. We conclude the paper by mentioning a few of them. They pertain to the embedding of disjunction in the scope of quantificational elements other than universal nominal quantifiers and to the parsing of exhaustified sentences more generally. First: We have not discussed distributive inferences in the scope of nominal quantifiers other than universal quantifiers. However, on the face of it, distributive inferences in the absence of corresponding plain negated inferences appear to be available with other quantifiers as well and can be derived in the framework described in this paper. For example, although the sentence in (72) is infelicitous, say, in a context in which none of my friends have a daughter, it is acceptable in a context in which many of my friends have a son and some of them also have a daughter. In parallel to our examples in this paper, this fact can be explained by recourse to embedded exhaustification.[27] It goes without saying that a more in-depth exploration of these issues will be necessary to test and possibly fine-tune our proposal.

(72)    Many of my friends have sons or daughters.

Second: The behavior of disjunction under modal quantifiers, both universal and existential ones, appears to differ from its behavior under nominal quantifiers. For example, sentences like (73) are judged as infelicitous in contexts in which plain negated inferences are false, say, in which there is a requirement to wear sneakers in the gym. We hope to tackle the differences between nominal and modal quantifiers in this respect, and their source, on another occasion.[28]

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[27] The sentence in (ia) can trigger distributive inferences, (ib), in the absence of corresponding plain negated inferences, (ic).

(i)     a.    Many of my friends have sons or daughters.
        b.    Some of my friends have sons $\wedge$ Some of my friends have daughters
        c.    $\neg$Many of my friends have sons $\wedge$ $\neg$Many of my friends have daughters

This reading follows from the parse in (ii), where there are three scope sites for the exhaustification operator: below the distributivity operator (cf. Schwarzschild 1996), above the distributivity operator but below the existential quantifier, and at the matrix level.

(ii)    $\mathrm{exh}(C_3)$(many friends$_x$ ($\mathrm{exh}(C_2)$(x DIST$_y$ ($\mathrm{exh}(C_1)$(y has sons or daughters)))))

If substitution alternatives are pruned from the domains of the embedded exhaustification operators and, say, all alternatives are pruned from the domain of the matrix exhaustification operator, the parse entails distributive inferences, (ib), in the absence of plain negated inferences, (ic).

[28] Standard SIs appear to us to be unavailable in the scope of modals (see Ippolito 2010, 2011 for discussion of the unavailability of embedded SIs in certain modal environments). Disjunction under modals giving rise to distributive inferences by way of plain negated inferences might, accordingly, follow from the unavailability of embedded exhaustification in this environment (recall that in the absence of embedded exhaustification, distributive inferences follow from plain negated inferences). An account along these lines would require a qualification of the principle of obligatoriness of exhaustification, stated in (61), effectively allowing for exhaustification to be obligatory only in specific environments, i.e., in environments in which it is available.

(73)     You are required to wear sneakers or running shorts.

Third: In Sect. 5 we have touched upon the fact that the grammatical approach to SIs predicts that exhaustified sentences are multiply ambiguous. We think that the results of our experiment suggest that this prediction is correct (see Sect. 5.4). However, there appear to be preferences among possible disambiguations of exhaustified sentences. In addition to the constraint on pruning in (37) and the principle of obligatory exhaustification in (61) – two grammatical principles – further factors may be involved. Although we have presented some speculative remarks pertaining to some of these factors and how they relate to our results, a more detailed exploration of these issues still remains to be undertaken.

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