Biological and Bio-Inspired Morphometry as a Route to Tunable and Enhanced Materials Design

by

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MPhil Micro- & Nanotechnology Enterprise, University of Cambridge (2011)

Submitted to the Department of Materials Science and Engineering
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Abstract

Structural materials in nature integrate classical materials selection rules with morphometry (geometry or shape-based rules) to create high-performance, multi-functional structures that exhibit tunable properties through extraordinary complexity, hierarchy, and precise structural control. This thesis explores the use of morphometry as a materials design parameter through the development of bio-inspired, flexible composite armor based on the articulated exoskeleton of an armored fish, Polypterus senegalus, which achieves uniform coverage and protection from predatory threats without restricting flexibility. First, the functional implications of shape and shape variation are examined as materials design parameters within the biological exoskeleton using a new method that integrates continuum strain analysis with landmark-based geometric morphometric analysis in 2D and 3D. Bioinspired flexible composite prototypes are fabricated using multi-material 3D printing and tested under passive loading (self-weight) and active loading (bending) to examine how the shape of scales contributes to local, interscale mobility mechanisms that generate anisotropic, global mechanical behavior. With one prototype design scheme, a wide array of mechanical behavior is generated with stiffness ranging over several orders of magnitude, including ‘mechanical invisibility’ of the scales, showing how morphometry can tune flexibility without varying the constituent materials. Finally, finite element models simulating the bending experiments are created to establish a computational framework for analyzing the mechanical response of the prototypes. The finite element models are then extended to examine the effect of different loading conditions, scale morphometry, multi-material architecture, and constituent material properties. The results show how morphometric-enabled materials design, inspired by structural biological materials, can allow for tunable behavior in flexible composites made of segmented scale assemblies to achieve enhanced user mobility, custom fit, and flexibility around joints for a variety of protective applications.

Thesis Supervisor: Mary C. Boyce
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Thesis Supervisor: Christine Ortiz
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For my parents
# Table of Contents

**LIST OF FIGURES** .................................................................................................................. 10

**LIST OF TABLES** .................................................................................................................. 22

1. **INTRODUCTION** ............................................................................................................... 23
   1.1. Motivation .......................................................................................................................... 23
   1.2. Overview .......................................................................................................................... 24
   1.3. Background ...................................................................................................................... 25
       1.3.1. Evolution of the biological model system: *Polypterus senegalus* ....................... 25
       1.3.2. Material composition of individual *P. senegalus* scales ..................................... 27
       1.3.3. Geometry and assembly of scales into the exoskeleton ....................................... 28
       1.3.4. Preliminary steps toward bio-inspired flexible armor .......................................... 29
   1.4. References ...................................................................................................................... 30

2. **2D MORPHOMETRIC HETEROGENEITY OF SCALES IN THE *P. senegalus* EXOSKELETON** ....................................................................................................................... 36
   2.1. Overview .......................................................................................................................... 36
   2.2. Materials and Methods .................................................................................................... 36
       2.2.1. X-ray microcomputed tomography (µCT) ............................................................... 36
       2.2.2. Geometric morphometric analysis in 2D ................................................................. 37
       2.2.3. Continuum strain model in 2D ............................................................................... 40
       2.2.4. Statistics ................................................................................................................... 41
   2.3. Results ............................................................................................................................. 44
       2.3.1. Morphometric variation of scales along the anteroposterior axis ......................... 45
       2.3.2. Morphometric variation of scales along the dorsoventral axis ........................... 47
       2.3.3. Morphometric variation of scales in the pectoral fin ............................................ 47
       2.3.4. Defining statistically heterogeneous scale variants .............................................. 50
   2.4. Discussion ....................................................................................................................... 51
   2.5. References ...................................................................................................................... 56

3. **3D MORPHOMETRIC HETEROGENEITY OF SCALES IN THE *P. senegalus* EXOSKELETON** ................................................................................................................................. 58
   3.1. Introduction ...................................................................................................................... 58
   3.2. Materials and Methods .................................................................................................... 58
       3.2.1. X-ray microcomputed tomography (µCT) ............................................................... 58
       3.2.2. Geometric morphometric analysis in 3D ................................................................. 58
       3.2.3. Continuum strain model in 3D ............................................................................... 60
3.3. Results ........................................................................................................... 63
3.3.1. 3D Morphometric variation along the anteroposterior axis ................. 64
3.3.2. 3D Morphometric variation along the dorsoventral axis .................... 68
3.3.3. 3D Morphometric variation along the pectoral fin transition seam ....... 69
3.4. Discussion ..................................................................................................... 71
3.5. References ................................................................................................... 72

4. Anisotropic Flexibility of Bio-inspired Flexible Composite Prototypes Under Passive Loading ................................................................. 73

4.1. Introduction ................................................................................................ 73
4.2. Materials and Methods ............................................................................. 73
4.2.1. Prototype design via associative modeling ........................................... 73
4.2.2. Prototype fabrication via multi-material 3D printing .......................... 73
4.2.3. Radius of curvature measurements ....................................................... 74
4.2.4. Joint degrees of freedom analysis ........................................................ 74
4.3. Results ......................................................................................................... 76
4.3.1. Anisotropy of flexibility through radius of curvature ......................... 76
4.3.2. Contribution of local mobility mechanisms to global behavior ......... 78
4.4. Discussion ................................................................................................... 79
4.5. References ................................................................................................... 80

5. Mechanical Behavior of Bio-inspired Flexible Composite Prototypes in Active Loading ................................................................. 82

5.1. Introduction ................................................................................................ 82
5.2. Materials and Methods ............................................................................. 82
5.2.1. Prototype design and fabrication ......................................................... 82
5.2.2. Bending experiment and analysis ......................................................... 84
5.3. Results ......................................................................................................... 84
5.3.1. Anisotropic stiffness in concave bending ............................................ 84
5.4. Discussion ................................................................................................... 89
5.5. References ................................................................................................... 91

6. Finite Element Simulations of Bio-inspired Flexible Composite Prototypes in Active Loading ........................................................................ 92

6.1. Introduction ................................................................................................ 92
6.2. Materials and Methods ............................................................................. 92
6.2.1. Model parts and assembly ................................................................. 92
6.2.2. Bending simulation and analysis ......................................................... 93
6.2.3. Tension simulation and analysis ......................................................... 93
6.3. Results ........................................................................................................................................................................94
6.3.1. Anisotropic stiffness in concave bending ....................................................................................................................94
6.3.2. Anisotropic stiffness in convex bending ......................................................................................................................103
6.3.3. Anisotropic stiffness in tension ...................................................................................................................................111
6.4. Discussion ........................................................................................................................................................................120
6.5. References ........................................................................................................................................................................122

7. Effect of Variation in Morphometric and Material Design on Mechanical Behavior ......................................................123
7.1. Introduction ......................................................................................................................................................................123
7.2. Materials and Methods ..................................................................................................................................................123
7.2.1. Building variation in finite element models ................................................................................................................123
7.2.2. Simulation of active loading ......................................................................................................................................124
7.3. Results ...............................................................................................................................................................................124
7.3.1. Effect of removing the paraserial interconnections .................................................................................................124
7.3.2. Effect of varying the stiffness ratio of constituent materials ......................................................................................129
7.3.3. Effect of removing the peg and socket joint ...............................................................................................................131
7.4. Discussion .........................................................................................................................................................................133
7.5. References ........................................................................................................................................................................135

8. Conclusion .............................................................................................................................................................................136
8.1. Summary ............................................................................................................................................................................136
8.2. Significance ........................................................................................................................................................................136
8.2.1. Morphometry as a materials design parameter ..........................................................................................................136
8.2.2. Multi-disciplinary framework for bioinspired engineering .........................................................................................136
8.2.3. Shattering the protection-flexibility tradeoff in armor design ....................................................................................137
8.3. Applications .....................................................................................................................................................................137
8.4. Future Directions .............................................................................................................................................................138

Appendix A: Derivation of Thin-Plate Spline Formulas for Generating Transformation Grids .............................................140
A.1. Calculating Thin-Plate Splines in 2D ............................................................................................................................140
A.2. Calculating Thin-Plate Splines in 3D ............................................................................................................................141
A.3. References ......................................................................................................................................................................142
List of Figures

Figure 1.1: The protection vs. flexibility tradeoff in current armor technologies [42-46, 52]... 24

Figure 1.2: The hierarchical materials-morphometric design principles of the *P. senegalus* exoskeleton. (a) X-ray microcomputed tomography (µCT) reconstruction of a *P. senegalus* specimen, adapted from [58]. (b) Anesthetized *P. senegalus* specimen (body length = 219 mm) exhibiting large bi-directional body curvature in axial bending [59]. (c) Optical micrographs of a single scale cross-section showing the quad-layered structure [30, 60]. (d) Scanning electron micrograph (SEM) of ganoine nanocrystals [30]. (e) SEM of the dentin layer [61]. (f) Backscatter SEM (BSEM) of isopedine cross-section [60]. (g) Atomic force micrograph (AFM), height-image, of a polished cross section of the bony plate [61]. (h) Optical micrographs of 1N and 2N microindentations on scale surface showing circumferential cracking [30]. (i) BSEM showing crack arrest at the ganoine-dentin junction [30]. (j) Schematic of the helical arrangement of scales showing the paraserial axis (red) and interserial axis (black) [55]. (k) Schematic showing the local articulation of scales. (l) µCT reconstruction of a single scale, interior (left) and exterior (right) views [62]. (m) SEM of outer scale surface [30]. (n) SEM of inner scale surface [30]. (o) Schematic of the articulated, paraserial peg-and-socket joint [62]. (p) Schematic of the sliding, interserial overlap joint [62]. (q) Schematic of the interior view of the local assembly of scales, showing sites of attachment of Sharpey’s fibers (asf) and attachment of stratum compactum (asc), as well as the fibers of the stratum compactum (sc) that run along the paraserial (ps) and interserial (is) axes [55]. (r) SEM of the interior of the scale showing the site of attachment of the stratum compactum to the axial ridge of the scale [55]. (s) SEM of stratum compactum [55]. (t) SEM showing Sharpey’s fibers connecting the peg and socket of adjacent scales [55].

Figure 1.3: Prior research in developing *P. senegalus* -inspired armor prototypes. (a) Two scales that have been unitized, enlarged, and 3D printed with plaster [59]. (b) Single scale incorporating material heterogeneity through the scale’s porous architecture [82]. (c) Monolithic semi-helical ring structures [59]. (d) Simplified scale design allowing two rotational degrees of freedom at the paraserial and interserial joints [59]. (e) Simplified scale design from (d) 3D-printed and assembled with rubber bands to mimic the fibrous interconnections [59]. (f) 1×2 arrays of scales incorporating compliant material elements in
the peg-and-socket interconnections (top) and the attachment site to the underlying substrate (bottom) using multimaterial 3D printing [59]. (g) 5x5 flexible scale assembly [59].

**Figure 2.1:** The *P. senegalus* exoskeleton. (a) μCT reconstruction (18 µm scan resolution) of the *P. senegalus* exoskeleton used in the morphometric analysis, and (b) μCT reconstruction of a *P. senegalus* specimen in a fully extended configuration (95 µm scan resolution) [10]. Images show the paraserial axis of scale articulation (column), the interserial axis of scale overlap, the anterior-posterior row, dorsal and ventral midlines of mirror symmetry, and pectoral fin. Select scales are numbered by column and row number (C#R#). The asterisked scale (C9R5) in (a) is used in Fig. 2.2.

**Figure 2.2:** μCT reconstruction (10 µm scan resolution) of a single *P. senegalus* scale (C9R5), (a) interior and (b) exterior views. Geometrical features are labeled: peg (P), socket (S), anterior process (AP), axial ridge (AR), and anterior margin (AM). The 20 LMs used for morphometric analysis are shown (red dots) and described in Table 2.1. The x-y coordinate axis is defined with the x-axis aligned with the scale’s paraserial axis, and y-axis defined 90° perpendicular to the x-axis in the plane of the scale. Two morphometric parameters are shown: peg tip angle (γ) and anterior process angle (θ).

**Figure 2.3:** Selection criteria for the number of LMs used in the morphometric analysis. (a) Single scale (C31R10) with 60 LMs placed around outer surface, numbered in order of their selection for calculating polygonal area. LMs in red represent the first 18 LMs shown in Fig. 2.2. (b) Plot of polygonal area versus number of landmarks selected to define the outline of the scale, for five different scales in different regions in the *P. senegalus* exoskeleton.

**Figure 2.4:** Landmark-based GM and continuum strain analysis of affine transformations. (a) Reference scale geometry (blue) mapped to an undeformed grid. (b-d) Transformation grids (T) and strain plots mapping the components of strain tensor ($\varepsilon_{11}$, $\varepsilon_{22}$, and $\varepsilon_{12}$), hydrostatic strain ($\varepsilon_{\text{hyd}}$) and the norm of the deviatoric strain tensor ($\varepsilon_{\text{dev}}$) for affine transformations (red) applied to the reference geometry (blue): (b) compression ($\varepsilon_{11} = -\varepsilon_{22} = -0.2$), (c) simple shear ($\varepsilon_{12} = +0.2$) and (d) uniform scaling ($\varepsilon_{11} = \varepsilon_{22} = +0.2$).

**Figure 2.5:** Transformation grids (T) and strain plots mapping the shear component of strain ($\varepsilon_{12}$) for a series of affine shear operations (red) applied to the reference geometry (blue): (a) $\varepsilon_{12} = -0.5$, (b) $\varepsilon_{12} = -0.4$, (c) $\varepsilon_{12} = -0.3$, (d) $\varepsilon_{12} = -0.2$, (e) $\varepsilon_{12} = -0.1$, (f) $\varepsilon_{12} = 0.0$, (g) $\varepsilon_{12} = +0.1$, (h) $\varepsilon_{12} = 0.5$.
(h) $\varepsilon_{12} = +0.2$, (i) $\varepsilon_{12} = +0.3$, (j) $\varepsilon_{12} = +0.4$, (k) $\varepsilon_{12} = +0.5$, and (l) plot of the total strain energy ($U$, normalized by maximum value) vs. shear strain. .........................................................

**Figure 2.6:** Scale shape variation from anterior to posterior. (a) μCT reconstruction of scales within a single row (R6) of the *P. senegalus* specimen, every fifth scale shown (15 μm scan resolution). (b) Schematic of the exoskeleton illustrating the row of scales used for the analysis. (c) Transformation grids (T) compare the geometry of three scales (C5R6, C25R6, and C45R6; in red) to the reference geometry (blue). Strain plots map the components of the strain tensor ($\varepsilon_{11}$, $\varepsilon_{22}$, and $\varepsilon_{12}$), hydrostatic strain ($\varepsilon_{\text{hyd}}$) and norm of the deviatoric strain tensor ($\varepsilon_{\text{dev}}$) for each grid element. .........................................................

**Figure 2.7:** Scale shape variation from dorsal to ventral. (a) μCT reconstruction of scales within a single column (C22) of the *P. senegalus* specimen, every other scale shown (15 μm scan resolution). (b) Schematic of the exoskeleton illustrating the column of scales used for the analysis. (c) Transformation grids (T) compare the geometry of three scales (C22R2, C22R8, and C22R14; in red) to the reference geometry (blue). Strain plots map the components of the strain tensor ($\varepsilon_{11}$, $\varepsilon_{22}$, and $\varepsilon_{12}$), hydrostatic strain ($\varepsilon_{\text{hyd}}$) and norm of the deviatoric strain tensor ($\varepsilon_{\text{dev}}$) at each grid point. .........................................................

**Figure 2.8:** The *P. senegalus* pectoral fin and scale shape variation along a transition seam. (a) μCT reconstruction of the left pectoral fin (8 μm scan resolution) highlighting a transition row segment. (b) Exterior (top) and interior (bottom) views of an articulated seam of scales connecting the main body to the pectoral fin. (c) Transformation grids (T) compare the geometry of three scales (P2, P4, and P6; in red) to the reference geometry (blue). Strain plots map the components of the strain tensor ($\varepsilon_{11}$, $\varepsilon_{22}$, and $\varepsilon_{12}$), hydrostatic strain ($\varepsilon_{\text{hyd}}$) and norm of the deviatoric strain tensor ($\varepsilon_{\text{dev}}$) at each grid point. .........................................................

**Figure 2.9:** Boxplots for the range of values of morphometric parameters over the row, column, and pectoral fin seam. $\gamma$ and $\theta$ are normalized by 180°, and V and U are normalized by their maximum value over all scales (5.44 mm³ and 2.1x10¹⁰ J, respectively) to generate unitless parameter values. Data are segmented into quartiles with no outliers. .........................

**Figure 2.10:** Schematic of interscale mobility mechanisms. (a) A quad of scales showing interserial and paraserial axis in native (resting) state. Motion of scales can be broken into four components: (b) interserial sliding, (c) interserial splay, (d) paraserial rotation, and (e) paraserial bending. .........................................................
Figure 2.11: Recomposing morphometric variation by applying strain. (a) Transformation grid (T) and strain tensor components ($\varepsilon_{11}$, $\varepsilon_{22}$, and $\varepsilon_{12}$) comparing a biological scale geometry (C10R6, in red) to the reference geometry (blue). (b) Transformation grids resulting from the application of each component of the strain tensor ($\varepsilon_{11}$, $\varepsilon_{22}$, and $\varepsilon_{12}$) found in (a) to the reference geometry. Applying all three components of the strain tensor to the reference geometry recovers the geometry of the biological scale.

Figure 3.1: $\mu$CT reconstruction (10 $\mu$m scan resolution) of a single P. senegalus scale (C9R5) (a) interior and (b) exterior views showing the 21 LMs (red dots) used in the 3D morphometric analysis, described in Table 3.1. The coordinate systems is defined from the centroid with the x-axis aligned with the scale’s paraserial axis, and y-axis defined 90° perpendicular to the x-axis in the plane of the scale, and the z-axis vertically through the thickness of the scale point to the exterior (top) surface of the scale.

Figure 3.2: Illustrations of the 3D morphometric parameters described in Table 3.2.

Figure 3.3: Morphometric parameters for the (a) row, (b) column, and (c) pectoral fin data series. Central thickness (T), curvature (L), % scale volume reduction due to concavity of the anterior margin (AM%), and 3D strain energy (U) are plotted as unitless parameters normalized by their maximum value (left axis). Angle of interserial overlap ($\lambda$) is plotted in degrees (right axis). Volume of the base of the scale (VB), volume of the peg (VP) and volume of the anterior shelf (VAS) are stacked as area plots as a percentage of the total scale volume. Calculations for these parameters are presented in Table 3.2.

Figure 3.4: 3D morphometric analysis of the average scale variant (C30R6). (a) $\mu$CT reconstruction of the scale. (b) Undeformed 3D grid. (c) Deformed 3D transformation grid showing scale geometry (red) compared to the reference geometry (blue). (d) 2D medial slices of the transformation grid in the XY, XZ, and YZ planes. (e) 3 XY slices. (f) 3 XZ slices. (g) 3 YZ slices. (h) Strain plots on medial XY, XZ, and YZ slides comparing the scale (red) to the reference (blue).

Figure 3.5: 3D morphometric analysis of the anterior scale variant (C10R6). (a) $\mu$CT reconstruction of the scale. (b) Undeformed 3D grid. (c) Deformed 3D transformation grid showing scale geometry (red) compared to the reference geometry (blue). (d) 2D medial slices of the transformation grid in the XY, XZ, and YZ planes. (e) 3 XY slices. (f) 3 XZ
slices. (g) 3 YZ slices. (h) Strain plots on medial XY, XZ, and YZ slides comparing the scale (red) to the reference (blue).

**Figure 3.6:** 3D morphometric analysis of the tail scale variant (C50R6). (a) µCT reconstruction of the scale. (b) Undeformed 3D grid. (c) Deformed 3D transformation grid showing scale geometry (red) compared to the reference geometry (blue). (d) 2D medial slices of the transformation grid in the XY, XZ, and YZ planes. (e) 3 XY slices. (f) 3 XZ slices. (g) 3 YZ slices. (h) Strain plots on medial XY, XZ, and YZ slides comparing the scale (red) to the reference (blue).

**Figure 3.7:** 3D morphometric analysis of the ventral scale variant (C22R14). (a) µCT reconstruction of the scale. (b) Undeformed 3D grid. (c) Deformed 3D transformation grid showing scale geometry (red) compared to the reference geometry (blue). (d) 2D medial slices of the transformation grid in the XY, XZ, and YZ planes. (e) 3 XY slices. (f) 3 XZ slices. (g) 3 YZ slices. (h) Strain plots on medial XY, XZ, and YZ slides comparing the scale (red) to the reference (blue).

**Figure 3.8:** 3D morphometric analysis of the pectoral fin scale variant (P6). (a) µCT reconstruction of the scale. (b) Undeformed 3D grid. (c) Deformed 3D transformation grid showing scale geometry (red) compared to the reference geometry (blue). (d) 2D medial slices of the transformation grid in the XY, XZ, and YZ planes. (e) 3 XY slices. (f) 3 XZ slices. (g) 3 YZ slices. (h) Strain plots on medial XY, XZ, and YZ slides comparing the scale (red) to the reference (blue).

**Figure 4.1:** Design and fabrication of *P. senegalus*-inspired flexible composite prototypes. (a) Rendering of an abstracted scale geometry, showing the orientation of the paraserial and interserial axes and two controllable parameters: scale length (L) and scale height (H). Adapted from [1]. (b) Schematic of the multimaterial components of the articulated assembly, bottom view, adapted from [1]. (c) Top view of the prototype design. (d) Cross-sectional view of the prototype design. (e) 3D printed flexible composite prototype. (f) Close-up of the 3D printed prototype showing the orientation of the paraserial and interserial axes.

**Figure 4.2:** Experimental methods to analyze prototype behavior under passive loading. (a) Setup of the curvature experiment. The prototype is draped over a curved mold. Rigid rods are inserted into three scales in a single line along the mold’s axis of curvature (y-axis).
camera along the mold’s line of zero curvature (x-axis) captures the image in order to calculate the radius of curvature of the prototype (R_p) vs. radius of curvature of the mold (R_m). (b) Setup of the joint degrees of freedom experiment. The prototype is draped over a curved mold. Rigid rods are inserted into three scales, two along the paraserial axis and two along the interserial axis of the assembly. Cameras along the mold’s line of zero curvature (x-axis) and line of curvature (y-axis) capture images to calculate interscale angles. (c-d) Top-view schematics illustrating interscale angles between Rods 1-2 and Rods 1-3.

**Figure 4.3:** Anisotropy of prototype flexibility measured by radius of curvature. (a) Prototype draped over a half-cylinder mold at orientation angle (α) = 0°. Lines show the curvature of the mold (white) and prototype (red). Inverse of curvature represents the radius of curvature of the prototype (R_p) and the radius of curvature of the mold (R_m = 120 mm). The prototype exhibits different curvatures when rotated to (b) α = 30°, (c) α = 60°, (d) α = 90°, and (e) α = 120°. (f) Top view of the prototype in the α = 0° orientation. (g) Top view of the prototype in the α = 75° orientation. (h) Relative radius of curvature (R_p/R_m) vs. α for the prototype, a variation with the halved length aspect ratio, a variation without the paraserial connections, and the substrate only without scales. Error bars represent standard deviation with N = 3 samples per prototype design.

**Figure 4.4:** Contribution of joint degrees of freedom to the prototype flexibility. (a) Schematics of four joint degrees of freedom and their corresponding interscale angles. (b) Interscale angles vs. rotation angle (α) of the prototype. Error bars represent standard deviation for N = 3 samples.

**Figure 5.1:** Design and testing of flexible composite prototypes in bending. (a-f) Schematics of the prototypes. Dashed lines represent the paraserial (ps) axis, interserial (is) axis, the angle (β) between the paraserial and interserial axes, and scale orientation angle (φ) of 0°, 30°, 60°, 90°, 120°, and 150°. (g) Optical photograph of the sample holder attached to the rods of the prototype and affixed to the plate of the Zwick. (h) Side-view of the φ = 60° sample loaded in concave bending at various vertical displacements (d), from left to right: d = 3 mm, d = 20 mm, d = 50 mm, and d = 80 mm.

**Figure 5.2:** Anisotropic stiffness of flexible composite prototypes in concave bending. (a) Force-displacement curves for prototypes with φ = 0°, 30°, 60°, 90°, 120°, and 150°. (b) Normalized stiffness (K) of each phase of each prototype’s loading curve. Error bars
represent standard deviation with N = 3 samples per prototype orientation........................................85

**Figure 5.3:** Schematics of the six interscale mobility mechanisms observed........................................85

**Figure 5.4:** Orientation-dependent behavior of the scaled prototypes in concave bending. (a) Photographs of φ = 0° sample: (i-ii) Vertical displacement (d) = 3 mm, side view and close up view of scales. (iii-vi) d = 20 mm, side view, close up view of scales, and close up view of substrate. (vi-vii) d = 60 mm, side view and close up view of scales. (b) Photographs of φ = 30° sample: (i-ii) d = 3 mm, side view and close up view of scales. (iii-iv) d = 15 mm, side view and close up view of scales. (v-vi) d = 80 mm, side view and close up view of scale. (c) Photographs of φ = 60° sample: (i-ii) d = 4 mm, side view and close up view of scales. (iii-vi) d = 50 mm and 80 mm, side view and close up view of scales. ........................................86

**Figure 5.5:** Orientation-dependent behavior of the scaled prototypes in concave bending. (a) Photographs of φ = 90° sample. (i-iii) Vertical displacement (d) = 15 mm, side view and corresponding close up views of scales. (iv-vi) d = 60 mm, side view, oblique view, and close up views of scales and substrate. (b) Photographs of φ = 120° sample. (i-ii) d = 4 mm, side view and close up view of scales. (iii-iv) d = 20 mm, side view and close up view of scales with d = 20 mm. (v-vi) d = 60 mm, side view and close up view of scales. (c) Photographs of φ = 150° sample. (i-ii) d = 4 mm, side view and close up view of scales. (iii-iv) d = 14 mm, side view and close up view of scales. (v-vi) d = 40 mm, side view and close up view of scales. ........................................................................................................87

**Figure 6.1:** Finite element model design. (a) Simplified scale, (i) top view and (ii) side view. (b) Grooved substrate, (i) top view for orientation angle (φ) = 60°, and (ii) side view with φ = 0° showing scales placed in the shallow groove, where red lines define the axial ridge of scales. (c) Assembly of parts for the model with φ = 60°.................................................................93

**Figure 6.2:** Finite element results for concave bending. (a) Normalized stiffness (K) of each phase of each finite element (FE) model and experiment (Exp.), N=3 and error bars represent standard deviation. (b) Interscale mobility mechanisms observed in the finite element models. From top to bottom, left: paraserial bending, interserial rotation, interserial splay; right: interserial sliding, paraserial rotation.................................................................94

**Figure 6.3:** Stress plots (Mises) from the finite element simulations of flexible composite prototypes in concave bending. (a) Stress plots from the φ = 0° model. (i) Front view at vertical displacement (d) = 3 mm. (ii) Front view at d = 9 mm. (iii) Back view at d = 9 mm.
Figure 6.4: Strain plots (logarithmic, max. principal) from the finite element simulations of flexible composite prototypes in concave bending. (a) Strain plots from the $\phi = 0^\circ$ model. (i) Back views at vertical displacement ($d = 3$ mm, (ii) $d = 9$ mm, and (iii) $d = 22$ mm. (iv) Front view at $d = 22$ mm. (v) Scale bar. (b) Strain plots from the $\phi = 30^\circ$ model. (i) Back views at $d = 3$ mm, (ii) $d = 14$ mm, and (iii) $d = 33$ mm. (iv) Close up view of the scale-substrate attachment at $d = 33$ mm. (v) Scale bar. (c) Strain plots from the $\phi = 60^\circ$ model. (i) Back views at $d = 5$ mm, (ii) $d = 8$ mm, and (iii) $d = 38$ mm. (iv) Close up view of the scale-substrate attachment at $d = 24$ mm. (v) Scale bar.

Figure 6.5: Strain plots (logarithmic, components ($LE_{ij}$)) from the finite element simulations in concave bending. (a) Strain plots from the $\phi = 0^\circ$ model, back view at vertical displacement ($d = 9$ mm): (i) $LE_{11}$, (ii) $LE_{22}$, (iii) $LE_{33}$, (iv) axial strain scale bar, (v) $LE_{12}$, (vi) $LE_{13}$, (vii) $LE_{23}$, and (viii) shear strain scale bar. (b) Strain plots from the $\phi = 30^\circ$ model, back view at $d = 14$ mm: (i) $LE_{11}$, (ii) $LE_{22}$, (iii) $LE_{33}$, (iv) axial strain scale bar, (v) $LE_{12}$, (vi) $LE_{13}$, (vii) $LE_{23}$, and (viii) shear strain scale bar. (c) Strain plots from the $\phi = 60^\circ$ model, back view at $d = 38$ mm: (i) $LE_{11}$, (ii) $LE_{22}$, (iii) $LE_{33}$, (iv) axial strain scale bar, (v) $LE_{12}$, (vi) $LE_{13}$, (vii) $LE_{23}$, and (viii) shear strain scale bar.

Figure 6.6: Stress plots (Mises) from the finite element simulations of flexible composite prototypes in concave bending. (a) Stress plots from the $\phi = 90^\circ$ model. (i) Front views at vertical displacement ($d = 12$ mm, (ii) $d = 43$ mm, and (iii) $d = 68$ mm. (iv) Scale bar. (b) Stress plots from the $\phi = 120^\circ$ model. (i) Front views at $d = 3$ mm, and (ii) $d = 12$ mm. (iii) Back view at $d = 12$ mm. (iv) Front view at $d = 52$ mm. (v) Scale bar. (c) Stress plots from the $\phi = 150^\circ$ model. (i) Back view at $d = 3$ mm. (ii) Front views at $d = 20$ mm, (iii) $d = 38$ mm, and (iv) $d = 55$ mm. (v) Scale bar.

Figure 6.7: Strain plots (logarithmic, max. principal) from the finite element simulations of flexible composite prototypes in concave bending. (a) Strain plots from the $\phi = 90^\circ$ model. (i) Back views at vertical displacement ($d = 12$ mm, (ii) $d = 43$ mm, and (iii) $d = 68$ mm. (iv) Front view at $d = 68$ mm, left side cropped. (v) Scale bar. (b) Strain plots from the $\phi =$
Figure 6.8: Strain plots (logarithmic, components (LEij)) from the finite element simulations in concave bending. (a) Strain plots from the φ = 90° model, back view at vertical displacement (d) = 43 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar. (b) Strain plots from the φ = 120° model, back view at d = 12 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar. (c) Strain plots from the φ = 150° model, back view at d = 12 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar. 

Figure 6.9: Normalized stiffness (K) of the finite element models in convex bending.

Figure 6.10: Stress plots (Mises) from the finite element simulations of flexible composite prototypes in convex bending. (a) Stress plots from the φ = 0° model. (i) Front view at vertical displacement (d) = 3 mm. (ii) Close up view at d = 3 mm. (iii) Front view at d = 15 mm. (iv) Scale bar. (b) Stress plots from the φ = 30° model. (i) Front views at d = 2 mm, (ii) d = 5 mm, and (iii) d = 16 mm. (iv) Back view at d = 24 mm, left side cropped. (v) Scale bar. (c) Stress plots from the φ = 60° model. (i) Front views at d = 3 mm, (ii) d = 10 mm, and (iii) d = 27 mm. (iv) Close up view at d = 40 mm. (v) Scale bar.

