Open Economy, Reform, and Learning.

by

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Abstract

This collection of papers study several aspects of the development process of Latin America.

Chapter two studies the optimal exchange rate policy when the economy is under a process of reform. During the last two decades, many Latin American countries engaged in disinflation programs based on both exchange rate management and fiscal reforms. However, in most instances, part of the fiscal reform was delayed or not implemented completely, so the fiscal deficit increased and the program had to be abandoned. The aftermath of these programs is not encouraging, since most of these policies turned out to be failures, lowering reserves and causing higher inflation rates. Given this record, it is worth asking why governments start a disinflation program even though the fiscal equilibrium is not guaranteed. In this chapter, I show that, if the reform process is uncertain and inflation has welfare costs, the optimal exchange rate policy implies the initiation of a disinflation program at the announcement of the fiscal reform.

Chapter three analyzes alternative sources of volatility in the region. It has been shown that Latin American economies are more volatile than industrialized countries. In this chapter, it is proved that when the source of uncertainty is the quality of the government, a capital account opening increases output variability. For suitable calibrations in the case of Latin American, it was possible to explain as much as 30 percent additional volatility.

Chapter four investigates the transition mechanism of international interest rates through the banking sector. It has been widely argued that learning is an important aspect of the credit market, both in the development of lender-firm relationships as well as in the formation of reputation. This chapter studies how the propagation mechanism of the interest rate changes when learning takes place in the credit market. I show that several facts reported in the credit market literature can be accounted by the model.

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Chapter 1

Introduction.

This collection of essays consist of three separate papers that study different aspects of the development process in Latin America.

The second chapter deals with the optimal exchange rate policy during a process of reform. It has been observed that many Latin American countries engaged in disinflation programs based on both exchange rate management and fiscal reforms. However, in most instances, part of the fiscal reform was delayed or not implemented completely, so the fiscal deficit increased and the program had to be abandoned. The aftermath of these programs was not encouraging, since most of these policies turned out to be failures, lowering reserves and causing higher inflation rates. Given this record, it is worth asking why governments started a disinflation program even though the fiscal equilibrium was not guaranteed. In this chapter, I show that under mild assumptions it is possible to rationalize the government's behavior. In particular, if the reform process is uncertain and the inflation rate has welfare costs, the optimal exchange rate policy implies the initiation of a disinflation program at the announcement of the fiscal reform.[1]

The third chapter studies sources of macroeconomic volatility in the region. As it is shown in the 1995 Economic and Social report of the Inter-American Development Bank, Latin American countries are subject to high volatility in comparison to industrialized economies. This higher

[1] As is shown in the chapter, this result is robust to alternative formulations of the problem. The most important extensions are: First, the consideration of balance of payments crisis possibilities. Second, the introduction of real exchange rate appreciations. Third, the possibility that several fiscal reforms are implemented through time.

variability in the region is reflected in every aspect of the economy: relative prices, quantities, and policies. In this chapter, I try to understand why Latin America is so volatile.[2] A simple theoretical model that generates large differences in countries' volatility is developed. It is assumed that there is two sources of uncertainty: government's quality and idiosyncratic output shocks. I compare what occurs when two economies that have the same output volatility when they are closed, engage in a capital liberalization. It is shown that the economy that has a larger share of uncertainty coming from the government's type experiences higher output and investment volatility. For suitable calibrations in the case of Latin American, it was possible to explain as much as 30 percent additional volatility. I argue that informational problems about the government's ability and quality are more frequent in the Latin American region than in Industrialized economies.

Finally, chapter four concentrates in the transmission mechanism of international interest rates.[3] The motivation of the model is based on the observation that bank-firm relationships are important in the availability of credit.[4] One of the important aspects of this relationship is the learning process that takes place between banks and firms. I concentrate on those aspects that are unknown to both the bank and the firm. This allows to abstract from asymmetric information problems which simplifies the optimal contract.[5] I study the dynamic behavior of the credit market to a permanent shock to the deposit's real interest rate. The model shows that when informational aspects are important, there is a strong bias against small firms during a recession. This behavior is consistent with the recent Mexican experience.[6] Moreover, if the increase in the deposit rate is interpreted as a tightening in monetary policy, then the model reproduce several facts reported in the credit market literature. First, it generates amplification of interest rate. Second, it produces cross sectional implications that are consistent with the flight to quality effect. Finally, it has persistence effects.

[2]The chapter concentrates in explaining differences in output volatility, however, the intuitions can be easily extended to other aspects of the economy.

[3]This chapter is also related to the literature of the credit market, and therefore, common interpretations are given.

[4]See Hoshi, Kashyap and Scharfstein (1990) and Petersen and Rajan (1994).

[5]If asymmetric information problems are considered, then ratchet effects are usually found. See Laffont and Tirole (1993). This complicates the analysis, however similar results are obtained.

[6]See Babatz and Conesa (1997).

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Chapter 2

Disinflation and Fiscal Reform: A Neoclassical Perspective.

2.1 Introduction

During the last two decades, many Latin American countries engaged in disinflation programs based on both exchange rate management and fiscal reforms. However, in most instances, part of the fiscal reform was delayed or not implemented completely, so the fiscal deficit increased and the program had to be abandoned. The aftermath of these programs is not encouraging: since most of these policies turned out to be failures, lowering reserves and causing higher inflation rates. Given this record, it is worth asking why governments start a disinflation program even though the fiscal equilibrium is not guaranteed. A more sensible strategy would be to stabilize the fiscal accounts first, and then reduce inflation.

In this paper we show that, under reasonable assumptions, it is possible to rationalize the government's behavior. In particular we show that, if the reform process is uncertain and inflation has welfare costs, the optimal exchange rate policy implies the initiation of a disinflation program at the announcement of a fiscal reform. Moreover, the reduction in seignorage is financed by reserves. Additionally, we show that even if there exists a possibility of a balance of payments crisis, it is still optimal to initiate a disinflation program. Finally, we show that it is optimal to engage in a sequence of stabilization programs until one of them is successful, or until a balance of payments crisis occurs.

The intuition of why the disinflation program is adopted is as follows: Consider an economy with only one tradable good and where the government finances its expenditures with inflation tax and reserves. Assume that the government maximizes the same consumer's utility and that both share the same information. Finally, assume that the level of expenditure has been constant and the economy is in steady state. At time zero, the government suddenly realizes that part of its expenditure is wasteful and decides to reduce it, although it has to convince other bureaucrats to do so. The outcome of the negotiations is unknown and therefore neither the government nor the consumers know when the expenditure is going to fall. However, the announcement of the willingness to reduce expenditure reduces the expected equivalent annuity of expenditures; thus, the net present value of taxes should decrease too. If taxation, and in particular seignorage, generates convex welfare costs, the consumers care about the path of inflation. In this context, the optimal policy is to spread the costs equally across time, which implies that inflation falls on the announcement of the fiscal reform. Moreover, if the reform never takes place and the disinflation program has to be abandoned, the ex-post inflation rate is higher than the one that existed before the program was initiated. This is because the disinflation was financed by increases in government debt and reductions in reserves. Therefore, it looks as if the government made a mistake when they implemented the program in the first place.

The three main ingredients of the model are the following: expenditure has a negative drift; inflation has welfare costs, and a disinflation program is costly. In the expenditure process the negative drift comes from the assumption that the government is implementing an expenditure reduction[1], while the uncertainty comes from the assumption that the reform requires negotiations with other agencies, the outcome of which is unknown.[2] In practice, governments are able to reduce expenditure and fiscal deficit in many different ways. There are short run measures that are relatively easy to implement, such as elimination of subsidies, reduction in public investment, delay in the increase of public sector wages, etc. However, some of these

[1] Note that we assume that the expenditure is wasteful, thus, its reduction is desirable from the consumer point of view. An alternative approach would be to assume that expenditure affects output and consumer's utility. However, if it still is desirable to reduce expenditure, then the results are going to be qualitatively the same. The assumption here adopted simplifies the problem.

[2] Alesina and Drazen (1991) provide a theoretical explanation of why these negotiations might require time. Also, see Alesina and Perotti (1996) and Guidotti and Vegh (19XX).

measures are not sustainable, and in a model of perfect foresight agents, ineffective. In other words, the present value of the deficit does not change and therefore, there is no effect on consumer's decisions. In this paper, we are interested in the long run and more permanent fiscal reforms, such as privatization, social security reforms, labor market liberalization, reduction in the size of the government, new tax laws, etc. In general, when these reforms are implemented there is a permanent change in the fiscal deficit process that will affect consumer's choices. However, these reforms require negotiations with congress, unions, and industries, and the experience of several Latin American countries has shown that their implementation is difficult, time-consuming, and sometimes unsuccessful.[3] .

In the model, we assume that inflation is the only available tax, and that it generates convex welfare costs. First, the assumption that inflation is the only available tax is capturing the fact that in Latin America, seignorage has been an important share of the government's revenue, especially before the reform. Moreover, it also reflects the fact that it is the marginal instrument used to raise revenue. In other words, the tax system is rigid and the only two sources the government has to finance a shock are reserves and seignorage.[4] The second assumption is that inflation has welfare costs, and in particular, that they are convex. In the literature, there are several papers that discuss the nature and measures of these costs.[5] In this paper, we simplify and capture them with a concave utility function and a cash in advance constraint. Additionally, this particular formulation can be interpreted as a tax smoothing problem, where inflation is a distortionary tax. Barro (1979) showed that when taxes are distortionary, the

[3]There are several examples of unsuccessful attempts at reform. In 1989, Venezuela started a disinflation program when the average income tax was four percent. That year, a new tax law (including a VAT) was introduced in congress. However, the VAT was not approved until 1993, long after the disinflation program was abandoned. Argentina is another example. Since 1991, it has been trying to obtain approval for a new labor law. However, it has not been successful yet.

[4]The assumption that inflation is the only tax eliminates issues of optimal composition of taxes, and the Olivera-Tanzi effect. In the literature on optimal inflation tax see Phelps (1973) for the first contribution. Additionally, Fischer (1983) studies optimal inflation tax in the context of different exchange rate regimes, Vegh (1989a) studies it in the context of currency substitution, and Aizenman (1987), De Gregorio (1993), and Vegh (1989b) study it in the context of different degrees of efficiency in the tax system. For the Olivera Tanzi effect see the seminal contributions by Olivera (1967) and Tanzi (1978). The exclusion of these issues simplifies the analysis; however, it is important to mention that if those aspects are introduced in the model, the results hold.

[5]Several authors had measured the welfare costs of inflation. The literature started with Bailey (1956), Fischer (1981) and Lucas (1981) where they argue that the welfare cost of moderate inflations is low. Recent constributions include Colley and Hansen (1989, 1991), İmrohoroğlu (1992), İmrohoroğlu and Prescott (1991), and Jones and Manuelli (1993). In general, the literature agrees on important welfare costs at high inflation rates.

optimal policy is to spread the tax burden across time: *tax smoothing*. In our case, the tax smoothing result implies inflation smoothing.[6] Note that the smoothing motive justifies the implementation of the disinflation program. In other words, the announcement of the fiscal reform implies that future welfare costs might be smaller. If the cost function is convex, then consumers want to transfer part of the future benefits to today, which implies reducing current inflation.

Finally, a disinflation program is costly because it deprives the government of a source of revenue, thus reducing reserves. This loss in reserves today leads to a higher level of inflation in the future, as the government seeks to recover revenue. This is the Sargent and Wallace (1980) effect, which in our case, appears as a reduction in reserves, rather than as an increase in debt. In practice, a disinflation program is costly, not only in loss of reserves, but in several and probably more important ways, such as recessions, loss in credibility in future programs, etc. In this paper, we oversimplify capturing those costs in the change in reserves.

The paper is organized as follows: In the next sub-sections we study some of the Latin American stabilization experiences and highlight the general aspects they share. Then we analyze what are the explanations the literature has advanced to justify the adoption of the programs.

In section two, we present the basic setup. In section three, we solve the model when there is no constraint on the level of reserves. We show that the optimal policy at the announcement of a fiscal reform, is a managed exchange rate with a depreciation rate smaller than the one implied by flexible exchange rate. In other words, an exchange rate based disinflation program has been initiated even though the fiscal accounts are not in equilibrium. This basic model captures two important facts: First, governments are willing to initiate a disinflation program at the announcement of a fiscal reform, financed with a reduction in reserves. Second, if the fiscal reform fails and the program had to be abandoned. There is a worsening of the economic situation of the country. The inflation rate is higher, and the level of reserves and consumption are lower compared to those that existed before the program was initiated.

In section four, we extend the model to include the possibility of a balance of payments

[6] It is important to mention that the assumption that the welfare costs are convex can be relaxed. It can be shown that if the concavity of seignorage is larger than the concavity of the welfare costs of inflation, then the optimal strategy is to smooth inflation.

crisis; we assume that there is a constraint on the minimum level of reserves. If the optimal unconstrained policy leads to a negative level of reserves, the balance of payments crisis limits the degree of smoothing that can be achieved. However, we show that the optimal policy still implies the implementation of a disinflation program at the announcement of a fiscal reform. The additional implication of this model is that there exists a positive probability of a balance of payments crisis. Notice that this does not mean that a balance of payments crisis is optimal, but rather that it is optimal to initiate a stabilization program even if there exists a probability of failure. In other words, avoiding the crisis at any cost is suboptimal.

In section five, we allow the government to initiate a new fiscal reform after the previous one had failed. We show that it is optimal to implement a sequence of stabilization programs until one of them is successful. An implication of the model is that every time a disinflation program fails, the inflation rate is higher than the initial one, and the next program is more costly to implement. Moreover, although each failed stabilization program is costly in terms of deteriorating the country's situation, it is still optimal to initiate a new one. This explains the experience of several Latin American countries, especially Brazil in the late 80's and Venezuela in the 90's.

In section six, we introduce real exchange rate considerations. It has been widely argued, in some of the unsuccessful stabilization programs, that the exchange rate was overvalued prior to the abandonment. In this section, we show that the optimal exchange rate regime implies a real exchange rate appreciation during the stabilization. Moreover, if expenditure is not adjusted and the program has to be abandoned, it looks as if the exchange rate was overvalued during the stabilization. Section seven concludes and offers recommendations for future research.

2.1.1 Latin American Experiences.

In this section we study some Latin American stabilization programs. We are interested in characterizing the typical stabilization experience; both from the fiscal and the inflation point of view.

In table 2.1, we classify some of the stabilization programs in the last 30 years, according to two criteria: whether it was successful or not in permanently reducing inflation, and whether the fiscal deficit was reduced before, during or never after the program was initiated. This list

is not exhaustive, although suggestive. We define that an stabilization is unsuccessful when inflation increases above the initial level or when another stabilization program is initiated. On the other hand, we decide that there was a fiscal effort is the fiscal deficit changed by more than ten percent of GDP or there is a fiscal surplus.[7] Two main points can be extracted from this table: First, notice that there are few successful cases. Second, notice that there are few cases where the fiscal deficit was reduced before the stabilization program. More importantly, these two cases (Chile 78 and Uruguay 79) were preceded by another stabilization program (Chile 75 and Uruguay 74).[8]

In figure 2-2, we show the path of the inflation rate, the expenditure (not including interests) and the fiscal deficit for several Latin American countries. The vertical dashed lines represent the years where a disinflation program was initiated. There are six points to be highlighted: First, in most of the cases, prior to a stabilization program there is a historically high level of inflation and fiscal deficit. Second, inflation falls relatively fast. Third, when the disinflation program is abandoned the inflation rate is usually higher than the initial one. Fourth, expenditure measured as a percentage of GDP falls relatively slowly, and in some cases it never falls. Fifth, in almost all cases, the fiscal deficit reduces on impact but it returns to the original level within 2 years of the program. In other words, the short term reductions in fiscal deficit are usually reversed.[9] Sixth, countries implement several disinflation programs.

[Figure 2-2]

Some evidence of the gradual process of reform can be obtained looking at the path of privatizations. In figure 2-1, we show the experience of four countries. Note that all the countries initiated their process of privatization several years after the implementation of the stabilization program. This is suggestive evidence that fiscal reforms take time.

[7] In other words, in Bolivia (1985) the fiscal deficit was reduced from 30 percent of GDP to 5 percent. Even though the country still had a considerably large fiscal deficit we assume that enough efforts were made. On the other hand, if the fiscal deficit decreases from 6 percent to 5 percent we decide that no effort was made.

[8] It is important to mention that not all unsuccessful programs were abandoned because a fiscal disequilibrium occurred. For example, in Chile 78 the program was abandoned because inflation was too inertial and the program was ineffective in reducing inflation, and not because there was an increase in the fiscal deficit.

[9] The fiscal deficit returns to its original level because, in general, the fiscal deficit is reduced in the short run by using transitory measures, such as reductions in public investment, delay in the increase of government wages, taxes on financial transactions, etc.

[Figure 2-1]

In summary, a typical Latin American country starts a disinflation program when there is a problem of high inflation and fiscal deficit. To take care of the inflation problem a nominal anchor is implemented, while to take care of the fiscal problem, a fiscal reform is initiated. However, the fiscal reform takes time and the deficit or the expenditure are not reduced at the speed the government thought. In the end, the monetary policy is abandoned and the economy returns to a higher level of inflation. In this situation, a new stabilization program is announced and the cycle starts all over again.

2.1.2 Related literature

This paper is mainly related to the literature on adoption of exchange rate regimes. There are several reasons that can justify the implementation of a disinflation program. First, governments can be irrational or ignorant. Even though this explanation has been advanced by several economists, we will try to explain as much as we can assuming that the government is rational.

Second, if the economy is working in the wrong side of the Laffer curve a reduction in inflation increases revenue. This is the Olivera-Tanzi effect (see Olivera (1967) and Tanzi (1978)). The idea is that there is a reduction in revenue due to the lag that exists between the realization of income and the time income tax is paid. In this context, there are reasons to initiate a stabilization to move the economy to the left hand side of the Laffer curve. Moreover, this effect implies that no fiscal effort is required. This is a very important aspect of disinflations, however, this effect implies that balance of payments crisis should be rare and that more programs should be successful.

Third, it can be thought that the disinflation is the result of an optimal tax choice. For example, consider an economy that has a high inflation tax and a low income tax. Moving toward the optimal tax portfolio implies a reduction in inflation and an increase in income tax. This kind of tax recompositions are common in the Latin American experience. However, this explanation implies that the disinflation programs should be financed by other taxes and not by reserves. Moreover, this explanation implies that balance of payments crisis should be rare. It is important to mention that this is a very important component of the reform process. In fact,

a new tax system is almost always part of the reform, however, the timing of the disinflation is difficult to reconcile with the path of the other taxes.

Fourth, the disinflation can be thought as a commitment or disciplinary device. If there is a conflict between the central bank and the government, and the central bank is the stronger one, then the monetary authority initiates a managed exchange rate to force the fiscal authority to reduce expenditure. The European disinflation experiences are an example. However, there are two reasons why we think this can not be all the story in Latin America: First, central bank independence is a relatively new concept for the continent. Second, in general, we observe that the monetary authority abandons the policy, and not the converse.

Finally, there are political economy models that concentrates on the adoption of the exchange rate regime. Tornell and Velasco (1995) study the political economy aspects of the adoption of fix versus flexible exchange rate. They have a political economy model that explains when a fixed exchange rate is more likely to be adopted. Political economy reasons are an important component of the story. However, in this paper, we want to abstract from these issues and explore additional explanations.

This paper is also related to the literature that studies the welfare costs of inflation, the literature on tax smoothing and the literature on the costs of disinflation programs. Finally, in some extent, this paper is related to the literature that studies disinflation programs based in exchange rate management. Due to constraints in space, it is impossible to do justice to these vast literatures.

In the welfare costs of inflation see Bailey (1956), Fischer (1981) and Lucas (1981) for the initial contributions. They argued that the welfare costs of low inflation rates are small: 0.5 percent of GDP. For recent contributions see Ball (1991), Chari, Christiano and Kehoe (1991), Colley and Hansen (1989, 1991), İmrohoroğlu (1992), İmrohoroğlu and Prescott (1991), and Jones and Manuelli (1993). In general, the literature agrees on finding important welfare costs at high inflation rates. On the other hand, less agreement occurs at low and moderate inflation rates. In particular, the measured costs do not justify the excess importance consumers give to the inflation. However, several steps had been given in order to understand this puzzle.

This paper is related to the literature on tax smoothing. See Barro (1979) for the seminal contribution. Additional literature on tax smoothing includes, Barro (1988), Calvo and Guidotti

(1992), Ball and Mankiw (1994), Mankiw (1984) and Saint-Paul (1994). In our model, we study the exchange rate implications of one of this particular taxes: inflation.

In this paper a disinflation program is costly because future inflations might increase. This is the Sargent and Wallace (1980) effect. The idea is that reducing inflation today implies an increase in debt that in the future requires a higher inflation rate. Liviatan (1984, 1986) presents a more general setup. Other costs of disinflations had been studied by van Wijnbergen (1988). In our model, we concentrate on the debt cost of disinflations.

Finally, this paper is related to the literature studying disinflation programs under exchange rate managements. The two main aspects where the literature had concentrated are: the boom-recession cycle, and the increase in real interest rates. First, there is boom-recession cycle after the implementation of an exchange rate based stabilization program. This has been explained by Rodriguez (1982) and Calvo and Vegh (1993). In both cases the cycle comes from inflation inertia. In the first model, it is exogenously assumed, while in the second one it is modelled as a lack of credibility in the program. Second, there is an increase in real interest rates on the implementation of the program. Two explanations has been advanced: First, when money is an additional anchor in the context of capital controls (Calvo and Vegh (1993)) a tight monetary policy implies an increase in real interest rates. Second, if the program is not fully credible, an increase in the risk premium increases the real interest rate. (Velasco (1993), see Agenor and Montiel (1996) for a survey).

2.2 Basic Model.

Consider an economy where there is a single tradable good and where PPP holds. Assume there is perfect capital mobility and zero foreign inflation. These assumptions imply that the domestic nominal inflation rate is equal to the rate of depreciation, and the domestic interest rate is equal to the depreciation rate plus the foreign real interest rate (assumed to be constant).[10] There are three agents: an infinitely lived representative consumer, the government and the central bank.

[10] We assume that there is no growth in the world economy and that it is in steady state, thus the international real interest rate is equal to the discount rate.

2.2.1 Consumers

Consumers choose their consumption path and portfolio holdings taking as given the exchange rate policy. Formally, the consumer's problem is,

$$\max_{\{c_t\}} \; E \int_0^{\infty} \ln c_t e^{-\rho t} dt \tag{2.1}$$

$$\begin{aligned} s.t. & \\ \dot{a}_t &= \rho a_t + y - c_t - i_t m_t \\ c_t &\leq \frac{1}{\alpha} m_t \\ \lim_{t \to \infty} a_t e^{-\rho t} &= 0 \end{aligned}$$

where c_t is consumption, y is output (assumed to be constant), a_t are the asset holdings denominated in tradables, m_t denotes money balances in terms of tradables, ρ is the discount rate (assumed to be constant), and i_t is the nominal domestic interest rate. The first equation is the consumer's objective function. The second one is the budget constraint in terms of tradables. The third one is the cash in advance constraint. And the fourth one is the transversality condition on consumer's assets.

There are four technical assumptions that simplify the model. First, we assume that consumers do not derive utility from government expenditure. Second, we assume that output is exogenously given.[11] Third, we adopt a cash in advance formulation.[12] And fourth, we assume log utility.[13] It can be shown that the qualitative results still hold if these assumptions are

[11] Relaxing these two assumptions does not change the results. If output depends on the level of expenditure or consumers derive utility from public expenditure, this makes the expenditure reduction less desirable. However, if reducing expenditure is welfare improving, then there is a reduction in tax requirements in the future and the results still hold.

[12] An equivalent formulation is one where money enters in the utility function. The same general results hold with the exception that the path of money holdings might be different. Cash in advance assumes that money and consumption are complements, and money in the utility function relaxes this assumption. We choose a cash in advance formulation because it captures the distortionary inflation tax in a simpler way.

[13] The choice of log utility simplifies the consumer's solution making current consumption independent of the future interest rate path. The reason is that the income and substitution effects cancel each other. A different utility function implies that current consumption is a function of the future path of interest rates, which unnecessarily complicates the central bank's problem. The log utility implies that the announcement of the reform does not have effect on current consumption unless current interest rates change. A different utility function implies that some intertemporal substitution will be made by the consumers. However, full smoothing

relaxed. In appendix 2.8.1 we show that the solution for the consumer's problem is,

$$c_t = \frac{y + \rho a_0}{1 + \alpha i_t} \tag{2.2}$$

$$m_t = \alpha \frac{y + \rho a_0}{1 + \alpha i_t} \tag{2.3}$$

2.2.2 Government

The government finances an exogenous expenditure on tradables, by inflationary tax and interest earnings on reserves. We assume that the government expenditure has no impact on output or the consumer's utility; it is wasteful expenditure. At time zero, the government announces an uncertain fiscal reform, in the sense that it is not sure when it can be implemented or if it will ever be. We assume that all agents have the same prior about the probability of success of the fiscal reform.[14]

Assume that the expenditure's process is the following,

$$g_t = \begin{cases} g_h & & & t < \tau \\ g_h & wp & 1-q & t \geq \tau \\ g_l & wp & q & \end{cases} \tag{2.4}$$

where q, τ and $g_h > g_l$ are exogenously given. Define the expenditure improvement as $\Delta g = g_h - g_l$. Define the bad state of the world as the state in which expenditure is not reduced, and the good state of the world as the state in which expenditure is permanently reduced. There are three technical remarks about the process: First, the timing of the adjustment is known, but not its outcome. Second, the expenditure process is exogenous. Third, the drift of the process is negative. The first remark can be relaxed and the results still hold (see the appendix). The second remark is made for simplicity. The question we are trying to address is why countries peg their exchange rates, conditional on having a fiscal reform in place. In other words, we

is only achieved if there is tax smoothing (this result comes from Barro (1979)). So, still it is the case that the optimal strategy will involve inflation smoothing.

[14] Notice that a lack of credibility can be modelled assuming that the consumer's prior about the probability of success is smaller than the government's one.

know that the typical advice given to a Latin American country is to take care of the fiscal accounts first, and then any management in the exchange rate is not risky. However, countries consistently do not follow this strategy, the question is why?[15] Finally, the third remark is crucial. The only reason to start a disinflation program is that good news are expected in the future. As we show in the appendix, a negative drift is a necessary condition for the initiation of a disinflation program.

The government's budget constraint is given by,

$$\dot{B}_t = e_t g_t - \Omega_t + i_t B_t \tag{2.5}$$

where B_t denotes the government debt held by the central bank and Ω_t represents the central bank's profits, discussed below. We assume that the government's debt is in nominal terms but indexed. This eliminates the incentives for discrete devaluations or surprise inflations to reduce its real value.

2.2.3 Central Bank

The central bank chooses the path of exchange rate depreciations that maximize consumer's utility, taking as given the government's expenditure path and the consumer's reaction function. The central bank's balance sheet and flow profits in nominal terms are given by:

$$\begin{aligned} M_t &= e_t r_t + B_t \\ \Omega_t &= i_t B_t + (i_t^* + \hat{e}_t)\, e_t r_t \end{aligned} \tag{2.6}$$

[15] Note that the assumption that the expenditure process is exogenous is forcing the monetary authority to adjust to changes in the fiscal deficit. This can be interpreted in two ways: The first one assumes that there are no conflicts between the monetary and the fiscal authority, and that the fiscal deficit is truly exogenous. The second interpretation assumes that there are conflicts between monetary and fiscal policy, and that the fiscal authority is the stronger. In other words, the expenditure process can be considered as exogenous by the central bank. There is a large literature that studies the tension between the fiscal and the monetary policy. We know that both policies must be coordinated to have a sustainable exchange rate regime and there are two ways in which this coordination can be achieved: On the one hand, it might be the case that the monetary authority is the stronger, thus the central bank chooses the amount of seignorage and the government adjusts its expenditure. The central bank might peg the exchange rate to force the fiscal authority to reduce expenditure. On the other hand, it might be the case that the fiscal authority is the stronger, and the monetary authority abandons its policy when the fiscal deficit is increased.

where M_t represents the nominal money holdings, B_t is the nominal value of government's debt, r_t is total reserves in foreign currency, and Ω_t is central bank's profits in nominal terms. i_t^* is the foreign nominal interest rate and $\hat{e}_t$ denotes the exchange rate depreciation. The first equation is the central bank's balance sheet. The second equation is the central bank's profits which consist of nominal interest earnings on government's debt, foreign interest earnings on reserves, and the capital gains on reserves due to a depreciation.

One implication of perfect capital mobility, the indexed government debt and the PPP assumptions is that choosing the exchange rate depreciation is the same as choosing the inflation rate or the nominal interest rate.[16] Given this equivalence we assume that the central bank chooses the nominal interest rate. Formally, the problem is,

$$\begin{aligned} & \max_{\{i_t\}} \quad E \int_0^\infty \ln\left(\frac{y+\rho a_0}{1+\alpha i_t}\right) e^{-\rho t} dt \qquad (2.7) \\ s.t. & \\ \dot{b}_t &= \rho b_t + g_t - i_t m_t \\ \lim_{t\to\infty} b_t e^{-\rho t} &= 0 \\ r_t &\geq \bar{r} \end{aligned}$$

The first constraint is the government's budget constraint in real terms. This is obtained by substituting (2.6) in (2.5), and rewriting it in terms of tradables. The second constraint is the transversality condition on the government's debt. It states that the central bank has to set an interest rate path that does not imply an exploding debt for the government. This captures the assumption that the central bank accommodates the fiscal policy (as we mentioned before). Finally, the third constraint is an international liquidity constraint reflected in a minimum level of reserves.

Note that we have simplified the model in several dimensions. First, there are no political economy issues: There is a representative consumer (thus no distributional issues) and all agents maximize the same utility function. Second, there are no Olivera-Tanzi effects and there is no

[16] Additionally, these assumptions imply that government foreign debt and reserves are perfect substitutes, so, a constraint on the level of reserves is also a constraint on the level of debt.

choice between inflation and other taxes. Therefore, under these assumptions, the explanations given in the literature would imply that a flexible exchange rate is the optimal policy. In the next section, we show that it is sufficient that the expenditure has a negative drift and that inflation has welfare costs, to justify the implementation of a disinflation program.

2.3 Why do governments initiate a disinflation program together with the announcement of a fiscal reform?

The main question of the paper is why do governments initiate a disinflation program even though the fiscal equilibrium is not guaranteed. Two alternative ways of possing the question are: First, why for some period of time the fiscal and monetary policy seemed to be inconsistent? Second, why, conditional on having a fiscal reform in place, countries decide to peg their exchange rates?

In this section, we show that if the fiscal reform is uncertain and inflation generates welfare costs then it is possible to rationalize the government's behavior.. To isolate the adoption question, we solve the simple case when there are no reserves constraints. The main result is that the optimal exchange rate path is a managed exchange rate regime with a depreciation rate lower than the one implied by flexible exchange rate. In other words, a disinflation (financed by reserves) is initiated at the announcement of a fiscal reform. A formal solution is shown in appendix 2.8.1.

The central bank's problem is to choose the path of nominal interest rates that solves (2.7) when $\bar{r} \rightarrow -\infty$. First, we solve the model for a flexible exchange rate as a benchmark. Second, we solve for the optimal exchange rate policy.

