18.466 midterm exam, Wednesday, April 2, 2003, 2:05-2:55 P. M.

Closed book exam. No books, notes, or calculators may be consulted during this exam. (Calculators are irrelevant since no questions involve numerical calculation.)

1. (a) Define the likelihood ratio $R_{Q/P}$ for two probability measures P and Q.

(b) Give a statement of the Neyman-Pearson lemma without any losses L_{PQ}, L_{QP}, or priors $\pi(P), \pi(Q)$. As suggested in the review session, you can give a formulation that makes little or no reference to randomized tests.

(c) Give a further statement with losses and priors.

2. (a) Define sequential probability ratio tests (SPRTs).

(b) State the main optimality theorem about SPRTs.

3. (a) Define what it means for a statistic to be pairwise sufficient for a family of laws.

(b) For a family dominated by a σ-finite measure, state the factorization theorem for the likelihood function in relation to a (pairwise) sufficient statistic.

(c) Define exponential families.

(b) For n i.i.d. observations from an exponential family as defined in part (c), give a sufficient statistic whose dimension doesn't depend on n.

4. (a) Given a prior distribution on a parameter space for a family of probability laws, define posterior distributions.

(b) For estimation of $g(\theta)$ having values in $\mathbb{R}^d$ with squared-error loss $\|T - g(\theta)\|^2$, if there is an estimator with finite risk, give a formula for a Bayes estimator.