ATTENUATION OF SOUND IN LINED CIRCULAR DUCTS

by

Young-chung Cho
K. Uno Ingard

GTL Report No. 120 April 1975

Addendum to
GTL Report No. 119

GAS TURBINE LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE, MASSACHUSETTS

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PREFACE

This is an addendum to Gas Turbine Laboratory Report No. 119. It covers additional work on sound propagation in lined ducts which was started under DOT Grant Agreement DOT-OS-30011 and continued under Supplement Agreements to the same grant.

The work is being monitored by Dr. Gordon Banerian, Office of Noise Abatement, Department of Transportation.

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A1. ATTENUATION OF SOUND IN LINED CIRCULAR DUCTS

In the previous report, we have used approximate expressions for the wall impedance for the discussion of sound attenuation in lined circular ducts. For instance, Eq. (2.5) has been used for the wall impedance of a circular duct lined with a resonator with a resistive screen, and Eq. (2.8) for a circular duct lined with a porous material.

If the ratio of the duct radius to the liner thickness (D/L) is large and the sound frequency is large, the impedance given in Eq. (2.5) or Eq. (2.8) is a good approximation for a lined circular duct. However, when either one of these conditions is not fulfilled, the radial spread of the wave in the liner imposes some effects on the sound attenuation, whereas no wave spreading takes place in the liner of a rectangular duct.

In this addendum we derive expressions for the wall impedance of lined circular ducts, accounting for the cylindrical spreading of the waves within the liner. The assumption of a locally reacting surface is still made. On the basis of the impedance thus obtained, the attenuation characteristics of a circular lined duct are computed for a wide range of parameters.

For direct comparison, the numerical results are presented in figures numbered similarly to the previous report. For example, "Figure 3.66" in the previous report is replaced by "Figure A3.66" for plotting the octave band TL vs kL in a circular duct lined with a porous material with the duct parameters: $\theta = 0.5$, $D/L = 1.094$ and area ratio = 1. (See page 124 and page A

A1.1 Resonator Liner

In the case of locally reacting duct walls, the sound propagation in the liner is negligible in the axial direction. Thus the pressure field in the liner is written as

$$p_L = A\left\{H_o^{(1)}(kr) + \alpha H_o^{(2)}(kr)\right\}, \qquad \text{(A1.1)}$$

where the H's are Hankel functions and A and α are constants.

The velocity field is then

$$\vec{u} = \hat{r}\,\frac{iA}{\rho c}\left\{H_1^{(1)}(kr) + \alpha H_1^{(2)}(kr)\right\}, \qquad \text{(A1.2)}$$

where $\hat{r}$ is the radial unit vector.

Since $u_r = 0$ at $r = b + L$, from Eq. (A1.2),

$$\alpha = -\frac{H_1^{(1)}[k(b+L)]}{H_1^{(2)}[k(b+L)]}\,. \qquad \text{(A1.3)}$$

The normalized wall reactance due to the cavity is

$$\lambda_c = \left.\frac{p}{\rho c u_r}\right|_{r=b} = -i\,\frac{H_o^{(1)}(kb) + \alpha H_o^{(2)}(kb)}{H_1^{(1)}(kb) + \alpha H_1^{(2)}(kb)}. \qquad (A1.4)$$

If $kb \gg 1$ and $L \ll b$, this equation reduces to

$$\chi_c \cong i\,\cot(kL). \qquad (A1.5)$$

This is the same as the cavity reactance in a rectangular duct lined with a resonator (cf. Eq. 2.5).

The total normalized impedance is

$$\zeta = \theta - i\,(kt' + \frac{H_o^{(1)}(kb) + \alpha H_o^{(2)}(kb)}{H_1^{(1)}(kb) + \alpha H_1^{(2)}(kb)}), \qquad (A1.6)$$

where θ is the dynamic resistance of the screen, kt' the reactance accounting for the inertia of the air in the screen, and α is given in Eq. (A1.3).

A1.2. Porous Liner

When the porous material is isotropic and homogeneous, the pressure field in the porous medium of an annular shape as shown in Fig. 2.20 is

$$p_L = A\left\{H_o^{(1)}(qr) + \alpha H_o^{(1)}(qr)\right\}, \qquad (A1.7)$$

where q is defined in Eqs. (2.9) and (2.10).

The velocity field is then

$$\vec{u}_L = \hat{r}\,\frac{ikA}{\rho c q}\left\{H_1^{(1)}(qr) + \alpha H_1^{(2)}(qr)\right\}. \qquad (A1.8)$$

It follows from $u_r\big|_{r=b+L} = 0$ that

$$\alpha = - \frac{H_1^{(1)}[q(b + L)]}{H_1^{(2)}[q(b + L)]} . \qquad \text{(A1.9)}$$

The normalized wall impedance is

$$\zeta = \frac{p_2}{\rho c u_r}\bigg|_{r=b} = - \frac{iq}{k} \frac{H^{(1)}(qb) + \alpha H_o^{(2)}(qb)}{H_1^{(1)}(qb) + \alpha H_1^{(2)}(qb)}. \qquad \text{(A1.10)}$$

If $qb \gg 1$ and $b \gg L$, this equation reduces to

$$\zeta \rightarrow \frac{iq}{k} \cot(qL). \qquad \text{(A1.11)}$$

This is the same as Eq. (2.8).

Figures A2.16 - A2.19: Real and imaginary parts of k_z for circular ducts lined with a resistive scrren type resonator liner. The real part of k_z is normalized by division by k. The imaginary part of k_z is presented in terms of $8.6859 \cdot Im(k_z)D$, which is the transmission loss in dB of a pure tone in a length D of the duct. Each figure corresponds to a different value of D/L. Each curve in a figure corresponds to a different value of the flow resistance θ (in units of ρc) of the screen.

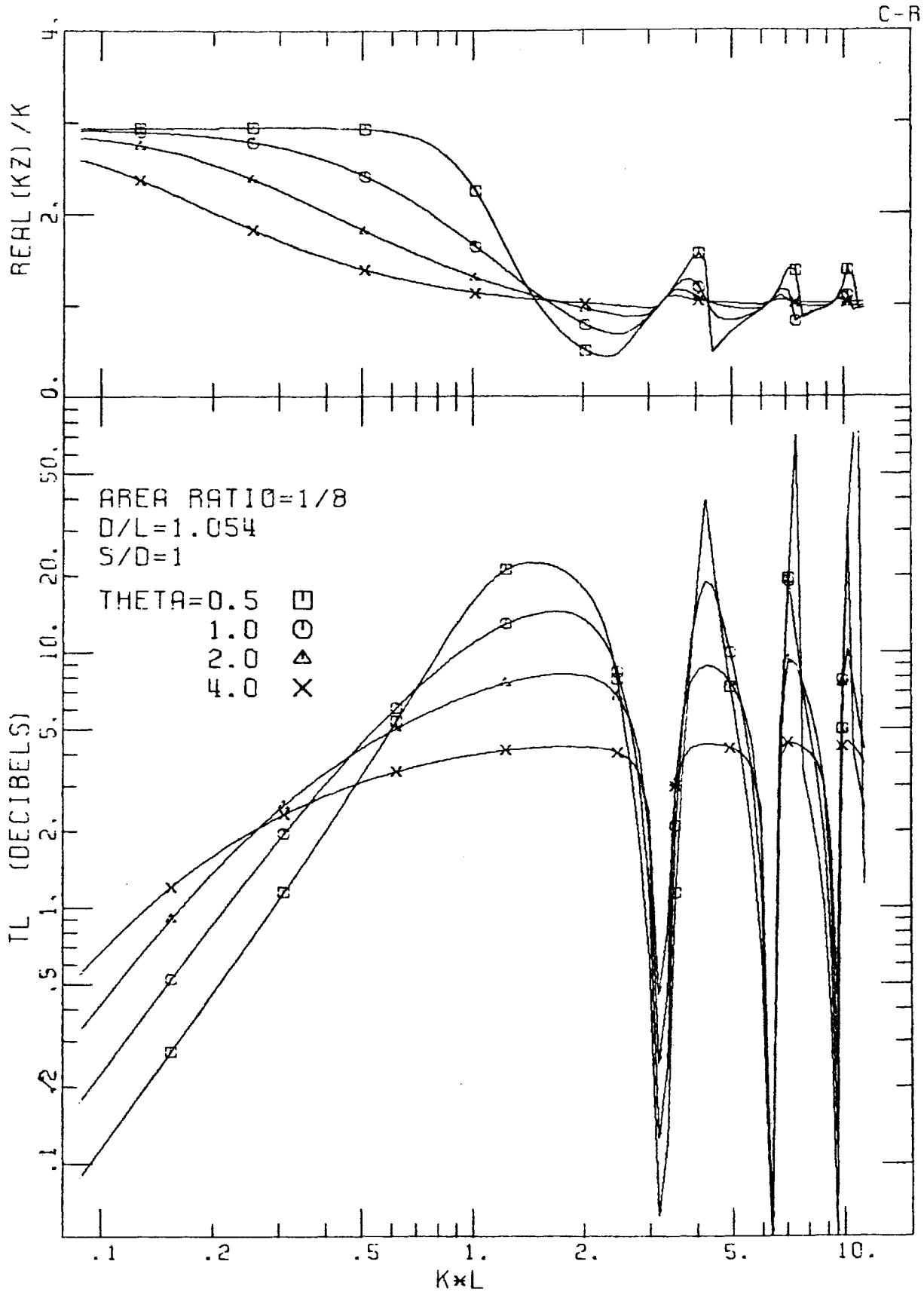

C-R
REAL (KZ) /K
4.
2.
0.
AREA RATIO=1/8
D/L=1.054
S/D=1
THETA=0.5
1.0
2.0
4.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

Figure A2.16

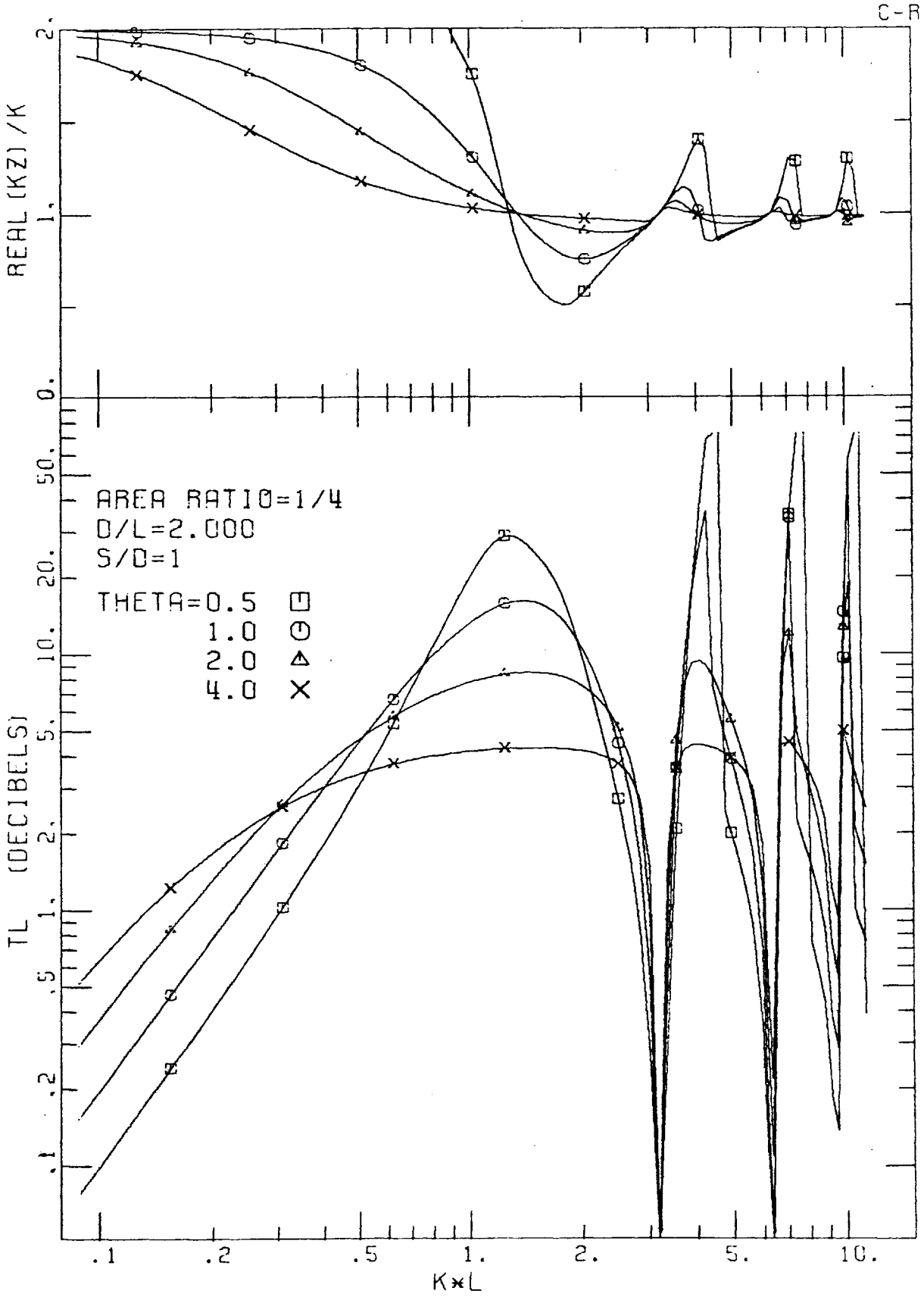
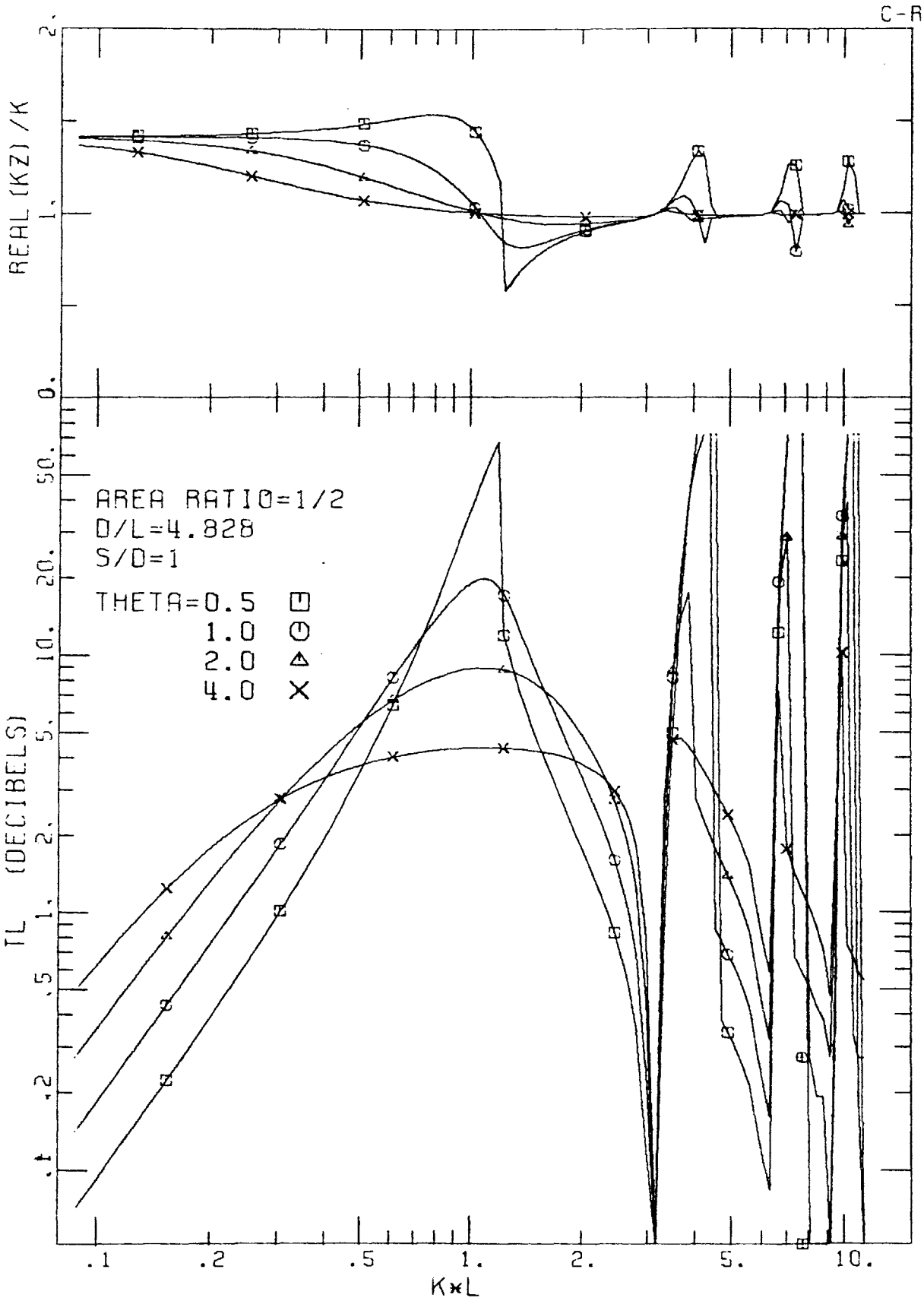

C-R
REAL (KZ) /K
2.
1.
0.
AREA RATIO=1/4
D/L=2.000
S/D=1
THETA=0.5
1.0
2.0
4.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

Figure A2.17


C-R
REAL (KZ) /K
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1.
0.
AREA RATIO=1/2
D/L=4.828
S/D=1
THETA=0.5
1.0
2.0
4.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

Figure A2.18

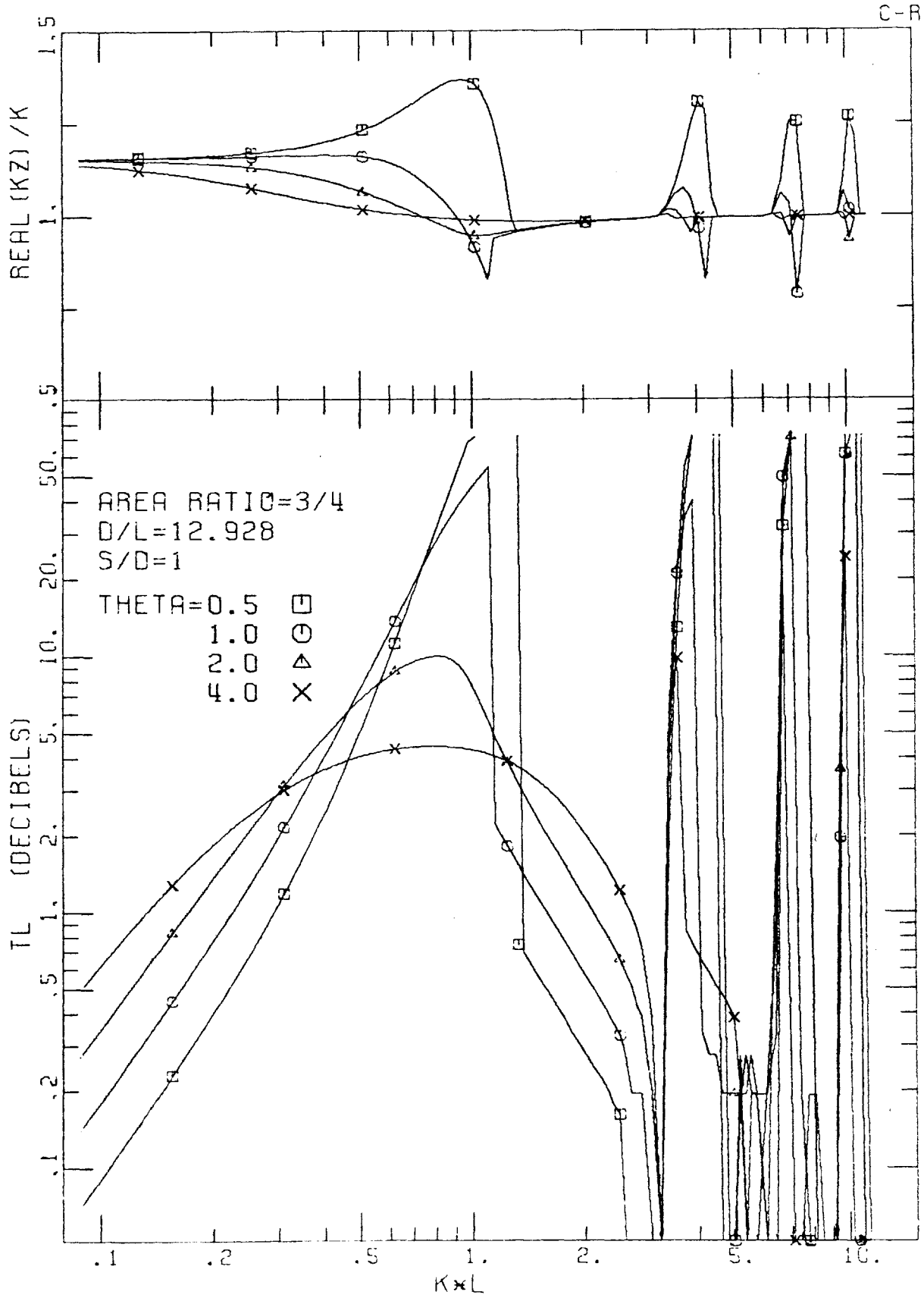
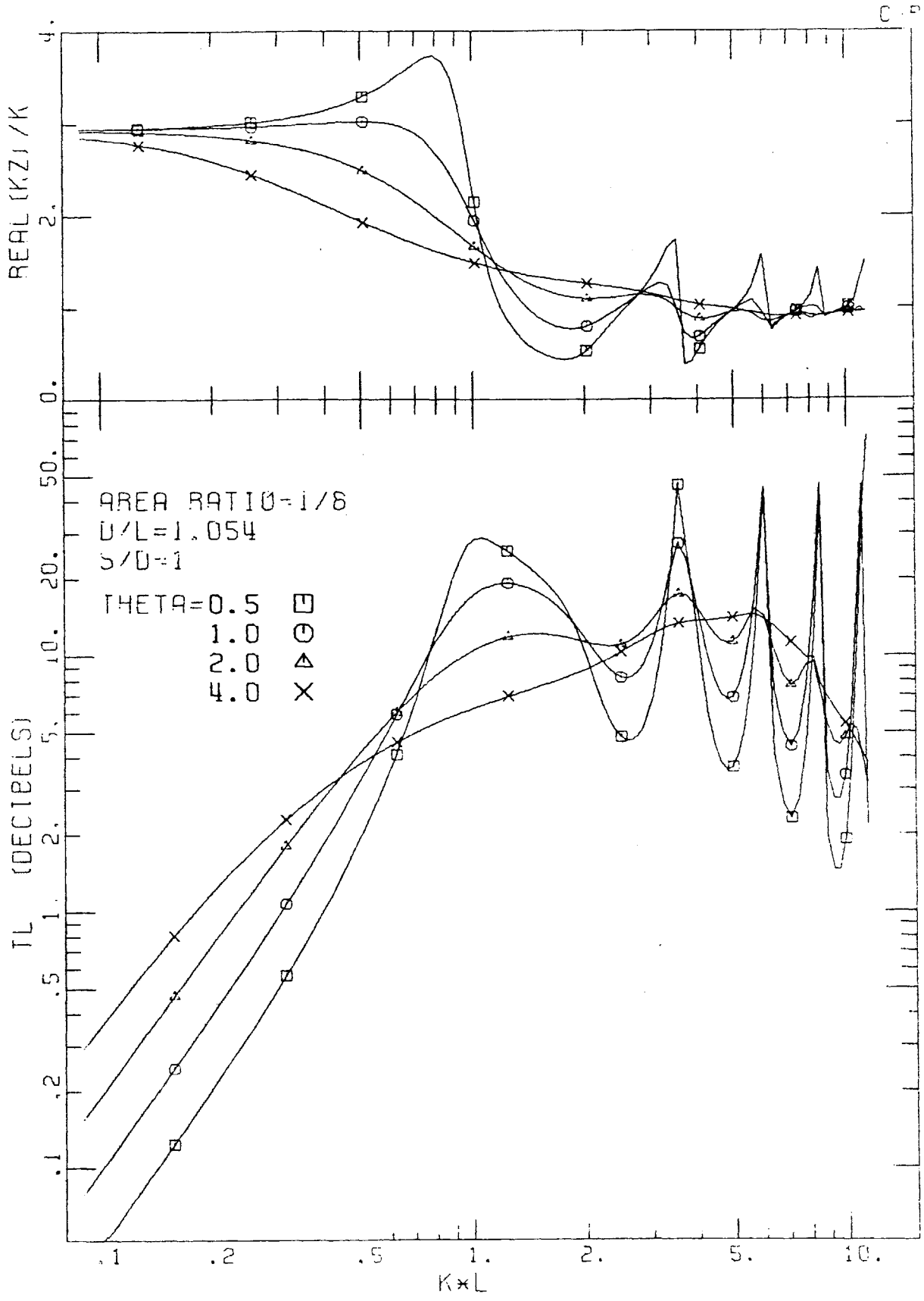

C-R
REAL(KZ)/K
1.5
1.
.5
AREA RATIO=3/4
D/L=12.928
S/D=1
THETA=0.5
1.0
2.0
4.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

Figure A2.19

Figures A2.21 - A2.28: Real and imaginary parts of k_z for circular ducts lined with a porous liner. The format is the same as in Figures A2.16 - A2.19.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4.
2.
0.
AREA RATIO=1/8
D/L=1.054
S/D=1
THETA=0.5
1.0
2.0
4.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

Figure A2.21

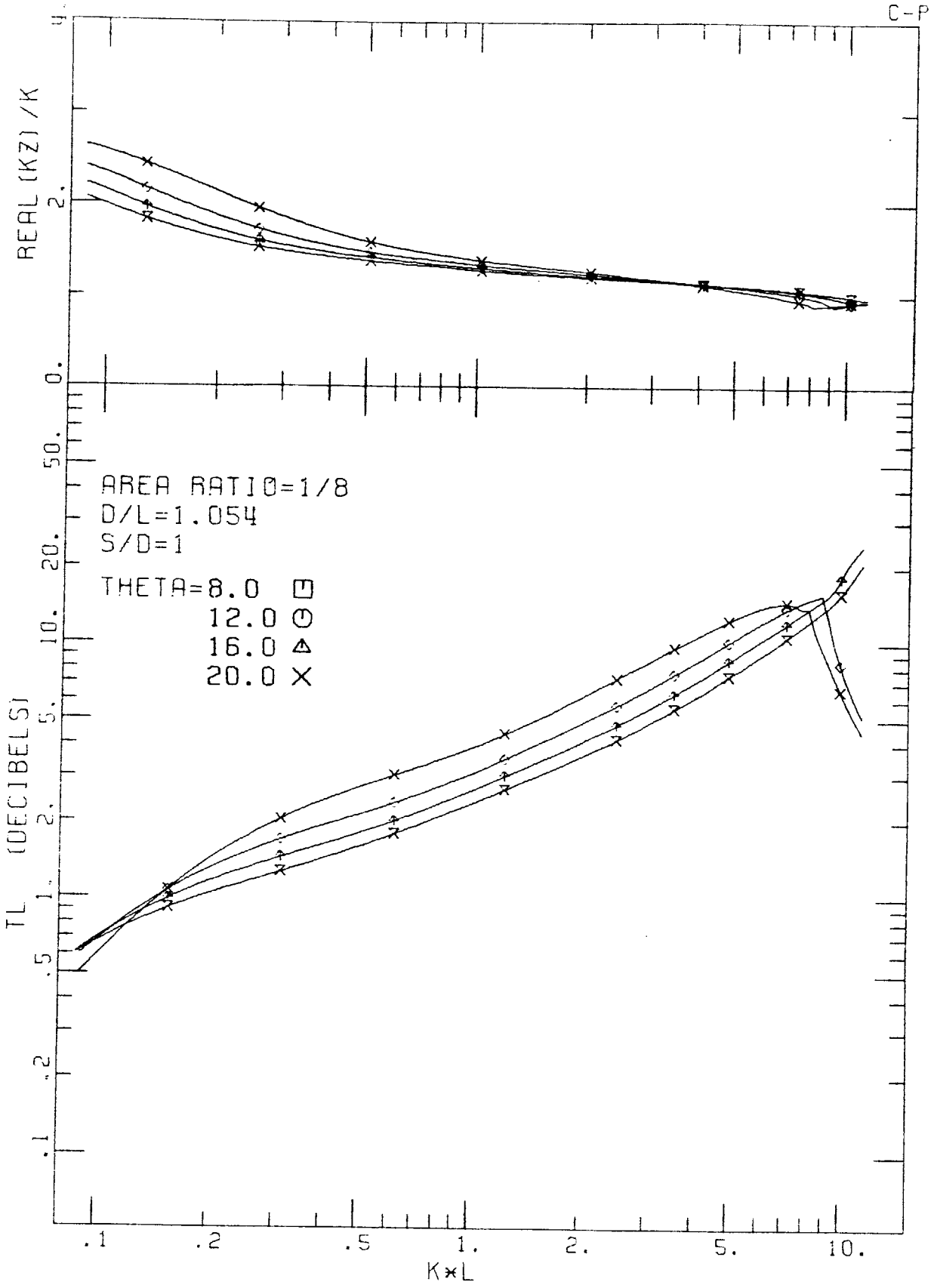

C-P
REAL (KZ) /K
4.
2.
0.
AREA RATIO=1/8
D/L=1.054
S/D=1
THETA=8.0
12.0
16.0
20.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