2.3.1 Flexible exchange rate.

We define the flexible exchange rate as the one that implies a constant level of reserves. Under this definition the government's debt is also constant. Imposing $\dot{b}_t = 0$ on the government's budget constraint we obtain,

$$\rho b_0 + g_t = i_t m_t$$

This equation implies that the seignorage has to be equal to the total government expenditures. In other words, the flexible exchange rate implies: First, that each period there is a balance budget. Second, that the rate of growth of nominal domestic credit is equal to the depreciation rate. It is easy to show that this is the unique constant rate of depreciation consistent with the transversality condition on government's debt. Given the money demand, equation (2.2), we can solve for the interest rate, which implicitly solve for the exchange rate depreciation.

$$\begin{aligned} \frac{1}{1+\alpha i_t} &= 1 - \frac{g_t + \rho b_0}{y + \rho a_0} \\ \hat{e}_t &= i_t - \rho \end{aligned} \tag{2.8}$$

In figure 2-5, we show the path of interest rates implied by the government expenditure process, where i_h^f denotes the interest rate when there is a high level of expenditure, and i_l^f when there is a low level of expenditure, under a flexible regime.

[Figure 2-5]

2.3.2 Optimal interest rate path

In this section we show that the optimal exchange rate path before τ, is a managed exchange rate with a depreciation rate smaller than the one implied by flexible exchange rate, and that after τ, the optimal regime is a flexible exchange rate. The problem is solved by backward induction.

We know that after τ expenditure is constant in each of the states of the world. In the appendix, we show that when expenditure is constant the optimal regime is a flexible exchange rate. Substituting $b\tau$ for b_0 in equation (2.8),

$$\frac{1}{1+\alpha i_h^1} = 1 - \frac{g_h + \rho b\tau}{y + \rho a_0} \tag{2.9}$$

$$\frac{1}{1+\alpha i_l^1} = 1 - \frac{g_l + \rho b\tau}{y + \rho a_0} \tag{2.10}$$

where i_h^1 is the interest rate consistent with the higher level of expenditure and i_l^1 is the one consistent with the lower level of expenditure.

The intuition of why the optimal regime after τ is a flexible regime is the following: After τ, the expenditure process is certain and constant. Therefore, at that time, the economy is not expecting any reduction in future taxations. Based on tax smoothing arguments, the optimal path of taxation involves a constant rate of inflation consistent with the transversality condition on government's debt. This implies that the budget has to be balanced every period. And this is our definition of flexible exchange rate regime. A second interpretation comes from arguments based on the Sargent and Wallace effect. It is desirable to reduce inflation, however, the initiation of a disinflation program will have future costs in terms of higher levels of debt. If there are no expected reductions in expenditure, higher future inflation rates will overcompensate the short term benefits of the disinflation.

The second step is to solve for the interest rate before τ. Writing the Hamiltonian and optimizing we obtain that the interest rate is constant prior to τ and that it satisfies the following constraint:

$$i^1 = (1-q)\, i_h^1 + q i_l^1 \tag{2.11}$$

where i^1 is the constant interest rate between $[0, \tau]$. Equation (2.11) comes from equating expected marginal utilities of consumption before and after τ.[17] Finally, we use the law of motion of debt to compute its value at time τ, given i^1.

$$b\tau = b_0 + \frac{e^{\rho\tau} - 1}{\rho}\left[g_h + \rho b_0 - (y + \rho a_0)\left[1 - \frac{1}{1+\alpha i^1}\right]\right] \tag{2.12}$$

Equations (2.9), (2.10), (2.11), and (2.12) constitute a system of four equations with four unknowns. The solution for the interest rate, debt, reserves and consumption are shown in figures 2-6 and 2-7.[18]

[17] This equation has this simple form because the log utility implies that marginal utility of consumption is linear with respect to the interest rate.

[18] We calibrate the model in the following way: First, the discount rate was computed as an average of nominal foreign interest rate, which implies $\rho = 0.10$. Second, the average debt was 40 percent of GDP before the disinflation programs were started. Third, M2 is approximately half of consumption, thus $\alpha = 0.50$. Fourth, wasteful expenditure is calibrated to be 20 percent of GDP and we assume that in two years it can drop to zero.

[Figure 2-6]

[Figure 2-7]

Proposition 1 *Along the optimal path, the exchange rate depreciation between* $[0, \tau]$ *is smaller than the one implied by flexible exchange rate. Moreover, foreign debt is increasing or, equivalently, reserves are falling.*

Proof. The proof is by contradiction. Suppose the proposition is false, assume that $i^1 \geq i_h^f$. Substituting in the intertemporal budget constraint of the government, we obtain $\dot{b}_t < 0$. This is because the larger interest rate implies a larger seignorage. Then, at time τ, the total debt is smaller than the initial debt b_0. This means that $i_h^1 < i_h^f$ and $i_l^1 < i_l^f$, according to equation (2.8), (2.9), and (2.10). Moreover, $i_h^1 > i_l^1$. However, using equation (2.11) the interest rate i^1 is a weighted average of the interest rates after τ. In particular, it is smaller than i_h^1 which is smaller than i_h^f. But this is a contradiction. ■

The proposition states that a disinflation program is initiate even though expenditure has not been adjusted. The disinflation causes an increase in debt due to the reduction in seignorage. If the fiscal adjustment fails, so the bad state of the world is realized, the new equilibrium depreciation rate is higher than the one that would prevail if a flexible exchange rate were adopted in the first place. As was mentioned in the introduction, ex-post, it looks as if the country made a mistake initiating the stabilization program.

The intuition of the result is the following: the announcement of the fiscal reform conveys good news in terms of future expected reductions in expenditure. At the announcement the expected equivalent annuity of expenditure falls. By the intertemporal budget constraint the expected equivalent annuity of taxation should fall too. Because inflation generates welfare costs the optimal path of inflation tax is to have a constant rate of inflation. This is the tax smoothing result. Notice that the smoothing incentives comes from the welfare costs of inflation and the fact that expected taxation has to fall.

This implies that $g_h = 0.20$, $g_l = 0$. Fifth, we normalize output to unit and impose current account equilibrium prior to the announcement ($y = c = 1$). This is not necessarily realistic, however we want to study how much deterioration in the current account is implied by the model. This normalization implies that initial assets are $a_0 = 2.5$. In figure 2-6 we show the path of the economy when there is an increase in the time of the reform, and in figure 2-7, when there is an increase in q. Note that we constructed this examples in such a way that reserves are negative for one of the cases.

There are three additional points to be highlighted about the solution: First, on impact, reserves go up and decrease thereafter. The reduction in the nominal interest rate implies an increase in demand for real balances, which is reflected in an increase in reserves on the implementation of the disinflation. Second, there is no guarantee that reserves are positive in the bad state of the world. If τ or the expected expenditure improvement are large enough, reserves can be negative, especially when the fiscal adjustment does not take place. Third, there is a consumption boom at the announcement of the reform and the trade balance deteriorates.[19]

There are two caveats worth to be mentioned. First, the optimal exchange rate regime is a managed exchange rate and not a fixed exchange rate. In our case, we explain a fall in the depreciation rate from 40 percent to 20 percent, but not to zero. Other reasons as visibility, credibility, or political economy have to be introduced to explain the adoption of a pure fixed exchange rate regime.

Second, the model implies that the fiscal deficit increases on impact. In several Latin American experiences this is not the case; the fiscal deficit do falls on impact. However, we know that unsustainable short term measures can be, and had been implemented to reduce the fiscal deficit. For example, reducing public investment and delaying wage increases in the public sector will show as reductions in the fiscal deficit. We know that these measures are unsustainable and therefore do not represent a real adjustment; it is only an accounting make up. In our model, consumers have perfect foresight and only permanent changes are taken into account. More specifically, consumers only care about the equivalent annuity of the governments expenditure (or fiscal deficit). To reconciliate our implication with the data we want to point out the following: The model implies that debt will be accumulated through out the disinflation program. We know that in most of the unsuccessful cases (the only exception is Mexico 87) the government debt was higher after the program was abandoned. In other words, the equivalent annuity of the fiscal deficit increased during the disinflation.[20]

[19] The comparative statics is analyzed in the appendix. An increase in q unambiguously increases debt at τ, and reduces current inflation. An increase in τ increases debt at time τ, and increases current inflation.

[20] An alternative view is to observe total domestic holdings of foreign assets. The model also implies that there is a reduction in foreign assets. In other words, that there is a current account deficit (on average). This is a standard fact observed in exchange rate based stabilization programs.

2.4 Balance of payments crisis: The Latin American case.

In this section, we capture the fact that most of the unsuccessful Latin American stabilization programs ended in a balance of payments crisis. In the basic framework, we introduce a lower limit on the level of reserves ($r_t \geq 0$). In the previous section, we solved the problem assuming that the government has no constraints on the level of reserves and obtain the following results: First, a disinflation program is initiated at the announcement of the fiscal reform. Second, if the fiscal reform is not successful, the ex-post inflation rate is larger than the one it would prevail if a flexible exchange rate were adopted. Third, there is a consumption boom and a deterioration of the current account. Fourth, reserves increase on impact and decrease afterwards.

In this section, we show that these main conclusions do not change in the presence of a balance of payments crisis. In addition, we show that the optimal path involves a positive probability of a balance of payments crisis, here defined as hitting the reserves constraint. Thus, still it is the case that in the presence of a balance of payments crisis the government is willing to initiate a disinflation program financed with reserves.

In the previous section, we showed that there are parameters that imply negative reserves. In these cases, if there exists a constraint on the level of reserves, the central bank is unable to implement the optimal unconstrained strategy. In those cases, the world is not willing to finance the consumption boom. Therefore, the solution for the constrained optimization problem is then a corner solution. The central bank sets the interest rate to the minimum one that guarantees that at time τ, in any event of the world, reserves are greater or equal than the minimum.

Formally, the central bank sets the interest rate such that in the bad state of the world reserves are zero.[21] Notice that the balance of payments crisis occurs *á la* Krugman (1979) with the twist that here the timing is given and not the fiscal deficit. The timing of the crisis is given by the expenditure process, and the inflation tax revenue adjusts to make the crisis rational at τ. In other words, the inflation tax is such that there is a fiscal deficit financed by reserves that makes optimal a speculative attack at τ. Notice that in Krugman's model

[21] This is because when expenditure is not adjusted interest rates increase and reserves fall. The converse occurs when the expenditure is adjusted. Thus, if the constraint is binding it has to be binding in the bad state of the world.

the fiscal deficit is exogenous and the timing is endogenous. Here, the timing is given and the deficit is endogenous. The formal solution is in appendix 2.8.2.

We know that after τ reserves are zero in the bad state; therefore domestic debt and money holdings are equal.

$$b\tau = m\tau \Rightarrow b\tau = \alpha\frac{y+\rho a_0}{1+\alpha i_h^c}$$

where i_h^c stands for the interest rate when the level of expenditure is high and there is a constraint on the level of reserves. The interest rate after τ also has to satisfy the transversality condition on the government debt, so, it is determined by equation (2.8). Solving for the maximum level of debt,

$$\bar{b}\tau = \frac{\alpha}{1+\alpha\rho}\left(y+\rho a_0 - g_h\right) \tag{2.13}$$

The interest rate prior to τ has to be consistent with a debt accumulation such that debt is equal to equation (2.13) at time τ. Using the equation for debt accumulation we solve for the interest rate (i^c) prior to τ.

$$\frac{1}{1+\alpha i^c} = \frac{1}{1+\alpha i^f} + \frac{\alpha\rho}{(1+\alpha\rho)\left(e^{\rho\tau}-1\right)}\left[1 - \frac{g_h + \rho b_0\left(1+\frac{1}{\alpha\rho}\right)}{y+\rho a_0}\right] \tag{2.14}$$

Proposition 2 *The optimal path implies that a disinflation program is initiated at the announcement of the fiscal reform. Most importantly, there is a positive probability of a balance of payments crisis.*[22]

Proof. i^1 implies an accumulation of debt that generates negative reserves, and we constructed i^c to have a lower rate of debt accumulation. Thus, $i^c > i^1$ by construction. To show that $i^f > i^c$ we follow the same proof by contradiction of proposition (1), or by inspection of equation (2.14).

Finally, the interest rate is computed such that the reserves reach their minimum in the case of not adjusting the expenditure. This means, that there is a balance of payments crisis at τ if there is no expenditure adjustment. ■

[22] *In our case, it is equal to the probability of not adjusting the fiscal expenditure* $(1-q)$.

Note that the proposition implies that a government initiates a disinflation program even though there is a risk of a balance of payments crisis.[23] The intuition is that the announcement of the fiscal reform conveys good news in the future and the government wants to transfer part of those future benefits to today in the form of higher real balances. The extent in which this transfer can be made is limited by the debt constraint. Therefore there is no full smoothing of consumption and money holdings; however some smoothing is achieved. This implies that reserves are falling. In the end, a balance of payments crisis occurs when expenditure is not adjusted. Notice that in this case, the only cost of the balance of payments crisis is the elimination of reserves. The proposition might not be true if additional costs are assumed.

The path for debt, interest rates, reserves and consumption are shown in figure 2-8. The thin continuous lines represent the solution when there are no reserves constraints, and the thick line represents the solution for the reserves constrained economy.

[Figure 2-8]

In summary, in this section we show that the nature of the solution does not change if a maximum level of debt exists. Still it is the case that the government initiates a disinflation program when a fiscal reform is announced. The important result is that the optimal policy implies that there exists a positive probability of a balance of payments crisis. Notice that this does not mean that a balance of payments crisis is optimal. Rather it means that it is optimal to initiate a disinflation program even if there exists a probability of a balance of payments crisis. In other words, avoiding the crisis at any cost is suboptimal.

2.5 Sequence of Stabilization Programs.

In figure 2-4, the Brazilian monthly inflation rate in the late 80's is plotted. The shaded area represents periods where disinflation programs where in place.[24] There are two facts that we can extract from this figure. First, notice that there is a sequence of unsuccessful stabilization

[23] This result is robust to alternative formulations of preferences and expenditure processes.

[24] In the particular case of Brazil, several of those cases involve price controls.

programs. Second, that every time the program fails, the inflation rate is higher than the inflation before the program was initiated. Therefore, the situation of the economy worsens after a stabilization program fails. This experience is not exclusively Brazilian. For example, Venezuela since 1983 had implemented five stabilization programs, and Argentina did the same in the 70's.

[Figure 2-4]

In this section, we show that it is optimal to implement a sequence of stabilization programs, even though each time it is harder to reduce inflation and it is more costly when the program fails. To capture this effect we change our basic framework and assume that the government is continuously trying to reduce expenditure: every time a fiscal reform fails, the government announces a new one. This behavior should raise naturally from the assumption that expenditure is wasteful. A very simple way of modelling it is to assume that expenditure follows a Poisson process.

$$g_{t+dt} = \begin{cases} g_h & w/p & 1-qdt & \text{if } g_t = g_h \\ g_l & w/p & qdt & \\ & & & \\ g_l & w/p & 1 & \text{if } g_t = g_l \end{cases}$$

where we define high state when there is a high level of expenditure, and low state when there is a low level of expenditure. This process implies that there is a fiscal reform at every point in time with probability qdt of being successful.[25]

The rest of the section is organized as follows: First, we solve the problem assuming no debt constraints. We show that the optimal policy implies a continuum of disinflation programs (thus a sequence of them). Second, we introduce a reserve constraint and show that, even though there exists the possibility of a balance of payments crisis, the optimal strategy is to implement

[25]There are two ways in which this process can be interpreted. First, at every time t the government announces a fiscal reform for time $t + dt$. If it fails, then the government announces a new one. Second, there is only one permanent fiscal reform, but the government is uncertain about when it is going to succeed. Thus, this section might also be interpreted as a relaxation of the expenditure process assumed in the text. Here we are relaxing the known timing of the reform.

a sequence of disinflation programs until one is successful. Finally, we introduce the possibility of foreign help (in the form of IMF and World Bank loans) and show that this implies that after a loan is made, the government implements a disinflation program until the balance of payments reappear. Moreover, we show that the loan is welfare improving.

The first result is that the optimal policy is to implement a continuum of disinflation programs. This result is shown by proving that when expenditure is high, the optimal nominal interest rate is always smaller than the one implied by flexible exchange rate. To solve the problem we define a value function in each of the states of the world.

$$\begin{aligned} \rho V^l(b_t) &= \max_{\theta_t} \left\{ \ln \theta_t + \left[\rho b_t + g_l - (y + \rho a_0)(1 - \theta_t) \right] V_b^l \right\} \\ \rho V^h(b_t) &= \max_{\theta_t} \left\{ \ln \theta_t + \left[\rho b_t + g_h - (y + \rho a_0)(1 - \theta_t) \right] V_b^h + q \left[V^l - V^h \right] \right\} \end{aligned} \tag{2.15}$$

where V^l is the value function when expenditure is low and V^h is the value function when expenditure is high. A formal solution is in appendix 2.8.3.

Proposition 3 *If expenditure is high, the optimal strategy involves a rate of depreciation smaller than the one implied by flexible exchange rate. Moreover, both inflation and government debt are increasing every unsuccessful fiscal reforms. Finally, the optimal strategy approaches the flexible exchange rate at high levels of debt.*

If expenditure is low, the optimal strategy is a flexible exchange regime.

Proof. The proof is in appendix 2.8.3 see propositions 5, 6, and 7. ■

The proposition implies that the optimal strategy when expenditure is high, is a managed exchange rate regime. Additionally, notice that it implies that the larger the level of debt, the smaller the disinflation effort. In other words, the difference in the nominal interest rate between the optimal and the implied by flexible exchange rate is a decreasing function of debt (see proposition 7).

The differential equations implied by the equation 2.15 do not have a close form solution, thus we solve them numerically. The solutions for the optimal policy when expenditure is high is shown in figure 2-9. Debt as a percentage of GDP is measured in the x-axis, i_l is the interest

rate implied by flexible exchange rate when expenditure is low (the bottom schedule), i_f is the interest rate implied by flexible exchange rate when expenditure is high (the top schedule), and i_h is the solution of the differential equation when expenditure is high. The interest rate is increasing with debt and is always smaller than the interest rate implied by flexible exchange rate in the high state.

[Figure 2-9]

We now introduce the possibility of a balance of payments crisis. Similarly as in the previous section, the maximum level of debt is given by equation (2.13). At this level of debt the optimal strategy is a flexible exchange rate regime; thus we use this constraint as a boundary condition for the differential equation. The solution is shown in figure 2-10, where i_{hc} is the interest rate of the constrained economy. Note that for low levels of debt, the solutions for the constrained and unconstrained economy are similar. On the other hand, when debt is increasing the constrained economy approaches the flexible exchange rate faster than the unconstrained economy. Finally, when the maximum level of debt is reached, the regime changes to a flexible exchange rate in the constrained economy. In other words, when reserves are zero there are no possibilities of financing a reduction in inflation, other than implementing the fiscal reform.

[Figure 2-10]

In figure 2-11, we compare the time path of debt and interest rates when the fiscal reform is unsuccessful for the constrained and unconstrained cases. b_t and i_t represent the debt and interest rate when the economy is unconstrained, while b_{tc} and i_{tc} are the debt and interest rate when the economy is constrained. As can be seen the unconstrained economy implies a faster accumulation of debt and a more aggressive disinflation program.

[Figure 2-11]

Finally, we show that if the country is close to the debt constraint a foreign loan is welfare improving and it implies an immediate implementation of a disinflation program. To clarify the intuition, assume that the economy has reached the maximum level of debt, so it has a flexible

exchange rate. Lets interpret the debt level net of foreign help. This means that a loan from the IMF or the World Bank increases the debt capacity of the country. In terms of our model, the economy jumps to the left in figure 2-10. Therefore, a more aggressive disinflation program is initiated, real balances increase, and the consumer's utility goes up.

In summary, the results in this section are the following: First, the optimal strategy implies a sequence of disinflation programs even though there exists the possibility of a balance of payments crisis. Second, the inflation rate is increasing through the path every time the disinflation program fails. Third, the larger the debt, the smaller the disinflation effort.

2.6 Real Exchange Rate

One of the most important aspects in exchange rate based stabilizations is the real exchange rate appreciation that accompanies the program. Moreover, when the disinflation program is abandoned it looks as if the exchange rate was overvalued prior to the crisis.[26] This problem occurs repeatedly in Latin America. In this section, we introduce non tradables in the model and analyze the path of the real exchange rate. We show that the optimal strategy implies an appreciation of the real exchange rate between time $[0, \tau]$. Moreover, this real appreciation looks as an overvaluation, if expenditure is not adjusted.

The introduction of real exchange rate considerations complicates the model significantly. To simplify the analysis we collapse all agents in one and do a welfare analysis.[27] We solve the problem in two steps: First, we solve for the total foreign holdings (private and public assets). Second, we solve for the money holdings and the real exchange rate. The maximization problem is the following,

[26] Several authors differentiate between an equilibrium real exchange rate appreciation and an overvaluation. The first one is an equilibrium situation, which means that little can, and should be done. On the other hand, the second one implies a disequilibrium situation, and therefore measures should be taken to return to equilibrium. However, these measures usually imply a prolonged recessions or the abandonment of the nominal anchor. See Dornbusch (1996).

[27] We know (Stockman (1981)) that a constant expected inflation rate achieves the first best allocation in a cash in advance model. Using this result we are able to recover the path of interest rates.

$$\max_{\{c_t\}} \quad E\int_0^\infty (1-\beta)\ln c_{N,t} + \beta \ln c_{T,t}\, e^{-\rho t} dt$$

$$\begin{aligned} s.t. & \\ \dot{x}_t &= \rho x_t + y_T + \frac{p_{N,t}}{e_t} y_N - c_{T,t} - \frac{p_{N,t}}{e_t} c_{N,t} - g_t \\ \lim_{t\to\infty} x_t e^{-\rho t} &= 0 \end{aligned}$$

where x_t are the domestic holdings of foreign assets, y_T is the output of tradables, y_N is the output of non tradable, $c_{T,t}$ is the consumption in tradables, $c_{N,t}$ is the consumption in tradables, e_t is the exchange rate level (and the price of tradables) and p_{Nt} is the price of non tradables. The first equation is the utility function, where β is the share of tradables. The second equation is the consolidated budget constraint.[28] Finally, the third equation is the transversality condition on total assets. Note that this problem is independent of the interest rate and money holdings (the solution is shown in appendix 2.8.5). The problem is solved by backward induction and it implies the following system of equations:

$$\begin{aligned} c_T^h &= y_T - g_h + \rho x_\tau \\ c_T^l &= y_T - g_l + \rho x_\tau \\ x_\tau &= x_0 + \left(\frac{e^{\rho\tau}-1}{\rho}\right)\left[y_T + \rho x_0 - g_h - c_T^1\right] \\ \frac{1}{c_T^1} &= \frac{1-q}{c_T^h} + \frac{q}{c_T^l} \end{aligned} \tag{2.16}$$

where c_T^1 is the consumption of tradables before τ, c_T^h is the consumption of tradables after τ when expenditure is high, and c_T^l is the consumption of tradables after τ when expenditure is

[28] The budget constraint is obtained by substracting the consumer's budget constraint and the government's budget constraint. This defines total foreign holdings by domestic agents. Formally,

$$\begin{aligned} x_t &= a_t - b_t \\ \dot{a}_t &= \rho a_t + y_T + \frac{p_{Nt}}{e_t} y_N - c_{T,t} - \frac{p_{Nt}}{e_t} c_{N,t} - i_t m_t \\ \dot{b}_t &= \rho b_t + g_t - i_t m_t \end{aligned}$$

low. This is a system of equations with four equations and four unknown that solves for the consumption path.

Additionally, from the maximization problem and the market clearing condition in non tradables we find the real exchange rate,

$$\frac{e_t}{P_{N,t}} = \frac{1-\beta}{\beta}\frac{y_N}{c_{T,t}} \tag{2.17}$$

Finally, to solve for the money holdings we use the cash in advance constraint.

$$m_t = \alpha\left[c_{T,t} + \frac{p_{N,t}}{e_t}c_{N,t}\right] = \frac{\alpha}{\beta}c_{T,t}$$

In figure 2-12, we show the solution for consumption of tradables, domestic holdings of foreign assets, the real exchange rate, and the money holdings. There are four results worth to be highlighted: First, notice that there is a consumption boom on the announcement of the fiscal reform, there is a current account deterioration and total foreign assets are declining.

[Figure 2-12]

Second, there is a real exchange rate appreciation on impact. There is no change in the production or in the government expenditure side, so, the real appreciation is driven by an unsustainable consumption boom that will be reversed if the expenditure is not adjusted.

Third, if the expenditure is adjusted there is a further real appreciation. On the other hand, if the expenditure is not adjusted there is a real depreciation. In other words, an unsuccessful stabilization program is abandoned with a nominal depreciation. Notice that the new real exchange rate is more depreciated than the one that existed before the fiscal reform was announced. Therefore, real wages and consumption are falling after every unsuccessful stabilization.

Finally, note that the model implies that total foreign holdings fall. It does not matter if the reduction occurs in the government (increase in government debt), in the central bank (reduction in reserves), or in the private sector (reduction in private foreign assets). The model implies that total assets fall. This captures the Mexican recent experience. It has been argued that the private sector increased its debt, not the government. In fact, this was one

of the arguments used by the Mexican government to justify why the current account deficit was *sustainable*. The model here presented implies that, indeed, the current account deficit is rational, but only sustainable if the expenditure is adjusted. The indeterminacy of which agent increases its debt is due to the fact that we have collapsed all the budget constraints in one. If we impose that the only tax is inflation, then there is a unique solution, and it implies that the government's debt is increasing.

In summary, in this section we show that the disinflation program conveys a real exchange rate appreciation. Moreover, that it looks as an overvaluation when the program is abandoned.

2.7 Conclusions

Several Latin American countries have initiated stabilization programs based on fiscal reforms and on exchange rate managements. In most of these cases, the program was abandoned, and *ex-post*, it seemed as if it was a bad idea to initiate it in the first place. This paper has shown that if inflation has convex welfare costs and the fiscal reform is uncertain, it is possible to explain the government's behavior. In particular, the paper shows why a government would implement a disinflation program at the same time it is initiating a fiscal reform. Additionally, the results show that it is optimal to lose reserves in the process of disinflation, even though there exists a possibility of a balance of payments crisis. Finally, the analysis implies that countries will implement a sequence of disinflation programs until one of them is successful, or until a balance of payments crisis occurs.

Two important aspects of stabilization programs have not been considered in the paper: First, we have not considered political economy and credibility issues to explain the adoption of these reforms. These are important aspects and a more complete story should include them. Notice that we are able to explain why a government should initiate a disinflation program. However, we predict that it should be implemented using a managed exchange rate and not a fixed exchange rate. Arguments such as visibility, signaling, and political economy have to be advanced in order to explain the observed behavior.

Second, almost all countries experienced an increase in real interest rate on the implementation of the program; however, in our model, the real interest rate is constant because we

assumed that government debt is indexed. Credibility and imperfect capital mobility could explain the path of the real interest rate. First, if credibility is associated with the process of reform, then risk premium can explain the changes in the real interest rate (see Velasco (1993)). Second, if the capital account is closed, and if the government is involved in a tight monetary policy, the interest rate on impact might increase. (see Calvo, Leiderman and Reinhart (1993)). These aspects should be included in future extensions.

Finally, in the model presented, the government and the supply side have been oversimplified. In future research these extensions should be studied. Especially, the process of the fiscal reform should be endogenized.

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2.8 Appendix

2.8.1 The basic model

In this section we solve the simple model presented in the text.

Consumer's problem

The consumer's problem is given by,

$$\begin{aligned} & \max_{\{c_t\}} \quad E \int_0^\infty \ln c_t \, e^{-\rho t} dt \\ s.t. & \\ \dot{a}_t &= \rho a_t + y - c_t - i_t m_t \\ c_t &\leq \frac{1}{\alpha} m_t \\ \lim_{t \to \infty} a_t e^{-\rho t} &= 0 \end{aligned}$$

where the first equation is the objective function. The second equation is the intertemporal budget constraint in terms of tradables. The third equation is the cash in advance constraint. And the fourth equation is the transversality condition on consumer's assets. Writing the Hamiltonian,

$$H = \ln c_t + \lambda_t \left(\rho a_t + y - c_t - i_t m_t\right) + \mu_t \left(\frac{1}{\alpha} m_t - c_t\right)$$

The first order conditions are,

$$\begin{aligned} \frac{1}{c_t} - \lambda_t - \mu_t &= 0 \\ -i_t \lambda_t + \frac{1}{\alpha} \mu_t &= 0 \\ \dot{\lambda}_t &= 0 \\ \alpha c_t &= m_t \end{aligned}$$

$$\rho a_t + y - c_t - i_t m_t = \dot{a}_t$$

where the first is the first order condition with respect to consumption. The second one is the derivative with respect to money holdings. The third one is the derivative with respect to assets. The fourth one is the derivative with respect to the multiplier of the cash in advance constraint. And the fifth one is the derivative with respect to λ_t. From the third first order condition,

$$\dot{\lambda}_t = 0 \Rightarrow \lambda_t = \lambda_0$$

From the second first order condition we have,

$$\mu_t = \lambda_0 \alpha i_t$$

Substituting in the consumption and money equations,

$$\begin{aligned} c_t &= \frac{1}{\lambda_0}\frac{1}{1+\alpha i_t} \\ m_t &= \frac{1}{\lambda_0}\frac{\alpha}{1+\alpha i_t} \end{aligned}$$

Substituting in the intertemporal budget constraint,

$$\dot{a}_t = \rho a_t + y - \frac{1}{\lambda_0}$$

Integrating and imposing the transversality condition we obtain,

$$\frac{1}{\lambda_0} = y + \rho a_0$$

Note that the consumer's consumption and money function does not depend on the future path of interest rates. This result comes from the log utility assumption.

Central Bank's problem

In this section we solve the central bank's problem. We derive the solution in two steps. First we solve the problem when there is no fiscal uncertainty (and expenditure is constant). Second, we solve the problem with the stochastic process assumed in the text.

Solution without fiscal uncertainty. We show that the solution when there is no risk in government expenditure is a flexible exchange rate. The problem is the following,

$$\begin{aligned} & \max_{\{i_t\}} \quad E \int_0^{\infty} \ln \frac{1}{1+\alpha i_t} e^{-\rho t} dt \\ s.t. & \\ \dot{b}_t &= \rho b_t + g_t - (y + \rho a_0)\left(1 - \frac{1}{1+\alpha i_t}\right) \\ \lim_{t\to\infty} b_t e^{-\rho t} &= 0 \end{aligned}$$

Define,

$$\theta_t = \frac{1}{1+\alpha i_t} \tag{2.18}$$

Writing the Hamiltonian,

$$H = \ln \theta_t - \lambda_t \left(\rho b_t + g_i - (y + \rho a_0)(1 - \theta_t)\right)$$

The first order conditions are,

$$\begin{aligned} \frac{1}{\theta_t} + (y + \rho a_0)\lambda_t &= 0 \\ \dot{\lambda}_t &= 0 \end{aligned}$$

Note that if $\dot{\lambda}_t = 0$, then $\dot{\theta}_t = 0$. This means that the multiplier is constant and that marginal consumption is constant. Which means that the optimal strategy for the government is to have a constant inflation; smooth the inflationary tax. To determine the level of the multiplier we

substitute in the budget constraint and impose the transversality condition. This implies that,

$$\theta_i = 1 - \frac{g_i + \rho b_0}{y + \rho a_0}$$

This means that the solution is a constant depreciation rate equal to the flexible exchange rate. Note that $\frac{\partial \theta_i}{\partial g_i} < 0$ and $\frac{\partial \theta_i}{\partial b} < 0$. Given the definition of θ this implies that $\frac{\partial i}{\partial g_i} > 0$ and $\frac{\partial i}{\partial b} > 0$. Thus, a larger level of expenditure or debt implies a larger nominal interest rate, which requires a larger rate of depreciation and inflation.

