Radiation Interactions with Matter: Energy Deposition

Biological effects are the end product of a long series of phenomena, set in motion by the passage of radiation through the medium.

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Chart: Chronology of events.

Interactions of Heavy Charged Particles

Energy-Loss Mechanisms

- The basic mechanism for the slowing down of a moving charged particle is **Coulombic interactions** between the particle and electrons in the medium. This is common to all charged particles

- A heavy charged particle traversing matter loses energy primarily through the **ionization** and **excitation** of atoms.

- The moving charged particle exerts **electromagnetic forces** on atomic electrons and imparts energy to them. The energy transferred may be sufficient to knock an electron out of an atom and thus **ionize** it, or it may leave the atom in an **excited, nonionized state**.

- A heavy charged particle can transfer only a **small fraction** of its energy in a single electronic collision. Its **deflection in the collision is negligible**.

- All heavy charged particles travel essentially **straight paths** in matter.

[Tubiana, 1990]

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Chart: Binding energy of electrons (eV).

W is the energy required to cause an ionization

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Maximum Energy Transfer in a Single Collision

The maximum energy transfer occurs if the collision is head-on.

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Fig. 5.2 in Turner J. E. *Atoms, Radiation, and Radiation Protection*, 2nd ed. New York, NY: Wiley-Interscience, 1995.

Assumptions:

- The particle moves rapidly compared with the electron.
- For maximum energy transfer, the collision is head-on.
- The energy transferred is large compared with the binding energy of the electron in the atom.
- Under these conditions the electron is considered to be initially free and at rest, and the collision is elastic.

Conservation of kinetic energy:

$$\frac{1}{2}MV^2 = \frac{1}{2}MV_1^2 + \frac{1}{2}mv_1^2$$

Conservation of momentum:

$$MV = MV_1 + mv_1.$$

$$Q_{max} = \frac{1}{2}MV^2 - \frac{1}{2}MV_1^2 = \frac{4mME}{(M+m)^2},$$

Where $E = MV^2/2$ is the initial kinetic energy of the incident particle.

Q_{max} values for a range of proton energies.

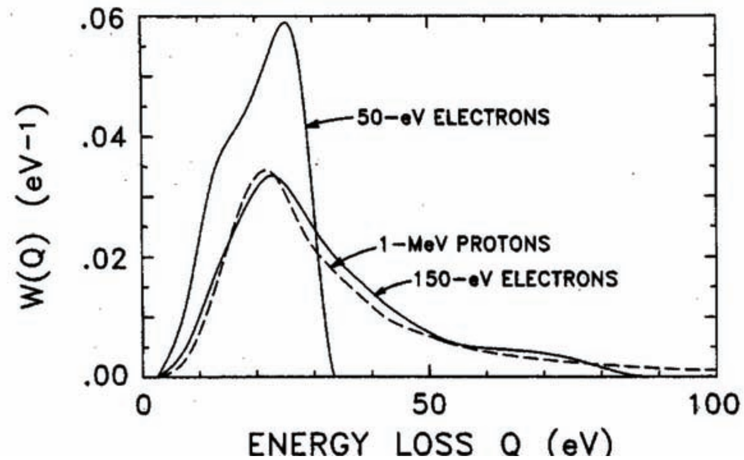
Except at extreme relativistic energies, the maximum fractional energy loss for a heavy charged particle is small.

Maximum Possible Energy Transfer, Q_{max}, in Proton Collision with Electron

Proton Kinetic Energy E (MeV)	Q_{max} (MeV)	Maximum Percentage Energy Transfer $100Q_{max}/E$
0.1	0.00022	0.22
1	0.0022	0.22
10	0.0219	0.22
100	0.229	0.23
10^3	3.33	0.33
10^4	136	1.4
10^5	1.06×10^4	10.6
10^6	5.38×10^5	53.8
10^7	9.21×10^6	92.1

$$Q_{\max} = \frac{4mME}{(M+m)^2}$$

Single Collision Energy Loss Spectra



Single-collision energy-loss spectra for 50-eV and 150-eV electrons and 1-MeV protons in liquid water. (Courtesy of Oak Ridge National Laboratory.)

