

## Lecture 2: The nucleus and nuclear instability

Nuclei are described using the following nomenclature:



**Z** is the atomic number, the number of protons: this defines the element.

**A** is called the “mass number”  $A = N + Z$ .

**N** is the number of neutrons ( $N = A - Z$ )

**Nuclide:** A species of nucleus of a given **Z** and **A**.

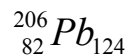
**Isotope:** Nuclides of an element (i.e. same **Z**) with different **N**.

**Isotone:** Nuclides having the same **N**.

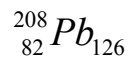
**Isobar:** Nuclides having the same **A**.

[A handy way to keep these straight is to note that isotope includes the letter “p” (same proton number), isotone the letter “n” (same neutron number), and isobar the letter “a” (same A).]

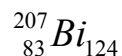
Example:



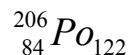
*is an isotope of*



*is an isotone of*



*and an isobar of*



## **Chart of the Nuclides**

Image removed due to copyright restrictions.

**90 natural elements**

**109 total elements**

**All elements with  $Z > 42$  are man-made**

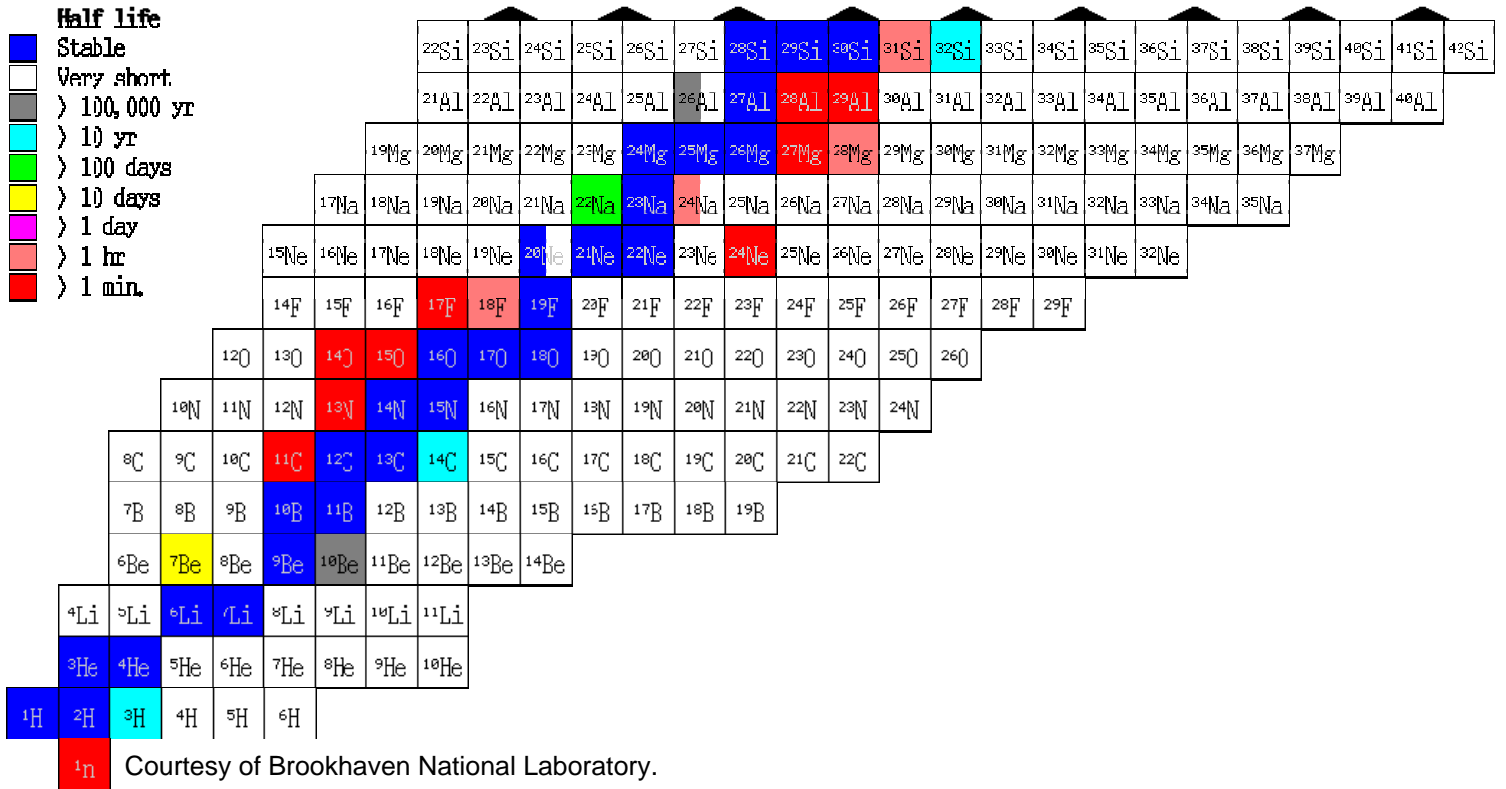
**Except for technetium  $Z=43$**

**Promethium  $Z = 61$**

**More than 800 nuclides are known (274 are stable)**

**“stable” unable to transform into another configuration  
without the addition of outside energy.**

**“unstable” = radioactive**



[[www2.bnl.gov/ton](http://www2.bnl.gov/ton)]

Image removed due to copyright restrictions.