Figure A2.22

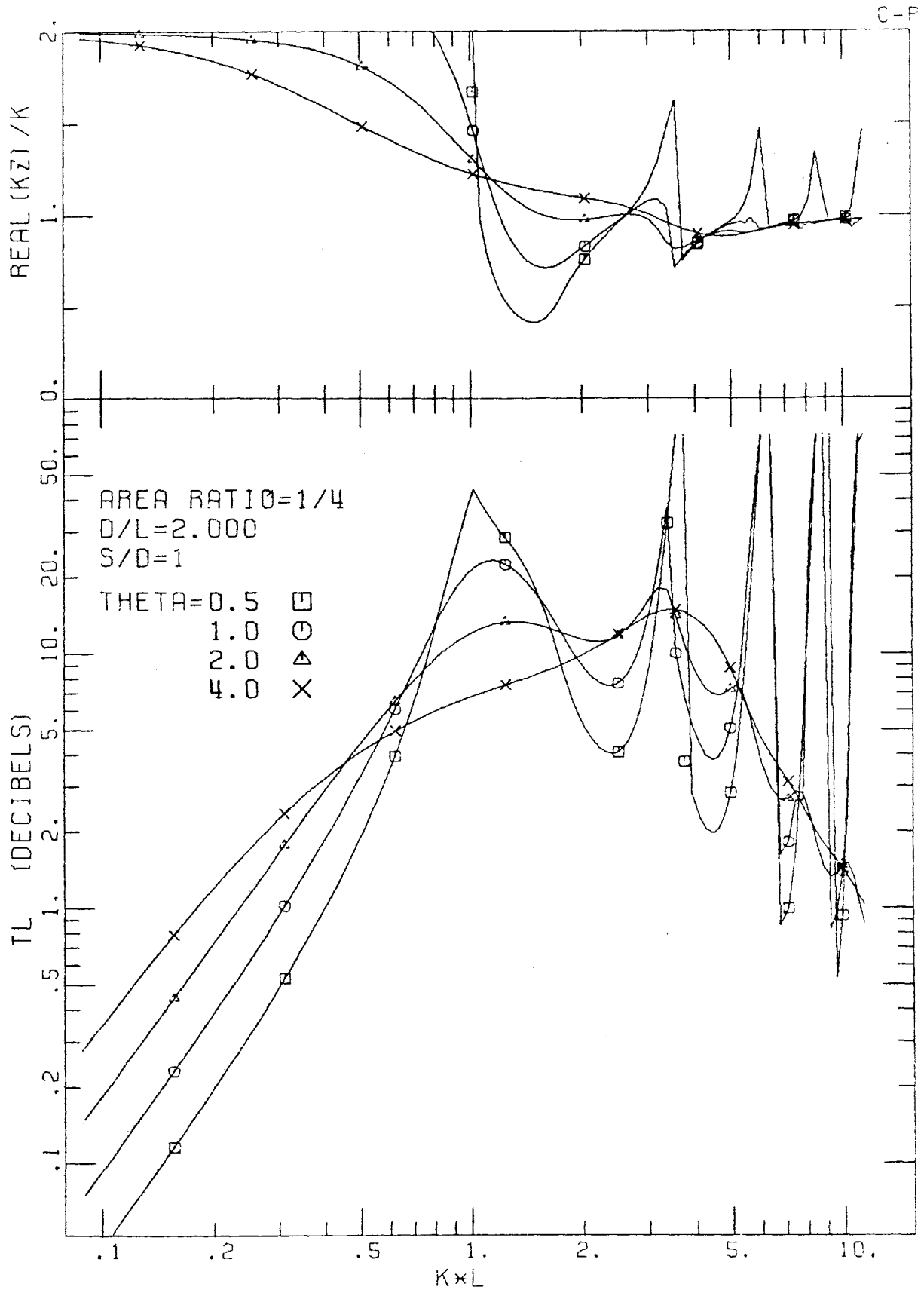

C-P
REAL (KZ) /K
2.
1.
0.
AREA RATIO=1/4
D/L=2.000
S/D=1
THETA=0.5
1.0
2.0
4.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

Figure A2.23

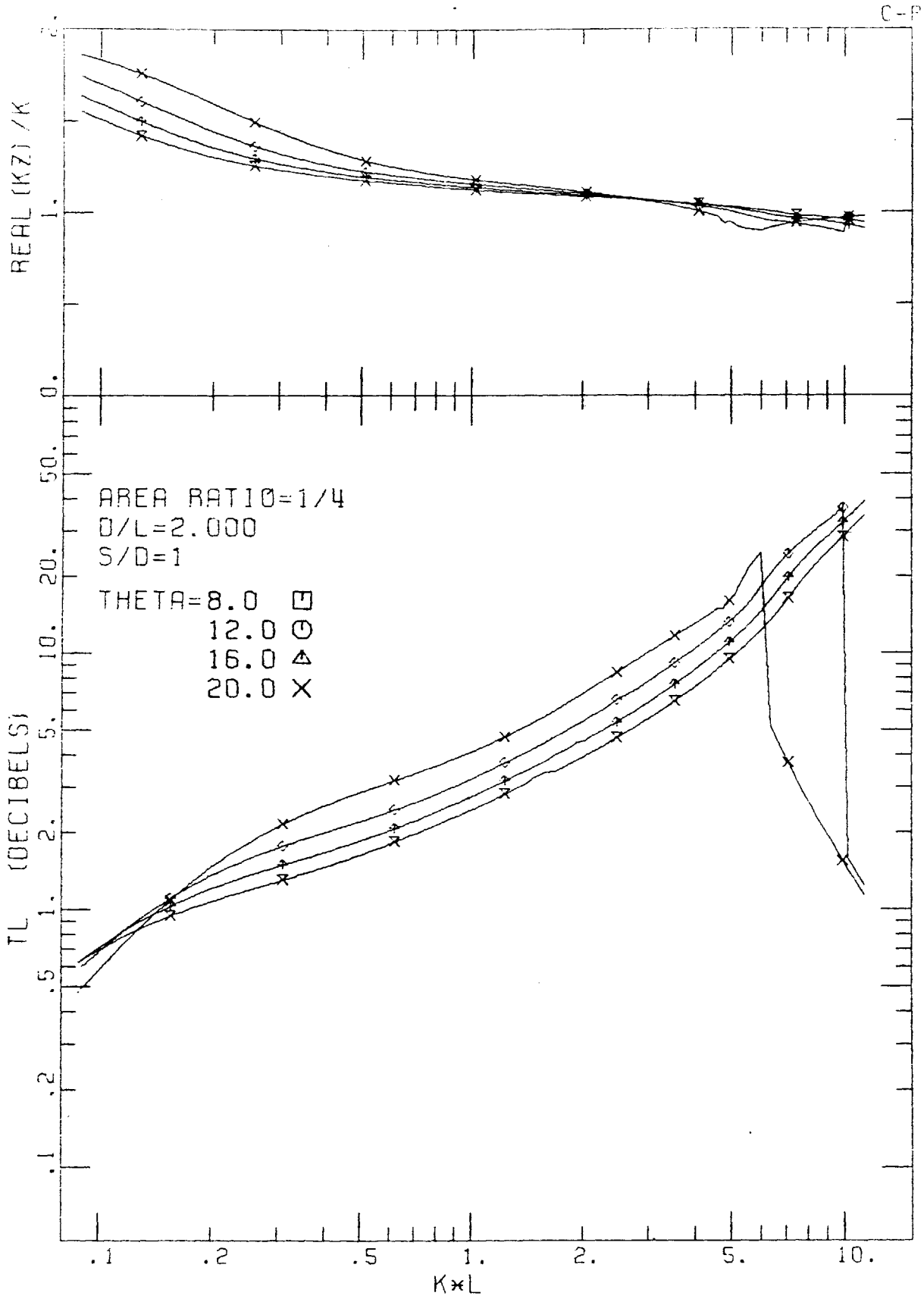

REAL (KZ) /K
2.
1.
0.
AREA RATIO=1/4
D/L=2.000
S/D=1
THETA=8.0
12.0
16.0
20.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

Figure A2.24

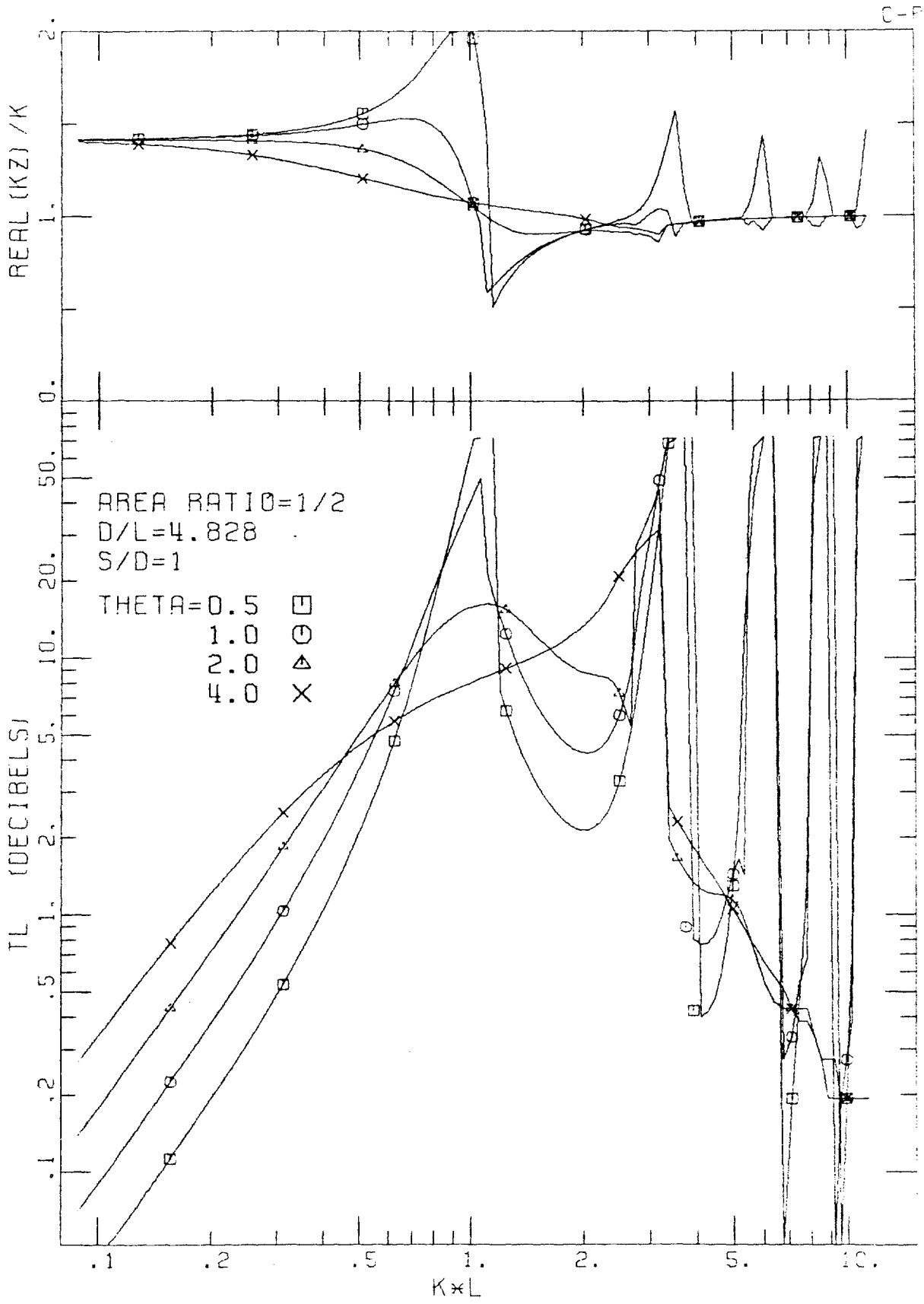

REAL(KZ)/K
2.
1.
0.
AREA RATIO=1/2
D/L=4.828
S/D=1
THETA=0.5
1.0
2.0
4.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

Figure A2.25

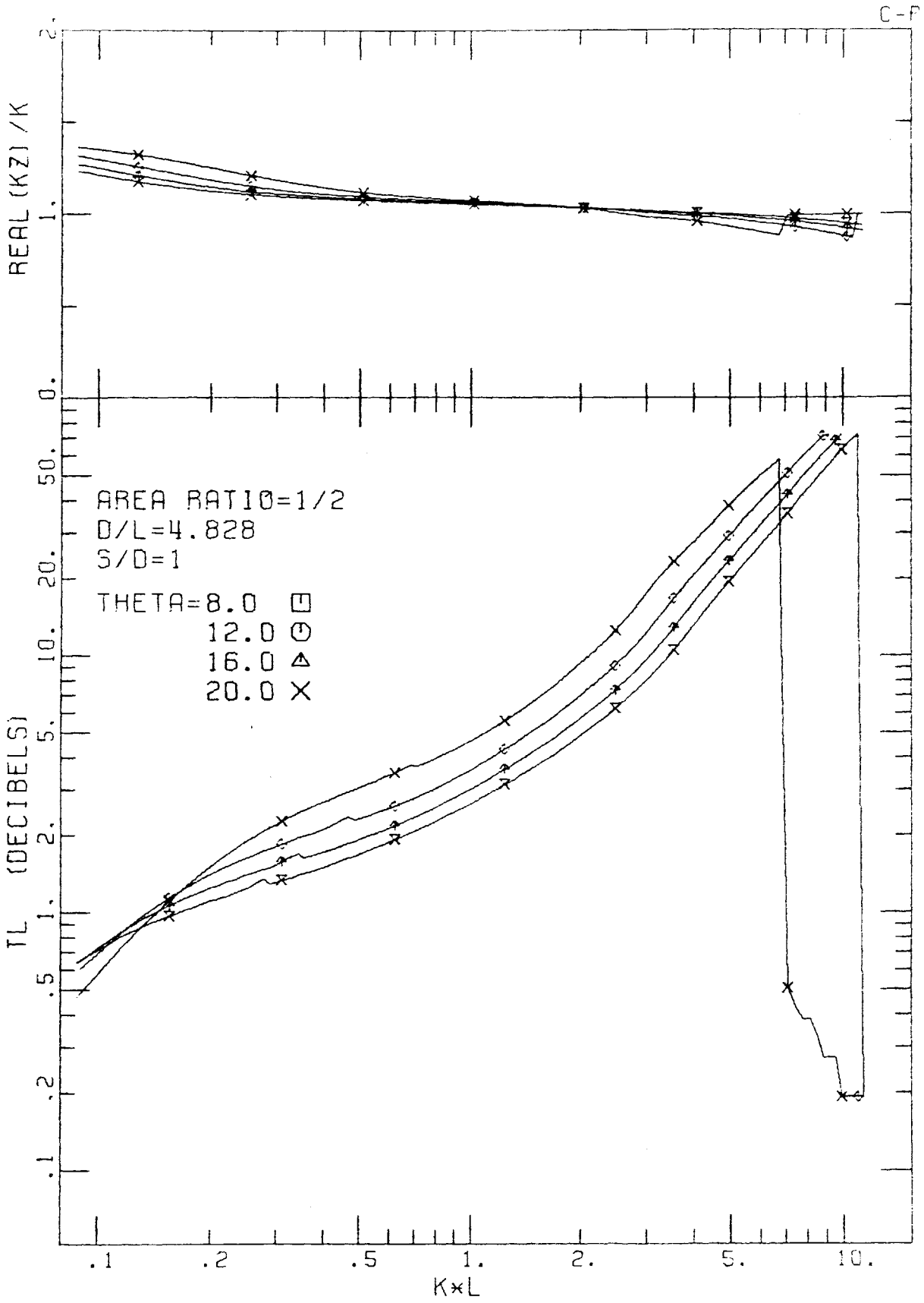

C-F
REAL (KZ) /K
2.
1.
0.
AREA RATIO=1/2
D/L=4.828
S/D=1
THETA=8.0
12.0
16.0
20.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

Figure A2.26

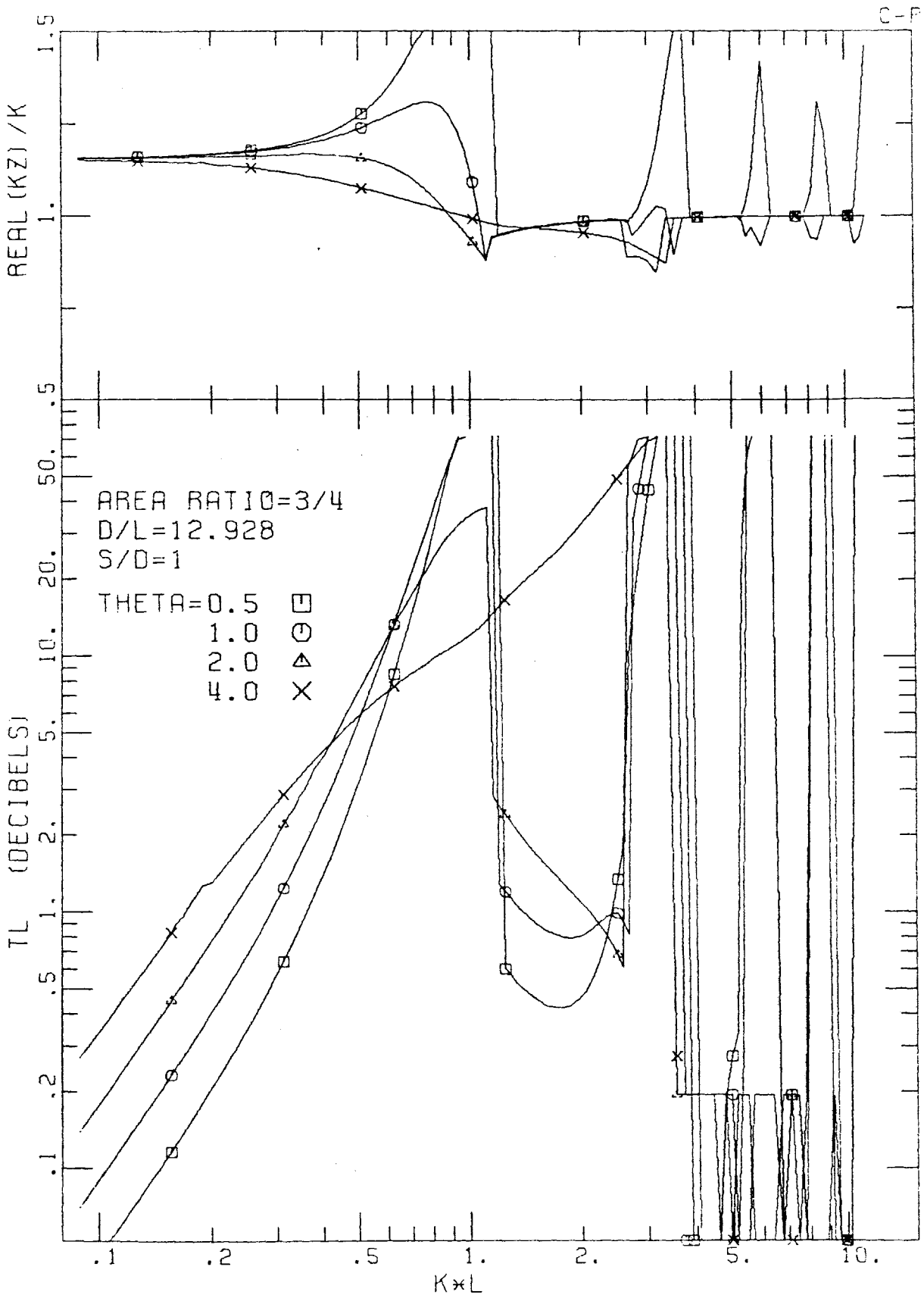

C-P
REAL (KZ) /K
1.5
1.
.5
AREA RATIO=3/4
D/L=12.928
S/D=1
THETA=0.5
1.0
2.0
4.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

Figure A2.27

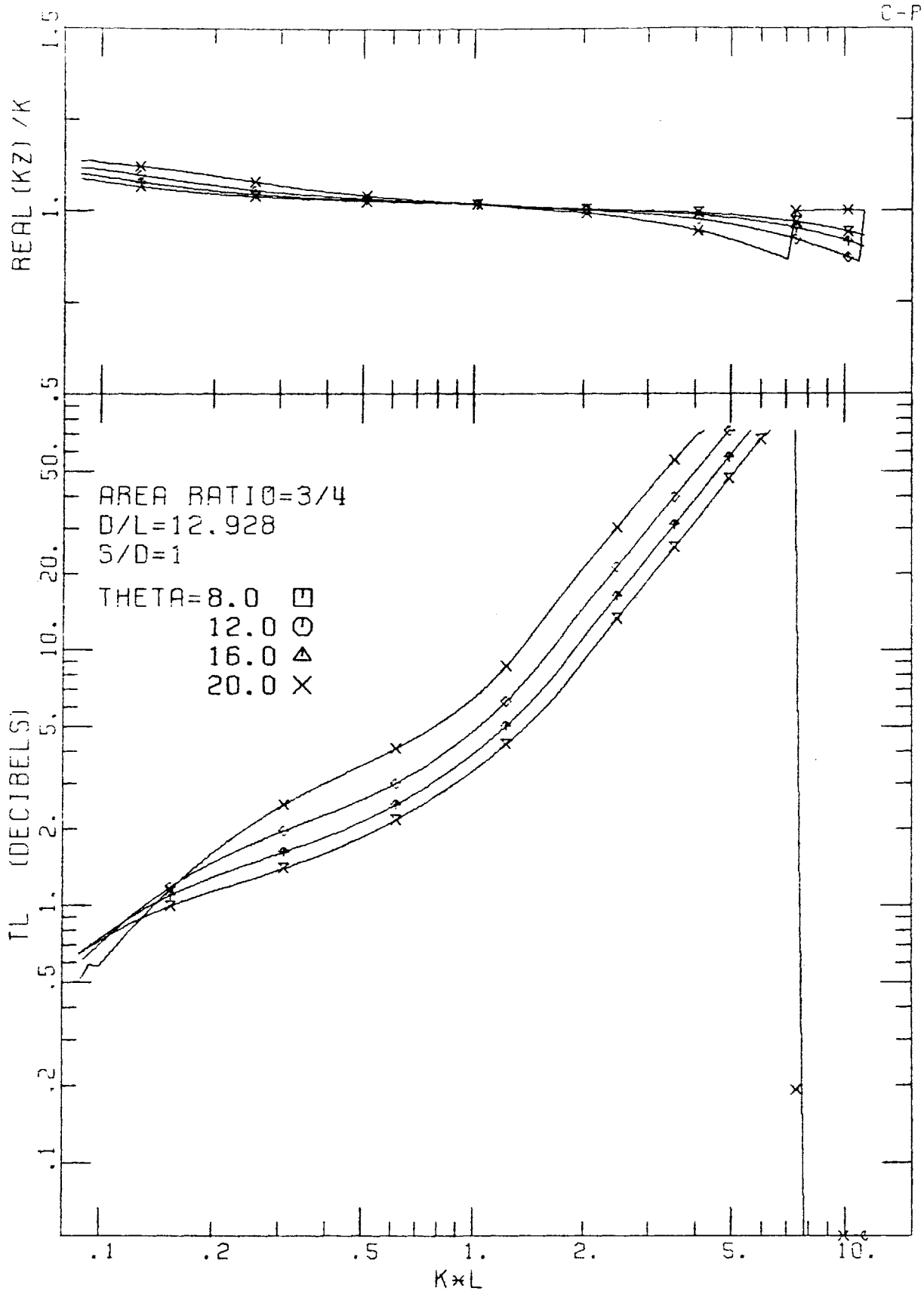

C-P
REAL (KZ) /K
1.5
1.
.5
AREA RATIO=3/4
D/L=12.928
S/D=1
THETA=8.0
12.0
16.0
20.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

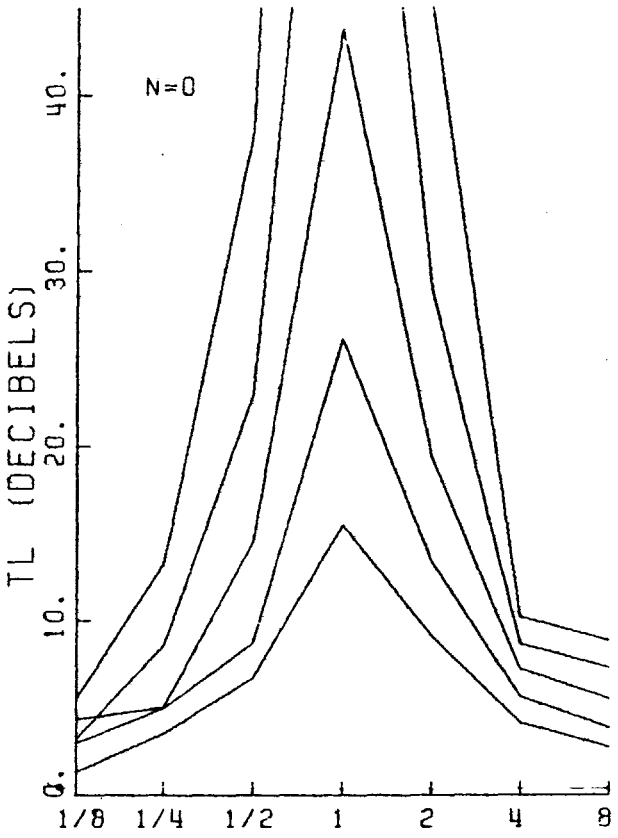
Figure A2.28

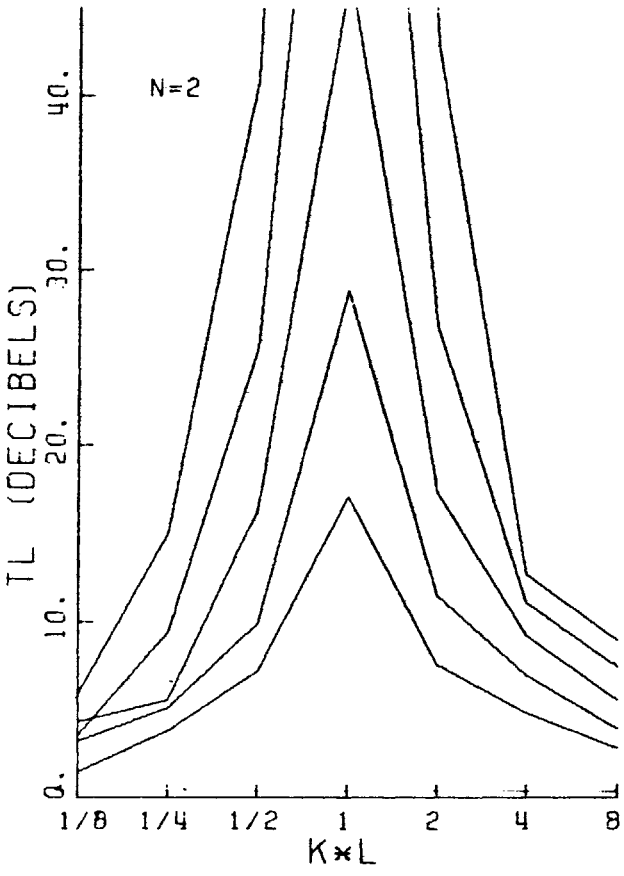
Figures A3.50 - A3.65: Octave band TL vs kL in circular ducts lined with a resistive screen type resonator liner. Each figure corresponds to a different combination of values of the screen resistance θ and D/L. Each figure contains three frames corresponding to different spectra of the incident wave as indicated by N. In each frame five curves are given corresponding to five values of the duct length parameter S/D, which are given at the corner of each figure.

Figure A3.50

THETA=0.5
D/L=1.094
AREA RATIO=1

S/D=16
8
4
2
1





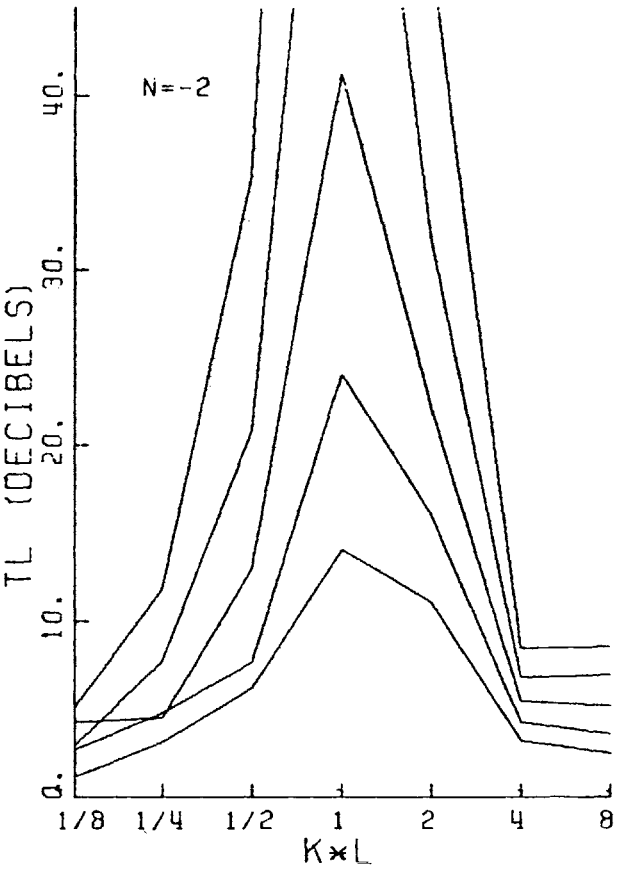
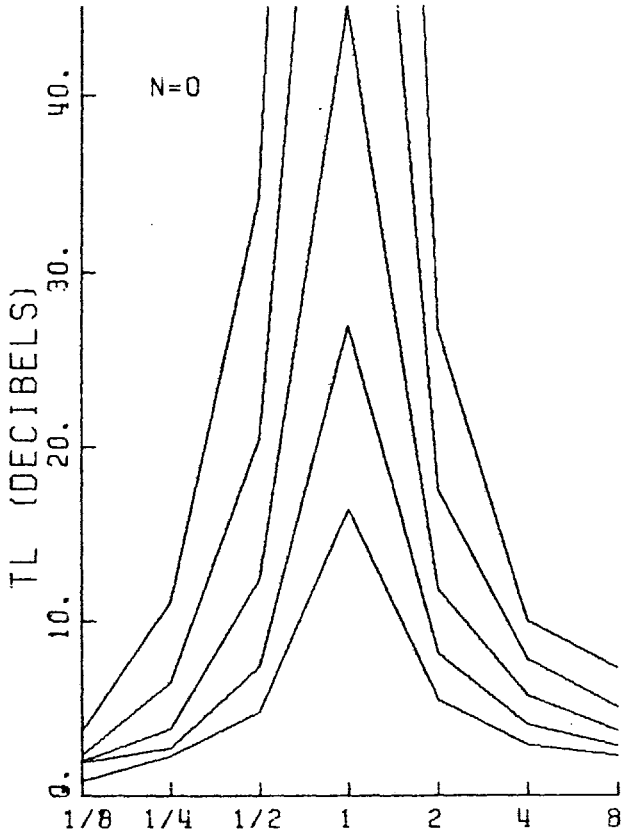
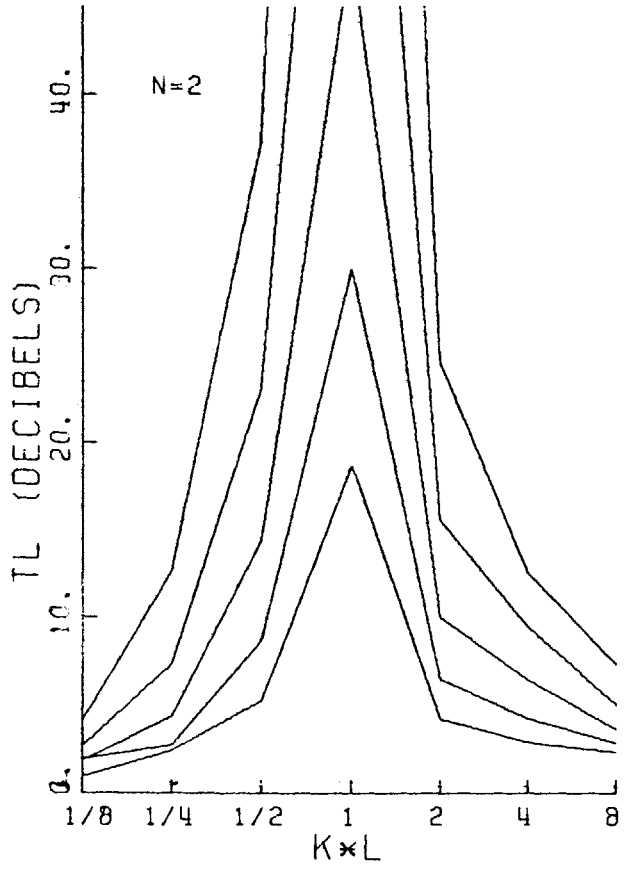