Fiscal uncertainty. We solve the problem by backward induction. Given that we know that without fiscal uncertainty the solution is a constant interest rate, then after τ, there should be a constant inflation rate consistent with a flexible exchange rate given the level of debt at τ. Formally,

$$\begin{aligned} \theta_h^1 &= 1 - \frac{g_h + \rho b\tau}{y + \rho a_0} \\ \theta_l^1 &= 1 - \frac{g_l + \rho b\tau}{y + \rho a_0} \end{aligned}$$

where θ_h^1 is the inverse of the interest rate when the level of expenditure is high, and θ_l^1 is the inverse of the interest rate when the level of expenditure is low. Substituting in the utility function and using the definition of the debt,

$$\begin{aligned} \max_{\{\theta_t\}} \quad & \left[\int_0^\tau \ln \theta_t e^{-\rho t} dt + \frac{1}{\rho} e^{-\rho t} \left(q \ln \theta_l^1 + (1-q) \ln \theta_h^1 \right) \right] \\ s.t. & \\ \dot{b}_t &= \rho b_t + g_i - (y + \rho a_0) \left(1 - \frac{1}{1 + \alpha i_t} \right) \end{aligned}$$

writing the Hamiltonian and solving the first order conditions we find (as before) that the optimal interest rate has to be constant between $[0, \tau]$. The first order conditions are,

$$\begin{aligned} \frac{1}{\theta_t} + (y + \rho a_0)\,\lambda_t &= 0 \\ \dot{\lambda}_t &= 0 \end{aligned}$$

Imposing that the interest rate has to be constant between $[0, \tau]$ the new optimization problem is,

$$\max_{\theta^1} \left[\frac{1}{\rho}\left(1 - e^{-\rho t}\right)\ln\theta^1 + \frac{1}{\rho}e^{-\rho t}\left(q\ln\theta_l^1 + (1-q)\ln\theta_h^1\right)\right]$$

s.t.

$$b\tau = b_0 + \frac{e^{\rho\tau} - 1}{\rho}\left[\rho b_0 + g_h - (y + \rho a_0)\left(1 - \theta^1\right)\right]$$

and subject to the definitions of θ_l^1 and θ_h^1. The first order condition implies (after some algebra),

$$\frac{1}{\theta^1} = \frac{q}{\theta_l^1} + \frac{1-q}{\theta_h^1}$$

This condition is saying that marginal utility of consumption before τ is equal to the expected marginal utility of consumption after τ. Which is the usual Euler condition on consumption. Substituting by the definitions of θ we obtain the equation (2.11) in the text for the interest rate.

To solve for the interest rate we have the following system of equations,

$$\begin{aligned} \frac{1}{1 + \alpha i_h^1} &= 1 - \frac{g_h + \rho b\tau}{y + \rho a_0} \\ \frac{1}{1 + \alpha i_l^1} &= 1 - \frac{g_l + \rho b\tau}{y + \rho a_0} \\ i^1 &= (1-q)\,i_h^1 + q i_l^1 \\ b\tau &= b_0 + \frac{e^{\rho\tau} - 1}{\rho}\left[g_h + \rho b_0 - (y + \rho a_0)\left[1 - \frac{1}{1 + \alpha i^1}\right]\right] \end{aligned}$$

Solving implicitly for the debt level and the interest rate,

$$(b\tau - b_0)\frac{\rho}{e^{\rho\tau}-1} = g_h + \rho b_0 - (y + \rho a_0) + \frac{[y + \rho a_0 - (g_h + \rho b\tau)]\,[y + \rho a_0 - (g_l + \rho b\tau)]}{[y + \rho a_0 - (qg_h + (1-q)g_l + \rho b\tau)]} \tag{2.19}$$

$$\begin{aligned} \frac{1 + \alpha i^1}{y + \rho a_0} &= \frac{1}{\Psi_{\tau,i}} - q\,(g_h - g_l)\,\frac{1}{\Psi_{\tau,i}\,[\Psi_{\tau,i} + (g_h - g_l)]} \\ \Psi_{\tau,i} &\equiv [y + \rho a_0 - (g_h + \rho b_0)]\,e^{\rho\tau} - \frac{y + \rho a_0}{1 + \alpha i^1}\,(e^{\rho\tau} - 1) \end{aligned} \tag{2.20}$$

Proposition 4 *$b\tau$ is increasing with τ and q, and i^1 is increasing with τ and decreasing with q.*

Proof. This is obtained by totally differentiating equation (2.19) and (2.20) ■

The intuition is that an increase in the probability of success increases the expected expenditure improvement and current interest rate falls by more, increasing the debt in the end. Similarly, an increase in τ increases the level of debt at time τ, and increases current inflation. In this case there are two effect: First, the reform occurs later. Second, there is more time to accumulate more debt. The first effect means that the interest rate increases and therefore the rate at which debt is accumulated is smaller. However, the second effect implies that the economy has to wait more until the good news come, thus more debt is accumulated. In our case, the second effect always dominates.

2.8.2 Model with debt constraint.

In this section we solve the problem when there is a constraint in the level of reserves or, equivalently, a constraint in the level of debt.

$$\max_{\{\theta_t\}} \; E\int_0^{\infty} \ln\theta_t e^{-\rho t} dt$$

$$
\begin{aligned}
& s.t. \\
\dot{b}_t &= \rho b_t + g_t - (y + \rho a_0)(1 - \theta_t) \\
\lim_{t \to \infty} b_t e^{-\rho t} &= 0 \\
\tau_t &\geq 0
\end{aligned}
$$

The problem is solved in two steps: First, we define the level of debt constraint. Second, we solve the Kuhn-Tucker problem.

We know that when the constraint is hit the expenditure is high. At that moment, money demand and total debt are equal. Moreover, the interest rate has to be one in which there is no change in the level of debt, thus it is the flexible exchange rate one. Formally,

$$
\begin{aligned}
b\tau &= m\tau \\
m\tau &= \alpha \frac{y + \rho a_0}{1 + \alpha i \tau} \\
\frac{1}{1 + \alpha i \tau} &= 1 - \frac{g_h + \rho b \tau}{y + \rho a_0}
\end{aligned}
$$

This system of equations uniquely determines the debt at τ,

$$
b\tau^c = \frac{\alpha}{1 + \alpha\rho}(y + \rho a_0 - g_h) \tag{2.21}
$$

Thus, if the constraint is binding, then the interest rate between $[0, \tau]$ has to be one such that the debt accumulated until time τ is equal to equation (2.21). The government's debt law of motion is,

$$
b\tau = b_0 + \frac{e^{\rho\tau} - 1}{\rho}\left[g_h + \rho b_0 - (y + \rho a_0)\left[1 - \frac{1}{1 + \alpha i^c}\right]\right]
$$

substituting in equation (2.21) we solve for the interest rate,

$$
\theta^c \equiv 1 + \Psi\tau\left[\alpha\rho - \frac{g_h\left((1 + \alpha\rho)e^{\rho\tau} - 1\right) + \rho b_0 (1 + \alpha\rho) e^{\rho\tau}}{y + \rho a_0}\right] \tag{2.22}
$$

$$\Psi_\tau \equiv \frac{1}{(1+\alpha\rho)(e^{\rho\tau}-1)}$$

After some algebra, we obtain equation (2.14) in the text.

The central bank's problem is the following,

$$\begin{aligned} & \max_{\{\theta_t\}} \; E\int_0^\infty \ln\theta_t e^{-\rho t}dt \\ s.t. & \\ \dot{b}_t &= \rho b_t + g_t - (y+\rho a_0)(1-\theta_t) \\ \lim_{t\to\infty} b_t e^{-\rho t} &= 0 \\ \theta_t &\leq \theta^c \text{ for all } t\epsilon[0,\tau] \end{aligned}$$

where the last constraint implies that the nominal interest has to be always larger or equal to i^c. Writing the Hamiltonian we solve the Kuhn-Tucker problem. If the constraint is not binding then the solution is the same as the previous section. If the constraint is binding, then the solution is,

$$\theta_t = \begin{cases} \theta^c & \tau < 0 \\ \theta_h^c = 1 - \frac{g_h+\rho b\tau^c}{y+\rho a_0} & g_t = g_h \quad \tau \geq 0 \\ \theta_l^c = 1 - \frac{g_l+\rho b\tau^c}{y+\rho a_0} & g_t = g_l \quad \tau \geq 0 \end{cases}$$

where $b\tau^c$ is given by equation (2.21) and θ^c is given by equation (2.22). Notice that by construction $b\tau^c$ is smaller than the debt obtained in the optimal unconstrained strategy, thus $\theta_h^c > \theta_h^1$ and $\theta_l^c > \theta_l^1$.

2.8.3 Expenditure follows a Poisson process

In this section we assume that the expenditure follows a Poisson process. First we solve the problem without reserves constraints. Second, we show the solution when there are reserves constraints.

No reserves constraint

Assume that expenditure follows,

$$g_{t+dt} = \begin{cases} g_h & w/p & 1-qdt & \text{if } g_t = g_h \\ g_l & w/p & qdt & \\ & & & \\ g_l & w/p & 1 & \text{if } g_t = g_l \end{cases}$$

In this case, we have two value functions. One for the low level of expenditure and one for the high level of expenditure. The Bellman's equations are,

$$\begin{aligned} \rho V^l(b_t) &= \max_{\theta_t} \left\{ \ln \theta_t + \left[\rho b_t + g_l - (y + \rho a_0)(1-\theta_t) \right] V_b^l \right\} \\ \rho V^h(b_t) &= \max_{\theta_t} \left\{ \ln \theta_t + \left[\rho b_t + g_h - (y + \rho a_0)(1-\theta_t) \right] V_b^h + q \left[V^l - V^h \right] \right\} \end{aligned}$$

The solution for the first one is the following: The first order condition and the envelope theorem equations are,

$$\begin{aligned} \frac{1}{\theta_t^l} + (y + \rho a_0) V_b^l &= 0 \\ \left[\rho b_t + g_l - (y + \rho a_0)\left(1 - \theta_t^l\right) \right] V_{bb}^l &= 0 \end{aligned} \tag{2.23}$$

This means that the solution is the flexible exchange rate regime. V_{bb}^l is different from zero and the second equation implies,

$$\dot{\theta}(b_t) = 1 - \frac{g_l + \rho b_t}{y + \rho a_0} \tag{2.24}$$

Substituting in the Bellman equation we solve for the value function,

$$V^l(b_t) = \frac{1}{\rho} \ln \theta^l(b_t)$$

Thus the solution implies a flexible exchange rate regime when expenditure is low. Notice that the value function is twice differentiable, decreasing and concave, and the interest rate policy function is increasing and convex.

Now, we solve the problem for the value function with high level of expenditure. The first order condition and the envelope theorem imply,

$$\frac{1}{\theta_t^h} + (y + \rho a_0) V_b^h = 0 \tag{2.25}$$

$$\left[\rho b_t + g_h - (y + \rho a_0)\left(1 - \theta_t^h\right)\right] V_{bb}^h = q\left[V_b^h - V_b^l\right] \tag{2.26}$$

Proposition 5 $\forall b_t < \infty \Rightarrow \theta^h(b_t) < \theta^l(b_t)$

Proof. Lets first show that they can not be equal, and then show that $\theta^h(b_t)$ can not be larger than $\theta^l(b_t)$. Assume $\theta^h(b_t) = \theta^l(b_t)$. Then equations (2.23) and (2.25) imply that, $V_b^l = V_b^h$. Substituting in the right hand side of equation (2.26) we obtain zero,

$$\left[\rho b_t + g_h - (y + \rho a_0)\left(1 - \theta_t^h\right)\right] = 0$$

which implies that the solution for $\theta^h(b_t)$ is,

$$\theta^h(b_t) = 1 - \frac{g_h + \rho b_t}{y + \rho a_0} \neq \theta^l(b_t) \ \forall\, b_t < \infty$$

Which is a contradiction, because for any finite level of debt, the policy functions are different (this is done by inspection).

Assume $\theta^h(b_t) > \theta^l(b_t)$. In this case, equations (2.23) and (2.25) imply,

$$(y + \rho a_0) V_b^h = -\frac{1}{\theta_t^h} > -\frac{1}{\theta_t^l} = (y + \rho a_0) V_b^l \ \Rightarrow \ V_b^h > V_b^l$$

This implies that the right hand side of equation (2.26) is always positive. Given the properties of the value function we know that V_{bb}^h is negative. Thus this would imply that the term in the brackets is negative.

$$\left[\rho b_t + g_h - (y + \rho a_0)\left(1 - \theta^h(b_t)\right)\right] < 0$$

Solving for $\theta^h(b_t)$

$$\theta^h(b_t) < 1 - \frac{g_h + \rho b_t}{y + \rho a_0} < \theta^l(b_t)$$

which is a contradiction. Therefore, $\theta^h(b_t) < \theta^l(b_t)$ for any finite level of debt. ■

Note that this proposition implies that $V_b^h < V_b^l$. Thus the right hand side of equation (2.26) is negative. The proposition is stating that the interest rate when the expenditure is high should be larger than the interest rate when the expenditure is low. Now lets show that the optimal solution implies a disinflation program.

Proposition 6 $\forall b_t < \infty \Rightarrow \theta^h(b_t) > \theta^f(b_t)$

Proof. Remember that we define $\theta^f(b_t)$ as the solution to

$$\left[\rho b_t + g_h - (y + \rho a_0)\left(1 - \theta^f(b_t)\right)\right] = 0$$

Given the concavity of the value function and proposition (5) $\left(V_b^h < V_b^l\right)$ we know that,

$$\left[\rho b_t + g_h - (y + \rho a_0)\left(1 - \theta^h(b_t)\right)\right] > 0$$

Therefore, the optimal path implies a reduction in reserves (increasing debt) and $\theta^h(b_t) > \theta^f(b_t)$ for any finite level of debt. ■

These two propositions imply that the disinflation program is initiated at the moment a reform program is announced. Note that we have relaxes the assumption of known time of the reform and still the characteristics of the solution are the same.

Substituting the definitions of θ and the value functions, we obtain the following differential equation for the interest rate when expenditure is high.

$$\left[\rho b_t + g_h - (y + \rho a_0)\left(1 - \frac{1}{1+\alpha i^h}\right)\right]\frac{\partial i^h}{\partial b_t} = q\left[i^h - i^l\right] \tag{2.27}$$

where $i^l(b_t)$ has a close form solution from equation (2.24). The boundary condition for the differential equation is,

$$\lim_{b\to-\infty} i^f = \lim_{b\to-\infty} i^h = \lim_{b\to-\infty} i^l = -\frac{1}{\alpha}$$

The solution is shown in figure 2-9. The schedule in the bottom is the interest rate when there is a low level of expenditure. The schedule on the top is the interest rate implied by a flexible exchange rate when expenditure is high. The schedule in the middle is the solution for the differential equation when expenditure is high. Some properties of the solution of the interest rate can be studied analytically,

Proposition 7 *$i^h(b_t)$ has the following properties: First, it is increasing, convex and continuous. Second, it is smaller than the interest rate implied by flexible exchange rate. And third, $\frac{di^h}{db_t} > \frac{di^l}{db_t}$, which implies that the disinflation effort decreases with the level of debt.*

Proof. Increasing, convex and continuous come from the properties of the solution. We know that our control problem is concave and continuous. Therefore, the policy function θ is continuous, non-increasing and concave. Solving for the interest rate as a function of θ, we obtain $i = \frac{1}{\alpha}\left(\frac{1}{\theta} - 1\right)$.

So, the proof of continuity and increasing follow directly from the definition. To prove convexity we differentiate two times the definition of the interest rate,

$$\frac{d^2 i^h}{db_t^2} = \frac{1}{\alpha}\left[-\frac{d^2\theta^h}{db_t^2}\frac{1}{\theta^{h2}} + \left(\frac{d\theta^h}{db_t}\right)^2\frac{2}{\theta^{h3}}\right] > 0$$

The second part of the proposition comes from proposition 6.

The third part of the proposition comes from differentiating equation 2.27. Differentiating with respect to b.

$$q\left[\frac{\partial i^h}{\partial b_t}-\frac{\partial i^l}{\partial b_t}\right] = \left[\rho b_t+g_h-(y+\rho a_0)\left(1-\frac{1}{1+\alpha i^h}\right)\right]\frac{\partial^2 i^h}{\partial b_t^2} \qquad (2.28)$$
$$-\alpha\left[\frac{\partial i^h}{\partial b_t}\right]^2\left[\frac{y+\rho a_0}{(1+\alpha i^h)^2}\right]$$

but we know from the definition of θ that the following is true (differentiating the definition)

$$\frac{d^2\theta}{db^2}=-\frac{\alpha}{(1+\alpha i)^2}\frac{d^2 i}{db^2}+\frac{\alpha^2}{(1+\alpha i)^3}\left[\frac{di}{db}\right]^2<0$$

reordering the second derivative,

$$\frac{-\alpha}{1+\alpha i}\left(\frac{1}{(1+\alpha i)}\frac{d^2 i}{db^2}-\frac{\alpha}{(1+\alpha i)^2}\left[\frac{di}{db}\right]^2\right)<0$$

This means that the right hand side of (2.28) is always positive. This means that the left hand side is positive. Implying that the slope of the solution in the high state is larger than the slope in the low state. ■

Reserves constraint

In this section we assume that reserves have to be positive. This imposes a limit on the maximum level of debt. As we did in appendix 2.8.2, the level of debt when the constraint is hit is,

$$b_t=\frac{\alpha}{1+\alpha\rho}(y+\rho a_0-g_h)$$

the solution for the constrained economy implies the same differential equation as before but with a different boundary condition. Formally,

$$\left[\rho b_t+g_h-(y+\rho a_0)\left(1-\frac{1}{1+\alpha i^h}\right)\right]\frac{\partial i^h}{\partial b_t} = q\left[i^h-i^l\right]$$

$$
\begin{aligned}
i^h(\bar{b}) &= \frac{1}{\alpha}\left(\frac{g_h + \rho\bar{b}}{y + \rho a_0 - g_h - \rho\bar{b}}\right) \\
\bar{b} &= \frac{\alpha}{1+\alpha\rho}(y + \rho a_0 - g_h)
\end{aligned}
$$

The properties of the solution are conserved. The numerical solutions is shown in figure 2-10. We compare the solution for the constrained and unconstrained economies. Notice that for low levels of debt the two solutions behave similarly. However, when the crisis is close the interest rate starts increasing faster in the constrained case.

2.8.4 Expenditure follows a Jump Diffusion Process

In the paper we solve the optimal interest rate when the process follows a particular case of a Poisson process. In this section we find the solution for a more general stochastic process. The process assumed in the text has two characteristics: First, its drift is negative. Second, the timing of the change is known. In this section we study how important is the assumption of negative drift. We analyze under which conditions a disinflation program is initiated.

Assume that the expenditure follows,

$$dg_t = \mu_g dt + \sigma_g dz_t + \lambda_g dq$$

where dz_t is the standard Weiner process, and dq is a Poisson process that takes value 0 with probability $1 - qdt$, and value 1 with probability qdt. The Bellman's equation in continuous time is,

$$\rho V(b_t, g_t) = \max_{\theta_t} \{\ln \theta_t + EdV\}$$

Using Itô's lemma we can show that,

$$\rho V(b_t, g_t) = \max_{\theta_t} \left\{ \begin{array}{c} \ln \theta_t + [\rho b_t + g_t - (y + \rho a_0)(1 - \theta_t)] V_b(b_t, g_t) + \\ \mu_g V_g(b_t, g_t) + \frac{1}{2}\sigma_g^2 V_{gg}(b_t, g_t) \\ -q[V(b_t, g_t + \lambda_g) - V(b_t, g_t)] \end{array} \right\}$$

where V_i represents the partial derivative with respect to argument i. The first order condition implies,

$$\frac{1}{\theta_t} + (y + \rho a_0) V_b = 0$$

Notice that the value function has the following properties. First, it is continuous and twice differentiable. Second, it is decreasing with respect to debt and government expenditure. Notice that this implies that the policy function (θ_t) is non-increasing with debt and expenditure. Finally, V is concave with respect to its arguments.

The envelope theorem implies,

$$\begin{aligned}\left[\rho b_t + g_t - (y + \rho a_0)(1 - \theta_t)\right] V_{bb} &= -\mu_g V_{gb} - \frac{1}{2}\sigma_g^2 V_{ggb} \\ &\quad + q\left[V_b(\cdot, g_t + \lambda_g) - V_b\right]\end{aligned} \tag{2.29}$$

From the first order condition and the definition of θ we have,

$$V_b = -(y + \rho a_0)(1 + \alpha i_t)$$

This implies that the derivatives of the value function can be written as $V_{bb} = -(y + \rho a_0)\alpha i_b$, $V_{gb} = -(y + \rho a_0)\alpha i_g$, and $V_{ggb} = -(y + \rho a_0)\alpha i_{gg}$. Substituting in (2.29),

$$\begin{aligned}&\left[-\frac{\partial i(b_t, g_t)}{\partial b_t}\right]\left[\rho b_t + g_t - (y + \rho a_0)\left(1 - \frac{1}{1 + \alpha i(b_t, g_t)}\right)\right] \\ &= \mu_g \frac{\partial i(b_t, g_t)}{\partial g_t} + \frac{1}{2}\sigma_g^2 \frac{\partial^2 i(b_t, g_t)}{\partial g_t^2} - q\left[i(b_t, g_t + \lambda_g) - i(b_t, g_t)\right]\end{aligned}$$

This is a delayed partial differential equation, and it does not have a close form solution. However, we are more interested in its characterization rather than in its solution. We want to understand when the interest rate implied by this equation is smaller than the interest rate from flexible exchange rate. In other words, what are the conditions on the stochastic process that generates a managed exchange rate with a loss in reserves.

Define i^f as the interest rate implied by flexible exchange rate. This means that i^f solves,

$$\rho b_t + g_t - (y + \rho a_0)\left(1 - \frac{1}{1 + \alpha i^f}\right) = 0$$

In order to characterize the solutions lets study several cases.

No uncertainty, no drift, no Poisson: $\mu_g = 0$, $\sigma_g^2 = 0$, $q = 0$.

In this case, the differential equation is,

$$\left[-\frac{\partial i_t}{\partial b_t}\right]\left[\rho b_t + g_t - (y+\rho a_0)\left(1 - \frac{1}{1+\alpha i_t}\right)\right] = 0$$

which implies that the optimal solution is equal to the flexible exchange rate. In this case, the government expenditure is constant and the optimal solution is a constant rate of depreciation consistent with the transversality condition on government debt.

No uncertainty with drift, no Poisson: $\mu_g \neq 0,\ \sigma_g^2 = 0,\ q = 0.$

In this case the differential equation is,

$$\left[-\frac{\partial i_t}{\partial b_t}\right]\left[\rho b_t + g_t - (y+\rho a_0)\left(1 - \frac{1}{1+\alpha i_t}\right)\right] = \mu_g \frac{\partial i_t}{\partial g_t}$$

Remember that we argued before that θ was a decreasing function of debt and expenditure. This means that the interest rate is an increasing function of both arguments.

On the one hand, assume that the drift is negative. The right hand side of the differential equation is negative. In the left hand side, the first term in the brackets is always negative, thus the second term has to be positive ($i_t < i^f$). The second term is the debt accumulation, and the solution implies that debt will be accumulated when there is a negative drift.

On the other hand, if the drift is positive, the second term has to be negative and debt should be decreasing ($i_t > i^f$). So, the central bank is accumulating reserves.

Notice that this is a case of certainty: If the government expenditure falls then future taxations are smaller, tax smoothing implies that current rate of inflation should be smaller. The converse is true when the drift is positive.

Uncertainty with no drift, no Poisson: $\mu_g = 0,\ \sigma_g^2 > 0,\ q = 0.$

In this case the differential equation is,

$$\left[-\frac{\partial i_t}{\partial b_t}\right]\left[\rho b_t + g_t - (y+\rho a_0)\left(1 - \frac{1}{1+\alpha i_t}\right)\right] = \frac{1}{2}\sigma_g^2 \frac{\partial^2 i_t}{\partial g_t^2}$$

We argued before that the value function is concave in its argument, therefore the interest rate

has to be a convex function on its argument. This implies that the right hand side is always positive. An increase in uncertainty implies a decreasing debt ($i_t > i^f$). This result comes from the prudence of the objective function.

Poisson with no uncertainty, no drift: $\mu_g = 0,\ \sigma_g^2 = 0,\ q > 0.$

In this case, the differential equation is,

$$\left[-\frac{\partial i_t}{\partial b_t}\right]\left[\rho b_t + g_t - (y + \rho a_0)\left(1 - \frac{1}{1+\alpha i_t}\right)\right] = -q\left[i_t\left(\cdot, g_t + \lambda_g\right) - i_t\right]$$

We know that the interest rate is increasing with the level of government expenditure thus the right hand side is negative and the solution implies that a disinflation program is initiated.

The conclusion that can be derived from studying these "cases" is that a negative drift is a necessary condition for obtaining an optimal exchange rate policy that run reserves down. However, it is not enough. If the drift is not large enough to make the right hand side negative, then the conclusion is reversed. In other words,

$$\mu_g \frac{\partial i_t}{\partial g_t} + \frac{1}{2}\sigma_g^2 \frac{\partial^2 i_t}{\partial g_t^2} - q\left[i_t\left(\cdot, g_t + \lambda_g\right) - i_t\right] < 0$$

Notice that this is an equilibrium condition, which makes it very difficult to characterize. Using numerical methods we can find the required drift to satisfy the condition, given certain level of uncertainty. We solve the problem for the case $q = 0$ and find the set of points $[\mu, \sigma^2]$ such that the solution for the differential equation implies a flexible exchange rate. The solution is shown in figure 2-13.

[Figure 2-13]

Solution when there is a debt constraint.

In the same way we did for the Poisson case, the debt constraint adds a constraint in the partial differential equation. In this case, the constraint is the following:

$$b_t \leq \frac{\alpha}{1+\alpha\rho}\left(y + \rho a_0 - g_t\right)$$

Moreover, we know that at that level of debt the interest rate is given by the flexible exchange rate one. We use this as a boundary condition for the partial differential equation.

The solution for the partial differential equation are shown in figure 2-14 and 2-15. Figure 2-14 is the solution for the unconstrained economy, and figure 2-15 is the solution for the constrained economy.

[Figure 2-14]

[Figure 2-15]

2.8.5 Model with tradables and non tradables.

In this section we solve the model assuming that there are tradables and non tradables. We solve the problem in two steps: First, we solve for the whole economy (consolidating all agents). Second, we solve for the money holdings. The consumer's and government's budget constraint are,

$$\begin{aligned}\dot{a}_t &= \rho a_t + y_T + \frac{p_{Nt}}{e_t} y_N - c_{T,t} - \frac{p_{Nt}}{e_t} c_{N,t} - i_t m_t \\ \dot{b}_t &= \rho b_t + g_t - i_t m_t\end{aligned}$$

where a_t are the consumer assets, b_t is the government's debt, y_T is the output of tradables, y_N represents the non tradable output, $c_{T,t}$ is the consumption in tradables, $c_{N,t}$ is the consumption in tradables, e_t is the exchange rate level (and the price of tradables), p_{Nt} is the price of non tradables, and β is the share of tradables. Define the net foreign holdings as, $x_t = a_t - b_t$. This implies that the problem is,

$$\begin{aligned}&\max_{\{c_t\}} \; E \int_0^\infty (1-\beta) \ln c_{N,t} + \beta \ln c_{T,t} \, e^{-\rho t} dt \\ s.t. & \\ \dot{x}_t &= \rho x_t + y_T + \frac{p_{N,t}}{e_t} y_N - c_{T,t} - \frac{p_{N,t}}{e_t} c_{N,t} - g_t \\ \lim_{t\to\infty} x_t e^{-\rho t} &= 0\end{aligned}$$

where the first equation is the objective function. The second equation is the intertemporal budget constraint in terms of tradables. And the third equation is the transversality condition on total assets. Writing the Hamiltonian,

$$H = (1-\beta) \ln c_{N,t} + \beta \ln c_{T,t} + \lambda_t \left(\rho x_t + y_T + \frac{p_{N,t}}{e_t} y_N - c_{T,t} - \frac{p_{N,t}}{e_t} c_{N,t} - g_t \right)$$

The first order conditions are,

$$\begin{aligned} \frac{\beta}{c_{T,t}} - \lambda_t &= 0 \\ \frac{1-\beta}{c_{N,t}} - \frac{p_{N,t}}{e_t}\lambda_t &= 0 \\ \dot{\lambda}_t &= 0 \end{aligned}$$

Substituting in the budget constraint and using the market clearing condition in the non tradable sector we obtain,

$$\dot{x}_t = \rho x_t + y_T - g_t - \frac{\beta}{\lambda_0}$$

Assume that the level of expenditure is constant, then integrating and imposing the transversality condition we obtain,[29]

$$\frac{\beta}{\lambda_0} = y_T - g_t + \rho x_0$$

Solution without fiscal uncertainty.

In this section, we show that if there is no fiscal uncertainty the optimal strategy implies a flexible exchange rate. If expenditure is constant the solution for the tradables and non tradables consumption is,

$$\begin{aligned} c_T &= y_T - g + \rho x_0 \\ c_N &= (y_T - g + \rho x_0)\frac{e_t}{p_{N,t}}\frac{1-\beta}{\beta} \end{aligned} \tag{2.30}$$

The cash in advance constraint implies,

[29] If the level of expenditure is not constant then

$$\frac{\beta}{\lambda_0} = y_T + \rho x_0 - \rho \int_0^\infty g_t e^{-\rho t} dt$$

In other words, the consumption level only cares about the equivalent annuity of the government's expenditure.

$$m_t = \alpha \left[c_T + \frac{p_{N,t}}{e_t} c_N \right] = \frac{\alpha}{\beta} c_T$$

Using the government's budget constraint we can compute the only constant depreciation rate consistent with the government's debt transversality condition: $i_t m_t = g + \rho b_0$. After some straight algebra,

$$\frac{1}{1 + \frac{\alpha}{\beta} i} = 1 - \frac{g + \rho b_0}{y + \rho a_0}$$

Finally, we use the market clearing condition in the non tradable sector to determine the real exchange rate,

$$z_t^f \equiv \frac{e_t}{P_{N,t}} = \frac{1 - \beta}{\beta} \frac{y_N}{y_T - g + \rho x_0} \tag{2.31}$$

We define that there is a real exchange rate appreciation when $z_t < z_t^f$.

Fiscal uncertainty.

In this section we solve the problem with expenditure uncertainty. Given that we know that without fiscal uncertainty the solution is a constant consumption, then after τ, there should be a constant inflation rate consistent with a flexible exchange rate. The consumption levels are,

$$\begin{aligned} c_T^h &= y_T - g_h + \rho x \tau \\ c_T^l &= y_T - g_l + \rho x \tau \end{aligned}$$

where c_T^h is consumption when the level of expenditure is high, and c_T^l is consumption when the level of expenditure is low. It is easy to show that consumption before τ also has to be constant. Substituting this in the utility function the new control problem is,

$$\max_{c_T^1} \left[\frac{\beta}{\rho}\left(1-e^{-\rho t}\right)\ln c_T^1 + \frac{\beta}{\rho}e^{-\rho t}\left(q\ln c_T^l + (1-q)\ln c_T^h\right) + \Psi_N\right]$$

s.t.

$$x_T = x_0 + \frac{e^{\rho\tau}-1}{\rho}\left[y_T + \rho x_0 - g_h - c_T^1\right]$$

where Ψ_N is a constant that involves the utility in non tradables and some constants. After some algebra we obtain the following system of equations,

$$\begin{aligned} c_T^h &= y_T - g_h + \rho x_T \\ c_T^l &= y_T - g_l + \rho x_T \\ x_T &= x_0 + \left(\frac{e^{\rho\tau}-1}{\rho}\right)\left[y_T + \rho x_0 - g_h - c_T^1\right] \\ \frac{1}{c_T^1} &= \frac{1-q}{c_T^h} + \frac{q}{c_T^l} \end{aligned}$$

The solution for the non tradable consumption is given by,

$$c_{N,t} = c_{T,t}\frac{e_t}{p_{N,t}}\frac{1-\beta}{\beta}$$

where we can solve for the real exchange rate as we did before.