																		91Pd	92Pd	93Pd	94Pd	95Pd	96Pd	97Pd	98Pd	99Pd	100Pd	101Pd	102Pd	103Pd	104Pd	105Pd	106Pd	107Pd	108Pd	109Pd	110Pd	111Pd	112Pd	113Pd	114Pd	115Pd			
																89Rh	90Rh	91Rh	92Rh	93Rh	94Rh	95Rh	96Rh	97Rh	98Rh	99Rh	100Rh	101Rh	102Rh	103Rh	104Rh	105Rh	106Rh	107Rh	108Rh	109Rh	110Rh	111Rh	112Rh	113Rh	114Rh	115Rh			
														87Ru	88Ru	89Ru	90Ru	91Ru	92Ru	93Ru	94Ru	95Ru	96Ru	97Ru	98Ru	99Ru	100Ru	101Ru	102Ru	103Ru	104Ru	105Ru	106Ru	107Ru	108Ru	109Ru	110Ru	111Ru	112Ru	113Ru	114Ru	115Ru			
												86Tc	87Tc	88Tc	89Tc	90Tc	91Tc	92Tc	93Tc	94Tc	95Tc	96Tc	97Tc	98Tc	99Tc	100Tc	101Tc	102Tc	103Tc	104Tc	105Tc	106Tc	107Tc	108Tc	109Tc	110Tc	111Tc	112Tc	113Tc	114Tc	115Tc				
								84Mo	85Mo	86Mo	87Mo	88Mo	89Mo	90Mo	91Mo	92Mo	93Mo	94Mo	95Mo	96Mo	97Mo	98Mo	99Mo	100Mo	101Mo	102Mo	103Mo	104Mo	105Mo	106Mo	107Mo	108Mo	109Mo	110Mo	111Mo	112Mo	113Mo	114Mo	115Mo						
						81Nb	82Nb	83Nb	84Nb	85Nb	86Nb	87Nb	88Nb	89Nb	90Nb	91Nb	92Nb	93Nb	94Nb	95Nb	96Nb	97Nb	98Nb	99Nb	100Nb	101Nb	102Nb	103Nb	104Nb	105Nb	106Nb	107Nb	108Nb	109Nb	110Nb	111Nb	112Nb	113Nb	114Nb	115Nb					
				80Zr	81Zr	82Zr	83Zr	84Zr	85Zr	86Zr	87Zr	88Zr	89Zr	90Zr	91Zr	92Zr	93Zr	94Zr	95Zr	96Zr	97Zr	98Zr	99Zr	100Zr	101Zr	102Zr	103Zr	104Zr	105Zr	106Zr	107Zr	108Zr	109Zr	110Zr	111Zr	112Zr	113Zr	114Zr	115Zr						
		77Y	78Y	79Y	80Y	81Y	82Y	83Y	84Y	85Y	86Y	87Y	88Y	89Y	90Y	91Y	92Y	93Y	94Y	95Y	96Y	97Y	98Y	99Y	100Y	101Y	102Y	103Y	104Y	105Y	106Y	107Y	108Y	109Y	110Y	111Y	112Y	113Y	114Y	115Y					
		73Sr	74Sr	75Sr	76Sr	77Sr	78Sr	79Sr	80Sr	81Sr	82Sr	83Sr	84Sr	85Sr	86Sr	87Sr	88Sr	89Sr	90Sr	91Sr	92Sr	93Sr	94Sr	95Sr	96Sr	97Sr	98Sr	99Sr	100Sr	101Sr	102Sr	103Sr	104Sr	105Sr	106Sr	107Sr	108Sr	109Sr	110Sr	111Sr	112Sr	113Sr	114Sr	115Sr	
		72Rb	73Rb	74Rb	75Rb	76Rb	77Rb	78Rb	79Rb	80Rb	81Rb	82Rb	83Rb	84Rb	85Rb	86Rb	87Rb	88Rb	89Rb	90Rb	91Rb	92Rb	93Rb	94Rb	95Rb	96Rb	97Rb	98Rb	99Rb	100Rb	101Rb	102Rb	103Rb	104Rb	105Rb	106Rb	107Rb	108Rb	109Rb	110Rb	111Rb	112Rb	113Rb	114Rb	115Rb
