Lecture 2: The nucleus and nuclear instability

Nuclei are described using the following nomenclature:

$$ {}^{A}_{Z}Element_{N} $$

Z is the atomic number, the number of protons: this defines the element.

A is called the "mass number" **A** = **N** + **Z**.

N is the number of neutrons (**N = A - Z**)

Nuclide: A species of nucleus of a given **Z** and **A.**

Isotope: Nuclides of an element (i.e. same **Z**) with different **N.**

Isotone: Nuclides having the same **N.**

Isobar: Nuclides having the same **A.**

[A handy way to keep these straight is to note that isoto<u>p</u>e includes the letter "p" (same <u>p</u>roton number), isoto<u>n</u>e the letter "n" (same <u>n</u>eutron number), and isob<u>a</u>r the letter "a" (same **<u>A</u>**).]

Example:

$$ {}^{206}_{82}Pb_{124} $$
$$ is\ \ an\ \ isotope\ of $$
$$ {}^{208}_{82}Pb_{126} $$
$$ is\ an\ isotone\ of $$
$$ {}^{207}_{83}Bi_{124} $$
$$ and\ an\ isobar\ of $$
$$ {}^{206}_{84}Po_{122} $$

Chart of the Nuclides

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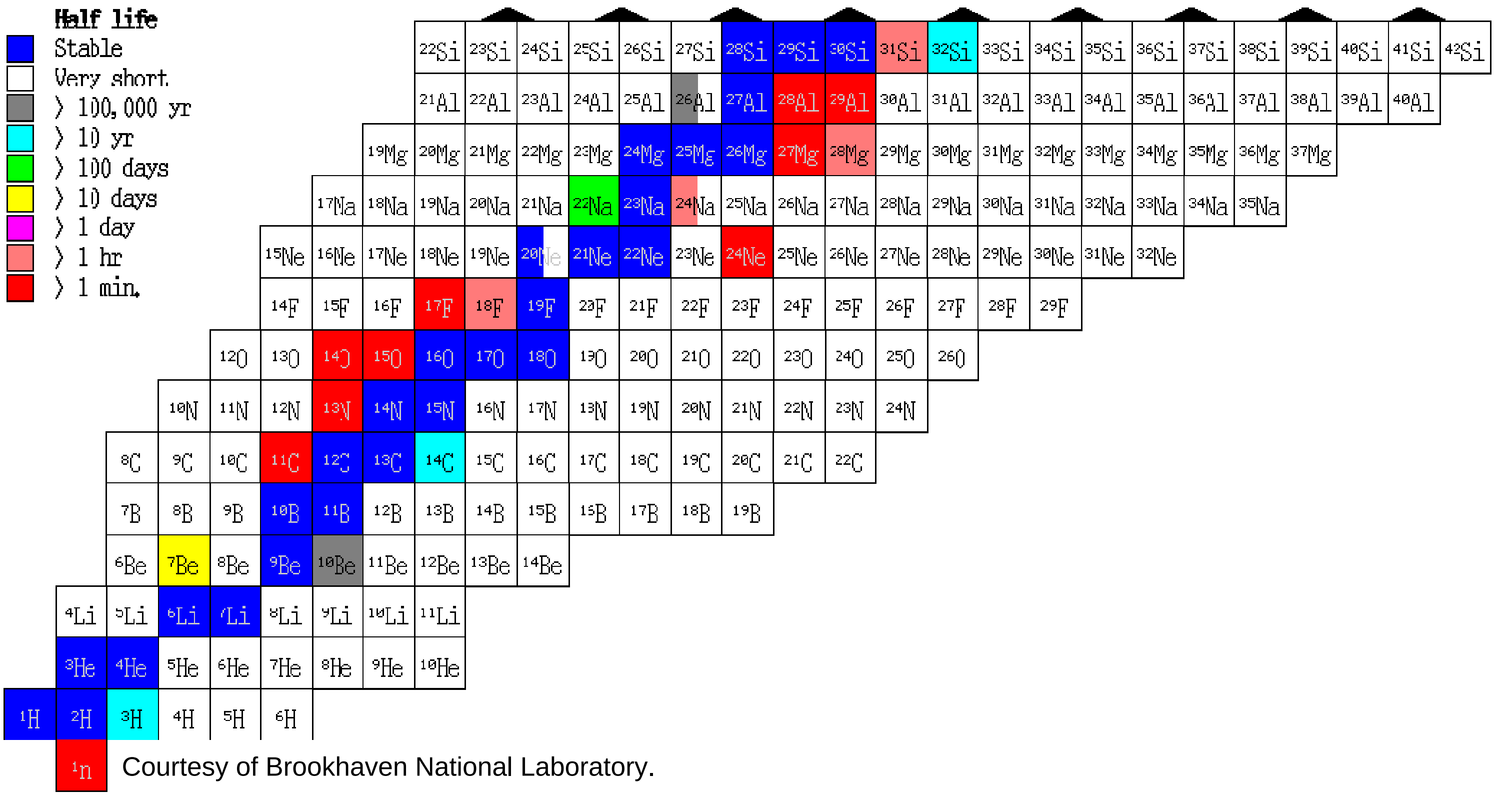
90 natural elements

