Hints on 18.466 PS 5 due Friday, March 14, 2003

1 = 2.4-1 <u>Clarification:</u> $0 < c < \infty$. If X has density f_c, $EX = \int_0^\infty x f_c(x)\,dx$ and $E(X^2) = \int_0^\infty x^2 f_c(x)\,dx$ are known or can be found via integrating by parts. We have $E(\bar{X}) = EX_1$ and $Var(\bar{X}) = Var(X_1)/n$ for any $X_1, \dots, X_n$ i.i.d. with finite variance.

2 = 2.4-3. $\int_0^1 x^{a-1}(1-x)^{b-1}dx = B(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$ for any $a, b > 0$ where $\Gamma(k) = (k-1)!$ for $k = 1, 2, \dots$. Thus find $EX_{(1)}$ and $Var(X_{(1)})$ for $U[0,1]$ and for $U[\theta, \theta+h]$. So find an unbiased estimator of θ and its variance.

3 = 2.4-4 (a) For the binomial(n,p) distribution the mean is np and the variance is $np(1-p)$. $f(p,j) = p^j(1-p)^{n-j}\binom{n}{j}$. Put the score function $\frac{\partial}{\partial p}\log f(p,j)$ over a common denominator.

(b) X with this geometric distribution has $EX = 1/p$ and $Var(X) = 1/p^2$?

4 = 2.5-2. Use (2.5.1) in the extended form stated just after it,

$$f(p,x) = C(\theta(p))\,h(x)\,e^{\sum_{j=1}^k \theta_j(p)T_j(x)}$$

k can be reduced from 2 to 1 if $\theta_1(p)$ and $\theta_2(p)$ are affinely dependent, <u>or</u> if $T_1(k)$ and $T_2(k)$ are.