Proposition 8 *Along the optimal path, the real exchange rate between* $[0,\tau]$ *is appreciated.*

Proof. To prove this result we show that there is a consumption boom prior to τ. Given this result and the definition of the real exchange rate we can prove the existence of an appreciation.

Lemma 9 *Consumption of tradables between* $[0,\tau]$ *is larger than the one implied by flexible exchange rate. In other words, there is a consumption boom.*

Proof. The lemma is proved by contradiction. The lemma claims that c_T^1 is larger than the implied consumption in a flexible exchange rate regime, denoted as $c_T^{f,h}$. Suppose the lemma is false. Given equation 2.30 we know that,

$$c_T^{f,h} = y_T - g_h + \rho x_0$$

If $c_T^1 < c_T^f$, then the assets accumulation equation implies,

$$x_\tau = e^{\rho\tau} x_0 + \frac{e^{\rho\tau} - 1}{\rho} \left[y_T - g_h - c_T^1 \right] > e^{\rho\tau} x_0 + \frac{e^{\rho\tau} - 1}{\rho} \left[y_T - g_h - c_T^{f,h} \right] = x_0$$

In other words, there is an accumulation of assets. This implies that consumptions after τ are larger than the ones implied by flexible exchange rate,

$$\begin{aligned} c_T^h &= y_T - g_h + \rho x_\tau > y_T - g_h + \rho x_0 = c_T^{f,h} \\ c_T^l &= y_T - g_l + \rho x_\tau > y_T - g_l + \rho x_0 = c_T^{l,h} \end{aligned}$$

But we know that consumption before τ is given by,

$$\frac{1}{c_T^1} = \frac{1-q}{c_T^h} + \frac{q}{c_T^l} < \frac{1-q}{c_T^{f,h}} + \frac{q}{c_T^{f,l}}$$

which is a contradiction. ■

The lemma implies that there is a consumption boom between $[0, \tau]$. Now, the proof of the proposition is just by inspection of equation 2.31. Notice that the real exchange rate is a monotone decreasing function of consumption of tradables. The optimal strategy implies a consumption boom $\left(c_T^1 > c_T^{f,h} \right)$ thus there is a real exchange rate appreciation $\left(z^1 < z^{f,h} \right)$.

■

	When the Expenditure was Reduced?		
	Before	During	Never
Successful	Chile 78	Argentina 91 Bolivia 85 Peru 90	
Not Successful	Mexico 87 Uruguay 79	Argentina 79 Argentina 85 Brazil 90 Chile 75 Mexico 82 Uruguay 74	Brazil 86 Venezuela 89 Venezuela 94

	When the Fiscal Deficit was Reduced?		
	Before	During	Never
Successful	Chile 78	Argentina 91 Bolivia 85 Peru 90	
Not Successful	Uruguay 79	Brazil 90 Chile 75 Mexico 82 Mexico 87 Uruguay 74 Venezuela 89 Venezuela 94	Argentina 79 Argentina 85 Brazil 86

Table 2.1: Latin American Stabilization Programs

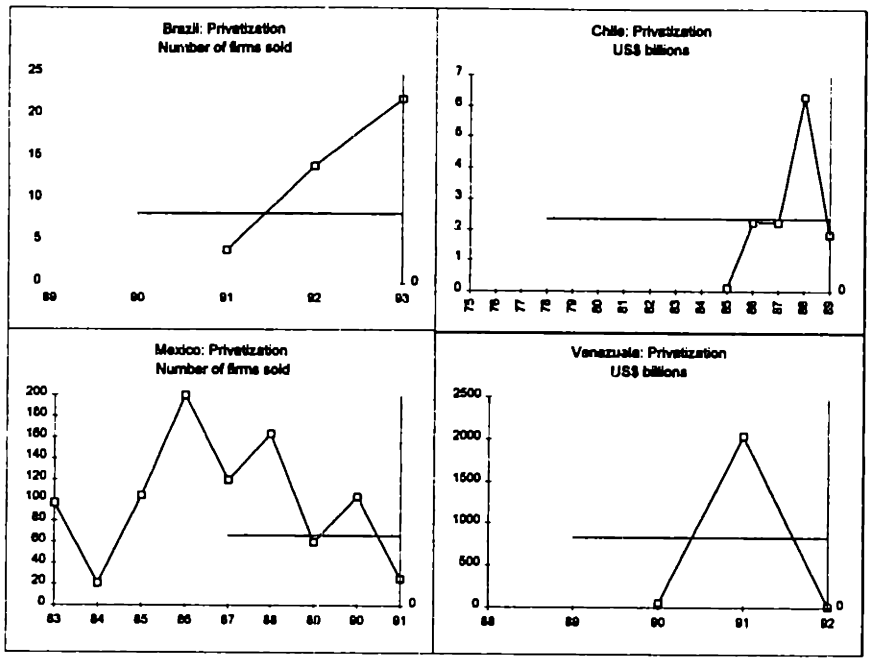

Figure 2-1: Stabilization programs and Privatization

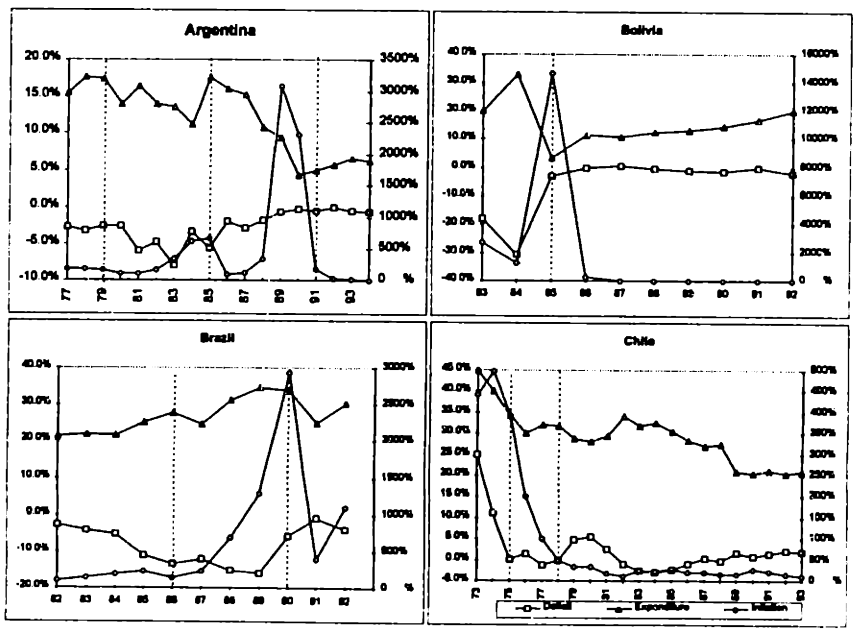

Figure 2-2: Latin American Experiences.

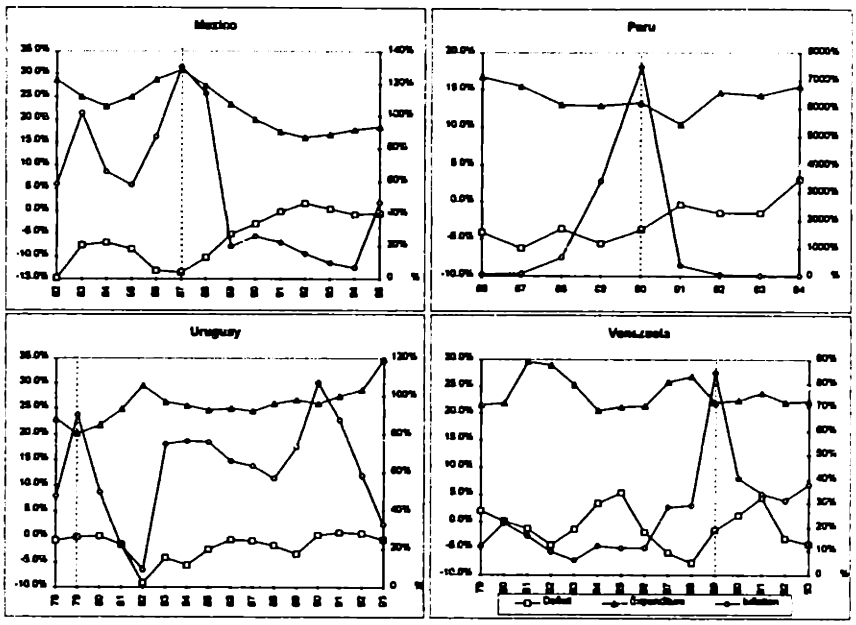

Figure 2-3: Latin American Experiences.

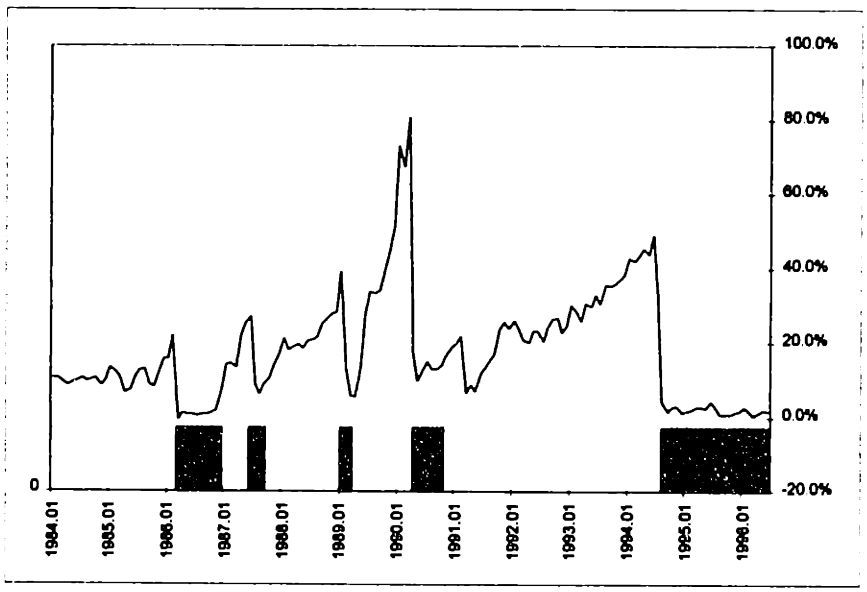

Figure 2-4: Stabilization programs and monthly inflation: Brazil.

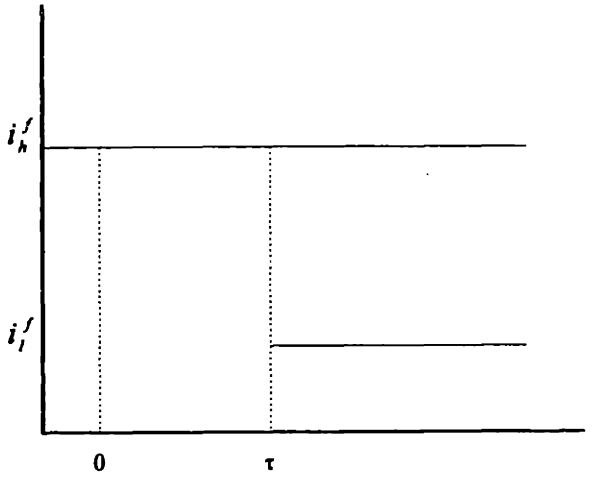

Figure 2-5: Interest rates implied by flexible exchange rate.

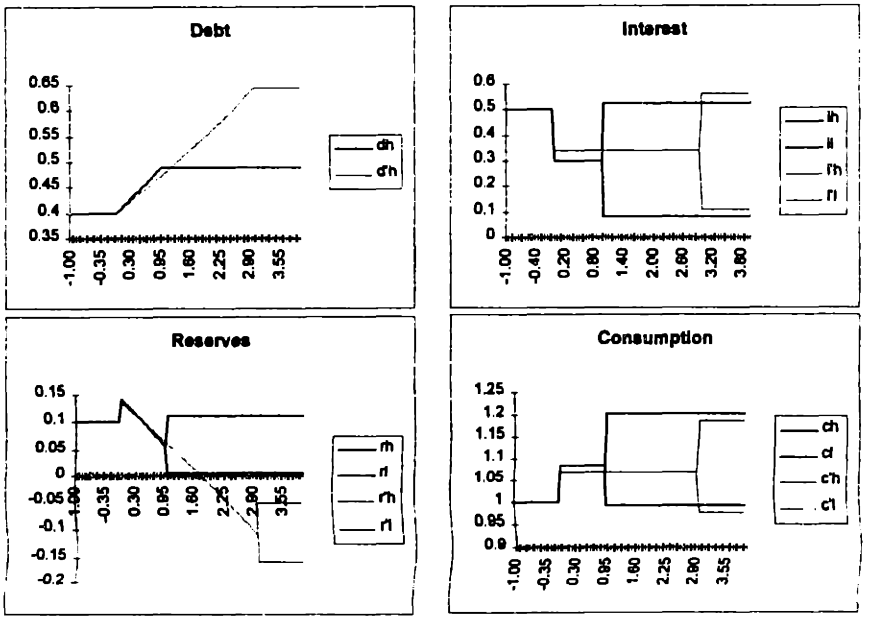

Figure 2-6: Simulations for the unconstrained economy. Increase in the time until reform.

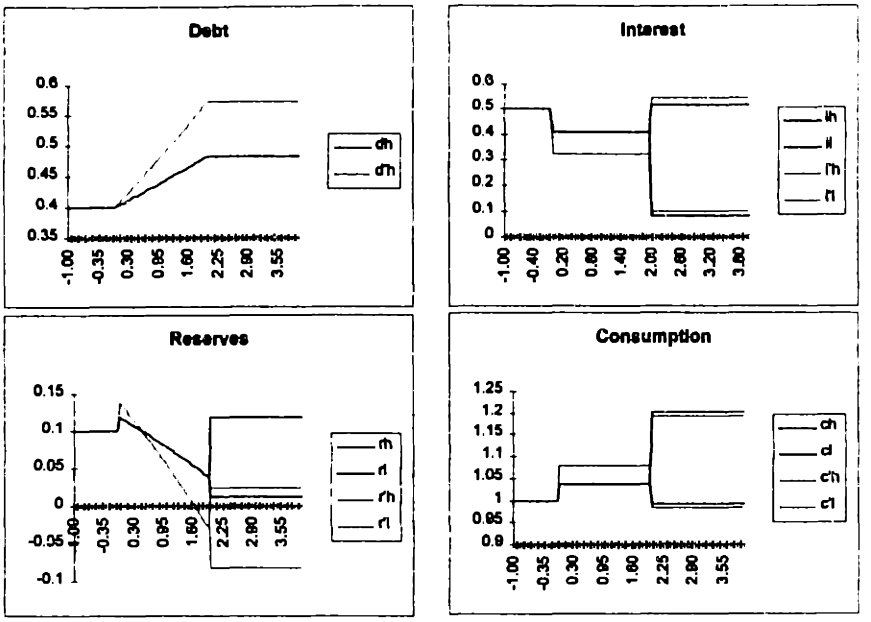

Figure 2-7: Simulations for the unconstrained economy. Increase in the probability of success of the reform.

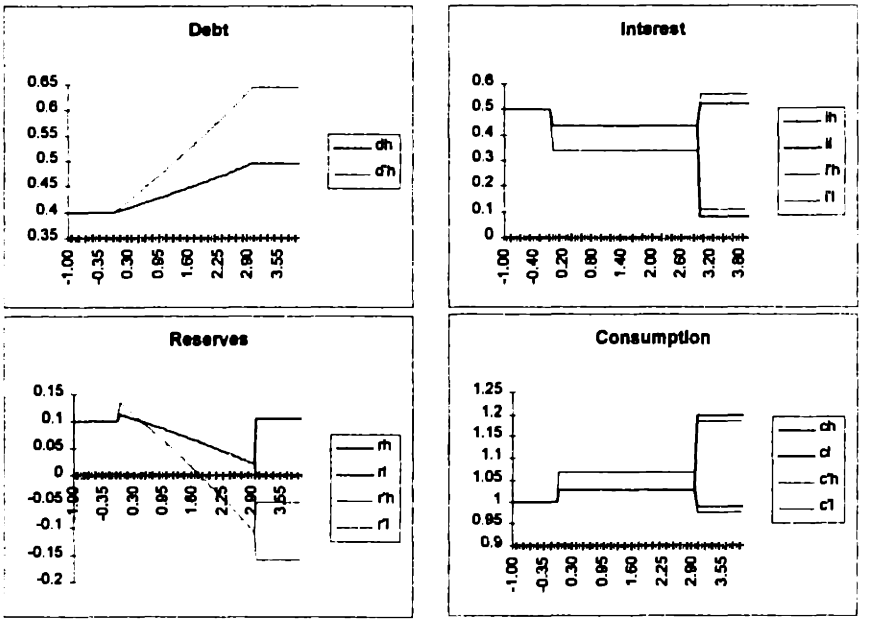

Figure 2-8: Comparison between the constrained and unconstrained economies.

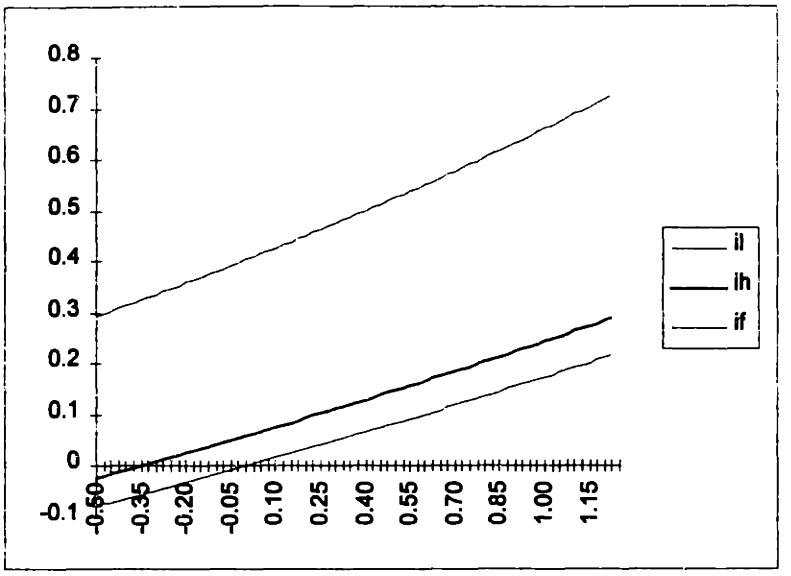

Figure 2-9: Solution to the Poisson Case.

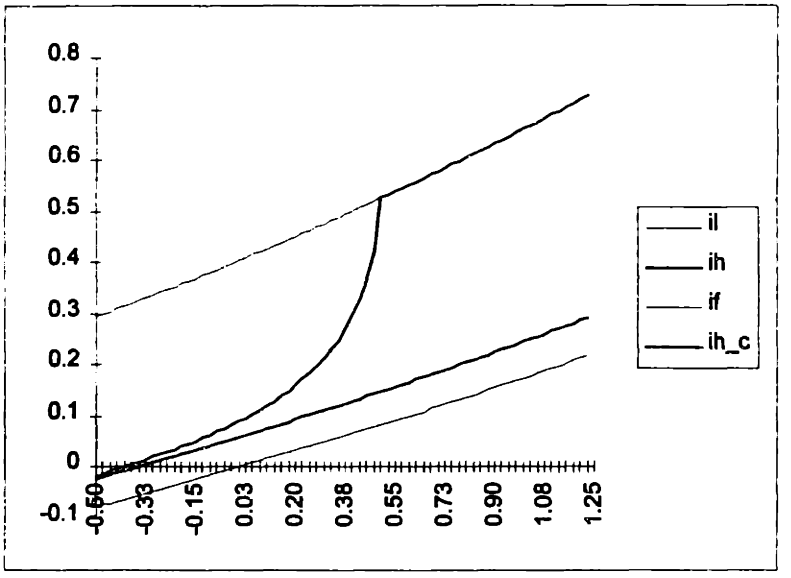

Figure 2-10: Solution to the Poison case. Comparison between the constrained and unconstrained economies.

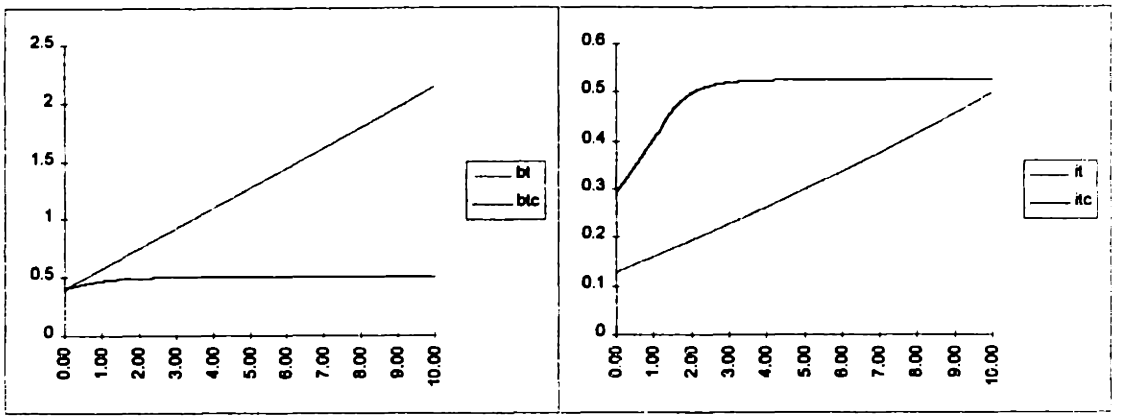

Figure 2-11: Solution to the Poisson case. Debt and Interest rate's path.

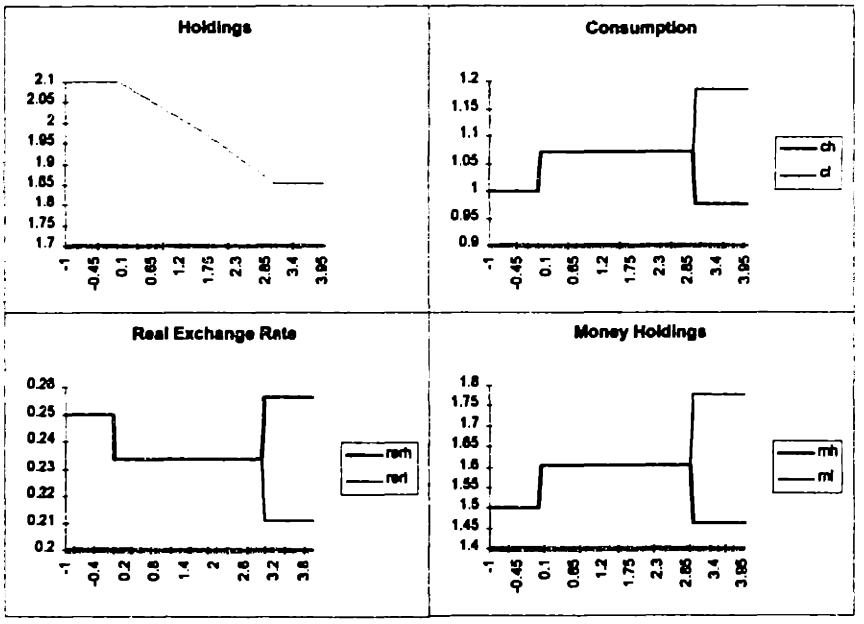

Figure 2-12: Real Exchange Rate Model.

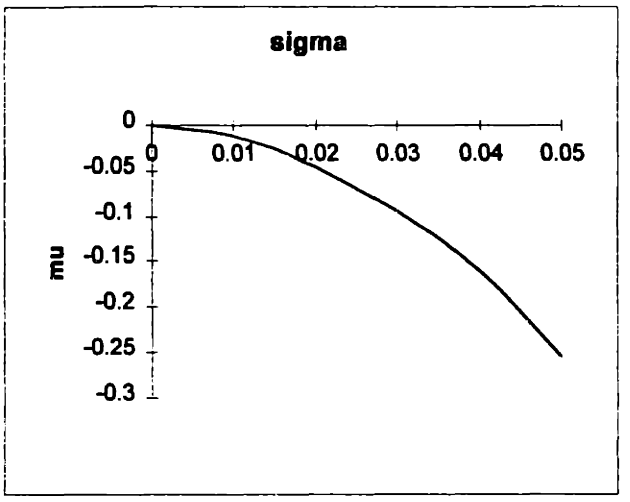

Figure 2-13: Solution to the Brownian Motion case. Determination of the region where current interest rates equal the pure flexible exchange rate.

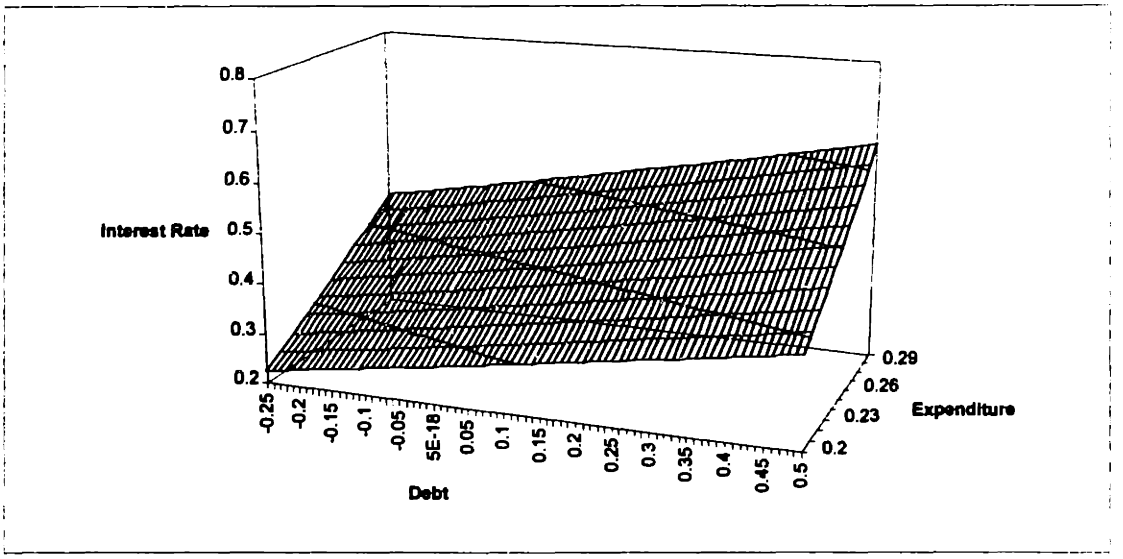

Figure 2-14: Solution to the Brownian Motion case under no debt constraints.

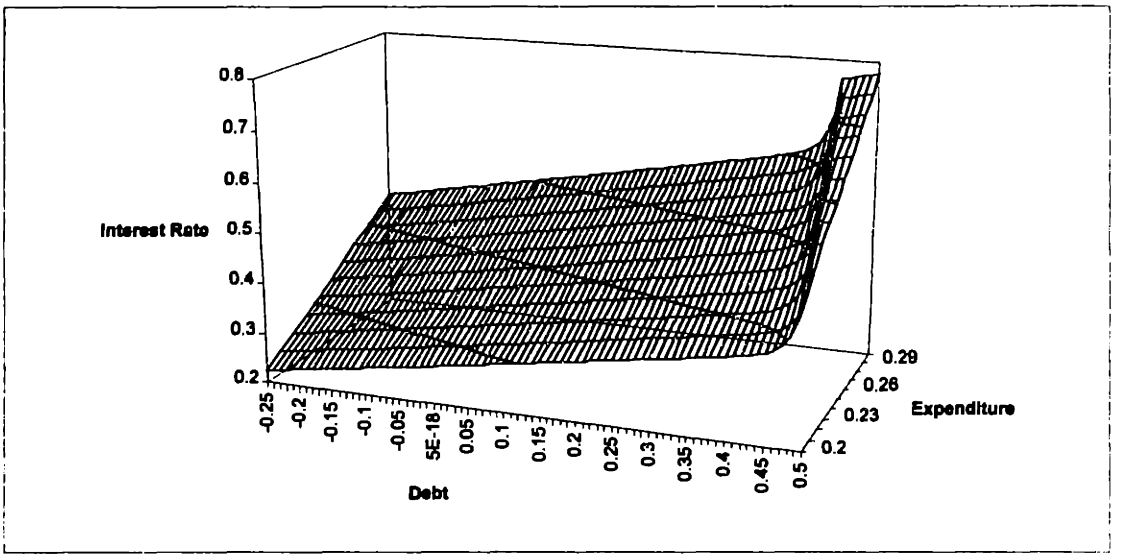

Figure 2-15: Solution to the Brownian Motion case under debt constraints.

Chapter 3

A source of volatility in Latin America: Changes in mood.

3.1 Introduction

The 1995 Economic and Social report of the Inter-American Development Bank (hereafter IDB 1995) argues that Latin American countries are subject to high volatility.[1] In the following table, a comparison between Latin America and Industrialized economies is shown.[2]

Standard Deviation of	Latin America	Industrialized countries
GDP Growth	4.7	2.2
Domestic Investment Growth	16.1	8.3
Change in Real Exchange Rate	13.4	4.8
Fiscal Deficit (% GDP)	4.7	2.4
Terms of Trade Growth	15.1	8.9
Capital Flows (% GDP)	2.8	1.7

As it can be seen, the volatility in Latin America is reflected in every aspect of the economy:

[1] As is reported in IDB (1995), there is a clear expectrum of volatilities: Industrialized economies are the less volatiles, Asia is next, Latin America is third, and Transition economies and Africa are the more volatile ones. In this paper, we concentrate in Latin America, because the problems described here better fit the region experience.

[2] The standard deviations were computed from 1970-1992. See IDB (1995).

relative prices, quantities, and policies. What causes this high volatility in the region? Can external factors account for it?

Calvo, Leiderman and Reinhart (1993) study the determinants of capital flow volatility in 10 Latin American economies. Due to problems in the measurement of capital flows, they concentrate on changes in real exchange rates and official reserves. The idea is that factors that increase capital inflows have to be ultimately reflected in a real exchange rate appreciation, or in an increase in reserves. They find that external factors can account for as much as 50 percent of the variance of the real exchange rate and of the official reserves. However, their methodology tends to overestimate the importance of these factors. First, there are other shocks that have not been considered in the regression, that do affect the real exchange rate and the official reserves (such as, oil discoveries). These shocks are transmitted into domestic policies, which are positively correlated with their explanatory variables. Thus, the omitted regressors bias is likely to be upward. Second, there might exist problems of endogeneity. They run a VAR in reduced form where policies have been solved out. If these policies amplify, rather than smooth, domestic and external shocks, then the estimates are upward biased.[3] Therefore, their estimate can be seen as an upper bound of how much external factors can explain.