109 total elements

All elements with Z > 42 are man-made
Except for technicium Z=43
Promethium Z = 61

More than 800 nuclides are known (274 are stable)
"stable" unable to transform into another configuration without the addition of outside energy.
"unstable" = radioactive



Courtesy of Brookhaven National Laboratory.

[www2.bnl.gov/ton]

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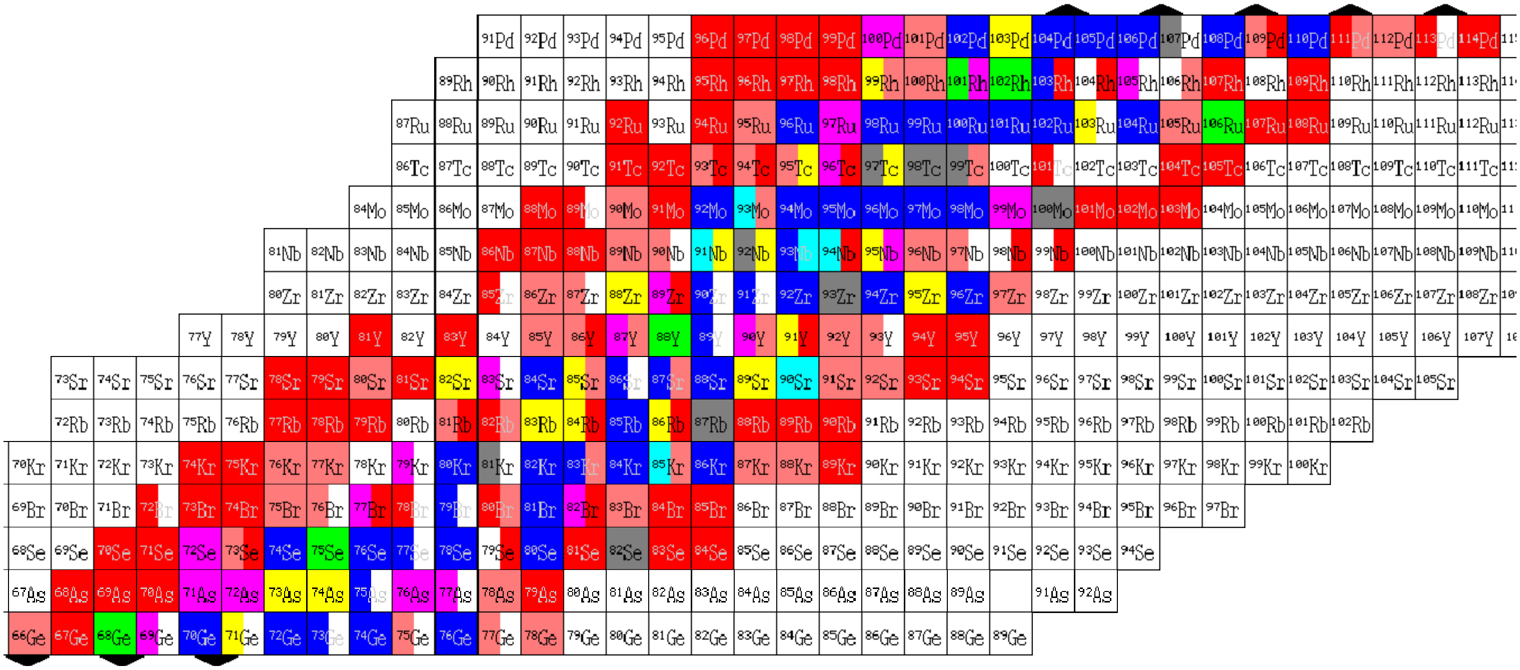