Figure A3.51

THETA=0.5
D/L=2.000
AREA RATIO=1

S/D=16
8
4
2
1





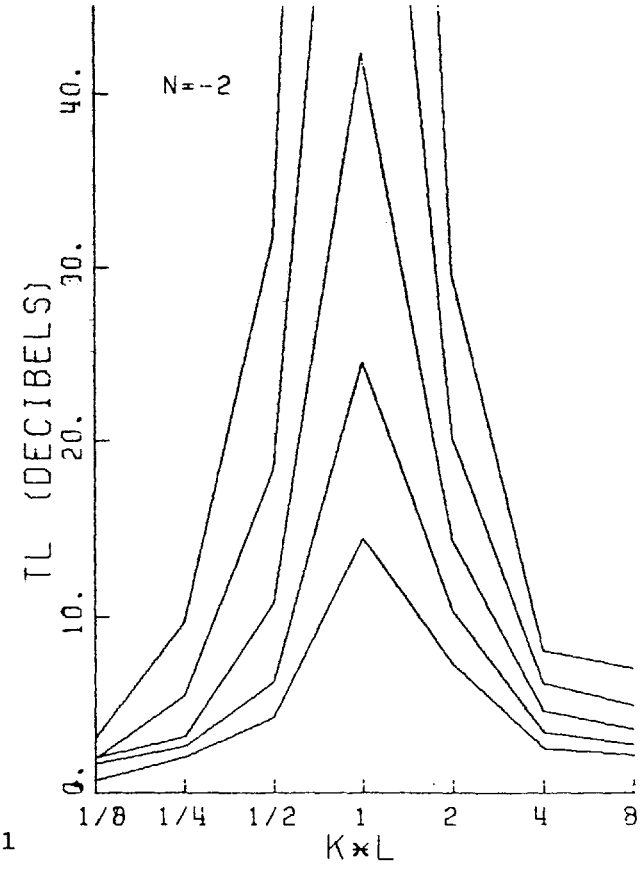
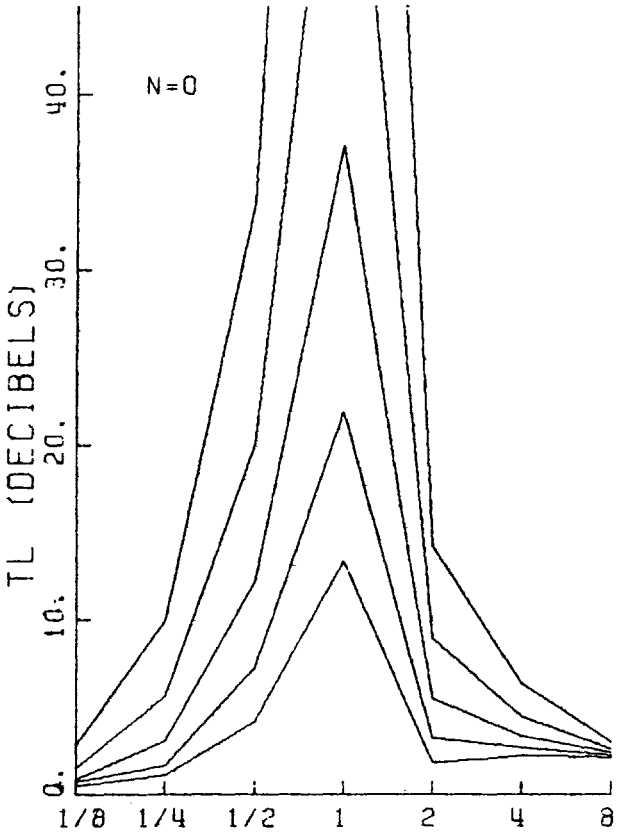
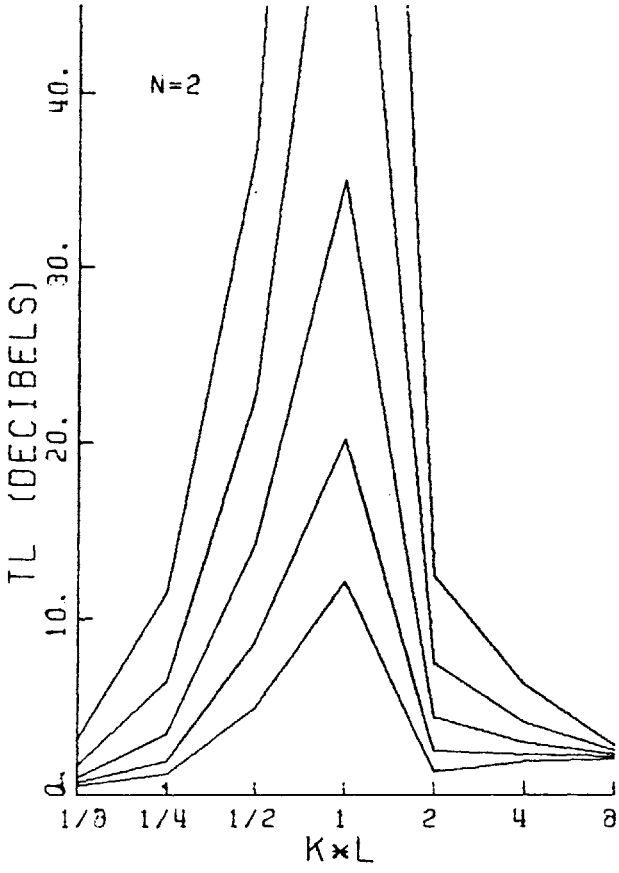


Figure A3.52

THETA=0.5
D/L=4.828
AREA RATIO=1

S/D=16
8
4
2
1





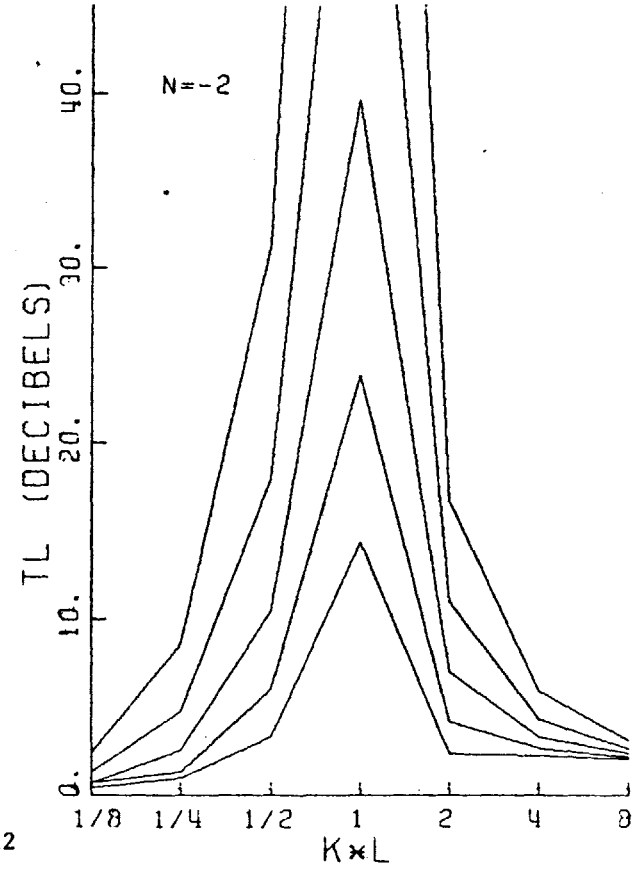
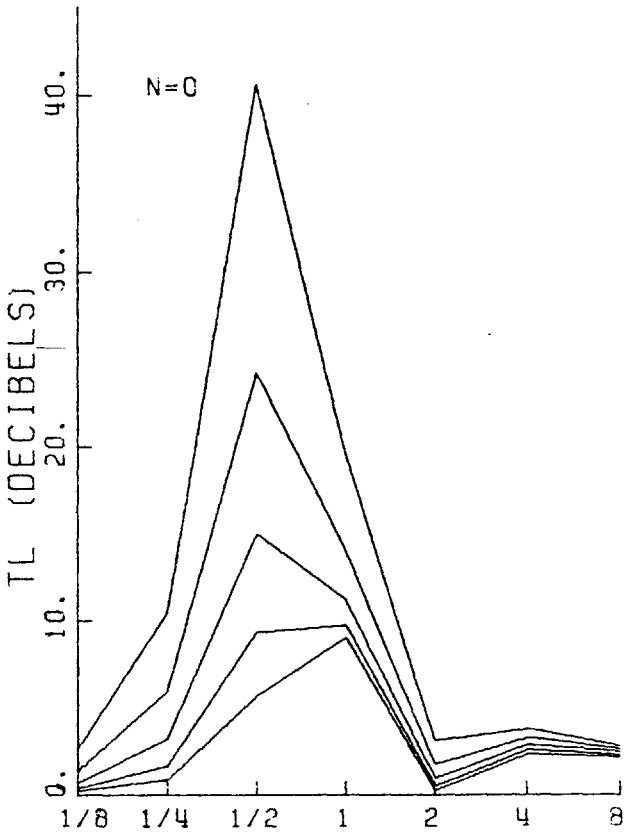
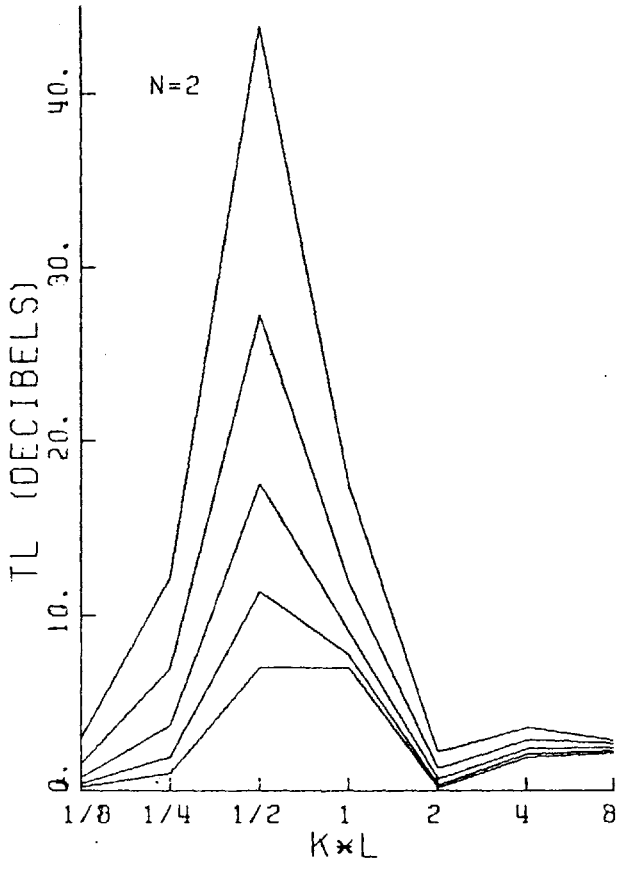


Figure A3.53

THETA=0.5
D/L=12.928
AREA RATIO=1

S/D=16
8
4
2
1





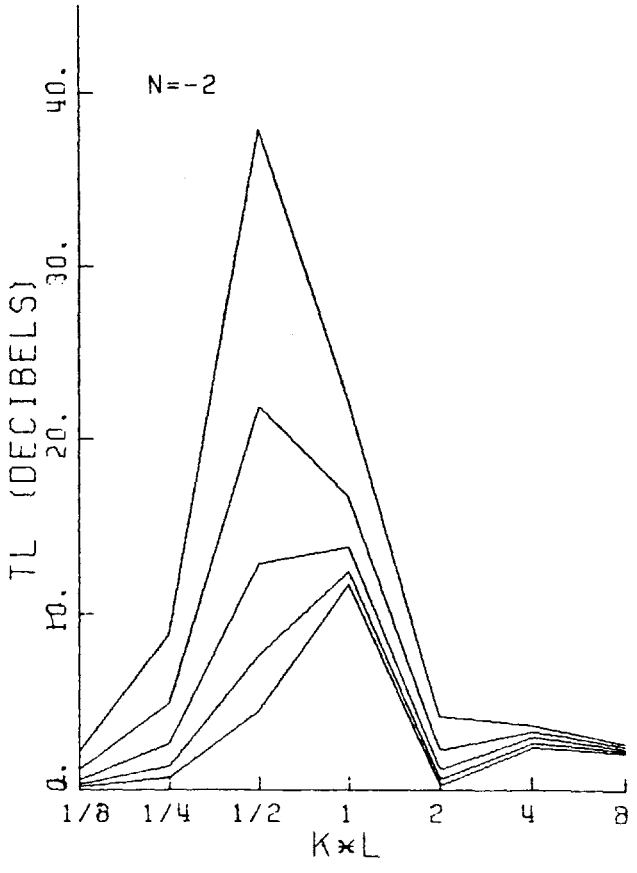
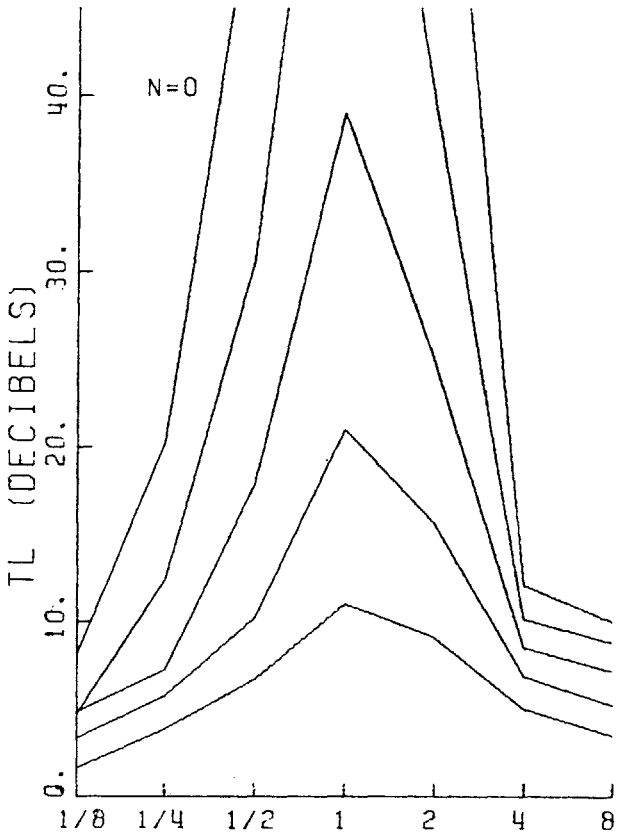
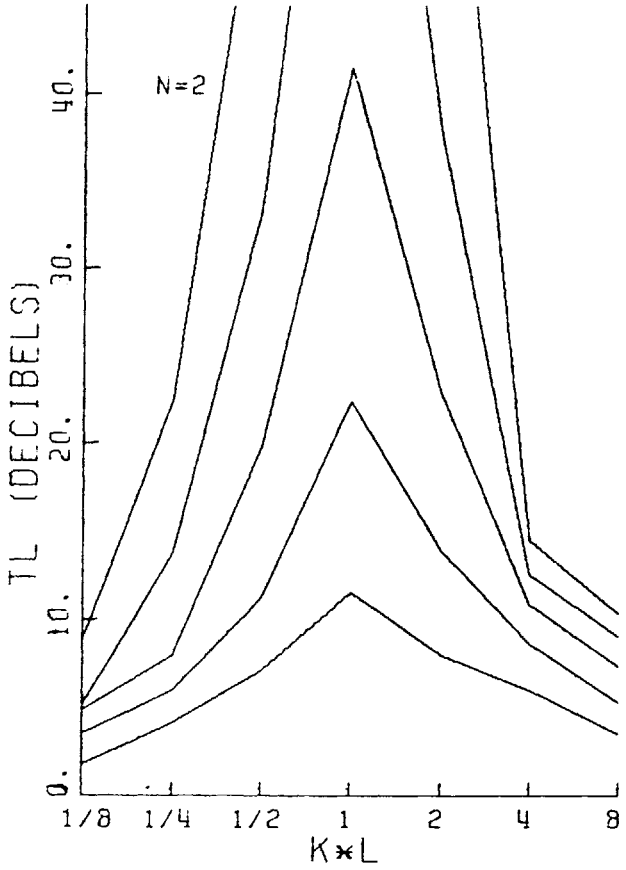


Figure A3.54

THETA=1.
D/L=1.094
AREA RATIO=1

S/D=16
8
4
2
1





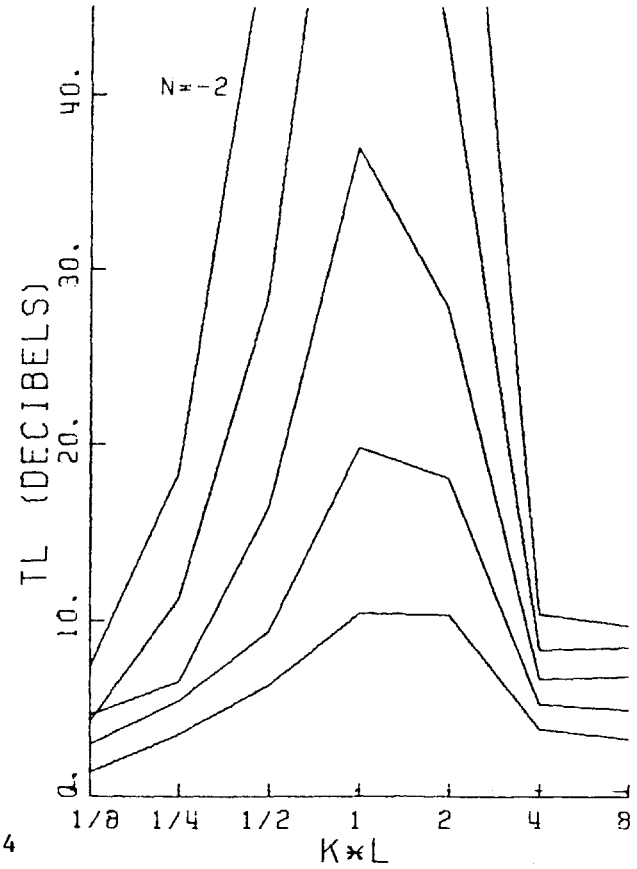
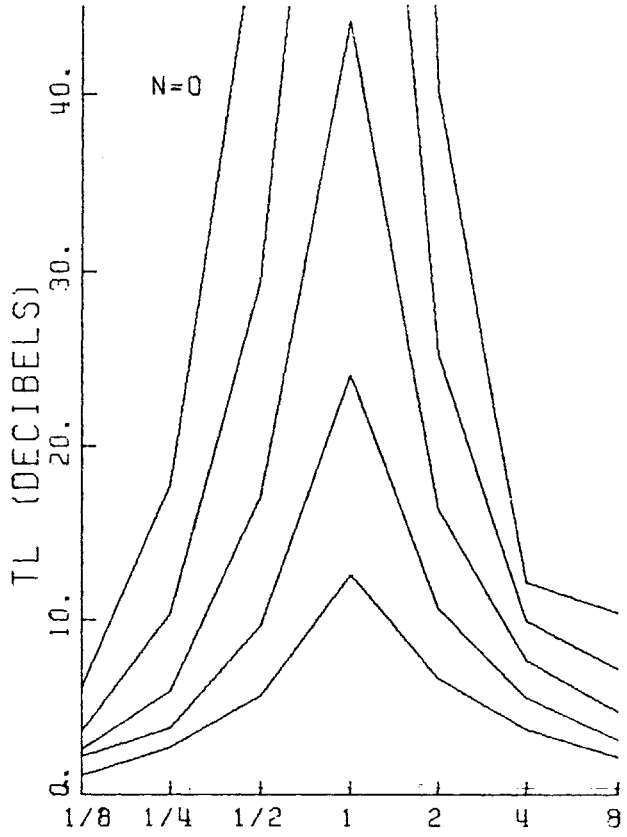
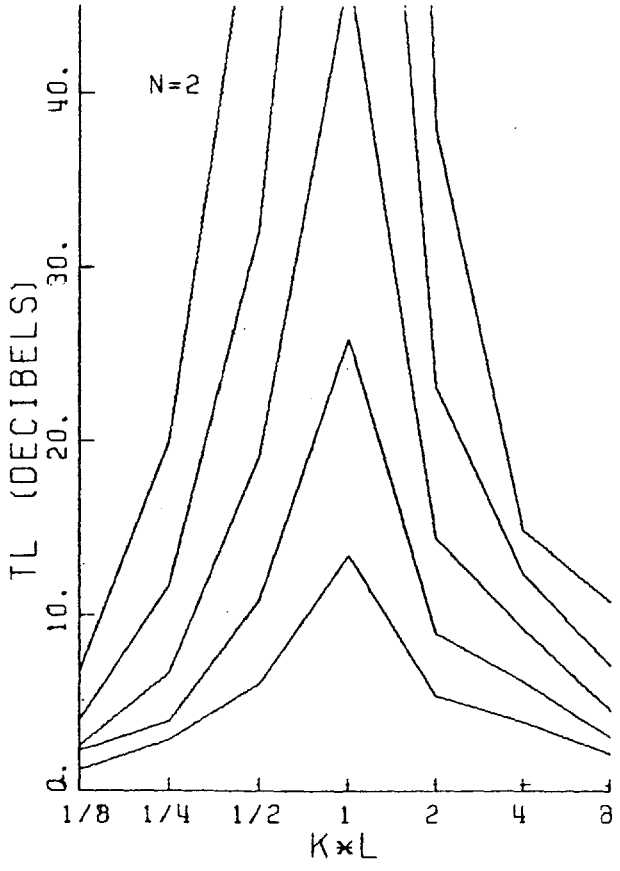


Figure A3.55

THETA=1.
D/L=2.000
AREA RATIO=1

S/D=16
8
4
2
1





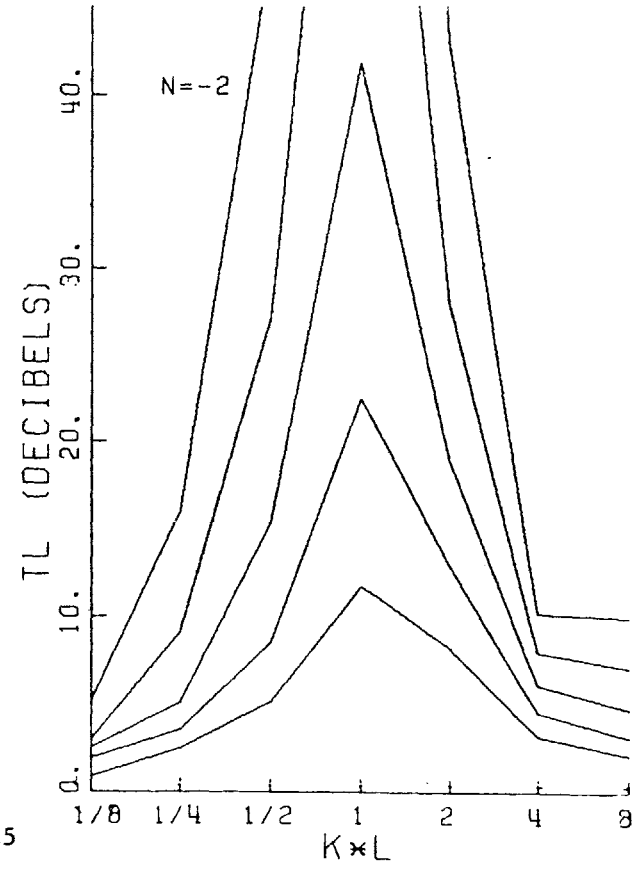
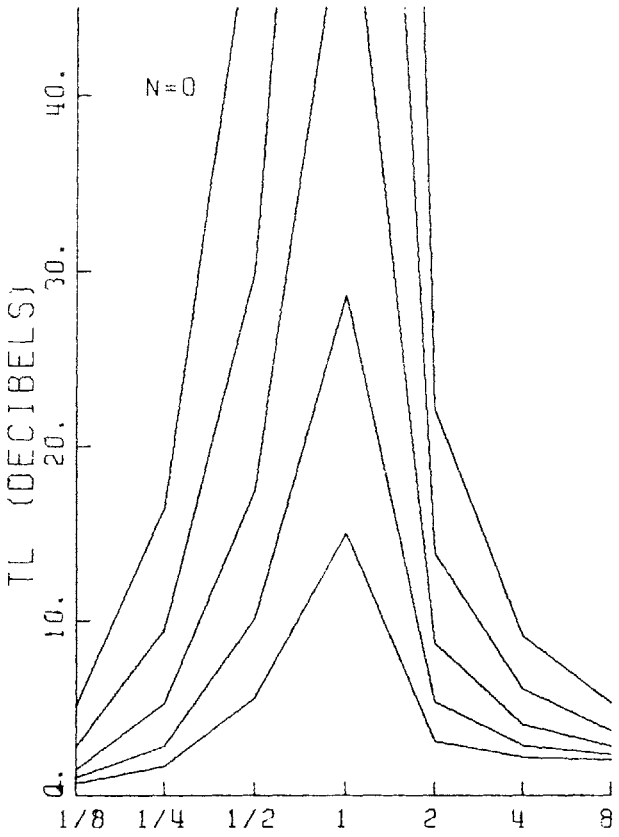
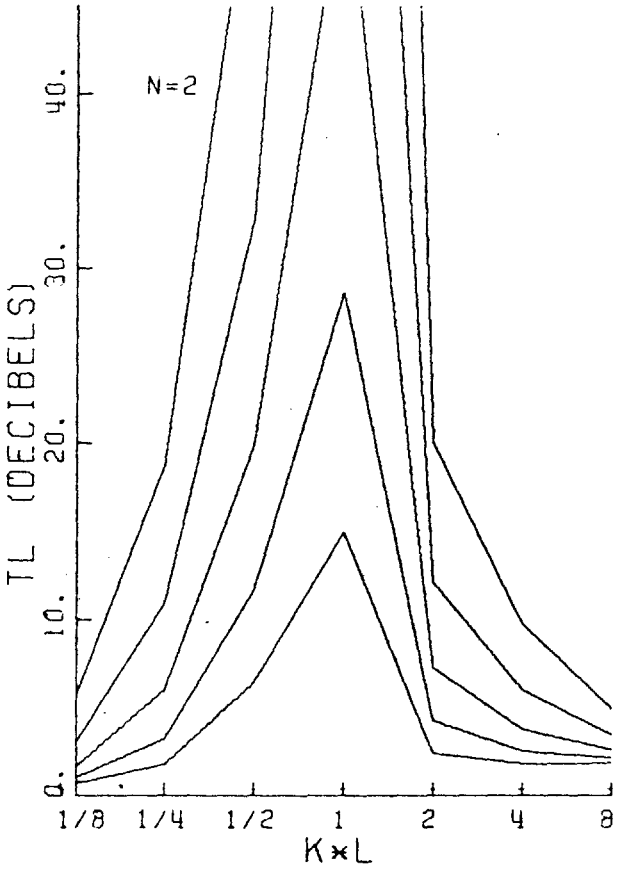


Figure A3.56

THETA=1.
D/L=4.828
AREA RATIO=1

S/D=16
8
4
2
1





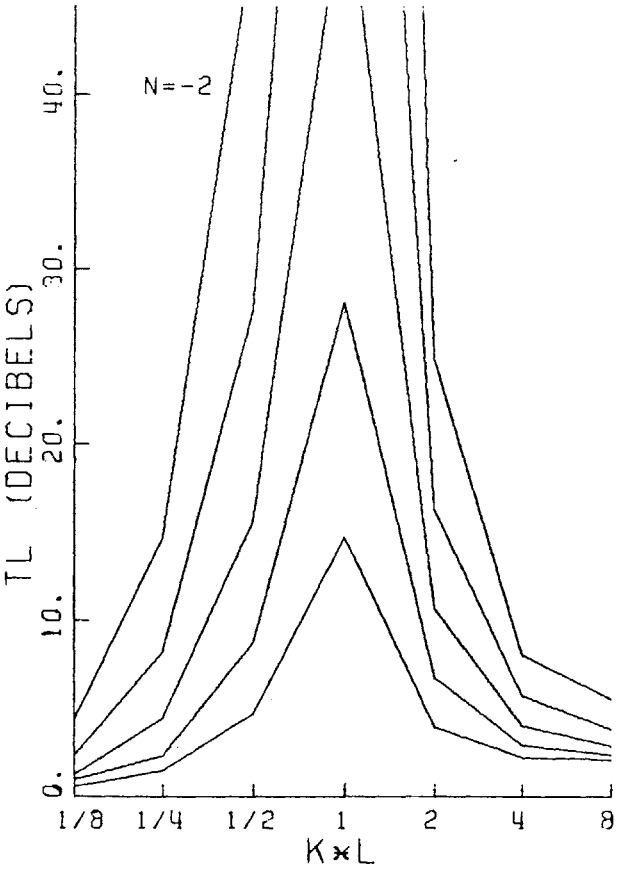
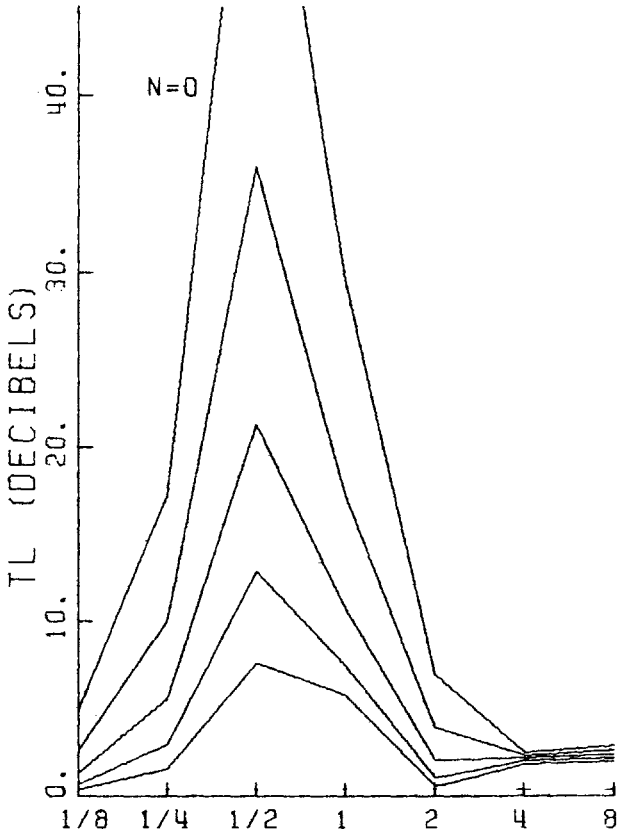