Figure 6.11: Strain plots (logarithmic, max. principal) from the finite element simulations of flexible composite prototypes in convex bending. (a) Strain plots from the φ = 0° model. (i) Back views at vertical displacement (d) = 3 mm, (ii) d = 12 mm, and (iii) d = 28 mm, and (iv) d = 53 mm. (v) Scale bar. (b) Strain plots from the φ = 30° model. (i) Front, close up view at d = 3 mm and (iii) d = 16 mm. (iv) Scale bar. (c) Strain plots from the φ = 60° model. (i) Back views at d = 10 mm, (ii) d = 27 mm, and (iii) d = 40 mm. (iv) Scale bar.

Figure 6.12: Strain plots (logarithmic, components (LEij)) from the finite element simulations in convex bending. (a) Strain plots from the φ = 0° model, back view at vertical displacement (d) = 28 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar. (b) Strain plots from the φ = 30° model, back view at d = 16 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi)
LE13, (vii) LE23, and (viii) shear strain scale bar. (c) Strain plots from the φ = 60° model, back view at d = 40 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar.

**Figure 6.13:** Stress plots (Mises) from the finite element simulations of flexible composite prototypes in convex bending. (a) Stress plots from the φ = 90° model. (i) Front views at vertical displacement (d) = 22 mm and (ii) 42 mm, left side cropped. (iii) Back view at d = 42 mm, right side cropped. (iv) Front view at d = 74 mm. (v) Scale bar. (b) Stress plots from the φ = 120° model. (i) Front views at d = 5 mm, (ii) d = 16 mm, and (iii) d = 41 mm. (iv) Side view at d = 55 mm. (v) Scale bar. (c) Stress plots from the φ = 150° model. (i) Front view at d = 2 mm. (ii) Close up view at d = 2 mm. (iii) Front view at d = 36 mm. (iv) Back view at d = 39 mm. (v) Scale bar.

**Figure 6.14:** Strain plots (logarithmic, max principal) from the finite element simulations of flexible composite prototypes in convex bending. (a) Strain plots from the φ = 90° model. (i) Front, oblique view at vertical displacement (d) = 22 mm, left side cropped. (ii) Back views at d = 22 mm, (iii) d = 42 mm, right side cropped, and (iv) d = 66 mm, right side cropped. (v) Scale bar. (b) Strain plots from the φ = 120° model. (i) Back views at d = 5 mm, (ii) d = 16 mm, (iii) d = 41 mm, and (iv) d = 55 mm. (v) Scale bar. (c) Strain plots from the φ = 150° model. (i) Back views at d = 2 mm, (ii) d = 10 mm, (iii) d = 20 mm, and (iv) d = 39 mm. (v) Scale bar.

**Figure 6.15:** Strain plots (logarithmic, components (LEij)) from the finite element simulations in convex bending. (a) Strain plots from the φ = 90° model, back view at vertical displacement (d) = 66 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar. (b) Strain plots from the φ = 120° model, back view at d = 16 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar. (c) Strain plots from the φ = 150° model, back view at d = 20 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar.

**Figure 6.16:** Normalized effective modulus (E) of the finite element models in tension.

**Figure 6.17:** Stress plots (Mises) from the finite element simulations of flexible composite prototypes in tension. (a) Stress plots from the φ = 0° model. (i) Front view at vertical displacement (d) = 1 mm. (ii) Close up view at d = 1 mm. (iii) Front view at d = 2.5 mm, left
side cropped. (iv) Front view at d = 6 mm, left side cropped. (v) Back view at d = 10 mm, left side cropped. (vi) Scale bar. (b) Stress plots from the $\phi = 30^\circ$ model. (i) Front view at d = 1 mm. (ii) Back view at d = 1 mm. (iii) Front view at d = 5 mm. (iv) Back view at d = 6 mm. (v) Scale bar. (c) Stress plots from the $\phi = 60^\circ$ model. (i) Front view at d = 5 mm. (ii) Back view at d = 15 mm. (iii) Close up view at d = 15 mm. (iv) Front view at d = 35 mm. (v) Back view at d = 45 mm. (vi) Scale bar.

Figure 6.18: Strain plots (logarithmic, max. principal) from the finite element simulations of flexible composite prototypes in tension. (a) Strain plots from the $\phi = 0^\circ$ model. (i) Back view at vertical displacement (d) = 3.8 mm. (ii) Front view at d = 3.8 mm, left side cropped. (iii) Close up view at d = 3.8 mm. (iv) Back view at d = 19 mm, left side cropped. (v) Front view at d = 19 mm. (vi) Scale bar. (b) Strain plots from the $\phi = 30^\circ$ model. (i) Back views at d = 2.5 mm, (ii) d = 4.4 mm, and (iii) d = 9 mm. (iv) Front view at d = 9 mm. (v) Back view at d = 14 mm. (vi) Scale bar. (c) Strain plots from the $\phi = 60^\circ$ model. (i) Back view at d = 5 mm. (ii) Close up view of scale edges at d = 9 mm. (iii) Back views at d = 15 mm and (iv) d = 29 mm. (v) Front view at d = 29 mm. (vi) Scale bar.

Figure 6.19: Strain plots (logarithmic, components (LEij)) from the finite element simulations in tension. (a) Strain plots from the $\phi = 0^\circ$ model, back view at vertical displacement (d) = 3.8 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar. (b) Strain plots from the $\phi = 30^\circ$ model, back view at d = 2.5 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar. (c) Strain plots from the $\phi = 0^\circ$ model, back view at d = 15 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar.

Figure 6.20: Stress plots (Mises) from the finite element simulations of flexible composite prototypes in tension. (a) Stress plots from the $\phi = 90^\circ$ model. (i) Front views at vertical displacement (d) = 22 mm, (ii) d = 36 mm, and (iii) d = 47 mm. (iv) Back view at d = 47 mm. (v) Scale bar. (b) Stress plots from the $\phi = 120^\circ$ model. (i) Front view at d = 5 mm. (ii) Back view at d = 5 mm. (iii) Close up view at d = 20 mm. (iv) Front view at d = 35 mm. (v) Back view at d = 49 mm, left side cropped. (vi) Scale bar. (c) Stress plots from the $\phi = 150^\circ$ model. (i) Close up view at d = 0.5 mm. (ii) Front views at d = 2 mm and (iii) d = 5 mm. (iv) Back view at d = 12 mm. (v) Scale bar.
Figure 6.21: Strain plots (logarithmic, max. principal) from the finite element simulations of flexible composite prototypes in tension. (a) Strain plots from the $\varphi = 90^\circ$ model. (i) Front view at vertical displacement ($d$) = 22 mm. (ii) Back views at $d = 22$ mm, (iii) $d = 36$ mm, and (iv) $d = 50$ mm. (v) Scale bar. (b) Strain plots from the $\varphi = 120^\circ$ model. (i) Back views at $d = 5$ mm and (ii) $d = 19$ mm. (iii) Front view at $d = 20$ mm. (iv) Back view at $d = 35$ mm. (v) Scale bar. (c) Strain plots from the $\varphi = 150^\circ$ model. (i) Back views at $d = 2$ mm and (ii) $d = 5$ mm. (iii-iv) Close up views of the paraserial peg and socket joint at $d = 5$ mm. (v) Back view at $d = 12$ mm. (vi) Front view at $d = 12$ mm, right side cropped. (vii) Scale bar........ 117

Figure 6.22: Strain plots (logarithmic, components ($LE_{ij}$)) from the finite element simulations in tension. (a) Strain plots from the $\varphi = 90^\circ$ model, back view at vertical displacement ($d$) = 50 mm: (i) $LE_{11}$, (ii) $LE_{22}$, (iii) $LE_{33}$, (iv) axial strain scale bar, (v) $LE_{12}$, (vi) $LE_{13}$, (vii) $LE_{23}$, and (viii) shear strain scale bar. (b) Strain plots from the $\varphi = 120^\circ$ model, back view at $d = 35$ mm: (i) $LE_{11}$, (ii) $LE_{22}$, (iii) $LE_{33}$, (iv) axial strain scale bar, (v) $LE_{12}$, (vi) $LE_{13}$, (vii) $LE_{23}$, and (viii) shear strain scale bar. (c) Strain plots from the $\varphi = 150^\circ$ model, back view at $d = 5$ mm: (i) $LE_{11}$, (ii) $LE_{22}$, (iii) $LE_{33}$, (iv) axial strain scale bar, (v) $LE_{12}$, (vi) $LE_{13}$, (vii) $LE_{23}$, and (viii) shear strain scale bar. ................................................................. 118

Figure 7.1: The effect of paraserial interconnections. Results from finite element models without paraserial interconnections (“No PS”, in red) compared to the original models with paraserial interconnections (“PS”, in black). (a) Normalized stiffness ($K$) by phase in concave bending. (b) Normalized stiffness ($K$) in convex bending. (c) Normalized effective modulus ($E$) in tension ........................................................................................................... 125

Figure 7.2: The effect of stiffness ratio ($E^*$). Normalized stiffness ($K$) of finite element models in concave bending by phase comparing varying stiffness ratios ($E^*$, in red) to the original model ($E^0$, in black) for: (a) $E^* = 10E^0$, (b) $E^* = 10E^0$, and (c) $E^* = 0.1E^0$ .................... 130

Figure 7.3: The effect of removing the peg and socket from the scale geometry in the finite element models in concave bending. (a) Scale design, front view (top) and side view (bottom). (b) Normalized stiffness ($K$) of finite element models in concave bending by phase comparing the scale geometry with no peg and socket (in red) to the original model (in black)................................................................................................................... 132
List of Tables

Table 2.1: Typology and descriptions for the 20 LMs defining the 2D scale geometry. ........... 38
Table 2.2: 2D Morphometric parameters, descriptions, and calculations.............................. 44
Table 2.3: P-values from student’s t-test of morphometric parameters of scales grouped by region in the fish exoskeleton: average (N = 30 scales), anterior (N = 36 scales), tail (N = 21 scales), ventral (N = 41 scales), and pectoral fin (N = 14 scales). Asterisks (*) represent p-values < 0.01. .................................................................................................................. 51
Table 3.1: Landmark typology and descriptions for the 21 LMs defining the 3D scale geometry. .................................................................................................................................................................................. 60
Table 3.2: Parameters used in the 3D morphometric analysis, illustrated in Fig. 3.2 .......... 62
Table 5.1: Displacement range, stiffness, and interscale mobility mechanisms for each loading phase of each prototype orientation in concave bending........................................... 85
1. Introduction

1.1. Motivation

This thesis explores the use of shape and shape variation as a materials design parameter to control the mechanical behavior of materials in nature and in bioinspired analogs with the goal of developing flexible armor. Structural biological materials incorporate classical materials design parameters with morphometry (geometry or shape-based rules) to create high-performance, multi-functional structures that exhibit tunable properties through extraordinary complexity, hierarchy, and precise structural control [1-8]. Geometric control of composite architectures can generate emergent material properties that outperform their base constituents such as high fracture toughness (e.g. in bone [9], nacre [10-12], and silica sponge skeletons [13]), extensive deformability (e.g. in skull sutures [14], wood cell walls [15], and shark cartilage [16]), actuation ability (e.g. in pine cones [17], seed capsules [18], and contractile roots [19]), or functional transition regimes between different materials (e.g. bone-ligament junctions [20], bone porosity gradients [21], and mussel byssus [22]). Selective pressures have led many soft-bodied animals to evolve hard, protective exoskeletons as natural armors (e.g. crustaceans [23], insects [24], mollusks [25-26], turtles [27-28], sea horses [29], and bony fish [30-33]) with intricate, hierarchical microstructures that exhibit excellent strength, toughness, and energy dissipation mechanisms, while also using geometrical design rules to introduce an array of additional functionalities such as multi-scale enhancement of mechanical properties [25, 28, 33-35], transparency [36], or flexibility [26, 37]. Armored fish in particular have evolved protective exoskeletons that provide penetration resistance while maintaining flexibility through scale geometry, arrangement, and interlocking mechanisms [30-32, 37-38]. Research on biological and bioinspired structural materials has largely focused on the inherent materials-based structure-property relationships including hierarchy [2, 11], spatial heterogeneity [39], anisotropy [40], mechanical property amplification [2], and threat-specific deformation mechanisms [30, 32]. With advent of high resolution materials characterization instrumentation, powerful computational simulation tools, and increasingly precise additive manufacturing techniques, it is becoming possible to study, replicate, and capitalize on the full range of hybrid materials-morphometric design principles found in nature.

Bioinspired engineering that integrates both materials and morphometric design principles shows promise for the development of flexible armor [38]. While the development of penetration resistant materials has progressed rapidly [41], there exists a tradeoff between protection and flexibility within current armor technology (Fig. 1.1). Today, large, monolithic plate armors are in use with various materials that are tailored to the application, such as metals like steel [42], ceramics like aluminum oxides, silicon carbides, and boron carbides [43], and Kevlar aramid fibers [44]. Flexibility can be introduced by segmenting plates into smaller units, such as in vest inserts [45] or elbow pads [46], leaving vulnerable weak spots in the gaps
between materials. Further attempts at engineering synthetic flexible armors have used simple geometries and assembly configurations [47-48], for example in mosaic tiled armor [49], articulating concrete mats [50], and flexible protective vests [51-52]. However, these designs tend to fail at the interfaces between materials, such as the adhesive attaching overlapping ceramic discs to the substrate in DragonSkin [52]. Using additive manufacturing as a fabrication method can eliminate the need for adhesives and mitigate interface failure while allowing for complex subunit shapes, multi-materiality, and tailorable material choice for target applications. Emerging biomimetic armors based on articulated fish armor designs can be useful for damage localization, flexibility, customizable design and fit, and selective replacement of damaged units.

![Figure 1.1: The protection vs. flexibility tradeoff in current armor technologies [42-46, 52].](image)

### 1.2. Overview

This thesis explores the use of morphometric-enabled materials design through the development of bio-inspired, flexible composite armor based on the articulated exoskeleton of an armored fish, *Polypterus senegalus*, which possesses a protective ganoid squamation that achieves uniform coverage and protection from predatory threats without restricting flexibility [53-55]. First, the functional implications of scale shape and shape variation are examined as materials design parameters within the biological exoskeleton using a new method that integrates continuum strain analysis with landmark-based geometric morphometric analysis in 2D (Chapter 2) and 3D (Chapter 3). Bioinspired flexible composite prototypes are then fabricated using multi-material 3D printing and tested under passive loading under self-weight (Chapter 4) and active loading in bending (Chapter 5) to examine how the shape of scales contributes to local, interscale mobility mechanisms that generate anisotropic, global mechanical behavior. With one prototype design scheme, a wide array of mechanical behavior is generated with stiffness ranging over several orders of magnitude, thus showing how morphometry can tune the flexibility of composite architectures without varying the constituent materials. In the orientation that exhibits the greatest flexibility, the scales are considered ‘mechanically invisible’ and do not add stiffness to the armor assembly. Finite element models simulating the bending experiments...
are then created to establish a computational framework for analyzing the mechanical response of the prototypes, and the models are extended to analyze behavior in other loading conditions, e.g. convex bending and tension (Chapter 6). The finite element simulations are further extended to examine the effect of the multi-material architecture, scale morphometry, and constituent material properties on the mechanical response of the bio-inspired, flexible composite materials (Chapter 7). Finally, conclusions and future directions stemming from the work presented in this thesis are discussed (Chapter 8). The results show how morphometric-enabled materials design, inspired by structural biological materials, can allow for tunable behavior in flexible composites made of segmented scale assemblies to achieve enhanced user mobility, custom fit, and flexibility around joints for a variety of protective applications.

1.3.  Background

The model system used in this thesis is the armored fish, *Polypterus senegalus* (bichir), which possesses a mineralized, full-coverage exoskeleton, shown in Fig. 1.2a-b, with flexibility for axial bending and torsion [53-55], escape maneuvers [56], and recoil aspiration [57]. The exoskeleton uses a hierarchy of design rules to enable multifunctional behavior (Fig. 1.2); heterogeneity in material structure and properties provides protection, while morphometry and articulation mechanisms contribute to flexibility [30, 55, 58-62]. The evolution of *P. senegalus* armor, its material composition, geometric design rules explored to date, and preliminary work toward creating bioinspired flexible composite prototypes are described below.

1.3.1. Evolution of the biological model system: *Polypterus senegalus*

*P. senegalus* (phylum Chordata, superclass Osteichthyes (bony fish), class Actinopterygii (ray-finned fish), order Polpteriformes, family Polypteridae) originated in the Cretaceous period approximately 95 million years ago [53-54]. The species has an exoskeleton comprised of armored, ganoid scales with a microstructure characteristic of ancient palaeoniscoid fish [63-64]. Dermal fish armor first appeared in Ostracoderms in the Paleozoic era approximately 500 million years ago [65] in response to predatory threats [66-67] with multilayered structures and geometries [68-69]. The large, heavy dermal armor plates broke into several smaller plates during the Devonian period (approximately 400 million years ago) as fish became predators and required faster swimming agility with light-weight armor and greater flexibility [66, 70]. The number of material layers in the armor decreased (from 4 to 1-3 layers), and the thickness of each material layer decreased (from 1500 to 100 µm thick) [53, 63]. Armored fish living today exhibit three major types of bony scales: ganoid, elasmoid, and placoid [54, 71]. While these categories of scales differ in shape, layered microstructure, and biomineralization processes, they are all made of the same mineral component, calcium phosphate, in the form of hydroxyapatite [72]. By contrast, many invertebrate exoskeletons, e.g. mollusks and sea urchins, are composed of calcium carbonate [73]. In the present day, *P. senegalus* lives in freshwater estuaries and shallow floodplains in Africa [74]. This species can be self-predatory [65], with a jaw structure and skull capable of powerful biting attacks during territorial fighting and feeding [74-77].
Figure 1.2: The hierarchical materials-morphometric design principles of the *P. senegalus* exoskeleton. (a) X-ray microcomputed tomography (µCT) reconstruction of a *P. senegalus* specimen, adapted from [58]. (b) Anesthetized *P. senegalus* specimen (body length = 219 mm) exhibiting large bi-directional body curvature in axial bending [59]. (c) Optical micrographs of a single scale cross-section showing
the quad-layered structure [30, 60]. (d) Scanning electron micrograph (SEM) of ganoine nanocrystals [30]. (e) SEM of the dentin layer [61]. (f) Backscatter SEM (BSEM) of isopedine cross-section [60]. (g) Atomic force micrograph (AFM), height-image, of a polished cross section of the bony plate [61]. (h) Optical micrographs of 1N and 2N microindentations on scale surface showing circumferential cracking [30]. (i) BSEM showing crack arrest at the ganoine-dentin junction [30]. (j) Schematic of the helical arrangement of scales showing the paraserial axis (red) and interserial axis (black) [55]. (k) Schematic showing the local articulation of scales. (l) µCT reconstruction of a single scale, interior (left) and exterior (right) views [62]. (m) SEM of outer scale surface [30]. (n) SEM of inner scale surface [30]. (o) Schematic of the articulated, paraserial peg-and-socket joint [62]. (p) Schematic of the sliding, interserial overlap joint [62]. (q) Schematic of the interior view of the local assembly of scales, showing sites of attachment of Sharpey’s fibers (asf) and attachment of stratum compactum (asc), as well as the fibers of the stratum compactum (sc) that run along the paraserial (ps) and interserial (is) axes [55]. (r) SEM of the interior of the scale showing the site of attachment of the stratum compactum to the axial ridge of the scale [55]. (s) SEM of stratum compactum [55]. (t) SEM showing Sharpey’s fibers connecting the peg and socket of adjacent scales [55].

1.3.2. Material composition of individual P. senegalus scales

The materiality of the mineralized ganoid scales underlies the protective functionality of the exoskeleton (Fig. 1.2c-i). The ganoid scales have a quad-layered microstructure of hydroxyapatite-organic composite materials, shown in Fig. 1.2c, composed of outermost ganoine (~10 µm thick, elastic modulus (E) = 55 GPa, >95% mineral content), dentin (~50 µm thick, E = 25 GPa, ~80% mineral content), isopedine (~40 µm thick, E = 14.5 GPa, 60-75% mineral content), and innermost bone (~300 µm thick, E = 13.5 GPa, 60-70% mineral content) with a porous architecture [30, 32, 54, 60-61, 78-82]. Each layer has a unique nanocomposite structure [30, 61]. The thin, outer ganoine layer (Fig. 1.2d) is a stratified, acellular, and highly mineralized collection of pseudoprismatic apatite crystallites (50 nm diameter) that twist in bundles toward the outer scale surface [30]. Dentin (Fig. 1.2e) consists of 1-2 µm diameter dentinal tubules consisting of type I collagen fibrils reinforced with nanocrystalline apatite (20-300 nm size) [30]. Isopedine (Fig. 1.2f) consists of orthogonal, collagenous layers alternating in thickness (3 µm and 6 µm), arranged in a plywood structure, and mineralized with apatite platelets (40-150 nm) [30]. The bony basal plate (Fig. 1.2g), which forms the bulk of the scale, is composed of vascularized bone lamellae with collagen fibrils aligned parallel to the scale surface [30]. The materials are interpenetrated with each other at the layer interfaces; for instance, at the dentin-ganoine junction, ganoine nanorods penetrate into the dentin, and organic ligaments from dentin are anchored within the ganoine structure [30, 54].

The multilayered design enables penetration resistance, toughness, and non-catastrophic pathways for energy dissipation while exhibiting load-dependent material properties, circumferential surface cracking, minimized weight, and microstructural length scale and material property length scale matching between the armor and the predatory teeth (threat matching) with optimized layer thickness [30, 32, 40, 83]. Each layer exhibits unique deformation and energy dissipation mechanisms, and the functionally-graded junctions between layers promote load transfer and stress redistribution while preventing delamination [30]. The hardness and stiffness of the mineralized ganoine work in conjunction with the compliance of the
dentin layer to dissipate energy through plasticity at high loads, to reduce the weight of the scale by ~20% while maintaining protective functionality, and to promote localized, circumferential cracking while suppressing radial crack propagation (Fig. 1.2h) [30]. The stratification of isopedine arrests microcracks that penetrate deeply into the scale at high loads, preventing catastrophic failure of the scale (Fig. 1.2i) [30]. The material properties and thickness of each layer are optimized to defend against the animal’s primary threat, tooth-biting attacks, by both deforming and fracturing the threat (tooth) and by also allowing for non-catastrophic avenues for energy dissipation through deformation within the scale microstructure [32, 40].

1.3.3. Geometry and assembly of scales into the exoskeleton

The multilayered materials design of the individual scales works in conjunction with scale geometry and interlocking mechanisms to enable body flexibility [55]. The scales are articulated over the surface of the dermis to form a protective exoskeleton assembly (Fig. 1.2j-l), rather than imbricated into the soft tissue of the fish. The squamation consists of columns of scales that wind around the body in two interwoven, semi-helical axes depicted in Fig. 1.2j: the oblique paraserial axis of scale articulation and the interserial axis of scale overlap [55]. The local articulation of scales (Fig. 1.2k) is derived from the complex shape of the individual scale (Fig. 1.2l). Morphometric design rules guide scale articulation (Fig. 1.2m-p). The scales possess distinct geometric features, as shown in Fig. 1.2m-n, such as the peg, socket, anterior process, and axial ridge [30, 55]. These features create two joint interconnections between neighboring scales: the articulated peg-and-socket joint in the paraserial direction within a column of scales (Fig. 1.2o), and the sliding overlap joint in the interserial direction between columns of scales (Fig. 1.2p). The combination of both joints allows for complex relative motion in both axes, such as shear, splay, and double curvature [59].

Two fibrous systems provide interscale connectivity in the exoskeleton (Fig. 1.2q-t) [54-55, 84]. The peg-and-socket joint is reinforced, supported, and aligned by collagenous Sharpey’s fibers. The scales are connected to the underlying dermis through the stratum compactum, an organic, fibrous tissue extending from the axial ridge [55]. Fig. 1.2q shows the sites of attachment of Sharpey’s fibers and attachment of stratum compactum on the interior of the scale assembly, as well as the fiber layers of the stratum compactum that run along the paraserial and interserial axes [55].

The shapes and sizes of scales vary throughout the fish exoskeleton, from large, heavily featured scales in the anterior region to small, featureless scales in the tail, yet the body maintains small radii of axial body curvature throughout [55, 84]. The dorsal and ventral midlines, comprised of scales with specialized double-socketed or double-pegged geometries, respectively, represent axes of mirror symmetry in scale geometry and hold the paraserial columns together [55]. Armored, limb-like pectoral fins extend from the main body exoskeleton.

It has been shown that the ganoid integuments of bony, actinopterygian fish do not limit body flexibility compared to nonganoid integuments [55, 85], as was hypothesized in the past.
The complex shape, orientation, joint articulation, and assembly of scales in the exoskeleton do not limit the body curvature of *P. senegalus* during steady state undulatory motion and do not resist torsion, but rather guide bending of the fish in the horizontal plane [55]. In Polypterids and Lepisosteids, body curvature was found to be limited by the trunk musculature in concave bending and strain on the stratum compactum in convex bending, rather than by the assembly of armored scales [55]. Experimentally, it was shown that cutting the dermis between scale rows in an armored Lepisosteid (*Lepisosteus osseus*, longnose gar) significantly reduces the flexural stiffness of the body, increases the neutral zone of curvature, and alters the swimming kinematics of the fish, whereas surgical removal of a scale row does not affect the body’s flexural stiffness [85].

### 1.3.4. Preliminary steps toward bio-inspired flexible armor

Prior research has made preliminary attempts at translating the components of the *P. senegalus* exoskeleton to synthetic prototypes to study the underlying kinematic schema. An individual scale was unitized, enlarged, and 3D printed with plaster to probe the scale geometry, morphology, and joint interconnections, shown in Fig. 1.3a [59-60]. Inclusion of material heterogeneity in a single scale architecture demonstrated the potential for tailoring the individual scale units, shown in Fig. 1.3b [82]. Monolithic ring structures based on the *P. senegalus* exoskeleton clarified the roles of the joints in axial bending and transverse stiffening of the exoskeleton (Fig. 1.3c) [59]. A simplified scale design was created to allow two rotational degrees of freedom at the paraserial and interserial joints (Fig. 1.3d) [59]. The simplified scale designs were fabricated by 3D printing and assembled with rubber bands to mimic the fibrous interconnections (Fig. 1.3e) [59]. Inclusion of compliant materials, e.g. the Sharpey’s fibers and stratum compactum in *P. senegalus*, within the prototype was achieved using multi-material 3D printing using TangoPlus (a soft, rubber-like elastomer) and VeroWhite (a hard, rigid polymer) on the Objet Connex 500 3D printer (Stratasys, USA). Small assemblies probing the local articulation of scales replicated the local scale-to-scale joint ranges of motion in the synthetic prototypes in Fig. 1.3f [59]. Arrays of large scales with a homogeneous geometry were printed in Fig. 1.3g [59]. However, a functional prototype that fully integrates both materials and morphometric design rules has yet to be created, and the mechanical behavior of *P. senegalus*-inspired scale assemblies has yet to be analyzed.
Figure 1.3: Prior research in developing *P. senegalus* -inspired armor prototypes. (a) Two scales that have been unitized, enlarged, and 3D printed with plaster [59]. (b) Single scale incorporating material heterogeneity through the scale’s porous architecture [82]. (c) Monolithic semi-helical ring structures [59]. (d) Simplified scale design allowing two rotational degrees of freedom at the paraserial and interserial joints [59]. (e) Simplified scale design from (d) 3D-printed and assembled with rubber bands to mimic the fibrous interconnections [59]. (f) 1×2 arrays of scales incorporating compliant material elements in the peg-and-socket interconnections (top) and the attachment site to the underlying substrate (bottom) using multimaterial 3D printing [59]. (g) 5x5 flexible scale assembly [59].

1.4. References


42. AR500 Armor Trauma Plate. Last accessed January 20, 2016 at
44. DuPont Kevlar, Tactical Vests. Last accessed January 20, 2016 at
45. Condor Outdoor Lightweight Plate Carrier. Last accessed January 20, 2016 at


2. 2D Morphometric Heterogeneity of Scales in the *P. senegalus* Exoskeleton

The contents of this chapter were published as an original article in the *Journal of Structural Biology* in 2015 [1].

2.1. Overview

This chapter examines shape and shape variation as a structural design element in the armored exoskeleton of *P. senegalus*. Scale geometry is analyzed along the anteroposterior and dorsoventral axes of the *P. senegalus* exoskeleton and into its pectoral fin. Digital 3D reconstructions of the mineralized scales are generated using X-ray microcomputed tomography (µCT). Geometric variation among scales is investigated using landmark-based geometric morphometrics (GM), and a set of geometric parameters describing shape variation are defined. Furthermore, a continuum mechanical strain formalism is developed to quantify the morphometric variation. GM transformation grids and strain plots illustrate the mechanisms of shape morphing by modeling shape variation through the complex loading conditions of heterogeneous strain fields. Five scale geometry variants are defined, and their functional implications are discussed in terms of the interscale mobility mechanisms that allow flexibility within the exoskeleton. The method is further discussed to show that the application of strain fields to a given geometry can achieve a target morphology. While GM is often used to visualize inter-organismal geometric variation as a metric for species differentiation [2], this intraspecies analysis probes morphometric heterogeneity of subunits within one specimen.

2.2. Materials and Methods

2.2.1. X-ray microcomputed tomography (µCT)

Scales from a deceased *P. senegalus* specimen (22 cm body length) were scanned by µCT (VivaCT40, Scanco Medical AG, Switzerland) operated at 45 kV and 177 µA with no filter on the incident x-rays [3]. Microtomographic slices were recorded every 8–18 µm with 360° rotation and were reconstructed with 8×8 µm to 18×18 µm volume elements (voxels) in plane. A constrained 3D Gaussian filter (σ = 0.8 and support = 1) was used to partially suppress noise in the volumes. Medical imaging software (MIMICS 15.1, Materialise, Belgium) was used to threshold the reconstructed transverse slices by white value with lower (7,500) and upper (30,000) values chosen manually through trial and error to isolate the mineralized scales from the organic connective tissue elements attached to the scales, and to build 3D polygonal meshes, using a bilinear and interplane interpolation algorithm, which were exported as stereolithography objects (STL). The specimen used in this study, shown in Fig. 2.1a, has 56 columns and 18 rows of scales identified by their column and row number (C#R#).
Figure 2.1: The *P. senegalus* exoskeleton. (a) μCT reconstruction (18 μm scan resolution) of the *P. senegalus* exoskeleton used in the morphometric analysis, and (b) μCT reconstruction of a *P. senegalus* specimen in a fully extended configuration (95 μm scan resolution) [10]. Images show the paraserial axis of scale articulation (column), the interserial axis of scale overlap, the anterior-posterior row, dorsal and ventral midlines of mirror symmetry, and pectoral fin. Select scales are numbered by column and row number (C#R#). The asterisked scale (C9R5) in (a) is used in Fig. 2.2.