- The y axis represents the **calculated** probability that a given collision will result in an energy loss Q.
- *N.B.*, the maximum energy loss calculated above for the 1 MeV proton, of 21.8 keV is off the scale.
- The most probable energy loss is on the order of 20 eV.
- *N.B.*, energy loss spectra for fast charged particles are very similar in the range of 10 – 70 eV.
- Energy loss spectra for slow charged particles differ, the most probable energy loss is closer to the Q_{max}.

Stopping Power

- The average **linear rate of energy loss** of a heavy charged particle in a medium (MeV cm^{-1}) is of fundamental importance in radiation physics, dosimetry and radiation biology.

- This quantity, designated $-dE/dx$, is called the **stopping power** of the medium for the particle.

- It is also referred to as the **linear energy transfer** (LET) of the particle, usually expressed as keV μm^{-1} in water.

- **Stopping power** and **LET** are closely associated with the dose and with the **biological effectiveness** of different kinds of radiation.

Stopping powers can be estimated from energy loss spectra.

- The "**macroscopic cross section**", μ, is the probability per unit distance of travel that an electronic collision takes place.

- The reciprocal of μ is the mean distance of travel or the **mean free path**, of a charged particle **between collisions**.

- **Stopping power** is the product of the macroscopic cross section and the average energy lost per collision.

$$-\frac{dE}{dx} = \mu Q_{ave}$$

Example:
The macroscopic cross section for a 1-MeV proton in water is 410 μm^{-1}, and the average energy lost per collision is 72 eV. What are the stopping power and the mean free path?

The stopping power, $-\frac{dE}{dx} = \mu Q_{ave}$

$$= 410\ \mu m^{-1} \times 72\ eV = 2.95 \times 10^4\ eV\ \mu m^{-1}$$

The mean free path of the 1-MeV proton is $1/\mu = 1(410\ \mu m^{-1}) = 0.0024\ \mu m = 24$ Å. [Atomic diameters are of the order of 1 Å to 2 Å.]

Calculations of Stopping Power

In 1913, Niels Bohr derived an explicit formula for the stopping power of heavy charged particles.

Bohr calculated the energy loss of a heavy charged particle in a collision with an electron, then averaged over all possible distances and energies.

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Fig. 5.4 in [Turner].

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Fig. 5.5 in [Turner].

The Bethe Formula for Stopping Power.

Using relativistic quantum mechanics, Bethe derived the following expression for the stopping power of a uniform medium for a heavy charged particle:

$$-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mc^2\beta^2}\left[\ln\frac{2mc^2\beta^2}{I(1-\beta^2)} - \beta^2\right].$$

$k_o = 8.99 \times 10^9$ N m^2 C^{-2} , (the Boltzman constant)
z = atomic number of the heavy particle,
e = magnitude of the electron charge,
n = number of electrons per unit volume in the medium,
m = electron rest mass,
c = speed of light in vacuum,
$\beta = V/c$ = speed of the particle relative to c,
I = mean excitation energy of the medium.

- Only the **charge** ze and **velocity** V of the heavy charged particle enter the expression for stopping power.

- For the medium, only the **electron density** n is important.

Tables for Computation of Stopping Powers

If the constants in the Bethe equation for stopping power, dE/dX, are combined, the equation reduces to the following form:

$$-\frac{dE}{dx} = \frac{5.08\,x\,10^{-31} z^2 n}{\beta^2}[F(\beta) - \ln I_{ev}] \quad \text{MeV cm}^{-1}$$

where, $F(\beta) = \ln \frac{1.02\,x\,10^6 \beta^2}{1-\beta^2} - \beta^2$

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Table 5.2 in [Turner].

Conveniently,…..

For a given value of β, the kinetic energy of a particle is proportional to the rest mass,

Table 5.2 can also be used for other heavy particles.