On the other hand, IDB (1995) runs growth regressions where the standard deviations of the growth rate and of the real exchange rate are the dependent variables. They find that external factors account for only 16 percent of the GDP variance, and six percent of the real exchange rate volatility. However, their methodology has severe problems of endogeneity. They include the volatility of domestic monetary and fiscal policies as regressors, which are obviously endogenous to external shocks. In their case, there is a downward bias, and therefore, they underestimate the explanatory power of external factors.

Both methodologies have some econometric problems and therefore the estimates should be taken cautiously; the true parameter lies somewhere in the middle. However, the results are still suggestive: external factors explain a sizable fraction of the Latin American variance, but there exists an important source of volatility that is explained by the government's policies and the agents' choices.

[3]Gavin, Hausmann, Perotti and Talvi (1996) show that domestic policies in Latin American are indeed pro-cyclical.

What makes the actions of the agents, and the policies of the government more volatile in Latin America than in industrialized economies? In this paper, we offer a simple theoretical model in which additional variance is generated through an informational problem.

To develop the intuition of the model, first assume that the ability of the government to manage the economy is uncertain.[4] Second, assume that the economic performance is the only measure of the government's type. Therefore, if a good realization is observed, the assessed ability of the government increases. Third, assume that foreign investors decide the amount of capital to invest, conditional on their beliefs on the government's type. Also assume that they invest more, when the expected ability of the government increases. Finally, assume that capital inflows reduce the real interest rate and increase the output.

In this setup, foreigners have incentives to invest more capital when good news come and performance improves. There is a reduction in the interest rate that increases the output and the probability that better outcomes are observed in the future. Therefore, a temporary shock is amplified and perpetuated until a new piece of information comes, and the expectations get reversed: The initial improvement in beliefs is confirmed with further better performances.

In this model, there are two effects working together: First, there is a *multiplier* effect. An increase in output leads to a reduction in the interest rate, which at the end, increases output even more.

Second, there is a *signal jamming* problem at low interest rates. A reduction in the interest rate increases the probability that better outcomes are observed for all types of government. This implies that after a good performance is observed, there is a higher probability that better outcomes will come in the future independently of the underlying type of government. Therefore, for some period, there might exist self-sustained regimes.

These two effects combined imply that, economies where the informational problems are more severe, are subjected to a higher volatility.

We assume that developing countries are more likely to face this type of signal extraction problem. These economies are subjected to uncertainty in the effectiveness of the structural

[4]This assumption can be justified if either the degree of corruption, or the managerial skills of the executive, or the effectiveness of the policies are uncertain. We argue that these circumstances are more frequent and severe in Latin America that in Industrialized economies.

program of reforms, in the political support of these reforms, in the ability and managerial skills of the executive, and finally, in the degree of corruption that exists. It can be argued that industrialized economies face the same type of uncertainties. However, the pace of reforms in developing countries and the variability in the quality of their governments in the last three decades, suggest that the informational problems are more severe in Latin America.

In this paper, we concentrate in the interaction between the government and the foreign investors. However, the central bank, the congress, and the private sector face a similar signal extraction problem. Thus, we could expect that their choices are subjected to the same fluctuations described above.

The paper is organized as follows: In section two, an overview of the model is presented. Particular attention is given to the *multiplier* effect and its time series implications. In section three, the general set up is developed, and the relative importance of the *signal jamming* problem is assessed. Finally, in section four, conclusions are discussed.

3.2 The model

Consider a small open economy with a single good that is perfectly tradable. Assume that there are two agents: the government and foreign investors. Assume that the output produced in the country depends on the government's ability and the real interest rate. The government's ability determines the output potential of the economy, while the real interest rate determines how close the economy is from its potential.

We assume that the government's ability is unknown, and that (through time) the agents learn about it by observing output. The economy is described then, by two equations: an output equation and an interest rate equation.

The output equation is a modified IS schedule. We assume that the higher the ability of the government is, the larger the expected output. Conversely, the higher the real interest rate is, the lower the realized output. Formally,

$$y_t = Y_t(\gamma_t) - \alpha r_t$$

where Y_t is the output potential, r_t is the expected real interest rate, y_t is the realized output,

and γ_t is the information that exists about the government's ability.

Assume that there are two types of government (good and bad), and that the potential output has the following process:

$$good \sim \begin{cases} y_h & q \\ y_l & 1-q \end{cases} \quad \text{and} \quad bad \sim \begin{cases} y_h & 1-q \\ y_l & q \end{cases} \tag{3.1}$$

where $y_h > y_l$, and q is assumed to be larger than $1/2$. Define the output gap as $\Delta y \equiv y_h - y_l$.

The properties of the potential output process are the following: The expected value when the type is good is $y_l + q\Delta y$, while when the type is bad it is $y_l + (1-q)\Delta y$. The variance of both types is the same and equal to $q(1-q)\Delta y^2$.

There are two sources of uncertainty in this model. First, conditional on a type, potential output fluctuates between good and bad outcomes. This is the *outcome* uncertainty and it is related to how large is Δy. Second, there is uncertainty about which type of government is in place, which depends on how different q is from $1/2$. This is the *government's type* uncertainty.

To illustrate this point consider the following examples: First, assume that $q = 1/2$. In this case, the two government types are identical, and the economy is facing only outcome uncertainty. Note that the output variance is equal to $\Delta y^2/4$ independently of the prior about the type.

Second, assume that $q = 1$. Conditional on the type, uncertainty is zero: if government's type is good (bad) then output potential is always equal to y_h (y_l). In this case, the uncertainty comes form the probability of facing a particular type of government. Denote γ_t as the probability that the government's type is good, then the variance of this economy is $\gamma_t(1-\gamma_t)\Delta y^2$.

The dynamic implications of these two economies are very different. Assume that $\gamma_0 = 1/2$. Conditional on time zero, both economies face the same uncertainty. However, after one period, output is realized and beliefs are updated. In the first case, nothing changes because output is independent of the prior. In the second case, γ_1 is either equal to one, or equal to zero. Therefore, after one period the conditional variance in the first case remains constant, while in the second case, it is zero.

This difference in the dynamic behavior of the two types of uncertainty combined with the agents' reaction function, is what generates additional volatility in the economy. We come back

to this point later.

In order to avoid asymmetric informational problems, assume that the government's ability is uncertain to both agents. This is a strong assumption, because it implies that not even the government knows its own type. This assumption simplifies the problem abstracting from optimal contracting issues.[5]

Assume that a new government has an exogenous prior equal to γ_0. Given the stochastic process of expenditure (equation 3.1), posteriors are updated according to,

$$\gamma_{t+1} = \begin{cases} \dfrac{q\gamma_t}{q\gamma_t + (1-q)(1-\gamma_t)} & \text{if } y_h \text{ is observed.} \\ \dfrac{(1-q)\gamma_t}{(1-q)\gamma_t + q(1-\gamma_t)} & \text{if } y_l \text{ is observed.} \end{cases} \tag{3.2}$$

Under these assumptions, the expected output conditional on information at time t, is given by:

$$Ey_t = \delta_t y_h + (1-\delta_t) y_l - \alpha r_t \tag{3.3}$$

where δ_t is the probability that the high output is observed:

$$\delta_t = q\,\gamma_t + (1-q)(1-\gamma_t)$$

Equation 3.3 is one of the equations that describes the economy. The second one is the foreign investors' reaction function.

Assume, for simplicity, that foreign investors choose the real interest rate, instead of the capital flows.[6] Moreover, assume it is given by the following relation,

$$r_t = \rho + x\,(Ey_t) \tag{3.4}$$

were ρ represents the international interest rate, which is assumed to be constant, and $x\,(.)$ is a risk premium, or default premium that foreign investors charge. For expositional convenience,

[5] In the presence asymmetric information problems, if there exists ratchet effects, similar results are obtained. See Laffont and Tirole (1993).

[6] This assumption is for expositional convenience given that in our model both variables are uniquely determined.

we adopt the default premium interpretation. Similar arguments can be advanced if $x(.)$ represents the risk premium.

Assume that if a bad outcome is realized, the foreigners do not receive the interest rate payments. Assume that they incur in a fix cost (bankruptcy costs) every time default occurs. Under these assumptions the expected real interest rate charged by foreigners satisfy an equation similar to 3.4.[7]

Assume that the properties of the default premium are the following: $x(.) > 0$, and $x'(.) < 0$. The intuition is that a larger expected output implies a lower probability that the debt is going to be defaulted. And therefore, the premium charged by the investors should be decreasing with respect to expected output.

Finally, to assure stability of equilibrium, assume that $|x'(.)| < \frac{1}{\alpha}$. This implies that the foreign investors' reaction function is flatter than the output equation.

Equations 3.3 and 3.4 form a system of equations that describes the economy. In figure 3-1, the two schedules are depicted. Given the assumptions on the default premium the foreigner's reaction function is flatter than the output equation.

[Figure 3-1]

Increases in γ_t shifts the output equation upward, and the foreigner's reaction function downward. The idea is that, for the same interest rate an increase in γ_t increases output potential shifting the Ey equation to the right. On the other hand, an increase in γ_t, reduces

[7] One way of justifying the foreigners' reaction function is to assume that they solve the following problem:

$$V(\gamma_t) = \max_{R_t} \left\{ (\delta_t R_t - (1-\delta_t)k - \rho) D(R_t) + \frac{1}{1+\rho} EV(\gamma_{t+1}) \right\}$$

where $D(R_t)$ is the downward slopping domestic demand for foreign capital, and k is the fixed bankruptcy cost that has to be paid in case of default. R_t is the interest rate paid in case of no default, and r_t is the expected interest rate. Therefore, in this case, $r_t = \delta_t R_t$. The first order condition of this problem is:

$$(\delta_t R_t - (1-\delta_t)k - \rho) D'(R_t) + \delta_t D(R_t) = 0$$

solving for the interest rate,

$$r_t = \delta_t R_t = \frac{\rho + (1-\delta_t)k}{1+\varepsilon_t}$$

where ε_t is the inverse of the elasticity of the demand with respect to interest rate. $\varepsilon_t = \frac{D(R_t)}{R_t} \frac{dR_t}{dD(R_t)}$. Note that a constant elasticity demand gives a reaction function that is equivalent to the one described in equation 3.4.

the default premium, conditional on the same output potential. This tends to shift downward the foreigners' schedule.

The rest of the section is organized as follows: First, the multiplier effect is derived. Second, the serial correlation in performances is shown. The multiplier effect and the serial correlation are the two first time series implications of the model. In the last part of the section, a simulation is run to assess their contribution to the additional volatility in output.

3.2.1 Multiplier effect

In this section, we determine the conditions under which multiplier effects are likely to be found. We discuss the intuition of these conditions in two contexts: First, in the benchmark case where the participants are the government and foreign investors. Second, when the participants are the government and the congress.

Totally differentiating equations 3.3 and 3.4 with respect to γ_t, and reordering we obtain the change in output due to an increase in the prior (a change in γ_t can be thought as an informational shock).

$$\frac{dEy_t}{d\gamma_t} = \frac{(2q-1)\,\Delta y}{1+\alpha x'\left(Ey_t\right)} > 0 \tag{3.5}$$

Equation 3.5 indicates how responsive output is to changes in information. Remember that in this economy agents are learning, thus, γ_t is an stochastic process. The more responsive output is to shocks in information, the larger the output variance is.

Note that when there is no uncertainty between governments there is no multiplier. In other words, if both types are identical ($q = 1/2$), then an informational shock has no impact on output.

A multiplier exists when the denominator in equation 3.5 is smaller than one (i.e., $-\frac{1}{\alpha} < x' < 0$). The first inequality is for stability reasons, and therefore is not discussed. The second one is an assumption.

How plausible is to find $x' < 0$? When $x\,(.)$ represents the default premium, it is reasonable to expect that it is decreasing with better information. In other words, an increase in the probability that the government's type is good implies a lower probability of default. Therefore, an increase in expected output reduces the premium.

However, even though for the foreigners' reaction function it is relatively easy to justify that $x(.)$ is decreasing, this is not necessarily the case in other circumstances. For example, assume that the players are the congress and the government. Reinterpret the model in the following way: First, assume that there is uncertainty about the ability of the government to implement reforms. Second, assume that every period the government submits a fixed number of new laws, and that r_t of those are rejected by the congress. Third, assume that it is costly for the congress to approve a reform, regardless of its implementation. These costs can be thought as political costs of dealing with the relevant constituency that is affected by the law. Finally, assume that the benefits of the implemented reforms are realized immediately.[8] Under this interpretation, the output and the r_t equations are unchanged. However, it is not clear that $x' < 0$.

On the one hand, the worse the situation of the economy is (Ey_t low) the larger the marginal benefit of reforms, so the congress has incentives to approve more laws.[9] This implies that x' should be positive. On the other hand, the smaller Ey_t is, the lower the probability that the government's ability is good. Because approving reforms is costly for the congress, then a reduction in the prior about the government's ability reduces the marginal benefits of reform, without changing its costs. Thus, there are no incentives to approve reforms.[10]

Note that if the informational aspect dominates, then there is a multiplier effect. Conversely, if the first effect dominates, then there is no multiplier effect.

It is possible to argue that informational problems about the true quality of the government are more important in Latin America than in industrialized economies. Not only the managerial skills of the executive, but the impact of the structural programs of reform, contribute to a more uncertain environment. We come back to this point at the end of the section.

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[8] This assumption is to abtract from business cycle issues. In general, reforms imply a recession-boom cycle. However, the net present value is positive (otherwise, they should have not been approved in the first place). We assume that this short run recesions are either full insured or that they do not exists. This assumption implies that reforms are always good independently of the state of the economy.

[9] Here we assume that the congress has a positive weight on the consumer's utility. This implications is consistent with the political economy literature that argues that reforms are more likely to be approved during recessions. See Aizenman and Yi (1997), Alesina and Drazen (1991), and Drazen and Grilli (1993).

[10] This explanation is related to the literature that studies the implementation of reforms when there is imperfect information about the effectiveness of the policy. See Rodrik (1992) and the references therein.

3.2.2 Serial correlation in performances.

The model implies that the serial correlation of output increases when a good performance is realized. Formally, the conditional probability that a good realization is observed is given by,

$$\Pr[y_h|\gamma_t] = q\gamma_t + (1-q)(1-\gamma_t)$$

Differentiating with respect to γ_t,

$$\frac{d}{d\gamma_t}\Pr[y_h|\gamma_t] = 2q - 1 > 0$$

Therefore, after a good performance is measured, the posterior distribution increases and there is a larger probability that better outcomes are observed.[11]

The higher q is, the larger the change in the serial correlation of performances. In other words, the more severe the signal extraction problem is, the stronger is the serial correlation.

Note that, in this case, this result is independent of the existence of a multiplier effect. The serial correlation comes from the assumed stochastic process of output and the updating in beliefs. It is a natural implication of the process of learning.

3.2.3 Output volatility.

In this section, we measure how the volatility of output is affected, when the multiplier effect and the serial correlations in performances are considered. We compare two different economies that have the same unconditional moments in potential output.

In equation 3.5, an increase in q increases the volatility of output. However, this is suggesting that a more volatile government's type implies a more volatile output, which should be expected. This is not a fair exercise. The idea is not to say that Latin America is more volatile, because it is more volatile.

In this section, q is changed maintaining the unconditional mean and variance of poten-

[11] This is an unconditional statement. This conclusion is not true when the probability is computed conditional on the underlying type of the government. In fact, in that case the probabilities are constant regardless of the prior. It is important that the statement is unconditional, because the empirical implications should be tested unconditional on the government's type.

tial output constant. This implies that if there is no foreign investors' reaction function, the economies are identical in their first two moments.

Given the government's process (equation 3.1) and the learning law of motion (equation 3.2) it is possible to compute the expectation and the variance of potential output at every point in time, conditional on time zero (conditional on γ_0).

$$E_0[y_t] = \delta_0 y_h + (1-\delta_0) y_l \qquad \text{and} \qquad V_0[y_t] = \delta_0(1-\delta_0)\Delta y^2 \tag{3.6}$$

In other words, the variance of output potential as of time zero is given by equation 3.6. However, after one period of output is realized, there is learning about the true type of the government, and the conditional variance of output changes. In the extreme, where $q = 1$, there is full learning after one period and the variance of output is zero after one period.

Note that the uncertainty the economy is facing is composed by: First, a permanent component that is reflected in the government's type uncertainty.. Second, a transitory shock, that is reflected in the fluctuation of output potential between high and low outcomes. We compare two economies that ex-ante have the same variance of output, but share different proportions of transitory and permanent shocks.[12]

Assume that $E_0[y_t]$ and $V_0[y_t]$ are fixed exogenously. Hence, the exercise is to increase q maintaining $E_0[y_t]$ and $V_0[y_t]$ constant by adjusting y_l and y_h.

The question is then, what is the variance of output taking into account the interest rate reaction function. Close form solutions to this problem does not exist therefore, simulations are computed.

The parameters chosen are the following: $E_0[y_t] = 2$, $V_0[y_t] = 0.5$, $\alpha = 0.5$. Three different initial priors (γ_0) where tested 0.25, 0.50, and 0.75, and q was varied from 0.60 to 0.85. The implied stochastic process for each q is given by,

[12] A different exercise would be one in which the ex-post variance of output potential is calibrated to be equal.

q	Δy	y_l	y_h
0.60	1.4213	1.2183	2.6396
0.70	1.4433	1.1340	2.5774
0.80	1.4825	1.0364	2.5189
0.85	1.5097	0.9810	2.4907

As it can be seen, an increase in q reduces the variance of each government' type and increases the variance across types.[13] This exercise changes the mixture of the type of uncertainty the economy is facing: from output uncertainty to government' type uncertainty. As was explained before, this has strong implications on the dynamic behavior of the economy.

The parameter α was chosen according to VAR estimates of the impact of monetary policy on output. This estimate is a lower bound, thus, it underestimates the importance of the multiplier effect. α represents how responsive is the economy to changes in real interest rates (default risk or risk premium), and not to movements in nominal interest rates.

The coefficient of variation ($\sqrt{V_0[y_t]}/E_0[y_t]$) was calibrated to be equal to the average coefficient of variation in the detrended output growth in Latin American countries. The international real interest rate was calibrated to be equal to the average long run US real interest rate from 1970 to 1996 ($\rho \sim 3$ percent).

Finally, the $x(.)$ function was assumed to come from a default risk formulation. We assumed that interest rates were not paid if y_l was realized. The reaction function was obtained by solving the following problem.

[13] An alternative exercise (that has the same implications) is to maintain constant the variance of each type. The variance is given by $q(1-q)\Delta y^2$. Totally differentiating with respect to q we obtain,

$$0 = dq \cdot (1-q)\Delta y^2 - dq \cdot q\Delta y^2 + d\Delta y \cdot 2q(1-q)\Delta y$$

this implies that,

$$\frac{d\Delta y}{dq} = \frac{(2q-1)\Delta y}{2q(1-q)} > 0$$

which is positive because the probability q is larger than 1/2. In conclusion, both methodologies have the same implications on Δy.

$$R_t = \arg\max \left[\delta_t R_t - (1-\delta_t)k - \rho\right] D\left(R_t\right)$$

where R_t is the interest rate paid when there is no default, k is the bankruptcy costs, which was calibrated to be equal to one percent of the total capital flows, and $D(.)$ is the demand for foreign capital measured as a percentage of GDP. Assumed that,

$$D\left(R_t\right) = \frac{1}{R_t \beta}$$

where β was calibrated with the average elasticity obtained in IDB (1995).[14]

The solution of this problem implies an interest rate reaction function,

$$r_t = \delta_t R_t = \frac{\beta}{\beta - 1}\left(\rho + (1-\delta_t)k\right) \tag{3.7}$$

where r_t is the expected interest rate paid by the country.

Note that equation 3.7 has a similar functional form as equation 3.4. Moreover, the implied default premium satisfies the assumptions previously made.[15]

The results of the simulation are presented in figures 3-2, 3-3, and 3-4, where the x-axis represents time. Figure 3-2 corresponds to the case when the initial prior γ_0 is equal to 0.50, figure 3-3 is the case when $\gamma_0 = 0.25$, and figure 3-4 is the case when $\gamma_0 = 0.75$.

[Figure 3-2]

[Figure 3-3]

[14] Their estimate is slightly smaller that four, so $\beta = 4$ was chosen.

[15] The implied premium is:

$$x\left(\gamma_t\right) = \frac{1}{\beta - 1}\left[\rho + \beta k\left(1 - \delta_t\right)\right]$$

Note that it is positive and decreasing with respect to information. Finally, we have to check that it is flatter than the output equation:

$$\frac{\beta k}{\beta - 1} < \frac{1}{\alpha}$$

Given the parameters calibrated from latin america the left hand side is close to 0.02, while the righ hand side is larger than 4.

[Figure 3-4]

In each figure, there are three panels: The first panel is the expected output, the second panel is the coefficient of variation in excess to the variance of potential output, and the third panel is the real interest rate. In each panel there are several schedules that represent changes in q. The arrow in the right hand of each panel shows the direction where q is increasing.

An increase in q reduces the expected output and increases the expected interest rate and variance. Remember that the parameters of the output process were computed such that changes in q are not reflected in the unconditional expectation or variance of the process. Thus, the impact of q on these variables comes from the interaction between the output equation and the interest rate reaction function.

Under these set of parameters, the model is able to explain an extra 30 percent (on average) standard deviation in output. The source of volatility in this economy is coming from the uncertainty about the type of the government and its interaction with capital flows' reaction function. The intuition is that larger $q's$ imply that the signal extraction problem is more severe (government's types are more different), and interest rates are more sensitive to the information about the government. Interest rates fluctuate through the path, and these movements are then transmitted into a higher volatility of output.

Finally, note that a reduction in the initial prior about the government, increases the importance of these effects in the short run.

In conclusion, this simple model highlights the increase in volatility due to a multiplier effect. The reaction function of foreign investors interacts with the signal extraction problem, generating additional volatility in the path. The question is then, how plausible these two ingredients are?

First, the existence of a reaction function for foreign investors, private sector or congress is relatively straight forward to rationalize, as was discussed above. The complication in this case, is to understand what are the conditions that imply a negative x'. In some instances, such as for foreign investors, this assumption is relatively easy to justify. In other cases, additional assumptions are required.

Second, how frequent can we expect to find economies subjected to signal extraction prob-

lems? and how important are these problems? There is no empirical evidence of the existence or not of these problems, therefore, no strong statement can be made. However, we discuss some circumstances in which it is reasonable to expect that signal extraction problems are present. And it is argued that these examples seem to fit better the experiences of Developing economies.

First, an economy that is engaged in a structural program of reform, is likely to have uncertainty about the effectiveness of those policies. This is unknown to both government and other agents. In this case, output growth would be a valid signal of the impact of the reforms. It can be argued that this is not a pure signal extraction problem. The government might have informational advantages over the rest of the economy.[16]

Second, the uncertainty might not be in how effective the reforms are, but in the political support they have. For example, assume that reforms have short term costs for the low income group of the society, and that it is uncertain how much political room the government has at any point in time. Output growth, social stability, and performance in general, are the only indications of the willingness of this constituency to continue with the program.

Finally, economies where the ability of the administration has been questioned for the last three decades, is likely to continue with problems of corruption and mismanagement. It is not clear that this characteristics are known in advance by the executive.

Given the experience of the last decades, it is possible to speculate that Latin American economies are more likely to be engaged in these type of situations. Therefore, it is in these countries, and not in the industrialized ones, where we expect to find multiplier effects. In other words, Latin American countries might have a larger q and therefore suffer a higher volatility.

3.3 Good news is no news

In this section we extend the previous model to include the *good news is no news* effect. In this case, we capture the idea that agents can take actions that enhance or facilitate the economic

[16] In all the examples, it can be argued that in some cases, the government faces uncertainty but it has an informational advantage over the rest of the economy, while in other cases, the agents have the informational advantage over the government. We argue, for simplicity, that these differences are small, or at least, that they last for short periods of time. Therefore, only the medium and long term uncertainty matters. Further research should extend the present results to include these aspects of the problem.

performance of the government. For example, if capital inflows and private investment are booming, or if the congressional approval for reforms is relaxed, then it is more likely to observe a good performance in the economy, independently of the ability of the government.

This effect is modeled by allowing the probability distribution of output to depend on r_t. Formally, assume the following output process:

$$good \sim \begin{cases} y_h & q(r_t) \\ y_l & 1-q(r_t) \end{cases} \quad \text{and} \quad bad \sim \begin{cases} y_h & p(r_t) \\ y_l & 1-p(r_t) \end{cases} \tag{3.8}$$

where $q(.)$ and $p(.)$ are continuous functions with properties: First, $q(.)$ and $p(.) \in [0,1]$. Second, the good type has always more probability that the good outcome is realized, thus $q(.) \geq p(.)$ for all interest rates. Third, both probabilities are decreasing with respect to the interest rate, so $q'(.) < 0$ and $p'(.) < 0$. Finally, both probabilities are equal to one when the interest rate is zero, $q(0) = p(0) = 1$.

The next three subsections study the implications of the model: the multiplier effect, the serial correlation of performances, and the signal jamming problem. In the last two subsections, these effects are quantified.

3.3.1 Multiplier effect

In this section, we compute the change in the multiplier effect when the probability distribution depends on the interest rate.

The output and the interest rate equations remain the same. Totally differentiating with respect to γ_t we obtain,

$$\frac{dEy_t}{d\gamma_t} = \frac{(q(r_t) - p(r_t))\,\Delta y}{1 + \alpha x'(Ey_t) - [\gamma_t q'(r_t) + (1-\gamma_t)\,p'(r_t)] \cdot \alpha x'(Ey_t)} \tag{3.9}$$

Note that the multiplier effect increases when the probabilities are more sensitive to the interest rate. To illustrate this point, note that if the probabilities are constant, then the change in output is given by,

$$\frac{dEy_t}{d\gamma_t} = \frac{(q-p)\,\Delta y}{1 + \alpha x'(Ey_t)} \tag{3.10}$$

Therefore, for the same p and q, equation 3.9 is larger that equation 3.10 if,

$$\alpha x'(Ey_t)\left[\gamma_t q'(r_t) + (1-\gamma_t)\, p'(r_t)\right] > 0$$

which is always satisfied, given the assumptions of x, q, and p.

The intuition is that an increase in the prior about the government increases output potential through two channels: First, there is higher probability that the government's type is good. Second, there is higher probability, for *both* types, that good performances are realized. This is the increase in the skewness of the distribution. Both effects work in the same direction, enhancing the multiplier effect.

3.3.2 Signal Jamming: Good news is no news.

In this section, we show that under the assumptions of q and p there is a signal jamming problem when the interest rate is low.

The signal jamming problem makes a good signal to be less informative when it is observed after a sequence of good signals. The intuition is as follows: an improvement in the prior about the government's type leads to a reduction in the interest rate. This increases the probability that good performances are observed for both types of government. Because the two probabilities are equal when the interest rate is zero, there is a region where the difference between the them is shrinking. In that region, observing a good performance is not very informative.

Formally, in the Bayes' rule the amount of updating when a good performance is observed depends on $p(r_t)/q(r_t)$. Conditional on the prior, if this fraction is close to one, then the prior and the posterior are similar. Note that when the interest rate is small, both probabilities are close to one and therefore this fraction tends to one.

On the other hand, if a bad outcomes is observed, the size of the updating depends on $[1-p(r_t)]/[1-q(r_t)]$. In this case, when the interest rate is small, both denominator and numerator are close to zero. Thus, the fraction can be very different from one.

To illustrate this point, assume that the probabilities are described by the following functional form:

$$q(r_t) = e^{-\beta_q \frac{r_t}{0.2-r_t}} \text{ and } p(r_t) = e^{-\beta_p \frac{r_t}{0.2-r_t}}$$

where $\beta_p > \beta_q$. Note that the functional forms satisfy the assumptions on p and q.[17]

If a good performance is observed, the updating is proportional to:

$$\frac{p(r_t)}{q(r_t)} = e^{-(\beta_p - \beta_q)\frac{r_t}{0.2-r_t}}$$

which tends to one when the interest goes to zero,

$$\lim_{r_t \to 0} \frac{p(r_t)}{q(r_t)} = 1$$

This means that the posterior distribution and the prior distribution are similar, and in the limit, they are identical.

On the other hand, if a bad performance is observed, the updating is proportional to,

$$\frac{1-p(r_t)}{1-q(r_t)} = \frac{1-e^{-\beta_p \frac{r_t}{0.2-r_t}}}{1-e^{-\beta_q \frac{r_t}{0.2-r_t}}}$$

computing the limit when the interest rate goes to zero,

$$\lim_{r_t \to 0} \frac{1-p(r_t)}{1-q(r_t)} = \frac{\beta_p}{\beta_q} > 1$$

This means that when the interest rate is close to zero, observing a good performance has a small contribution to the updating of priors, while observing a bad performance could imply a large change in posteriors.

In order to provide a more precise measure of how important the updating is, a simulation is run. We assume that $\beta_p = 2$ and $\beta_q = 1$.[18] The schedules are shown in figure 3-5, where the x-axis is the interest rate.

[17] Both probabilities start at one when the interest rate is zero, they are decreasing, and $q > p$ for all interest rates. The functional form assumed implies that the probabilities are zero at 0.2. This is an expositional simplification that allows to concentrate the interest rate within the $[0, 0.2]$ interval.

[18] Note that β_p and β_q are unobservable in the data, thus the numbers chosen here do not follow any calibration.

[Figure 3-5]

In figure 3-6, the posterior probabilities are shown. The top panel is the posterior when a good performance is observed, while the bottom panel is the posterior when a bad outcome is observed. There are five different initial priors (0.5 to 0.9) which are represented by the schedules in each panel.

[Figure 3-6]

As can be seen, when the interest rate is small and a good outcome is observed, the posterior is similar to the prior.[19] However, under the same interest rate, when a bad outcome is realized, the posterior and the prior are very different. Opposite implications are obtained when the interest rate is large. In figure 3-7, the absolute change in the prior is reported.

[Figure 3-7]

3.3.3 Serial correlation in performances.

In this section, we show that the serial correlation in the performances is exacerbated when the probabilities are sensitive to the interest rate. Formally,

$$\Pr\left[y_h|\,\gamma_t\right] = \gamma_t\, q\left(r_t\right) + \left(1-\gamma_t\right) p\left(r_t\right)$$

Differentiating with respect to γ_t,

$$\begin{aligned} \frac{d}{d\gamma_t}\Pr\left[y_h|\,\gamma_t\right] &= \left[q\left(r_t\right) - p\left(r_t\right)\right] + \\ &\quad \left[\left[\gamma_t\, q'\left(r_t\right) + \left(1-\gamma_t\right) p'\left(r_t\right)\right] \cdot \alpha x'\left(Ey_t\right) \cdot \frac{dEy_t}{d\gamma_t}\right] \end{aligned} \tag{3.11}$$

[19] The panels have to be read in the following way: Assume that the prior is 0.50 and the interest rate is 2.6 percent. If a good performance is observed, we go to the top panel (good performance observed), the dotted line (the 0.50 prior), and obtain its value when the interest rate is 2.6 percent. The posterior is 0.57. Thus, the updating changes the probability that the type of government is good from 0.50 to 0.57. On the other hand, if a bad outcome is observed, we go to the bottom panel (the dotted line). In this case, the posterior is 0.17. This means that a bad performance updates priors from 0.50 to 0.17. This example illustrates the magnitud of the updating when a bad outcome is observed and the interest rates are small.

If the probabilities are independent of the interest rate, the changes in the serial correlation are given by the difference in the probabilities: $q(r_t) - p(r_t)$. This is independent of the existence, or not, of multiplier effects, which is the result obtained in section 3.2.2.