2.2.2. Geometric morphometric analysis in 2D

A custom-written visual basic script extracted the 3D spatial coordinates of mouse-clicked landmarks (LM) on the STL objects in CAD software (RHINOCEROS, Robert McNeel and Associates, USA). Twenty LMs, eighteen to define the 2D outline plus two interior points, were chosen to represent the 2D scale geometry, shown on a single scale (C9R5) in Fig. 2.2 and described in Table 2.1, based on their consistency of relative position, adequate coverage of form, and repeatability across scales [4-5]. LM coordinates were then: (i) translated to set the centroid (calculated as the 3D mean of all LM coordinates) as the origin of the local coordinate system, (ii) scaled by normalizing the vectors from the centroid to each LM by least squares fit to leave size-independent scale geometry, and (iii) rotated about the centroid to define a consistent coordinate system with LM18-LM6 defining the x-axis, the y-axis defined 90° perpendicular to the x-axis in the plane of the scale with LM14 having a +y value, and the z-axis through the thickness of the scale with LM 19 having a +z value.
Figure 2.2: μCT reconstruction (10 µm scan resolution) of a single *P. senegalus* scale (C9R5), (a) interior and (b) exterior views. Geometrical features are labeled: peg (P), socket (S), anterior process (AP), axial ridge (AR), and anterior margin (AM). The 20 LMs used for morphometric analysis are shown (red dots) and described in Table 2.1. The x-y coordinate axis is defined with the x-axis aligned with the scale’s paraserial axis, and y-axis defined 90° perpendicular to the x-axis in the plane of the scale. Two morphometric parameters are shown: peg tip angle (γ) and anterior process angle (θ).

Table 2.1: Typology and descriptions for the 20 LMs defining the 2D scale geometry.

<table>
<thead>
<tr>
<th>Landmark</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I</td>
<td>Posterior corner</td>
</tr>
<tr>
<td>2</td>
<td>III</td>
<td>Midpoint between dorsal and posterior corners</td>
</tr>
<tr>
<td>3</td>
<td>I</td>
<td>Dorsal corner</td>
</tr>
<tr>
<td>4</td>
<td>II</td>
<td>Dorsal base of peg</td>
</tr>
<tr>
<td>5</td>
<td>III</td>
<td>Midpoint between dorsal base and tip of peg</td>
</tr>
<tr>
<td>6</td>
<td>I</td>
<td>Tip of peg</td>
</tr>
<tr>
<td>7</td>
<td>III</td>
<td>Midpoint between anterior base and tip of peg</td>
</tr>
<tr>
<td>8</td>
<td>II</td>
<td>Anterior base of peg</td>
</tr>
<tr>
<td>9</td>
<td>II</td>
<td>Anterior point of anterior margin transition (interserial overlap)</td>
</tr>
<tr>
<td>10</td>
<td>I</td>
<td>Tip of anterior process (most anterior point)</td>
</tr>
<tr>
<td>11</td>
<td>II</td>
<td>Anterior point of paraserial overlap transition</td>
</tr>
<tr>
<td>12</td>
<td>II</td>
<td>Anterior margin valley</td>
</tr>
<tr>
<td>13</td>
<td>III</td>
<td>Midpoint between anterior margin valley and anterior margin peak</td>
</tr>
<tr>
<td>14</td>
<td>II</td>
<td>Anterior margin peak</td>
</tr>
<tr>
<td>15</td>
<td>II</td>
<td>Ventral point of paraserial overlap transition</td>
</tr>
<tr>
<td>16</td>
<td>I</td>
<td>Posterior point of anterior margin transition (interserial overlap)</td>
</tr>
<tr>
<td>17</td>
<td>I</td>
<td>Ventral corner</td>
</tr>
<tr>
<td>18</td>
<td>I</td>
<td>Midpoint of socket on edge of scale</td>
</tr>
<tr>
<td>19</td>
<td>II</td>
<td>Point between tip of peg and socket on the top side of scale</td>
</tr>
<tr>
<td>20</td>
<td>II</td>
<td>Central base of peg</td>
</tr>
</tbody>
</table>
The 18 LMs defining the 2D outline converged on the 2D polygonal area of the scale. **Fig. 2.3** shows a representative scale (C30R10) with 60 LMs numbered in order of their selection to define the outline of a scale; the first 18 LM used for the morphometric analysis are shown in red. **Fig. 2.3b** plots the 2D (x-y) polygonal area defined by a given number of LMs for five different scales. The polygonal area did not change significantly with increasing the density of LMs beyond the 18 used to define the outline of the scale in this morphometric analysis. Two additional LMs were chosen to define a central point on the top of the scale in the +z direction close to the origin (LM19 in **Fig. 2.2**) and the length of the peg (LM20 in **Fig. 2.2**). LM typology was assigned in accordance with Bookstein’s descriptions [5], where:

(i) Type I LMs have discrete position that are clearly identifiable with minimal possibility of error and can be defined independently from other LMs. They mark distinct vertexes in the scale geometry at LMs 1, 3, 6, 10, 16, 17, and 18.

(ii) Type II LMs have a possibility of placement error and may be positioned relative to other LMs, e.g. local minima or maxima in curvature. They describe loci that shift in relation to other geometric features, at LMs 4, 8, 9, 11, 12, 14, 15, 19, and 20.

(iii) Type III LMs describe geometric features that may be difficult to identify despite a morphological importance. They define LMs that capture scale shape between Type I and Type II LMs at LMs 2, 5, 7, and 13.

**Figure 2.3:** Selection criteria for the number of LMs used in the morphometric analysis. (a) Single scale (C31R10) with 60 LMs placed around outer surface, numbered in order of their selection for calculating polygonal area. LMs in red represent the first 18 LMs shown in **Fig. 2.2**. (b) Plot of polygonal area versus number of landmarks selected to define the outline of the scale, for five different scales in different regions in the *P. senegalus* exoskeleton.
A reference scale geometry was defined from the mean LM coordinates of all scales in the specimen. A custom-written MATLAB script generated transformation grids by first mapping the x-y coordinates of the reference geometry to a 2D grid with the centroid at the origin (x = 0, y = 0) in the center of the grid with grid spacing (t) = 0.1. Each grid element was labeled by (m, q) coordinates, starting from (m = 0, q = 0) in the lower left corner of Quadrant III (x < 0, y < 0), with m incrementing in the +y direction and q incrementing in the +x direction. For each transformation grid, a single scale was selected as the comparison geometry and placed on the grid. The grid was then deformed so that the reference geometry’s LMs are fitted to the comparison geometry’s LMs using thin-plate splines. The positional coordinates for vertices of each grid element in the resulting transformation grid were calculated as
\[ f(m, q) = \begin{bmatrix} f_x(m, q) \\ f_y(m, q) \end{bmatrix} \] according to Bookstein’s formulation [6]. The derivation of thin-plate spline formulas for generating 2D transformation grids is presented in Appendix A.

### 2.2.3. Continuum strain model in 2D

Strain contours were plotted onto the transformation grids in MATLAB. The GM deformations were taken as linear elastic in 2D (plane strain in the XY plane), and the components of the strain tensor for each grid element were computed as:

\[
\begin{align*}
\varepsilon_{11}(m, q) &= \frac{1}{dx} \left( \frac{\partial u_x}{\partial x} \right) dx = \frac{1}{t} (f_x(m, q + 1) - f_x(m, q) - t) \\
\varepsilon_{22}(m, q) &= \frac{1}{dy} \left( \frac{\partial u_y}{\partial y} \right) dy = \frac{1}{t} (f_y(m + 1, q) - f_y(m, q) - t) \\
\varepsilon_{12}(m, q) &= \frac{1}{2} \left( \frac{1}{dx} \left( \frac{\partial u_y}{\partial x} \right) dx \right) + \frac{1}{dy} \left( \frac{\partial u_x}{\partial y} \right) dy \\
&= \frac{1}{2t} (f_y(m, q + 1) - f_y(m, q) + f_x(m + 1, q) - f_x(m, q))
\end{align*}
\]

where t is the grid spacing in both the x- and y-directions. Axial strains measure length changes along a given direction; for instance, positive and negative values of \( \varepsilon_{11} \) correspond to expansion and compression along the x-axis, respectively. Shear strain (\( \varepsilon_{12} \)) measures changes in angles with respect to both x- and y-axes in plane. The hydrostatic strain for each grid element, representing the strain associated with grid element area change, was computed as:

\[ \varepsilon_{hyd}(m, q) = \frac{1}{2} (\varepsilon_{11}(m, q) + \varepsilon_{22}(m, q)) \]

The deviatoric strain tensor was computed as:

\[ \varepsilon_{dev}(m, q) = \begin{bmatrix} \varepsilon_{11}(m, q) - \varepsilon_{hyd}(m, q) & \varepsilon_{12}(m, q) \\ \varepsilon_{12}(m, q) & \varepsilon_{22}(m, q) - \varepsilon_{hyd}(m, q) \end{bmatrix} \]
and the Euclidean norm was taken to provide a scalar value representing the shape change of the grid elements. Considering the shape change as plane strain in the XY plane, the 2D strain energy density for each grid element was calculated as:

\[ u(m, q) = \frac{1}{2} \sigma^T \varepsilon = \frac{1}{2} [(2\mu + \lambda)(\varepsilon_{11}(m, q)^2 + \varepsilon_{22}(m, q)^2) + 2\lambda(\varepsilon_{11}(m, q) \cdot \varepsilon_{22}(m, q)) + \mu (\gamma_{12}(m, q)^2)], \]

where \( \mu = \frac{E}{2(1-\nu)} \), \( \lambda = \frac{E \nu}{(1+\nu)(1-2\nu)} \), \( \gamma_{12} = 2\varepsilon_{12} \), elastic modulus \( E = 55 \text{ GPa} \) (modulus of ganoine, the outer layer of ganoid scales [7-8]), and Poisson ratio \( \nu = 0.3 \). Total strain energy (\( U \)) over the 2D transformation grid was calculated as:

\[ U = \sum_{m,q} u(m, q) \cdot t^2, \]

a scalar value representing the energy needed to morph the reference geometry into the comparison geometry, essentially describing how different a comparison geometry is from the reference geometry.

To illustrate the meaning of these strain components and their correlation with the deformation of grid elements in the geometric morphometric analysis, the strain associated with simple affine transformations applied to the reference \( P. \) senegalus geometry in shape space (compression with equal and opposite extension, simple shear) and form space (scaling) were calculated and plotted as strain contours on their associated transformation grids in Fig. 2.4. Transformation grids deform uniformly with all grid element edges remaining parallel for affine transformations, and the resulting strain contours are uniform. In compression (Fig. 2.4b), axial strains are nonzero (\( \varepsilon_{11} = -\varepsilon_{22} = -0.2 \)) and shear strain is zero; hydrostatic strain is also zero since there is no grid element area change associated with this deformation, however the deviatoric strain is nonzero (\( \varepsilon_{\text{dev}} = +0.2 \)) because the grid elements change shape. In simple shear (Fig. 2.4c), axial strains are zero while shear strain is nonzero (\( \varepsilon_{12} = +0.2 \)); there is no hydrostatic strain as the grid element area does not change, however the grid element shape change causes deviatoric strain to be nonzero (\( \varepsilon_{\text{dev}} = +0.2 \)). In uniform scaling (Fig. 2.4d), axial strains are nonzero (\( \varepsilon_{11} = \varepsilon_{22} = +0.2 \)), shear strain is zero, hydrostatic strain is nonzero (\( \varepsilon_{\text{hyd}} = +0.2 \)) since all grid elements grow in area, and deviatoric strain is zero since no elements deform in shape. Furthermore, the gradual morphing of a shape and total strain energy through a series of affine shear operations applied to the reference geometry is illustrated in Fig. 2.5. The transformation grids and shear strain plots show that the geometry morphs uniformly with shear operations from \( \varepsilon_{12} = -0.5 \) to +0.5. The total strain energy (\( U \), normalized by maximum value) increases linearly as the grid elements distort with the magnitude of shear.

2.2.4. Statistics

Boxplots of unitless parameter values were generated in MATLAB with the Plotly toolbox. Statistical software (JMP Pro 11, SAS Institute, USA) was used for the student’s t-test.
Figure 2.4: Landmark-based GM and continuum strain analysis of affine transformations. (a) Reference scale geometry (blue) mapped to an undeformed grid. (b-d) Transformation grids (T) and strain plots mapping the components of strain tensor (\(\varepsilon_{11}, \varepsilon_{22}, \) and \(\varepsilon_{12}\)), hydrostatic strain (\(\varepsilon_{\text{hyd}}\)) and the norm of the deviatoric strain tensor (\(\varepsilon_{\text{dev}}\)) for affine transformations (red) applied to the reference geometry (blue): (b) compression (\(\varepsilon_{11} = -\varepsilon_{22} = -0.2\)), (c) simple shear (\(\varepsilon_{12} = +0.2\)) and (d) uniform scaling (\(\varepsilon_{11} = \varepsilon_{22} = +0.2\)).
Figure 2.5: Transformation grids (T) and strain plots mapping the shear component of strain ($\varepsilon_{12}$) for a series of affine shear operations (red) applied to the reference geometry (blue): (a) $\varepsilon_{12} = -0.5$, (b) $\varepsilon_{12} = -0.4$, (c) $\varepsilon_{12} = -0.3$, (d) $\varepsilon_{12} = -0.2$, (e) $\varepsilon_{12} = -0.1$, (f) $\varepsilon_{12} = 0.0$, (g) $\varepsilon_{12} = +0.1$, (h) $\varepsilon_{12} = +0.2$, (i) $\varepsilon_{12} = +0.3$, (j) $\varepsilon_{12} = +0.4$, (k) $\varepsilon_{12} = +0.5$, and (l) plot of the total strain energy ($U$, normalized by maximum value) vs. shear strain.
2.3. Results

The mineralized exoskeleton of *P. senegalus* provides full body coverage. The specimen used in this study, shown in the µCT reconstruction in Fig. 2.1a, has 56 columns and 18 rows of scales identified by column and row number (C#R#). Another specimen in an extended configuration (Fig. 2.1b) illustrates the arrangement of scales in the integument along two interwoven, semi-helical axes: the oblique paraserial axis of scale articulation and the interserial axis of scale overlap [9-10]. The dorsal and ventral midlines, comprised of scales with specialized double-socketed or double-pegged geometries, respectively, represent axes of mirror symmetry in scale geometry and hold the paraserial columns together; these scales are excluded from the subsequent analysis. Armored, limb-like pectoral fins extend from the main body exoskeleton.

The scale geometry is complex. Fig. 2.2 shows interior and exterior view of a single scale (C9R5) with geometric features highlighted: peg (P) and socket (S) which define the peg-and-socket joint along the paraserial axis, the anterior process (AP) and anterior margin (AM) which form the sliding overlap joint, and the axial ridge (AR) which serves as the site of attachment for the stratum compactum to the dermis. The 2D geometry of the scale is represented by 20 LMs, shown in Fig. 2.2 and described in Table 2.1. Five size-independent parameters, listed in Table 2.2, are calculated from LM coordinates: peg length (PL), AP length (APL), diagonal aspect ratio (DAR), peg tip angle (γ), and AP angle (θ). Scale volume (V) is a size dependent parameter extracted from the µCT reconstruction of the scale. Total strain energy (U) is a size independent parameter calculated from strain formulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PL</td>
<td>Peg length</td>
<td>Distance between LM6-LM20</td>
</tr>
<tr>
<td>APL</td>
<td>Anterior process length</td>
<td>Distance between LM10-LM12</td>
</tr>
<tr>
<td>DAR</td>
<td>Diagonal aspect ratio</td>
<td>Distance between LM3-LM16 / Distance between LM1-LM10</td>
</tr>
<tr>
<td>γ</td>
<td>Peg tip angle</td>
<td>Angle between LM3-LM16 / LM1-LM10</td>
</tr>
<tr>
<td>θ</td>
<td>Anterior process angle</td>
<td>Angle between LM10-LM19-LM6</td>
</tr>
<tr>
<td>V</td>
<td>Volume of scale</td>
<td>Volume of the STL object of the scale</td>
</tr>
<tr>
<td>U</td>
<td>Strain energy</td>
<td>$U = \sum_{m,q} u(m,q) * t^2$, where $u(m,q) = \frac{1}{2} [(2\mu + \lambda) * (\varepsilon_{11}(m,q))^2 + \varepsilon_{22}(m,q)^2) + 2\lambda*(\varepsilon_{11}(m,q) * \varepsilon_{22}(m,q)) + \mu \gamma_{12}(m,q)^2]$</td>
</tr>
</tbody>
</table>
2.3.1. Morphometric variation of scales along the anteroposterior axis

The scale geometry morphs gradually along the anteroposterior axis. Fig. 2.6a shows μCT images of scales across a single row (R6), depicted in the sketch in Fig. 2.6b. Scale volume (V, normalized by maximum value over all scales) increases from 0.30–1.0 from the anterior region to midsection of the fish (C1R6-C24R6) and then decreases to 0.069 in the tail (C25R6-C56R6). The global shape morphs from a complex polygon to a simplified rhombus in the tail. In Fig. 2.6c, GM transformation grids show the deformation of a square grid fit to the reference geometry (in blue) to generate the biological geometry (in red) of three representative scales from the anterior (C5R6), midsection (C25R6), and tail (C45R6). These plots show that shape change is dominated by shearing of the global geometry as the DAR decreases from 0.69 (C5R6) to 0.59 (C25R6) to 0.54 (C45R6), supplemented by localized compression and expansion around geometric features.

Components of the 2D strain tensor (ɛ_{11}, ɛ_{22}, and ɛ_{12}) were calculated for each grid element in the transformation grid and plotted in Fig. 2.6c to provide enhanced visual contrast of the mechanisms governing shape change. Shear strain (ɛ_{12}) plots show the global shearing phenomenon: for C5R6 the grid shears to the left and ɛ_{12} is predominantly negative, C25R6 shows low magnitudes of shear strain, and C45R6 has several regions of high, positive shear strain. Localized regions of axial and shear strain correlate with length scale changes in geometric features of scales. The peg is longer anteriorly with a similar sharpness (PL = 0.12, γ = 71.7° for C5R6) compared to the reference geometry, which is closely represented by C25R6 (PL = 0.11, γ = 71.4°), while shorter and flatter in the tail (PL = 0.048, γ = 79.6° for C45R6), visualized through localized axial and shear strain around the peg. The peg disappears caudally (PL = 0.02, γ = 173° for C56R6). In anterior scales, the AP has a similar length to the midsection scales (APL = 0.26 for C5R6, APL = 0.25 for C25R6), but the concave AM is more pronounced as visualized by the negative axial strains around LM12 for C5R6. Tail scales have shorter APs and flat AMs (APL = 0.12 for C45R6), visualized by the negative axial strain around LM10 and positive axial strain around LM12. Orientation of the AP relative to the peg remains constant along the row of scales (θ = 47.8° for C5R6, θ = 49.6° for C25R6, θ = 49.3° for C45R6).

Total strain energy (U, normalized by maximum value over all scales) represents the energy needed to morph the reference geometry into the comparison geometry. Lower values of U correspond to scale geometries that are most similar to the reference geometry, e.g. midsection scales (C25R6-C40R6) with U < 0.1. Anterior scales have U ranging from 0.1-0.2, representing geometries that vary moderately from the reference geometry. Caudal scales see U climbing from 0.1-1.0 (C42R6-C56R6) representing geometries with large variance from the reference geometry. Hydrostatic strain (ɛ_{hyd}) and norm of the deviatoric strain (ɛ_{dev}) are plotted in Fig. 2.6c, and show that the anterior scales differ from the average scale geometry by AM shape and shearing of the global geometry, while the tail scales differ by the AM and AP shape.
Figure 2.6: Scale shape variation from anterior to posterior. (a) μCT reconstruction of scales within a single row (R6) of the *P. senegalus* specimen, every fifth scale shown (15 μm scan resolution). (b) Schematic of the exoskeleton illustrating the row of scales used for the analysis. (c) Transformation grids (T) compare the geometry of three scales (C5R6, C25R6, and C45R6; in red) to the reference geometry (blue). Strain plots map the components of the strain tensor (\(\varepsilon_{11}, \varepsilon_{22}, \text{ and } \varepsilon_{12}\)), hydrostatic strain (\(\varepsilon_{\text{hyd}}\)) and norm of the deviatoric strain tensor (\(\varepsilon_{\text{dev}}\)) for each grid element.
2.3.2. Morphometric variation of scales along the dorsoventral axis

Scale geometry morphs rapidly dorsoventrally through the transformation grids. Fig. 2.7a shows every other scale down a single, paraserial column of scales (C22) in the specimen’s midsection, depicted in the corresponding sketch in Fig. 2.7b. Scale volume increases from 0.36-0.88 in the dorsal region (C22R2-C22R7) and decreases to 0.15 ventrally (C22R8-C22R17). AP length and orientation and AM shape change with scale volume. Global shape change is characterized by a shrinking aspect ratio and the skewing of geometric features relative to each other. Transformation grids for three representative scales (C22R2, C22R8, and C22R14) in Fig. 2.7c show that scale shape change is dominated by a combination of y-directional compression and shearing of the global geometry as the DAR decreases from 0.60 (C22R2) to 0.58 (C22R8) to 0.36 (C22R14), supplemented by localized compression and expansion.

Strain plots in Fig. 2.7c show global compression: dorsal (C22R2) and ventral (C22R14) scales are compressed in the y-direction compared to the reference geometry, while middle scales (C22R8) are expanded in the y-direction. Localized regions of axial and shear strain correlate with distortion of geometric features. The peg is sharper dorsally (PL = 0.10, γ = 59.2° for C22R2) and flatter ventrally (PL = 0.11, γ = 69.0° for C22R8; PL = 0.092, γ = 94.0° for C22R14) through localized axial and shear strain about the peg. The socket disappears in ventral scales (interior view of scales not shown). The AP rotates away from the peg as θ increases from 33.7° (C22R2) to 57.8° (C22R8) to 61.4° (C22R14) while APL increases from 0.18 (C22R2) to 0.26 (C22R8) to 0.28 (C22R14), correlated with positive axial and shear strain in the ventral scale between the peg and AP. The AM shape changes drastically. Middle scales have a more pronounced AM, evidenced by axial and shear strains that alternate between negative and positive values between LM11-LM17. The AM flattens in dorsal and ventral scales. Middle scales (C22R6-C22R9) have U ≤ 0.1, representing geometries that vary moderately from the reference geometry. Dorsal scales have U ranging from 0.1-0.2, while ventral scales have U ranging from 0.1-0.4. Plots of ε\text{dev} and ε\text{hyd} show that C22R2 differs from the average scale geometry by peg and AM shape, C22R8 differs by AM and AP shape, and C22R14 differs by θ in addition to peg and AM shape.

2.3.3. Morphometric variation of scales in the pectoral fin

The exterior surface of the pectoral fin, shown in Fig. 2.8a, is covered with mineralized scales. The connection between the main exoskeleton and pectoral fin is fully armored and articulated. A row of seven scales (labeled P1-P7) transitioning from the anterior of the main exoskeleton into the pectoral fin is shown in Fig. 2.8b. The global shape morphs rapidly over ~7 mm from a complex geometry to simplified quadrilateral with diminished features. Scale volume decreases from 0.11-0.035 over the seam. The transformation grids for three representative scales (P2, P4, P6) in Fig. 2.8c show that shape change is dominated by a combination of y-directional expansion and shearing of the global geometry as DAR increases from 0.72 (P2) to 0.75 (P4) to 1.0 (P6), supplemented by localized compression and expansion.
Figure 2.7: Scale shape variation from dorsal to ventral. (a) μCT reconstruction of scales within a single column (C22) of the P. senegalus specimen, every other scale shown (15 µm scan resolution). (b) Schematic of the exoskeleton illustrating the column of scales used for the analysis. (c) Transformation grids (T) compare the geometry of three scales (C22R2, C22R8, and C22R14; in red) to the reference geometry (blue). Strain plots map the components of the strain tensor (ε₁₁, ε₂₂, and ε₁₂), hydrostatic strain (εʰyd) and norm of the deviatoric strain tensor (εdev) at each grid point.
Figure 2.8: The *P. senegalus* pectoral fin and scale shape variation along a transition seam. (a) μCT reconstruction of the left pectoral fin (8 µm scan resolution) highlighting a transition row segment. (b) Exterior (top) and interior (bottom) views of an articulated seam of scales connecting the main body to the pectoral fin. (c) Transformation grids (T) compare the geometry of three scales (P2, P4, and P6; in red) to the reference geometry (blue). Strain plots map the components of the strain tensor ($\varepsilon_{11}$, $\varepsilon_{22}$, and $\varepsilon_{12}$), hydrostatic strain ($\varepsilon_{\text{hyd}}$) and norm of the deviatoric strain tensor ($\varepsilon_{\text{dev}}$) at each grid point.
Strain plots in Fig. 2.8c illustrate that the almost-average geometry (P2) first expands in the +y direction (P4), and then shears in the -xy direction (P6). Locally, the peg shortens and flattens over the seam (PL = 0.098, γ = 79.0° for P2; PL = 0.040, γ = 130° for P4; PL = 0.034, γ = 148° for P6) through regional axial and shear strain into an overlap joint and the socket disappears. The AP rotates away from the peg along the seam while the APL remains relatively unchanged (θ = 60.6°, APL = 0.14 for P2; θ = 67.1°, APL = 0.13 for P4; θ = 84.1°, APL = 0.12 for P6). The AM flattens, shown by alternating negative and positive axial and shear strains between LM11-LM17. Scales in the beginning of the seam (P1-P3) that resemble the average scale geometry have U ranging from 0.15-0.25. Remaining scales (P4-P7) have U ranging from 0.3-0.55. Plots of ε_{dev} and ε_{hyd} show that P2 differs from the average scale geometry by AP and AM shape, P4 differs by AM, AP, and peg shape, and P6 differs by peg and AP shape.

### 2.3.4. Defining statistically heterogeneous scale variants

The range of morphometric parameters for the row, column, and pectoral fin seam are plotted as normalized quantities in Fig. 2.9. The row exhibits the widest spread in all parameter values, perhaps due to containing more scales and spanning a greater distance in the exoskeleton. The column exhibits a wide spread in DAR, V, and U. The seam, despite containing only seven scales with similar volumes, exhibits a wide spread in PL, DAR, γ, and U. Five zones of scale geometry variants were defined: anterior, average (midsection), tail, ventral, and pectoral fin. Parameter values of scales in these five zones were pooled from the three data sets plus additional scales in the exoskeleton, and the student’s t-test was used to determine statistical significance in parameter value differences by zone. P-values from the student’s t-test are presented in Table 2.3. The five scale variants differ from each other through combinations of the morphometric parameters, and their morphometries are statistically heterogeneous.

**Figure 2.9:** Boxplots for the range of values of morphometric parameters over the row, column, and pectoral fin seam. γ and θ are normalized by 180°, and V and U are normalized by their maximum value over all scales (5.44 mm³ and 2.1x10¹⁰ J, respectively) to generate unitless parameter values. Data are segmented into quartiles with no outliers.
Table 2.3: P-values from student’s t-test of morphometric parameters of scales grouped by region in the fish exoskeleton: average (N = 30 scales), anterior (N = 36 scales), tail (N = 21 scales), ventral (N = 41 scales), and pectoral fin (N = 14 scales). Asterisks (*) represent p-values < 0.01.

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2.4. Discussion

The *P. senegalus* integument utilizes heterogeneity in scale morphometry to generate a protective exoskeleton that provides uniform coverage and preserves biomechanical flexibility. The trends in morphometric variation observed in this study are consistent with literature reports along the dorsoventral [11] and anteroposterior [9] axes. The *P. senegalus* integument is capable of achieving the same extreme curvature in axial bending in every location along the entire body, however the mechanisms by which these curvatures are achieved vary depending on the morphometry of the scale [9]. Constraints on the allowable ranges of motion between *P. senegalus* scales are defined by joint type along the paraserial and interserial axes. A schematic of four scales in their native state (straight fish body) are shown in Fig. 2.10a. The interserial overlap joint allows for sliding (Fig. 2.10b) as the AM and AP slide under the adjacent scale;
sliding is limited by the width of the AR, which serves as the site of connection to the stratum compactum and underlying dermis. The scales can also splay interserially (Fig. 2.10c) as paraserial columns rotate away from each other. Small paraserial rotations (Fig. 2.10d) and bending (Fig. 2.10e) may be allowed within a column of scales, limited by the dorsal (peg and surrounding area) and ventral (socket and surrounding area) surfaces of connection and the collagenous Sharpey’s fibers that reinforce, support, and align the joint. The interscale mobility mechanisms that are used by the defined five scale variants are described below:

1. **Average** scales are large and shaped similar the reference geometry with pronounced features. The peg and socket provide tight articulation for structural rigidity down a column of scales to resist paraserial rotation, paraserial bending, and interserial splay in order to sustain compressive and tensile strains along the paraserial axis [9, 11]. The long AP and concave AM allow for interserial sliding in axial bending and torsion. Gemballa and Bartsch [9] measured the degree of interserial sliding in axial bending on live specimens; they define ‘lateral strain and ‘lateral stress’ as percent change in scale overlap between adjacent scales on concave and convex sides of the body, respectively, normalized to 1.0 representing a straight body [9]. When the fish trunk is bent, scales on the concave side of the exoskeleton increase their scale overlap via sliding, while scales on the convex side decrease overlap by sliding apart. Scales in the mid-section of the trunk can achieve lateral strains ranging from 0.61-0.80 and lateral stresses ranging from 1.21-1.24 [9].

2. **Anterior** scales are smaller than the average scale, and their pure shape is negatively sheared from the average scale. Expansion of the peg reinforces a tight peg-and-socket joint to further restrict paraserial rotation, paraserial bending, and interserial splay enabling the scales to sustain the helical component of torsional strain in the integument [9, 11]. The AR is narrower and negative axial and shear strain contours around LM12 yield a more pronounced AM alongside a large AP to achieve greater degrees of interserial sliding (lateral strains of 0.42-0.46 and lateral stresses of 1.34-1.40 [9]), although the fish rarely flexes the anterior region of its trunk during steady state swimming.

3. **Tail** scales are smaller with a less complex geometry that is positively sheared from the average scale geometry. Negative axial strain contours around LM10 in conjunction with positive axial strain around LM12 yield a short AP, flat AM, and wide AR so that the tail scales achieve small degrees of interserial sliding (lateral strains of 0.92-0.96 and lateral stresses of 1.00-1.01 [9]). The fish flexes its tail rapidly during steady state swimming and achieves high curvatures. Localized axial and shear strain shorten and flatten the peg to relax paraserial rotational and bending constraints, while the smaller subunit size of tail scales allows for smaller radii of curvature before scales touch in the segmented assembly. These geometric modifications allow the tail to accommodate flexure with small changes in scale overlap.
4. **Ventral** scales are globally compressed in the y-direction and positively sheared from the average geometry. Regional positive axial and shear strains flatten the pronounced pegs without changing their length, so that the peg-and-socket joint continues to restrict paraserial rotation. However, the vanishing socket in ventral scales allows them to paraserially bend and interserially splay within the plane of the integument. Since the plane of the integument in the ventral region of the body is no longer in the same plane as axial bending and torsion, interscale bending and splay are necessary to allow the exoskeleton curve. Alternating negative and positive axial and shear strains between LM10-LM17 lengthen the AP and flatten the AM to assist splay between scales while providing full body coverage during body flexion.