Example:
The ratio of kinetic energies of a deuteron and a proton **traveling at the same speed** is

$$\frac{\frac{1}{2}M_d V^2}{\frac{1}{2}M_p V^2} = \frac{M_d}{M_p} = 2$$

Therefore the value of F(β) of 9.972 for a 10 MeV proton, also applies to a 20 MeV deuteron.

Mean Excitation Energies

Mean excitation energies, *I*, have been calculated using the quantum mechanical approach or measured in experiments. The following approximate empirical formulas can be used to estimate the I value in eV for an element with atomic number Z:

$I \approx 19.0$ eV; $Z = 1$ (hydrogen)

$I \approx 11.2$ eV + (11.7)(Z) eV; $2 \leq Z \leq 13$

$I \approx 52.8$ eV + (8.71)(Z) eV; $Z > 13$

For compounds or mixtures, the contributions from the individual components must be added.

In this way a composite lnI value can be obtained that is weighted by the electron densities of the various elements.

The following example is for water (and is probably **sufficient for tissue**).

$$n \ln I = \sum_i N_i Z_i \ln I_i,$$

Where n is the total number of electrons in the material ($n = \Sigma_i N_i Z_i$)

When the material is a pure compound, the electron densities can be replaced by the electron numbers in a single molecule.

Example:
Calculate the mean excitation energy of H_2O

Solution:
I values are obtained from the empirical relations above.
For H, $I_H = 19.0$ eV, for O, $I_O = 11.2 + 11.7 \times 8 = 105$ eV.
Only the ratios, $N_i Z_i / n$ are needed to calculate the composite I.
Since H_2O has 10 electrons, 2 from H and 8 from O, the equation becomes

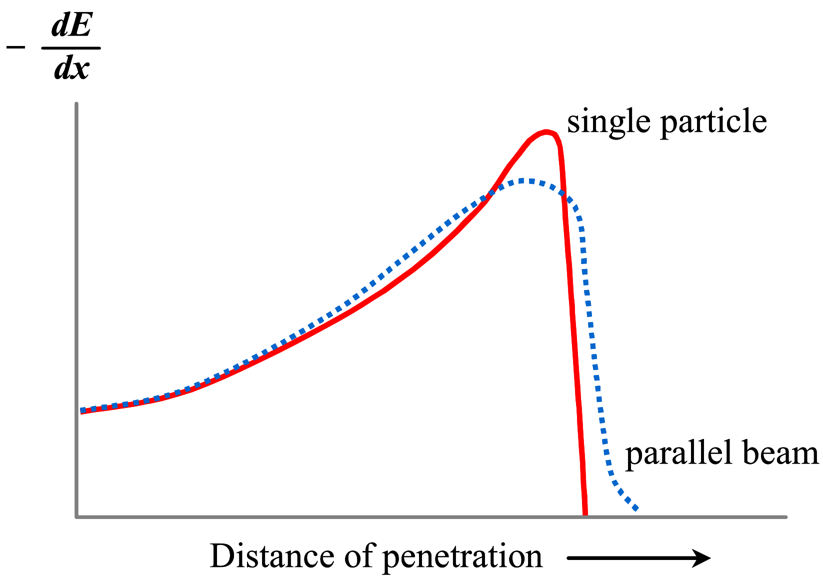
$$\ln I = \frac{2x1}{10}\ln 19.0 + \frac{1x8}{10}\ln 105 = 4.312 \qquad \text{giving } I = 74.6 \text{ eV}$$

Stopping power versus distance: the Bragg Peak

$$-\frac{dE}{dx} = \frac{5.08 x 10^{-31} z^2 n}{\beta^2}[F(\beta) - \ln I_{ev}] \quad \text{MeV cm}^{-1}$$

- At low energies, the factor in front of the bracket increases as $\beta \rightarrow 0$, causing a peak (called the Bragg peak) to occur.

