

22.01 Problem Set 5 Homework Solutions

1) (3 points) A narrow beam of 10^8 photons per second is normally incident on a 6 mm aluminum sheet. The beam consists of equal numbers of 200 keV photons and 2 MeV photons.

a) Calculate the number of photons per second of each energy that are transmitted without interaction through the sheet.

Since we are asked to calculate the number of photons transmitted through the sheet we need to use the attenuation coefficients for the 200 keV and 2 MeV photons, which we can find in Figure 8.8 of Turner. For Aluminum these are roughly $0.12 \text{ cm}^2/\text{g}$ for 200 keV photons and $0.045 \text{ cm}^2/\text{g}$ for 2 MeV photons. Solving for the uncollided flux:

For 200 keV photons:

$$I = I_0 e^{-\left(\frac{\mu}{\rho}\right)\rho x} = 5 \times 10^7 \cdot e^{-0.12 \cdot 2.7 \cdot 0.6} = 4.11 \times 10^7 \text{ s}^{-1}$$

For 2 MeV photons:

$$I = I_0 e^{-\left(\frac{\mu}{\rho}\right)\rho x} = 5 \times 10^7 \cdot e^{-0.045 \cdot 2.7 \cdot 0.6} = 4.64 \times 10^7 \text{ s}^{-1}$$

b) Calculate the transmitted intensities for the same two photon beams striking a lead sheet of the same thickness.

Instead of aluminum we now have lead. Again using figure 8.8, the attenuation coefficients are roughly $0.90 \text{ cm}^2/\text{g}$ for 200 keV photons and $0.058 \text{ cm}^2/\text{g}$ for 2 MeV photons. Solving for the uncollided flux:

For 200 keV photons:

$$I = I_0 e^{-\left(\frac{\mu}{\rho}\right)\rho x} = 5 \times 10^7 \cdot e^{-0.9 \cdot 11.4 \cdot 0.6} = 1.1 \times 10^5 \text{ s}^{-1}$$

For 2 MeV photons:

$$I = I_0 e^{-\left(\frac{\mu}{\rho}\right)\rho x} = 5 \times 10^7 \cdot e^{-0.058 \cdot 2.7 \cdot 0.6} = 3.37 \times 10^7 \text{ s}^{-1}$$

c) Calculate the atomic cross sections of aluminum and lead for the 200 keV and the 2 MeV photons.

Atomic cross section is the effective area of interaction probability per atom, or in the units of cm^2/atom . Therefore using the attenuation coefficients and converting to cm^2/atom :

$$\frac{\mu}{\rho} \Rightarrow \frac{cm^2}{g} \cdot \frac{\# g}{mol} \cdot \frac{1 mol}{6.02 \times 10^{23} atoms} = \frac{cm^2}{atom}$$

For aluminum: 200 keV atomic cross section: **5.38 barns**
 2 MeV atomic cross section: **2.02 barns**

For lead: 200 keV atomic cross section: **310.0 barns**
 2 MeV atomic cross section: **20.0 barns**

2. (3 points) A narrow beam of 200 keV photons strikes a target consisting of a 1.4-cm-thick sheet of aluminum followed by a 2-mm-thick sheet of lead.

(a) What fraction of the incident photons penetrates both sheets without interacting?

At 200 keV, the attenuation coefficients from figure 8.8 is 0.12 cm²/g for aluminum and 0.90 cm²/g for lead.

Calculating the photon intensity leaving the aluminum:

$$I = I_0 e^{-\left(\frac{\mu}{\rho}\right)\rho x} = I_0 \cdot e^{-0.12 \cdot 2.7 \cdot 1.4} = 0.635 I_0$$

This is the fraction of intensity entering the lead shield. The amount transmitted through the lead shield is then:

$$I = I_{Al} e^{-\left(\frac{\mu}{\rho}\right)\rho x} = I_0 \cdot e^{-\mu_{Al} x_{Al}} e^{-\mu_{Pb} x_{Pb}} = 0.635 I_0 \cdot e^{-0.9 \cdot 1.4 \cdot 0.2} = 0.0816 I_0$$

(b) Would there be any difference if the photons came from the other direction, entering the lead first and then the aluminum?

From the above equation, it shows that it does not make a difference if you place the aluminum or the lead first.