5. **Pectoral fin** scales are small, quadrilateral, with variable shapes that are globally compressed in the x-direction, expanded in the y-direction, and negatively sheared from the average scale geometry. Complex 3D oscillations of the pectoral fin [9] are enabled by the scale morphometry. Axial expansion and compression of the peg in opposite directions flatten the peg-and-socket joint into a second overlap joint, thereby allowing for sliding in both the interserial and paraserial axes while removing restrictions on paraserial rotation and bending. Alternating negative and positive axial and shear strains between LM10-LM17 shorten the AP and flatten the AM to promote small radii of curvature during pectoral fin flexure. The transition between the anterior region of the exoskeleton and the pectoral fin is a materially homogeneous seam that utilizes a rapid morphometric gradient, which allows for uniform coverage and eliminates weak spots that often occur in materially heterogeneous seams.

![Figure 2.10](image)

**Figure 2.10:** Schematic of interscale mobility mechanisms. (a) A quad of scales showing interserial and paraserial axis in native (resting) state. Motion of scales can be broken into four components: (b) interserial sliding, (c) interserial splay, (d) paraserial rotation, and (e) paraserial bending.
This analysis of pure shape change shows how a starting geometry can be stretched or compressed to achieve a new geometry through the application of localized strain fields. This analysis ignores the implications of scale size and orientation discussed by Gemballa and Bartsch [9]. Landmark-based GM assesses shape variation by comparing landmarks placed on distinct and consistently identifiable anatomical loci on all specimens after removing rigid body transformations [4-5, 12-13] and then visualizing statistical patterns in the deformation of these loci by fitting thin-plate splines [6]. Splines are a computationally simple method for curve and surface interpolations [14]. Unlike smoothing algorithms (e.g. Gaussian) which output a weighted average of pixel distortion around each pixel, splines construct a curve of minimum curvature that passes through a set of reference points and then bend that curve to fit the landmarks of a comparison geometry. The warped image of the transformation grids is constructed by interpolating the curves throughout the plane [15]. The thin-plate spline method provides flexibility to adapt to any suitable subset of landmarks defining a shape, and it is easily extendable from 2D to 3D [16], for instance to study parameters such as thickness, curvature, and angle of overlap in the *P. senegalus* scales.

Here, morphometric heterogeneity is assessed with strain plots calculated using continuum element formulations. Recent literature postulated that computing strain may serve as a way to show how and in which ways geometries differ from each other [16-19]. GM deformations are solutions of the biharmonic equation to minimize bending energy, similar to finite element analysis with explicit bending or integrals of the deformation [18]. By using continuum element formulations to compute 2D strain fields from the thin-plate spline transformations, GM is used to turn observed deformations into computed fictitious forces to better assess how a shape can morph through distinct geometries along a sequence.

To validate the finite element computation of strain, the computed strain fields comparing a biological scale to the reference geometry can be applied as a heterogeneous strain field to the reference geometry to recompose the biological scale geometry. Essentially, a heterogeneous strain field can be applied to a shape in order to morph it into a target geometry. First, the transformation grid and associated strain fields comparing a biological scale (C10R6) to the reference geometry are computed, shown in Fig. 2.11a. The values for each component of the strain tensor ($\varepsilon_{11}$, $\varepsilon_{22}$, and $\varepsilon_{12}$) at each landmark are extracted. Then, returning to the reference geometry mapped onto an undeformed grid in Fig. 2.11b, the components of the heterogeneous strain field are applied to the landmarks of the reference geometry. After applying all three components, the strained reference geometry is equivalent to the target biological geometry. It is concluded that fitting thin-plate splines to deform a reference geometry to a comparison geometry is mathematically equivalent to applying the heterogeneous strain field.

The *P. senegalus* exoskeleton combines geometry-based and materials-based structural design principles to generate a protective structure that provides uniform coverage and preserves biomechanical flexibility. In this study, landmark-based GM was used to analyze the spatial morphometric variation of *P. senegalus* scales. Transformation grids based on thin-plate splines
visualize the difference in scale geometries. 2D strain plots based on continuum mechanics formulations further illustrate the mechanisms by which geometries morph from each other. Total strain energy provides a quantitative basis to determine how different given geometries are from each other. Geometric heterogeneity in scale shape divides the *P. senegalus* exoskeleton into five variants (average, anterior, tail, ventral, and pectoral fin) which utilize different shape-based joint articulation mechanism to move relative to each other within the integument. The technique can also be used to morph a shape into a target geometry with the application of a heterogeneous strain field. In the future, this technique may be used to analyze the geometry change and concomitant changes in interscale mobility mechanisms during the fish’s growth and development, for instance comparing the juvenile elasmoid scales of *P. senegalus* to the adult ganoid scales [20]. The results can be directly used to develop computational shape morphing algorithms such as our MetaMesh model [21], which defines local, regional, and global hierarchies of design rules of an articulated assembly of *P. senegalus*-inspired sub-units that adapts to complex hosting surfaces.

![Figure 2.11: Recomposing morphometric variation by applying strain. (a) Transformation grid (T) and strain tensor components (ε₁₁, ε₂₂, and ε₁₂) comparing a biological scale geometry (C10R6, in red) to the reference geometry (blue). (b) Transformation grids resulting from the application of each component of the strain tensor (ε₁₁, ε₂₂, and ε₁₂) found in (a) to the reference geometry. Applying all three components of the strain tensor to the reference geometry recovers the geometry of the biological scale.](image-url)
2.5. References


3. 3D Morphometric Heterogeneity of Scales in the 
*P. senegalus* Exoskeleton

3.1. Introduction

The scale shape, shape variation, and interlocking mechanisms in the *P. senegalus* exoskeleton are three-dimensional in nature. In this chapter, the 2D morphometric analysis presented in Chapter 2 is extended to 3D in order to study the effects of 3D parameters. Scale geometry is analyzed along the anteroposterior and dorsoventral axes of the *P. senegalus* exoskeleton and into its pectoral fin, and the results are analyzed with regard to the five heterogeneous scale variants defined in Chapter 2. The landmarks defining the geometry of the scale, the geometric morphometric analysis, and continuum strain model are updated to include z-components. Additionally, a new set of parameters are defined to analyze the scale geometry in 3D, including thickness, curvature, and angle of overlap. Shape variation within the *P. senegalus* exoskeleton is further discussed in a broader context of computational design and modeling material properties, adding complexity to the understanding of the 3D joints and articulation mechanisms enabling mobility in the *P. senegalus* exoskeleton.

3.2. Materials and Methods

3.2.1. X-ray microcomputed tomography (µCT)

Scales from a deceased *P. senegalus* specimen (22 cm body length) were scanned by µCT (VivaCT40, Scanco Medical AG, Switzerland) and reconstructed into digital 3D stereolithography (STL) objects as described in Chapter 2.2.1. The specimen used in this study has 56 columns and 18 rows of scales identified by their column and row number (C#R#).

3.2.2. Geometric morphometric analysis in 3D

A custom-written visual basic script extracted the 3D spatial coordinates of mouse-clicked landmarks (LM) on the STL objects in CAD software (RHINOCEROUS, Robert McNeel and Associates, USA). The LM definitions and transformation grid computations were updated for 3D calculations. 21 LMs were chosen to represent the 3D scale shape: the 20 LMs defined in Chapter 2.2.2 were assigned a z-component based on their location through the thickness of the scale, and a new LM21 was chosen to represent a central point on the bottom of the scale opposite LM19. These LMs are shown on a single scale (C9R5) in Fig. 3.1 and described in Table 3.1 with typologies assigned according to Bookstein’s definitions [1]. LM coordinates were then: (i) translated to set the centroid (calculated as the 3D mean of all LM coordinates) as the origin of the local coordinate system, (ii) scaled by normalizing the vectors from the centroid to each LM by least squares fit to leave size-independent scale geometry, and (iii) rotated about the centroid to define a consistent coordinate system with LM18-LM6 defining
Figure 3.1: μCT reconstruction (10 µm scan resolution) of a single P. senegalus scale (C9R5) (a) interior and (b) exterior views showing the 21 LMs (red dots) used in the 3D morphometric analysis, described in Table 3.1. The coordinate systems is defined from the centroid with the x-axis aligned with the scale’s paraserial axis, and y-axis defined 90° perpendicular to the x-axis in the plane of the scale, and the z-axis vertically through the thickness of the scale point to the exterior (top) surface of the scale.

The x-axis, the y-axis defined 90° perpendicular to the x-axis in the plane of the scale with LM14 having a +y value, and the z-axis through the thickness of the scale with LM 19 having a +z value.

A reference scale geometry was defined from the mean LM coordinates of all scales in the specimen. A custom-written MATLAB script generated transformation grids by first mapping the x-y-z coordinates of the reference geometry to a 3D grid with the centroid at the origin (x = 0, y = 0, z = 0) in the center of the grid with grid spacing (t) = 0.1. Each grid element was labeled by (m, q, s) coordinates, starting from (m = 0, q = 0, s = 0) in the lower left corner of Quadrant III (x < 0, y < 0, z < 0), with m incrementing in the +y direction, q incrementing in the +x direction, and s incrementing in the +z direction. For each transformation grid, a single scale was selected as the comparison geometry and placed on the grid. The grid was then deformed so that the reference geometry’s LMs are fitted to the comparison geometry’s LMs using thin-plate splines. The positional coordinates for vertices of each grid element in the resulting transformation grid were calculated as \( f(m, q, s) = [f_x(m, q, s), f_y(m, q, s), f_z(m, q, s)] \) as defined by Weber’s (2011) formulation [2]. The derivation of thin-plate spline formulas for generating 3D transformation grids is presented in Appendix A. Additionally, 2D slices of the 3D grid were calculated to supplement the analysis.
Table 3.1: Landmark typology and descriptions for the 21 LMs defining the 3D scale geometry.

<table>
<thead>
<tr>
<th>Landmark</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>Posterior corner, bottom of scale</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>Midpoint between dorsal and posterior corners, top of scale</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>Dorsal corner, top of scale</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>Dorsal base of peg, edge of scale (from above)</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>Midpoint between dorsal base and tip of peg, edge of scale (from above)</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>Tip of peg, edge of scale (from above)</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>Midpoint between ventral base and tip of peg edge of scale (from above)</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>Ventral base of peg edge of scale (from above)</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>Anterior point of paraserial overlap transition surrounding the peg, edge of scale (from bottom)</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>Tip of anterior process (most anterior point), edge of scale (from above)</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>Anterior point of paraserial overlap transition surrounding the peg, edge of scale (from bottom)</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>Anterior margin valley, edge of scale (from bottom)</td>
</tr>
<tr>
<td>13</td>
<td>3</td>
<td>Midpoint between anterior margin valley and anterior margin peak, edge of scale (from bottom)</td>
</tr>
<tr>
<td>14</td>
<td>2</td>
<td>Anterior margin peak, edge of scale (from bottom)</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>Ventral point of paraserial overlap transition surrounding the socket, edge of scale (from bottom)</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>Posterior point of axial margin transition (interserial overlap), top of scale</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>Ventral corner, top of scale</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>Midpoint of socket on edge of scale, top of scale</td>
</tr>
<tr>
<td>19</td>
<td>2</td>
<td>Midpoint between tip of peg and socket, top of scale</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>Base of peg, top of scale</td>
</tr>
<tr>
<td>21</td>
<td>2</td>
<td>Midpoint opposite of LM 19 on bottom of scale</td>
</tr>
</tbody>
</table>

3.2.3. Continuum strain model in 3D

Strain contours were plotted onto the medial 2D slices of the transformation grids in each plane using MATLAB. The GM deformations were taken as linear elastic, and the components of the strain tensor for each grid element were computed as:

\[
\varepsilon_{11}(m, q, s) = \frac{1}{dx} \left( \frac{\partial u_x}{\partial x} dx \right) = \frac{1}{t} (f_x(m, q + 1, s) - f_x(m, q, s) - t)
\]

\[
\varepsilon_{22}(m, q, s) = \frac{1}{dy} \left( \frac{\partial u_y}{\partial y} dy \right) = \frac{1}{t} (f_y(m + 1, q, s) - f_y(m, q, s) - t)
\]

\[
\varepsilon_{33}(m, q, s) = \frac{1}{dz} \left( \frac{\partial u_z}{\partial z} dz \right) = \frac{1}{t} (f_z(m, q, s + 1) - f_z(m, q, s) - t)
\]

60
\[ \varepsilon_{12}(m,q,s) = \frac{1}{2} \left( \frac{1}{dx} \left( \frac{\partial u_y}{\partial x} \right) dx + \frac{1}{dy} \left( \frac{\partial u_x}{\partial y} \right) dy \right) \]
\[ = \frac{1}{2t} (f_y(m, q + 1, s) - f_y(m, q, s) + f_x(m + 1, q, s) - f_x(m, q, s)) \]
\[ \varepsilon_{13}(m,q,s) = \frac{1}{2} \left( \frac{1}{dx} \left( \frac{\partial u_x}{\partial x} \right) dx + \frac{1}{dz} \left( \frac{\partial u_x}{\partial z} \right) dz \right) \]
\[ = \frac{1}{2t} (f_z(m, q + 1, s) - f_z(m, q, s) + f_x(m, q, s + 1) - f_x(m, q, s)) \]
\[ \varepsilon_{23}(m,q,s) = \frac{1}{2} \left( \frac{1}{dy} \left( \frac{\partial u_z}{\partial y} \right) dy + \frac{1}{dz} \left( \frac{\partial u_y}{\partial z} \right) dz \right) \]
\[ = \frac{1}{2t} (f_z(m + 1, q, s) - f_z(m, q, s) + f_y(m, q, s + 1) - f_y(m, q, s)) \]

where \( t \) is the grid spacing in the \( x \)-, \( y \)-, and \( z \)-directions. The hydrostatic strain for each grid element, representing the strain associated with grid element area change, was computed as:

\[ \varepsilon_{hyd}(m,q,s) = \frac{1}{3} (\varepsilon_{11}(m,q,s) + \varepsilon_{22}(m,q,s) + \varepsilon_{33}(m,q,s)) \]

The deviatoric strain tensor was computed as:

\[ \varepsilon_{dev}(m,q,s) = \begin{bmatrix} \varepsilon_{11}(m,q,s) - \varepsilon_{hyd}(m,q,s) & \varepsilon_{12}(m,q,s) & \varepsilon_{13}(m,q,s) \\ \varepsilon_{12}(m,q,s) & \varepsilon_{22}(m,q,s) - \varepsilon_{hyd}(m,q,s) & \varepsilon_{23}(m,q,s) \\ \varepsilon_{13}(m,q,s) & \varepsilon_{23}(m,q,s) & \varepsilon_{33}(m,q,s) - \varepsilon_{hyd}(m,q,s) \end{bmatrix} \]

and the Euclidean norm was taken to provide a scalar value representing the shape change of the grid elements. The 3D strain energy density for each grid element was calculated as:

\[ u(m,q,s) = \frac{1}{2} \sigma^T \varepsilon = \frac{1}{2} [(2\mu + \lambda)(\varepsilon_{11}(m,q,s) + \varepsilon_{22}(m,q,s) + \varepsilon_{33}(m,q,s))^2 + 2\lambda(\varepsilon_{11}(m,q,s) \varepsilon_{22}(m,q,s) + \varepsilon_{11}(m,q,s) \varepsilon_{33}(m,q,s) + \varepsilon_{22}(m,q,s) \varepsilon_{33}(m,q,s)) + \lambda \varepsilon_{12}(m,q,s) + \lambda \varepsilon_{13}(m,q,s) + \varepsilon_{23}(m,q,s)] \]

where \( \mu = \frac{E}{2(1-v)} \), \( \lambda = \frac{E v}{(1+v)(1-2v)} \), \( \gamma_{ij} = 2\epsilon_{ij} \) for \( i \neq j \), elastic modulus \( E = 55 \) GPa (modulus of ganoin, the outer layer of ganoid scales [3-4]), and Poisson ratio \( v = 0.3 \). Total strain energy \( (U) \) over the 3D transformation grid was calculated as:

\[ U = \sum_{m,q,s} u(m,q,s) \cdot \frac{t^3}{3}. \]

### 3.2.4. Morphometric parameters in 3D

A set of size-independent parameters were calculated from the LM coordinates: central scale thickness (\( T \)), angle of interserial overlap on the anterior shelf (\( \lambda \)), scale curvature (\( K \)),
volume of the scale (VC), from volume of peg (VP), volume of base (VB), volume of anterior shelf (VAS), and percent volume reduction from the concavity of the anterior margin (AM%). These parameters are visualized in Fig. 3.2 and their calculations are described in Table 3.2. Central thickness, curvature, and percent reduction due to concavity of the anterior shelf were normalized by their maximum value over all scales (T = 0.10, K = 5.67, and AM% = 3.06%) for unitless parameter values. Total 3D strain energy (U) was also normalized by its maximum value over all scales (U = 1.34 x 10^9). Angle of interserial overlap (λ) is represented in degrees. Volume of the base of the scale (VB), volume of the peg (VP) and volume of the anterior shelf (VAS) are stacked as area plots representing their percentage of the total scale volume (VC).

![Illustrations of the 3D morphometric parameters](image)

**Figure 3.2**: Illustrations of the 3D morphometric parameters described in Table 3.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>Central thickness</td>
<td>Z- Distance between LM19 and LM21</td>
</tr>
<tr>
<td>λ</td>
<td>Angle of overlap</td>
<td>YZ angle between LM16, LM14, LM21</td>
</tr>
<tr>
<td>K</td>
<td>Curvature</td>
<td>Curvature through LM1, LM19, LM14</td>
</tr>
<tr>
<td>VP</td>
<td>Volume of peg</td>
<td>Volume of convex hull of LM4-8, LM20</td>
</tr>
<tr>
<td>VB</td>
<td>Volume of base</td>
<td>Volume of convex hull of LM1-4, LM8, LM9, LM16-21</td>
</tr>
<tr>
<td>VAS</td>
<td>Volume of anterior shelf</td>
<td>Volume of convex hull of LM9-16 minus VNS</td>
</tr>
<tr>
<td>VNS</td>
<td>Volume of negative space</td>
<td>Volume of convex hull of LM11-14</td>
</tr>
<tr>
<td>VC</td>
<td>Volume of scale</td>
<td>VB + VAS + VP</td>
</tr>
<tr>
<td>AM%</td>
<td>% Volume reduction from VNS</td>
<td>(VNS / VC) * 100</td>
</tr>
</tbody>
</table>
3.3. Results

The 3D morphometric parameter values characterizing *P. senegalus* scale shape are plotted in Fig. 3.3 to along the anterior-posterior row (S6) in Fig. 3.3a, dorsoventral column (C22) in Fig. 3.3b, and pectoral fin seam in Fig 3.3c. The results are further described below.

**Figure 3.3:** Morphometric parameters for the (a) row, (b) column, and (c) pectoral fin data series. Central thickness (T), curvature (L), % scale volume reduction due to concavity of the anterior margin (AM%), and 3D strain energy (U) are plotted as unitless parameters normalized by their maximum value (left axis). Angle of interserial overlap (λ) is plotted in degrees (right axis). Volume of the base of the scale (VB), volume of the peg (VP) and volume of the anterior shelf (VAS) are stacked as area plots as a percentage of the total scale volume. Calculations for these parameters are presented in Table 3.2.
3.3.1. 3D Morphometric variation along the anteroposterior axis

In 3D, as in 2D, scale geometry morphs gradually from head to tail on the fish (Fig. 3.3a). Examining three scales from anterior (C10R6) to midsection (C30R6) to tail (C50R6), scale thickness generally increases (T = 0.55 to 0.79 to 0.84), volume reduction from the anterior margin decreases (AM% = 1.00 to 0.30 to 0.017), angle of overlap decreases (\( \lambda = 58^\circ \) to \( 51^\circ \) to \( 47^\circ \)). Curvature of scales is greater for anterior (K = 0.71) and tail (K = 0.67) scales compared to the midsection (K = 0.62). The percent volume of the scale comprising the base of the scale increases from anterior to midsection to tail (VB = 79.7\% to 81.1\% to 91.8\%), while peg volume decreases (VP = 2.3\% to 1.2\% to 0.4\%) and volume of anterior shelf decreases (VAS = 18\% to 17.7\% to 7.8\%). The 3D strain energy is lowest for anterior scale (U = 0.13) and increases through the midsection and tail (U = 0.18 to 0.32), seeming to be affected more greatly by the thickness in the Z-direction compared to the 2D geometry in the XY plane.

3D transformation grids and strain plots for these three representative scales along the anteroposterior axis for the average (C30R6), anterior (C10R6), and tail (C50R6) scale variants are presented in Fig. 3.4, Fig. 3.5, and Fig. 3.6 respectively. The undeformed 3D grid and transformation grid for a single scale representing the ‘average’ scale (C30R6; Fig. 3.4a), compared to the reference geometry (in blue) are shown in Fig. 3.4b-c, indicating global expansion in the +Y direction and shearing in the -YZ direction. 2D slices can more clearly illustrate the deformations within the 3D grid; Fig. 3.4d shows medial slices through each plane, and Fig. 3.4e-g shows three slices at different locations within the XY, XZ, and YZ planes, illustrating how shape deformations are not necessarily equivalent through all planes in 3D space. The components of axial strain (\( \varepsilon_{11} \), \( \varepsilon_{22} \), \( \varepsilon_{33} \)) and shear strain (\( \varepsilon_{12} \), \( \varepsilon_{13} \), \( \varepsilon_{23} \)) for each grid element are plotted in Fig. 3.4h on medial slices through the XY, XZ, and YZ planes. The plots of \( \varepsilon_{33} \) show the global expansion in the +Z direction through the entire scale, representing a scale that is thicker than the reference geometry. The plots of \( \varepsilon_{23} \) show –YZ shear. The remaining plots show localized pockets of low-magnitude axial and shear strain around specific geometric features, generally indicating that this geometry closely matches the reference geometry.

Transformation grids and strain plots for the anterior scale variant (C10R6) are presented in Fig. 3.5. The anterior scale is globally sheared in the –XY and –XZ directions compared to the average geometry, with localized pockets of axial strain around specific geometric features. Low magnitudes of axial strain in the Z-direction indicate a thickness that closely matches the reference geometry, which helps to minimize the value for 3D strain energy. While the size and shape of the base of the scale is similar to the average geometry, the curvature of the anterior scale can be seen through the shape of the XZ slices. In the 2D analysis, the peg was found to be longer than the average geometry; in 3D, the peg has a greater volume (VP = 2.3\%) compared to the average geometry (VP = 1.2\%), evidenced by pockets of axial expansion in the +X, +Y, and +Z directions. The anterior scale shows the highest degree of concavity in the anterior margin compared to all scales (AM% = 1), depicted through pockets of negative XY and YZ shear around LM14-16.
Transformation grids and strain plots for the tail scale variant (C50R6) are presented in Fig. 3.6. The global geometry of the tail scale is heavily sheared in the +XY direction. Half of the scale, in the +X halfspace containing the peg and anterior process, are negatively sheared as the peg volume (VP = 0.4%) decreases, and the shorted anterior process contributes to a smaller volume of the anterior shelf (AS = 7.8%). Localized regions of alternating positive and negative axial and shear strain flatten the concavity of the anterior margin (AM% = 0.02) and decrease the overlap angle (λ = 47°).

Figure 3.4: 3D morphometric analysis of the average scale variant (C30R6). (a) µCT reconstruction of the scale. (b) Undeformed 3D grid. (c) Deformed 3D transformation grid showing scale geometry (red) compared to the reference geometry (blue). (d) 2D medial slices of the transformation grid in the XY, XZ, and YZ planes. (e) 3 XY slices. (f) 3 XZ slices. (g) 3 YZ slices. (h) Strain plots on medial XY, XZ, and YZ slices comparing the scale (red) to the reference (blue).
Figure 3.5: 3D morphometric analysis of the anterior scale variant (C10R6). (a) µCT reconstruction of the scale. (b) Undeformed 3D grid. (c) Deformed 3D transformation grid showing scale geometry (red) compared to the reference geometry (blue). (d) 2D medial slices of the transformation grid in the XY, XZ, and YZ planes. (e) 3 XY slices. (f) 3 XZ slices. (g) 3 YZ slices. (h) Strain plots on medial XY, XZ, and YZ slides comparing the scale (red) to the reference (blue).
Figure 3.6: 3D morphometric analysis of the tail scale variant (C50R6). (a) µCT reconstruction of the scale. (b) Undeformed 3D grid. (c) Deformed 3D transformation grid showing scale geometry (red) compared to the reference geometry (blue). (d) 2D medial slices of the transformation grid in the XY, XZ, and YZ planes. (e) 3 XY slices. (f) 3 XZ slices. (g) 3 YZ slices. (h) Strain plots on medial XY, XZ, and YZ slides comparing the scale (red) to the reference (blue).
3.3.2. 3D Morphometric variation along the dorsoventral axis

The scale geometry morphs rapidly from the dorsal to ventral midlines of the fish, analyzed down the C22 column of scales (Fig. 3.3b). Morphometric parameters along three representative scales at the top (C22R4), middle (C22R8), and bottom (C22R14) show that, in general, the scale thickness decreases down the column (T = 0.99 to 0.60 to 0.48) alongside the interserial overlap angle ($\lambda$ = 49° to 44° to 31°). Curvature is greater in the dorsal scale (0.679), while equivalent between the middle and ventral scales (K = 0.52). Concavity of the anterior margin is greatest in the middle scale (AM% = 0.63) compared to the dorsal (AM% = 0.13) and ventral (AM% = 0.02) scales, while strain energy follows the opposite trend being least for the middle scale (U = 0.072) and greater for the dorsal (U = 0.56) and ventral (U = 1.0) scales. The percent volume of the scale comprising the base of the scale increases from the middle scale (VB = 75.9%) to both the dorsal (VB = 82.7%) and ventral (VB = 78%) ends of the column, while

Figure 3.7: 3D morphometric analysis of the ventral scale variant (C22R14). (a) µCT reconstruction of the scale. (b) Undeformed 3D grid. (c) Deformed 3D transformation grid showing scale geometry (red) compared to the reference geometry (blue). (d) 2D medial slices of the transformation grid in the XY, XZ, and YZ planes. (e) 3 XY slices. (f) 3 XZ slices. (g) 3 YZ slices. (h) Strain plots on medial XY, XZ, and YZ slides comparing the scale (red) to the reference (blue).
volume of the anterior shelf follows the opposite trend being greatest for the middle scale (VAS = 22.3%) and decreasing to the dorsal (VB = 15.7%) and ventral (VB = 19.5%) ends. The peg volume increases down the column (VP = 1.6% to 1.8% to 2.5%).

Transformation grids and strain plots for the ventral scale variant (C22R14) are presented in Fig. 3.7. The ventral scale is globally compressed in the Z-direction, positively sheared in the XY and YZ directions, and mildly negatively sheared in the XZ direction. The ventral scale is significantly thinner (T = 0.48) compared to the reference geometry, which is closely resembled by the midsection ‘average’ scale geometry (C30R6; T = 0.79), and the strain plots for the ventral scale show Z-compression ($\varepsilon_{33}$) in each plane. The ventral scale is flatter in curvature (K = 0.52) compared to the midsection geometry (K = 0.62), as can be seen in the XZ slices of the transformation grids and strain plots. While the ventral scale has a similar base volume (VB = 78%) compared to the midsection scales (VB = 81%) the peg volume is much larger in the ventral scale (VP = 2.5%, compared to VP = 1.2% for C30R6). The 2D analysis showed that anterior processes in the ventral scales are longer, however the volume of the anterior shelf (VAS = 19%) is comparable to the midsection scale (VAS = 18%) due to the shallower overlap angle ($\lambda = 31^\circ$ vs. $\lambda = 51^\circ$). The lack of concavity in the anterior margin does not save volume for the ventral scale (AM% = 0.02).

### 3.3.3. 3D Morphometric variation along the pectoral fin transition seam

The scale morphometry morphs very rapidly along the row of seven scales (labeled P1-P7) transitioning from the main body exoskeleton into the pectoral fin (Fig. 3.3c). Examining three scales along the seam (P2, P4, and P6), the scale thickness increases (T = 0.59 to 0.86 to 1.0) alongside strain energy (U = 0.10 to 0.43 to 0.79). Curvature is greater in the middle of the seam for P4 (K = 0.21) compared to the P2 (0.15) and P6 (0.20), while angle of overlap and percent volume reduction due to the concavity of the anterior margin follow the opposite trend being least in the middle of the seam ($\lambda = 24^\circ$, AM% = 0.02) compared to P2 ($\lambda = 38^\circ$, AM% = 0.04) and P6 ($\lambda = 64^\circ$, AM% = 0.08). For the scale in the middle of the seam, the percent volume comprising the base of the scale is the least (VB = 75.3%) and volume of the anterior shelf (VS = 22.6%) and peg (VP = 2.1%) are greatest, compared to the earlier in the seam in the main body exoskeleton (VB = 81.9%, VAS = 16.5%, and VP = 1.6% for P2) and later in the seam into the pectoral fin (VB = 94.3%, VAS = 5.3%, and VP = 0.4% for P6).

Transformation grids and strain plots for the pectoral fin variant (P6) are presented in Fig. 3.8. Globally, the pectoral fin scale is expanded in the Y- and Z-directions while sheared in the –XY and +XZ directions. Additionally, the region of the scale in the +X halfspace sees compression in the X-direction and shear in the –YZ direction, while the region in the –X halfspace sees expansion in the X-direction and positive shear in the +YZ direction. These strain plots show how the complex average scale geometry is transformed into a square, relatively featureless body with two sliding overlap joints. The base of the scale comprises most of its volume (VB = 94.3%) with very small anterior shelf (VAS = 5.3%) and peg (VP = 0.4%).
Figure 3.8: 3D morphometric analysis of the pectoral fin scale variant (P6). (a) µCT reconstruction of the scale. (b) Undeformed 3D grid. (c) Deformed 3D transformation grid showing scale geometry (red) compared to the reference geometry (blue). (d) 2D medial slices of the transformation grid in the XY, XZ, and YZ planes. (e) 3 XY slices. (f) 3 XZ slices. (g) 3 YZ slices. (h) Strain plots on medial XY, XZ, and YZ slices comparing the scale (red) to the reference (blue).
3.4. Discussion

These results add a layer of complexity to the understanding of shape variation within the *P. senegalus* exoskeleton. Since the joint articulation mechanisms and geometric features of the scales are 3D in nature, 3D design parameters are necessary to develop bio-inspired designs based on the ganoid exoskeleton in order to mimic the interscale mechanical interactions that enable simultaneous protection and flexibility. For example, the morphometric analyses presented in Chapter 2-3 were directly integrated into the *MetaMesh* model, which adapts the design of the *P. senegalus* exoskeleton to fit complex, curved, and arbitrary ‘host surfaces’ [5]. The *MetaMesh* model utilizes computational design parameters at three levels of resolution: locally, in terms of the individual scale shape which may vary throughout the mesh of the host surface depending on the targeted functionality; regionally, in terms of the complex, interscale connectivity and overlap joints to preserve operational articulation mechanisms of scales with their neighbors as they vary in geometry; and globally, in terms of generatively producing the scale array by both optimizing the mesh and tailoring the scale geometry to functional requirements of the surface. The results of this computational model provide a basis for the future integration of physiological and kinematic schema of the target application to create biomimetic protective surfaces using segmented, articulated components that maintain mobility alongside full body coverage.