- The linear rate of energy loss is a maximum as the particle energy approaches 0.



Rate of energy loss along an alpha particle track.

- The peak in energy loss at low energies is exemplified in the Figure, above, which plots -dE/dx of an alpha particle as a function of distance in a material.

- For most of the alpha particle track, the charge on the alpha is two electron charges, and the rate of energy loss increases roughly as 1/E as predicted by the equation for stopping power.

- Near the end of the track, the charge is reduced through electron pickup and the curve falls off.

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Fig. 24.6 in Hall, Eric J. *Radiobiology for the Radiologist*, 5th ed. Philadephia PA: Lippincott Williams & Wilkins, 2000.

Stopping Power of Water for Protons

$$-\frac{dE}{dx}=\frac{5.08x10^{-31}z^2n}{\beta^2}[F(\beta)-\ln I_{ev}] \quad \text{MeV cm}^{-1}$$

What is needed to calculate **stopping power**, - dE/dX?

n	the electron density
z	the atomic number
lnI	the mean excitation energy

For protons, z = 1,

The gram molecular weight of water is 18.0 g/mole and the number of electrons per molecule is 10.
One m^3 of water has a mass of 10^6 g.

The density of electrons, n, is:

$$n = 6.02 \times 10^{23} \text{ molecules/mole} \times \frac{10^6\, g\, m^{-3}}{18.0\, g/mole} \times 10\ e^-/\text{molecule} =$$

$$3.34 \times 10^{29} \text{ electrons/m}^3$$

As found above, for water, ln I_{ev} = 4.312. Therefore, eq (1) gives

$$-\frac{dE}{dx}=\frac{0.170}{\beta^2}[F(\beta)-4.31] \quad \text{MeV cm}^{-1}$$

At 1 MeV, from Table 5.2, β^2 = 0.00213 and F(β) = 7.69, therefore,

$$-\frac{dE}{dx}=\frac{0.170}{0.00213}[7.69-4.31] \ = \ 270 \text{ MeV cm}^{-1}$$

The stopping power of water for a 1 MeV proton is 270 MeV cm^{-1}

Mass Stopping Power

- The **mass stopping power** of a material is obtained by dividing the stopping power by the density ρ.
- Common units for mass stopping power, -dE/ρdx, are MeV cm^2 g^{-1}.
- The mass stopping power is a useful quantity because it expresses the rate of energy loss of the charged particle per g cm^{-2} of the medium traversed.
- In a gas, for example, -d*E*/dx depends on pressure, but –d*E*/ρdx does not, because dividing by the density exactly compensates for the pressure.
- Mass stopping power does not differ greatly for materials with similar atomic composition.
- Mass stopping powers for water can be scaled by density and used for tissue, plastics, hydrocarbons, and other materials that consist primarily of light elements.

For Pb (Z=82), on the other hand, -d*E*/ρdx = 17.5 MeV cm^2g^{-1} for 10-MeV protons. (water ~ 47 MeV cm^2 g^{-1} for 10 MeV protons)

**Generally, heavy atoms are less efficient on a g cm^{-2} basis for slowing down heavy charged particles, because many of their electrons are too tightly bound in the inner shells to participate effectively in the absorption of energy.

Range

The **range** of a charged particle is the distance it travels before coming to rest.

The range is **NOT** equal to the energy divided by the stopping power.

Table 5.3 gives the mass stopping power and range of protons in water. The range is expressed in g cm^{-2}; that is, the range in cm multiplied by the density of water (ρ = 1 g cm^{-3}).

Like mass stopping power, the range in g cm^{-2} applies to all materials of similar atomic composition.

A useful relationship…..