3. (2 points) Food irradiation is carried out with very intense ⁶⁰Co sources. Calculate the dose rate in air (Gy/sec) at distances of 1 meter and 10 meters from a 50,000 Ci source of ⁶⁰Co.

We use equation 12.26 to calculate dose rate in air from a gamma point source, with C in Ci and we estimate E as the average of the two emitted photons, which is 1.25 MeV. Then we multiply the activity by two to reflect that there are two emitted gammas of this average energy :

At 1 meter:

$$\dot{D} = \frac{1.27 \times 10^{-6} CE}{r^2} = \frac{1.27 \times 10^{-6} \cdot 50000 \cdot 1.25 MeV}{1} = .079 Gy/s \cdot 2 = 0.16 Gy/s$$

At 10 meters the dose rate is 1.59×10^{-3} Gy/s. This illustrates the importance of distance to shield oneself from a radiation source. You could also do the dose rate for each gamma and sum the two and it should come out to approximately the same since the energies of the photons are similar.

4. (4 points) A gold foil weighing 3.500 mg is irradiated with thermal neutrons for exactly 10 minutes. Forty-eight hours after the end of the irradiation, the foil is placed in a gamma spectrometer with 100% counting efficiency and an activity of 2750 Bq is recorded.

Given:

^{197}Au (100% abundance)

thermal neutron capture cross section, $\sigma = 98.8$ barns

$^{197}\text{Au} + n \rightarrow ^{198}\text{Au}$

^{198}Au $t_{1/2} = 2.7$ days

(a) What was the thermal neutron flux (neutrons/cm²/sec) to which the foil was exposed?

Since 48 hours passed from the irradiation time, we need to correct for any decay with $\lambda = 2.97 \times 10^{-6} \text{ s}^{-1}$:

$$A = A_0 e^{-\lambda t}$$

$$2750 = A_0 e^{-2.97 \times 10^{-6} \cdot 48 \cdot 60 \cdot 60}$$

$$A_0 = 4598 \text{ Bq}$$

Using equation 9.36 for neutron activation:

$$N_T = 3.5 \times 10^{-3} \cdot \frac{1 \text{ mol}}{197 \text{ g}} \cdot \frac{6.02 \times 10^{23} \text{ atoms}}{1 \text{ mol}} = 1.07 \times 10^{19} \text{ atoms}$$

$$A = \lambda N = \Phi \sigma N_T (1 - e^{-\lambda t}) \Rightarrow 4598 = \Phi (98.8 \times 10^{-24}) (1.07 \times 10^{19}) (1 - e^{-2.97 \times 10^{-6} \cdot 600})$$

$$\Phi = 2.17 \times 10^9 \text{ n/cm}^2 \text{ s}$$

(b) What would be the saturation activity of this gold foil when exposed to this neutron flux?

The saturation activity is:

$$A = \Phi \sigma N_T = (2.17 \times 10^9) (98.8 \times 10^{-24}) (1.07 \times 10^{19}) = 2.29 \times 10^6 \text{ Bq}$$

5. (3 points) A narrow beam of 100 MeV neutrons, with a fluence of 10^5 n/cm²/sec, is normally incident on an aluminum [$^{27}_{13}\text{Al}$] plate. The elastic scattering cross section of aluminum for 100 MeV neutrons is 0.95 barns. The density of aluminum is 2.7 g/cm³. (a) How thick must the aluminum plate be in order to reduce the number of unscattered neutrons emerging from the plate by three orders of magnitude?

The equation to find the intensity of neutrons out of a shield of thickness x is:

$$I = I_0 e^{-\Sigma x} = I_0 e^{-N_T \sigma x}$$

Calculating for the macroscopic cross section and then solving for x:

$$\Sigma = N\sigma = \frac{6.02 \times 10^{23} \text{ atoms}}{27 \text{ g}} \cdot \frac{0.95 \times 10^{-24} \text{ cm}^2}{\text{atom}} = 0.02 \frac{\text{cm}^2}{\text{g}}$$

$$\frac{I}{I_0} = 0.001 = e^{-\Sigma x} = e^{-0.02 \cdot 2.7 \cdot x} \Rightarrow x = 127 \text{ cm}$$

(b) How much would this plate attenuate a narrow beam of 100 MeV **photons**?