Courtesy of Brookhaven National Laboratory.

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Nuclear Structure: Forces in the nucleus

Coulomb Force

Force between two point charges, q, separated by distance, r (Coulomb's Law)

$$F(N) = \frac{k_0 q_1 q_2}{r^2}$$

k_0 = 8.98755 x 10^9 N m^2 C^{-2} (Boltzman constant)

Potential energy (MeV) of one particle relative to the other

$$PE(MeV) = \frac{k_0 q_1 q_2}{r}$$

Strong Nuclear Force

- **Acts over short distances**
- **~ 10^{-15} m**
- **can overcome Coulomb repulsion**
- **acts on protons and neutrons**

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Fig 3.1 in Turner J. E. *Atoms, Radiation, and Radiation Protection*, 2nd ed. New York: Wiley-Interscience, 1995.

Summary of Nuclear Forces:
Nuclei give off energy (i.e., radiation) in an attempt to become more stable

Nuclear instability can be traced to the interaction of i) Coulomb and ii) strong nuclear force.

Coulomb	**Strong Nuclear**
repulsive	attractive
$p^+ - p^+$	$p^+ - p^+$, $n - n$, $p^+ - n$
doesn't saturate	short range; falls off quickly
weak (eg. e^- to nucleus, ~ few eV to .1 MeV)	very strong (several decades of MeV)
atom is mostly empty space	nucleus is densely packed

Due to the Coulomb-nuclear force balance, nuclei exhibit a roughly constant density and radius.

Energy-Mass Equivalence

Atomic Mass Units (amu, or AMU)

By definition: Atomic masses are measured on a scale in which a $^{12}C_6$ atom is exactly 12 atomic mass units.

Gram atomic weight of any element contains N_0 atoms (N_0 = Avogadro's number).

12 grams of carbon = 6.02×10^{23} carbon atoms

$$\frac{12\, g\, carbon}{6.02\, x 10^{23}\ atoms} = \frac{1.99\, x 10^{-23}\, g}{carbon\ atom} = 12\ \ AMU$$

$$1\ amu\ = \frac{1.99\, X 10^{-23}\ g}{12} = 1.66\, x 10^{-24}\ g\ = 1.66\, x 10^{-27}\ kg$$

Using Einstein's mass-energy equivalence formula: $E=m_oc^2$,

1.660531×10^{-27} kg x $(3.0 \times 10^{8}\ m/s)^2 = 1.49448 \times 10^{-10}\ kg\ m^2/s^2 = 1.49448 \times 10^{-10}$ Joule

Given: 1.6022×10^{-19} Joules = 1 eV

1 AMU is equivalent to 931.48 MeV

Rest mass energies and mass equivalences:

electron mass:	0.000549 amu	= 0.511 MeV
proton mass:	1.007277 amu	= 938.28 MeV
neutron mass:	1.008665 amu	= 939.57 MeV
hydrogen atom:	1.007825 amu	

Mass Differences, Δ

The mass of a nuclide is LESS than the sum of its parts…

- Energy released when all constituents come together.
- Nuclear force so strong that the mass of the bound system is smaller than the sum of the components.

$\Delta = M - A$, or $M = \Delta + A$

M is the true atomic mass
A is the atomic number

Image removed due to copyright restrictions.
Appendix D in [Turner]

Nuclear reactions release energy

$$ {}^{1}_{0}n + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + {}^{0}_{0}\gamma $$

How much energy is released?