70Kr	71Kr	72Kr	73Kr	74Kr	75Kr	76Kr	77Kr	78Kr	79Kr	80Kr	81Kr	82Kr	83Kr	84Kr	85Kr	86Kr	87Kr	88Kr	89Kr	90Kr	91Kr	92Kr	93Kr	94Kr	95Kr	96Kr	97Kr	98Kr	99Kr	100Kr															
69Br	70Br	71Br	72Br	73Br	74Br	75Br	76Br	77Br	78Br	79Br	80Br	81Br	82Br	83Br	84Br	85Br	86Br	87Br	88Br	89Br	90Br	91Br	92Br	93Br	94Br	95Br	96Br	97Br																	
68Se	69Se	70Se	71Se	72Se	73Se	74Se	75Se	76Se	77Se	78Se	79Se	80Se	81Se	82Se	83Se	84Se	85Se	86Se	87Se	88Se	89Se	90Se	91Se	92Se	93Se	94Se																			
67As	68As	69As	70As	71As	72As	73As	74As	75As	76As	77As	78As	79As	80As	81As	82As	83As	84As	85As	86As	87As	88As	89As	90As	91As	92As																				
66Ge	67Ge	68Ge	69Ge	70Ge	71Ge	72Ge	73Ge	74Ge	75Ge	76Ge	77Ge	78Ge	79Ge	80Ge	81Ge	82Ge	83Ge	84Ge	85Ge	86Ge	87Ge	88Ge	89Ge																						

Courtesy of Brookhaven National Laboratory.

Image removed due to copyright restrictions.

## **Nuclear Structure: Forces in the nucleus**

### **Coulomb Force**

**Force between two point charges, q, separated by distance, r (Coulomb's Law)**

$$F(N) = \frac{k_0 q_1 q_2}{r^2} \quad k_0 = 8.98755 \times 10^9 \text{ N m}^2 \text{ C}^{-2} \text{ (Boltzman constant)}$$

**Potential energy (MeV) of one particle relative to the other**

$$PE(MeV) = \frac{k_0 q_1 q_2}{r}$$

### **Strong Nuclear Force**

- **Acts over short distances**
- **$\sim 10^{-15}$  m**
- **can overcome Coulomb repulsion**
- **acts on protons and neutrons**

Image removed due to copyright restrictions.

Fig 3.1 in Turner J. E. *Atoms, Radiation, and Radiation Protection*, 2<sup>nd</sup> ed. New York: Wiley-Interscience, 1995.

### **Summary of Nuclear Forces:**

Nuclei give off energy (i.e., radiation) in an attempt to become more stable

Nuclear instability can be traced to the interaction of i) Coulomb and ii) strong nuclear force.