Figure A3.57

THETA=1.
D/L=12.928
AREA RATIO=1

S/D=16
8
4
2
1



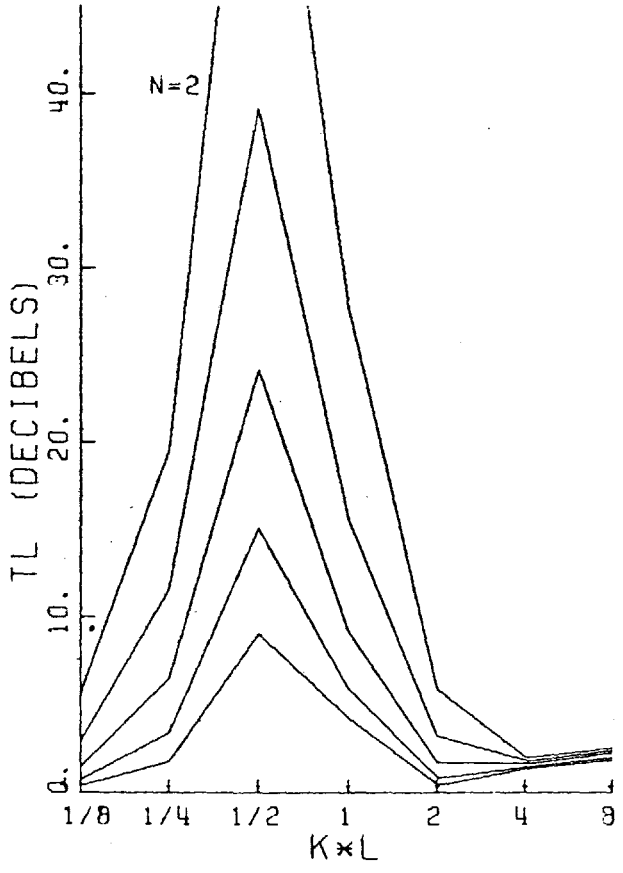
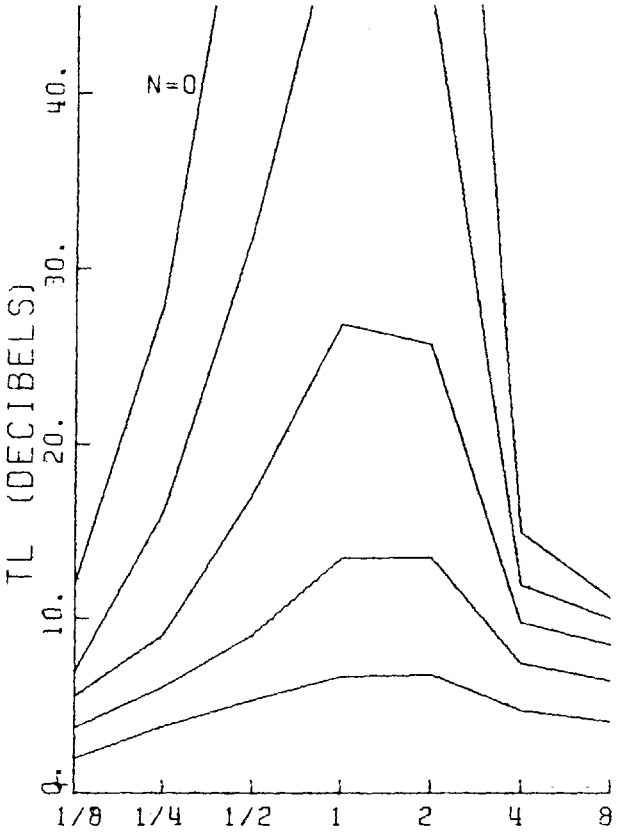
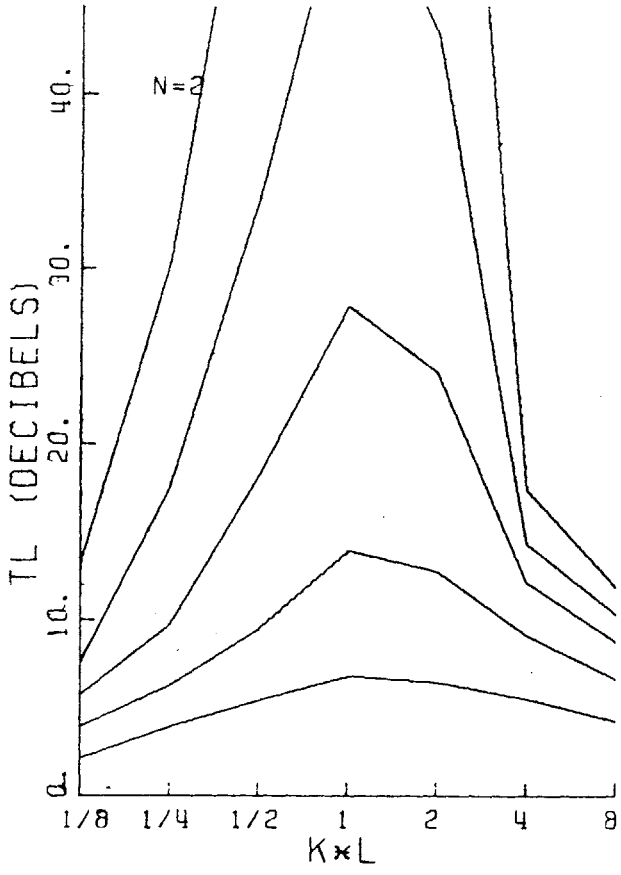


Figure A3.58

THETA=2.
D/L=1.094
AREA RATIO=1

S/D=16
8
4
2
1





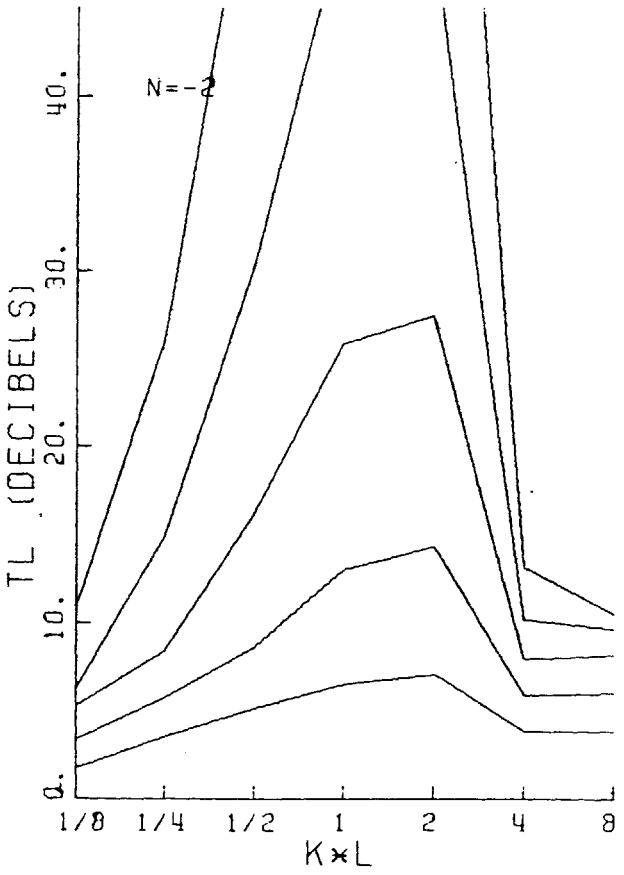
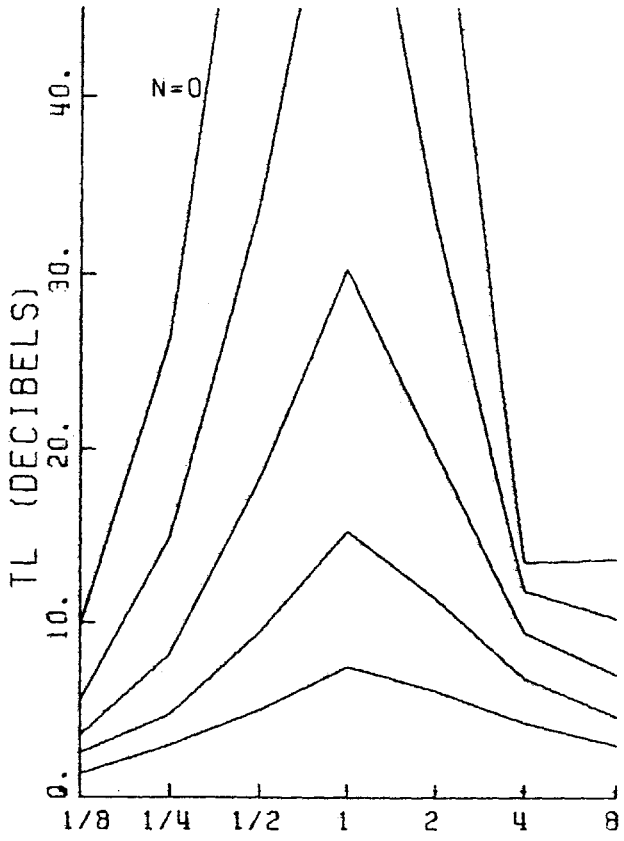
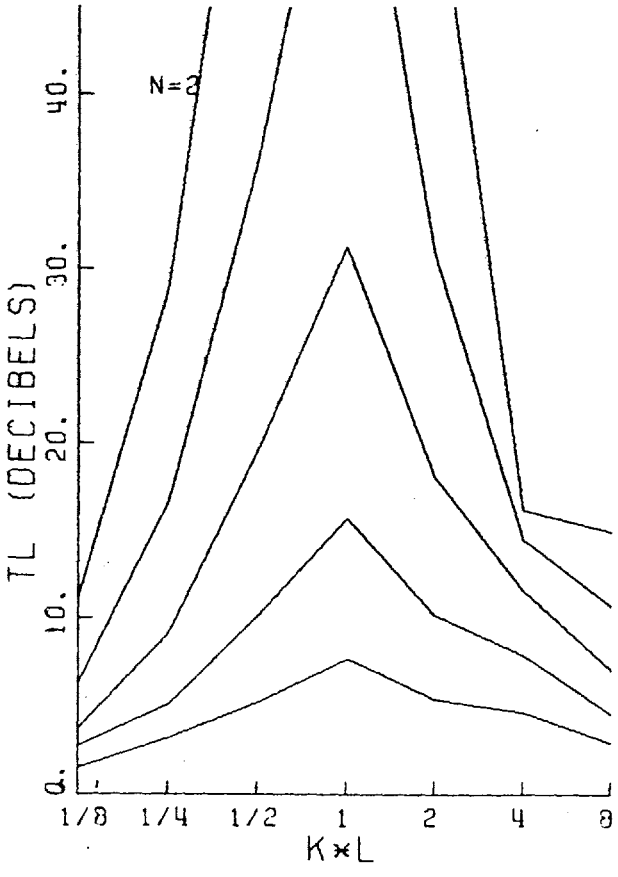


Figure A3.59

THETA=2.
D/L=2.000
AREA RATIO=1

S/D=16
8
4
2
1





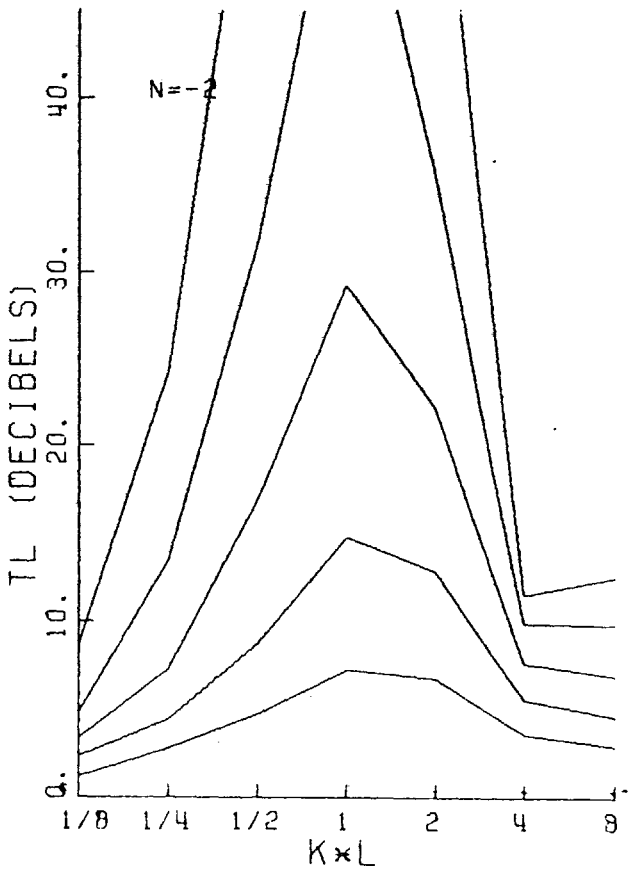
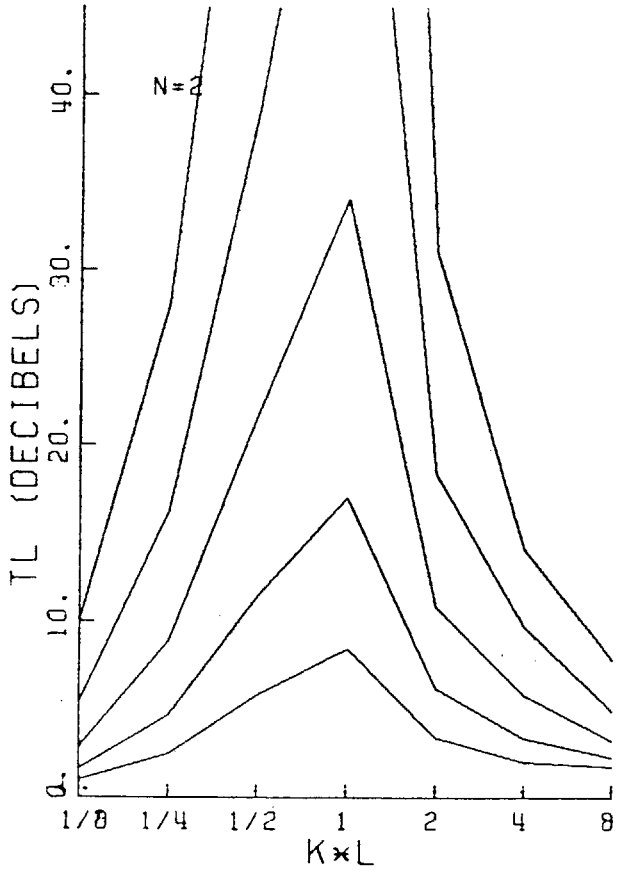


Figure A3.60

THETA=2.
D/L=4.828
AREA RATIO=1

S/D=16
8
4
2
1



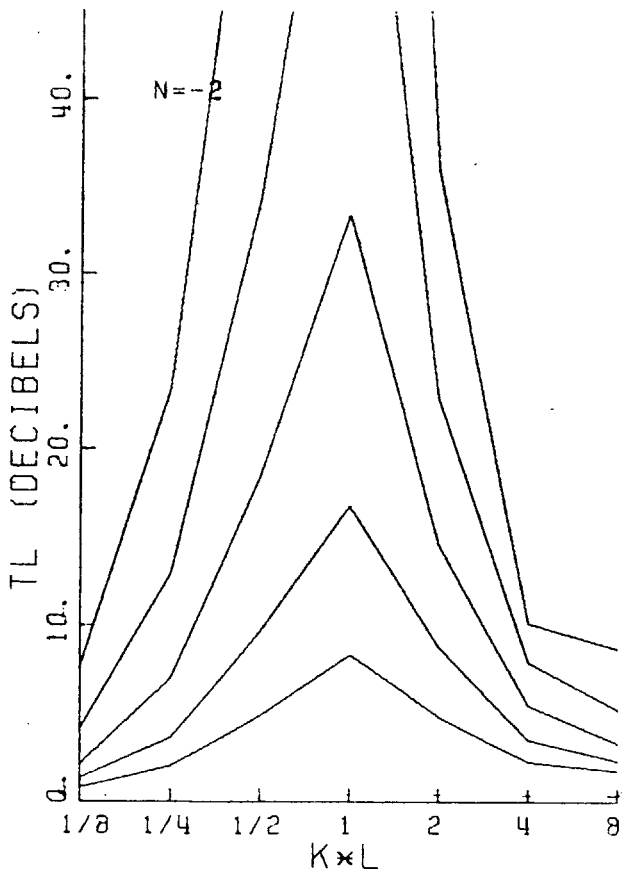
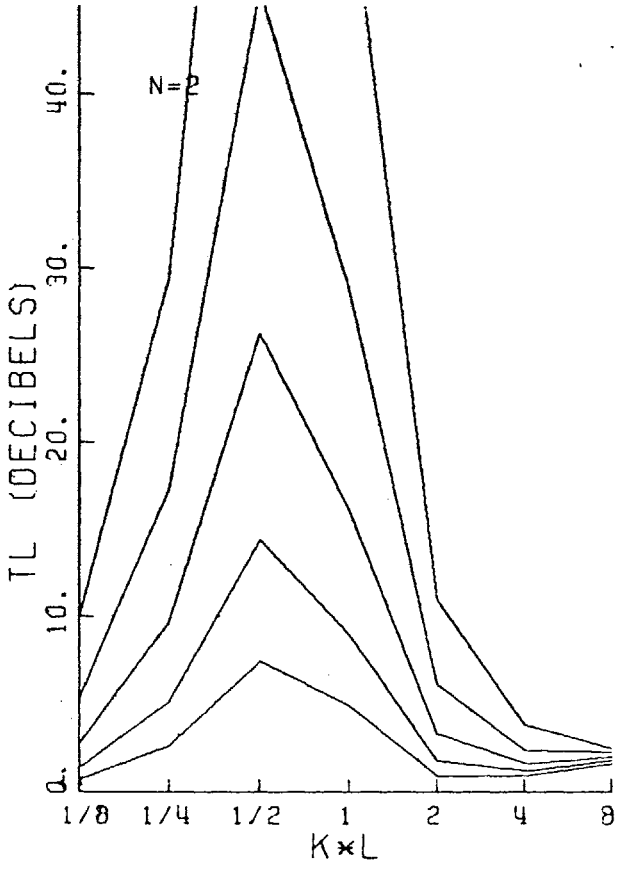
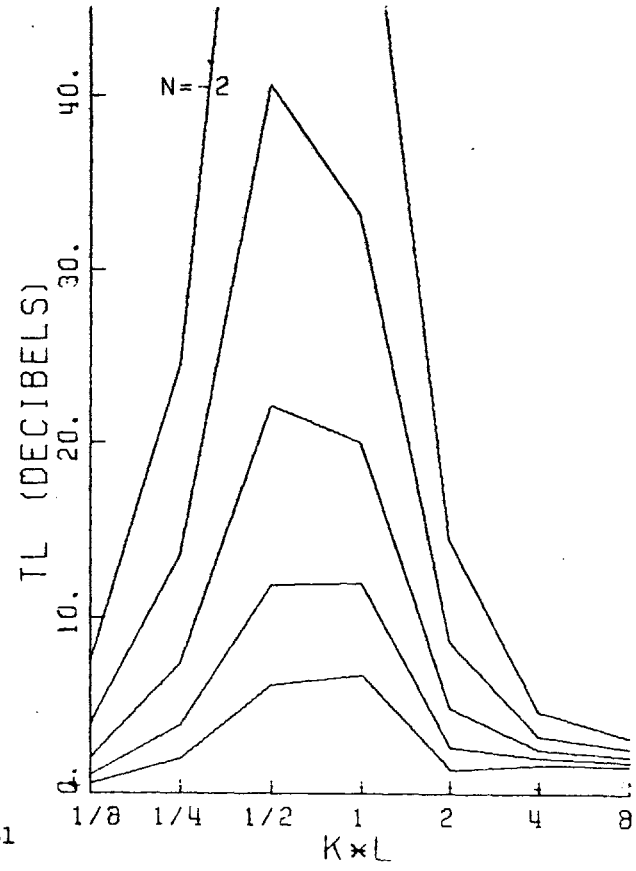


Figure A3.61

THETA=2.
D/L=12.928
AREA RATIO=1

S/D=16
8
4
2
1





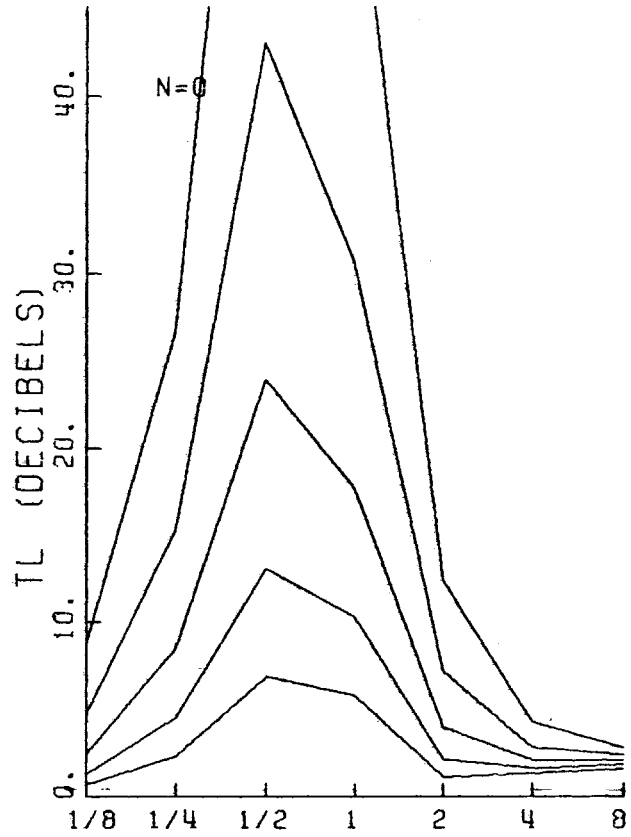
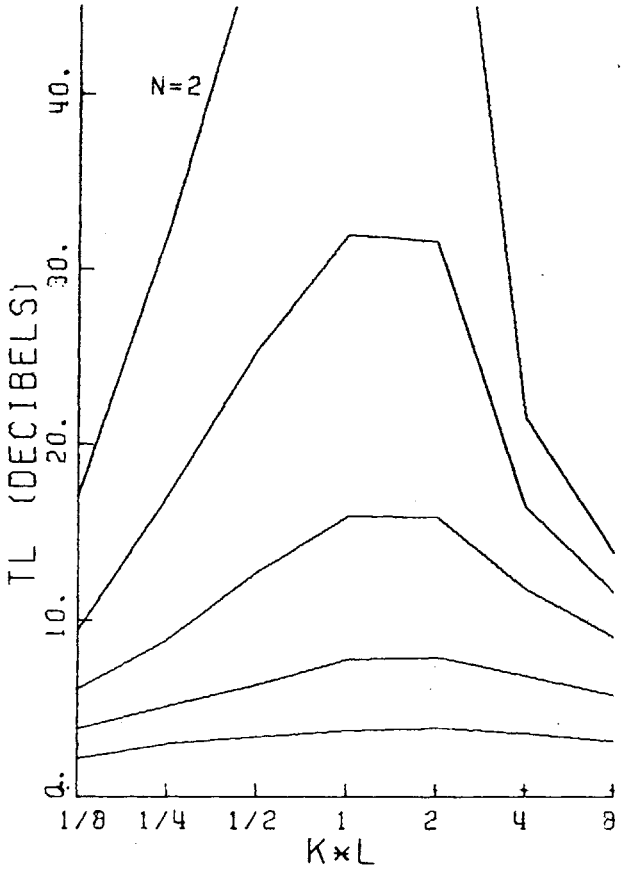
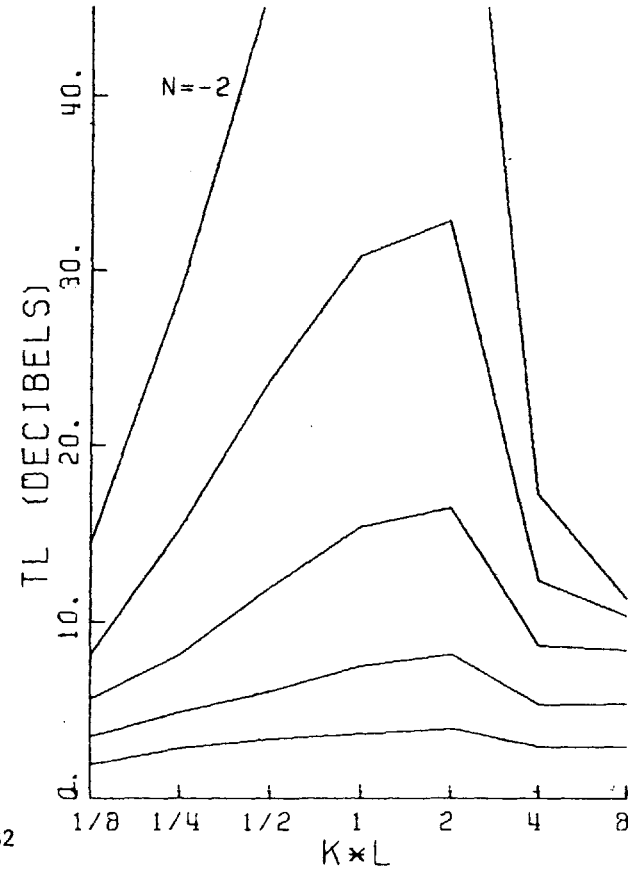


Figure A3.62

THETA=4.
D/L=1.094
AREA RATIO=1

S/D=16
8
4
2
1





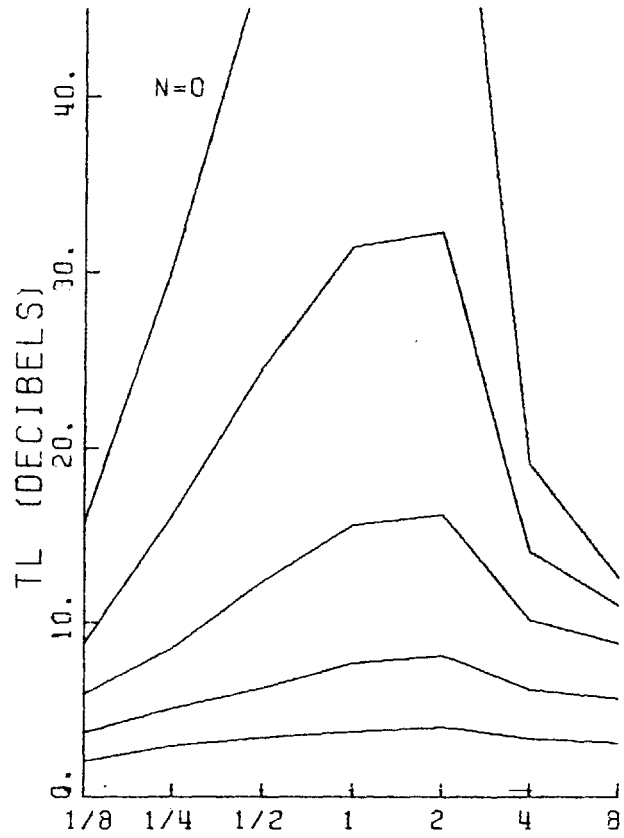
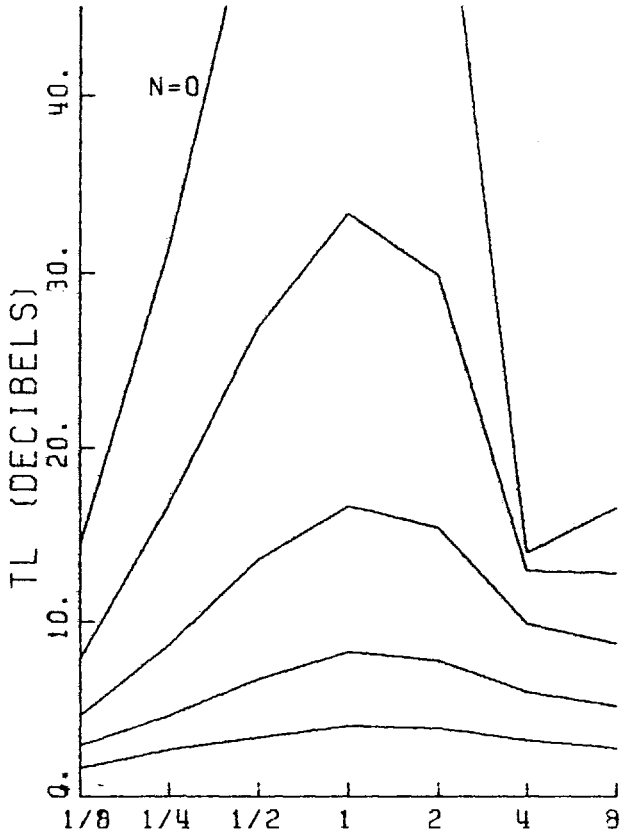
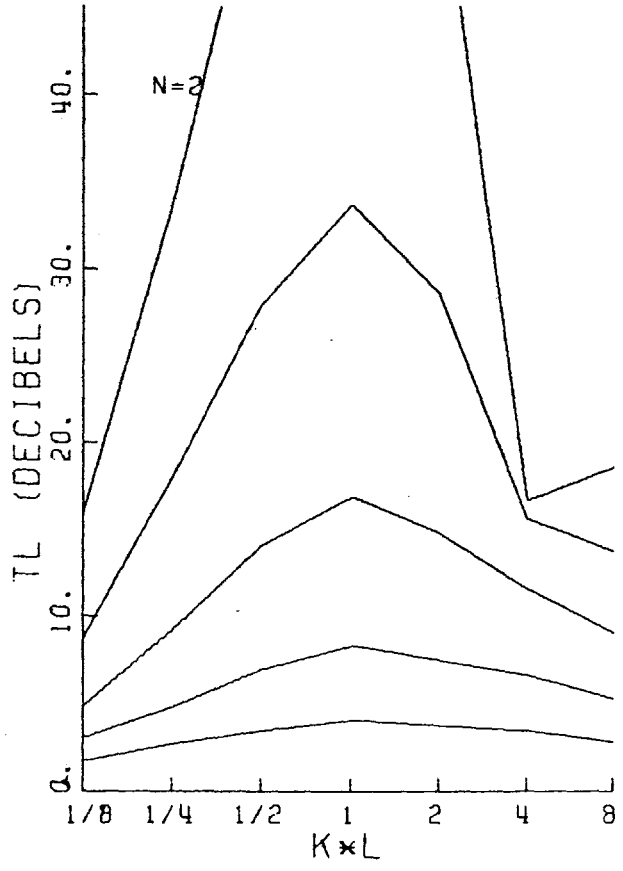