Compare the total masses on both sides of the arrow.

Nuclear Binding Energies

The *difference in mass* between a given nucleus and the sum of the same number of individual protons and neutrons is the *binding energy*.

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Fig 3.3 in [Turner]

Nuclear Stability/Instability

- **Strong nuclear force, operates over short range**
- **"saturates" quickly**
- **neutrons interact only with neighbors**
- **protons interact (repulse via Coulomb interaction) throughout the entire nucleus.**

In heavier nuclei, the #neutrons must increase faster than the number of protons to maintain stability.

- **N/Z ratio = 1 at low A**
- **e.g., Mg Z=12, but N=12,13 or 14 (isotopes)**
- **N/Z ratio approaches 1.5 when Z~80**

"Line of stability" Z = N

Any nucleus far from the "line of stability" will be unstable.

The position of a nucleus relative to the line of stability will define the mode of nuclear instability (radioactive decay mode).

Radioactive decay tends towards the line of stability

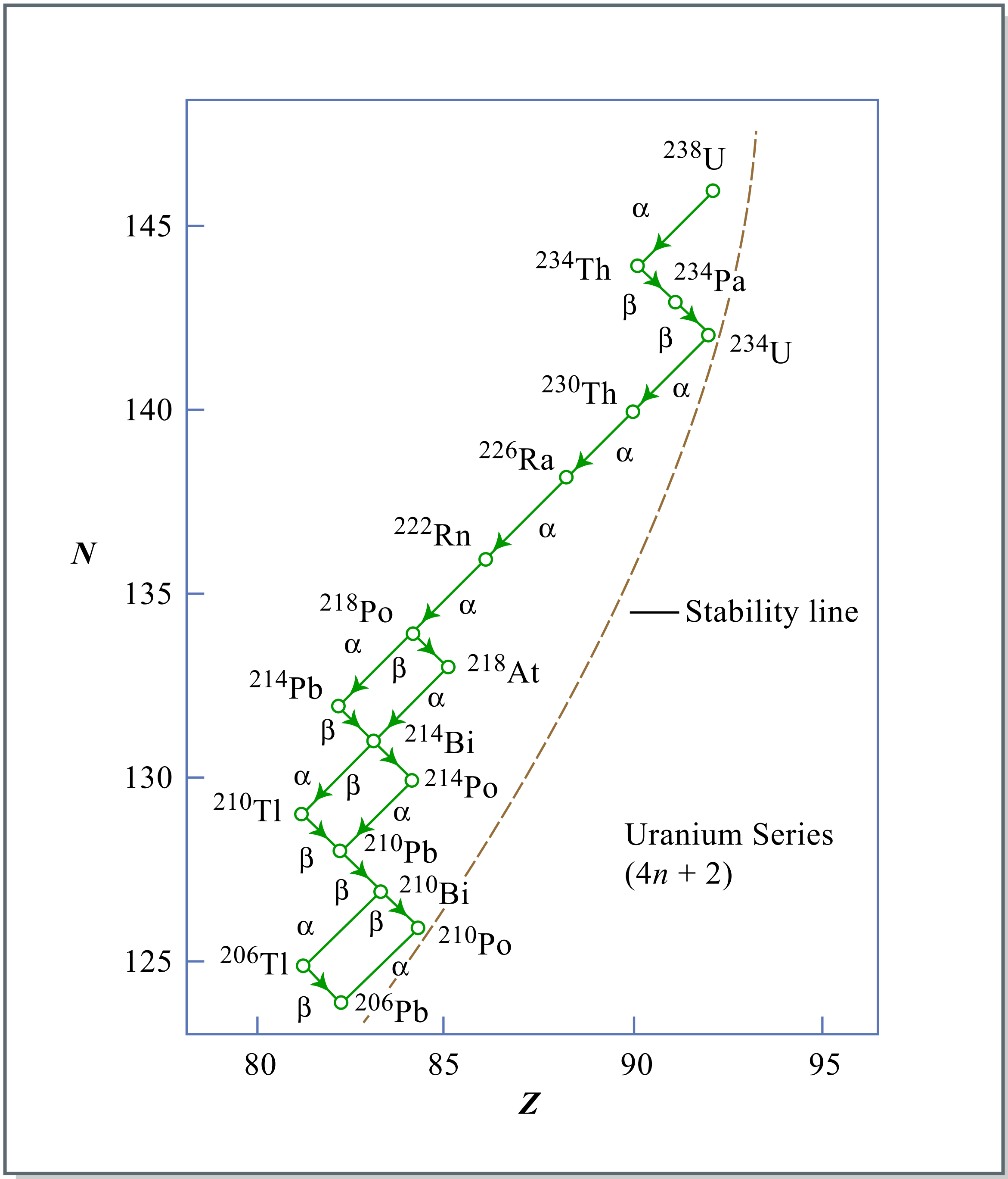

Figure by MIT OCW.

Alpha decay

Natural alpha emitters: Z > 83

$$ {}^{A}_{Z}P \rightarrow {}^{A-4}_{Z-2}D + {}^{4}_{2}He + Q $$

- conservation of electric charges
- conservation of nucleons

E.g., $$ {}^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He $$

How much energy, Q, is released?

Compare the masses on both sides of the arrow.

$Q = M_{Ra} - M_{Rn} - M_{He}$