From figure 8.8 $(\mu/\rho) = 0.025 \text{ cm}^2/\text{g}$, solving:

$$I = I_0 e^{-\left(\frac{\mu}{\rho}\right) \rho x} \Rightarrow \frac{I}{I_0} = e^{-0.025 \cdot 2.7 \cdot 127} = 1.89 \times 10^{-4}$$

We see that neutrons are more penetrating than photons.

6. (4 points) A 1 mCi source of ^{60}Co is placed in the center of a cylindrical water-filled tank with an inside diameter of 20 cm and a depth of 100 cm. The tank is made of iron with a wall thickness of 1 cm. What is the uncollided energy fluence rate at the outer surface of the tank nearest the source?

The energy fluence is defined as:

$$\dot{\Psi} = \frac{CE}{4\pi r^2}$$

where C is the activity of the source at that point. The activity will decrease as the gammas go through the water and the iron so we need to find out what intensity or activity makes it out of the shielding. Let average the energy of the ^{60}Co to simply things at 1.25 MeV. The attenuation coefficient of 1.25 MeV through water in Figure 8.9 is approximately 0.06 cm²/g and from Figure 8.8 is approximately 0.058 cm²/g. Solving for the intensity out of the water and then the iron:

$$I = I_0 \cdot e^{-\mu_{\text{H}_2\text{O}} x_{\text{H}_2\text{O}}} e^{-\mu_{\text{Fe}} x_{\text{Fe}}} = 1 \times 10^{-3} \cdot 3.7 \times 10^{10} \cdot e^{-0.06 \cdot 1 \cdot 10} e^{-0.058 \cdot 7.86 \cdot 1} = 1.3 \times 10^7 \text{ Bq}$$

That is the activity coming out of the cylinder. Calculating the uncollided energy fluence with the total radius of 11 cm:

$$\dot{\Psi} = \frac{CE}{4\pi r^2} = \frac{1.3 \times 10^7 \cdot 2 \cdot 1.25}{4\pi(11^2)} = 2.14 \times 10^4 \frac{\text{MeV}}{\text{cm}^2 \text{s}}$$

7. (7 points)

(a) The energy of the Compton scattered photon is given by 8.12:

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_e c^2} (1 - \cos \theta)} = \frac{1 \text{ MeV}}{1 + \frac{1 \text{ MeV}}{0.511 \text{ MeV}} (1 - \cos 55^\circ)} = 0.545 \text{ MeV}$$

(b) The change in wavelength is given by equation 8.13:

$$\Delta\lambda = \lambda' - \lambda = c \left(\frac{1}{\nu'} - \frac{1}{\nu} \right) = \frac{h}{mc} (1 - \cos \theta) = 2.43 \times 10^{-12} \text{ m} (1 - \cos 55^\circ) = 1.03 \times 10^{-12} \text{ m}$$

(c) The angle of recoil of the electron

A photon of energy $h\nu$ and momentum $h\nu/c$ is incident on a stationary, free electron. After the collision, the photon is scattered at an angle θ with energy $h\nu'$ and momentum $h\nu'/c$. The struck electron recoils at an angle ϕ with total energy E' and momentum P' .

Conservation of total energy in the collision requires that:

$$h\nu + mc^2 = h\nu' + E'$$

Conservation of components of momentum in the horizontal and vertical directions gives:

$$\frac{h\nu}{c} = \frac{h\nu'}{c} \cos \theta + P' \cos \phi$$

$$\frac{h\nu'}{c} \sin \theta = P' \sin \phi$$

From this, you can divide the two equations and get (equation 8.21):

$$\tan \phi = \frac{\sin \theta}{\left(1 + \frac{h\nu}{mc^2}\right)(1 - \cos \theta)}$$

When you solve for ϕ you find that $\phi = 33^\circ$

(d) (not graded) The recoil energy of the electron

The kinetic energy acquired by the secondary electron is given by:

$$T = h\nu - h\nu'$$

And therefore,

$$T = h\nu \frac{1 - \cos \theta}{\frac{mc^2}{h\nu} + 1 - \cos \theta} = 0.454 \text{ MeV}$$

8) (1 point) A 4-MeV photon creates an electron-positron pair in the field of a nucleus. What is the total kinetic energy of the pair?