Note that, on the other hand, the second bracket in equation 3.11 is always positive: First, q', p' and x' are negative. Second, $-\frac{1}{\alpha} < x'$, therefore $\frac{dEy_t}{d\gamma_t}$ exists and is positive. Thus, the serial correlation is larger in this case.

Moreover, an increase in the multiplier (x' smaller) or an increase in the responsiveness of the probabilities (q' or p' are larger in absolute value) implies an increase in the change in serial correlation.

3.3.4 Excess optimism and excess pessimism.

In the previous subsections, the implications of the model were derived: the multiplier effect, the serial correlation in performances, and the good news is no news effect. In this section and the next one, we compute the relative importance of these effects.

One of the time series implications of the model is the existence of periods where a good (bad) performance is likely to be followed by a high (low) outcome. This implication comes from the serial correlation of performances described above. However, we argued that the serial correlation problem is exacerbated, when the probability distribution of potential output depends on the agents choices. In this section, we compare the properties of two economies: one where the probabilities change with the agents actions, and one where the probabilities are constant.

First, we show that when the interest rate affects the probability distribution, then there exists the possibility of bad regimes that are self-sustained for some period of time. These bad regimes represent periods in which the agents are *confused* about the true underlying type of government.

Second, we show that the distribution of actions implies that incorrectly specified econometric model might lead the agents' reaction function to overreact to fundamentals. In other words, it looks as if there are periods of excess optimism and periods of excess pessimism.

The first implication requires that the probability distribution of agents' choices is bimodal. Beliefs and actions move between these two regimes, where one is the long run (true) equilib-

rium, while the other one is only sustainable for a finite period of time.

The following exercise was run: 1000 random stories of 20 periods each were generated. At the beginning of each story we assume that the prior about the government's type is γ_0. Because the bimodal property is both a conditional and unconditional implication, we assume that the government's type is always good. This implies that the mode with the smaller interest rate is the long run equilibrium.

The interest rate path was computed and the histogram is shown in figure 3-8. A second case was run assuming that the probabilities were unaffected by the interest rate. The probabilities were calibrated to be equal to the time series average of the first case. See figure 3-9.

[Figure 3-8]

[Figure 3-9]

Note that the distribution of interest rates, in the first case, has two modes, while in the second case, it seems to have only one mode.

In other words, the serial correlation in the first case is exacerbated up to a point where there is a regime of high interest rates that is self-sustained. Agents believe that the government has low ability, and therefore they are not willing to take actions that enhance the government's performance. Thus, bad outcomes are likely to occur, and the priors reinforced.

We studied the case where the government's type is good. This implies that the wrong regime occurs at a high interest rate. On the other hand, if it is assumed that the type is bad, then the period when agents are confused occurs at a low interest rate.

It is important to notice, that the probability that the wrong regimes exist forever is zero. In fact, these are wrong regimes because agents are *confused* about the true type of the government. At some point in time, agents will obtain enough signal that will lead them to learn the true type.

Lets turn our attention to the overreaction of agents. The mean interest rate in the first simulation is 2.77 percent while it is 2.76 percent in the second one: the mean interest rate is the same for both economies. This result comes from the fact that we calibrated the probability distribution for the second simulation to coincide with the average in the first one.

The average interest rate when there is a good performance (y_h is observed) is 2.624 percent in the first case, and 2.738 percent in the second case. On the other hand, when a bad output is observed, the interest rates are 2.942 percent and 2.863 percent, respectively. Note that in the first case the interest rate is lower (higher) than in the second case, when a good (bad) performance is observed.

The intuition of these results is the following: The probability that the government's type is good increases when good performances are observed. This implies that the interest rate tend to reduce. In the case of the constant probability economy, this is the end of the story. However, when the probability distribution depends on the agents' actions, then the reduction in the interest rate increases the probability that good performances are observed for all types. This reduces the probability of default, and therefore there is a further drop in the premium.[20] On the other hand, when bad outcomes are observed, the interest rate increases and the signal jamming problem forces the premium to go up.

This means that if the two economies are compared, in the first one when the interest rate is high, it is too high, and when the interest rate is low, it is too low: Agents seem to suffer from excess optimism and excess pessimism.

The empirical implication of this effect is that agents responses look as if they overreact to changes in fundamentals, when the same model is estimated in two different countries. In other words, if a single econometric model is estimated in these two economies, then there is an *unconditional* overreaction of the agents in the first case, and an *unconditional* underreaction in the second case.

3.3.5 Output volatility.

Finally, the second time series implication of the model is that the volatility of output is larger when these effects are considered. Moreover, in this case, we are also interested in determine what is the contribution of each one.

Under the parameters described above, the output volatility increases by 28.40 percent when the multiplier effect is considered, and an additional 16.16 percent variance is generated when

[20]Note that this model implies that premiums are pro-cyclical. This empirical implication should be further developed.

the effect on the probabilities is included.

As was mentioned before, this model is able to explain a large difference in volatility between two economies that share the same unconditional moments in output potential.

3.4 Conclusion.

Why is Latin America so volatile? In this paper, we offer a simple theoretical model that explains large differences in countries' volatility as a result of informational problems.

Agents face a signal extraction problem about the government's ability, and their actions affect the distribution of output. We show that the additional volatility comes from two sources: a multiplier effect generated by the agents' reaction function, and a signal jamming problem when good performances are observed. These effects have the following implications:

1. There is a higher volatility in output in those economies that are subjected to severe signal extraction problems, even though the unconditional moments of the government process are the same. For suitable calibrations in the Latin American case, it was possible to explain as much as 30 percent additional volatility.[21]

2. The model implies that performances (as well as agents actions) are serially correlated, and that their distribution is bimodal. [22]

   The bimodal property of the distribution comes from the possibility of self-sustained regimes. This offers a different interpretation to speculative attacks[23], which can be driven by informational issues. To illustrate this point assume that the type of government is bad. In this case, a sequence of good signals increases the probability that the government's type is good. Agents take actions that favor the realization of good performances. Therefore, the low interest rate regimes is self-sustained. Agents are confused about the true type of

[21] In the model there are two parameteres that are unobservable β_p and β_q. However, in section two, where the multiplier effect is documented, all parameters can be calibrated with actual figures in Latin America. In that section, the model is able to explain a substantial source of variance.

[22] There is some evidence that capital flows and reforms in Latin America have this bimodal property. For capital flows stilized facts, see Calvo, Leiderman, and Reinhart (1993) and the references therein. For the facts of the reform processes in Latin America see Dornbusch and Edwards (1995), Easterly, Loaiza and Montiel (1997), Edwards (1995), and IDB (1996).

[23] See Krugman (1979) for the seminal contribution.

the government. A small piece of information (a bad realization) produces large changes in expectations and in behavior. Thus the economy moves from the low interest rate regime to a higher one.

Note that the agents do not make permanent mistakes: the low interest rate regime is unsustainable. However, what this model predicts is that it might take time for the agents to assess the true fundamental of the economy, and through this process mistakes are possible.

Finally, we argue that Latin American countries are more likely to be subjected to this type of informational problem. In particular, the fact that these economies are engaged in ambitious programs of reform, and that their government's ability has been questioned for the last three decades, contribute to the existence of signal extraction problems.

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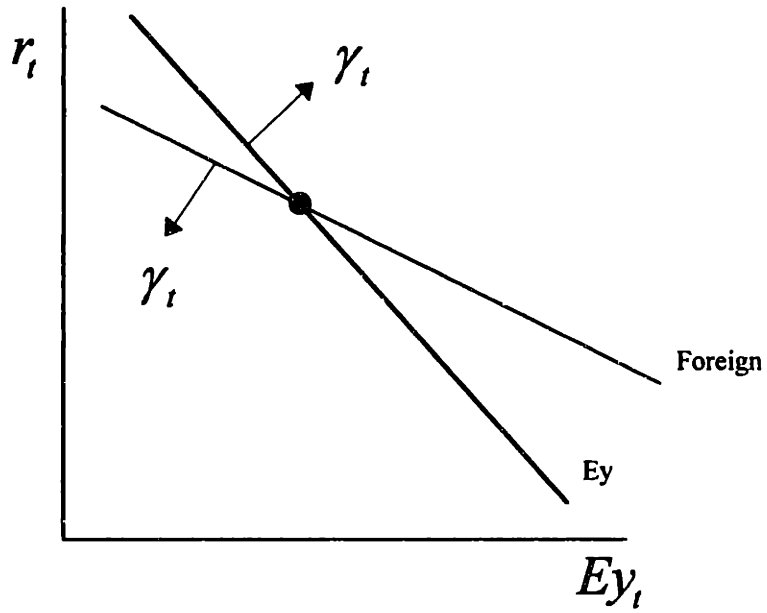

Figure 3-1: Reaction Functions.

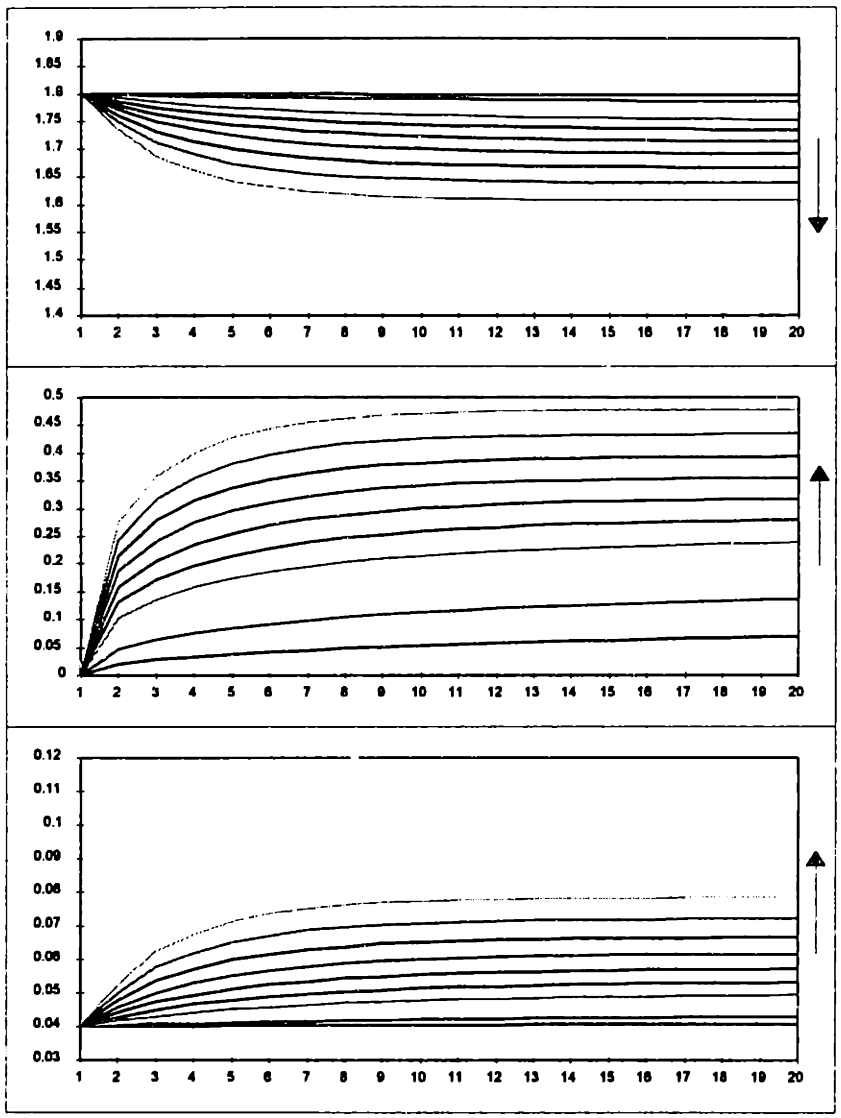

Figure 3-2: Prior is equal to 0.50.

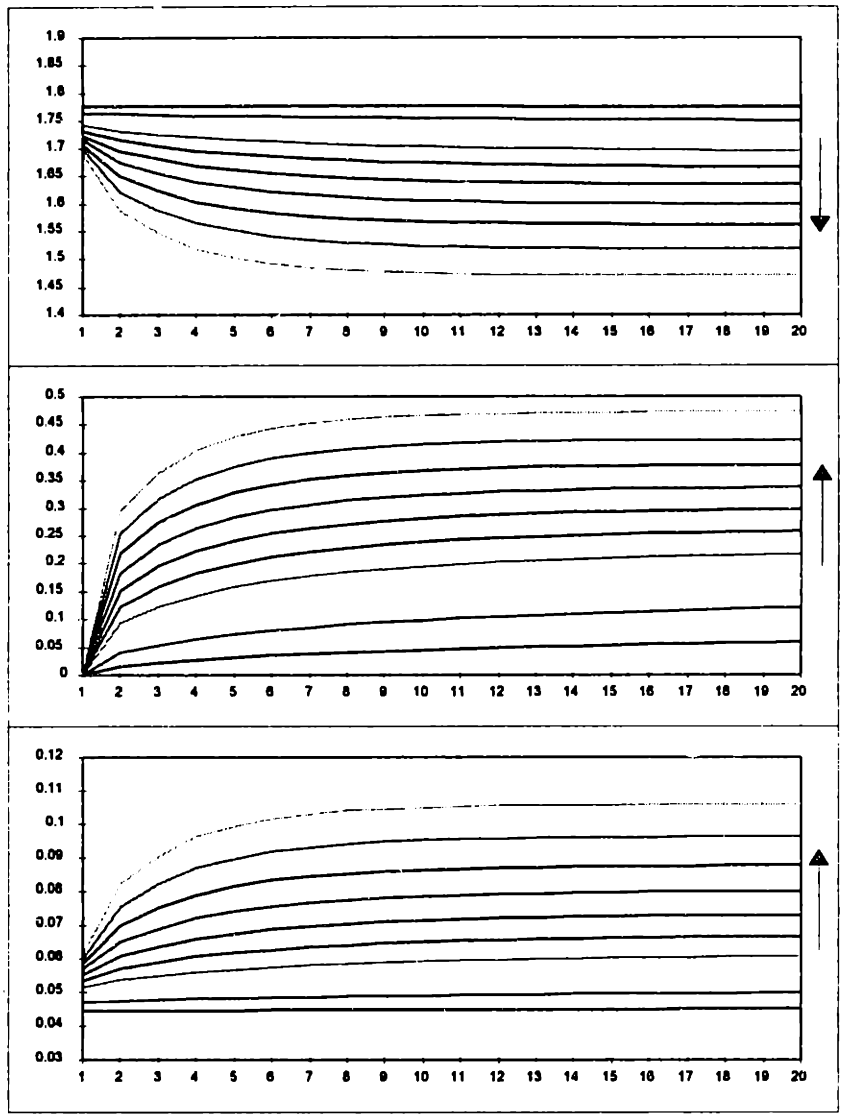

Figure 3-3: Prior is equal to 0.25.

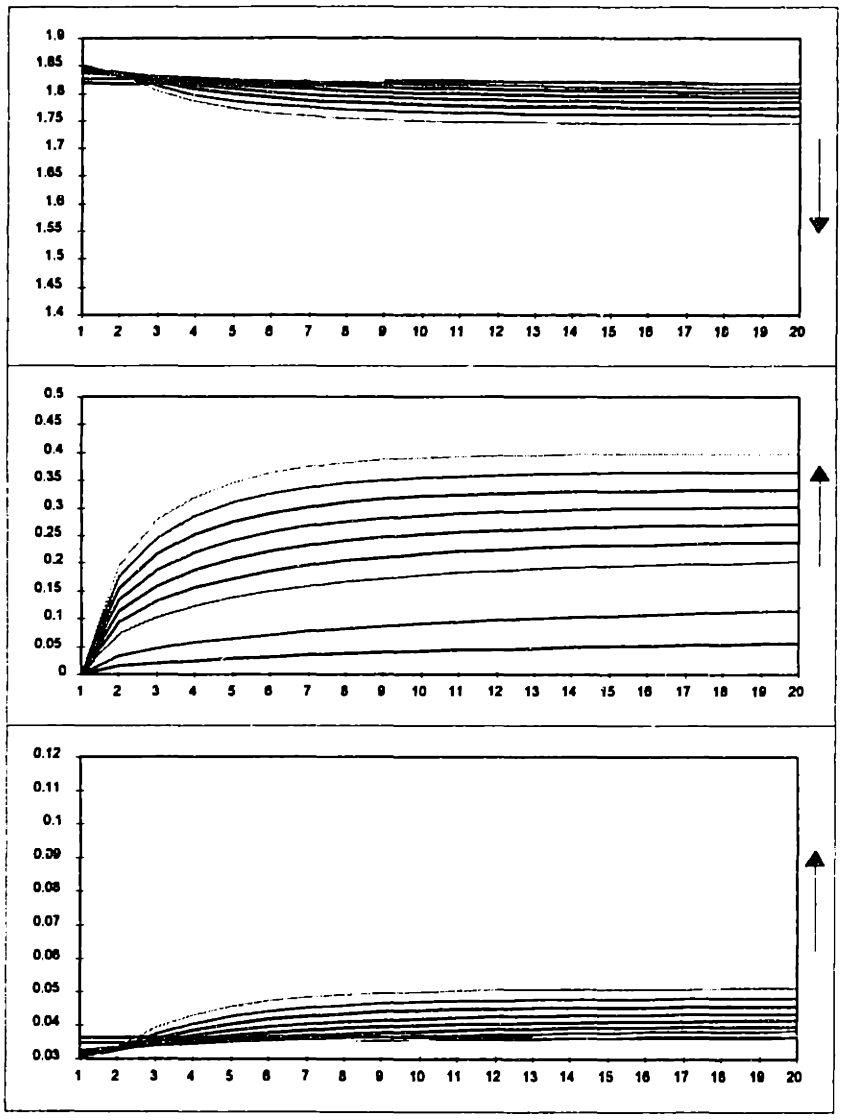

Figure 3-4: Prior is equal to 0.75.

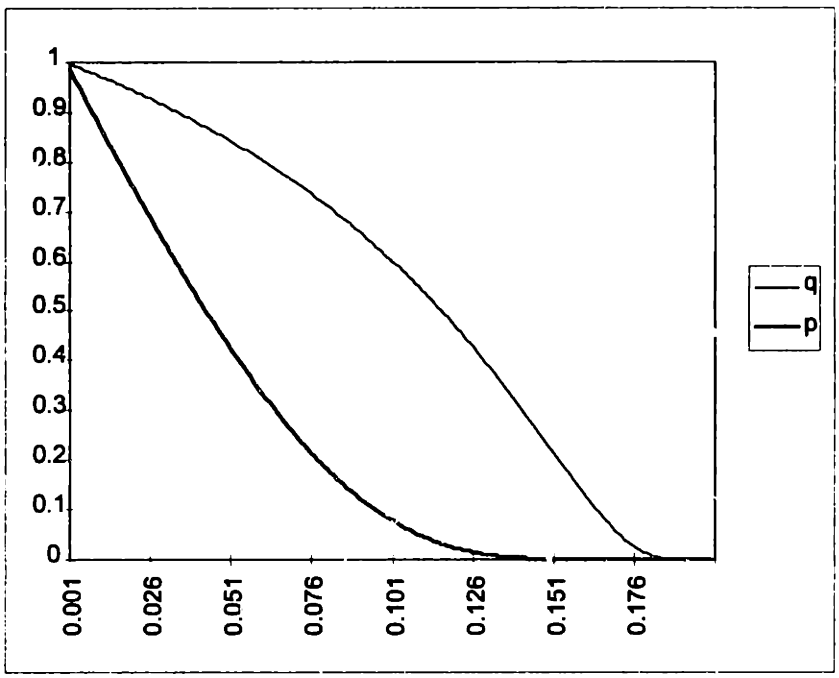

Figure 3-5: Probability functions.

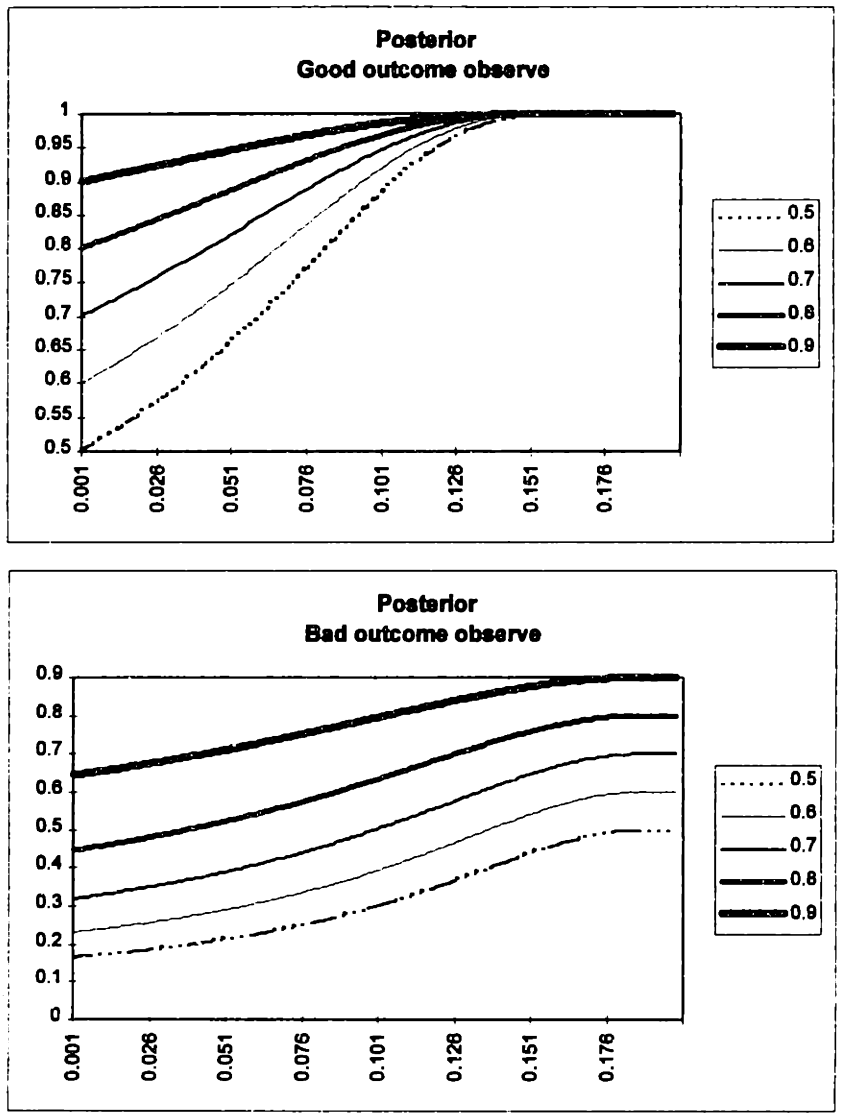

Figure 3-6: Posterior distributions depending on outcome observed and interest rate.

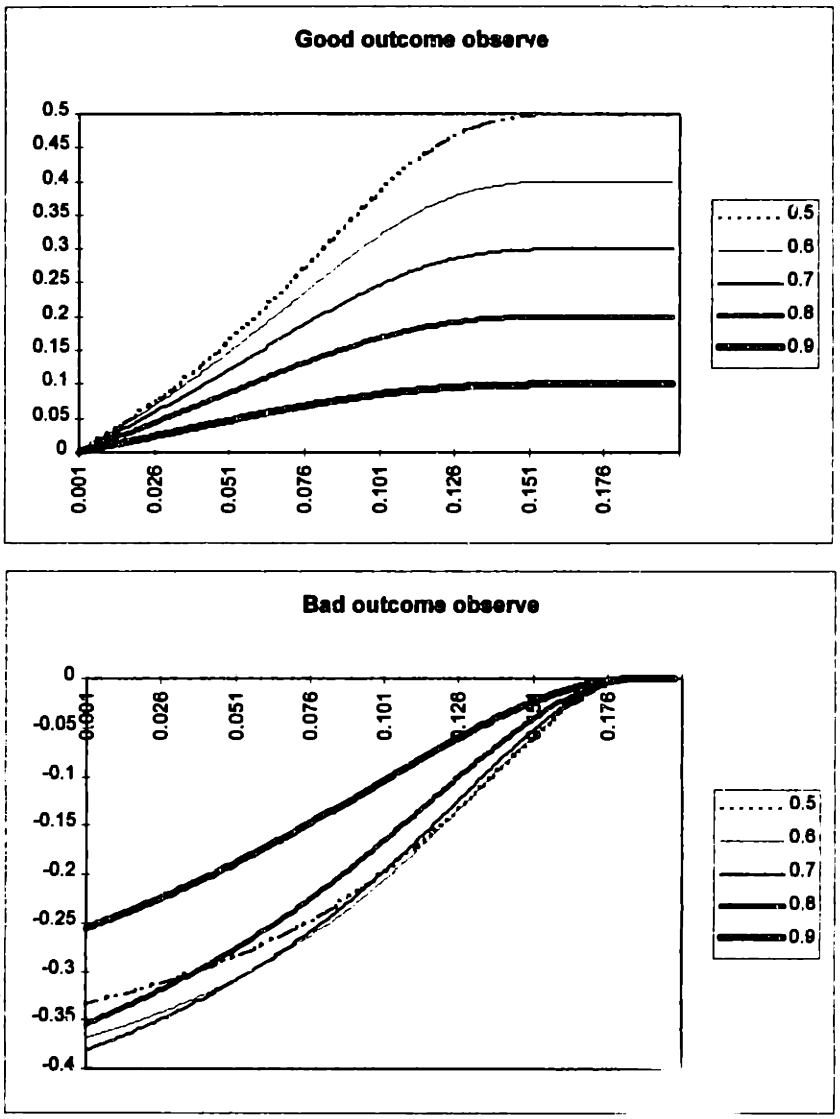

Figure 3-7: Absolute change in the prior.

Figure 3-8: Interest rate distribution: Probabilities are affected by interest rates.

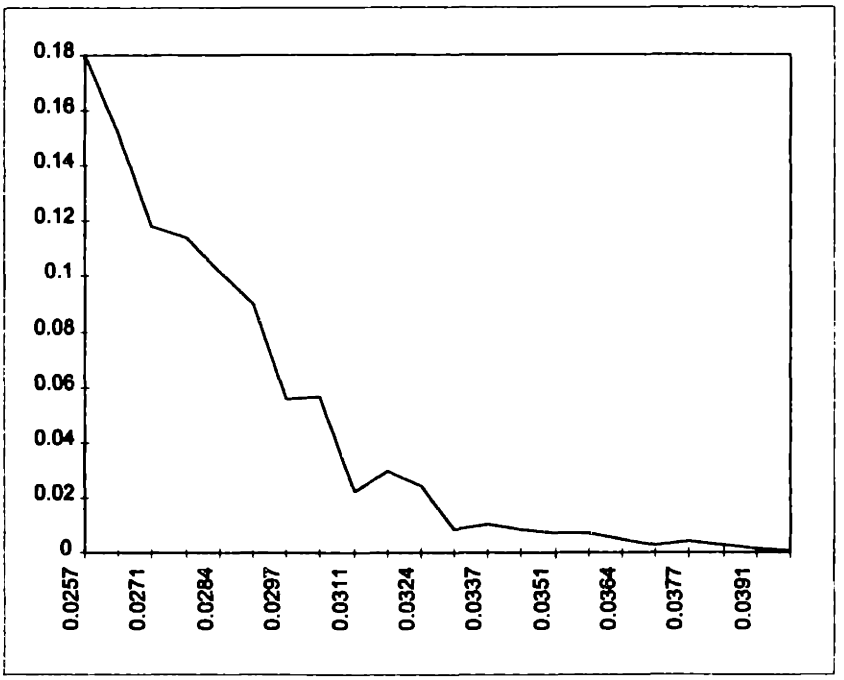

Figure 3-9: Interest rate distribution: Constant probabilities.

Chapter 4

Learning by Lending.

4.1 Introduction

It has been widely argued that learning is an important aspect of the credit market, both in the development of lender-firm relationships as well as in the formation of reputation. In particular, Petersen and Rajan (1994) show that longer relationships imply larger availabilities of credit, while Hoshi, Kashyap and Scharfstein (1990) find that firms that belong to a Keiretsu are less liquidity constraint. Both results are explained based on the fact that the credit relationship ameliorates the informational problems between lenders and firms.

This paper studies how the propagation mechanism of the interest rate changes when learning takes place in the credit market. Learning reduces the degree of imperfect information, and the question is how this impacts the allocation of credit. In the literature, it has been shown that the lender-firm relationship is important in the establishment of quantities and prices of loans. In this paper, we concentrate on how the flow of information is determined, and what are the aggregate implications of this process.

We show that several facts reported in the credit market literature can be accounted by the model.[1] First, it generates amplification of interest rate and productivity shocks. Second, it produces cross sectional implications that are consistent with the flight to quality effect. Finally, it has persistence effects.

[1] The facts of the credit market are summarized in Bernanke and Gertler (1995).

Learning occurs at different levels in the lender-firm relationship. In one extreme, the firm has perfect information and the lender faces a pure asymmetric information problem. On the other extreme, the firm has imperfect information about its own type, and has no informational advantages over the lender.

In the first case, learning reduces the degree of asymmetry in the relationship. Learning is possible when either lenders are unable to implement a full separating contract or it is too costly to do so. In the literature, this is known as the Ratchet effect; there exists an incompleteness (in general, full commitment contracts are not available) and the full separating contract cannot be implemented. Moreover, there are conditions in which the full separation is not even desirable.[2] Therefore, the optimal contract involves partial separation through time and this is how the learning process is characterized. In this approach, asymmetric information and learning aspects have to be considered together.

In the second case, there is a pure signal extraction problem. Asymmetric information problems do not exist, and learning reduces the degree of imperfect information. This is the case studied in this paper. There are advantages and disadvantages of this approach. On the one hand, it implies that the credit market plays no role in the economy (other than supplying funds). This leaves aside important aspects of the contracting problem, such as the determination of collateral requirements, decisions of commitment, among others. On the other hand, it has three advantages: First, learning aspects can be studied separately. Second, it simplifies the optimal contract given that there is no principal-agent problem. Third, and more importantly, it allows the information to be public. If there exists asymmetric information, then learning has to be specific to the lender-firm relationship; otherwise, there would be free riding problems. However, in practice, this assumption is not trivial to impose. An outsider (lender) that observes the price and the quantities of a particular loan, and the history of repayments of the firm, should be able to reverse engineering and determine the priors the incumbent lender has. In the signal extraction problem, these issues are simplified by the fact that the information is public.

We consider an economy composed by a continuum of small monopolistic firms facing an unknown linear downward slopping demand. We assume that the process of learning occurs

[2]See Laffont and Tirole (1993).

through time and that it is proportional to the level of production. This assumption has two implications: First, newer firms are more likely to face a larger uncertainty in comparison to older firms. Second, the larger the level of investment, the larger the information disclosed.

In this setup, investment has an additional role as gatherer of information. Increasing investment, or equivalently output, improves the information that exists on the firm. If information is valuable, then there are incentives to invest more. This extra investment comes from what has been called in the literature the *experimentational incentive.* It depends on two main aspects: how easy it is to learn and how valuable is the learning. The first one is related to how the posterior distribution is affected by current decisions. In other words, how much the information improves with an increase in investment. As we would expect under rational learning, the better the information on a firm is, the harder it is to improve it. In the limit, when the posterior is either one or zero, the information cannot be refined. The second aspect is a function of how different is the net present value of the project with changes in the information. In this case, conditional on the new information, the benefit of learning is related to the marginal increase in the net present value of the project. We assume that for a well known firm it is harder to learn and the marginal increase is small. Therefore, we can expect that the experimentational incentive is stronger for newer firms.