For two heavy charged particles ***at the same initial speed β***, the ratio of their ranges is simply

$$\frac{R_1(\beta)}{R_2(\beta)} = \frac{z_2^2 M_1}{z_1^2 M_2},$$

where:

R_1 and R_2 are the ranges
M_1 and M_2 are the rest masses and
z_1 and z_2 are the charges

If particle number 2 is a proton (M_2 = 1 and z_2 = 1), then the range R of the other particle is given by:

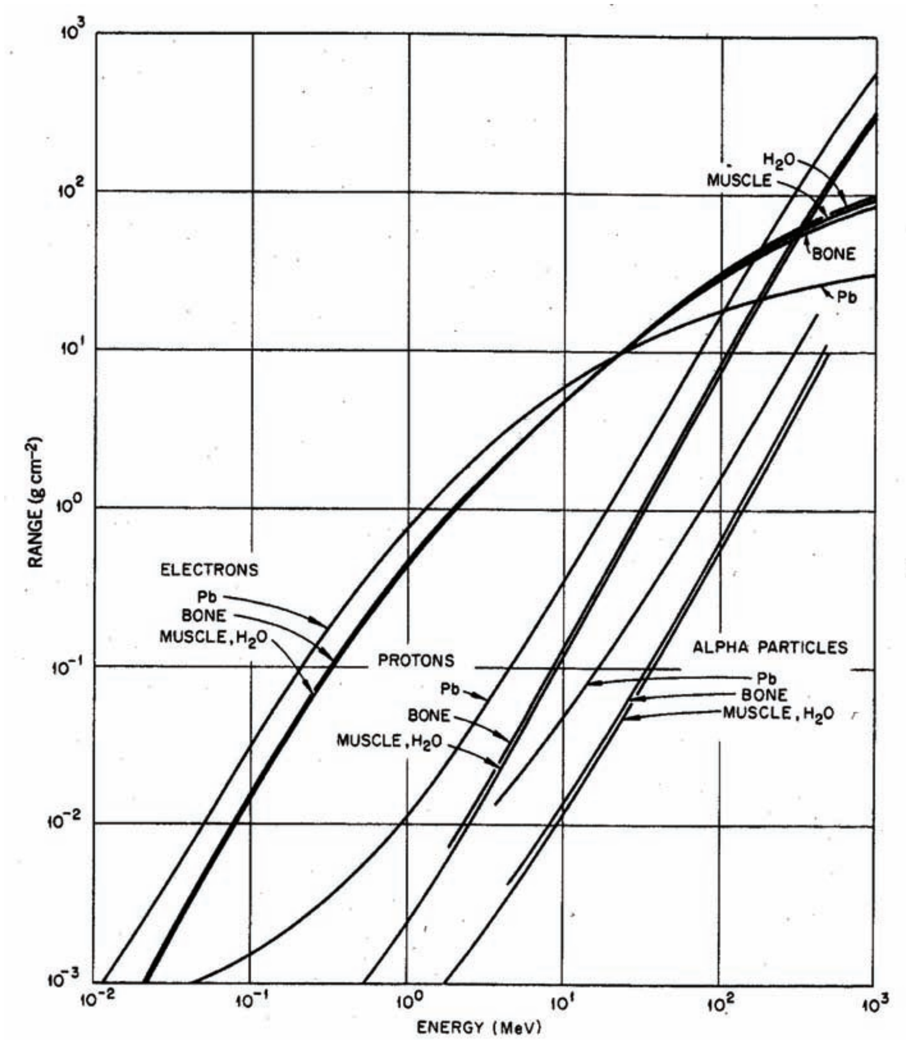
$$R(\beta) = \frac{M}{z^2} R_p(\beta),$$

where $R_p(\beta)$ is the proton range.

Image removed due to copyright restrictions.
Table 5.3 in [Turner].

Figure 5.7 shows the ranges in g cm^{-2} of protons, alpha particles, and electrons in water or muscle (virtually the same), bone, and lead.

For a given proton energy, the range in g cm^{-2} is greater in Pb than in H_2O, consistent with the smaller mass stopping power of Pb.



Ranges of protons, alpha particles, and electrons in water, muscle, bone, and lead, expressed in g cm^{-2}. (Courtesy of Oak Ridge National Laboratory. Used with permission.)