#### **Coulomb**

repulsive

$p^+ - p^+$

doesn't saturate

weak (eg.  $e^-$  to nucleus,

~ few eV to .1 MeV)

atom is mostly empty space

#### **Strong Nuclear**

attractive

$p^+ - p^+$ ,  $n - n$ ,  $p^+ - n$

short range; falls off quickly

very strong (several decades of MeV)

nucleus is densely packed

Due to the Coulomb-nuclear force balance, nuclei exhibit a roughly constant density and radius.

## Energy-Mass Equivalence

*Atomic Mass Units* (amu, or AMU)

By definition: Atomic masses are measured on a scale in which a  $^{12}\text{C}_6$  atom is exactly 12 atomic mass units.

**Gram atomic weight** of any element contains  $N_0$  atoms ( $N_0 = \text{Avogadro's number}$ ).

12 grams of carbon =  $6.02 \times 10^{23}$  carbon atoms

$$\frac{12 \text{ g carbon}}{6.02 \times 10^{23} \text{ atoms}} = \frac{1.99 \times 10^{-23} \text{ g}}{\text{carbon atom}} = 12 \text{ AMU}$$

$$1 \text{ amu} = \frac{1.99 \times 10^{-23} \text{ g}}{12} = 1.66 \times 10^{-24} \text{ g} = 1.66 \times 10^{-27} \text{ kg}$$

Using Einstein's mass-energy equivalence formula:  $E = m_0 c^2$ ,

$$1.660531 \times 10^{-27} \text{ kg} \times (3.0 \times 10^8 \text{ m/s})^2 = 1.49448 \times 10^{-10} \text{ kg m}^2/\text{s}^2 = 1.49448 \times 10^{-10} \text{ Joule}$$

Given:  $1.6022 \times 10^{-19} \text{ Joules} = 1 \text{ eV}$

***1 AMU is equivalent to 931.48 MeV***

Rest mass energies and mass equivalences:

electron mass:	0.000549 amu = 0.511 MeV
proton mass:	1.007277 amu = 938.28 MeV
neutron mass:	1.008665 amu = 939.57 MeV
hydrogen atom:	1.007825 amu

## Mass Differences, $\Delta$

**The mass of a nuclide is LESS than the sum of its parts...**

- Energy released when all constituents come together.
- Nuclear force so strong that the mass of the bound system is smaller than the sum of the components.

$$\Delta = M - A, \text{ or } M = \Delta + A$$

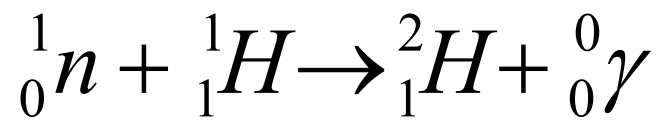
M is the true atomic mass

A is the atomic number

Image removed due to copyright restrictions.  
Appendix D in [Turner]



## Nuclear reactions release energy



How much energy is released?

Compare the total masses on both sides of the arrow.

## Nuclear Binding Energies

The *difference in mass* between a given nucleus and the sum of the same number of individual protons and neutrons is the *binding energy*.

Image removed due to copyright restrictions.  
Fig 3.3 in [Turner]

## **Nuclear Stability/Instability**

- **Strong nuclear force, operates over short range**
- **“saturates” quickly**
- **neutrons interact only with neighbors**
- **protons interact (repulse via Coulomb interaction) throughout the entire nucleus.**

**In heavier nuclei, the #neutrons must increase faster than the number of protons to maintain stability.**

- **N/Z ratio = 1 at low A**
- **e.g., Mg Z=12, but N=12,13 or 14 (isotopes)**
- **N/Z ratio approaches 1.5 when Z~80**

**“Line of stability”  $Z = N$**

**Any nucleus far from the “line of stability” will be unstable.**

**The position of a nucleus relative to the line of stability will define the mode of nuclear instability (radioactive decay mode).**

Image removed due to copyright restrictions.