Figure A3.63

THETA=4.
D/L=2.000
AREA RATIO=1

S/D=16
8
4
2
1





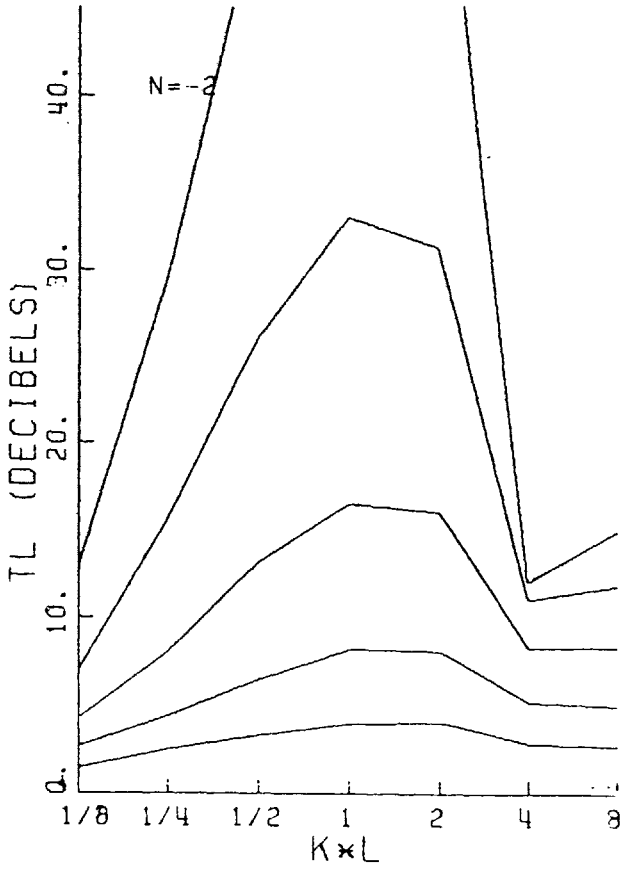
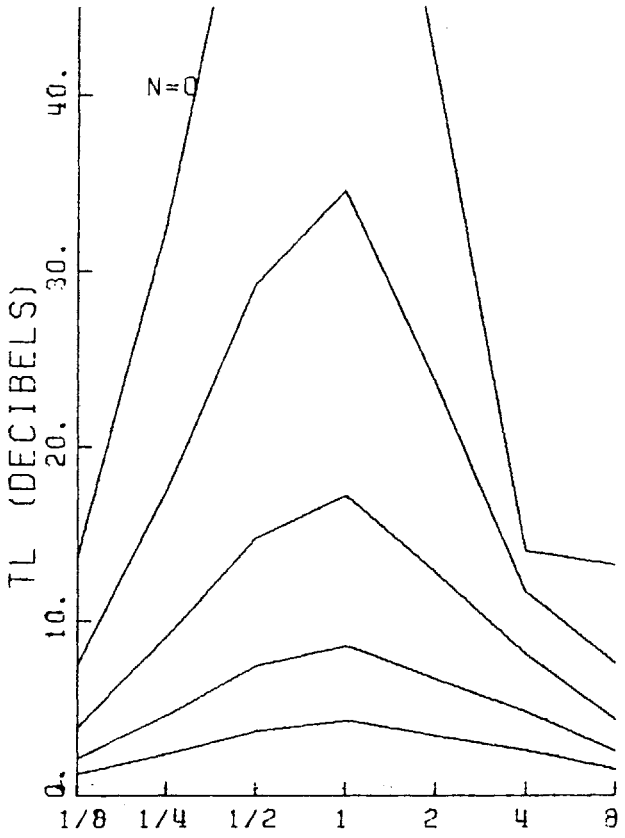
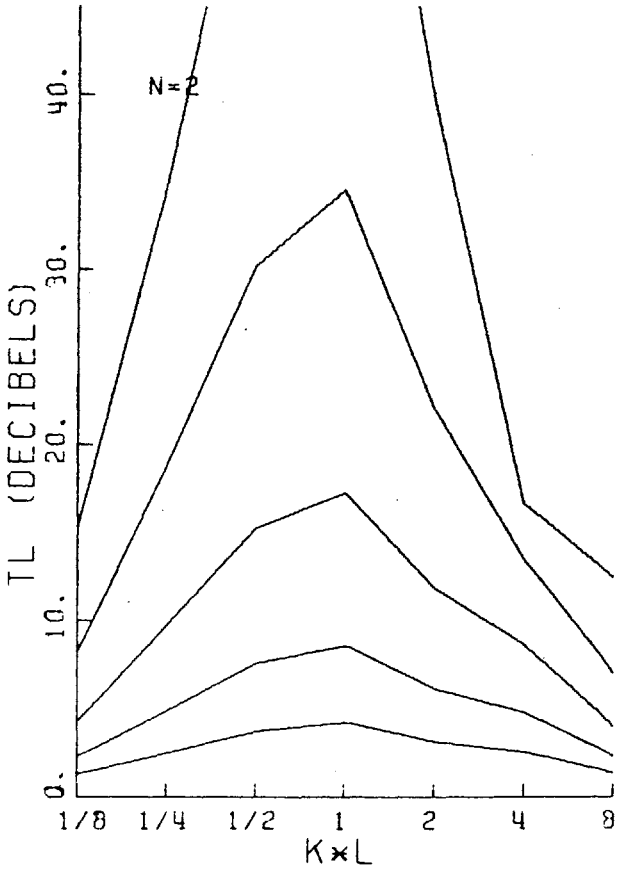


Figure A3.64

THETA=4.
D/L=4.828
AREA RATIO=1

S/D=16
8
4
2
1





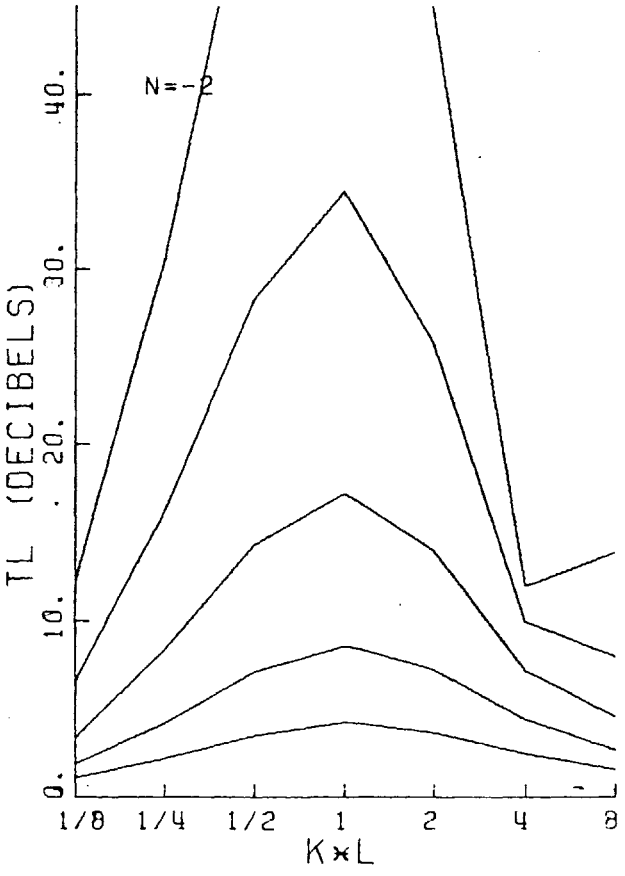
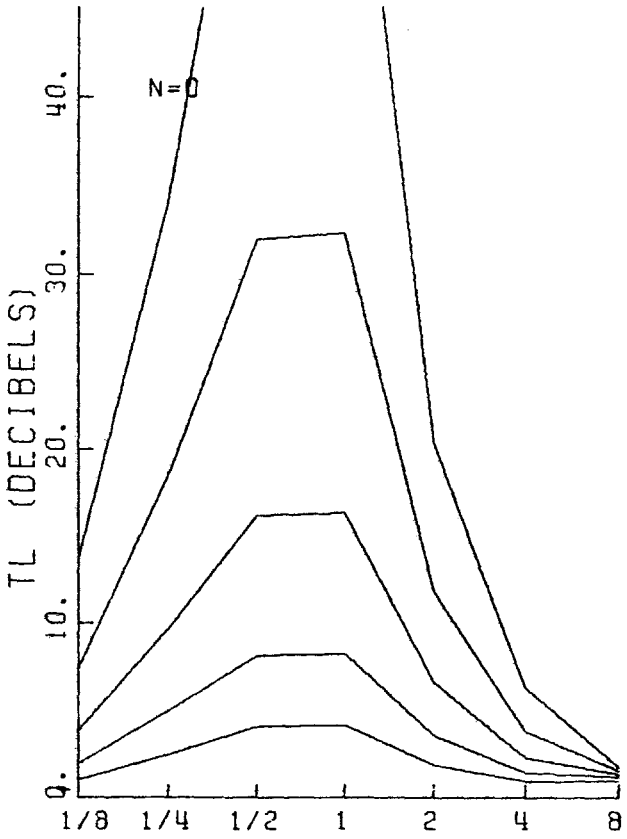
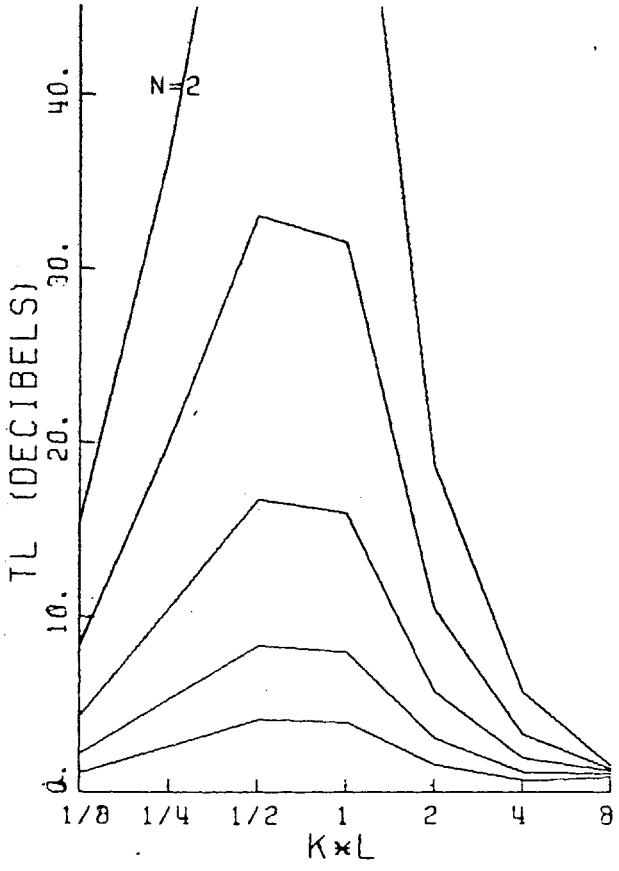


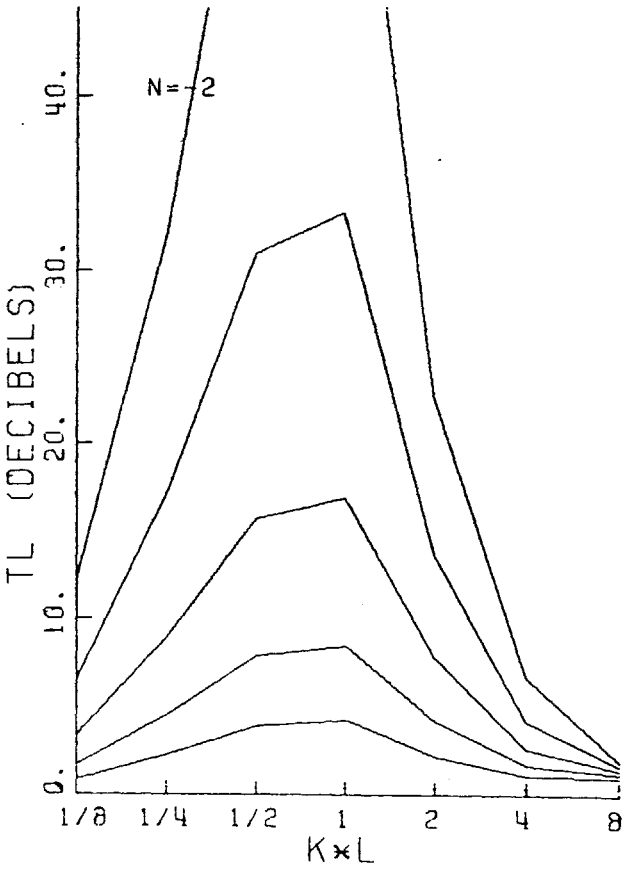
Figure A3.65

THETA=4.
D/L=12.928
AREA RATIO=1

S/D=16
8
4
2
1





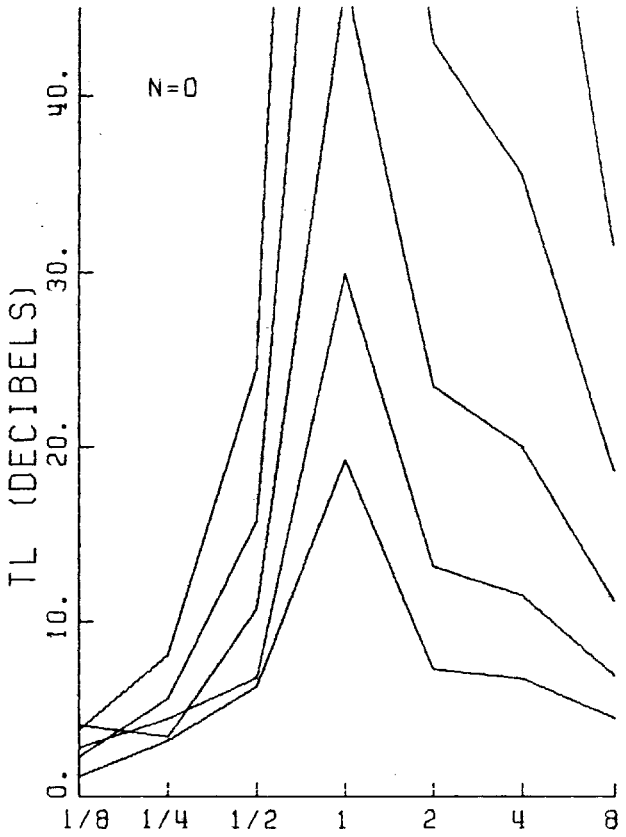


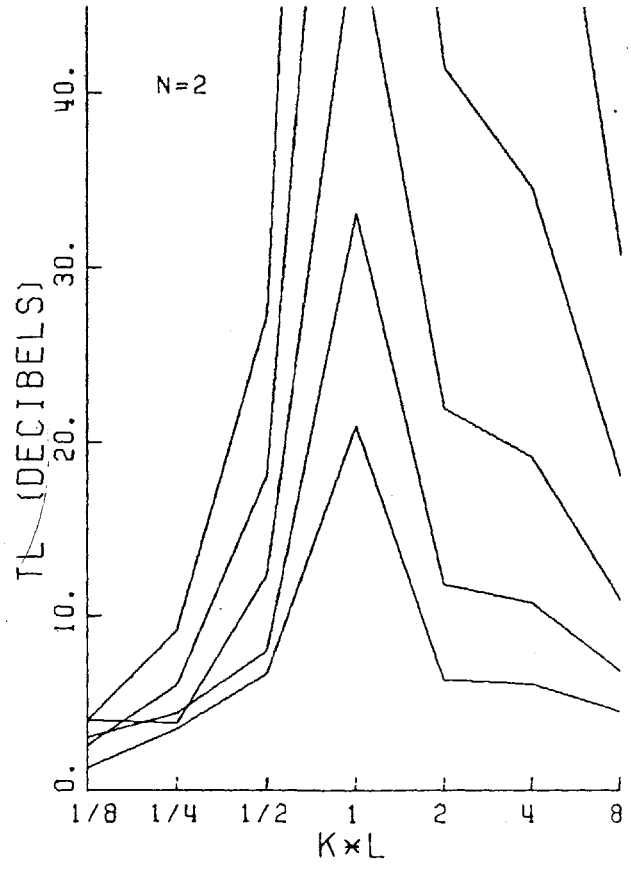
Figures A3.66 - A3.97: Octave band TL vs kL for a circular duct lined with a porous liner. The format is the same as in Figures A3.50 - A3.65.

Figure A3.66

THETA=0.5
D/L=1.094
AREA RATIO=1

S/D=16
8
4
2
1





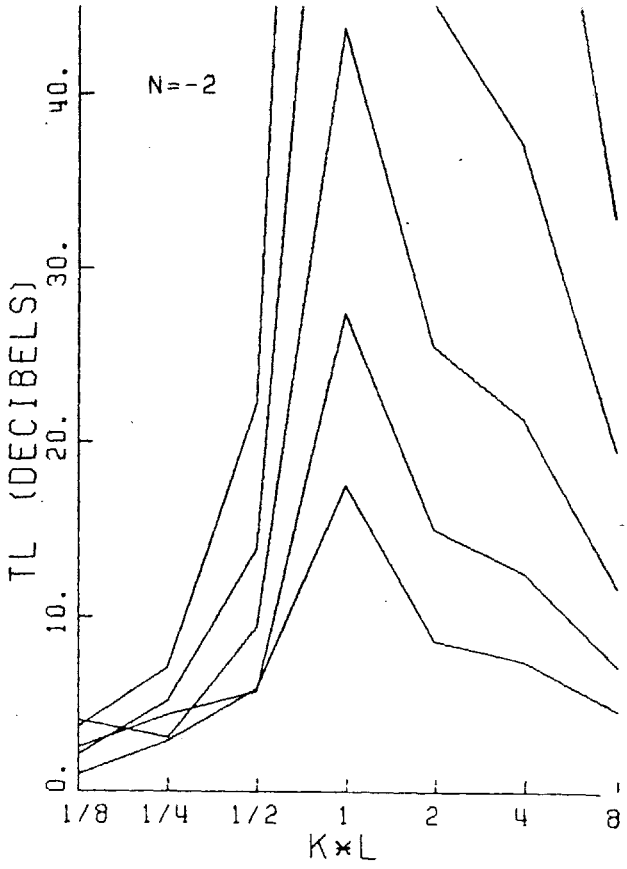
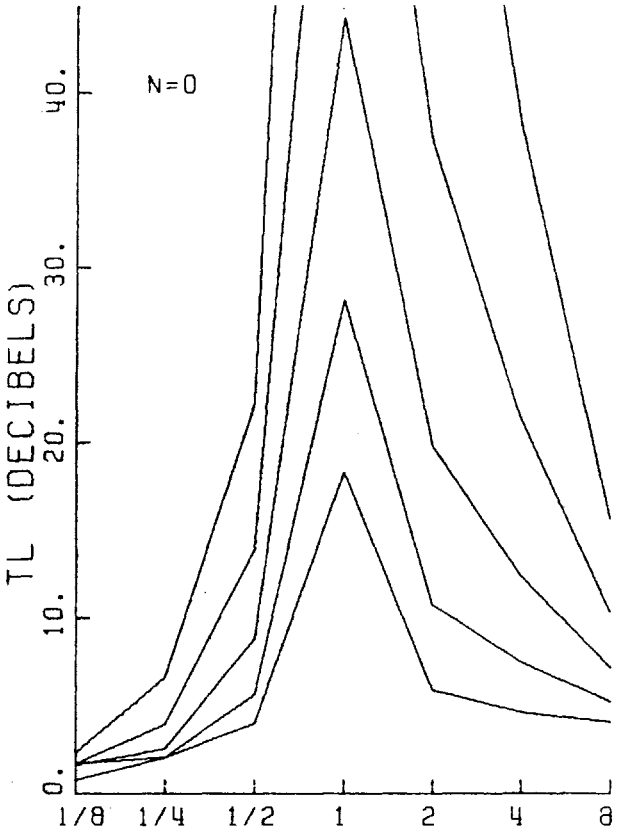
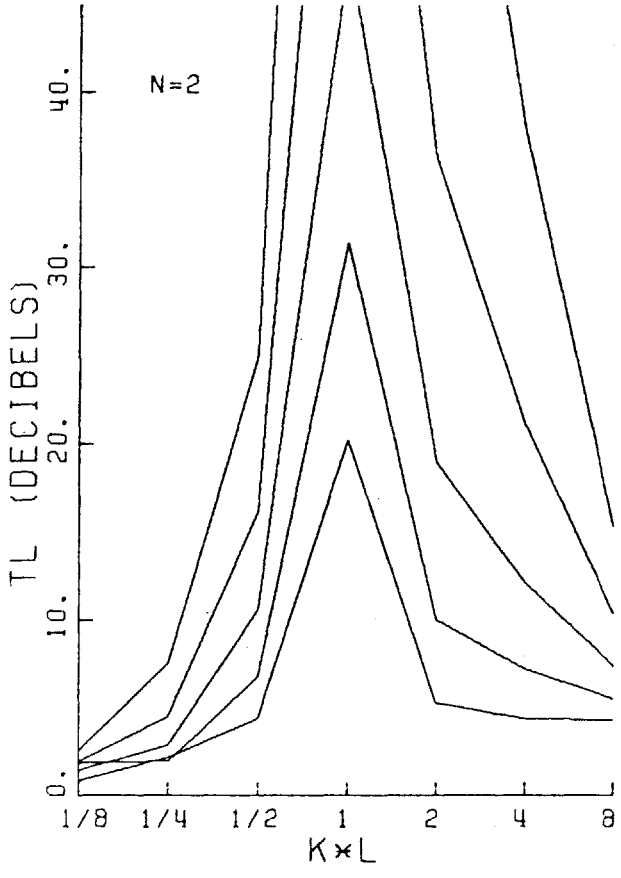


Figure A3.67

THETA=0.5
D/L=2.000
AREA RATIO=1

S/D=16
8
4
2
1





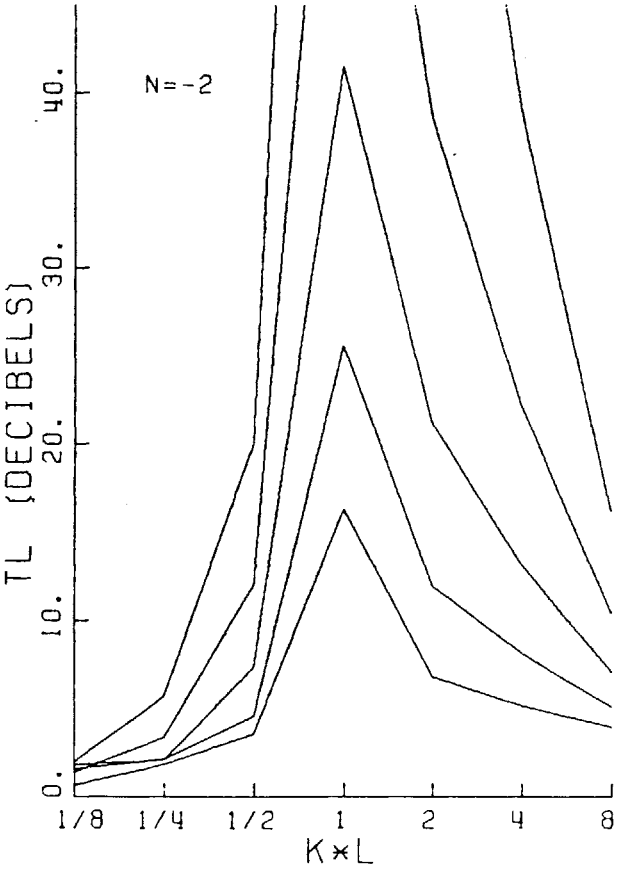
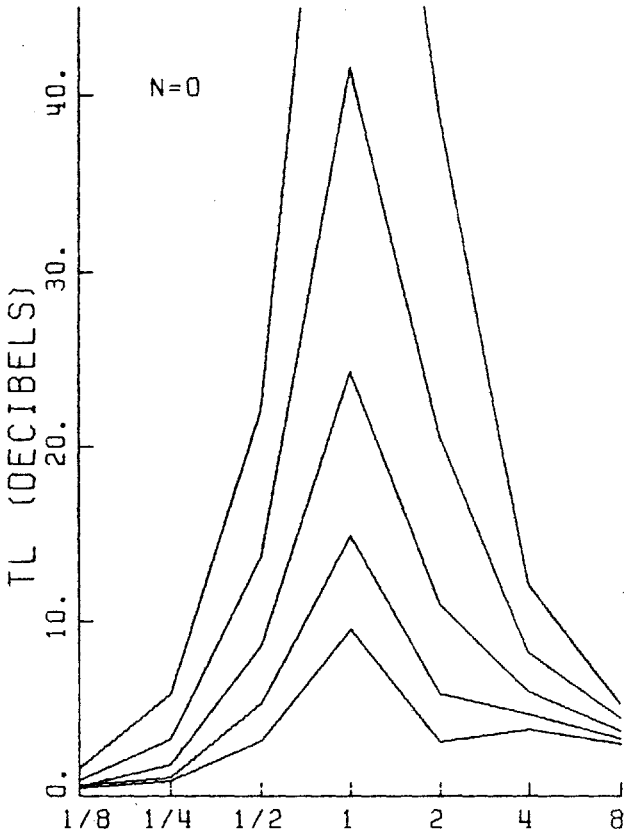
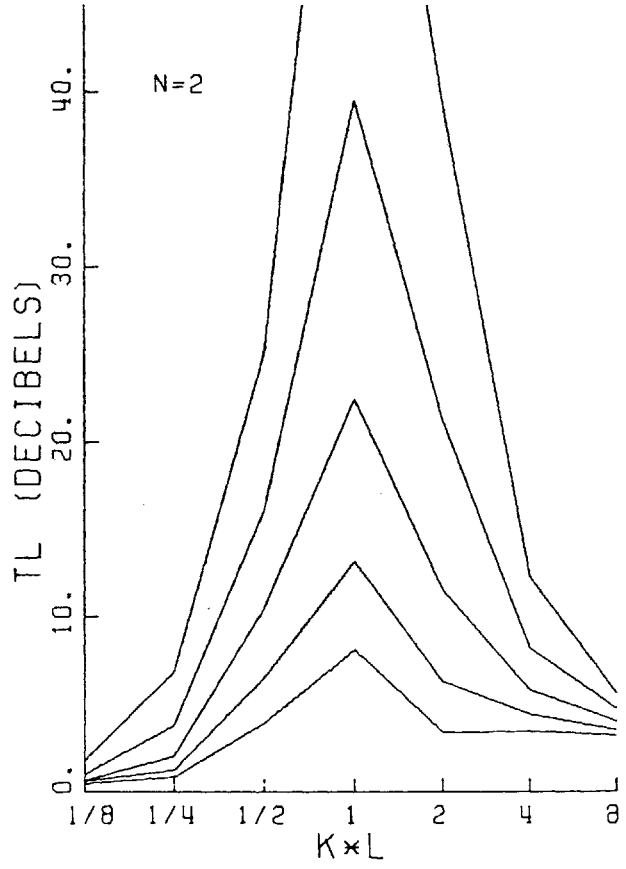


Figure A3.68

THETA=0.5
D/L=4.828
AREA RATIO=1

S/D=16
8
4
2
1





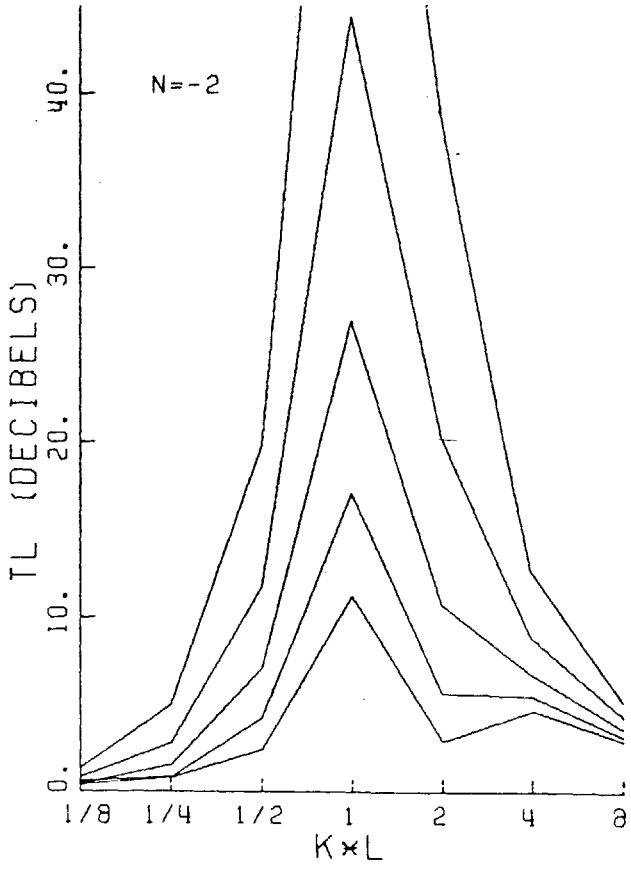
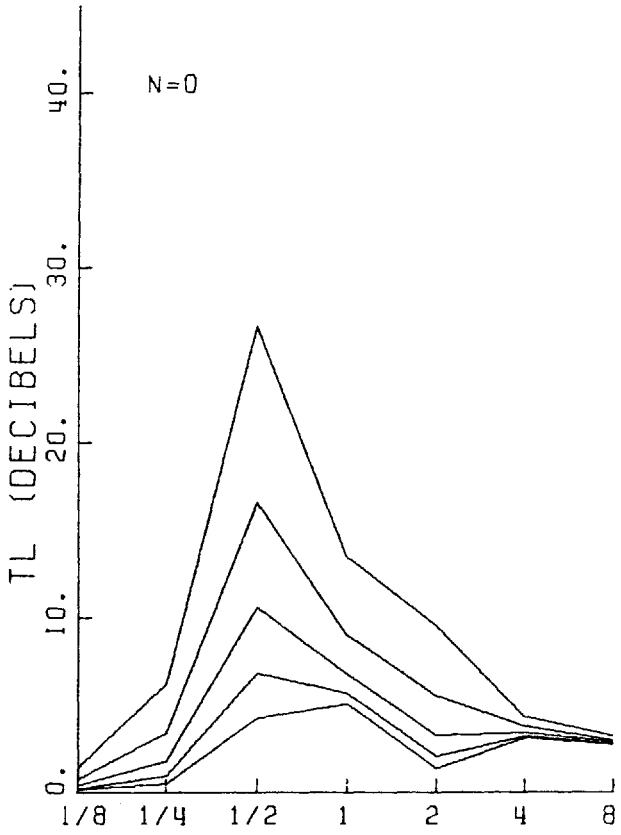
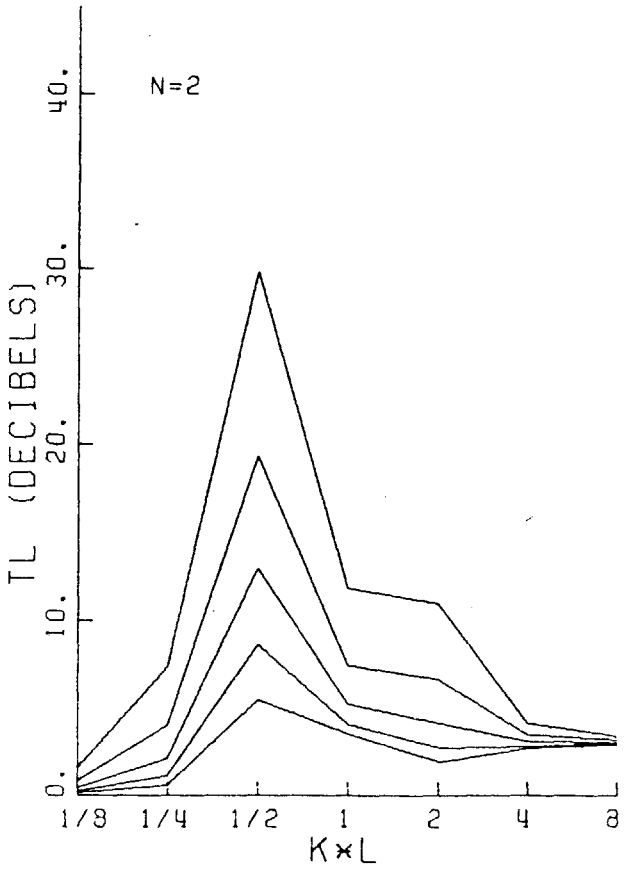