Use Δ values in Turner Appendix D.

$Q = \Delta_{Ra} - \Delta_{Rn} - \Delta_{He}$

$Q = 23.69 - 16.39 - 2.42 = 4.88$ MeV

How is this energy, Q, distributed?

Shared by the daughters, the Rn nucleus and the alpha particle.

- Momentum is conserved: mv = MV
- Kinetic energy of the 2 products = Q

$$\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = Q$$

The energy of the alpha particle:

$$E_\alpha = \frac{1}{2}mv^2 = \frac{MQ}{m+M}$$

The energy of the Rn nucleus:

$$E_N = \frac{1}{2}MV^2 = \frac{mQ}{m+M}$$

Alpha decay results in a ***2-particle emission***.
Q is fixed by the mass balance
E_α is fixed by the conservation laws (energy, momentum)

Therefore, ***alpha particles must have discrete energies.***

Nuclear Decay Scheme Diagrams

Graphical display of *nuclear* transformations

- **Decay mode**
- **Energy transitions**
- **Abundances (branching ratios)**

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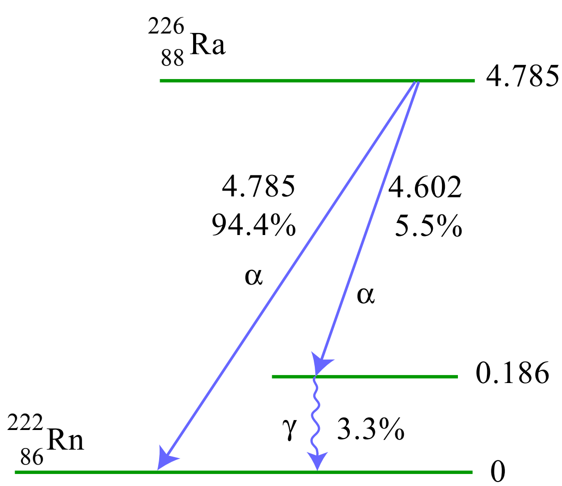
Fig. 3.4 in [Turner]

Conventions:

- Arrows slanting to the left indicate ***decrease in Z***
- Arrows slanting to the right indicate an ***increase in Z***
- Wavy lines going straight down indicate a ***gamma emission*** from the nucleus.



Nuclear Decay Scheme of $^{226}_{88}$Ra.

Figure by MIT OCW.

Decay Scheme Exercise

$$^{219}_{86}Rn \rightarrow {}^{215}_{82}Po + {}^{4}_{2}He$$

α	**6.82 (80%)**
	6.55 (12%)
	6.42 (7%)
γ	**0.271 (10%)**
	0.402 (7%)