Radioactive decay tends towards the line of stability

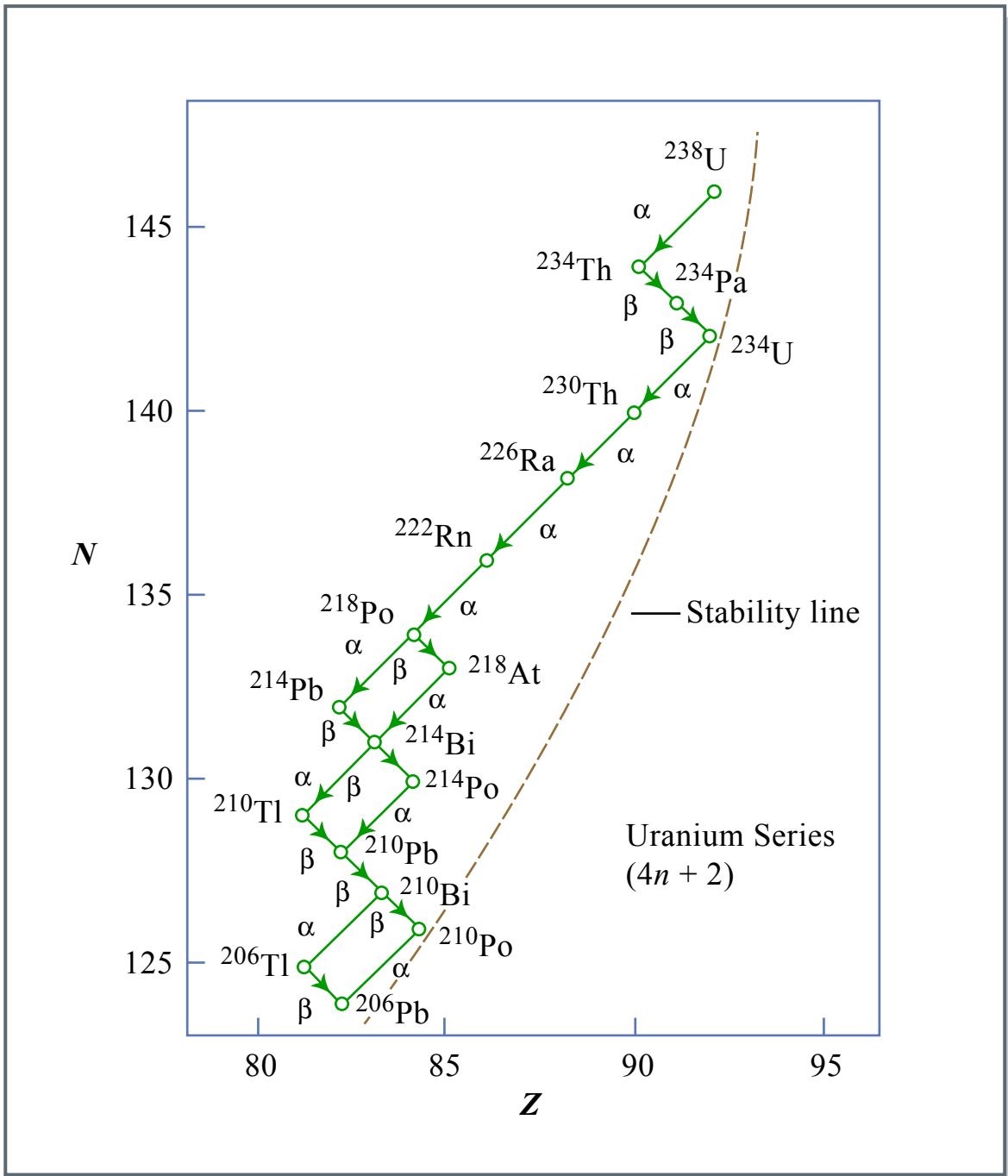
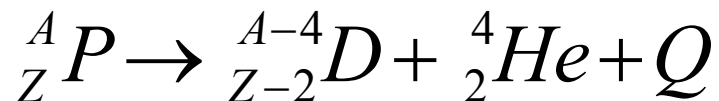


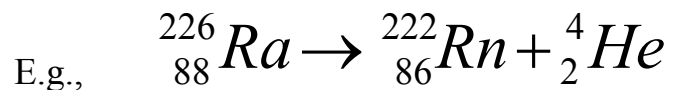
Figure by MIT OCW.