In this context, an increase in the interest rate has two effects on the investment decision: First, there is an increase in the cost of capital that reduces total investment. This is the standard interest rate channel. Second, there is a reduction of the value of the firm in all the future states of the world, which decreases the net present value of improving information and the incentives to experiment. Both effects go in the same direction reducing current investment. The first effect is common to all firms, while the second one varies across them.

The flight to quality effect is explained by the differences in the valuations of learning. In an old and well known firm there are no incentives to learn and therefore, it is only subjected to the interest rate channel. A newer and unknown firm faces both. Thus, it suffers relatively more during recessions, which is how the flight to quality effect has been characterized.

The model also predicts aggregate amplification effects. The aggregate elasticity with respect to the interest rate in the learning economy is larger than the one with full information, which in the model coincides with the elasticity of the oldest and well known firm. The amplifi-

cation effect comes from two sources: First, there is a larger reaction in small firm's investment. This means that the aggregate elasticity, which is a weighted average of individual elasticities, is larger than the elasticity from the oldest firm, and therefore larger than the one implied by the full information case. Second, there is a shift in the steady state distribution toward newer firms. The change in the distribution function is a consequence of a reduction in the speed of learning. The intuition is that an increase in the interest rate lowers current investment, that leads to a decrease in the information disclosed. The learning process is slowed down and the distribution shifts toward less known firms. Note that both effects exacerbate the aggregate response.

Finally, the persistence effect is due to the change in the distribution of knowledge of the firms. In this case, a transitory shock affects current investment decisions and next period's distribution of priors. The aggregate effect lasts until the steady state distribution is reached again.

The paper is organized as follows. In section two, we present the basic setup. In section three, we study the two period case. This simple model allows for closed form solutions, and therefore, the intuitions is easily developed. In section four, we solve the infinite horizon case and derive the model's aggregate implications (*amplification effect*), cross sectional implications (*flight to quality effect*), and time series implications (*persistence effect*). In section five, we conclude and discuss further extensions.

4.1.1 Related literature

This paper is related to three different strands of the literature. The credit market, the lending relationship, and the active exploration literatures.

The credit market literature is too extensive to make it justice here. However, the main aspects are discussed in Bernanke (1983, 1993), Bernanke and Blinder (1982), Bernanke and Gertler (1995), Bernanke, Gertler and Gilchrist (1994), Calomiris (1993), Dimsdale (1994), Gertler (1988), Kashyap and Stein (1994), King and Levine (1993). From the theoretical point of view the most closely related papers are the following: First, Bernanke and Gertler (1988) explain the persistence in the credit market through the balance sheet effect. The idea is that current changes in the interest rate affect the firms balance sheet, which determine the next

period's credit availability. Our model generates persistence in a similar way. In our case, the firm's asset is its prior distribution and this is affected by the shocks. Second, Kiyotaki and Moore (1995) explain the amplification effect through the existence of collateral requirements. In this case, an increase in the interest rate decreases the current value of the firm. This reduces the value of the collateral, which implies that credit falls even more. These papers concentrate on the asymmetric information problems between the credit market and investors. In this paper, we show that similar implications are obtained in a model where only imperfect information problems exist.

Lang and Nakamura (1995), and Nakamura (1993) study learning in the banking sector. Their idea is that bankers infer the quality of one client given the information of past clients in the same industry. They characterize the flight to quality more as an industry effect rather than as firm specific effect.[3] In this paper, the information is specific to the firm.

In the corporate finance literature, Sharpe (1990), Rajan (1992), Petersen and Rajan (1995) study theoretical models of the banker-client relationship. They show that if the knowledge is specific to the bank, then there exists lock-in effects. In other words, given that the bank has improved its information about the firm, now it is in the position to extract more rents, but the firm does not leave because it is obtaining better terms in its credit. This literature is mainly concerned with studying competition in the banking sector. On the empirical side there are several papers showing that lending relationships are important for credit allocation at the micro level. It is impossible to summarize them here, however, the most closely related papers are Petersen and Rajan (1994) and Hoshi, Kashyap and Scharfstein (1990). Petersen and Rajan (1994) find that bank loans increase with the age of the banker-client relationship. They look at the probability that trade credit is used by firms. The idea is that trade credit is costly (the effective annual interest rate is between 30-40 percent), thus, firms should borrow and take advantage of the trade discount. They measure the probability that this loan is made and find that it increases with the age of the client.[4] Hoshi, Kashyap and Scharfstein (1990) compare the correlation between investment and cash flow for Japanese firms. They show that investment

[3]The empirical evidence shows that the flight to quality effect occurs at the firm level and not at the industry level. See Bernanke, Gertler, and Gilchrist (1994).

[4]For example, a firm 10 years older has approximately 15 percent more probability that the loan is made. Given that trade credit involves premiums above 20 percent, the increase in probability represents a large saving in expected working capital cost of funding.

is less correlated to cash flow for those firms that belong to a Keiretsu. The assumption is that those firms are subject to small informational problems, therefore less liquidity constrained. Notice that this is consistent with the theory that relationships (in this case between the firm and the Keiretsu) determine the availability of credit.[5]

Finally, the literature on active exploration models highlights the difference between Bayesian passive learning and active learning.[6] In our model, changes in the credit level change the information released, in the same way as it is done in an active exploration model.

4.2 Model

Consider an economy with a continuum of differentiated goods, one factor of production, and two types of agents: Managers and Lenders. Both types consist of a continuum of infinitely lived individuals with mass one. Managers are the only types that can own and run a project, but, they have zero endowment. Each lender owns an exogenous endowment flow of M units every period, that is used as the factor of production. Neither the endowment nor the goods are storable.

The timing of each period is the following: First, loans and investments are made at the beginning of the period. Second, production takes place and prices are realized. Third, investment is fully depreciated, firms pay loans plus interests, and consumption takes place. Finally, some projects are destroyed and an equal mass is created in the next period.

Lenders decide to lend to the managers or to use an exogenous technology. We assume that the exogenous technology has constant returns to scale and yields $1 + \rho$ with certainty every period.

We assume that a manager can own only one indivisible project in each period and that a firm is created when a project is randomly picked from a pool. We assume that the creation and destruction of such projects has no costs, and that their production function is the following:

[5] For more empirical evidence see Davis (1992, 1994), Dimsdale (1994), and Sushka, Slovin and Polonchek (1993).

[6] See Pakes and Ericson (1989a, 1989b) for active exploration models. The standard pasive learning model is Jovanovic (1982). For active learning see, Aghion, Bolton and Bruno (1988), Drezé (1972), Prescott (1972), and Trefler (1993).

$$y_t = AI_t$$

where y_t is output, A is the aggregate state of technology, known and permanent, and I_t is investment, which consists of endowment and is fully depreciated at the end of the period.

We assume that there exists a constant exogenous probability of destruction equal to $1-\beta$. Additionally, we assume that the manager can decide to destroy a project when it considers profitable to do so. In both cases, we assume that the manager picks a new one instantaneously.

Each project faces an unknown downward slopping demand, that is linear and given by:

$$P = b + u - \theta y$$

where P is the price, $b+u$ is the intersect, where the first one in known and the second one is uniformly distributed between $[0,1]$. The slope $\theta = \{\theta_g, \theta_b\}$ is unknown. We assume that $\theta_g < \theta_b$ and define $\Delta\theta \equiv \theta_b - \theta_g$. Assume there are two types of demand: the *good* demand has a constant coefficient $\theta = \theta_g$, and the *bad* demand has a probabilistic slope, $\theta = \theta_g$ with probability δ_b and $\theta = \theta_b$ with probability $1-\delta_b$. We assume that all projects in the pool have the same prior probability (γ_0) of facing a good demand.

This problem has been analyzed in the literature and it is known that the monopoly experiments to learn about the demand. The direction of experimentation depends on what instrument the monopoly uses. If the monopoly post prices and quantities are chosen by the agents, then the monopoly increases prices above its static solution. Conversely, if the monopoly sets the quantity and observes the realized price, then it offers a quantity larger than the static solution.[7] The timing imposed implies that credits are made before prices are realized. Therefore, it is assumed that firms offer quantities and observe prices. This assumption is not crucial for the results presented here, however, it makes the timing issue simpler.[8]

[7] See Trefler (1993) for a thorough discussion of the subject.

[8] If the monopoly announces prices, then credit decisions should be taken after prices are observed. This requires a strong commitment in the credit market, to supply any loan the firm demands conditional on the observed price. This commitment is reflected in a schedule of credit-prices, which complicates the optimization but generates the same qualitative results.

Finally, assume that the discount rate is equal to the return in the exogenous technology. Assume that it is a small open economy with perfect capital mobility, and the rest of the world is in steady state with zero growth. Thus, the exogenous technology can be interpreted as foreign bonds that are repaid within the period.

4.2.1 Learning process.

Notice that in this set up there is no asymmetric information problem between lenders and firms, there is only an imperfect information problem. As was mentioned in the introduction, we are looking at the allocation of credit in a market in which both sides are learning at the same time. Define γ_a as the prior probability that the demand is of good type at the beginning of the period, where a is some index explained below. δ_a is the probability that θ_g occurs, and it is given by $\delta_a = \gamma_a + (1-\gamma_a)\cdot\delta_b$. Conditional on the level of output, the observed prices are going to be distributed in the following manner:

$$P_g \in [b-\theta_g y, b+1-\theta_g y] \text{ if } \theta_g \text{ occurred}$$
$$P_b \in [b-\theta_b y, b+1-\theta_b y] \text{ if } \theta_b \text{ occurred}$$

Define three regions: *Top*, where prices were generated by a demand with slope θ_g for sure. This region occurs when the realized price is above the maximum of the distribution under θ_b. In other words, when $b+u-\theta_g y > b+1-\theta_b y$. The probability that the price falls in this region is given by $\delta_a \Delta\theta y$, where δ_a is defined above, and $\Delta\theta y$ is the conditional probability that the realized price under θ_g is larger than the maximum under θ_b.

The *Bottom* region is when θ_b has occurred for sure. Conversely as in the top region, the realized price is smaller than the minimum of the distribution under θ_g. In this case, the probability that the price falls in this region is given by $(1-\delta_a)\Delta\theta y$

Finally, the *Middle* region is when the realized price could be generated by both demands. The probability that the price falls in the middle region is given by $1-\Delta\theta y$.

In table 4.1, the posterior beliefs and the probabilities are summarized. The first column shows the probability that prices might fall in a particular region. The second column shows

	Probability	Posterior Beliefs	λ_{next}
top	$\delta_a \Delta\theta y_a$	$\frac{\gamma_a}{\gamma_a+(1-\gamma_a)\delta_a}$	$\delta_a \lambda_a$
middle	$1-\Delta\theta y_a$	γ_a	λ_a
bottom	$(1-\delta_a)\Delta\theta y_a$	0	$\lambda_{bad} \equiv \infty$

Table 4.1: Learning

how the posterior beliefs evolve. The third column is a transformation of γ that is used as the state variable in the maximization problem, where $\lambda \equiv \frac{1-\gamma}{\gamma}$.[9] Note that this transformation implies that the smaller λ is, the better the prior about the firm.

The posterior beliefs are computed accordingly to Bayes rule. Note that the model implies that when signals fall in the middle region thay are not informative: the posterior beliefs are equal to the prior ones. On the other hand, the bottom region is full informative: If prices fall in this region then it was generated by a demand with slope θ_b, which implies that the project is facing a bad demand for sure. Finally, the top region is informative and priors are updated upwards.

In our model, the updating of the posterior beliefs is independent of the level of investment. In other words, an increase in output (y_a) increases the probabilities of learning but not the *size* of the information released[10]. This is a particular feature of the distribution function and uncertainty chosen. If the distribution function is not uniform and there is a continuum of types then the level of investment also affects how much information is released. In this case, the experimentational incentive comes from both the changes in the distribution of posterior beliefs and changes in the amount of information released. In this paper, we analyze a simpler version of the general case, however, the qualitative implications are the same.[11]

[9] The simplicity of this state variable comes from the fact that the learning process is a linear function of the current state. The benefits of using this state variable will become evident when the infinite horizon case is analyzed.

[10] The size of the information is related to the updating in the Bayes' rule. The larger the updating is (how different the prior and the posterior are) the more infromation was released. Note that in our model, the change in the priors is always δ_a, which is independent of the level of investment. However, this is not the case in more general setups.

[11] See Dreze (1972) and Prescott (1972) for general conditions in which experimentational incentives exist.

4.2.2 Credit Market

We assume that the credit market is organized as a set of lenders that pool their endowments and decide their credit allocations in a competitive way. There are two important assumptions about the behavior of the market. First, we assume that M is large enough so that all credit demands are satisfied in equilibrium. This assumption implies that the maximization problem can be solved for each firm separately. In other words, we can treat each lending problem as if the opportunity cost of those funds is the exogenous technology. Thus, the problem is equivalent to one of partial equilibrium.

The second assumption is that loans are always paid in all states of the world; there are no limited liability problems, even though managers have no endowment. Note that managers that hold projects with the same priors can fully hedge against price shocks and always produce the expected profits. This is because the space of priors is a countable space, while firms are represented by a continuum. Thus, there is always a continuum of firms with positive mass that share the same priors. Therefore, if there exists a perfect insurance market, then they can hedge against idiosyncratic shocks.

These two assumptions imply that the lending rate is always equal to the return on the exogenous technology.

4.3 Two period case

In this section, we assume that the world is an overlapping generations model where projects last for two periods. This simple version of the model allows for a close form solution and therefore, the intuitions are easily developed. Additionally, we assume that there is no destruction (either exogenous $\beta = 0$ or endogenous) in order to concentrate on the learning aspects of the problem.

The instantaneous profits of each firm is given by:

$$E\pi_t = E\left[P_t A I_t - (1+\rho) I_t\right] \tag{4.1}$$

where

$$EP_t = b + \tfrac{1}{2} - \theta_t A I_t$$

and where θ_t represents the expected slope of demand.

4.3.1 Full information case.

In this section, we study the credit allocation under perfect information. We determine the level of investment and how it changes with increases in ρ and A. We use this case as the benchmark where amplification effects and flight to quality effects are going to be compared.

Assume that the manager knows the expected slope of the demand (θ_i). If the project faces a good demand then $\theta_i = \theta_g$, and if the project faces a bad demand then $\theta_i = \theta_b - \Delta\theta\,\delta_b$. Because there is no value of learning the maximization of both periods is the same, and equivalent to the static problem. Formally,

$$\max_{I_i} \left(b + \tfrac{1}{2} - \theta_i A I_i\right) A I_i - (1+\rho) I_i$$

Taking the first order condition we obtain the following investment policy function:

$$I_i = \frac{1}{2A\theta_i}\left[b + \tfrac{1}{2} - \left(\frac{1+\rho}{A}\right)\right] \tag{4.2}$$

Assumption Assume that the parameters of the model satisfy the following inequalities: $b + \frac{1}{2} - \left(\frac{1+\rho}{A}\right) \geq 0$ and $b + \frac{1}{2} - 2\left(\frac{1+\rho}{A}\right) \leq 0$

The first inequality guaranties that in equilibrium investment is always positive. Thus, the monopoly is working in the elastic part of the demand. Moreover, this implies that an increase in the interest rate decreases investment. The second inequality guaranties that an increase in productivity (A) leads to an increase in investment. The first assumption is crucial for the results, while the second one is a technical assumption that simplifies the proofs.

Computing the elasticities with respect to changes in $(1+\rho)$ and A,

$$\begin{aligned} \xi\rho^{FI} &\equiv -\tfrac{1+\rho}{I}\tfrac{\partial I}{\partial(1+\rho)} = \frac{\left(\frac{1+\rho}{A}\right)}{\eta} \\ \xi_A^{FI} &\equiv \tfrac{A}{I}\tfrac{\partial I}{\partial A} = \frac{\left(\frac{1+\rho}{A}\right) - \eta}{\eta} \end{aligned} \tag{4.3}$$

where

$$\eta \equiv b + \tfrac{1}{2} - \left(\frac{1+\rho}{A}\right)$$

Note that the elasticities in the static maximization problem are independent of the type. So, in the full information case there is no cross sectional differences in the responses of investment, and the aggregate implications are identical to the individual ones.

4.3.2 Signal extraction problem.

In this section, we solve the imperfect information problem: We assume that the project's type is unobservable.

In the first period, all firms have the same prior about the probability of facing a good demand. Denote this prior as λ_1. In the second period, there are three different posterior beliefs depending on the first period's realization: Either, the price in the first period falls in the top region and the prior is updated upward ($\lambda_2 = \delta_b \lambda_1$), or it falls in the middle region where no learning occurs (λ_1), or it falls in the bottom region and the firm is facing a bad demand for sure (λ_{bad}). Lets solve the problem by backward induction.

Second period

Conditional on the prior at the beginning of the second period, there are three expected slopes of demand: First, if the signal in the first period was not informative, then $\theta_1 = \theta_b - \Delta\theta \left(\delta_b + \frac{1-\delta_b}{1+\lambda_1}\right)$. Second, if the price felt in the top region then posterior beliefs are updated upward and $\theta_2 = \theta_b - \Delta\theta \left(\delta_b + \frac{1-\delta_b}{1+\delta_b\lambda_1}\right)$. Finally, if the price felt in the bottom region the manager is sure that it is facing a bad demand, then $\theta_{bad} = \theta_b - \Delta\theta\,\delta_b$. Optimizing profits conditional on θ_i, implies that the investment level and the value of a firm are given by:

$$I_{2,i} = \frac{\eta}{2A\theta_i} \quad , \quad V_{2,i} = \frac{\eta^2}{4\theta_i} \tag{4.4}$$

Proposition 10 *The elasticities in the full information case and in the second period of the imperfect information case are equal:* $\xi\rho^{FI} = \xi\rho^2$ *and* $\xi_A^{FI} = \xi_A^2$

Proof. See the appendix. □

This result is due to the fact that the static elasticities are independent of the types (θ_i). In this model, there is no action in the second period because it is equivalent to a static maximization. In the next section, we study the first period problem and show the effects of the experimentational incentive.

First period

The maximization problem in the first period is the following:

$$\max_{I_1} EP_1\, AI_1 - (1+\rho)I_1 + \frac{1}{1+\rho}\left[\begin{array}{c} \delta_1 \Delta\theta\, AI_1\, V_{2,\theta_2} \\ +(1-\delta_1)\Delta\theta\, AI_1\, V_{2,\theta_{bad}} \\ +(1-\Delta\theta\, AI_1)\, V_{2,\theta_1} \end{array}\right] \tag{4.5}$$

where the term in the brackets is the expected value of the firm in the second period. The first order condition of this problem is the following:

$$\left[\frac{\partial EP_1}{\partial I_1}\, AI_1 + A\, EP_1 - (1+\rho)\right] + \frac{\Delta\theta\, A}{1+\rho}\left[\delta_1 V_{2,\theta_2} + (1-\delta_1)V_{2,\theta_{bad}} - V_{2,\theta_1}\right] = 0$$

The second bracket in the first order condition is the marginal value of information. Note that it depends on the expected present value of learning "something" (good or bad): $\delta_1 V_{2,\theta_2} + (1-\delta_1)V_{2,\theta_{bad}}$, minus the value of not learning and maintaining the same prior: V_{2,θ_1}. As it was discussed in the introduction, the value of information is larger the easier is to learn ($\Delta\theta\, A$) plus how valuable it is to learn ($\delta_1 V_{2,\theta_2} + (1-\delta_1)V_{2,\theta_{bad}} - V_{2,\theta_1}$).

Substituting by the definitions of the demand and the value functions, the first order condition is,

$$\left[\frac{\partial EP_1}{\partial I_1}\, I_1 + EP_1 - \frac{1+\rho}{A}\right] + \left[\frac{\Delta\theta}{1+\rho}\eta^2\Psi\right] = 0$$

where the marginal value of information is,

$$\Psi \equiv \frac{1}{4}\left[\frac{\delta_1}{\theta_2} + \frac{1-\delta_1}{\theta_{bad}} - \frac{1}{\theta_1}\right]$$

The first term in the first order condition is the static maximization, while the second term depends on the marginal value of information. The first order condition implies that the marginal benefit of investment today is equated to the expected marginal benefit of improving information in the next period. Solving for the investment we obtain equation (4.6).

$$I_1 = \frac{\eta}{2A\theta_1}\left[1 + \frac{\Delta\theta}{1+\rho}\eta\Psi\right] \tag{4.6}$$

Proposition 11 *Conditional on information, investment in the first period is larger than the one implied by static maximization. In other words,* $\Psi > 0$.

Proof. See the appendix. □

Ψ is always positive except when it is zero at $\lambda = 0$ or $\lambda = \infty$. In other words, it is always valuable to learn more about the project, except when everything is already known.

Investment plays, in the first period, an additional role as gatherer of information. The proposition states that the marginal value of information is positive, which implies a further increase in investment. In the next proposition, we formalize the amplification and the flight to quality effect.

Proposition 12 *There are amplification effects in the first period. In other words, the elasticities in the first period are always larger than the elasticities in the second period (or equivalently, larger than those obtained in the full information case).*

Proof. See the appendix. □

The intuition behind the result of proposition 12 is the following: When the interest rate increases the firm's decision is affected through two mechanisms. First, the increase in the cost of capital tends to decrease current investment, which is the usual interest rate channel. This effect is common to all firms, and as we showed before, it is independent of the type. Second, the increase in the interest rate leads to a fall in the net present value of future improvements in information. Thus, investment is further reduced. This extra reaction is what causes the elasticity to be larger for newer firms. Note that the reduction in the value of information comes from two sources. First, the increase in the interest rate implies an increase in the discount rate ($\frac{1}{1+\rho}\downarrow$). Second, and more importantly, the value of the project is reduced in each state of the world ($\eta\downarrow$).

Proposition 12 implies that there are amplification effects at the aggregate level. Investment in the second period has the same elasticity as the full information case, while in the first period the elasticity is larger. Therefore, the aggregate elasticity is larger that the full information case.

Additionally, the model implies that there are flight to quality effects in the sense that new firms suffer disproportionately more during recessions. This is a direct implication of the proposition: The elasticity is larger for newer firms.

Finally, there are persistence effects. In this model, the persistence effect last for one period because firms live for only two periods. However, the intuition can be easily extended for longer horizons. A transitory change in the interest rate reduces current investment. There is a reduction in the speed of learning and the distribution is shifted toward newer firms. The change in the distribution of beliefs has persistence at the aggregate level until the steady state is reached again. The persistence in this model comes from changes in the information the credit market has about the firms. This effect looks similar to the balance sheet effect.[12] Indeed, in this model, the only asset the firm has is its information. A change in the interest rate affects the distribution of the information and therefore of the implicit asset. In figure (4-1), we show a simulation of the aggregate response to a temporary shock in the interest rate.

[Figure 4-1]

4.4 The infinite horizon case.

In this section, we study the case when projects can live for infinite periods. We allow for both exogenous and endogenous destruction. Endogenous destruction occurs when the posterior probability that a project is facing a good demand is lower than the prior that a project has when it is born (i.e. the project is destroyed if $\gamma < \gamma_0$).

It is important to note, that the information is project specific and not manager specific. In other words, when a project is destroyed it does not say anything about the manager's skills. Therefore, all new projects have the same prior γ_0, regardless of the manager's record.

[12] See Bernanke and Gertler (1988).

4.4.1 Characterization of the solution

Each manager has a signal extraction problem that can be written as:

$$V_t(\gamma_a) = \max_{I_a} \left\{ E\pi_a + \frac{1}{1+\rho} EV_{t+1} \right\} \tag{4.7}$$

where a denotes the number of times the prices has fallen in the top region, ρ is the interest rate and $E\pi_a$ represents the expected flow profits conditional on the prior information. As we did before, we change the state variable to λ_a in which the properties of the flow profits and the learning process are simpler.

$$V(\lambda_a) = \max_{I_a} \left\{ E\pi_a + \frac{\beta}{1+\rho} \left[\begin{array}{l} \delta_a \Delta\theta AI_a V(\delta_b \lambda_a) + \\ (1 - \Delta\theta AI_a) V(\lambda_a) + \\ (1-\delta_a) \Delta\theta AI_a V(\lambda_0) \end{array} \right] + \frac{1-\beta}{1+\rho} V(\lambda_0) \right\}$$

where we define $\lambda_{a+1} = \delta_b \lambda_a$. Note that the better the prior (larger γ_a) the smaller the state variable (λ_a). We have assumed that projects are exogenously destroyed at rate $1-\beta$, and that they are endogenously destroyed every time the posterior probability is smaller than γ_0 (i.e., when the bottom region is reached). In the next proposition we formalize the properties of V.

Proposition 13 *$V(.)$ is unique, decreasing and convex.*[13]

Proof. See the appendix. □

This proposition implies that when λ increases (γ decreases) the net present value of the project is decreasing. As we would expect, this is saying that the net present value of a project improves when the probability that it is facing a good demand increases: The better the information the credit market has about a firm, the larger its net present value.

Operating inside the maximization operator, and identifying the marginal value of information, we obtain the following Bellman equation:

$$\frac{1+\rho-\beta}{1+\rho} V(\lambda_a) = \max_{I_a} \left\{ E\pi_a + \frac{\beta \Delta\theta AI_a}{1+\rho} C_f(\lambda_a) \right\} + \frac{1-\beta}{1+\rho} V(\lambda_0) \tag{4.8}$$

[13] *The proof of this proposition depends on the state variable definition. In this case, profits are a monotonic function of the state variable and its convexity properties do not change. This is not necessarily true under γ as the state variable. In particular, the convexity is not assured.*

where

$$C_f(\lambda_a) = \delta_a V(\delta_b \lambda_a) + (1-\delta_a) V(\lambda_0) - V(\lambda_a) \tag{4.9}$$

C_f represents the expected marginal value of information. As in the two period case, it is equal to the expected value of learning "something" minus the value of maintaining the same information. Note that it depends only on value functions, thus, if the credit market is allocating investment independently, its derivative with respect to the current level of investment is zero.[14] This simplifies the first order condition of the problem, given by:

$$\frac{\partial E\pi_a}{\partial I_a} + \frac{\beta \Delta\theta A}{1+\rho} C_f = 0$$

where the first term is the static maximization and the second one is related to the marginal value of information. Later we show that C_f is non negative, and in particular that it is zero when $\lambda = 0$.

Solving for the investment policy function the following equation is obtained,

$$I_a = \frac{1}{2\theta_a A}\left[\eta + \tfrac{\beta\Delta\theta}{1+\rho} C_f(\lambda_a)\right] \tag{4.10}$$

and the value function is,

$$\tfrac{1+\rho-\beta}{1+\rho} V(\lambda_a) = \frac{1}{4\theta_a}\left[\eta + \tfrac{\beta\Delta\theta}{1+\rho} C_f(\lambda_a)\right]^2 + \tfrac{1-\beta}{1+\rho} V(\lambda_0) \tag{4.11}$$

In the next proposition we characterize the investment policy function as a decreasing function of the state variable. Subsequently, we prove that the level of investment is always larger than the *myopic* solution. In other words, that C_f is non negative. Finally, we show that if the value of information is concave, investment is convex.

The first two results are related to the optimal level of investment. On the one hand, the better the information the credit market has about a firm, the larger the expected productivity and therefore, the larger the optimal level of investment. On the other hand, the better the prior the credit market has about a project, the less valuable the learning process; thus, less

[14] This is just an application of the envelope thecrem.

incentives to experiment. In the first two propositions, we show that the productivity incentive dominates the experimentational incentive and investment is increasing with information.

Proposition 14 *Investment is decreasing in* λ_a.

Proof. See the appendix. □

In other words, regardless of the importance of the learning benefits, the productivity incentive for investment dominates the experimentational incentives. This result is a direct implication of the properties of the value function. The value function of a firm is increasing with the better information, and so is its investment.

In the next proposition we show that the experimentational incentives exist, and therefore we can expect the level of investment to be higher than the one implied by static maximization, which we called *myopic*.

Proposition 15 *Investment in the learning economy is always larger than the myopic case.*

Proof. See the appendix. □

In other words $C_f \geq 0$. In this model, changes in the valuation of C_f generate the amplification and flight to quality effects. Remember that the static solution implies an elasticity of investment to interest rate and productivity shocks that is independent of the type. Thus, if there is any departure from this benchmark it comes from the changes in C_f.

Proposition 16 *If the value of information is concave with respect to* γ_t, *then investment is also convex with respect to* γ_t.

Proof. See the appendix. □

Given the characterization of the solutions lets look at the empirical implications of the model. Unfortunately, these implications can not be derived analytically and therefore a numerical solution is presented.

4.4.2 Numerical solution.

We choose two sets of parameters to illustrate the implications of the model. The first set involves an economy where learning is not to valuable. This is because the good and the bad

demand are similar. The parameters are: $\theta_b = 0.2$, $\theta_g = 0.1$, $\delta_b = 0.75$, $\rho = 0.1$, $\beta = 0.90$, $b = 1$, $A = 1$, and $\gamma_0 = 0.25$, and the solution is shown in figure (4-2). The second set of parameters, imply high values of learning. The parameters are $\theta_b = 1.0$, $\theta_g = 0.1$, $\delta_b = 0.50$, and the results are shown in figure (4-3). The x-axis represents the prior that the firm is facing a good demand (γ).

Both figures are composed by six panels: The first one is the value function, the second panel is the investment policy, the third one is the value of information C_f, the forth one is the steady state distribution, and the last two panels are elasticity of investment with respect to a change in the interest rate, and the change in the steady state distribution to an increase in the interest rate.

[Figure 4-2]

[Figure 4-3]

As can be seen, the value function and the investment policy are an increasing and convex function of information. Under the parameters chosen, the properties of the value function and the investment policy function are invariant to the transformation in the state variable. This is not necessarily true for all parameters. In particular, the convexity of the value function is generally violated. Under the second set of parameters the value function is reduced in each state of the world, this is because the firm is facing a worse demand on average. Additionally, the investment policy tilts upwards. For old and well known firms the investment policy is close in both cases. This is because the changes in the parameters only affect those projects that face a bad demand. However, for all firms with priors less than one, the investment policy is lower under the second set of parameters.

The third panel shows that the value of information is always positive and that it reaches zero at $\gamma = 1$, with a slope different from zero. The function is not necessarily monotonic, as shown in figure (4-3). The value of information is small under the first set of parameters, but is relatively large under the second set of parameters. In particular, it represents less than one percent of the value of the firm in the first case, while it is around 10 percent of the value of the firm in the second case.

In the fourth panel, the distribution of firms in steady state is shown. The distribution function was computed solving the following problem:

$$g_{t+1}(\lambda_a) = \begin{cases} \beta \begin{bmatrix} (1-\Delta\theta\, AI_a)\; g_t(\lambda_a) \\ +\delta_{a-1}\,\Delta\theta\, AI_{a-1}\; g_t(\lambda_{a-1}) \end{bmatrix} & if \quad \lambda_a \neq \lambda_0 \\ & \quad\; \text{or} \neq 0 \\ \beta \begin{bmatrix} (1-\Delta\theta\, AI_a)\; g_t(\lambda_a) \\ +\delta_a\,\Delta\theta\, AI_a\; g_t(\lambda_a) \\ +\delta_{a-1}\,\Delta\theta\, AI_{a-1}\; g_t(\lambda_{a-1}) \end{bmatrix} & if \quad \lambda_a = 0 \\ \beta\left[(1-\Delta\theta\, AI_0)\; g_t(\lambda_0)\right] \\ +(1-\beta) \\ +\int_0^\infty (1-\delta_a)\;\Delta\theta\, AI_a\; g_t(\lambda_a)\; da & if \quad \lambda_a = \lambda_0 \end{cases}$$

where t represents the iteration. This equation is similar to a forward Kolmogorov equation when the underlying process follows a Poisson.