Figure A3.69

THETA=0.5
D/L=12.928
AREA RATIO=1

S/D=16
8
4
2
1





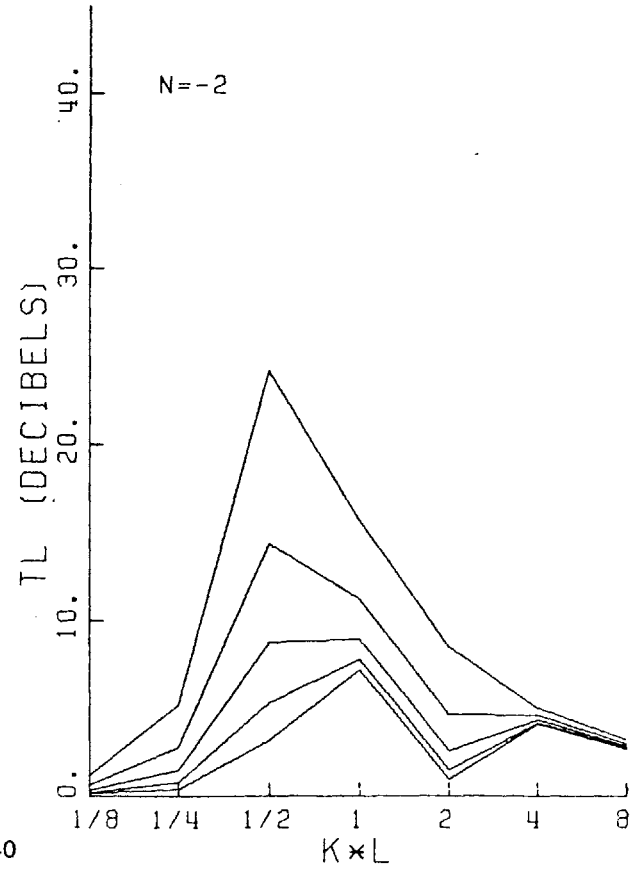
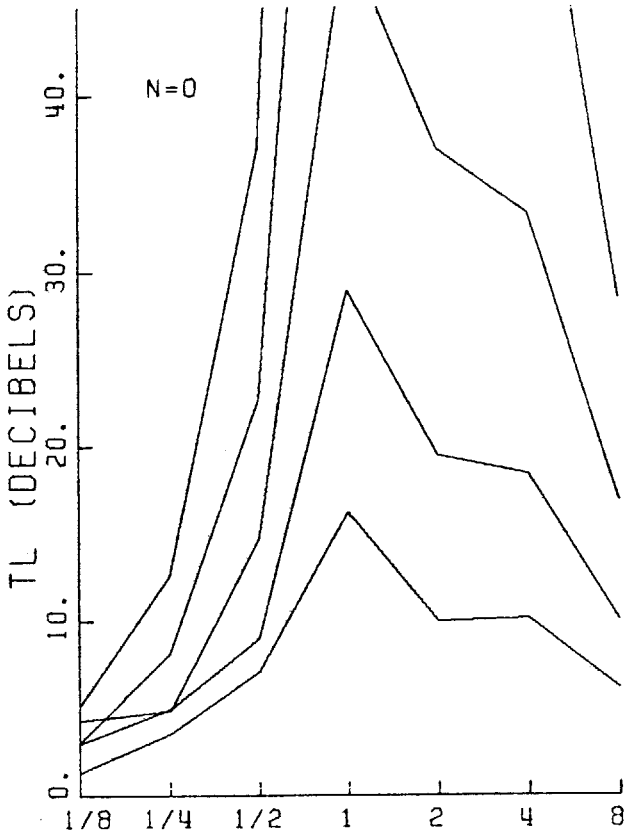
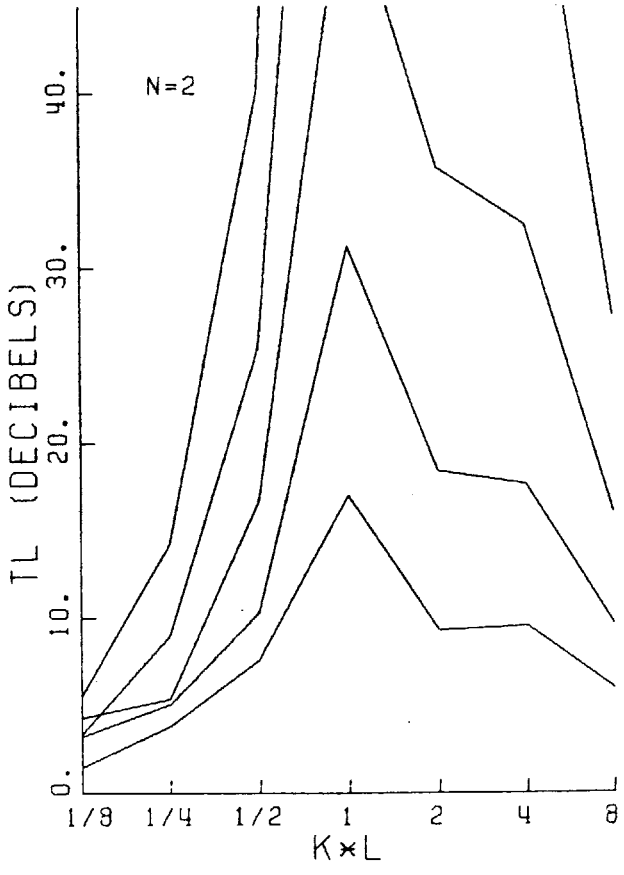


Figure A3.70

THETA=1.
D/L=1.094
AREA RATIO=1

S/D=16
8
4
2
1





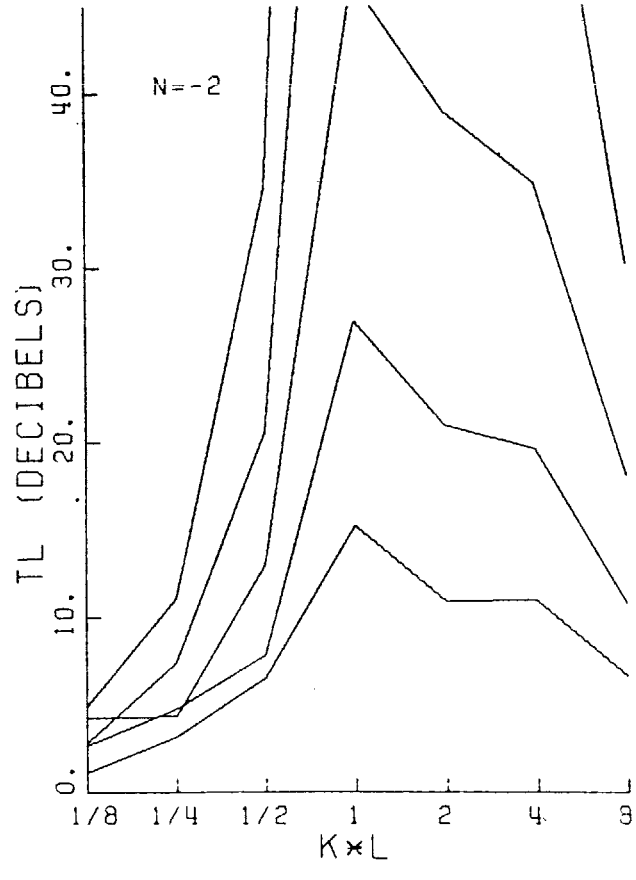
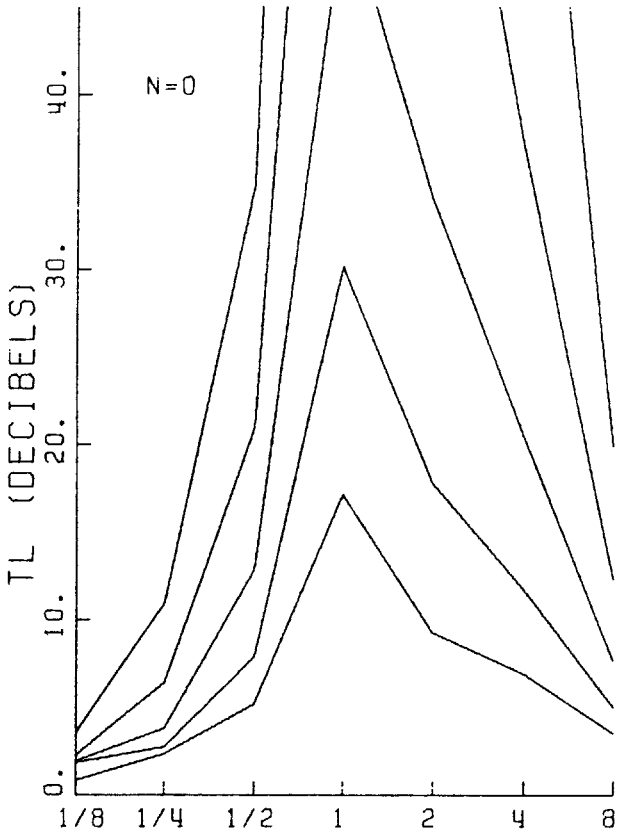
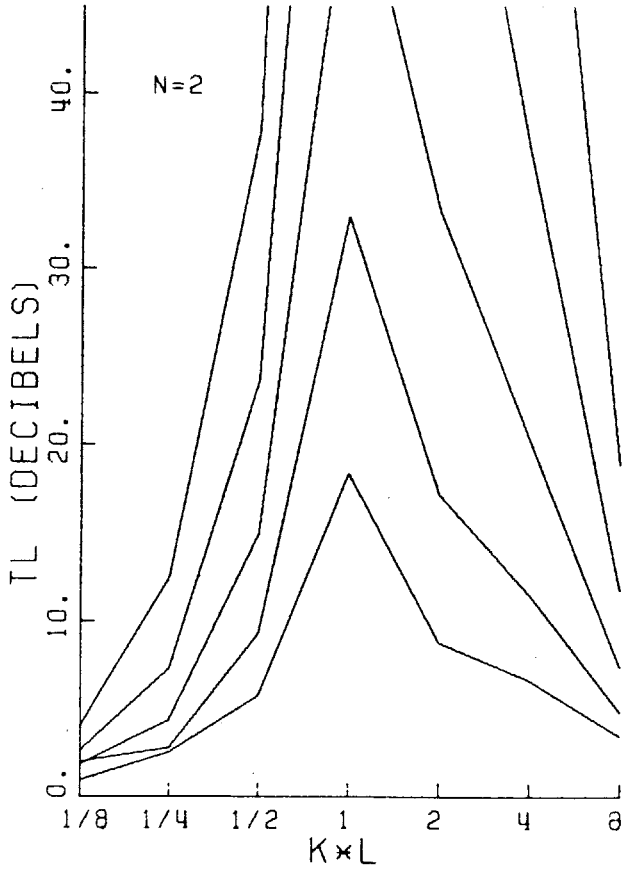


Figure A3.71

THETA=1.
D/L=2.000
AREA RATIO=1

S/D=16
8
4
2
1





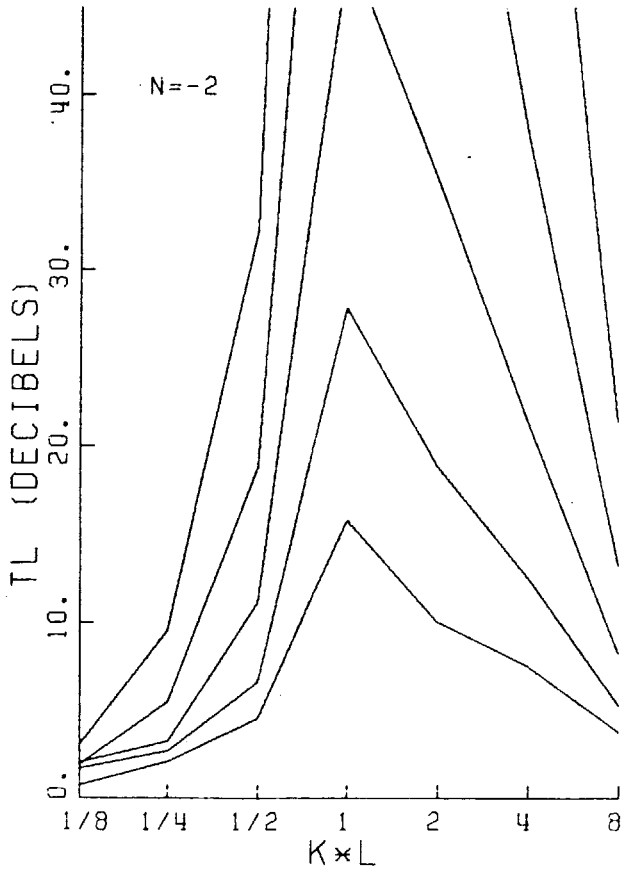
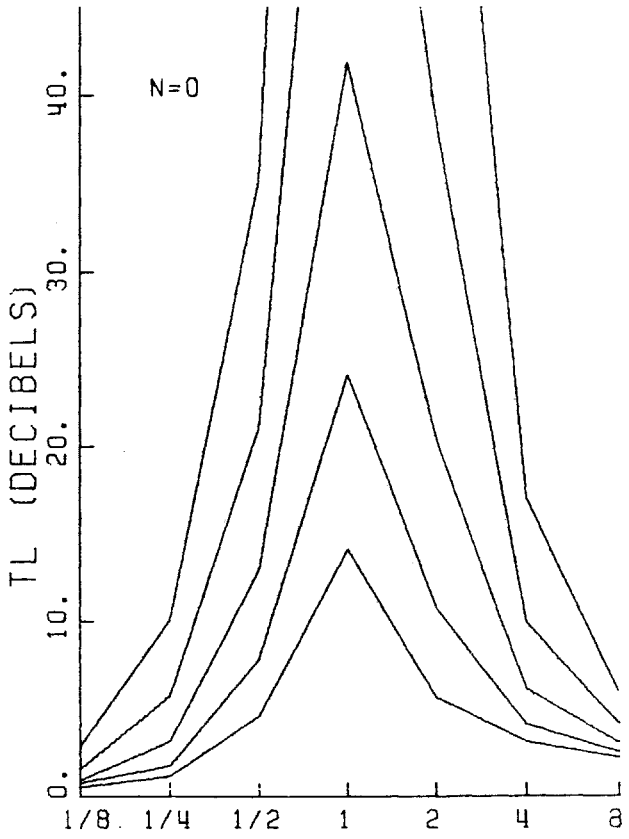
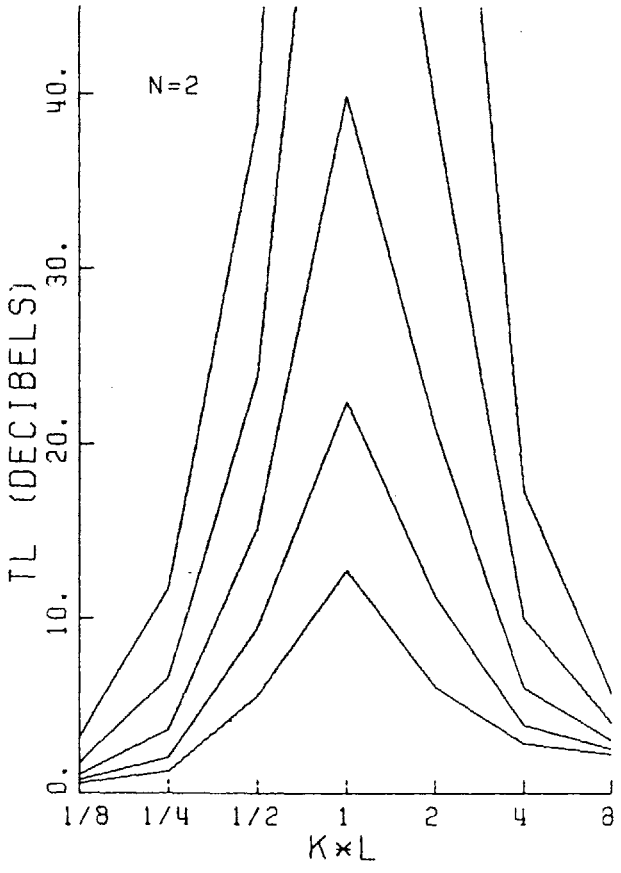


Figure A3.72

THETA=1.
D/L=4.828
AREA RATIO=1

S/D=16
8
4
2
1





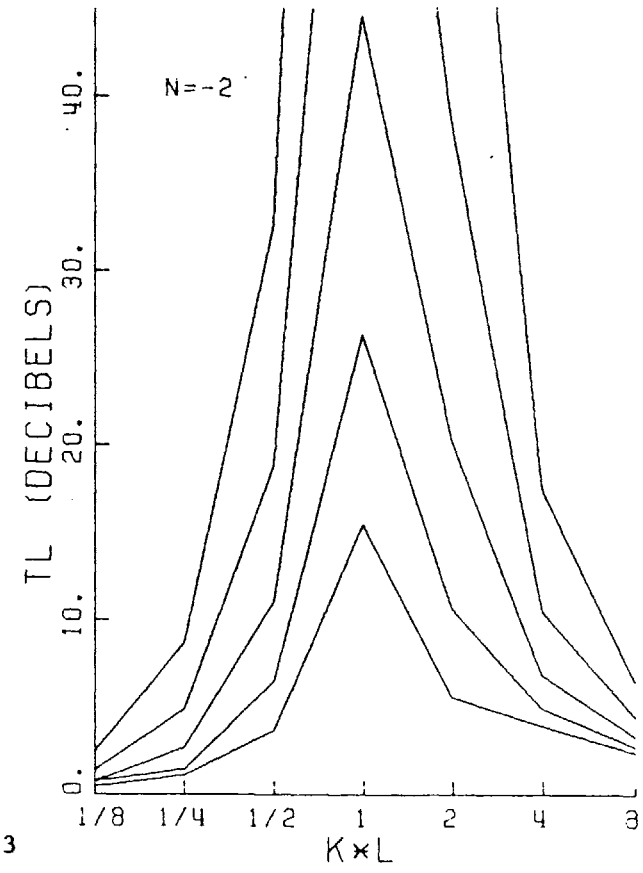
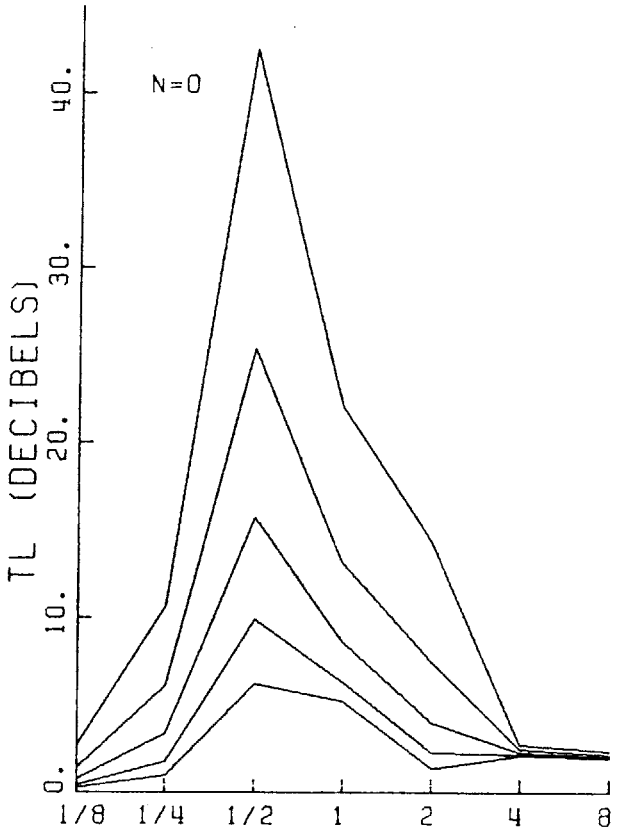
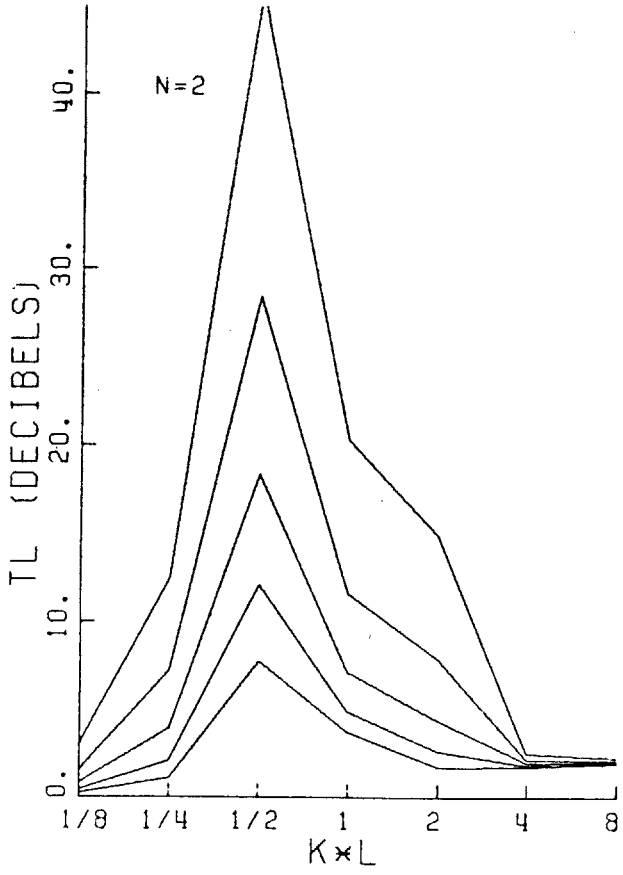


Figure A3.73

THETA=1.
D/L=12.928
AREA RATIO=1

S/D=16
8
4
2
1





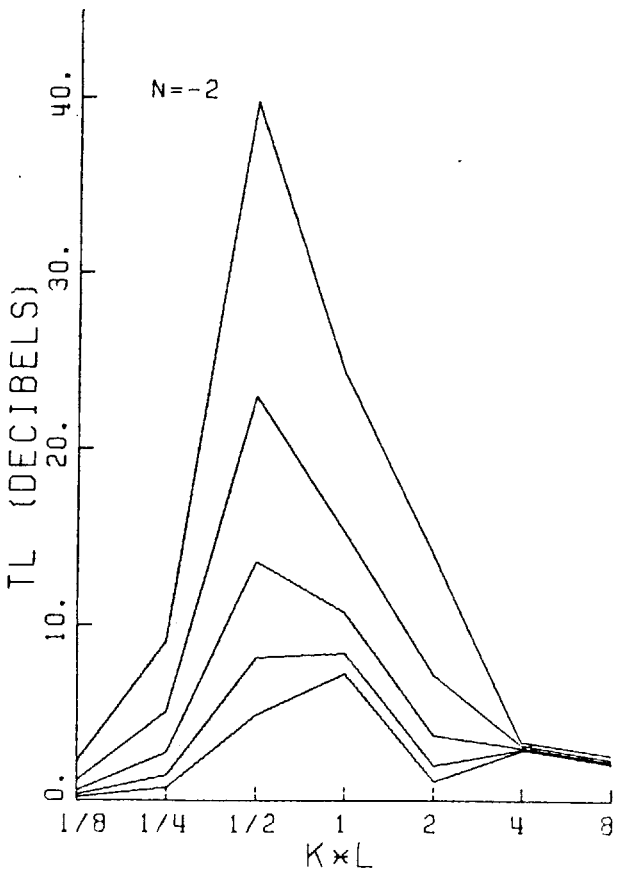
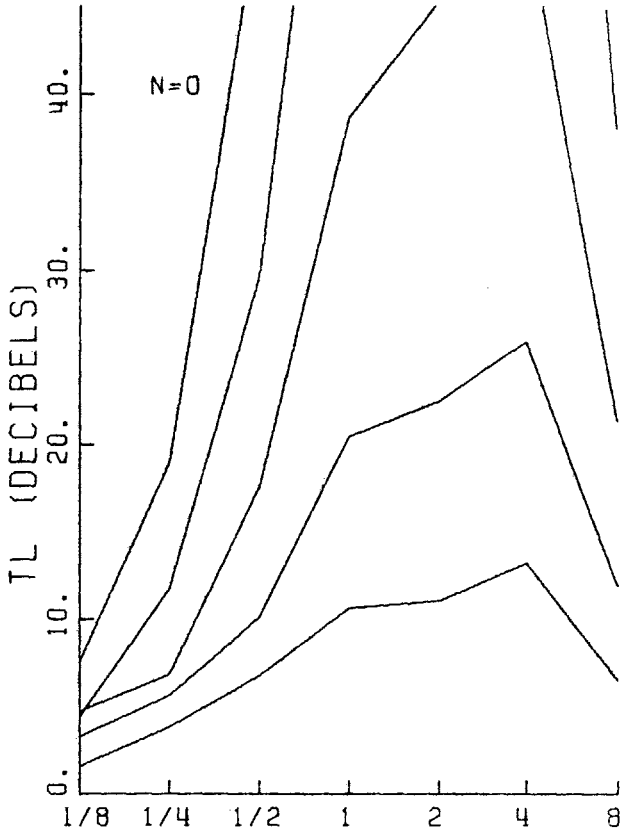


Figure A3.74

THETA=2.
D/L=1.094
AREA RATIO=1

S/D=16
8
4
2
1



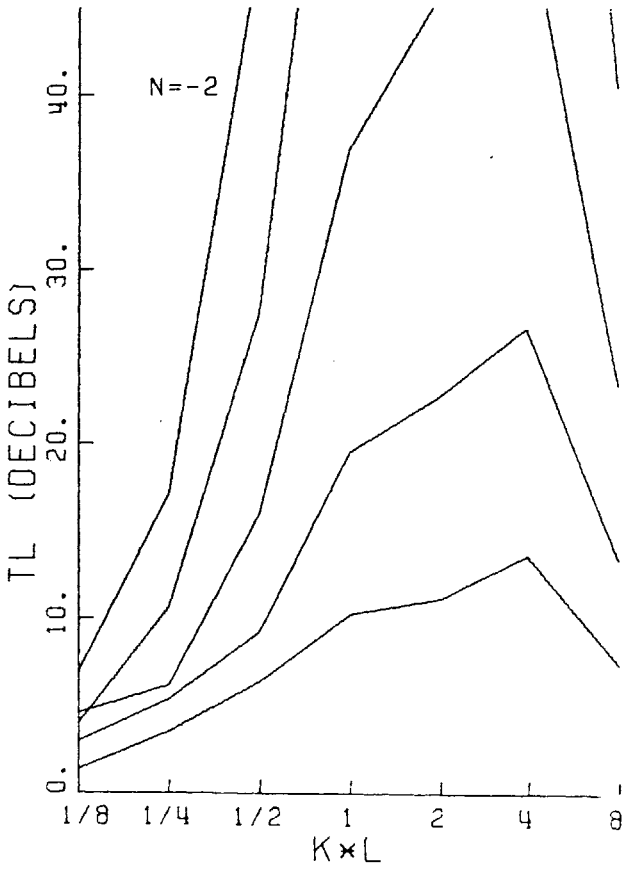
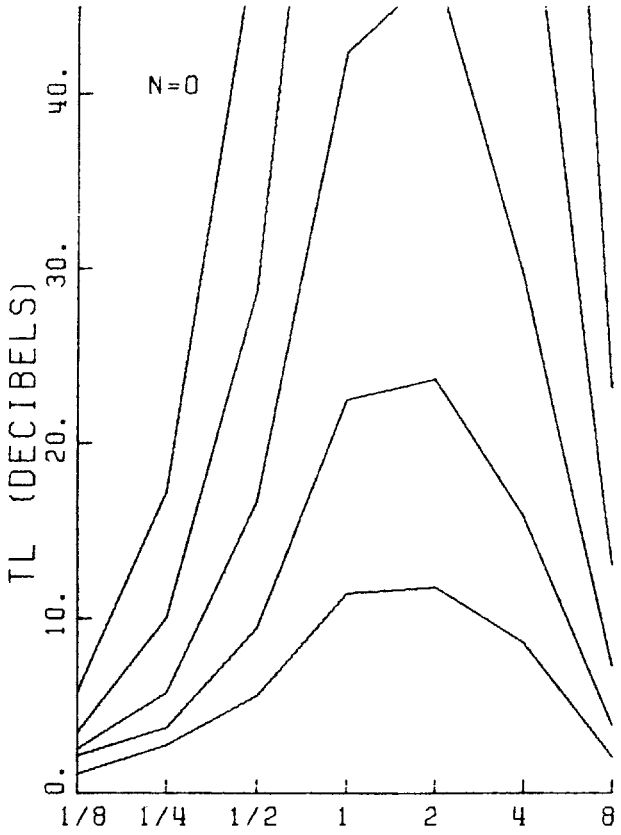
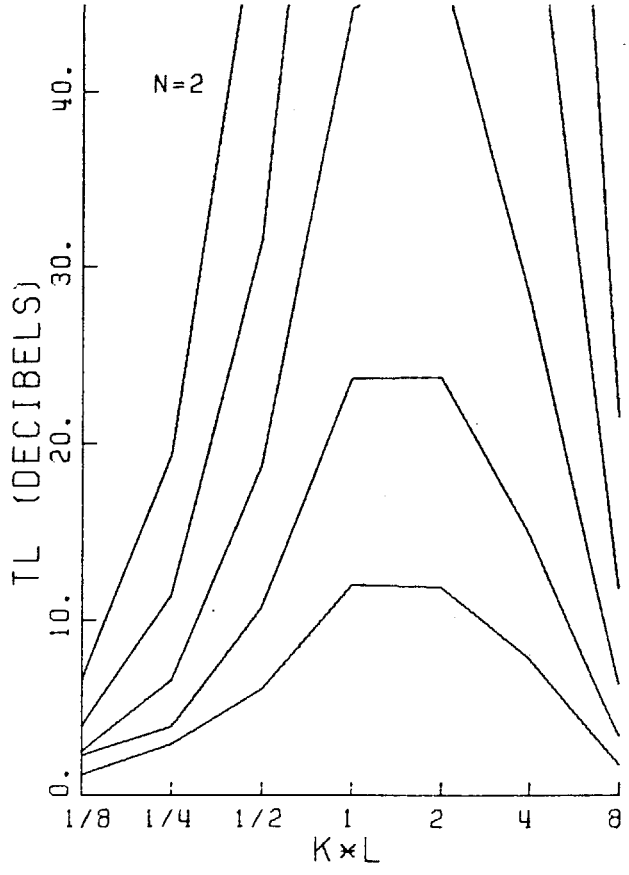


