

18.466 PS9, due Wednesday, April 30, 2003

1. Problem 3.6.1.
2. Problem 3.6.2.
3. Problem 3.6.3.
4. Consider $\psi(\theta, x) = \rho'(x - \theta)$ for ρ equal to wide-sense Huber function (b) on p. 6 of section 3.4, $\rho(x) = (c^2 + x^2)^{1/2}$ for some $c > 0$. Take $c = 1$. Verify that in section 3.5, conditions (B-1) through (B-4) all hold for any law P . *Hints:* for (B-3), show that $\lambda'(\theta) < 0$ for all θ , and find limits of $\lambda(\theta)$ as $\theta \rightarrow -\infty$ or $+\infty$.
5. Consider the narrow-sense Huber functions, (c) on p. 6 of section 3.4, where for some $b > 0$, $\rho(x) = x^2$ for $|x| \leq b$ and $c|x| + d$ otherwise, where c and d are chosen to make ρ a C^1 function. Show that in this case there is always a θ_0 such that $\lambda(\theta_0) = 0$ (by the intermediate value theorem: show that $\lambda(\cdot)$ is continuous, positive at some θ and negative at some other θ). Show however that if a law P has an interval of medians longer than $2b$, in other words its distribution function $F(x) = P((-\infty, x])$ is equal to $1/2$ on such an interval, then $\lambda(\theta)$ is not 0 at a unique point θ_0 but is zero on some interval (u, v) with $u < v$.