Using equation 8.41:

$$h\nu = 2m_e c^2 + T_+ + T_- \Rightarrow T_+ + T_- = 4 - 1.02 = 2.98 \text{ MeV}$$

9) (4 points) An experiment is carried out with monoenergetic photons in “good” geometry. The relative count rate of the detector is measured with different thicknesses x of tin (Sn) used as absorber. The following data are measured:

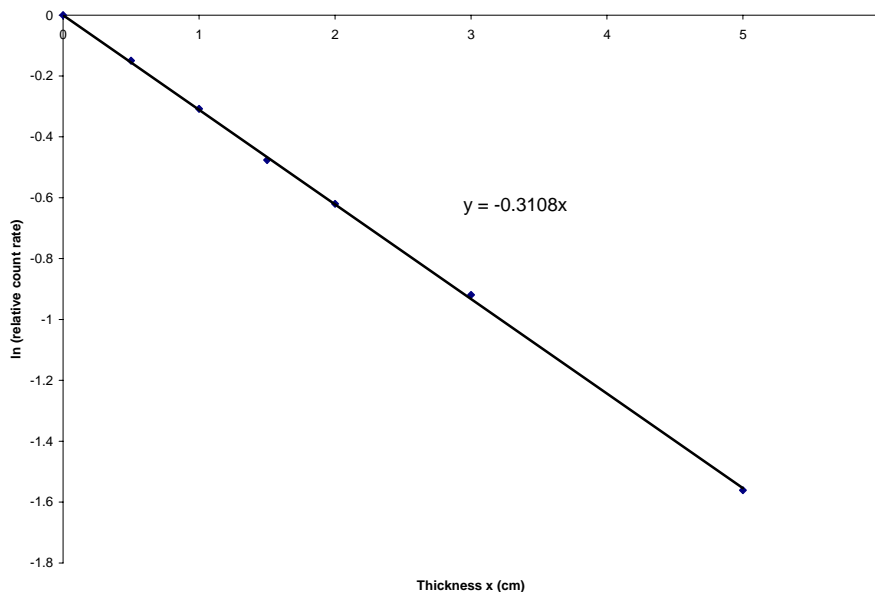
$x(\text{cm})$	0	0.50	1.0	1.5	2.0	3.0	5.0
Relative count rate	1.00	0.861	0.735	0.621	0.538	0.399	0.210

(a) What is the value of the linear attenuation coefficient?

Taking the interaction equation and solving for μ :

$$\ln\left(\frac{I}{I_0}\right) = -\mu x$$

This is a $y = mx$ equation where μ is the slope and the intercept is zero. We are given the ratio of intensities and the thickness. Plotting the data and the linear equation:



Therefore the linear attenuation coefficient is **0.311 cm⁻¹**.

(b) What is the value of the mass attenuation coefficient?

$$\left(\frac{\mu}{\rho}\right) = .311 \frac{1}{cm} \cdot \frac{cm^3}{7.3g} = 0.043 \frac{cm^2}{g}$$

(c) What is the atomic cross section?

$$\text{Atomic cross section} = 0.043 \frac{cm^2}{g} \cdot \frac{118.7g}{mol} \cdot \frac{1mol}{6.02 \times 10^{23} \text{ atoms}} = 8.48 \text{ barns}$$

(d) What is the photon energy?

Looking at figure 8.8 with a 0.043 cm²/g attenuation coefficient gives a photon energy of approximately **1.9 MeV**.

10) (2 points) A sample containing 62 grams of ³¹P (100% abundant) is exposed to 2 x 10¹¹ thermal neutrons cm⁻² sec⁻¹. If the thermal neutron absorption cross section is 0.19 barns, for how long must the sample be irradiated to produce 3.7 x 10¹⁰ Bq of ³²P?

Using Equation 9.36 for neutron activation and solving for t:

$$\lambda N = \phi \sigma N_T (1 - e^{-\lambda t})$$

$$t = \frac{-\ln\left(1 - \frac{\lambda N}{\phi \sigma N_T}\right)}{\lambda} = \frac{-\ln\left(1 - \frac{3.7 \times 10^{10} \text{ s}^{-1}}{(2 \times 10^{11} \text{ n/cm}^2 \cdot \text{s}) (0.19 \times 10^{-24} \text{ cm}^2) \left(\frac{62 \text{ g}}{31 \text{ g/mole}} \cdot 6.02 \times 10^{23} \text{ mole}^{-1}\right)}\right)}{\ln 2 / (14.28 \text{ d} \times \frac{24 \text{ hour}}{\text{day}})}$$

$$t = 817.8 \text{ hours}$$