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Erratum

Erratum to: Classical $\mathcal{W}$-Algebras and Generalized Drinfeld–Sokolov Hierarchies for Minimal and Short Nilpotents

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There is an error in Sect. 6.2 of the original article in the computation of equations of the generalized Drinfeld–Sokolov hierarchy associated to the minimal nilpotent element f of the Lie algebra $\mathfrak{g} = \mathfrak{sl}_n$, $n \geq 3$, corresponding to the unique (up to a constant factor) non-zero central element $c \in \mathfrak{g}_0^f$ (such c exists for minimal f only in the case $\mathfrak{g} = \mathfrak{sl}_n$, $n \geq 3$).

Namely, equation $\frac{dL}{dt_1} = 0$ in the second line of page 652 should be replaced by

$$\frac{dL}{dt_1} = \sum_{k \in J_{\frac{1}{2}}} \Big(\psi([f, v_k]) \psi([c, [f, v^k]]) \Big)' .$$

Consequently, one should add to Eq. (6.17) of the original article the following equations

$$\begin{aligned} \frac{d\psi(u)}{dt_{\tilde{1}}} &= \psi([c, u])'' - \frac{1}{2(x|x)} L \psi([c, u]), \\ \frac{dL}{dt_{\tilde{1}}} &= \sum_{k \in J_{\frac{1}{2}}} \Big(\psi([f, v_k]) \psi([c, [f, v^k]]) \Big)' . \end{aligned} \tag{1}$$

This error affected the example of $\mathfrak{g} = \mathfrak{sl}_3$ in Sect. 8 of our paper [DSKV14a]. Namely, to (8.7) one should add two conserved densities:

$$g_{\tilde{0}} = \varphi \, , \qquad g_{\tilde{1}} = 6\psi_+ \psi_- \, ,$$

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to Eq. (8.9) one should add the equation

$$\frac{d}{dt_{\tilde{0}}}\begin{pmatrix} L \\ \psi_+ \\ \psi_- \\ \varphi \end{pmatrix} = \begin{pmatrix} 0 \\ -3\psi_+ \\ 3\psi_- \\ 0 \end{pmatrix},$$

to Eq. (8.10) one should add equations

$$\begin{aligned} \frac{dL}{dt_{\tilde{1}}} &= 6\,(\psi_+\psi_-)', \\ \frac{d\psi_\pm}{dt_{\tilde{1}}} &= \mp 3\psi_\pm'' \pm 3L\psi_\pm \mp \varphi^2\psi_\pm - \frac{3}{2}\psi_\pm\varphi' - 3\varphi\psi_\pm', \\ \frac{d\varphi}{dt_{\tilde{1}}} &= 0, \end{aligned}$$

and to Eq. (8.20) one should add equations

$$\frac{d}{dt_{\tilde{0}}}\begin{pmatrix} L \\ \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} 0 \\ -3\psi_+ \\ 3\psi_- \end{pmatrix}, \qquad \frac{d}{dt_{\tilde{1}}}\begin{pmatrix} L \\ \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} 6\,(\psi_+\psi_-)' \\ -3\psi_+'' + 3L\psi_+ \\ 3\psi_-'' - 3L\psi_- \end{pmatrix}.$$

The latter is the well known Yajima–Oikawa (YO) equation [YO76].

Thus, in [DSKV14a] we proved that the YO hierarchy is obtained by Dirac reduction from the minimal $\mathfrak{sl}_3$ generalized Drinfeld–Sokolov hierarchy, and, as a result, we gave in formulas (8.4) and (8.5) two compatible Poisson structures for the YO hierarchy. The latter were found in [Che92].

Furthermore, this error affected the example considered in Sect. 2.5 of our paper [DSKV14b]: one should add Eqs. (1) to the equations at the end of Sect. 2.5 there in the case of $\mathfrak{g} = \mathfrak{sl}_n$, $n \geq 3$. Thus, in Sect. 2.5 of [DSKV14b] we proved that the $n-2$-component YO hierarchy is obtained by Dirac reduction from the minimal $\mathfrak{sl}_n$ generalized Drinfeld–Sokolov hierarchy. As a result, we gave two compatible Poisson structures for the $n-2$-component YO hierarchy, which we denoted by $\overline{H}_0$ and $\overline{H}_1^D$ in Sect. 2.5 of [DSKV14b].

The new type of reduced Kadomtsev–Petviashvili (KP) hierarchy, called the constrained KP hierarchy, was introduced in [KSS91,KS92], where it was observed that the YO hierarchy can be obtained as a constrained KP hierarchy (and this was used in [Che92] to find its bi-Poisson structure). A more general s-vector constrained KP hierarchy was introduced in [SS93], where it was shown that the multi-component YO hierarchy, studied in [Ma81], can be obtained by this construction. After that, in [ZC94] the more general s-vector m-constrained KP hierarchy was introduced (the constrained KP hierarchy of [KS92,Che92,SS93] corresponds to $m = 2$).

Our main observation in this regard is that the s-vector m-constrained KP hierarchy is isomorphic to the Dirac reduction by conformal weight 1 fields of the generalized Drinfeld–Sokolov hierarchy [DSKV14a,DSKV13], associated to the Lie algebra $\mathfrak{g} = \mathfrak{sl}_{m+s}$ and its nilpotent element f corresponding to the partition $(m, 1, \ldots, 1)$ of $m+s$. (Note that for both hierarchies the number of fields is equal to $m-1+2s = \dim(\mathfrak{g}^f/\mathfrak{g}_0^f)$.) For $m = 2$ this observation is proved in [DSKV14b].

We are grateful to Professor Takayuki Tsuchida, who pointed out to us that our Eq. (8.10) in [DSKV14a] is a higher symmetry of the YO equation in the YO hierarchy [Che92], which led us to the discovery of the error in the original article.

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