The trends of spatially-dependent morphometric variation presented in this thesis may be generalized across the species of fish. Pearson qualitatively assessed scale shape variation in the *P. senegalus* integument; Pearson’s sketches of scales down a single column (dorsal, medial, and ventral scales) and corresponding text describe dorsoventral trends in morphometric variation similar to what is observed in this thesis: shrinking diagonal aspect ratio, pegs that are sharper dorsally and flatten ventrally, anterior process orientations that rotate away from the body of the scale ventrally while increasing in length, and anterior margins that are most pronounced in the medial scales [6]. Pearson also describes the ‘reduced’ caudal (tail) scales: smaller pegs, shorter anterior processes, and flattened anterior margins [6]. Gemballa and Bartsch also examined the shape variation of scales in Polypterids with SEM micrographs of scales from the anterior and posterior region of the fish along the lateral line [7]. They describe the same trends in morphometric variation throughout the exoskeleton as observed in this thesis: anterior scales have the longest pegs and large anterior processes; tail scales have shorter pegs with the similar peg tip angles to anterior scales, as well as shorter anterior processes, flat anterior margins, and smaller diagonal aspect ratios; dorsoventrally, the orientation of the anterior process to the body of the scale changes, and the socket disappears in ventral scales [7]. Furthermore, Gemballa and Bartsch generalize their observations in scale shape variation across many species of genus *Polypterus* as they found little difference among various Polypterids including *P. senegalus*, *P. palmas*, *P. weeksii*, and *P. delhezi* [7]. While the specific parameter values characterizing the shape of individual scales vary between specimens, the trends of shape variation reported throughout the exoskeleton may be generally applicable to the species.
The deformation of the scale landmarks, which are a virtual representation of the geometry, is assumed to be isotropic both in the 2D analysis and 3D analysis. Real P. senegalus scales in 3D can considered as transversely isotropic at the macroscopic level due to their stratified, quad-layer composition through the thickness of the scale, with isotropic properties in the 2D XY plane and different material properties in the transverse Z-direction normal to the scale surface. The modulus of each material in the scale has been experimentally determined through the scale cross section [3]. Additionally, the scales have been modeled with isotropic linear elastic-perfectly plastic material elements with axisymmetry about the z-axis for each layer [3-4]. Furthermore, the calculation of strain energy uses the modulus for ganoine, which is the stiffest material in the scale as an upper bound for the values that strain energy could take. Ganoine has been shown to be mechanically anisotropic at the micro- and nanoscale due to the orientation of hydroxyapatite crystallites within the organic matrix [8]. While this anisotropy has implications on stress-dissipation pathways for indentation (i.e. biting attacks) on the scale, it was shown that this constitutive behavior does not result in a direction-dependent indentation modulus or hardness [8]. Thus the assumption of isotropy for modulus and Poisson ratio within the 2D XY plane and extended through the cross-section can be considered to be valid.

3.5. References

4. Anisotropic Flexibility of Bio-inspired Flexible Composite Prototypes under Passive Loading

4.1. Introduction

In this chapter, bioinspired flexible composite prototypes based on the exoskeleton of *P. senegalus* are fabricated and tested using novel methods to examine how scale shape contributes to local interscale mobility mechanisms and generates anisotropic mechanical behavior. First, the hierarchical geometric rules of assembly in the *P. senegalus* exoskeleton are translated to synthetic, flexible composite prototypes which are then fabricated by multi-material 3D printing. A new method is developed to experimentally assess the flexibility of the prototypes under passive loading (self-weight) by examining the orientation-dependence of the prototype’s radius of curvature and the local contributions of scale shape and joint articulation mechanisms to the global behavior of the sample. The results show how the complex scale shape contributes to local interscale mobility mechanisms that contribute to anisotropic flexibility.

4.2. Materials and Methods

4.2.1. Prototype design via associative modeling

The prototypes were designed using parametric and associative modeling techniques by K. Zolotovsky using methods from Reichert (2010) and Zolotovsky (2012) [1-2]. Parametric CAD software (SOLIDWORKS®, Dassault Systèmes SolidWorks Corp., France) was used to design an abstracted 3D model of the scale geometry (Fig. 4.1a) with an overall rhomboid shape allowing for a tapered overlap area between scales, a tetrahedral peg, a corresponding inverted concave socket, schema for the paraserial and interserial axis, and with controllable geometric parameters such as scale length (L) and height (H). Associative modeling was used to replicate the individual scale geometry into square arrays with 1 mm spacing in the paraserial and interserial directions and with an angle of 60° between the paraserial and interserial axes (Fig. 4.1b-d). The soft tissue substrate (akin to the dermis of the fish) was modeled as a separate layer. Connective elements between the peg and socket of adjacent scales (mimicking the Sharpey’s fibers in the fish exoskeleton) and between the scales and the substrate (mimicking the stratum compactum in the fish exoskeleton) were modeled as additional layers.

4.2.2. Prototype fabrication via multi-material 3D printing

The translated prototypes were fabricated as a flexible array of scales (Fig. 4.1e-f) via multi-material 3D printing (OBJET Connex500™, Stratasys, USA) by K. Zolotovsky. The STL files for the prototype were imported into OBJET Studio software and assigned to commercially available UV-cured polymer materials: the rigid components were printed with VeroWhite (hard plastic with elastic modulus (E) = 2.0 GPa [3-4]), and the soft components were printed with
TangoPlus (rubber-like elastomer with $E = 0.63$ MPa [3-4]). The print jobs were submitted using the digital printing mode at 30 µm resolution. Print support material was removed with a water jet and manual brushing.

4.2.3. **Radius of curvature measurements**

The radius of curvature of the prototype under self-weight was examined by draping the prototype over a curved, half-cylinder mold (radius $R_m = 120$ mm) without the application of an external load, with a camera situated along the mold’s axis of zero curvature (global x-axis) (**Fig. 4.2a**). The prototype was rotated over the mold by an angle $\alpha = 0-180^\circ$ in $15^\circ$ increments, where $\alpha = 0^\circ$ corresponds to the paraserial peg-and-socket axis in line with the mold’s axis of zero curvature. In each orientation, normal projection rods were inserted into three scales on a single line parallel to the mold’s axis of curvature (global y-axis), and the radius of curvature of the prototype ($R_p$) was measured by drawing a circle amongst points of connection between the normal rod and the scales. The experiment was repeated with three samples ($N = 3$) per prototype design. Three prototype variations were tested:

i. 13×13 array of scales with $L = H = 20$ mm,

ii. 26×13 array of scales with halved aspect ratio, $L/2 = H = 20$ mm,

iii. 13×13 array of scales with $L = H = 20$ mm, without paraserial connective material.

4.2.4. **Joint degrees of freedom analysis**

A new rod-displacement method was developed to determine the contributions of the paraserial and interserial joint degrees of freedom to the mechanical behavior of the sample. The prototype was draped over a curved, half-cylinder mold (radius $R_m = 120$ mm) without the application of an external load, with cameras situated along the mold’s axis of zero curvature (global x-axis) and axis of curvature (global y-axis) (**Fig. 4.2b**). Three normal projection rods were inserted into rigid subunits of the prototype along the paraserial axis (rods 1 and 2) and interserial axis (rods 1 and 3) (**Fig. 4.3c-d**). The prototype was rotated about the mold by an angle $\alpha = 0-180^\circ$ in $30^\circ$ increments. At each orientation, interscale angles ($\theta_{ij}^k$) were measured as the angle ($\theta$) between two normal rods of adjacent scales $(ij)$ as viewed from a camera on positioned on the x- or y-axis ($k$). The surface alignment in the global coordinate system tangent to the projection rods was calculated as $\tan(\theta_x^i) = \frac{z_i}{y_i}$ and $\tan(\theta_y^i) = \frac{z_i}{x_i}$. A 3D-coordinate transformation was used to extract information about the relative displacement of adjacent scales in local u-v coordinates with $X' = A_{3D} \cdot X$ and $A_{3D} = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}$. 

74
Figure 4.1: Design and fabrication of *P. senegalus*-inspired flexible composite prototypes. (a) Rendering of an abstracted scale geometry, showing the orientation of the paraserial and interserial axes and two controllable parameters: scale length (L) and scale height (H). Adapted from [1]. (b) Schematic of the multimaterial components of the articulated assembly, bottom view, adapted from [1]. (c) Top view of the prototype design. (d) Cross-sectional view of the prototype design. (e) 3D printed flexible composite prototype. (f) Close-up of the 3D printed prototype showing the orientation of the paraserial and interserial axes.

Figure 4.2: Experimental methods to analyze prototype behavior under passive loading. (a) Setup of the curvature experiment. The prototype is draped over a curved mold. Rigid rods are inserted into three scales in a single line along the mold’s axis of curvature (y-axis). A camera along the mold’s line of zero curvature (x-axis) captures the image in order to calculate the radius of curvature of the prototype (R_p) vs. radius of curvature of the mold (R_m). (b) Setup of the joint degrees of freedom experiment. The prototype is draped over a curved mold. Rigid rods are inserted into three scales, two along the paraserial axis and two along the interserial axis of the assembly. Cameras along the mold’s line of zero curvature (x-axis) and line of curvature (y-axis) capture images to calculate interscale angles. (c-d) Top-view schematics illustrating interscale angles between Rods 1-2 and Rods 1-3.
4.3. Results

The synthetic flexible composite prototype translates the design principles from the *P. senegalus* exoskeleton from a closed-ring structure to a flat surface, and it includes the morphological features that underpin the direction-dependent flexibility of the exoskeleton: the abstracted single scale geometry, the interlocking peg-and-socket joints with reinforcing compliant material in the paraserial direction mimicking Sharpey’s fibers, the angular arrangement of overlap joints in the interserial direction, a compliant substrate analogous to the underlying dermal tissue, and compliant connections between the substrate and the scales at the axial ridges mimicking the stratum compactum. Prototypes were designed using parametric and associative modeling in computer-aided design (CAD) software and fabricated by multi-material 3D printing, using VeroWhite for the scales and TangoPlus for the soft components.

4.3.1. Anisotropy of flexibility through radius of curvature

The innate flexibility of the prototype under passive loading (self-weight) is examined by draping a prototype consisting of a 13×13 array of 20 mm scales over a half-cylinder mold without the application of an external load (*Fig. 4.3*). The flexibility of the prototype varies with the orientation of the scales over the mold (*Fig. 4.3a-e*). The orientation angle (α) is defined as the angle between the mold’s axis of zero curvature and the prototype’s rigid paraserial axis. The prototype exhibits maximum flexibility at α = 0° where the paraserial peg-and-socket axis is aligned with the mold’s axis of zero curvature (global x-axis), and the overlap joint aligns with the axis of mold curvature (global y-axis) (*Fig. 4.3a, f*). In this orientation, the overlap joint allows the columns of scales to slide away from each other without resistance, and the prototype completely conforms to the mold. The prototype exhibits maximum rigidity at α = 90°, where the peg-and-socket joint is aligned along the axis of mold curvature (global y-axis) (*Fig. 4.3d, g*). In this orientation, the articulated peg-and-socket joint restricts the motion of scales within the column of scales, and thus resists flexure of the prototype so that it lies flat above the mold.

The radius of curvature of the prototype relative to the radius of curvature of the mold (R_p/R_m) is plotted against orientation angle (α) in *Fig. 4.3h*. At α = 0°, the prototype completely conforms to the mold with R_p = R_m. The radius of curvature of the prototype increased with α until a maximum rigidity α = 90° with R_p= 4.2R_m. As the prototype was rotated past α = 90°, the peg-and-socket joint returns to alignment with the mold’s line of zero curvature, and the global flexibility returns. A second prototype was fabricated using scales with a halved aspect ratio, shown in the schematic in *Fig. 4.3h* (right). This assembly exhibited lower flexural rigidity than the original design while exhibiting the same anisotropic trend of R_p/R_m increasing to a maximum over α = 0-90°, and decreasing from α = 90-165°.

A third prototype was designed without the paraserial connective elements between the peg and socket of neighboring scales. This prototype assembly exhibited uniformly maximal flexibility in all orientations, mechanically equivalent to the flexible substrate without scales (*Fig. 4.3h*). The results suggest that collagenous Sharpey’s fibers are a critical structural
Figure 4.3: Anisotropy of prototype flexibility measured by radius of curvature. (a) Prototype draped over a half-cylinder mold at orientation angle (\(\alpha\)) = 0°. Lines show the curvature of the mold (white) and prototype (red). Inverse of curvature represents the radius of curvature of the prototype (\(R_p\)) and the radius of curvature of the mold (\(R_m = 120\) mm). The prototype exhibits different curvatures when rotated to (b) \(\alpha = 30°\), (c) \(\alpha = 60°\), (d) \(\alpha = 90°\), and (e) \(\alpha = 120°\). (f) Top view of the prototype in the \(\alpha = 0°\) orientation. (g) Top view of the prototype in the \(\alpha = 75°\) orientation. (h) Relative radius of curvature (\(R_p/R_m\)) vs. \(\alpha\) for the prototype, a variation with the halved length aspect ratio, a variation without the paraserial connections, and the substrate only without scales. Error bars represent standard deviation with \(N = 3\) samples per prototype design.
component in restricting the ranges of motion between scales at the peg-and-socket joint, thereby resisting flexure along the paraserial axis and introducing mechanical anisotropy to the system.

4.3.2. Contribution of local mobility mechanisms to global behavior

Anisotropy in global flexibility is dictated by the local contributions of the paraserial and interserial joint structures to the larger assembly. Normal projection rods are inserted into rigid subunits of the prototype along the paraserial axis (rods 1 and 2) and interserial axis (rods 1 and 3) visible to a camera positioned along the global axes. The prototype is rotated about the mold in 30° increments. At each orientation, interscale angles ($\theta^{ij}_k$) are measured as the angle ($\theta$) between two normal rods of adjacent scales (ij) as viewed from a camera on positioned on the x- or y-axis (k). These interscale angles correspond to local interscale mobility mechanisms, depicted in Fig. 4.4a, and their values are plotted in Fig. 4.4b. The data indicates that $\theta^{12}_x$ (paraserial rotation) is small and varies over a very limited range; the paraserial axis is a rigid axis. $\theta^{13}_x$ (interserial sliding) is largest for $\alpha = 0\text{–}30^\circ$, where the prototype conforms to the curvature of the mold, and then decreases through $\alpha = 60\text{–}90^\circ$ as the prototype’s radius of curvature increases and the interserial sliding mechanism is constrained. $\theta^{12}_y$ (paraserial bending) rapidly increases with rotation over $\alpha = 30\text{–}90^\circ$. The compliant Sharpey’s fibers that restrain paraserial rotation and bending contribute greatly to the resistance to bending in the $\alpha = 90^\circ$ orientation. Finally, values for $\theta^{13}_y$ (interserial rotation) are small, but contribute to the prototype deformation in the $\alpha = 30^\circ$, 90°, and 120° orientations.

**Figure 4.4:** Contribution of joint degrees of freedom to the prototype flexibility. (a) Schematics of four joint degrees of freedom and their corresponding interscale angles. (b) Interscale angles vs. rotation angle ($\alpha$) of the prototype. Error bars represent standard deviation for $N = 3$ samples.
4.4. Discussion

These results demonstrate how a combination of scientific analysis and architectural design tools can be used to translate the complex design principles of a biological exoskeleton to a synthetic prototype and to characterize its flexibility. Throughout the translation process, the geometric components of the natural system are geometrically abstracted while preserving the rules of assembly underpinning the biomechanical behavior. Associative 3D modeling allows for variation of the scale shape to tune the mechanical properties to local functional requirements. Additionally, these results provide a new experimental method to evaluate the structural flexibility of synthetic segmented assemblies that are highly adapted to their function. The rod-displacement method quantifies the direction-dependent passive flexibility of homogeneous synthetic assemblies and establishes a connection between two types of information: the individual geometric features of the scale unit shape and the global anisotropic flexibility of the assembly. Establishing this link between local shape (geometry) and global performance (biomechanics) is the first step in applying the design principles of natural system to synthetic designs with local tailorable for functional fitness.

The experiments show that the paraserial connective elements within the peg-and-socket are critical for generating anisotropy in flexibility, meaning that they restrict flexibility in certain orientations. When the goal is to develop flexible materials, it may seem counterintuitive to include these elements. However, without them, neighboring scales along the paraserial direction slide apart from each other during flexure; without an overlap joint in this direction, the substrate material between the scales is exposed and the armors system becomes vulnerable to threats. Consequently, it seems that the complex architecture of the *P. senegalus* exoskeleton, including scale shape, scale-to-scale joint interactions, and multi-material connective tissue, is tailored to the fish’s needs in order to enable flexibility in certain directions while restricting flexibility in other orientations in order to maintain full-body coverage for protection.

Additive manufacturing techniques, like 3D printing, enable a one-step fabrication method to produce multi-material structures with complex 3D architectures. The prototypes developed in this chapter utilize a combination of 3D scale shape, topology definition in the soft substrate, compliant connective elements buried within the scales, and a selective use of empty (negative) space. The connective tissue between the peg-and-socket, mimicking Sharpey’s fibers, is the most challenging element to include in the prototypes; without it, perhaps the prototypes could be fabricated with simpler or faster methods such as machining, casting, injection molding, or sintering techniques, which have been done with 3D topologically interlocking materials [5], by simply creating the rigid scales and embedding them a soft substrate. However, the inclusion of the paraserial interconnections necessitates a technique like 3D printing which can include complicated multi-material elements through layer-by-layer fabrication processes.

An additional benefit of multi-material 3D printing is the ability to create multi-material structures without adhesives or glues at interfaces, thereby eliminating a source of failure common in composite materials [6-9]. 3D printing materials, e.g. VeroWhite and TangoPlus
used here, mix at their interfaces during the print process and cure in situ. As a result, the constituent materials are described in the literature to “adhere to each other perfectly” with a strong interfacial adhesion so that the 3D printed prototypes do not fail at the interfaces [10]. Thus, the behavior of the prototypes is solely dependent on the complex architecture and the material properties of the components, and not governed by the interfaces.

Finally, the techniques used to design and fabricate the flexible, composite scale assembly prototypes lends themselves to future development of fully customizable armor. Advances in computational design methods, e.g. the MetaMesh model, can adapt the prototype design based on the P. senegalus exoskeleton to fit complex, curved, and arbitrary host surfaces [11]. Moreover, advances in additive manufacturing processes are beginning to print structural materials such as metals [12-13], ceramics (like hydroxyapatite and glass [14-15]), and carbon fiber (e.g. Kevlar [16]). Future armor designs can be envisioned that are fully tailorable both to the fit of the user through design algorithms and to the threat through materials selection.

4.5. References


5. Mechanical Behavior of Bio-Inspired Flexible Composite Prototypes in Active Loading

5.1. Introduction

In this chapter, bioinspired flexible composite prototypes based on the exoskeleton of *P. senegalus* are further tested to examine how scale shape contributes to local interscale mobility mechanisms and generates anisotropic mechanical behavior under active loading (bending). The prototypes used in Chapter 4 are redesigned for testing in bending and fabricated by multi-material 3D printing. A new method is developed to experimentally characterize the mechanical behavior of the prototypes using a mechanical tester in order to determine the orientation-dependent (anisotropic) bending stiffness of the samples. The most flexible orientations, including one in which the scales are ‘mechanically invisible’ without adding stiffness to the armor assembly, correspond to physiologically relevant bending modes in the fish. With one prototype design scheme, a wide array of mechanical behavior is generated with stiffness ranging over several orders of magnitude, thus showing how morphometry can tune the flexibility of protective, composite architectures without varying constituent materials.

5.2. Materials and Methods

5.2.1. Prototype design and fabrication

New prototypes were designed using associative modeling for testing in bending, shown in Fig. 5.1. The prototypes consisted of 124 mm × 112 mm 2D array of 20 mm scales with an angle $\beta = 60^\circ$ between the paraserial and interserial axes. The scales were aligned in orientation angles ($\varphi$) of 0°, 30°, 60°, 90°, 120°, and 150°, where $\varphi$ is defined as the angle between the paraserial peg-and-socket axis and the loading direction (+y) (Fig. 5.1a-f). The multimaterial components (soft substrate, paraserial connections, and connections between scales and substrate) were integrated into the prototype design. The scales have a maximum thickness of 4.1 mm at the axial ridge (site of attachment to substrate) and taper to points at the sides; the substrate has a maximum thickness of 4.4 mm at the ridges surrounding the scale’s axial ridge, and minimum thickness of 2.57 mm at the site of the axial ridge; the total sample thickness is 7.2 mm. Rigid rods (10 mm diameter) were added to the top and bottom of the assembly with a 21 mm tolerance extending past the sample on either side. Sample holders were designed to fit around the rigid rods and transmit the stress to the plates of the mechanical tester through a flat surface (Fig. 5.1g). The prototype designs were exported as separate STL files for the rigid components (scale, rods) and soft components (substrate, connective elements).

The prototypes were fabricated via multi-material 3D printing (OBJET Connex500™, Stratasys, USA) by K. Zolotovsky. The STL files for the prototype were imported into OBJET Studio software and assigned to commercially available UV-cured polymer materials: the rigid
Figure 5.1: Design and testing of flexible composite prototypes in bending. (a-f) Schematics of the prototypes. Dashed lines represent the paraserial (ps) axis, interserial (is) axis, the angle ($\beta$) between the paraserial and interserial axes, and scale orientation angle ($\phi$) of $0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$, $120^\circ$, and $150^\circ$. (g) Optical photograph of the sample holder attached to the rods of the prototype and affixed to the plate of the Zwick. (h) Side-view of the $\phi = 60^\circ$ sample loaded in concave bending at various vertical displacements (d), from left to right: d = 3 mm, d = 20 mm, d = 50 mm, and d = 80 mm.
components were printed with VeroWhite (hard plastic with elastic modulus (E) = 2.0 GPa [1-2]), and the soft components were printed with TangoPlus (rubber-like elastomer with E = 0.63 MPa [1-2]). Sample holders were printed with VeroWhite. The print jobs were submitted using the digital printing mode at 30 µm resolution. Print support material was removed with a water jet and manual brushing.

5.2.2. Bending experiment and analysis

The prototypes were experimentally tested in bending induced by axial compression on a mechanical tester (Zwick Z010, Zwick Roell, Germany) using load cells ranging from 20-2500 N. Sample holders were affixed to the load cell with Permacel (Nitto) tape for pin-pin boundary conditions that allow rod rotation about the x-axis and constrain rotation about the y- and z-axes to prevent global twisting of the sample (Fig. 5.1g). The samples were subjected to post-buckling bending deform concavely (scales facing in) by setting an initial lateral deflection of 1 mm and zero-ing the force before displacement-controlled compressive loading at a strain rate of 1 mm/s (Fig. 5.1h). The reaction force (F) vs. vertical displacement (d) for the prototypes was measured, and the experiment was performed with three samples (N = 3) per orientation. Sample stiffness (K) was calculated as the slope of the loading curve and normalized by the stiffness of the control sample consisting of a 4.4 mm sheet of TangoPlus without scales (K = 7.22 N/m).

5.3. Results

5.3.1. Anisotropic stiffness in concave bending

Under the application of external load, the flexible composite prototypes exhibit anisotropic mechanical response in concave bending (Fig. 5.2). During bending, the complex scale shape contributes to local interscale mobility mechanisms, which in turn determine the bending stiffness of the global sample. Reaction force (F) vs. vertical displacement (d) for the prototypes with φ = 0°, 30°, 60°, 90°, 120°, and 150° is plotted in Fig. 5.2a. Each orientation has a characteristic loading response which can be divided into phases. Stiffness (K) for each phase, or region, plotted in Fig. 5.2b, is calculated as the slope of the loading curve and normalized by the stiffness of a control sample with no scales. Various interscale mobility mechanisms, depicted in Fig. 5.3, were observed to contribute to the mechanical response of the assembly. Stiffness and interscale mechanisms observed in each loading phase of each orientation are tabulated in Table 5.1, and pictures of the experiments are shown in Fig. 5.4 and Fig. 5.5.

In the φ = 0° orientation (Fig. 5.4a), the paraserial axis of the scales is aligned with the loading direction. The scales first undergo paraserial bending as the compliant TangoPlus connection within the peg-and-socket joint resists deformation (K = 47.8) until the scales interlock paraserially (Fig. 5.4a.i-ii). The paraserial interlock causes paraserial rotation as the anterior margin rotates toward the substrate, coupled with interserial rotation when the anterior process pushes into the substrate (into plane) and the back of the anterior margin and socket lifts up (out of plane) (Fig. 5.4a.iii-iv). While the scale rotation relieves the paraserial interlock,
Figure 5.2: Anisotropic stiffness of flexible composite prototypes in concave bending. (a) Force-displacement curves for prototypes with $\phi = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$, $120^\circ$, and $150^\circ$. (b) Normalized stiffness ($K$) of each phase of each prototype’s loading curve. Error bars represent standard deviation with $N = 3$ samples per prototype orientation.

Figure 5.3: Schematics of the six interscale mobility mechanisms observed.

Table 5.1: Displacement range, stiffness, and interscale mobility mechanisms for each loading phase of each prototype orientation in concave bending.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>Phase</th>
<th>Displacement</th>
<th>$K$</th>
<th>Mechanism(s)</th>
</tr>
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<tr>
<td>$0^\circ$</td>
<td>I</td>
<td>0-5 mm</td>
<td>47.8</td>
<td>paraserial bending</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>5-85 mm</td>
<td>15.8</td>
<td>paraserial + interserial rotation</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>0-5 mm</td>
<td>126</td>
<td>paraserial bending</td>
</tr>
<tr>
<td>$30^\circ$</td>
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<td>5-15 mm</td>
<td>21.7</td>
<td>paraserial rotation</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>15-85 mm</td>
<td>$&lt;0$</td>
<td>paraserial failure</td>
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<td>$60^\circ$</td>
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<td>0-5 mm</td>
<td>31.3</td>
<td>interserial sliding</td>
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<td></td>
<td>II</td>
<td>5-85 mm</td>
<td>10.5</td>
<td>paraserial + interserial rotation</td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>I</td>
<td>0-45 mm</td>
<td>0.99</td>
<td>interserial sliding</td>
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<td></td>
<td>II</td>
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</tr>
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<td>I</td>
<td>35-85 mm</td>
<td>21.4</td>
<td>paraserial bending</td>
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Figure 5.4: Orientation-dependent behavior of the scaled prototypes in concave bending. (a) Photographs of $\phi = 0^\circ$ sample: (i-ii) Vertical displacement ($d$) = 3 mm, side view and close up view of scales. (iii-v) $d$ = 20 mm, side view, close up view of scales, and close up view of substrate. (vii) $d$ = 60 mm, side view and close up view of scales. (b) Photographs of $\phi = 30^\circ$ sample: (i-ii) $d$ = 3 mm, side view and close up view of scales. (iii-v) $d$ = 15 mm, side view and close up view of scales. (v-vi) $d$ = 80 mm, side view and close up view of scale. (c) Photographs of $\phi = 60^\circ$ sample: (i-ii) $d$ = 4 mm, side view and close up view of scales. (iii-v) $d$ = 50 mm and 80 mm, side view and close up view of scales.
Figure 5.5: Orientation-dependent behavior of the scaled prototypes in concave bending. (a) Photographs of $\varphi = 90^\circ$ sample. (i-iii) Vertical displacement ($d$) = 15 mm, side view and corresponding close up views of scales. (iv-vi) $d$ = 60 mm, side view, oblique view, and close up views of scales and substrate. (b) Photographs of $\varphi = 120^\circ$ sample. (i-ii) $d$ = 4 mm, side view and close up view of scales. (iii-iv) $d$ = 20 mm, side view and close up view of scales with $d$ = 20 mm. (v-vi) $d$ = 60 mm, side view and close up view of scales. (c) Photographs of $\varphi = 150^\circ$ sample. (i-ii) $d$ = 4 mm, side view and close up view of scales. (iii-iv) $d$ = 14 mm, side view and close up view of scales. (v-vi) $d$ = 40 mm, side view and close up view of scales.
resistance to bending \((K = 15.8)\) originates from new interserial interlock, where the back end of the anterior margin contacts the scale in the adjacent column, and the anterior process making contact with the substrate (Fig. 5.4a.v). The compliant connective material in the peg-and-socket connections continue to be strained and resist scale rotation and bending. At large deformations, paraserial failure occurs as the peg-and-socket interconnections tear (Fig. 5.4a.v-vii), which removes resistance to paraserial bending, paraserial rotation, and interserial rotation. These connections break periodically (not all at once) so that the sample continues to bear load.

In the \(\phi = 30^\circ\) orientation (Fig. 5.4b), the scales initially undergo paraserial bending \((K = 126)\) until paraserial interlock (Fig. 5.4b.i-ii). The interlock causes small paraserial rotation alongside paraserial bending as the anterior margin rotates into plane (Fig. 5.4.b.iii-iv). The orientation of the scales gives tolerance to the anterior process so that it is not pushed into substrate; there is no coupling with interserial rotation, and the stiffness drops \((K = 21.7)\). At large deformations, paraserial failures occur simultaneously throughout the sample (Fig. 5.4b.v-vi), after which the prototype offers no resistance to global bending \((K < 0)\).

In the \(\phi = 60^\circ\) orientation (Fig. 5.4c), the scales first undergo interserial sliding as the anterior shelf slides under the scale in the adjacent column (Fig. 5.4c.i-ii). The oblique angle of orientation then causes the scales to touch interserially and resist deformation \((K = 31.3)\). As the sample continues to bend, the interserial interlock causes small paraserial rotation (anterior margin moves out of plane) coupled with large interserial rotation (anterior process moves out of plane; back of anterior margin and socket move into plane) to accommodate interserial sliding (Fig. 5.4c.iii-v). The compliant paraserial connections resist the rotations but do not break, so the prototype is able to bear load as it bends \((K = 10.5)\).

In the \(\phi = 90^\circ\) orientation (Fig. 5.5a), the paraserial axis is perpendicular to the loading direction. In bending, the columns of scales move relative to each other via interserial sliding (Fig. 5.5a.i-iii), and the only resistance to bending comes from bending of the substrate. Here, the rigid scales are “mechanically invisible” as they do not contribute any resistance to bending, and the sample’s stiffness matches that of the substrate \((K = 0.99)\). At large degrees of deformation, the scales begin to touch interserially (Fig. 5.5a.iv-vi). The scale contacts cause the columns of scales to rotate about the axial ridge, further straining the compliant material at the site of attachment to the substrate, and the stiffness of the sample increases \((K = 4.23)\).

When the \(\phi = 120^\circ\) prototype bends (Fig. 5.5b), the scales first move via interserial sliding (Fig. 5.5b.i-ii). However, the anterior process is pushed into the substrate immediately, and these contacts induce paraserial rotation (anterior margin moves into plane) coupled with interserial rotation (anterior process and peg move into plane; back of anterior margin and socket move out of plane) to generate additional paraserial and interserial scale contacts (Fig. 5.5b.iii-iv) that in turn generate high stiffness \((K = 124)\). These scale interlocks contribute to the high stiffness with further bending \((K = 46.8)\). At very large deformations, the scales splay interserially (Fig. 5.5b.v-vi) to further accommodate sliding with reduced stiffness \((K = 24.1)\).
The $\phi = 150^\circ$ prototype is the stiffest of all orientations (Fig. 5.5c). At the onset of bending, small degrees of paraserial bending causes the anterior process to push into the substrate immediately while the back end of the anterior margin makes contact with the scale in the adjacent column (Fig. 5.5c.i-ii). These scale contacts do not allow for further paraserial bending, provide no tolerance for interserial sliding, and cause high stiffness ($K = 450$). With further bending, the scale contacts cause paraserial rotation (anterior margin moves into plane) coupled with interserial rotation (back of anterior margin and socket move out of plane); however, the interlocked scales (Fig. 5.5c.iii-iv) continue to resist bending with high stiffness ($K = 80.9$). At very large deformations, the scale interlocks generate such high resistance to bending that the rods at the top and bottom of the sample begin bow out to accommodate the global compression (Fig. 5.5c.v-vi). Eventually, the compliant paraserial interconnections start to crack, and small degrees of paraserial bending are observed while the scale interlocks allow the prototype to sustain high loads ($K = 21.4$).

