

1. Problem 1.5.6.

2. Problem 1.7.1.

3-4. On the two-point sample space $\{0, 1\}$, let $P(1) = Q(0) = 1/(t+1)$ and $P(0) = Q(1) = t/(t+1)$ for some $t > 1$. Let the losses when one law is true and the other is chosen be equal, $L_{PQ} = L_{QP} = A$ for some $A > 0$ and let the cost per observation be $A/4$.

(a) Let the prior $p = \pi(P)$. If $p \leq 1/4$ or $p \geq 3/4$ evaluate the risk $R(p)$.

(b) What is the Bayes sequential test of P vs. Q in each range of p in part (a)?

(c) From here on suppose that $t > 9$. Then among all sequential tests taking at least one observation, find one minimizing the risk for each p with $1/4 < p < 3/4$. *Hint:* in this case the posterior p_x given one observation $x = 0$ or 1 will be $< 1/4$ or $> 3/4$ and then parts (a) and (b) suggest what the decision should be.

(d) Evaluate $r(p)$, the minimal risk if one takes at least one observation, for $1/4 < p < 3/4$ (this is the risk of the test found in part (c)).

(e) Find the unrestricted minimal risk $R(p)$ and a Bayes test for every p , $0 < p < 1$ (see the diagram in Fig. 1.7A).

(f) Show that $SPRT(A, B)$ is inadmissible in this case (for the given losses and cost per observation, with no prior specified) if $A < 1/t < 1 < t < B$.