## Alpha decay

Natural alpha emitters:  $Z > 83$



- conservation of electric charges
- conservation of nucleons



*How much energy,  $Q$ , is released?*

Compare the masses on both sides of the arrow.

$$Q = M_{Ra} - M_{Rn} - M_{He}$$

Use  $\Delta$  values in Turner Appendix D.

$$Q = \Delta_{Ra} - \Delta_{Rn} - \Delta_{He}$$

$$Q = 23.69 - 16.39 - 2.42 = 4.88 \text{ MeV}$$

***How is this energy,  $Q$ , distributed?***

Shared by the daughters, the Rn nucleus and the alpha particle.

- Momentum is conserved:  $mv = MV$
- Kinetic energy of the 2 products =  $Q$

$$\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = Q$$

The energy of the alpha particle:

$$E_{\alpha} = \frac{1}{2}mv^2 = \frac{MQ}{m+M}$$

The energy of the Rn nucleus:

$$E_N = \frac{1}{2}MV^2 = \frac{mQ}{m+M}$$

Alpha decay results in a ***2-particle emission***.

$Q$  is fixed by the mass balance

$E_{\alpha}$  is fixed by the conservation laws (energy, momentum)

Therefore, ***alpha particles must have discrete energies***.

## Nuclear Decay Scheme Diagrams

### Graphical display of *nuclear* transformations

- **Decay mode**
- **Energy transitions**
- **Abundances (branching ratios)**

Image removed due to copyright restrictions.

Fig. 3.4 in [Turner]

Conventions:

- Arrows slanting to the left indicate *decrease in Z*
- Arrows slanting to the right indicate an *increase in Z*
- Wavy lines going straight down indicate a *gamma emission* from the nucleus.

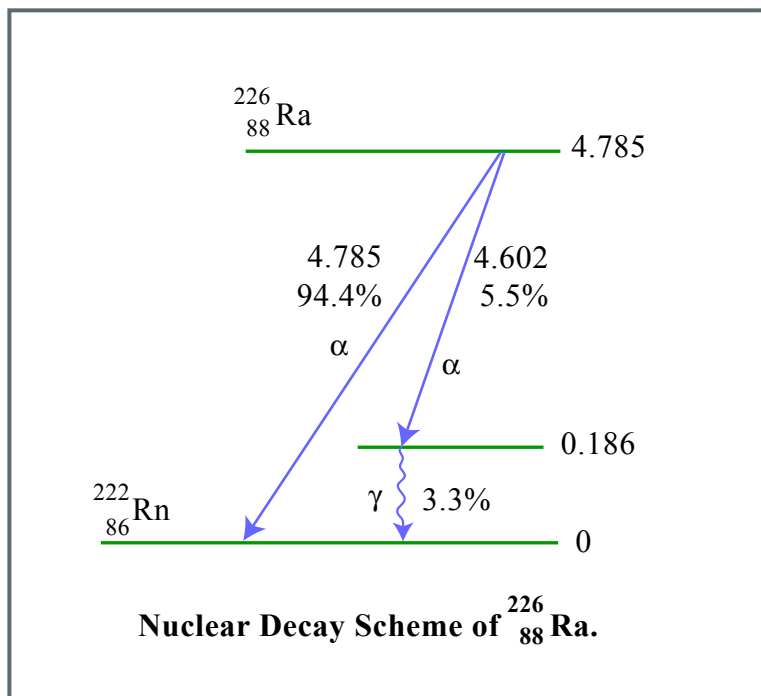
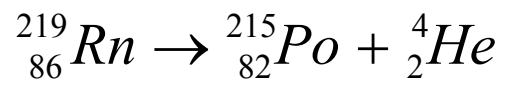


Figure by MIT OCW.

## Decay Scheme Exercise



<b><math>\alpha</math></b>	<b>6.82 (80%)</b>
	<b>6.55 (12%)</b>
	<b>6.42 (7%)</b>
<b><math>\gamma</math></b>	<b>0.271 (10%)</b>
	<b>0.402 (7%)</b>