The interpretation of the equation is the following: When λ is not in the extremes the mass of firms that is going to be at any λ_a is equal to the proportion of firms that survived and their price signal was non informative $[\beta(1-\Delta\theta\, AI_a)\; g_t(\lambda_a)]$ plus the mass of firms that were in the previous stage of learning, survived the exogenous destruction and received a good price signal $[\beta\delta_{a-1}\,\Delta\theta\, AI_{a-1}\; g_t(\lambda_{a-1})]$.

Second, when we know that the firm is of good type ($\lambda = 0$) then we have to add an extra term that corresponds to those that received a good price signal but now it is not informative, or in other words, that the updating gives a posterior equal to the prior.

Finally, the new firms mass is equal to those that were new in the previous period, that survived and the price signal was not informative, plus the mass of firms that was exogenously destroyed $(1-\beta)$, plus the mass of firms that was endogenously destroyed $\left[\int_0^\infty (1-\delta_a)\;\Delta\theta\, AI_a\; g_t(\lambda_a)\; da\right]$. In this case, we have to add across all types and compute their endogenous destructions.

In general, the distribution in steady state is dominated by the exogenous destruction: it looks as if it is a geometric distribution. However, as we show below, changes in interest rates

change the distribution function, and this has strong impact on the aggregate response.

The last two panels show the response of the economy to a change in the interest rate. An increase in the real interest rate is a shock to the time preferences of all agents. In this model, there could be two additional interpretation. It can be a shock to the international interest rate, or, if we assume that the central bank can control the real interest rate, it can be a shock to monetary policy. This later interpretation requires the assumption that the central bank is able to control the real interest rate, which is only possible if there are sticky prices. The exercise run is a permanent change in the discount rate and therefore, the monetary interpretation requires strong assumptions about the degree of stickiness.

In the fifth panel, we show the elasticity of investment with respect to the interest rate. For all firms, the elasticity in the learning economy is always larger than the elasticity implied by the static maximization. Note that the elasticity is a monotonically decreasing function, which is the flight to quality effect. An increase in the interest rate implies a larger contraction (in relative terms) for smaller firms than for larger firms. In other words, the credit market is less willing to invest in learning for the new firms and therefore there is an extra reduction in the investment.

The flight to quality requires the elasticities to be a monotonically decreasing function of information. However, in order to have aggregate amplification effects, we require that the average response of the economy is larger than the one implied by the static maximization. In order to show this result we have to look not only at the individual response, but also at the change in the distribution of firms.

In the sixth panel, we show the change in the steady state distribution of firms. An increase in interest rate biases the distribution toward more new firms. The idea is that an increase in the cost of funds reduces investment, which in the end reduces the speed at which information is released. The aggregate effect is then composed by two effects. The direct effect that comes from the change in individual investment, plus the indirect effect that comes from the shift in the distribution. Both effects go in the same direction, both tend to reduce aggregate investment. For example, for the first set of parameters, the elasticity of the newest firm is 0.2525 while the elasticity of the oldest firm is 0.2500. The aggregate elasticity without taking into consideration distributional effects is 0.2523. Which, as we would expect, is between

the maximum and minimum of the individual elasticities. The aggregate effect, taking into consideration changes in the distribution, is 0.2557. Comparing the contribution of each effect in the amplification, the first one increases the elasticity in one percent, while the distribution effect increases it in an additional one percent. In this simulation, the distributional effect is of the same order of magnitude than the pure individual investment reaction. In the second set of parameters, the oldest firm elasticity is 0.2500, while the newest firm elasticity is 0.3925. This implies an aggregate elasticity of 0.4401. In other words, the experimentational incentive can have strong amplification effects. Note that in both cases the aggregate elasticity is larger than all the individual ones. This is an immediate implication of the change in the distribution.

The impact on the aggregate investment through the distribution takes time to be reflected. In the simulation, 50 percent of the total impact was realized after 10 periods of the shock, and 90 percent of the impact required almost 20 periods.

Finally, the persistence effect is presented in figure (4-4). The exercise run is the following: we compute the impulse response to a transitory shock (one period increase) in the interest rate. We assume that it is common knowledge that the shock is transitory, therefore, the learning incentive is not affected by the change in the interest rate.

[Figure 4-4]

Similar results are obtained when changes in productivity (A) are analyzed. The same cross sectional and aggregate implications are shown.

4.5 Conclusion and extensions

It has been widely argued that learning is an important aspect of the credit market, both in the development of lender-firm relationships as well as in the formation of reputation.

In this paper, we show that the inclusion of learning in a model of the credit market has implications that are consistent with the facts reported in the literature. In particular, we find that the model generates amplification effects, flight to quality effects, and persistence effects. These implications come from changes in the marginal value of information. An increase in the interest rate or in aggregate productivity, changes the value of gathering information.

This model allows to study the aggregate implications of other aspects of the credit market, such as bankruptcy costs and bank competition.

First, this model can predict that bankruptcy costs have large implications on the aggregate level of investment. It has been argued that because bankruptcy costs are small (less than 2 percent of the value of the firm) they have a small impact on the individual availability of credit. In the model presented here, this implications will continue to be true. However, the steady state distribution will be shifted toward newer firms, and the impact on the aggregate level of investment could be large.

Second, the impact of different degrees of competition in the credit market can be analyzed. There is a large literature studying the subject at the firm level, and this model could give some lights of its implications at the aggregate level. It is possible to analyze the impact of a more volatile monetary policy on the credit allocation, as well as the implications of a financial opening.

The credit market literature has mainly concentrate on inefficiencies in the demand side of credit, and generally oversimplifying the supply side. In this paper, we have tried to improve the understanding of the macroeconomic implications of a model that allows for more complex treatments of the supply side of credit. More research is needed in this direction.

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4.6 Appendix.

Proof. Proposition 10.

This is obtained by directly differentiating the investment equations (4.2) and (4.4). The elasticities are given in equation (4.3)□

Proof. Proposition 11.

The sign of Ψ depends on the sign of $\delta_1\theta_1\theta_{bad} + (1-\delta_1)\theta_2\theta_1 - \theta_2\theta_{bad}$. Which is equal to $\delta_1\theta_1(\theta_{bad} - \theta_2) + \theta_2(\theta_1 - \theta_{bad})$. Substituting by the definitions of θ_i, we obtain the following expression,

$$\theta_b\left[\delta_b - \delta_1 + \delta_1(\delta_2 - \delta_b)\right] - \Delta\theta\left[\delta_2(\delta_b - \delta_1) + \delta_1^2(\delta_2 - \delta_b)\right]$$

It is easy to show that the first bracket is equal to zero while the second bracket is negative. Thus,

$$\Psi = \frac{1}{4}\frac{\lambda_1}{1+\lambda_1}\frac{(1-\delta_b)^3}{(1+\lambda_1)(1+\delta_b\lambda_1)}\frac{\Delta\theta^2}{\theta_1\theta_2\vartheta_{bad}} \geq 0$$

Which is always positive.□

Proof. Proposition 12.

Note that Ψ is independent of the interest rate and the state of technology. Computing the elasticities in the first period, amplification occurs if and only if the following inequality is satisfied.

$$\xi\rho^1 = \frac{\frac{1+\rho}{A} + \frac{\Delta\theta}{1+\rho}\Psi\eta^2 + 2\frac{\Delta\theta}{A}\Psi\eta}{\eta + \frac{\Delta\theta}{1+\rho}\Psi\eta^2} \overset{?}{>} \frac{\frac{1+\rho}{A}}{\eta} \tag{4.12}$$

After some algebra it is possible to show that,

$$\frac{\Delta\theta}{1+\rho}\Psi\eta^2\left[\eta + \frac{1+\rho}{A}\right] > 0$$

Which is true given that Ψ is positive.

The second step of the proof is to show that there is also amplification with respect to productivity shocks. Solving for the elasticity with respect to productivity changes, similar results are obtained.

$$\xi_A^1 = \frac{-\left[\eta + \frac{\Delta\theta}{1+\rho}\Psi\eta^2\right] + \frac{1+\rho}{A}\left[1 + 2\frac{\Delta\theta}{A}\Psi\eta\right]}{\eta + \frac{\Delta\theta}{1+\rho}\Psi\eta^2} \stackrel{?}{>} \frac{\frac{1+\rho}{A} - \eta}{\eta} \tag{4.13}$$

It can be shown that if and only if Ψ is positive, there are amplification effects. □

Proof. Proposition 13.

The proof is based on Lucas, Prescott, and Stokey (1989) (Theorem 9.7). The sufficient conditions for uniqueness, decreasing and convexity are the following: First, the state variable belongs to the positive real line with an euclidean norm (which is a convex Borel set). Second, the shocks belong to a compact set (both the u and θ are compact). Third, the control variable (I_a) belongs to a non-empty, compact valued, and continuous correspondence, which in our case it belogs to the real line. Forth, the profit function is continuous, and strictly decreasing in λ. Fifth, ρ is positive and finite, thus the discount rate is between zero and one. Under this conditions it is possible to show that the Bellman equation describes an operator satisfying the Blackwell's sufficient conditions for a contraction, and since the space of continuous functions is Banach, then the solution to the operator has a unique fix point. Moreover, it is strictly decreasing and convex because $E\pi_a$ is strictly decreasing and convex in λ.□

Proof. Proposition 14.

Differentiating the investment rule and the value function (equations 4.10 and 4.11) with respect to λ we obtain:

$$\begin{aligned}
\frac{1}{I_{t,a}}\frac{\partial I_{t,a}}{\partial \lambda_a} &= -\frac{1}{\theta_a}\frac{\partial \theta_a}{\partial \lambda_a} + \phi \\
\frac{1}{V}\frac{\partial V}{\partial \lambda_a} &= -\frac{1}{\theta_a}\frac{\partial \theta_a}{\partial \lambda_a} + 2\phi \\
\phi &\equiv \frac{\frac{\Delta\theta\beta}{1+\rho}\frac{\partial C_f}{\partial \lambda_a}}{b + \frac{1}{2} - \frac{1+\rho}{A} + \frac{\Delta\theta\beta}{1+\rho}C_f(\lambda_a)}
\end{aligned}$$

The first term in the differentiations is negative. Additionally, we known that the value function is decreasing, so: $-\frac{1}{\theta_a}\frac{\partial\theta_a}{\partial\lambda_a}+2\phi<0$. This means that $\phi<\frac{1}{2}\frac{1}{\theta_a}\frac{\partial\theta_a}{\partial\lambda_a}$, which implies that the investment derivative is $<-\frac{1}{2}\frac{1}{\theta_a}\frac{\partial\theta_a}{\partial\lambda_a}$, which is always negative.

Proof. Proposition 15.

The proposition requires C_f to be non-negative. The proof is done in four steps. First, we show that not all C_f's can be zero. Second, we show that if $C_f(\lambda_0)$ and $C_f(\lambda_{a+1})$ are positive then $C_f(\lambda_a)$ has to be positive. In other words, if the value of information for the new projects is positive, and if the value of information for the next state of information is also positive, then current value of information is positive. Third, we show that $C_f(\lambda_0)$ is always positive. And finally, we show that the derivative of C_f is positive when $\lambda=0$. This concludes the proof.

Lemma 17 *Not all C_f's are zero.*

Proof. Lets show that if all C_f's then we have a contradiction. If all C_f's are zero then the value functions are equal to: $\frac{1+\rho-\beta}{1+\rho}V(\lambda_a)=\frac{1}{4\theta_a}\eta^2+\frac{1-\beta}{1+\rho}V(\lambda_0)$. Thus, the marginal value function given by equation (4.9) is,

$$C_f(\lambda_a)=\frac{1+\rho}{1+\rho-\beta}\frac{\eta^2}{4}\left[\frac{\delta_a}{\theta_{a+1}}+\frac{1-\delta_a}{\theta_0}-\frac{1}{\theta_a}\right]$$

We know that $\theta_0=\theta_b-\Delta\theta\delta_0<\theta_b-\Delta\theta\delta_b$, which we define as θ_{bad}. Thus, it is sufficient to show that $\frac{\delta_a}{\theta_{a+1}}+\frac{1-\delta_a}{\theta_{bad}}-\frac{1}{\theta_a}>0$ in order to prove the contradiction. Substituting by the definitions we obtain that the sign of this term depends on the sign of,

$$\theta_b\left[\delta_a(\delta_{a+1}-\delta_b)-(\delta_a-\delta_b)\right]+\Delta\theta\left[-\delta_a^2(\delta_{a+1}-\delta_b)+\delta_{a+1}(\delta_a-\delta_b)\right]$$

It is easy to show that the first bracket is always zero, and that the second bracket is always positive.□

Lemma 18 *If $C_f(\lambda_{a+1})$ and $C_f(\lambda_0)$ are both positives, then $C_f(\lambda_a)>0$.*

Proof. If $C_f(\lambda_{a+1}) > 0$ then $\frac{1+\rho-\beta}{1+\rho}V(\lambda_{a+1}) > \frac{1}{4\theta_{a+1}}\eta^2 + \frac{1-\beta}{1+\rho}V(\lambda_0)$, and if $C_f(\lambda_0) > 0$ then $\frac{1+\rho-\beta}{1+\rho}V(\lambda_0) > \frac{1}{4\theta_0}\eta^2 + \frac{1-\beta}{1+\rho}V(\lambda_0)$. Lets show that $C_f(\lambda_a)$ has to be positive. Lets assume that it is non-positive, then $\frac{1+\rho-\beta}{1+\rho}V(\lambda_a) \leq \frac{1}{4\theta_a}\eta^2 + \frac{1-\beta}{1+\rho}V(\lambda_0)$. Substituting this inequalities in the definition of $C_f(\lambda_a)$, we obtain the following inequality.

$$\begin{aligned} C_f(\lambda_a) &= \delta_a[V(\delta_b\lambda_a) - V(\lambda_a)] + (1-\delta_a)[V(\lambda_0) - V(\lambda_a)] \\ &> \frac{1+\rho}{1+\rho-\beta}\frac{\eta^2}{4}\left[\frac{\delta_a}{\theta_{a+1}} + \frac{1-\delta_a}{\theta_0} - \frac{1}{\theta_a}\right] > 0 \end{aligned}$$

The first inequality comes from the inequalities in the value functions, while the second one comes from the previous lemma.□

Lemma 19 $C_f(\lambda_0) > 0$.

Proof. This is just by inspection of equation (4.9).

$$\begin{aligned} C_f(\lambda_0) &= \delta_0[V(\delta_b\lambda_0) - V(\lambda_0)] + (1-\delta_0)[V(\lambda_0) - V(\lambda_0)] \\ &= \delta_0[V(\delta_b\lambda_0) - V(\lambda_0)] > 0 \end{aligned}$$

This is strictly larger than zero because δ_0 is strictly positive and $V(.)$ is strictly decreasing in λ.□

Lemma 20 *C_f is non-negative for all λ.*

Proof. In order to prove this proposition we only need to show that $C_f(0) = 0$, and that the slope of C_f at $\lambda = 0$ is positive. Then using the previous lemmas we show that the function has to be positive everywhere. The first part of the proof comes from substituting in (4.9) and using the fact that $\delta_{\lambda=0} = 1$. In other words, when the firm is known to be of good type it always faces a good demand.

$$\begin{aligned} C_f(\lambda_{a+1}) - C_f(\lambda_a) &= \delta_{a+1}\left[V\left(\delta_b^2\lambda_a\right) - V(\delta_b\lambda_a)\right] \\ &\quad - \left[V(\delta_b\lambda_a) - V(\lambda_a)\right] \\ &\quad + (\delta_{a+1} - \delta_a)\left[V(\delta_b\lambda_a) - V(\lambda_0)\right] \end{aligned}$$

Lets approximate $\lambda_a \sim \delta_b\lambda_a + d\lambda$, which implies $\delta_b\lambda_a \sim \delta_b^2\lambda_a + \delta_b d\lambda$. Notice that this approximation is only valid when λ is close to zero. Substituting, and defining $-dC_f \equiv C_f(\lambda_{a+1}) - C_f(\lambda_a)$,

$$\begin{aligned} -dC_f &= -V'\left(\delta_b^2\lambda_a\right)\delta_{a+1}\,\delta_b d\lambda \\ &\quad + V'(\delta_b\lambda_a)\,d\lambda \\ &\quad + \frac{1-\delta_b}{(1+\lambda_a)(1+\delta_b\lambda_a)}\left[V(\delta_b\lambda_a) - V(\lambda_0)\right]d\lambda \end{aligned}$$

Evaluating at $\lambda_a = 0$, we have that $\delta_a = \delta_{a+1} = 1$, and

$$\begin{aligned} -dC_f &= -V'(0)\,(1-\delta_b)\,d\lambda + (1-\delta_b)\left[V(0) - V(\lambda_0)\right]d\lambda \\ &= (1-\delta_b)\,d\lambda\left[-V'(0) + \left[V(0) - V(\lambda_0)\right]\right] \end{aligned}$$

The term in the bracket is positive because V is strictly decreasing and strictly convex. This means that the derivative of the value of information is positive at the origin.□

This is the end of the proof.□

Proof. Proposition 16.

To prove that investment is concave we do the following procedure. First, we compute the second derivative of the value function, which we know is negative. Second, we compute the second derivative of the investment policy and show under which conditions it is negative.

$$\frac{1+\rho-\beta}{1+\rho}\frac{\partial^2 V}{\partial\lambda^2} = \frac{1}{4\theta_a}\left[\eta + \frac{\Delta\theta\beta}{1+\rho}C_f(\lambda_a)\right]^2\left\{\begin{array}{c} -\frac{1}{\theta_a}\frac{\partial^2\theta_a}{\partial\lambda^2} + 2\left(\frac{1}{\theta_a}\frac{\partial\theta_a}{\partial\lambda}\right)^2 - 4\left(\frac{1}{\theta_a}\frac{\partial\theta_a}{\partial\lambda}\right)\Phi_1 \\ +2\Phi_2 + 2\Phi_1^2 \end{array}\right\}$$

$$\frac{\partial^2 I}{\partial \lambda^2} = \frac{1}{2A\theta_a}\left[\eta + \frac{\Delta\theta\beta}{1+\rho}C_f(\lambda_a)\right]\left\{\begin{array}{c} -\frac{1}{\theta_a}\frac{\partial^2\theta_a}{\partial\lambda^2} + 2\left(\frac{1}{\theta_a}\frac{\partial\theta_a}{\partial\lambda}\right)^2 - 2\left(\frac{1}{\theta_a}\frac{\partial\theta_a}{\partial\lambda}\right)\Phi_1 \\ +\Phi_2 \end{array}\right\}$$

where

$$\begin{aligned}\Phi_1 &\equiv \frac{\Delta\theta\beta}{1+\rho}\frac{\frac{\partial C_f(\lambda_a)}{\partial\lambda}}{\eta + \frac{\Delta\theta\beta}{1+\rho}C_f(\lambda_a)} \\ \Phi_2 &\equiv \frac{\Delta\theta\beta}{1+\rho}\frac{\frac{\partial^2 C_f(\lambda_a)}{\partial\lambda^2}}{\eta + \frac{\Delta\theta\beta}{1+\rho}C_f(\lambda_a)}\end{aligned}$$

We know that the term in the brackets of the derivative of V is positive. So,

$$-\frac{1}{\theta_a}\frac{\partial^2\theta_a}{\partial\lambda^2} + 2\left(\frac{1}{\theta_a}\frac{\partial\theta_a}{\partial\lambda}\right)^2 - 4\left(\frac{1}{\theta_a}\frac{\partial\theta_a}{\partial\lambda}\right)\Phi_1 + 2\Phi_2 + 2\Phi_1^2 > 0$$

Which implies that

$$-\frac{1}{\theta_a}\frac{\partial^2\theta_a}{\partial\lambda^2} + 2\left(\frac{1}{\theta_a}\frac{\partial\theta_a}{\partial\lambda}\right)^2 - 2\left(\frac{1}{\theta_a}\frac{\partial\theta_a}{\partial\lambda}\right)\Phi_1 + \Phi_2 > 2\left(\frac{1}{\theta_a}\frac{\partial\theta_a}{\partial\lambda}\right)\Phi_1 - \Phi_2 - 2\Phi_1^2$$

Thus, if we can show that the right hand side term is positive then we have a sufficient condition for convexity of investment. Notice that we can rewrite the term in the following way: $2\Phi_1\left(\frac{1}{\theta_a}\frac{\partial\theta_a}{\partial\lambda} - \Phi_1\right) - \Phi_2$. Which the term in the bracket is related to the first derivative of the investment policy.

$$\frac{\partial V}{\partial\lambda} = \frac{1}{V}\left[-\frac{1}{\theta_a}\frac{\partial\theta_a}{\partial\lambda} + 2\Phi_1\right] < 0 \Longrightarrow \frac{1}{\theta_a}\frac{\partial\theta_a}{\partial\lambda} - \Phi_1 > \Phi_1$$

So, $2\Phi_1\left(\frac{1}{\theta_a}\frac{\partial\theta_a}{\partial\lambda} - \Phi_1\right) - \Phi_2 > 2\Phi_1^2 - \Phi_2$. Note that the first term is always positive. The second term is positive if and only if C_f is concave. In other words, Φ_2 is negative. □

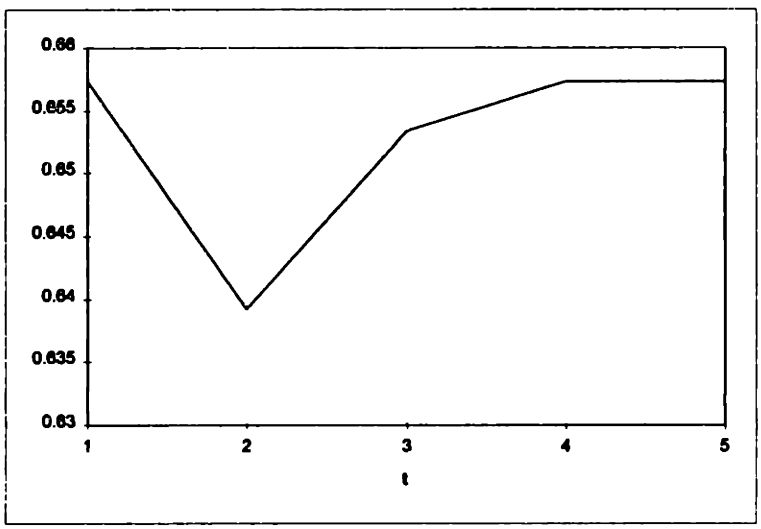

Figure 4-1: Aggregate investment evolution. Transitory shock.

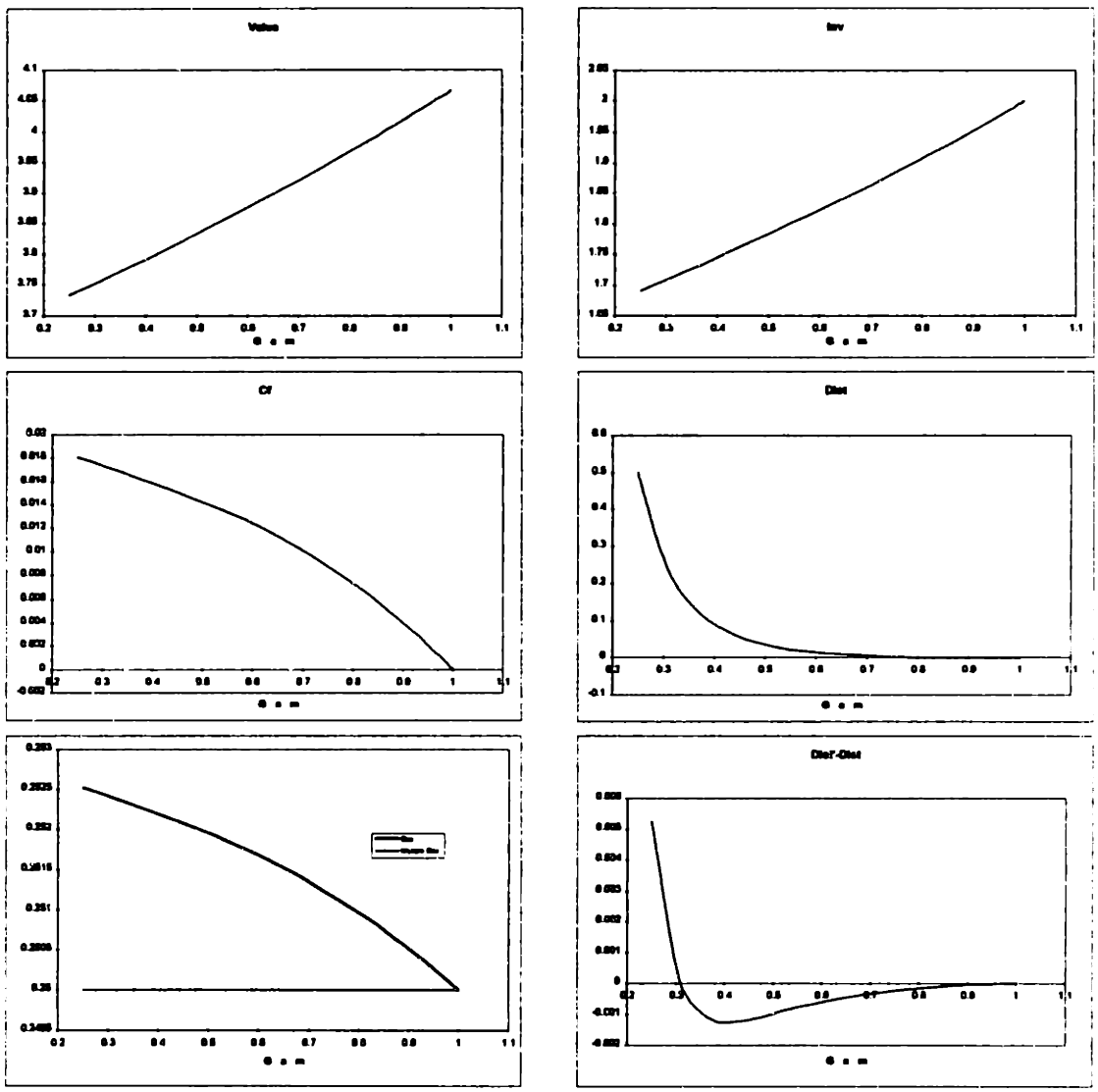

Figure 4-2: Solution for the learning economy. First set of parameters.

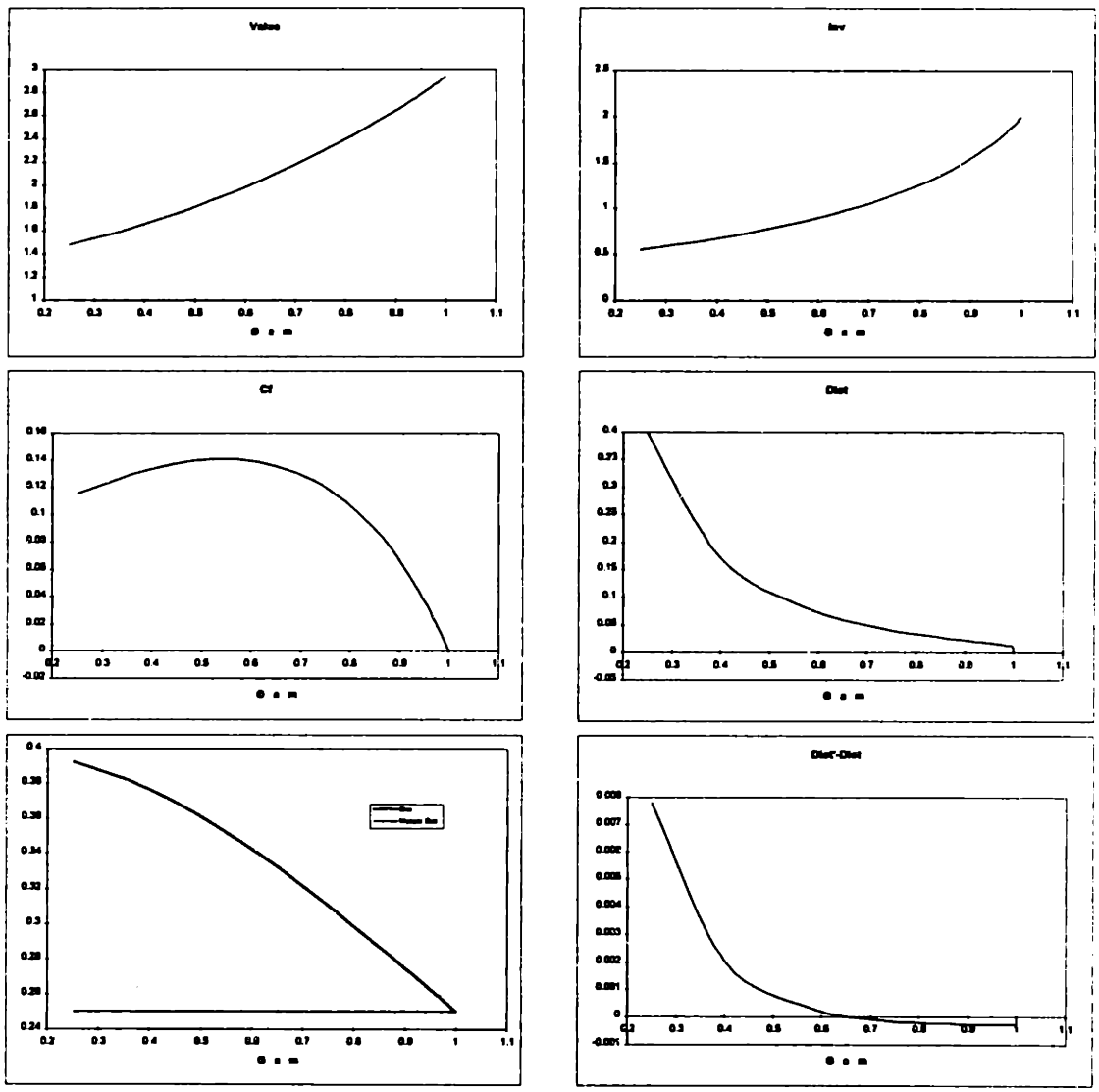

Figure 4-3: Solution for the learning economy. Second set of parameters.

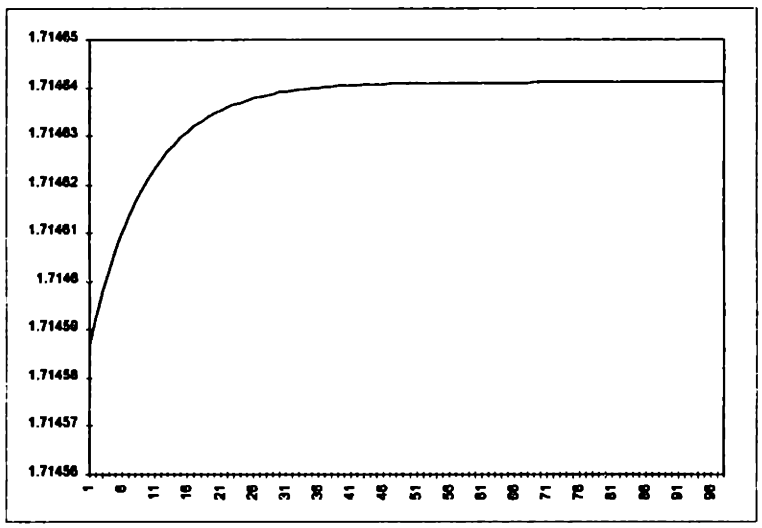

Figure 4-4: Aggregate Investment. Response to a transitory shock in the infinite horizon case.