5.4. Discussion

The hierarchy of shape- and materials-based design principles are translated from the biological exoskeleton of $P. senegalus$ and integrated into the bioinspired flexible composite prototypes, including the complex shape of rigid scales, interscale joint articulation structure, assembly of scales into an armored surface, and soft connective components (substrate, scale-to-substrate attachment, and paraserial connections). The prototypes are able to replicate the biomechanical behavior of the biological exoskeleton, where the complex scale shape and joint articulations contribute to local, interscale mobility mechanisms which in turn determine the bending response of the global sample, and also generate global anisotropic (orientation-dependent) mechanical behavior in the scale assembly.

The fish engages in both convex and concave bending during its normal undulatory motion [3]. In concave bending, scale-to-scale contacts generate greater resistance to bending with complex local mechanisms. In convex bending and tension, the stiffness of the compliant materials, e.g. the substrate, paraserial connections, and scale-to-substrate connections, is hypothesized to dominate the response of the sample. The two lowest stiffness orientations, $\phi = 90^\circ$ and $\phi = 60^\circ$, correspond to the two commonly observed bending modes in the fish: axial bending and torsion, respectively [3-4]. In these orientations, the interserial sliding mechanism allows the assembly to bend under a small applied load without generating interscale contacts or introducing stress concentrations within the scales in the initial phase of deformation. In the $90^\circ$ orientation, all strains are sustained in the substrate up to 36% deformation (40 vertical mm / 112 mm total sample height), and sample stiffness matches that of the control sample without any scales. Since the rigid scales are “mechanically invisible” and do not contribute any resistance to bending, we show that it is possible to use shape as a materials design parameter to create composite materials that provide added protection from the stiffer material (e.g. scales) while maintaining the flexibility of the compliant material (e.g. substrate). In the $60^\circ$ orientation, all strains are sustained in the substrate up to 4.5% deformation, after which the orientation of scales
relative to the loading direction allows the interscale contacts to utilize low stiffness interscale mobility mechanisms, i.e. coupled paraserial and interserial rotation, to continue to enable interserial sliding of scales.

The high stiffness orientations correspond to bending modes in which the fish does not engage; for instance, 150° represents dorsoventral bending about a horizontal plane through the middle of the fish. Thus, the complex geometry and orientation of scales enables flexibility of the integument in directions that it uses for axial bending and torsion, and restricts flexibility in the directions it does not need. Features such as the oblong anterior process and concave anterior margin that make contact with the substrate and interlock with neighboring scales provide full-body coverage of scales over the dermis as the scales move apart from each other, e.g. in ventral scales that splay apart from each other since the plane of bending is different than the plane of axial bending or torsion [5], and in scales that slide apart during convex bending. Moreover, the complex geometry, orientation of scales, interlocking joints, and paraserial interconnections are further hypothesized to stiffen the scale armor system against penetrating loads normal to the surface of the scale assembly. Resistance to flexure prevents back deflection of the scales into the dermis of the fish, wherein morphometry-enabled protection mechanisms work alongside materials-based penetration resistance (e.g. circumferential surface cracking, minimized weight, and microstructural length scale and material property length scale matching between the armor and the predatory teeth (threat matching) with optimized layer thickness [10-14]) to add impact protection to the scale assembly without sacrificing flexibility in the directions necessary for axial bending and torsion.

Understanding complex materials-morphometric design rules in natural exoskeletons and translating them to synthetic designs holds tremendous application for bio-inspired flexible armor [6-7]. In the flexible composite prototypes inspired by the scale armor of P. senegalus, one design scheme exhibits a wide array of mechanical behavior with bending stiffness ranging over several orders of magnitude ($K = 1-320$), thus showing how morphometry can tune the flexibility of protective, composite architectures without varying the constituent materials. Extensions of this work include the integration of morphometric heterogeneity [5, 8], ability to conform to arbitrary curved surfaces [8], and intrascale material heterogeneity [9] into the prototype design for a truly hierarchical design that replicates all aspects of the biological armor. Biomimetic armors utilizing a segmented, squamation design hold enormous potential for a wide variety of applications through damage localization, flexibility, reduced cost of fabrication, and selective replacement of damaged units.
5.5. References


6. Finite Element Simulations of Bio-Inspired Flexible Composite Prototypes in Active Loading

6.1. Introduction

In this chapter, a computational framework is established for simulating bending and tension of the bioinspired flexible composite prototypes based on the exoskeleton of *P. senegalus*. The prototypes tested in Chapter 5 are modeled in finite element software, and their mechanical behavior is simulated in concave bending. The results show excellent agreement with the experimental results, including stiffness values and local, interscale deformation mechanisms that contribute to the global mechanical response of the sample. Stress and strain within the models are captured and correlated with the deformation mechanisms. The model is then extended to simulate the mechanical behavior of the prototypes in convex bending and tension. By capturing the complex deformation mechanisms observed experimentally, a computational framework is established for analyzing the behavior of these bioinspired flexible composite materials in accordance with the scale morphometry, material composition, orientation, and loading condition. The model allows for future work to study the effect of variations in morphometry and material composition on the behavior of the flexible composite assembly.

6.2. Materials and Methods

6.2.1. Model parts and assembly

Three parts were designed for the model using finite element software (ABAQUS, Dassault Systemes, France): a simplified scale geometry, a substrate material, and rigid rods. The scale was designed to include the characteristic geometric features of the *P. senegalus* scale including a rhomboid base (thickness = 4.1 mm), tapered anterior shelf (overlap angle = 32°), conical peg (length = 4.5 mm, tip angle = 31°), socket, anterior process, and axial ridge (width = 2.5 mm) and simplified with flattened surfaces on the body of the scale (Fig. 6.1a). The substrate was modeled as a 112 mm × 124 mm rectangle with a thickness of 3.8 mm and with shallow grooves to fit the axial ridges of scales with 1 mm interserial spacing between scales (Fig. 6.1b). The groove depth was 0.7 mm, giving a total model thickness of 7.2 mm to match the experimental prototypes. The scale and substrate were meshed with and meshed with tetrahedral C3D4 elements (standard, linear, 3D stress). Cylindrical rods were modeled as rigid bodies to fit at the top and bottom edges of the assembly and meshed with R3D4 elements. VeroWhite was modeled as an isotropic linear elastic material (E = 2.0 GPa, v = 0.43, and ρ = 1.175 g/cm³) and assigned to the scales and rigid rods. TangoPlus was modeled as a Neohookian hyperelastic material (C11 = 0.63 MPa, D1 = 10⁻⁵, and ρ = 1.120 g/cm³) and assigned to the substrate [1-3].

In the assembly of parts, scales were placed in the grooves of the substrate with 1 mm paraserial spacing. Tie constraints were modeled between contact surfaces: the scale’s axial ridge
to the substrate grooves, the rods to the top and bottom of the substrate, and the rods to scales at the top and bottom of the assembly. The TangoPlus interconnections between the peg and socket of adjacent scales were modeled as linear elastic springs with a stiffness $k_{sp} = C11 = 0.63 \text{ N/m}$. Separate models were developed for the six orientation angles ($\varphi$) of 0°, 30°, 60°, 90°, 120°, and 150°, where $\varphi$ is defined as the angle between the paraseral peg-and-socket axis and the loading direction (-y). A control model with a flat substrate and no scales was created to match the experimental TangoPlus control sample.

![Figure 6.1: Finite element model design.](image)

(a) Simplified scale, (i) top view and (ii) side view. (b) Grooved substrate, (i) top view for orientation angle ($\varphi$) = 60°, and (ii) side view with $\varphi = 0°$ showing scales placed in the shallow groove, where red lines define the axial ridge of scales. (c) Assembly of parts for the model with $\varphi = 60°$.

### 6.2.2. Bending simulation and analysis

Bending was simulated with pin-pin boundary conditions allowing rotation of rods about the x-axis, translation of the top rod in the y-axis only, and free corresponding movement of the model. Three steps were applied: (1) Rotation of both rods to set a concave or convex bending mode shape with lateral deflection = 10 mm (width of a rod) with free translation of the top rod. (2) Relaxation allowing free rotation of rods with fixed top rod displacement to relieve internal stresses. (3) Translation (-y) of the top rod at 1 mm/s with free rod rotation to bend the model. Force-displacement (F-d) curves were generated as a measure of the reaction force vs. vertical displacement of the top rod at every increment. Values for stiffness ($K$) were calculated as the slope of the F-d data and normalized by the stiffness of a control model consisting of a 4.4 mm sheet of TangoPlus without scales ($K = 10.7 \text{ N/m}$). Stresses (Mises) and strains (logarithmic, max. principal and components) were captured through the whole model at every 10 increments.

### 6.2.3. Tension simulation and analysis

Tension was simulated under uniaxial loading conditions in the +y direction at a strain rate of 1 mm/s constraining rod translation in the x- and z-axes and rod rotation in the x-, y-, and
z-axes with free corresponding movement of elements within the assembly. Force-displacement (F-d) curves were generated as a measure of the reaction force vs. vertical displacement of the top rod at every increment. An effective tensile modulus (E [MPa]) was calculated as the slope of the stress-strain curve, where engineering stress (σ) = F / A, A = 0.124 m × 0.0072 m (cross-sectional area of the model in the loading direction) and engineering strain (ε) = d / 0.112 m (height of the assembly), and normalized by the stiffness of a control model consisting of a 4.4 mm sheet of TangoPlus without scales (E = 1.29 MPa). Stresses (Mises) and strains (logarithmic, max. principal and components) were captured through the whole model at every 10 increments.

6.3. Results

6.3.1. Anisotropic stiffness in concave bending

The finite element simulations of the flexible composite prototypes in concave bending exhibit anisotropic mechanical behavior depending on the orientation angle (φ). For the range of models with φ = 0°, 30°, 60°, 90°, 120°, and 150°, the simulations show a good match to the experimental results in terms of the global stiffness and corresponding local, interscale deformation mechanisms. As in the experiment, the simulations for each orientation exhibit a characteristic loading response which can be divided into phases. Stiffness (K) for each phase, plotted in Fig. 6.2a against the experimental values, is calculated as the slope of the loading curve and normalized by the stiffness of a control model with no scales. Stiffness values match the experiments well, and the minor differences are attributed to the simplified scale geometry in the finite element model. Various interscale mobility mechanisms, depicted in Fig. 6.2b, were also observed in the simulations. Stresses (Mises) and strains (logarithmic, max. principal and components) are plotted for each of the models during bending in Figs. 6.3 - 6.8.
Figure 6.3: Stress plots (Mises) from the finite element simulations of flexible composite prototypes in concave bending. (a) Stress plots from the φ = 0° model. (i) Front view at vertical displacement (d) = 3 mm. (ii) Front view at d = 9 mm. (iii) Back view at d = 9 mm. (iv) Front view at d = 22 mm. (v) Scale bar. (b) Stress plots from the φ = 30° model. (i) Front views at d = 3 mm, (ii) d = 14 mm, and (iii) d = 33 mm. (iv) Scale bar. (c) Stress plots from the φ = 60° model. (i) Front views at d = 5 mm, (ii) d = 8 mm, and (iii) d = 38 mm. (iv) Scale bar.
Figure 6.4: Strain plots (logarithmic, max. principal) from the finite element simulations of flexible composite prototypes in concave bending. (a) Strain plots from the \( \phi = 0^\circ \) model. (i) Back views at vertical displacement \( d = 3 \text{ mm} \), (ii) \( d = 9 \text{ mm} \), and (iii) \( d = 22 \text{ mm} \). (iv) Front view at \( d = 22 \text{ mm} \). (v) Scale bar. (b) Strain plots from the \( \phi = 30^\circ \) model. (i) Back views at \( d = 3 \text{ mm} \), (ii) \( d = 14 \text{ mm} \), and (iii) \( d = 33 \text{ mm} \). (iv) Close up view of the scale-substrate attachment at \( d = 33 \text{ mm} \). (v) Scale bar. (c) Strain plots from the \( \phi = 60^\circ \) model. (i) Back views at \( d = 5 \text{ mm} \), (ii) \( d = 8 \text{ mm} \), and (iii) \( d = 38 \text{ mm} \). (iv) Close up view of the scale-substrate attachment at \( d = 24 \text{ mm} \). (v) Scale bar.
Figure 6.5: Strain plots (logarithmic, components (LEij)) from the finite element simulations in concave bending. (a) Strain plots from the $\phi = 0^\circ$ model, back view at vertical displacement (d) = 9 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar. (b) Strain plots from the $\phi = 30^\circ$ model, back view at d = 14 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar. (c) Strain plots from the $\phi = 60^\circ$ model, back view at d = 38 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar.
Figure 6.6: Stress plots (Mises) from the finite element simulations of flexible composite prototypes in concave bending. (a) Stress plots from the $\varphi = 90^\circ$ model. (i) Front views at vertical displacement ($d$) = 12 mm, (ii) $d$ = 43 mm, and (iii) $d$ = 68 mm. (iv) Scale bar. (b) Stress plots from the $\varphi = 120^\circ$ model. (i) Front views at $d$ = 3 mm, and (ii) $d$ = 12 mm. (iii) Back view at $d$ = 12 mm. (iv) Front view at $d$ = 52 mm. (v) Scale bar. (c) Stress plots from the $\varphi = 150^\circ$ model. (i) Back view at $d$ = 3 mm. (ii) Front views at $d$ = 20 mm, (iii) $d$ = 38 mm, and (iv) $d$ = 55 mm. (v) Scale bar.
Figure 6.7: Strain plots (logarithmic, max. principal) from the finite element simulations of flexible composite prototypes in concave bending. (a) Strain plots from the $\phi = 90^\circ$ model. (i) Back views at vertical displacement $d = 12$ mm, (ii) $d = 43$ mm, and (iii) $d = 68$ mm. (iv) Front view at $d = 68$ mm, left side cropped. (v) Scale bar. (b) Strain plots from the $\phi = 120^\circ$ model. (i) Back views at $d = 3$ mm, (ii) $d = 12$ mm, and (iii) $d = 26$ mm. (iv) Scale bar. (c) Strain plots from the $\phi = 150^\circ$ model. (i) Back views at $d = 3$ mm, (ii) $d = 12$ mm, and (iii) $d = 38$ mm. (iv) Scale bar.
Figure 6.8: Strain plots (logarithmic, components \( (LE_{ij}) \)) from the finite element simulations in concave bending. (a) Strain plots from the \( \phi = 90^\circ \) model, back view at vertical displacement \( (d) = 43 \) mm: (i) \( LE_{11} \), (ii) \( LE_{22} \), (iii) \( LE_{33} \), (iv) axial strain scale bar, (v) \( LE_{12} \), (vi) \( LE_{13} \), (vii) \( LE_{23} \), and (viii) shear strain scale bar. (b) Strain plots from the \( \phi = 120^\circ \) model, back view at \( d = 12 \) mm: (i) \( LE_{11} \), (ii) \( LE_{22} \), (iii) \( LE_{33} \), (iv) axial strain scale bar, (v) \( LE_{12} \), (vi) \( LE_{13} \), (vii) \( LE_{23} \), and (viii) shear strain scale bar. (c) Strain plots from the \( \phi = 150^\circ \) model, back view at \( d = 12 \) mm: (i) \( LE_{11} \), (ii) \( LE_{22} \), (iii) \( LE_{33} \), (iv) axial strain scale bar, (v) \( LE_{12} \), (vi) \( LE_{13} \), (vii) \( LE_{23} \), and (viii) shear strain scale bar.
In the $\phi = 0^\circ$ model (Fig. 6.3a, Fig. 6.4a, Fig. 6.5a), the scales undergo paraserial bending until scale interlock. In this phase, the compliant paraserial interconnections resist deformation in the model ($K = 45.4$), and stress concentrations are generated on the scale surrounding the peg and socket (Fig. 6.3a.i). As in the experiment, the scales then undergo paraserial rotation until interserial interlock where the back end of the anterior margin makes contact with the scale in the adjacent column causing stresses within the scales to grow (Fig. 6.3a.ii); meanwhile, stresses are generated in the substrate material when the anterior process pushes into the substrate (Fig. 6.3a.iii). The compliant, paraserial interconnections continue to be strained with greater degrees of bending ($K = 13.4$). At large deformations, the high internal stresses are distributed throughout the body of the scales (Fig. 6.3a.iv) before the paraserial interconnections fail. Throughout bending, strains accumulate and grow in the substrate. Tensile strains form on the back side of the substrate, and compressive strains form on the front of the substrate beneath the scales with large principal strains at sites where the anterior process of the scale makes contact with the substrate (Fig. 6.4a.i-iii). The individual axial and shear components of strain are shown in Fig. 6.5a. The scales act like rigid bodies, and their elements do not strain during bending (Fig. 6.4a.iv).

In the $\phi = 30^\circ$ model (Fig. 6.3b, Fig. 6.4b, Fig. 6.5b), the scales initially undergo paraserial bending until paraserial interlock ($K = 109$); stresses form both at the sites of interlock on the scales and around the peg-and-socket joint of paraserial scales due to the strain on the material in the paraserial interconnections (Fig 6.3b.i). With the onset of the paraserial rotation mechanism alongside paraserial bending, stresses grow larger and become distributed over the scale surfaces (Fig. 6.3b.ii). Strain and corresponding stress on the compliant paraserial interconnections accommodate both the paraserial bending and paraserial rotation mechanisms ($K = 15.0$). At large deformations, stresses spread over the entire scale surface with the greatest magnitude surrounding the peg-and-socket joint of neighboring scales (Fig. 6.3b.iii) before the paraserial connections fail. During bending, strains due to the scale rotations start to accumulate in the substrate where the axial ridge of the scale is attached to the groove of the substrate (Fig. 6.4b.i). As the model continues to bend, strains grow and spread throughout the neighboring elements in the substrate (Fig. 6.4b.ii-iii). Individual axial and shear components of strain are shown in Fig. 6.5b. While the elements of the substrate attached to the scales are strained, the elements of the scales do not sustain any strain (Fig. 6.4b.iv).

In the $\phi = 60^\circ$ model (Fig. 6.3c, Fig. 6.4c, Fig. 6.5c), the scales first undergo interserial sliding as the anterior shelf slides under the scale in the adjacent column ($K = 27.5$) without generating any stresses on the scales (Fig. 6.3c.i). When the scales begin to touch interserially, stresses are generated at the site of contact; paraserial rotations, coupled with large interserial rotations, introduce strain on the compliant, paraserial connective material, generating additional stress on the scales surrounding the peg-and-socket joint (Fig. 6.3c.ii). Stresses continue to grow at the sites of interscale contact and spread throughout the body of the scales ($K = 6.9$) (Fig. 6.3c.iii). During bending, strains accumulate in the substrate regions behind the scales, as the elements of the substrate are stretched to accommodate global bending while the scales undergo
interserial sliding (Fig. 6.4c.i). Strains in the substrate grow as the model bends further (Fig. 6.4c.ii-iii). Individual axial and shear components of strain are shown in Fig. 6.5c. Large principal strains are observed at the site of attachment of the axial ridge to the substrate due to the coupled paraserial and interserial rotations of the scales (Fig. 6.4c.iv).

In the $\phi = 90^\circ$ model (Fig. 6.6a, Fig. 6.7a, Fig. 6.8a), the columns of scales move relative to each other via interserial sliding; without scale rotations or contacts, the only resistance to bending comes from tensile stresses in the substrate (Fig. 6.6a.i). As observed in the experiment, the scales in the model are ‘mechanically invisible’ in this region, since they do not contribute to the stiffness of the model ($K = 1.0$). At larger deformations, the scales touch interserially; as the scales rotate about their axial ridge, the substrate material surrounding the axial ridges becomes strained; stresses in the substrate are then transmitted into the center of the scales ($K = 3.4$) (Fig. 6.6a.ii) and slowly grow as the model bends (Fig. 6.6a.iii). All strains are also sustained in the substrate during the bending of the model; principal strains grow in columns along the paraserial axis of the scale assembly and spread throughout the substrate (Fig. 6.7a.i-iii). Individual axial and shear components of strain are shown in Fig. 6.8a. Larger magnitudes of strains are seen on the back of the substrate. Without scale rotations, the elements around the axial ridge of scales are not strained greater than the rest of the substrate elements in this orientation (Fig. 6.7a.iv).

In the $\phi = 120^\circ$ model (Fig. 6.6b, Fig. 6.7b, Fig. 6.8b), the scales first move by a small degree of interserial sliding until the anterior process makes contact with the substrate, inducing paraserial and interserial rotations that in turn generate paraserial and interserial scale contacts with high stiffness ($K = 89.4$). Stress concentrations form on the scales in regions surrounding the peg and socket where paraserial contacts occur (Fig. 6.6b.i), at the sites of interserial contact between the columns of scales (Fig. 6.6b.ii), and throughout the substrate where the anterior process touches ($K = 40.7$) (Fig. 6.6b.iii). At very large deformations, high stress throughout the model causes the scales splay interserially (Fig. 6.6b.iv) in order to accommodate further bending ($K = 7.79$). Due to scale rotations, principal strains accumulate and grow in the substrate where the axial ridge of the scale is attached to the groove of the substrate (Fig. 6.7b.i-ii). Individual axial and shear components of strain are shown in Fig. 6.8b. At large deformations, the anterior margins of the scales push into the substrate and cause it to tear (Fig. 6.7b.iii).

In the $\phi = 150^\circ$ model (Fig. 6.6c, Fig. 6.7c, Fig. 6.8c), small degrees of initial paraserial rotation cause the anterior process to push into the substrate (Fig. 6.6c.i). Meanwhile, the scales interlock with each other and high stresses are sustained at the sites of contact between scales ($K = 276$) and begin to spread through the columns of scales ($K = 64.3$) (Fig. 6.6c.ii). At large deformations, scale interlocks continue to generate high stresses throughout the model ($K = 29.7$) (Fig. 6.6c.iii-iv). The substrate material bears the strain in the model; high magnitudes of principal strains accumulate and grow in the substrate where the anterior margin makes contact with the substrate (Fig. 6.7c.i-iii). Individual axial and shear components of strain are shown in Fig. 6.8c. At large deformations, the sharp point on the anterior margin of the scales tears through the substrate (Fig. 6.7c.iii).
6.3.2. Anisotropic stiffness in convex bending

In convex bending, the finite element simulations of the flexible composite prototypes also exhibit orientation-dependent (anisotropic) behavior. Each of the models with orientation angle (φ) ranging between 0°, 30°, 60°, 90°, 120°, and 150°, exhibits a characteristic loading behavior. Stiffness (K) for each model, plotted in Fig. 6.9, is calculated as the slope of the loading curve and normalized by the stiffness of a control model with no scales. Unlike in concave bending, each model exhibits a single stiffness value which remains constant during loading in convex bending. The φ = 30°, 60°, 120°, and 150° models exhibit similar stiffness to each other (K ~ 3.5-5), while the φ = 0° model is much stiffer (K = 84.4) and the φ = 90° is the least stiff (K = 0.93). Stresses (Mises) and strains (logarithmic, max. principal and components) are plotted for each of the models during bending in Figs. 6.10 - 6.15.

In φ = 0° model (Fig. 6.10a, Fig. 6.11a, Fig. 6.12a), the paraserial axis of the assembly of scales is aligned with the loading direction. Since the scales are rigid compared to the compliant materials, strain in the model is sustained by the soft substrate and the paraserial interconnections between the peg-and-socket of the scales. The scales accommodate global convex bending via the paraserial bending mechanism through the duration of loading. Stress concentrations initially form within the soft material in the paraserial interconnections, and stress is transmitted to the peg, socket, and surrounding regions of the scales (Fig. 6.10a.i-ii). As the model continues to bend, the strains on the connective material grow along with the stresses, and the stresses become distributed over the body of the scales (Fig. 6.10a.iii), leading to high stiffness (K = 84.4) until the paraserial interconnections break. During bending, strains are sustained in the substrate material. In this orientation, principal strains form in the spaces between the scales, as the substrate elements between the scales (paraserially and interserially) stretch to allow global deformation of the model (Fig. 6.11a.i-iii). Compressive strains form on the back side of the substrate, and tensile strains on the front of the substrate beneath the scales.

![Figure 6.9: Normalized stiffness (K) of the finite element models in convex bending.](image)
Figure 6.10: Stress plots (Mises) from the finite element simulations of flexible composite prototypes in convex bending. (a) Stress plots from the φ = 0° model. (i) Front view at vertical displacement (d) = 3 mm. (ii) Close up view at d = 3 mm. (iii) Front view at d = 15 mm. (iv) Scale bar. (b) Stress plots from the φ = 30° model. (i) Front views at d = 2 mm, (ii) d = 5 mm, and (iii) d = 16 mm. (iv) Back view at d = 24 mm, left side cropped. (v) Scale bar. (c) Stress plots from the φ = 60° model. (i) Front views at d = 3 mm, (ii) d = 10 mm, and (iii) d = 27 mm. (iv) Close up view at d = 40 mm. (v) Scale bar.
Figure 6.11: Strain plots (logarithmic, max. principal) from the finite element simulations of flexible composite prototypes in convex bending. (a) Strain plots from the $\phi = 0^\circ$ model. (i) Back views at vertical displacement ($d$) = 3 mm, (ii) $d$ = 12 mm, (iii) $d$ = 28 mm, and (iv) $d$ = 53 mm. (v) Scale bar. (b) Strain plots from the $\phi = 30^\circ$ model. (i) Front, close up view at $d$ = 15 mm. (ii) Back view at $d$ = 3 mm and (iii) $d$ = 16 mm. (iv) Scale bar. (c) Strain plots from the $\phi = 60^\circ$ model. (i) Back views at $d$ = 10 mm, (ii) $d$ = 27 mm, and (iii) $d$ = 40 mm. (iv) Scale bar.
Figure 6.12: Strain plots (logarithmic, components (LEij)) from the finite element simulations in convex bending. (a) Strain plots from the φ = 0° model, back view at vertical displacement (d) = 28 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar. (b) Strain plots from the φ = 30° model, back view at d = 16 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar. (c) Strain plots from the φ = 60° model, back view at d = 40 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar.
Figure 6.13: Stress plots (Mises) from the finite element simulations of flexible composite prototypes in convex bending. (a) Stress plots from the $\phi = 90^\circ$ model. (i) Front views at vertical displacement (d) = 22 mm and (ii) 42 mm, left side cropped. (iii) Back view at d = 42 mm, right side cropped. (iv) Front view at d = 74 mm. (v) Scale bar. (b) Stress plots from the $\phi = 120^\circ$ model. (i) Front views at d = 5 mm, (ii) d = 16 mm, and (iii) d = 41 mm. (iv) Side view at d = 55 mm. (v) Scale bar. (c) Stress plots from the $\phi = 150^\circ$ model. (i) Front view at d = 2 mm. (ii) Close up view at d = 2 mm. (iii) Front view at d = 36 mm. (iv) Back view at d = 39 mm. (v) Scale bar.
Figure 6.14: Strain plots (logarithmic, max principal) from the finite element simulations of flexible composite prototypes in convex bending. (a) Strain plots from the $\phi = 90^\circ$ model. (i) Front, oblique view at vertical displacement $(d) = 22$ mm, left side cropped. (ii) Back views at $d = 22$ mm, (iii) $d = 42$ mm, right side cropped, and (iv) $d = 66$ mm, right side cropped. (v) Scale bar. (b) Strain plots from the $\phi = 120^\circ$ model. (i) Back views at $d = 5$ mm, (ii) $d = 16$ mm, (iii) $d = 41$ mm, and (iv) $d = 55$ mm. (v) Scale bar. (c) Strain plots from the $\phi = 150^\circ$ model. (i) Back views at $d = 2$ mm, (ii) $d = 10$ mm, (iii) $d = 20$ mm, and (iv) $d = 39$ mm. (v) Scale bar.
Figure 6.15: Strain plots (logarithmic, components (LEij)) from the finite element simulations in convex bending. (a) Strain plots from the φ = 90° model, back view at vertical displacement (d) = 66 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar. (b) Strain plots from the φ = 120° model, back view at d = 16 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar. (c) Strain plots from the φ = 150° model, back view at d = 20 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar.
The individual axial and shear components of strain are shown in Fig. 6.12a. Like in concave bending, the scales are rigid and do not deform. After the paraserial connections break (Fig. 6.11a.iv), the substrate wrinkles, particularly evident at the edges, to accommodate global bending of the model alongside scale rigidity.

In $\varphi = 30^\circ$ model (Fig. 6.10b, Fig. 6.11b, Fig. 6.12b), strain in the model is initially sustained by the soft substrate and paraserial interconnections as the scales move relative to each other via paraserial bending. Stresses in the model are initially concentrated in the paraserial interconnections and the surrounding regions of the scales (Fig. 6.10b.i-ii). The oblique angle of the scales allows them to rotate paraserially to relieve stress on the assembly, resulting in a stiffness lower by an order of magnitude ($K = 5.02$) than the $\varphi = 0^\circ$ model. Paraserial rotation is coupled with interserial rotation, and stresses on the scales grow as the model bends further (Fig. 6.10b.iii). At large deformations, the large degree of interserial rotation begins to cause high strains on the substrate material attached to the axial ridges of the scales. As the substrate bends and the individual, rigid scales do not deform, stresses build up on the substrate material in the regions surrounding the axial ridge (Fig. 6.10b.iv). Corresponding principal strains develop on the substrate at the site of attachment of the scales’ axial ridges (Fig. 6.11b.i), as was observed in concave bending, and they continue to grow as the model bends (Fig. 6.11b.ii-iii). Individual axial and shear components of strain are shown in Fig. 6.12b.

In $\varphi = 60^\circ$ model (Fig. 6.10c, Fig. 6.11c, 6.12c), low stresses are observed throughout the model as the scales initially move relative to each other via interserial sliding and small degrees of paraserial bending (Fig. 6.10c.i). As the model continues to bend, stresses on the paraserial interconnections grow, and the scales relieve these stresses through paraserial and interserial rotation (Fig. 6.10c.ii) yielding a low stiffness ($K = 4.29$). The rotation of the scale accommodates further interserial sliding. At larger deformations, the scales undergo small degrees of paraserial bending, which adds strain to the paraserial interconnections, and stresses are transferred to the surrounding regions of the scales (Fig. 6.10c.iii-iv). In the substrate, principal strains are concentrated around the sites of attachment to the scales’ axial ridges due to the coupled paraserial and interserial of the scales (Fig. 6.11c.i). The strains grow in magnitude as the model continues to bend (Fig.6.11c.ii-iii). Individual axial and shear components of strain are shown in Fig. 6.12c.