Figure A3.75

THETA=2.
D/L=2.000
AREA RATIO=1

S/D=16
8
4
2
1





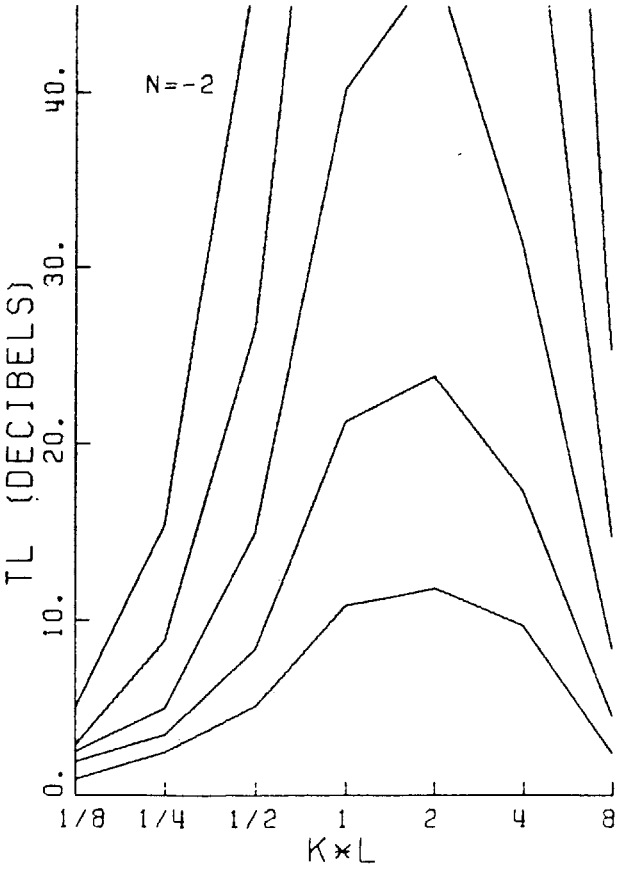
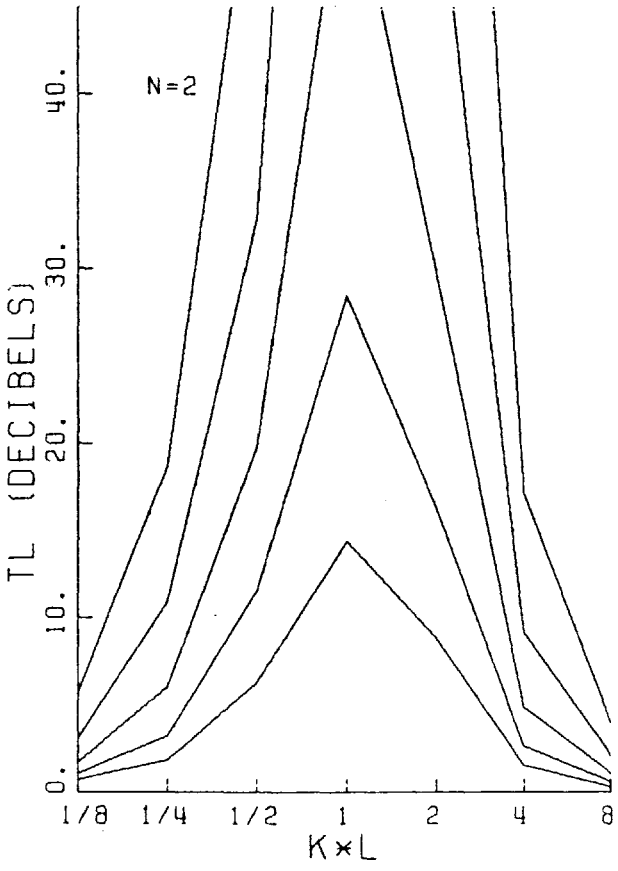


Figure A3.76

THETA=2.
D/L=4.828
AREA RATIO=1

S/D=16
8
4
2
1



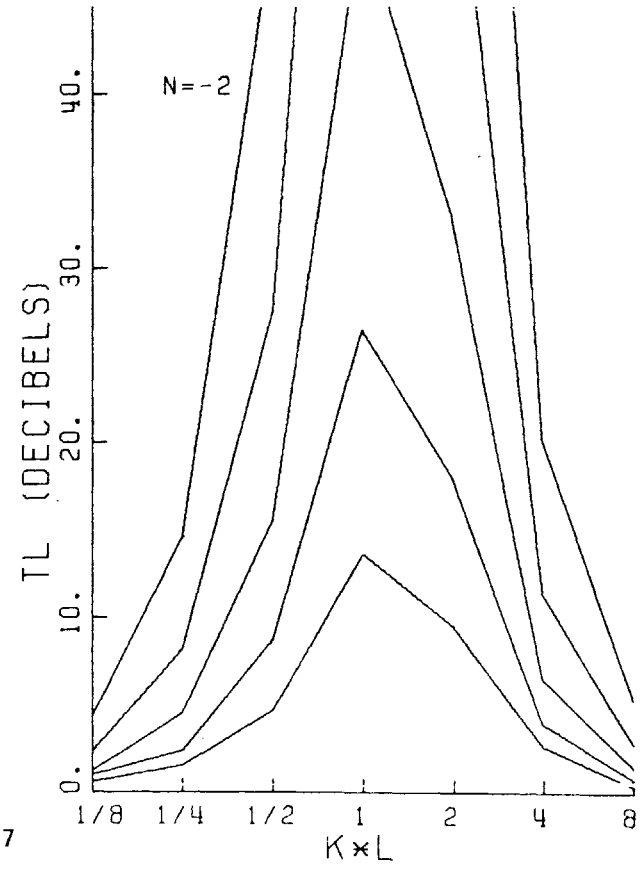
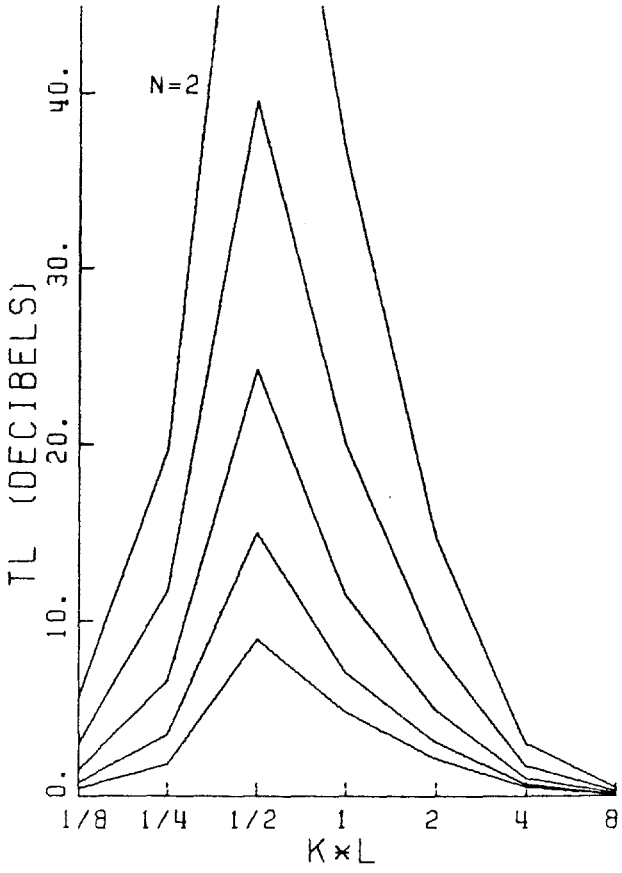
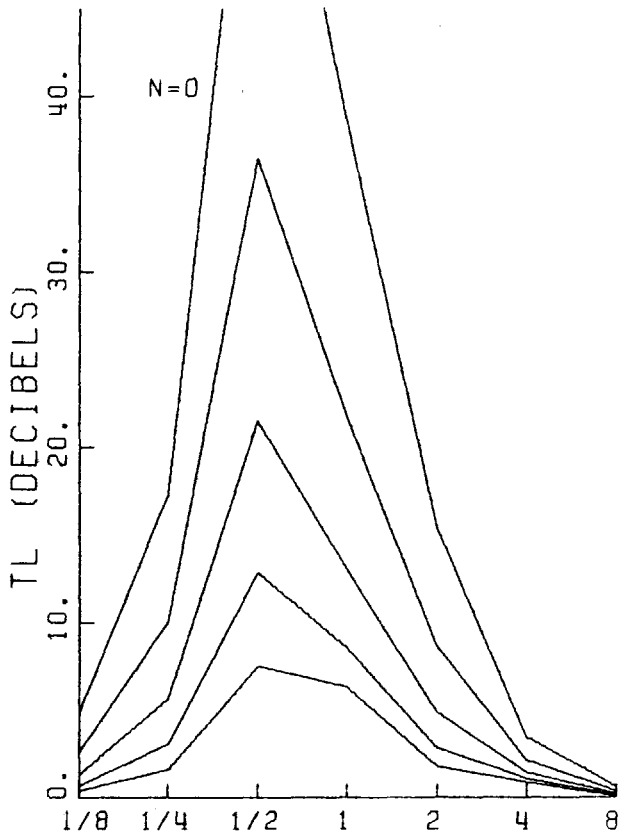


Figure A3.77

THETA=2.
D/L=12.928
AREA RATIO=1

S/D=16
8
4
2
1





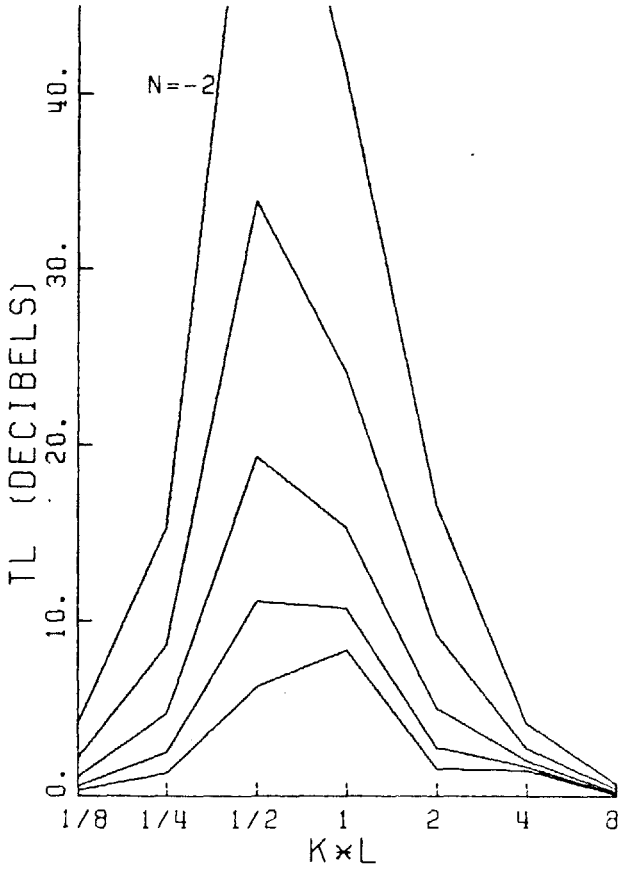
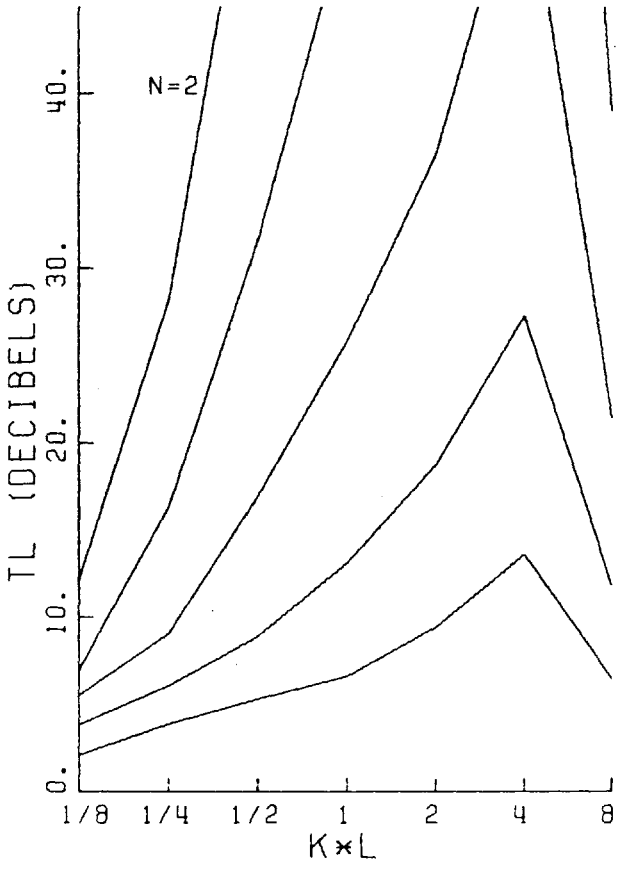
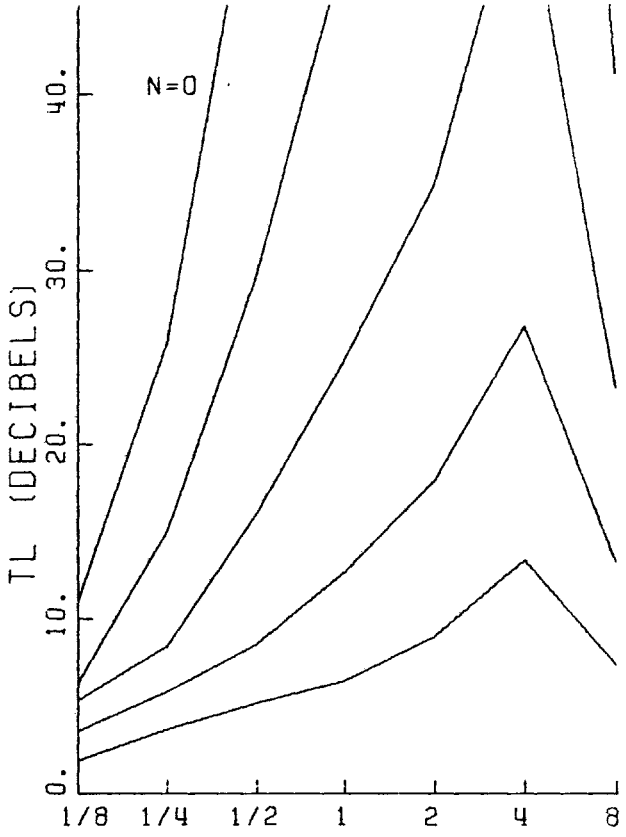


Figure A3.78

THETA=4.
D/L=1.094
AREA RATIO=1

S/D=16
8
4
2
1





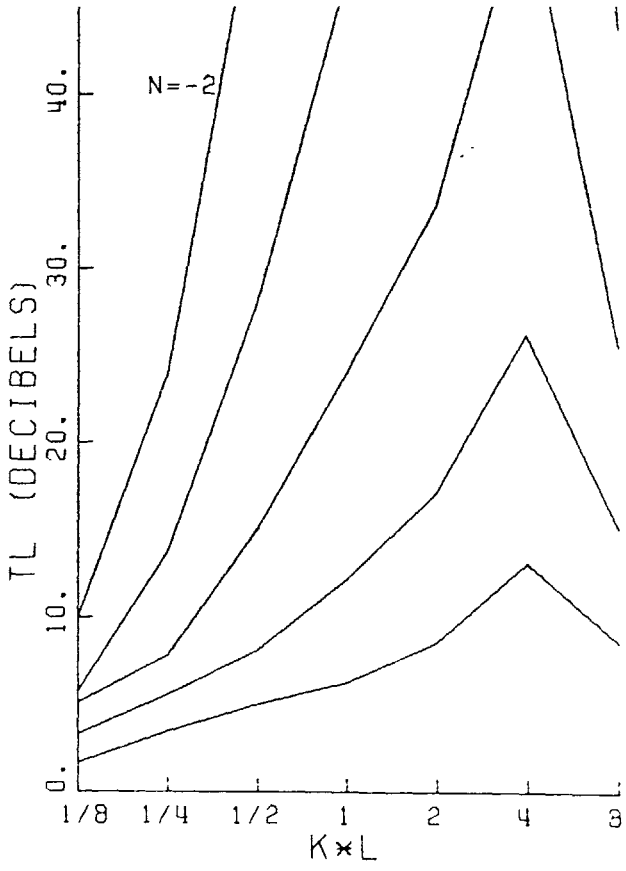
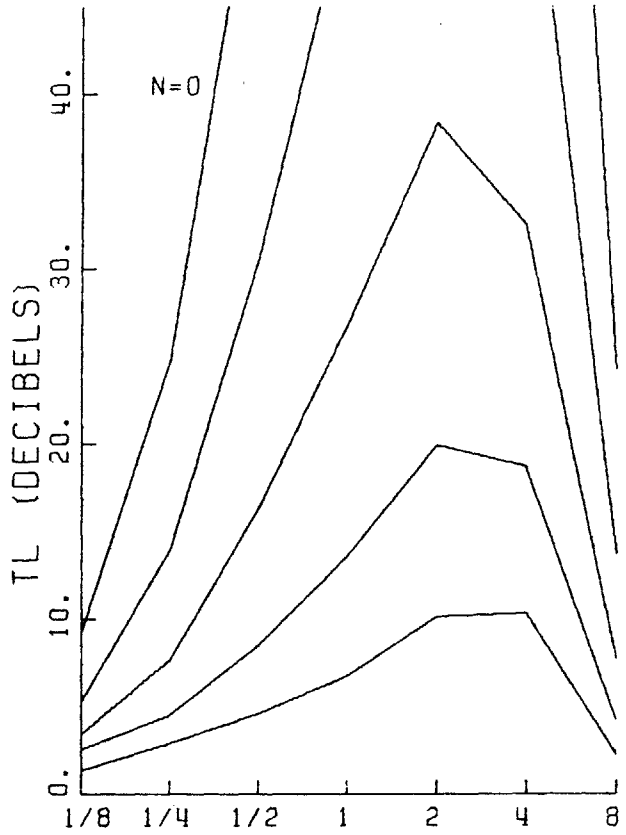
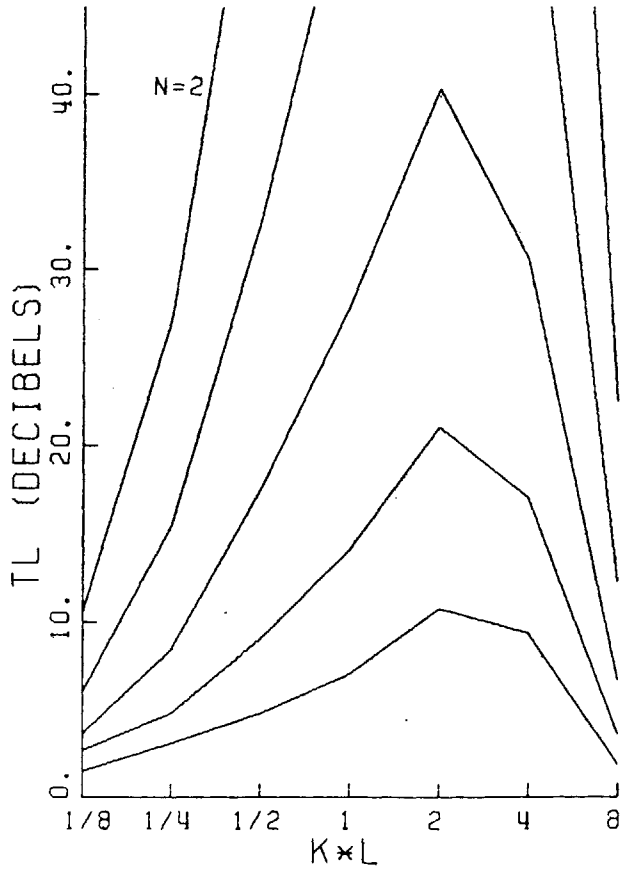


Figure A3.79

THETA=4.
D/L=2.000
AREA RATIO=1

S/D=16
8
4
2
1





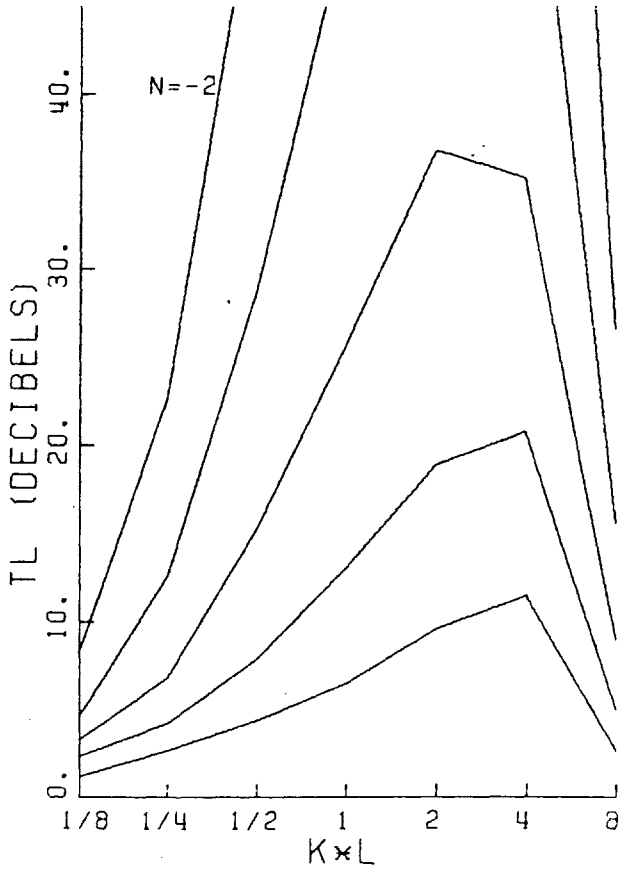
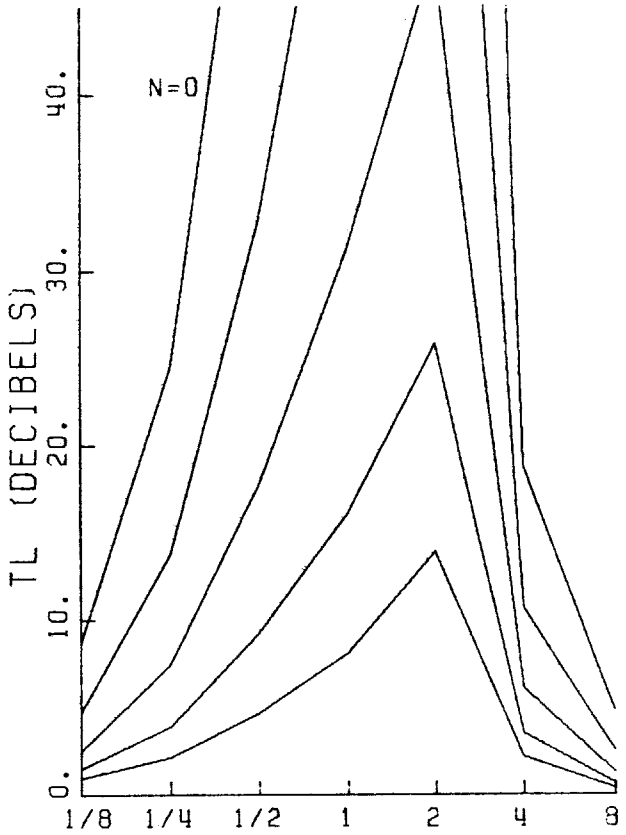
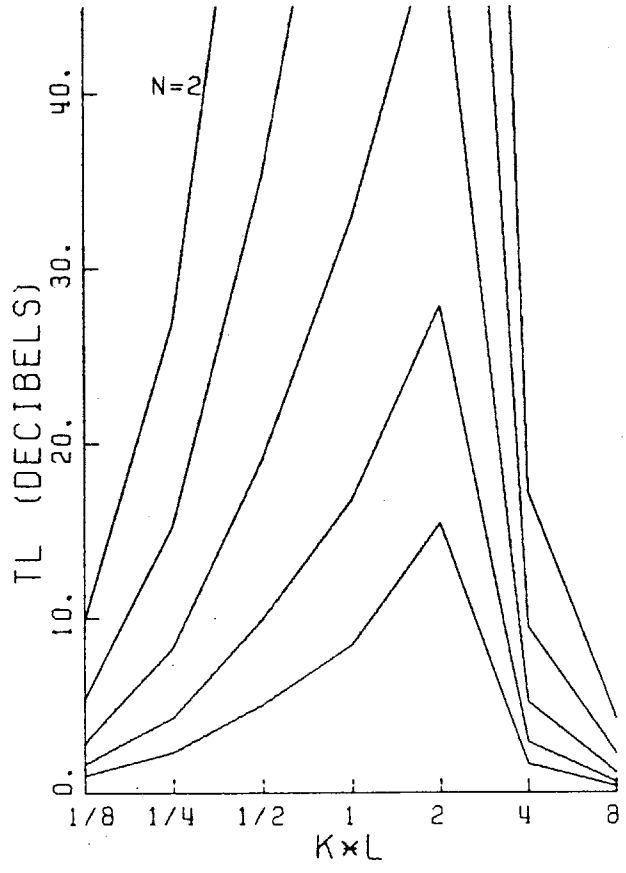


Figure A3.80

THETA=4.
D/L=4.828
AREA RATIO=1

S/D=16
8
4
2
1





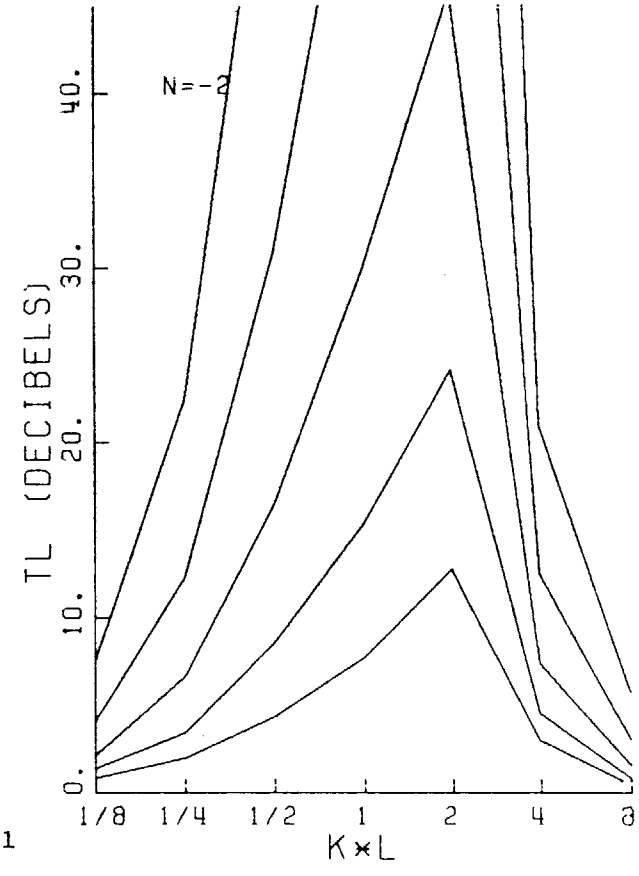
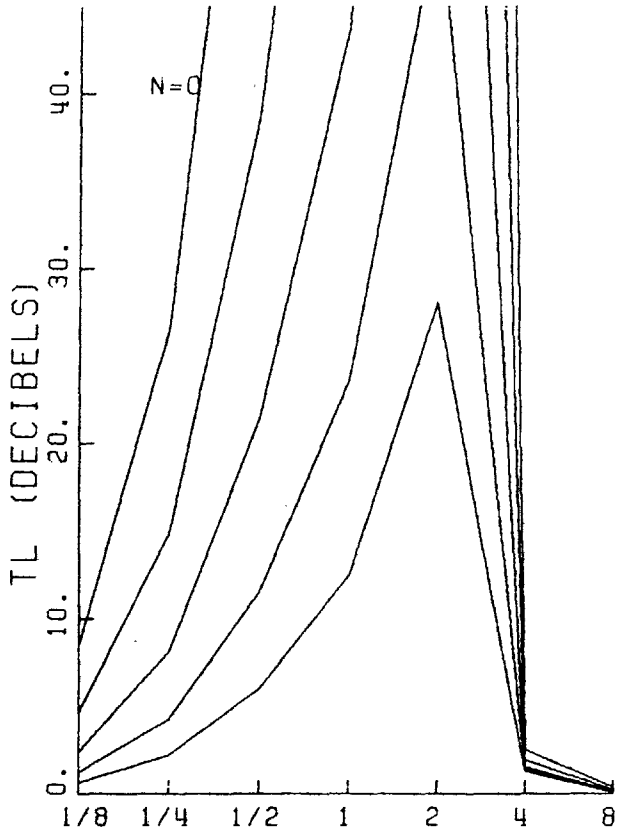
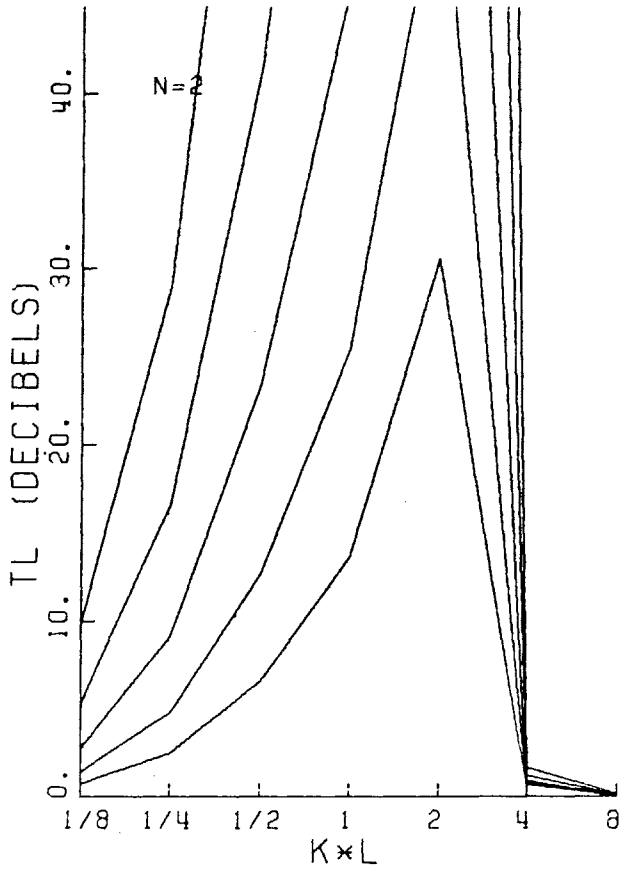