In $\varphi = 90^\circ$ model (Fig. 6.13a, Fig. 6.14a, 6.15a), the overlap joint is aligned with the loading direction, and the paraserial axis is perpendicular to the loading direction. Stresses and strains are sustained on the substrate only, as the scales move apart via interserial sliding (Fig. 6.13a.i, Fig. 6.14a.i). Since the scales do not offer any resistance to bending, they can be considered ‘mechanically invisible’ in this orientation, and the stiffness is low ($K = 0.93$). At large deformations beyond vertical displacement ($d$) ~ 40 mm, the high degree of curvature generates larger stresses in the region surrounding the axial ridge of the scales, since the scales remain rigid and do not deform. Lower magnitudes of stress relative are transmitted into the scales (Fig. 6.13a.ii-iii). Even at very large deformations, the substrate continues to bend easily.
due to its hyperelasticity, the scales continue to slide apart and offer no resistance to bending, and the stiffness remains unchanged (Fig. 6.13a.iv). The compressive principal strains on the backside of the substrate are initially homogenous in magnitude (Fig. 6.14a.ii); with greater deformation, certain elements carry more principal strain (Fig. 6.14a.iii-iv). Individual axial and shear components of strain are shown in Fig. 6.15a. Larger values of strain are observed on the front, tensile side of the substrate, particularly at the site of attachment to the scales.

In $\phi = 120^\circ$ model (Fig. 6.13b, Fig. 6.14b, 6.15b), low stresses are initially observed through the model as the scales start to move via interserial sliding and small degrees of paraserial rotation (Fig. 6.13b.i). As the model continues to deform, the stresses on the paraserial interconnections grow, and the scales relieve these stresses through paraserial and interserial rotation (Fig. 6.13b.ii), similar to the $\phi = 60^\circ$ model, yielding a low stiffness ($K = 3.79$). At larger deformations, the scales undergo paraserial bending, which generates greater stresses on the paraserial interconnections which are transmitted through the body of the scales (Fig. 6.13b.iii). At very large deformations, the high degree of scale rotation cause the anterior process of the scales to stick out from the model’s surface (Fig. 6.13b.iv). In the substrate, principal strains form at the sites of attachment to the axial ridges due to the coupled paraserial and interserial of the scales (Fig. 6.14b.i). The strains grow in magnitude as the model continues to bend (Fig.6.14b.ii-iv). Individual axial and shear components of strain are shown in Fig. 6.15b.

In $\phi = 150^\circ$ model (Fig. 6.13c, Fig. 6.14c, 6.15c), the scales initially move relative to each other via paraserial bending, straining the paraserial interconnections. Stresses in the model are initially concentrated in the paraserial interconnections and the surrounding regions of the scales (Fig. 6.13c.i-ii). As the model continues to bend, the scales rotate paraserially and interserially to accommodate the deformation with low stiffness ($K = 4.99$), and stresses within the body of the scales grows (Fig. 6.13c.iii). At large deformations, stresses accumulate in the substrate in the regions attached to the axial ridges of scales, again due to the rigid scales not deforming along with the substrate (Fig. 6.13c.iv). Corresponding principal strains develop on the substrate at the site of attachment of the scales’ axial ridges (Fig. 6.14c.i), as observed in concave bending, and they continue to grow as the model bends (Fig. 6.14c.ii-iv). Individual axial and shear components of strain are shown in Fig. 6.15c.

6.3.3. Anisotropic stiffness in tension

In tension, the finite element simulations of the flexible composite prototypes also exhibit orientation-dependent (anisotropic) behavior. Each of the models with orientation angle ($\phi$) ranging between $0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$, $120^\circ$, and $150^\circ$, exhibits a characteristic loading behavior. The effective modulus ($E$) for each model, plotted in Fig. 6.16, is calculated as the slope of the stress-strain curve and normalized by the modulus of a control model with no scales. Unlike in concave bending, each model exhibit a single stiffness in tension which remains constant during loading. The $\phi = 90^\circ$ model exhibits the lowest modulus ($E = 0.57$), followed by the $\phi = 60^\circ$ and $\phi = 120^\circ$ models which exhibit similar moduli to each other ($E \sim 1$), followed by the $\phi = 30^\circ$ and $150^\circ$ models ($E \sim 13$), and the $\phi = 0^\circ$ model exhibits the greatest modulus ($E = 64$). Stresses (Mises)
and strains (logarithmic, max. principal and components) are plotted for each of the models during tension in Figs. 6.17 - 6.22.

In $\phi = 0^\circ$ model (Fig. 6.17a, Fig. 6.18a, Fig. 6.19a), the paraserial axis is aligned with the loading direction. As in a bilayer composite, tensile strains are sustained by both the substrate and scale armor layers; in the scale layer, the compliant, paraserial interconnections bear the strain, since the scales are segmented and stiff. Stresses first build up in the paraserial interconnections and the regions of the scales surrounding the peg-and-socket joint (Fig. 6.17a.i-ii). Principal strains build in the substrate material (Fig. 6.18a.i); the individual axial and shear components of strain are shown in Fig. 6.19a. In the scale armor layer, the scales do not deform while the paraserial interconnections are strained (Fig. 6.18a.ii-iii). As the model is stretched further, the stresses continue to be transferred through the body of the scales (Fig. 6.17a.iii-iv) and the modulus remains high ($E = 64.3$). Eventually, the paraserial connective material fails; all strains, and subsequently stresses, are then sustained in the substrate material (Fig. 6.17a.v, Fig. 6.18a.iv). Greater magnitudes of principal strain are observed on the front of the substrate, between the scales in the grooves that fit the axial ridges of the scales, since these elements are free to deform without compatibility requirements from the axial ridge elements, compared to elements directly underneath the scales and on the back side of the substrate (Fig. 6.18a.v).

In $\phi = 30^\circ$ model (Fig. 6.17b, Fig. 6.18b, Fig. 6.19b), vertical tension initially puts strain on the paraserial interconnections in the scale layer, generating stress concentrations around the peg and socket of the scales (Fig. 6.17b.i). Corresponding principal strains in the substrate develop between the scales in the paraserial direction (Fig. 6.18b.i). The individual axial and shear components of strain are shown in Fig. 6.19b. With further load, the oblique orientation of the scales allows for paraserial rotation to accommodate the vertical expansion of the global assembly with a lower modulus ($E = 13.6$) than the $\phi = 0^\circ$ model. As the scales rotate, the substrate material attached to the axial ridge of the scales also rotates, and stress concentrations

![Figure 6.16: Normalized effective modulus ($E$) of the finite element models in tension.](image)

112
**Figure 6.17**: Stress plots (Mises) from the finite element simulations of flexible composite prototypes in tension. (a) Stress plots from the $\varphi = 0^\circ$ model. (i) Front view at vertical displacement ($d$) = 1 mm. (ii) Close up view at $d$ = 1 mm. (iii) Front view at $d$ = 2.5 mm, left side cropped. (iv) Front view at $d$ = 6 mm, left side cropped. (v) Back view at $d$ = 10 mm, left side cropped. (vi) Scale bar. (b) Stress plots from the $\varphi = 30^\circ$ model. (i) Front view at $d$ = 1 mm. (ii) Back view at $d$ = 1 mm. (iii) Front view at $d$ = 5 mm. (iv) Back view at $d$ = 6 mm. (v) Scale bar. (c) Stress plots from the $\varphi = 60^\circ$ model. (i) Front view at $d$ = 5 mm. (ii) Back view at $d$ = 15 mm. (iii) Close up view at $d$ = 15 mm. (iv) Front view at $d$ = 35 mm. (v) Back view at $d$ = 45 mm. (vi) Scale bar.
Figure 6.18: Strain plots (logarithmic, max. principal) from the finite element simulations of flexible composite prototypes in tension. (a) Strain plots from the φ = 0° model. (i) Back view at vertical displacement (d) = 3.8 mm. (ii) Front view at d = 3.8 mm, left side cropped. (iii) Close up view at d = 3.8 mm. (iv) Back view at d = 19 mm, left side cropped. (v) Front view at d = 19 mm. (vi) Scale bar. (b) Strain plots from the φ = 30° model. (i) Back views at d = 2.5 mm, (ii) d = 4.4 mm, and (iii) d = 9 mm. (iv) Front view at d = 9 mm. (v) Back view at d = 14 mm. (vi) Scale bar. (c) Strain plots from the φ = 60° model. (i) Back view at d = 5 mm. (ii) Close up view of scale edges at d = 9 mm. (iii) Back views at d = 15 mm and (iv) d = 29 mm. (v) Front view at d = 29 mm. (vi) Scale bar.
Figure 6.19: Strain plots (logarithmic, components (LEij)) from the finite element simulations in tension. (a) Strain plots from the $\phi = 0^\circ$ model, back view at vertical displacement (d) = 3.8 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar. (b) Strain plots from the $\phi = 30^\circ$ model, back view at d = 2.5 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar. (c) Strain plots from the $\phi = 0^\circ$ model, back view at d = 15 mm: (i) LE11, (ii) LE22, (iii) LE33, (iv) axial strain scale bar, (v) LE12, (vi) LE13, (vii) LE23, and (viii) shear strain scale bar.
Figure 6.20: Stress plots (Mises) from the finite element simulations of flexible composite prototypes in tension. (a) Stress plots from the $\phi = 90^\circ$ model. (i) Front views at vertical displacement ($d$) = 22 mm, (ii) $d = 36$ mm, and (iii) $d = 47$ mm. (iv) Back view at $d = 47$ mm. (v) Scale bar. (b) Stress plots from the $\phi = 120^\circ$ model. (i) Front view at $d = 5$ mm. (ii) Back view at $d = 5$ mm. (iii) Close up view at $d = 20$ mm. (iv) Front view at $d = 35$ mm. (v) Back view at $d = 49$ mm, left side cropped. (vi) Scale bar. (c) Stress plots from the $\phi = 150^\circ$ model. (i) Close up view at $d = 0.5$ mm. (ii) Front views at $d = 2$ mm and (iii) $d = 5$ mm. (iv) Back view at $d = 12$ mm. (v) Scale bar.
Figure 6.21: Strain plots (logarithmic, max. principal) from the finite element simulations of flexible composite prototypes in tension. (a) Strain plots from the $\phi = 90^\circ$ model. (i) Front view at vertical displacement ($d$) = 22 mm. (ii) Back views at $d = 22$ mm, (iii) $d = 36$ mm, and (iv) $d = 50$ mm. (v) Scale bar. (b) Strain plots from the $\phi = 120^\circ$ model. (i) Back views at $d = 5$ mm and (ii) $d = 19$ mm. (iii) Front view at $d = 20$ mm. (iv) Back view at $d = 35$ mm. (v) Scale bar. (c) Strain plots from the $\phi = 150^\circ$ model. (i) Back views at $d = 2$ mm and (ii) $d = 5$ mm. (iii-iv) Close up views of the paraserial peg and socket joint at $d = 5$ mm. (v) Back view at $d = 12$ mm. (vi) Front view at $d = 12$ mm, right side cropped. (vii) Scale bar.
**Figure 6.22:** Strain plots (logarithmic, components (LE\(ij\))) from the finite element simulations in tension. (a) Strain plots from the \(\phi = 90^\circ\) model, back view at vertical displacement (d) = 50 mm: (i) LE\(11\), (ii) LE\(22\), (iii) LE\(33\), (iv) axial strain scale bar, (v) LE\(12\), (vi) LE\(13\), (vii) LE\(23\), and (viii) shear strain scale bar. (b) Strain plots from the \(\phi = 120^\circ\) model, back view at d = 35 mm: (i) LE\(11\), (ii) LE\(22\), (iii) LE\(33\), (iv) axial strain scale bar, (v) LE\(12\), (vi) LE\(13\), (vii) LE\(23\), and (viii) shear strain scale bar. (c) Strain plots from the \(\phi = 150^\circ\) model, back view at d = 5 mm: (i) LE\(11\), (ii) LE\(22\), (iii) LE\(33\), (iv) axial strain scale bar, (v) LE\(12\), (vi) LE\(13\), (vii) LE\(23\), and (viii) shear strain scale bar.
build up there (Fig. 6.17b.ii). Principal strains grow in the substrate at an oblique angle along the columns of scales spanning from top to bottom of the sample (Fig. 6.18b.ii-iii), as well as within the grooves attached to the axial ridges (Fig. 6.18b.iv). The stresses continue to grow until they are distributed over the entire scale body (Fig. 6.17b.iii), and throughout the substrate material (Fig. 6.17b.iv). After the paraserial interconnections break, strains in the substrate continue to grow (Fig. 6.18b.v) as the model stretches without resistance from the scale assembly.

In $\phi = 60^\circ$ model (Fig. 6.17c, Fig. 6.18c, Fig. 6.19c), the scale orientation yields a design where certain columns of scales are not attached to either the top or bottom rod. The interserial segmentation causes the substrate material to bear the bulk of the tensile strain (and corresponding stresses) with small stress concentrations transmitted into scales attached to the top and bottom rods (Fig. 6.17c.i-ii, Fig. 6.18c.i) resulting in a low modulus similar to the reference model without scales ($E = 0.96$). As the load increases, the columns of scales shear relative to each other, causing corresponding wrinkling of the substrate material (Fig. 6.17c.iii, Fig. 6.18c.ii). Since global lateral movement of the scale columns is restricted, reaction forces (and stresses) build within the scales (Fig. 6.17c.iv). The substrate material between the scales continues to strain with additional load (Fig. 6.18c.iii-v); the individual axial and shear components of strain are shown in Fig. 6.19c. Stress bands corresponding to the scale shearing are apparent at large deformations (Fig. 6.17c.v).

In $\phi = 90^\circ$ model (Fig. 6.20a, Fig. 6.21a, Fig. 6.22a), the paraserial axis is perpendicular to the loading direction, yielding full vertical segmentation between the columns of scales. The substrate bears all of the tensile strain and stress in the model, with no stresses on the scales or the paraserial interconnections (Fig. 6.20a.i, Fig. 6.21a.i-ii). In this orientation, the scales are again ‘mechanically invisible’ as they do not contribute to the stiffness of the model. The modulus ($E = 0.57$) is lower than the control, since the TangoPlus substrate that bears the load is thinner (3.8 mm) than the through-sample thickness (7.2 mm) used in calculating stress ($3.8/7.2 = 0.53$). At large vertical deformations (d > 20 mm), low magnitudes of stress concentrations begin to appear within the scales (Fig. 6.20a.ii-iii), negligible compared to the substrate (Fig. 6.20a.iv). Also, higher-magnitude bands of principal strain arise in the substrate regions between the paraserial columns of scales (Fig. 6.21a.iii-iv). The individual axial and shear components of strain are shown in Fig. 6.22a.

In $\phi = 120^\circ$ model (Fig. 6.20b, Fig. 6.21b, Fig. 6.22b), the orientation of scales again yields full segmentation between the columns of scales, and the substrate bears the bulk of the tensile strain and corresponding stresses (Fig. 6.20b.i-ii, Fig. 6.21b.i). The model’s behavior and subsequently low modulus ($E = 0.99$) are similar to the $\phi = 60^\circ$ model. As the columns of scales shear relative to each other, shear bands of stress and principal strain appear in the substrate to accommodate vertical expansion of the assembly (Fig. 6.20b.ii, Fig. 6.21b.ii), and the edges of the substrate are observed to wrinkle (Fig. 6.20b.iii-iv, Fig. 6.21b.iii). At large deformations, high strains within the elements of the substrate between the columns of scales allow for vertical expansion (Fig. 6.21b.iv). The individual axial and shear components of strain are shown in Fig.
Meanwhile stresses grow within the columns of scales (Fig. 6.20b.iv) and throughout the underlying substrate (Fig. 6.20b.v).

The $\phi = 150^\circ$ model (Fig. 6.20c, Fig. 6.21c, Fig. 6.22c) behaves similarly to the $\phi = 30^\circ$ model with a comparable intermediate modulus ($E = 13.3$). The onset of uniaxial tension induces strain on the paraserial interconnections within the columns of scales, generating stress concentrations around the peg and socket of the scales (Fig. 6.20c.i). With further load, the stresses within the scales grow, and the oblique orientation of the scales allows for paraserial rotation to accommodate the vertical expansion of the global assembly (Fig. 6.20c.ii-iii). The corresponding principal strains grow in the substrate along the columns of scales spanning the height of the sample (Fig. 6.21c.i-ii). The individual axial and shear components of strain are shown in Fig. 6.22c. At large deformations, it is possible to see the elements at the tip of the peg straining along with the paraserial interconnections (Fig. 6.21c.iii). As the scales rotate, the substrate material attached to the axial ridge is strained (Fig. 6.21c.iv), and stress concentrations arise in these regions of the substrate (Fig. 6.20c.iv). After the paraserial interconnections break, strains in the substrate continue to grow (Fig. 6.21c.v) as the model stretches without resistance from the scale assembly (Fig. 6.21c.vi).

### 6.4. Discussion

This chapter establishes finite element models for simulating the mechanical behavior of the bioinspired, flexible composite prototypes developed in Chapter 5 in bending and tension. In concave bending (scale facing in), each orientation ($\phi = 0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ, \text{and} 150^\circ$) exhibits a different, multi-phase loading response governed by the complex scale geometry. A unique set of interscale mobility mechanisms are observed for each orientation, and the observed mechanisms are correlated with the global stiffness of the model. The results from the finite element simulations in concave bending match the experimental results, reaffirming the main conclusion from Chapter 5 that one design scheme can exhibit a wide array of mechanical behavior with stiffness ranging over several orders of magnitude ($K = 1-276$), thus showing how morphometry can tune the flexibility of protective, composite architectures without varying the constituent materials. Minor differences between the experiment and the simulations are attributed to the simplified scale geometry in the finite element model. By capturing the complex deformation mechanisms observed experimentally, a computational framework is established for analyzing the behavior of these bioinspired flexible composite materials in accordance with the scale morphometry, material composition, orientation, and loading condition.

In this chapter, the model was extended to examine the effect of different loading conditions (convex bending and tension) on the mechanical behavior of the scale assemblies. In convex bending (scales facing out), each model exhibits a single stiffness value which remains constant during loading. The soft materials (substrate, paraserial interconnections) govern the anisotropic mechanical response with small contributions interscale mobility mechanisms, resulting in global stiffness ranging from $K = 0.93-84$. Extreme values of stiffness are seen when
the joint axes are aligned with the loading direction. Maximum stiffness arises when the paraserial axis of the peg-and-socket joint is aligned with the loading direction in the $\varphi = 0^\circ$ orientation; the scales only deform by paraserial bending, which puts high tensile stresses on the paraserial interconnections between the peg and socket of adjacent scales. Minimum stiffness is seen when the axis of the overlap joint is aligned with the loading direction in the $\varphi = 90^\circ$ orientation; the scales move apart by interserial sliding, with all strains sustained in the substrate, and no stresses on the scales. Four orientations ($\varphi = 30^\circ$, $60^\circ$, $120^\circ$, and $150^\circ$) exhibit similar stiffness to each other ($K \sim 3.5\)\,5$); in each of these orientations, a unique combination of scale rotation mechanisms relieve stresses on the paraserial interconnections to allow global bending.

In uniaxial tension, the soft materials (substrate and paraserial interconnections) continue to govern the anisotropic mechanical response of the scale assemblies with small contributions from interscale mobility mechanisms. The effective tensile modulus ranges from $E = 0.57$-64 depending on the sample orientation. The $\varphi = 90^\circ$ orientation exhibits the lowest modulus ($E = 0.57$) as substrate material sustains all of the tensile strains, with no stresses on the scales. The control sample with no scales has a modulus of $E_{TP} = 1.0$; in the $\varphi = 90^\circ$ orientation, the scale assembly is a more compliant material than the control sample with the added benefit of the protective scales. The $\varphi = 0^\circ$ orientation exhibits the highest modulus ($E = 64.3$), where the paraserial interconnections resist vertical expansion of the global $\varphi = 90^\circ$ assembly. Even in the stiffest orientation ($\varphi = 0^\circ$), the effective modulus of the scale assembly is far below the modulus of VeroWhite ($E_{VW} = 2046$), and also below a calculated rule-of-mixtures modulus for a simple bilayer composite, where $V_f$ is the volume fraction of the TangoPlus and VeroWhite materials:

$$E_{\text{bilayer}} = V_{f,TP} * E_{TP} + V_{f,VW} * E_{VW} + \frac{3.8 \, mm}{7.2 \, mm} * 1.0 + \frac{(7.2 \, mm - 3.8 \, mm)}{7.2 \, mm} * 2046 = 966$$

The intermediate orientations exhibit intermediate moduli. In the $\varphi = 30^\circ$ and $\varphi = 150^\circ$ orientations ($E \sim 13$), paraserial rotations relieve the stresses on the paraserial interconnections and allow the sample to expand. In the $\varphi = 60^\circ$ and $\varphi = 120^\circ$ orientations ($E \sim 1$), the columns of scales shear relative to each other while the substrate bears the bulk of the vertical tensile strains.

This chapter establishes a comprehensive computational model to assess the mechanical behavior of flexible composite prototypes inspired by the exoskeleton of *P. senegalus* that were developed in Chapter 5. By replicating the experimental results for concave bending and expanding the model to examine different loading conditions (convex bending and tension), the development of the model has generated avenues for studying the effect of variations, such as scale morphometry or the properties of the constituent materials, on the mechanical behavior of the scale assembly computationally. Simulations that parametrically examine variations in both materials and morphometric properties can help to identify the most promising armor designs, saving the need to fabricate and experimentally test all intermediate variations.
6.5. References


7. Effect of Variation in Morphometric and Material Design on Mechanical Behavior

7.1. Introduction

In this chapter, variations in both morphometric and material design are introduced into the finite element simulations of the flexible composite prototypes inspired by the *P. senegalus* exoskeleton. The finite element models developed in Chapter 6 are extended to examine the effect of variations in the multi-material architecture, scale morphometry, and constituent material properties on the mechanical behavior of the bio-inspired, flexible composite materials. Here, the models are re-designed to remove the paraserial interconnections, vary the stiffness ratio between the composite material constituents, and alter scale shape through removal of the peg-and-socket joint, and the simulations are run under active loading conditions (concave bending, convex bending, and tension). The results show how a combination of design elements, such as scale geometry, scale-to-scale joints and articulation mechanisms, composite material architecture, and the material properties of the constituent materials can be used to tune the mechanical behavior of bio-inspired flexible composite materials.

7.2. Materials and Methods

7.2.1. Building variation in finite element models

The basis models of the scale assembly prototypes with orientation angles (φ) of 0°, 30°, 60°, 90°, 120°, and 150°, were created in finite element software (ABAQUS, Dassault Systemes, France) as described in Chapter 6.2. The models consisted of three parts (scales, grooved substrate, and rigid rods) assembled into an array of scales with 1 mm paraserial and interserial spacing. VeroWhite was modeled as an isotropic linear elastic material (E = 2.0 GPa, ν = 0.43, and ρ = 1.175 g/cm³) and assigned to the scales and rigid rods [1-3]. TangoPlus was modeled as a Neo-hookean hyperelastic material (C11 = 0.63 MPa, D1 = 10⁻⁵, and ρ = 1.120 g/cm³) and assigned to the substrate [1-3]. The TangoPlus interconnections between the peg and socket of adjacent scales were modeled as linear elastic springs with a stiffness k_sp = C11 = 0.63 N/m. In the basis model, the stiffness ratio between the soft and hard material components is: $E^0 = \frac{C_{11,TP}}{E_{VW}} = \frac{0.63 \text{ MPa}}{2000 \text{ MPa}} = 3.15 \times 10^{-4}$.

Three sets of variations in material composition and morphometry were introduced into the model during the design process:

1. To examine the effect of removing the paraserial interconnections, the linear elastic springs connecting the peg and socket of adjacent scales were suppressed in the model. The remaining model components, assembly, and material properties were unchanged.
To examine the effect of stiffness ratio ($E^*$) between the constituent material components, the material constants for the soft components (TangoPlus substrate and linear elastic springs) were varied:

i. $E^* = 10E^0$, where $C_{11,TP} = k_{sp} = 6.3$.

ii. $E^* = 100E^0$, where $C_{11,TP} = k_{sp} = 63$.

iii. $E^* = 0.1E^0$, where $C_{11,TP} = k_{sp} = 0.063$.

The model components, assembly, and material properties for VeroWhite were unchanged.

To examine the effect of the peg and socket joint, the scale geometry was redesigned without the conical peg and corresponding inverted socket. The assembly was generated in the same fashion as the basis model, without the inclusion of the paraserial interconnections. The material properties for VeroWhite and TangoPlus were unchanged.

### 7.2.2. Simulation of active loading

Active loading conditions (concave bending, convex bending, and tension) were simulated on the models as described in Chapter 6.2. Bending was simulated with pin-pin boundary conditions for axial loading at 1 mm/s strain to induce lateral bending (concave and convex). Tension was simulated under uniaxial loading conditions in the +y direction at a strain rate of 1 mm/s constraining rod translation in the x- and z-axes and rod rotation in the x-, y-, and z-axes with free corresponding movement of elements within the sample. Force-displacement (F-d) curves were generated from the reaction force vs. vertical displacement of the top rod at every increment. Values for stiffness ($K$) were calculated as the slope of the F-d data and normalized by the stiffness of a control model consisting of a 4.4 mm sheet of TangoPlus without scales ($K = 10.7 \text{ N/m}$). For the tension simulations, an effective tensile modulus ($E [\text{MPa}]$) was calculated as the slope of the stress-strain curve, where engineering stress $\sigma = F / A$, $A = 0.124 \text{ m} \times 0.0072 \text{ m}$ (cross-sectional area of the sample in the loading direction) and engineering strain $\varepsilon = d / 0.112 \text{ m}$ (height of the sample), and normalized by the stiffness of a control model consisting of a 4.4 mm sheet of TangoPlus without scales ($E = 1.29 \text{ MPa}$).

### 7.3. Results

#### 7.3.1. Effect of removing the paraserial interconnections

This set of finite element models investigates the effect of modifying the composite material architecture by removing the paraserial interconnections. The finite element simulations of the prototypes without the paraserial interconnections exhibit anisotropic mechanical behavior in concave bending, convex bending, and tension over the range of models with orientation angle ($\varphi$) = 0°, 30°, 60°, 90°, 120°, and 150° (Fig. 7.1).
Figure 7.1: The effect of paraserial interconnections. Results from finite element models without paraserial interconnections (“No PS”, in red) compared to the original models with paraserial interconnections (“PS”, in black). (a) Normalized stiffness ($K$) by phase in concave bending. (b) Normalized stiffness ($K$) in convex bending. (c) Normalized effective modulus ($E$) in tension.
Concave Bending

In concave bending, the models exhibit anisotropic, multi-phase loading behavior where each phase has a stiffness \( (K) \) calculated as the slope of the loading curve and normalized by the stiffness of a control model without scales. Stiffness for each phase of each orientation is plotted in Fig. 7.1a and compared to the original model with the paraserial interconnections from Chapter 6.3.1.

In the \( \phi = 0^\circ \) orientation, the scales first undergo paraserial bending with a much lower stiffness without the compliant paraserial interconnections \( (K = 5.5) \) compared to the original models \( (K_0 = 45.4) \), where the interconnections provided resistance to the paraserial bending mechanism. After scale interlock, the second phase behavior exhibits the same mechanisms and similar stiffness values with and without the paraserial interconnections \( (K = 13.1; K_0 = 13.4) \); the scales undergo paraserial rotation until interserial interlock, and the resistance to bending due to scale-to-scale contacts (the back end of the anterior margin makes contact with the scale in the adjacent column) and scale-to-substrate contacts (anterior margin pushes into the substrate).

In the \( \phi = 30^\circ \) orientation, the scales again undergo paraserial bending until interlock. The first phase exhibits a lower stiffness \( (K = 3.9) \) than the original model \( (K_0 = 109) \), since there are no paraserial interconnections to resist paraserial bending. In the second phase of bending, the stiffness of the model remains low \( (K = 5.0) \) compared to the original model \( (K_0 = 15.0) \). While the interscale mobility mechanisms are the same with and without the paraserial interconnections (paraserial rotation due to interscale contacts), the model without the paraserial interconnections allows each scale to rotate freely within its column without resistance. Hence, resistance to global bending only originates from interscale contacts and stiffness remains low.

The \( \phi = 60^\circ \) model exhibits a single phase loading curve with a lower stiffness \( (K = 3.1) \) compared to each of the two phases of the original model \( (\text{Phase I}, K_0 = 27.5; \text{Phase II}, K_0 = 6.9) \). The scales first deform by interserial sliding, where the behavior of the compliant substrate dominates the bending response with a low stiffness. After the columns of scales begin to touch interserially, the scales rotate paraserially and interserially in response. Without the paraserial interconnections, each scale rotates freely within its column without resistance and the stiffness remains unchanged.

The \( \phi = 90^\circ \) model exhibits the same behavior with and without the paraserial interconnections. In the first phase, the columns of scales move relative to each other via interserial sliding, and the only resistance to bending comes from strains on the substrate beneath the scales \( (K = K_0 = 1.0) \). The scales are ‘mechanically invisible’ in this region. At larger deformations, the scales begin to touch interserially and the scales rotate about their axial ridge. Since each scale rotates simultaneously with its neighbors within the same column, the paraserial interconnections do not provide any resistance within the model, and therefore the stiffness values are the same with and without them \( (K = K_0 = 3.4) \).

In the \( \phi = 120^\circ \) model, the scales first move by small degrees of interserial sliding until
the anterior process makes contact with the substrate to induce paraserial and interserial rotations that in turn generate paraserial and interserial scale contacts. Without the paraserial interconnections providing resistance to scale rotation, the stiffness is lower ($K = 14.9$) compared to the original model ($K_0 = 89.4$). In the second phase, resistance to bending increases ($K = 4.8$) due to the scale-to-scale contacts (interserial contact between the columns of scales) and scale-to-substrate contacts (the anterior process touches the substrate); however, the lack of paraserial interconnections allows the scales to rotate independently so that the stiffness is lower than the original model ($K_0 = 40.7$ to 7.8).

In the $\phi = 150^\circ$ model, the scales first undergo small degrees of paraserial rotation until the anterior process pushes into the substrate and scales interlock with their neighbors. The lack of paraserial interconnection to provide resistance to paraserial rotation reduces the stiffness of the model in this phase ($K = 18.7$) compared to the original model ($K_0 = 276$), while the scale interlocks continue to make this model stiffer than the other orientations. As scale interlocks generate a greater resistance to bending ($K = 29.1$), the model exhibits a stiffness similar to the third phase of the original model after the paraserial interconnections begin to fail ($K_0 = 29.7$).

### 7.3.1.2. Convex Bending

In convex bending, the finite element models without paraserial interconnections continue to exhibit anisotropic behavior. Each orientation exhibits a single-phase loading curve with a characteristic stiffness ($K$) value, calculated as the slope of the loading curve and normalized by the stiffness of a control model with no scales. Stiffness for each model is plotted in Fig. 7.1b and compared to the original model with the paraserial interconnections described in Chapter 6.3.2. Following a similar trend as the models with paraserial interconnections, the models without paraserial interconnections exhibit the highest stiffness for $\phi = 0^\circ$ ($K = 20.9$), lowest stiffness for $\phi = 90^\circ$ ($K = 0.93$), and intermediate stiffness for the remaining models.

In the $\phi = 0^\circ$ model (Fig. 6.6a), scales accommodate global convex bending through paraserial bending through the entire duration of bending. Without the paraserial interconnections, the scales are segmented from each other in the loading direction; since the strains in the model are sustained by the substrate material while the scales slide apart, the stiffness is lower ($K = 20.9$) than the original model with paraserial interconnections ($K_0 = 84.4$). While the model bends, the scales make contact with each other in two locations: paraserially as pegs rub against the side of the socket of the neighboring scale, and interserially as the anterior process hits the bottom edge of the adjacent scale. Moreover, as the substrate continues to bend while the scales stay rigid, large local strains are seen in the regions of the substrate surrounding the axial ridges of scales, with large stresses transmitted to the axial ridges of scales, contributing to the model’s resistance to bending.

In the $\phi = 30^\circ$ model, the scales move apart by paraserial bending until they make contact with each other. Then, the oblique orientation of the scales allows for paraserial rotation to relieve the contact stresses, resulting in a lower stiffness ($K = 4.8$) than the $\phi = 0^\circ$ model.
Paraserial rotation is coupled with interserial rotation; at large deformations, the large degrees of interserial rotation begin to cause high strain on the substrate material attached to the axial ridges of the scales. In the model with the paraserial interconnections, this was the primary source of resistance to bending, with a small contribution from the paraserial interconnections. Hence, the model without paraserial interconnections has only a slightly lower stiffness than the original model ($K_0 = 5.0$).

In $\phi = 60^\circ$ model, the scales initially move relative to each other via interserial sliding and small degrees of paraserial bending. Without the paraserial interconnections, the scales can rotate freely, both paraserially and interserially, to accommodate further interserial sliding with a lower stiffness ($K = 2.0$) than the original model ($K_0 = 4.3$). Even at large deformations, there is no resistance to paraserial bending as the model continues to bend.

The $\phi = 90^\circ$ orientation exhibits the same behavior with and without the paraserial interconnections. As the scales move apart via interserial sliding, stresses and strains are sustained on the substrate only. The scales are again considered ‘mechanically invisible’ with a low stiffness matching the substrate ($K = K_0 = 0.93$). The hyperelasticity of the TangoPlus material allows the substrate to continue to bend easily without resistance while the scales continue to slide apart, and the stiffness remains low.

In the $\phi = 120^\circ$ model, the scales initially move relative to each other via interserial sliding and small degrees of paraserial rotation, and then utilize small degrees of both paraserial and interserial rotations to accommodate further sliding. Without the paraserial interconnections, the scales can rotate freely about each other within the column of scales. However, since the degrees of rotations are small in this orientation, it has only a slightly lower stiffness ($K = 3.1$) than the original model ($K_0 = 3.8$).

The $\phi = 150^\circ$ model without the paraserial interconnections exhibits a higher stiffness ($K = 13.7$) than the original model with paraserial interconnections ($K_0 = 4.0$). As the model bends, the scales initially move relative to each other via paraserial bending. Without the paraserial interconnections, the scales rotate freely and make contact with each other in several locations: paraserially as pegs rub against the side of the socket of the next scale, interserially as the anterior process hits the bottom edge of the adjacent scale, and again interserially as the back of the anterior margin hits the corner for the adjacent scale. As a result of these scale interlocks, the stiffness increases. At large deformations, stresses accumulate in the substrate regions attached to the axial ridges of scales, again due to the rigid scales not bending along with the substrate.

### 7.3.1.3. Tension

In tension, the finite element models without paraserial interconnections continue to exhibit anisotropic behavior. Each orientation exhibits a single-phase loading curve with an effective modulus ($E$) value, calculated as the slope of the stress-strain curve and normalized by the modulus of a control model with no scales. The effective modulus for each model is plotted in Fig. 7.1c and compared to the original model described in Chapter 6.3.3.
The behavior of the φ = 60°, 90°, and 120° models in tension relies on the substrate only since the columns of scales are segmented from each other. As a result, the loading behavior in tension for the models with and without paraserial interconnections is the same as described in Chapter 6.3.3: in the 90° orientation, the columns of scales slide apart while the substrate stretches, and in 60° and 120° orientations, the columns of scales shear relative to each other. Since the tensile behavior does not rely on the paraserial interconnections to resist scale rotations or bending, the modulus values are the same between the models with and without them.

In the φ = 0°, 30°, and 150° models, the substrate also bears the strain from the tensile loads; without the paraserial interconnections, the scales are segmented within their columns and can slide apart. In the 0° orientation, the scales do not rotate or bend; however, as the sample stretches, the substrates deform while the scales do not, generate high stresses in the substrate material surrounding the axial ridge of scales. This is the primary source of resistance to tension, and the modulus of the model without paraserial interconnections (E = 52.0) is only slightly lower than the original model (E₀ = 64.3). In the 30° and 150° models, the stresses on the substrate material surrounding the axial ridge are also the primary source of resistance to bending; again, the moduli are only slightly lower than the original model (for φ = 30°, E = 6.8, E₀ = 13.6; for φ = 150°, E = 9.3, E₀ = 13.3). However, the oblique orientation of the scales allows for paraserial rotation to accommodate vertical expansion of the assembly, for a lower modulus than the φ = 0° model.

7.3.2. Effect of varying the stiffness ratio of constituent materials

This set of finite element models investigates the effect of modifying the material properties by varying the stiffness ratio of the constituent materials (E*). Three sets of models were developed with E* = 10E₀, 100E₀, and 0.1E₀, where E₀ is the stiffness ratio of the constituent materials in the original models developed in Chapter 6. The simulations of the models with varying stiffness ratios exhibited anisotropic mechanical behavior in concave bending over the range of models with orientation angle (φ) = 0°, 30°, 60°, 90°, 120°, and 150°. Each set of models exhibits a multi-phase loading curve where each phase has a characteristic stiffness (K) calculated as the slope of the loading curve and normalized by the stiffness of a control model with no scales. Stiffness values for each phase of each model with the varying stiffness ratios is plotted in Fig. 7.2, and compared to the original models in Chapter 6.3.1.

For E* = 10E₀, the models exhibit multi-phase loading curves with associated interscale mechanisms. Stiffness values by phase for all orientations are plotted in Fig. 7.2a and compared to the original model. While the loading behavior is similar to the original model, the magnitude of stiffness for each phase in each orientation is shifted upward in magnitude. Certain orientations shift by exactly one order of magnitude, matching the shift in stiffness ratio; this occurs when the behavior of the model relies strictly on the behavior of the compliant materials whose elasticity constants were changed. This includes the φ = 90° orientation where the columns of scales move via interserial sliding and all strains are sustained in the substrate (Phase I: K = 9.7 ≈ 10K₀ = 10; Phase II: K = 31.9 ≈ 10K₀ = 34). This also includes the φ = 60° and 120°
Figure 7.2: The effect of stiffness ratio (E*). Normalized stiffness (K) of finite element models in concave bending by phase comparing varying stiffness ratios (E*, in red) to the original model (E₀, in black) for: (a) E* = 10E₀, (b) E* = 10E₀, and (c) E* = 0.1E₀.
models where the scales move via a combination of interserial sliding, paraserial rotation, and interserial rotations. The other models ($\phi = 0^\circ, 30^\circ,$ and $150^\circ$) see stiffness values that shift upwards by other amounts, since they rely on a combination of soft material properties and scale interactions through interscale contacts.

For $E^* = 100E^0$, the new models again exhibited similar loading curves and associated interscale mechanisms for all orientations to the original models. The stiffness values by phase for all orientations are plotted in Fig. 7.2b. While the loading behavior is similar to the original model, the magnitude of stiffness for each phase in each orientation is shifted upward by a greater order of magnitude than for the $E^* = 10E^0$ models. Again, certain orientations ($\phi = 90^\circ$ and $\phi = 60^\circ$) shift by exactly two orders of magnitude, matching the shift in stiffness ratio; this occurs when the behavior of the model relies strictly on the behavior of the compliant materials whose elasticity constants were changed. As the stiffness of the soft material increases to become closer to the stiffness of the rigid scales, the interplay between the two materials becomes more complex. For example, the substrate may not necessarily carry all the strains in the model as elements on the scales might begin to deform. Additionally, the degree and contribution of the interscale mobility mechanisms to global bending change, thus affecting stiffness.

For $E^* = 0.1E^0$, the new models again exhibited similar loading curves with associated interscale mobility mechanisms to the original models; however, the contribution of the interscale mobility mechanisms to the global bending behavior changes. Stiffness values by phase for all orientations are plotted in Fig. 7.2c. In general, the magnitude of stiffness shifts downward with the decreased elasticity constants of the substrate materials. The reduction of the spring constant for the paraserial interconnections leads to lower resistance to paraserial bending, paraserial rotation, and interserial rotation mechanisms that allow the global model to bend. As observed with the models without paraserial interconnections described in Chapter 7.3.1, the paraserial separation of scales can both allow these interscale mechanisms to occur more readily, but also introduce a complicated array of new interscale contacts. As a result, the stiffness values for the models with $E^* = 0.1E^0$ range over four orders of magnitude, while the $E^* = 10E^0$, $E^* = 100E^0$, and original models span three orders of magnitude.

### 7.3.3. Effect of removing the peg and socket joint

This set of finite element models investigates the effect of modifying the scale geometry through the removal of two characteristic features, the peg and socket. The revised scale geometry used in the models is shown in Fig. 7.3a. Without the peg and socket, the paraserial interconnections between the peg and socket of the original model were also removed. The simulations without the peg and socket exhibit anisotropic mechanical behavior in concave bending over the range of models with orientation angle ($\phi = 0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ,$ and $150^\circ$). The models exhibit multi-phase bending behavior where each phase has a characteristic stiffness ($K$) calculated as the slope of the loading curve and normalized by the stiffness of a control model with no scales. Stiffness for each phase of each model is plotted in Fig. 7.3b and compared to the original models described in Chapter 6.3.1.
Figure 7.3: The effect of removing the peg and socket from the scale geometry in the finite element models in concave bending. (a) Scale design, front view (top) and side view (bottom). (b) Normalized stiffness ($K$) of finite element models in concave bending by phase comparing the scale geometry with no peg and socket (in red) to the original model (in black).

In the $\phi = 0^\circ$ orientation, the scales first undergo paraserial bending with a much lower stiffness without the peg, socket, and compliant paraserial interconnections ($K = 10.0$) compared to the original models ($K_0 = 45.4$); without the joint and the interconnection, there is no resistance to the paraserial bending mechanism. At larger degrees of bending, the scales interlock paraserially where the surface of the scale around the former peg touches the surface around the former socket of the adjacent scale, and the stiffness of the model increases ($K = 21.1$). After interlock, the scales see resistance to paraserial bending and rotation mechanisms since there is no compliance from a soft interconnection. Additional scale-to-scale contacts (the back end of the anterior margin makes contact with the scale in the adjacent column) and scale-to-substrate contacts (anterior margin pushes into the substrate) result in a Phase II stiffness that is higher than the original model ($K_0 = 13.4$).

In the $\phi = 30^\circ$ orientation, the scale again undergo paraserial bending until interlock with a lower stiffness ($K = 34.1$) than the original model ($K_0 = 109.0$) due to the full paraserial segmentation between the scales. In the second phase of bending, the stiffness of the model increases ($K = 40.9$) and is higher than in the original model ($K_0 = 15.0$). Without the peg and socket joint, the scales face a greater resistance to the paraserial rotation mechanism due to the interscale contacts. In the original model, paraserial rotation allowed the original model to relieve the stresses from the interscale contacts; without the peg and socket joint, the scales can no longer rotate freely due to the interscale contacts, and resistance to bending increases.

The $\phi = 60^\circ$ orientation exhibits a single phase loading curve with a lower stiffness ($K = 5.5$) compared to either of the two phases of the original model (Phase I, $K_0 = 27.5$; Phase II, $K_0 = 6.9$). The scales first deform by interserial sliding, where the compliant substrate dominates the bending response with a low stiffness. After the columns of scales begin to touch interserially,
the oblique orientation of the scales allows them to rotate paraserially and interserially, even without the peg-and-socket joint, and the stiffness remains unchanged.

The $\phi = 90^\circ$ orientation exhibits the same behavior with and without the peg-and-socket joint on the scale geometry. In the first phase, the columns of scales move relative to each other via interserial sliding, and the only resistance to bending comes from the substrate beneath the scales ($K = K_0 = 1.0$). The scales are ‘mechanically invisible’ in this region. At larger deformations, the scales begin to touch interserially and the scales rotate about their axial ridge. Since each scale rotates simultaneously with its neighbors within the same column, the peg and socket joint contributed nothing additional to the original model; thus, the stiffness of the model is the same with and without the joint ($K = K_0 = 3.4$).

In the $\phi = 120^\circ$ orientation, the scales first move by small degrees of interserial sliding until the anterior process makes contact with the substrate, inducing paraserial and interserial rotations which in turn general paraserial and interserial scale contacts. In the initial stages of bending, before paraserial contacts are formed, the paraserial spacing between scales allows them to rotate freely, generating a lower stiffness ($K = 28.0$) compared to the original model ($K_0 = 89.4$). In the second phase, resistance to bending decreases ($K = 9.7$) as the oblique orientation of the scales allows them to continue rotating despite the interscale contacts and the lack of peg-and-socket joint, similar to the $\phi = 60^\circ$ model.

In the $\phi = 150^\circ$ model, the scales first undergo small degrees of paraserial rotation until the anterior process pushes into the substrate and scales interlock with their neighbors. Without the peg-and-socket joint and paraserial interconnections, the resistance to paraserial rotation is mitigated and the stiffness is smaller in this phase ($K = 53.7$) compared to the original model ($K_0 = 276$). As scale interlocks generate a greater resistance to bending ($K = 62.7$), the model exhibits a stiffness similar to the second phase of the original model ($K_0 = 64.3$), as the stresses that build up from the interscale contacts spread through the columns of scales.

7.4. Discussion

In this chapter, variations were introduced into the finite element models to simulate their effect on the mechanical behavior of the flexible composites under active loading. The results show how multiple elements of the prototype design, such as the multi-material components of the composite architecture, the properties of the constituent materials, and the morphometry of the scale subunits, can tune the flexibility of scale armor prototypes.

The effect of modifying the composite material architecture was investigated by removing the paraserial interconnections. In Chapter 4, the paraserial interconnections mimicking the Sharpey’s fibers of the biological exoskeleton were shown to be critical in controlling the anisotropic flexibility of the prototypes under passive loading. In this chapter, the paraserial interconnections are shown to act as the primary source of resistance to global bending, especially in the initial stages of bending, by resisting certain interscale mobility mechanisms such as paraserial bending, paraserial rotation, and interserial rotation. Unlike in
passive loading, the behavior of the models remains anisotropic without the paraserial interconnections in active loading for concave bending, convex bending, and tension. However, in each loading mode, the paraserial interconnections do not affect the behavior of the 90° orientation, where the paraserial axis is perpendicular to the loading direction and the peg-and-joint does not play a role in interscale mobility mechanisms that contribute to the global bending of the sample. In concave bending, the degree of anisotropy is lower without the paraserial connections where stiffness ranges over two orders of magnitude, rather than over three orders of magnitude with the paraserial connections. In convex bending, the degree of anisotropy also decreases, while in tension the degree of anisotropy stays the same.

The effect of modifying the properties of the constituent materials was investigated by varying the stiffness ratio between the compliant components (substrate and paraserial interconnections) and the rigid scales. As the stiffness ratio changed, the magnitude of stiffness values observed in the finite element models during concave bending also changed accordingly; when the soft materials were made more rigid, the global bending response of the models was stiffer. When the stiffness of the soft and rigid materials become more similar to each other, the interplay between the two materials becomes more complex as elements on the rigid scales begin to deform and the interscale mobility mechanisms change. However, when the soft materials were made softer, new interscale mechanisms became apparent in the model, and the degree of anisotropy among the various scale orientations increased. The results show how material choice in the prototypes can be used to target the desired stiffness of the samples in prototype to tailor it to a specific application.

Finally, the effect of modifying the scale morphometry on the prototype behavior was investigated by removing the peg and socket from each scale. While the models continued to exhibit complex, anisotropic behavior in concave bending, the interscale mobility mechanisms and associated stiffness values changed for the loading profile in each orientation. The results show how a small change in the morphometric design of flexible composite materials can affect mechanical behavior. Extensions of this work can look at the effect of other characteristic geometric features of the *P. senegalus* scale, such as the anterior process, axial ridge, and concavity of the anterior margin. Furthermore, models can be designed to parametrically investigate the effect of morphometric parameters that were found to vary through the biological exoskeleton in Chapter 2 and Chapter 3, such as peg tip angle, peg length, angle of interserial overlap, anterior process length, and central thickness. To conclude, the results in this chapter show how both materials and morphometric design principles can be used to tailor the behavior of the flexible, scale armor prototypes through the use of finite element simulations.
7.5. References


8. Conclusion

8.1. Summary

Structural biological materials in nature give insight into the use of morphometric-enabled materials design to create synthetic armor systems that are both protective and flexible. This thesis explores the use of morphometry as a materials design parameter through the development of flexible composite armor prototypes inspired by the articulated exoskeleton of an armored fish, *Polypterus senegalus*. The research presented in this thesis draws upon methods from materials science, mechanical engineering, biology, and architecture to create bio-inspired flexible, protective materials and to establish an experimental and computational framework for both analyzing and optimizing their mechanical behavior. Scale shape and shape variation in the fish exoskeleton are studied in Chapter 2 and Chapter 3. Bioinspired flexible composite prototypes are designed, fabricated, and experimentally tested in Chapter 4 and Chapter 5. Finite element models simulating the mechanical behavior of the synthetic, flexible scale assemblies are developed in Chapter 6 and expanded upon in Chapter 7. Through this work, this thesis shows how morphometric-enabled materials design, inspired by structural biological materials, can allow for tunable behavior in flexible composites made of segmented scale assemblies to achieve enhanced user mobility, custom fit, and flexibility around joints for a variety of protective applications.

8.2. Significance

8.2.1. Morphometry as a materials design parameter

By using morphometry as a materials design parameter, this thesis shows how shape, or form, of a material can be used to change the properties of composite systems. In Chapter 4 and Chapter 5, analysis of bioinspired flexible scale armor prototypes shows how the complex shape of scales contributes to local, interscale mobility mechanisms that result in anisotropic, global mechanical behavior. With one prototype design scheme, a wide array of mechanical behavior is generated with stiffness ranging over several orders of magnitude, thus showing how morphometry can tune the flexibility of composite materials without changing the volume fraction or composition of the constituent materials. Moreover, this thesis shows that morphometry can turn a stiff system into a flexible system without changing its material architecture. In Chapter 5, the scale shape and joint articulation mechanisms yield a structure in which the rigid scales are ‘mechanically invisible’ in a certain orientation; the prototype behaves with the flexibility of the underlying soft material substrate but maintains the protective functionality of the rigid scales.

8.2.2. Multi-disciplinary framework for bioinspired engineering

This thesis demonstrates that a combination of scientific analysis and architectural design
tools can be used to translate the hierarchical, morphometric-materials design principles of a biological exoskeleton to a synthetic prototype and to characterize its mechanical behavior. This research uses an interdisciplinary approach involving biological characterization methods, computational design, additive manufacturing, experimental testing of mechanical properties, and finite element simulations of mechanical behavior. Algorithmic design, form-finding computational tools, and additive manufacturing allow unprecedented opportunities to control both materiality and morphometry in the design of flexible composite systems with complex architectures. Additionally, new experimental methods characterize the mechanical behavior of the flexible composite prototypes alongside an intricate computational model that replicates the complex joint interactions observed experimentally. Herein a connection between two types of information, the individual geometric features of the scale unit shape and the global anisotropic flexibility of the assembly, has been established. This link between local shape (morphometry) and global performance (biomechanics) is the first step in applying the design principles of natural system to synthetic designs with local tailorable for functional fitness through bioinspired engineering. This approach combines inherent materiality that has been a subject of traditional material science research with morphometry, e.g. control over geometry, shape, and topological variation, to create new class of tunable hybrid materials that are adjustable to the specific needs of a given application, e.g. body armor for athletes and military personnel, protective skins on buildings and machinery, or packaging for consumer products.

8.2.3. Shattering the protection-flexibility tradeoff in armor design

Currently, armor design is subject to a tradeoff between flexibility and protection. As discussed in Chapter 1, materials in use today with high protective capabilities are typically stiff, monolithic units with little flexibility. Attempts at designing flexible armor have tried to segment the armored plate materials, leaving penetrable gaps between subunits, or to use overlapping structures in composite designs, which tend to fail at adhesive interfaces. The flexible scale armor prototypes developed in this thesis based on the P. senegalus exoskeleton represent a new step toward shattering the protection-flexibility tradeoff in armor design, wherein a shape-based design system has been established that utilizes segmented, articulated subunits to selectively enable flexibility in certain directions and also maintain uniform coverage of the underlying tissue. As discussed in Chapter 4, the use of 3D printing enables the fabrication of multi-material structures without adhesives or glue at interfaces, thereby eliminating a source of failure common in composite materials.

8.3. Applications

The primary application for this thesis research is the development of flexible body armor, in order to create flexible composite materials that offer both protection and mobility for a wide array of commercial applications, such as athletic body armor, athletic training gear, thermal protection for firefighters, soldier armor, instrument casings, and consumer product packaging. Furthermore, segmented flexible composite materials in particular are advantageous
due to modularity, ease and reduced cost of fabrication for customized host surfaces, ability to localize damage to individual subunits, ability to allow the selective replacement of damaged units, and ability to permit the use of a wide range of constituent materials.

8.4. Future Directions

The multidisciplinary nature of this thesis opens numerous pathways along which to extend the future directions of research. For materials scientists, the systematic combination of structure and morphometry at multiple length scales opens new paradigms in the constitutive laws that govern the properties of materials. For mechanical engineers, this research enables the fabrication of flexible composite structures with spatially tailorable mechanical behavior, such as stiffness, strength, toughness, flexural rigidity, and failure mechanisms. For architects, this project offers the opportunity to use methods of algorithmic design and 3D printing to create new classes of hybrid materials.

The finite element simulations developed in Chapter 6 and Chapter 7 can be used to parametrically investigate the effect of geometric variation on the mechanical behavior of flexible composite prototypes. Morphometric features, which were found in Chapter 2 and Chapter 3 to vary within the biological exoskeleton, may be changed within the models, e.g. scale aspect ratio, peg length and shape, anterior process length and orientation, thickness, and overlap angle. Furthermore, finite element models can be developed based on the five scale geometry variants defined in Chapter 2 to prove their functional significance in flexible composite assemblies in terms of the local, interscale mobility mechanisms that their geometries enable. Geometries that show promising results in simulations may be integrated into physical prototypes to be fabricated and experimentally tested.

Another extension of this work is to optimize the structure-property-function relationships in the flexible composite materials by incorporating both heterogeneous morphometries (gradation in scale shape) and heterogeneous materials (e.g. multi-layering, porosity, and gradation in material properties) at multiple length scales. At the macroscale, spatial heterogeneity in scale shape can be incorporated into the prototype design to mimic the scale shape gradation found in the fish exoskeleton. At the micro- and meso-scale, heterogeneity in material composition can be introduced through blends of the constituent materials at varying volume fractions. At the nanoscale, the material properties of the constituent materials can be enhanced through the incorporation of nanofibers and nanoparticles within the 3D printing process to create hierarchical composite structures that exhibit mechanical property amplification, e.g. improved stiffness, strength, and toughness of the composites. The aim would be to utilize a hierarchy of spatially-heterogeneous materials and morphometric design elements to generate enhanced composite materials with varying local material properties and mechanical behaviors tuned to the specific needs of the hosting surface while affording global protection, flexibility, and coverage.

A final extension of this thesis is to fabricate structurally-robust flexible composite
materials with increased penetration resistance while maintaining flexibility. This may require the invention of new additive co-manufacturing methods that combine different classes of materials, e.g. ceramics with polymers, to expand the design space of flexible composite materials with superior properties. A few commercially available 3D printers have started to additively manufacture ceramics (e.g. certain selective laser sintering printers) and could potentially be modified to print low-sintering temperature ceramics alongside high-curing temperature polymer materials in a new co-manufacturing process. The behavior of the ‘high-performance’ co-manufactured flexible composite materials may then be tested at realistic mechanical loads such as high strain rates in tension, compression, and bending or high indentation loads to approximate physiological threats. A further challenge may be to measure the back-deflection of the prototypes and the force transmitted to the underlying surface to assess the system’s ability to protect a soft body underneath. In summary, the future directions of the work in this thesis would aim to make flexible body armor that is customizable to both the wearer and the threat into a reality through the use of morphometric-enabled materials design.
Appendix A: Derivation of Thin-Plate Spline Formulas for Generating Transformation Grids

This appendix describes the mathematical basis of thin-plate splines to generate transformation grids for geometric morphometric analysis in 2D and 3D, based on the derivations presented by Bookstein (1989) and Weber (2011) [1-2]. The deformation of a grid mapped to a set of reference landmarks to generate a transformation grid mapped to the comparison geometry’s landmarks is achieved through the use of thin-plate splines.

A.1. Calculating Thin-Plate Splines in 2D

The vector \( P_i = (x_i, y_i) \) represents the coordinates of the reference geometry landmarks, where \( x_i, y_i \) represents the x- and y- components of landmark \( i \) for \( i = 1 ... k \) and \( k = \# \) landmarks (here, \( k = 20 \)). The function \( r_{ij} = |P_i - P_j| = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \) represents the distance between points \( i \) and \( j \). The kernel function \( U(r_{ij}) = r^2 \log r^2 \), which can be represented as \( U_{ij} = U(P_i - P_j) \), builds the following matrices:

\[
K = \begin{pmatrix}
0 & U_{12} & \cdots & U_{1k} \\
U_{21} & 0 & \cdots & U_{2k} \\
\vdots & \vdots & \ddots & \vdots \\
U_{k1} & U_{k2} & \cdots & 0
\end{pmatrix},
\]

\[
Q = \begin{pmatrix}
1 & x_1 & y_1 \\
1 & x_2 & y_2 \\
\vdots & \vdots & \vdots \\
1 & x_k & y_k
\end{pmatrix}
\]

of dimension \( 3 \times k \),

\[
L = \begin{pmatrix}
K \\
Q^T
\end{pmatrix}
\]

of dimension \((k + 3) \times (k + 3)\).

The column vector \( V_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix} \) holds the coordinates of the comparison geometry’s landmarks, where \( x_i, y_i \) represents the x- and y- components of landmark for \( i = 1 ... k \) and \( k = \# \) landmarks (here, \( k = 20 \)). Then, build the following matrices:

\[
H = (V_i \ldots V_k | 0 \ 0 \ 0)^T = \begin{pmatrix} x_1 & \cdots & x_k \\ y_1 & \cdots & y_k \\ 0 & \cdots & 0
\end{pmatrix}^T
\]

\[
W = L^{-1}H = (w_{1x} \ldots w_{kx} | a_0 \ a_x \ a_y)^T
\]

The elements of \( W \) define a function \( f(x, y) \) that calculates the new position of each point \((x, y)\) everywhere in the plane according to the thin-plate spline mapping:

\[
f(x, y) = a_0 + a_x x + a_y y + \sum_{i=1}^k w_i U(|P_i - (x, y)|)
\]
In this thesis, the function \( f(x, y) \) translates the positional coordinates for vertices of each grid element in the reference grid \((x, y)\) into their corresponding positional coordinates in the transformation grid \( f(x, y) = [f_x(x, y), f_y(x, y)] \).

There are three key properties of the function \( f \):

1) The function maps the coordinates of each landmark in the reference geometry to the position of the landmark in the comparison geometry. That is, \( f(P_i) = V_i \).

2) The function minimizes the bending energy \( I_f \) out of all possible interpolation functions that can map the reference geometry landmarks to the comparison geometry landmarks in plane. That is, it minimizes the nonnegative quantity:

\[
I_f = \int_{\mathbb{R}^2} \left( \frac{\partial^2 f}{\partial x^2} \right)^2 + 2 \left( \frac{\partial^2 f}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 f}{\partial y^2} \right)^2 \, dx \, dy.
\]

3) The value of the bending energy is, where \( L_k^{-1} \) is the upper-left \( k \times k \) submatrix of \( L^{-1} \):

\[
I_f = \frac{1}{8 \pi} W K W^T = \frac{1}{8 \pi} V (L_k^{-1} K L_k^{-1}) V^T
\]

A.2. Calculating Thin-Plate Splines in 3D

The vector \( P_i = (x_i, y_i, z_i) \) represents the coordinates of the reference geometry landmarks, where \( x_i, y_i, z_i \) represents the x-, y-, and z- components of landmark \( i \) for \( i = 1 \ldots k \) and \( k = \# \) landmarks (here, \( k = 21 \)). The distance between points \( i \) and \( j \) is:

\[
r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}.
\]

The kernel function \( U(r_{ij}) = |r_{ij}| \), which can be represented as \( U_{ij} = U(P_i - P_j) \), builds the following matrices:

\[
K = \begin{pmatrix}
0 & U_{12} & \ldots & U_{1k} \\
U_{21} & 0 & \ldots & U_{2k} \\
\vdots & \vdots & \ddots & \vdots \\
U_{k1} & U_{k2} & \ldots & 0
\end{pmatrix},
\]

\[
Q = \begin{pmatrix}
1 & x_1 & y_1 & z_1 \\
1 & x_2 & y_2 & z_2 \\
\vdots & \vdots & \vdots & \vdots \\
1 & x_k & y_k & z_k
\end{pmatrix}
\]

of dimension \( 4 \times k \),

\[
L = \begin{pmatrix}
K \\
Q^T \\
0
\end{pmatrix}
\]

of dimension \( (k + 4) \times (k + 4) \).

The column vector \( V_i = (x_i \ y_i \ z_i)^T \) holds the coordinates of the comparison geometry’s landmarks, where \( x_i, y_i, z_i \) represents the x-, y-, and z- components of landmark \( i \) for \( i = 1 \ldots k \) and \( k = \# \) landmarks (here, \( k = 21 \)). Then, build the following matrices:

\[
H = (V_i \ \ldots \ V_k \ | \ 0 \ 0 \ 0)^T = \begin{pmatrix}
x_1 & \ldots & x_k & 0 & 0 & 0 & 0 \\
y_1 & \ldots & y_k & 0 & 0 & 0 & 0 \\
z_1 & \ldots & z_k & 0 & 0 & 0 & 0
\end{pmatrix}^T,
\]

141
\[ W = L^{-1}H = (w_{1x} \ldots w_{kx} \mid a_0 \ a_x \ a_y \ a_z)^T. \]

The elements of \( W \) define a function \( f(x, y, z) \) that calculates the new position of each point \((x, y, z)\) everywhere in the space according to the thin-plate spline mapping:

\[
f(x, y, z) = a_0 + a_xx + a_yy + a_zz + \sum_{i=1}^{k} w_i U(|P_i - (x, y, z)|).
\]

In this thesis, the function \( f(x, y, z) \) translates the positional coordinates for vertices of each grid element in the reference grid \((x, y, z)\) into their corresponding positional coordinates in the transformation grid \( f(x, y, z) = [f_x(x, y, z), f_y(x, y, z), f_z(x, y, z)] \). The same key properties for \( f \) hold in 3D as in 2D.

A.3. References