Figure A3.81

THETA=4.
D/L=12.928
AREA RATIO=1

S/D=16
8
4
2
1





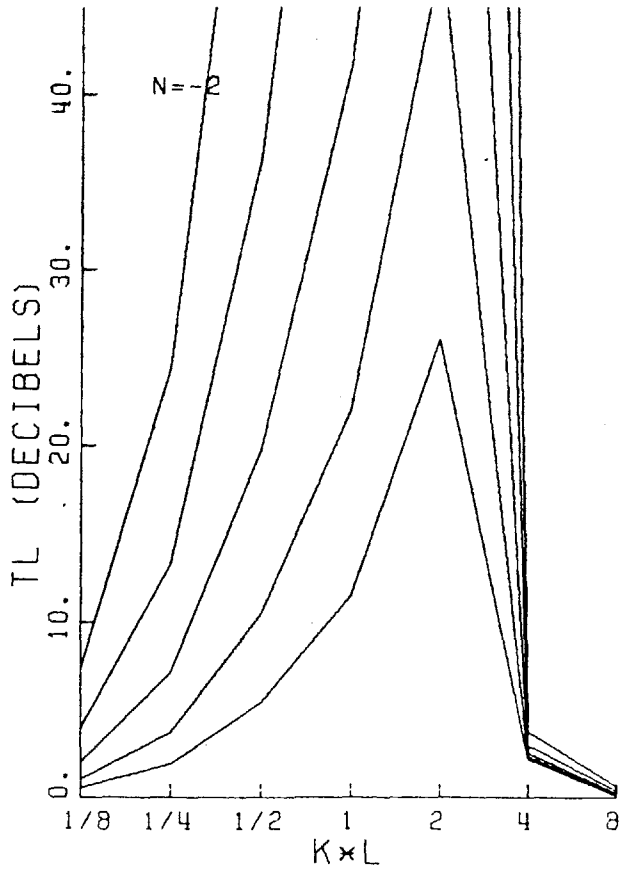
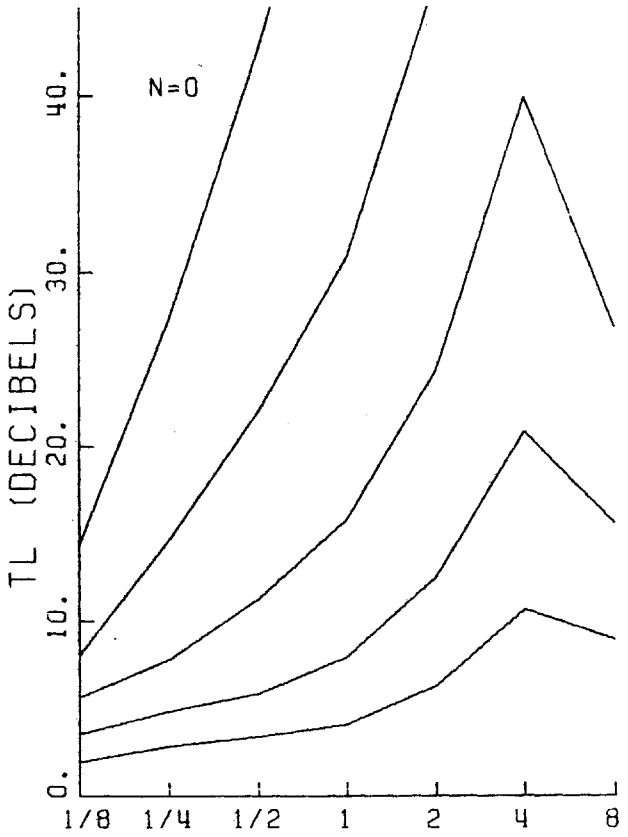
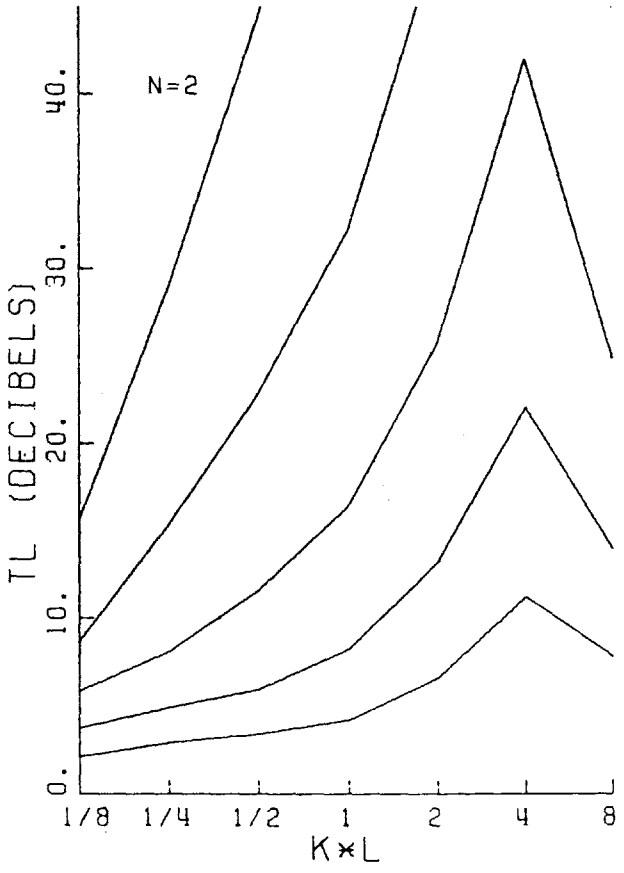


Figure A3.82

THETA=8.
D/L=1.094
AREA RATIO=1

S/D=16
8
4
2
1





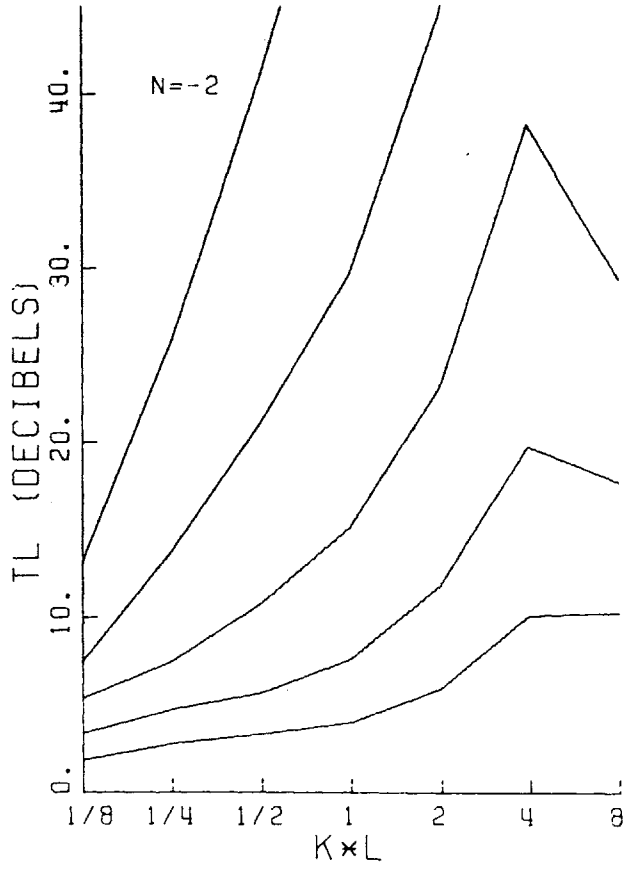
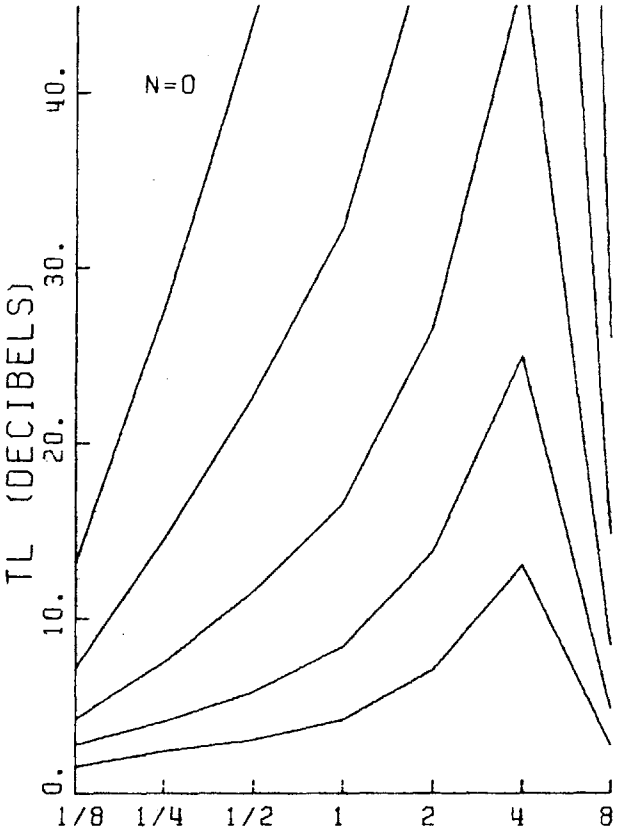
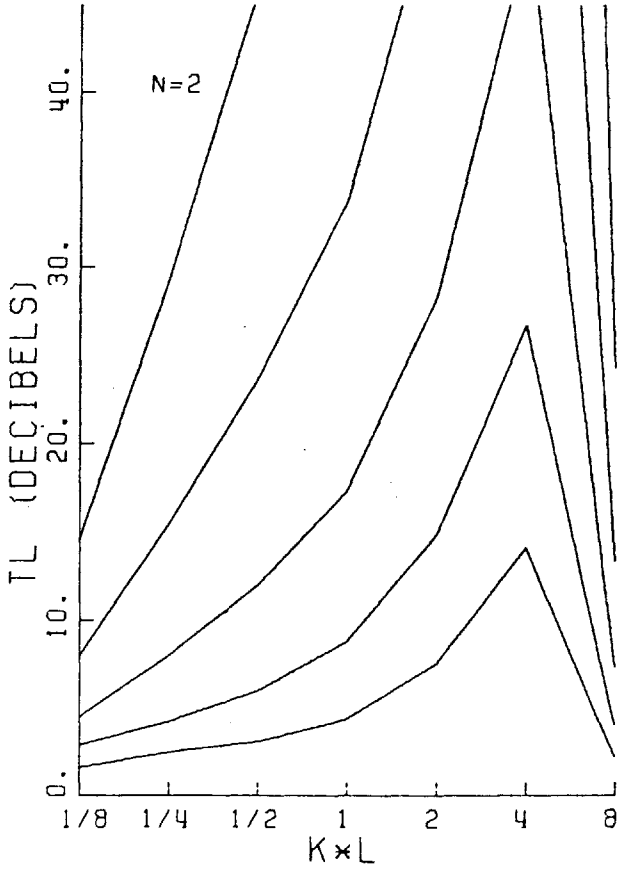


Figure A3.83

THETA=8.
D/L=2.000
AREA RATIO=1

S/D=16
8
4
2
1





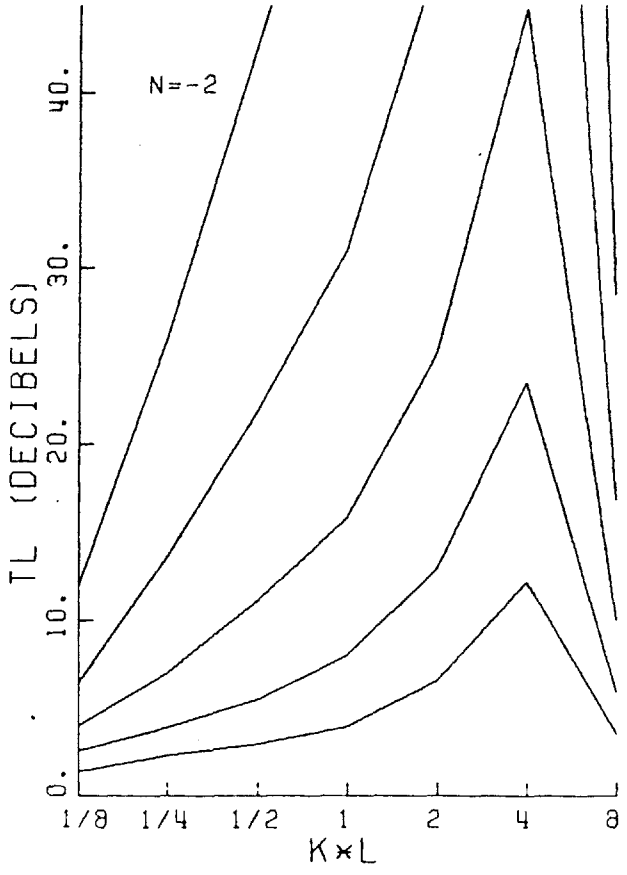
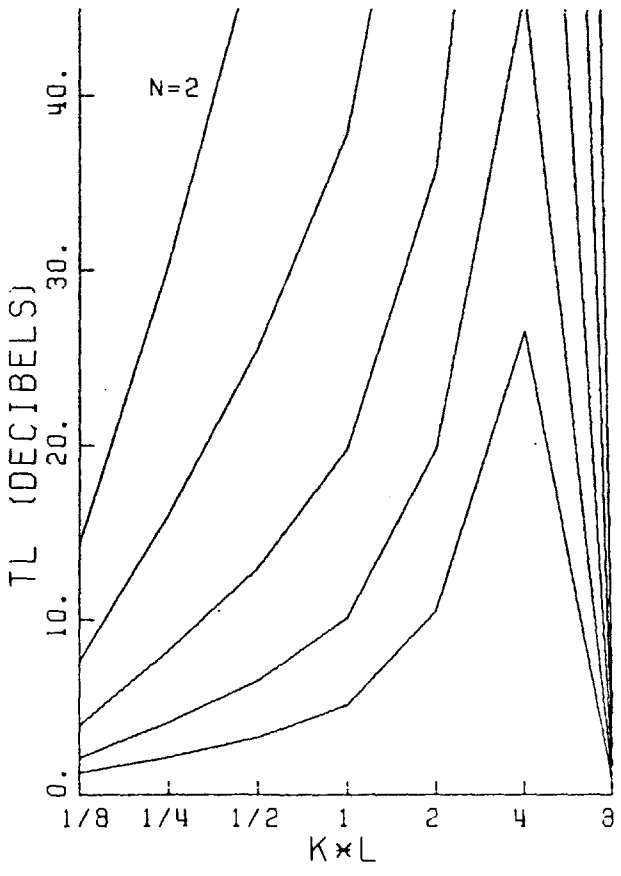
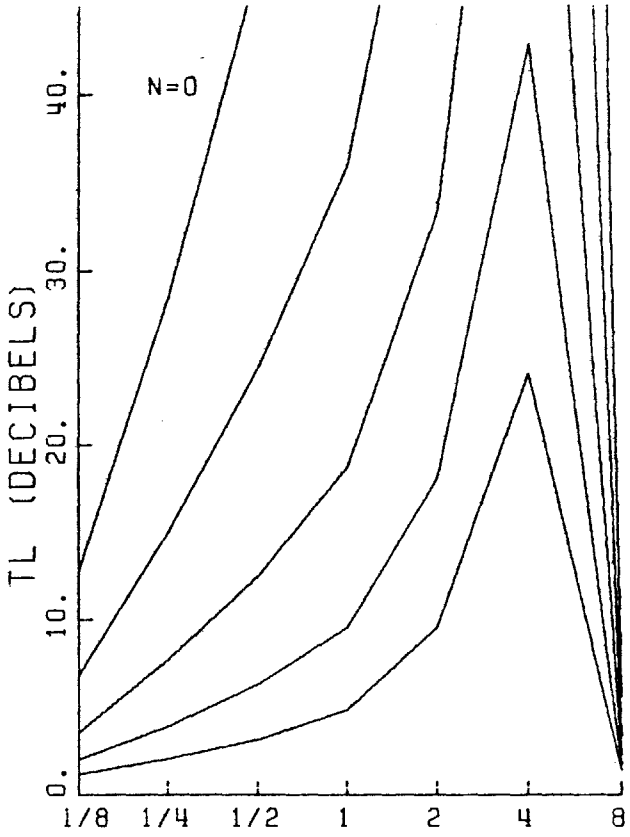


Figure A3.84

THETA=8.
D/L=4.828
AREA RATIO=1

S/D=16
8
4
2
1





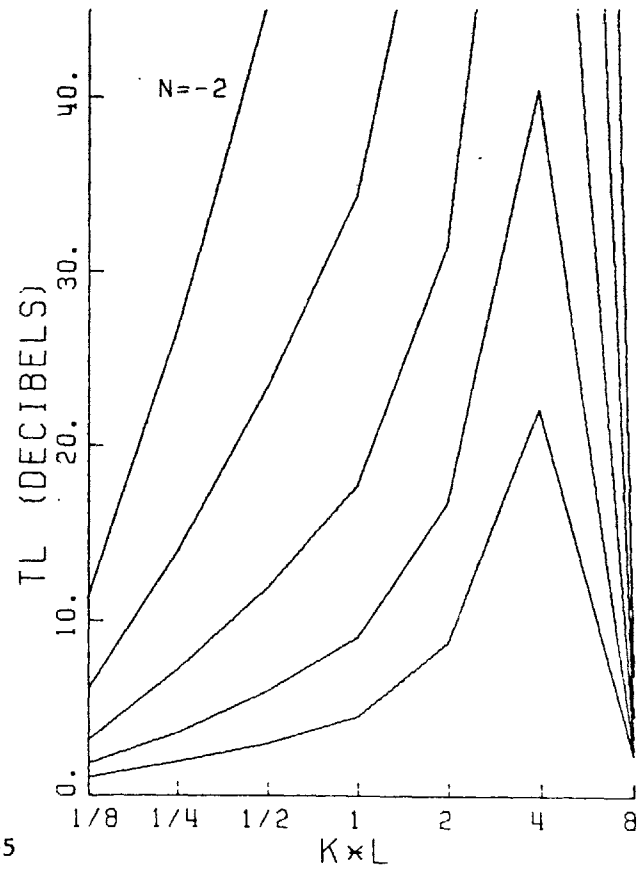
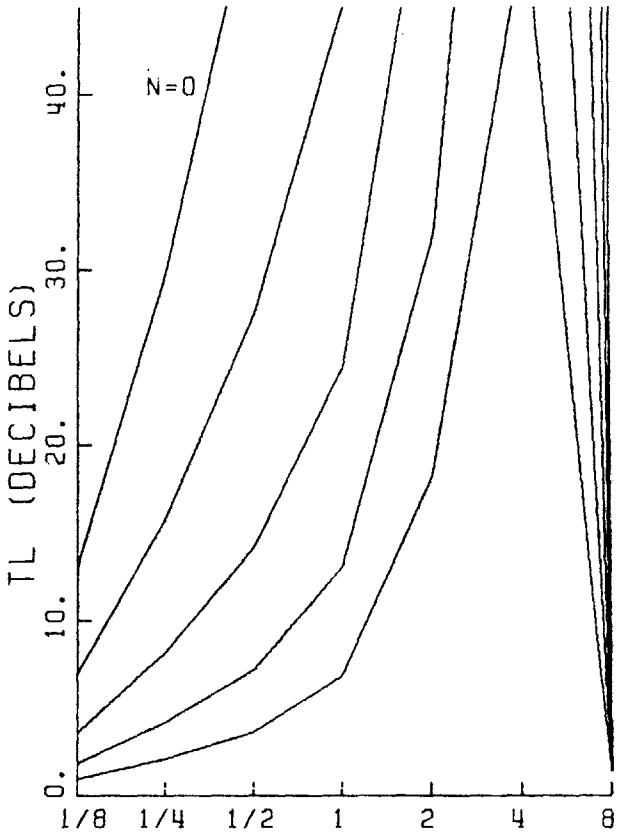
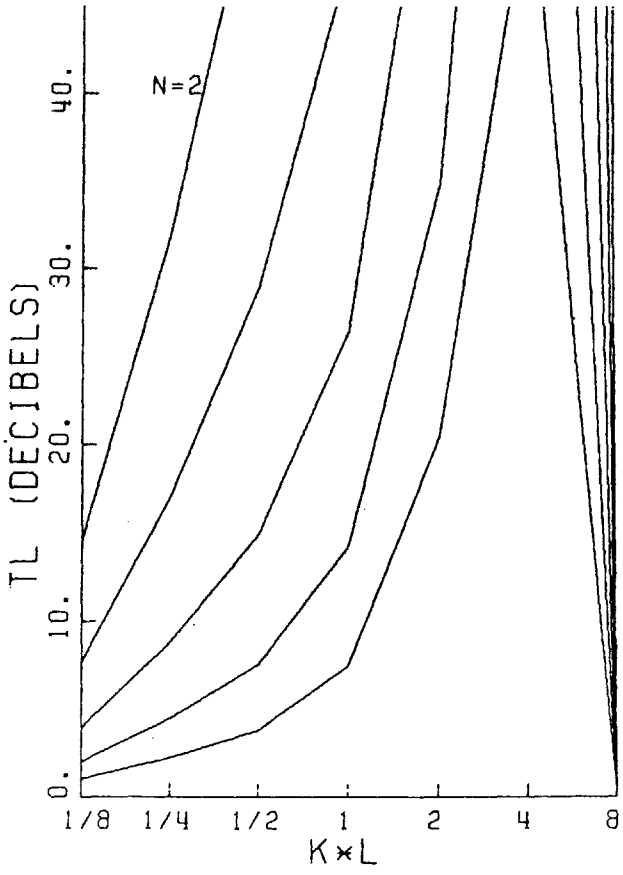


Figure A3.85

THETA=8.
D/L=12.928
AREA RATIO=1

S/D=16
8
4
2
1





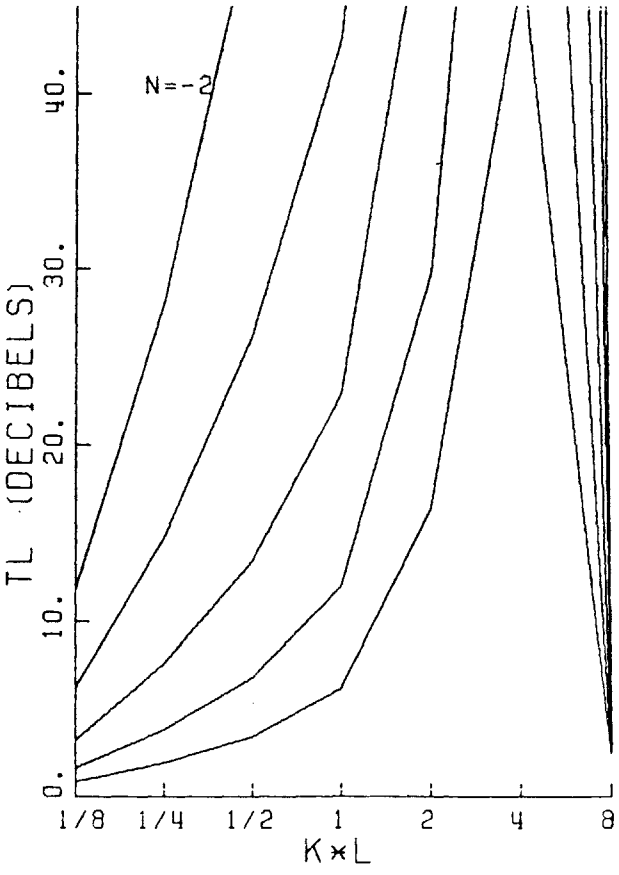
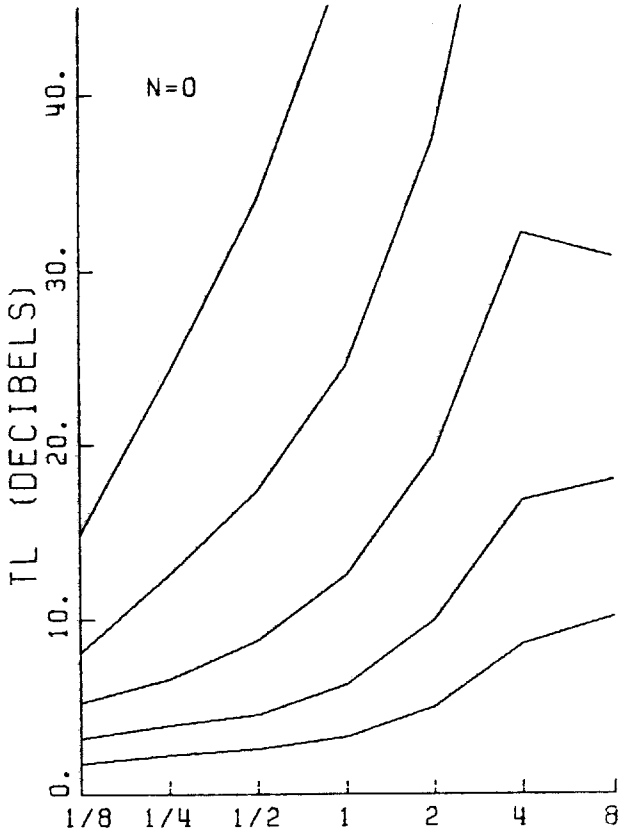
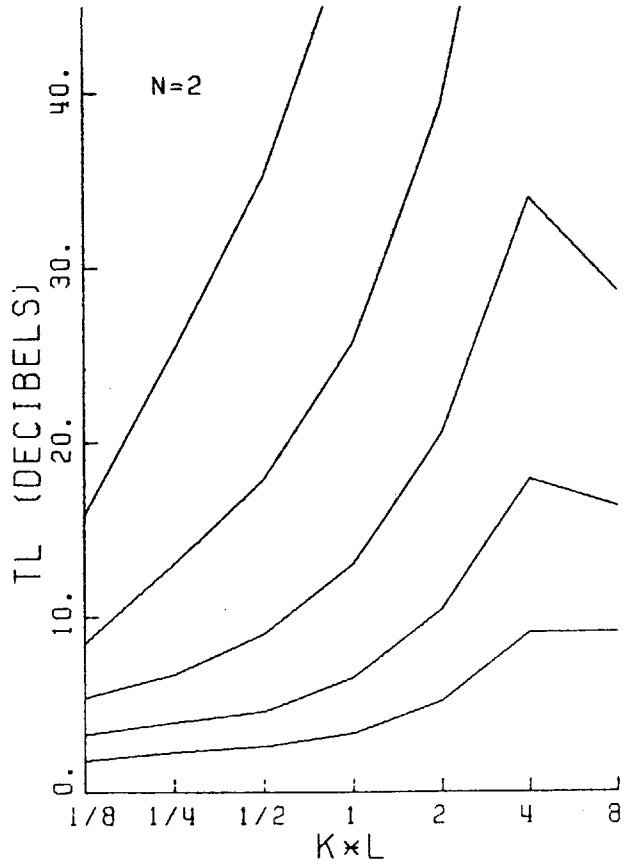


Figure A3.86

THETA=12.
D/L=1.094
AREA RATIO=1

S/D=16
8
4
2
1





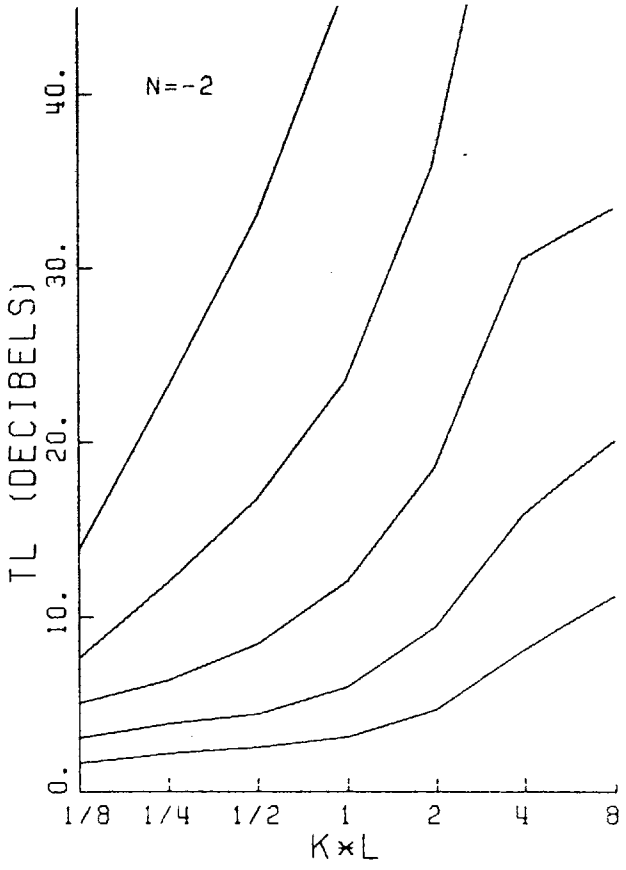
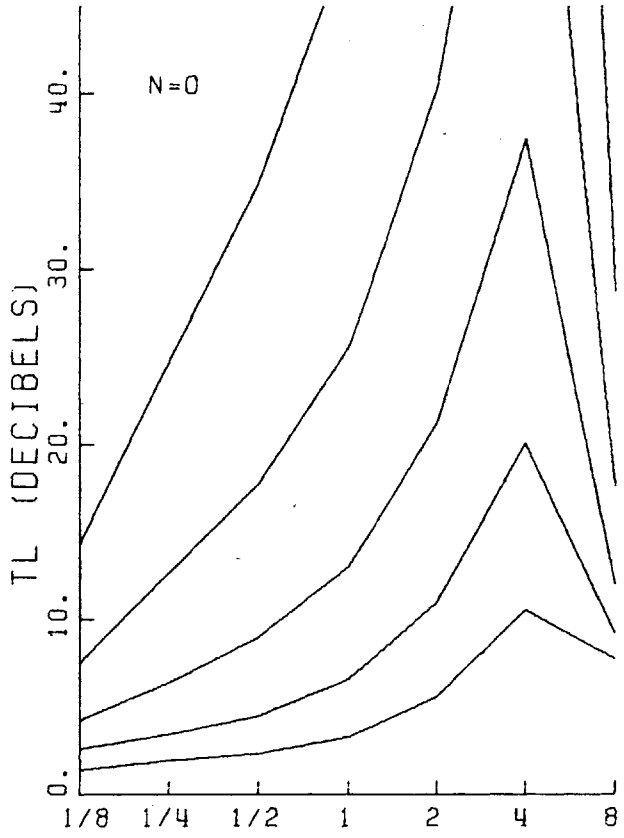
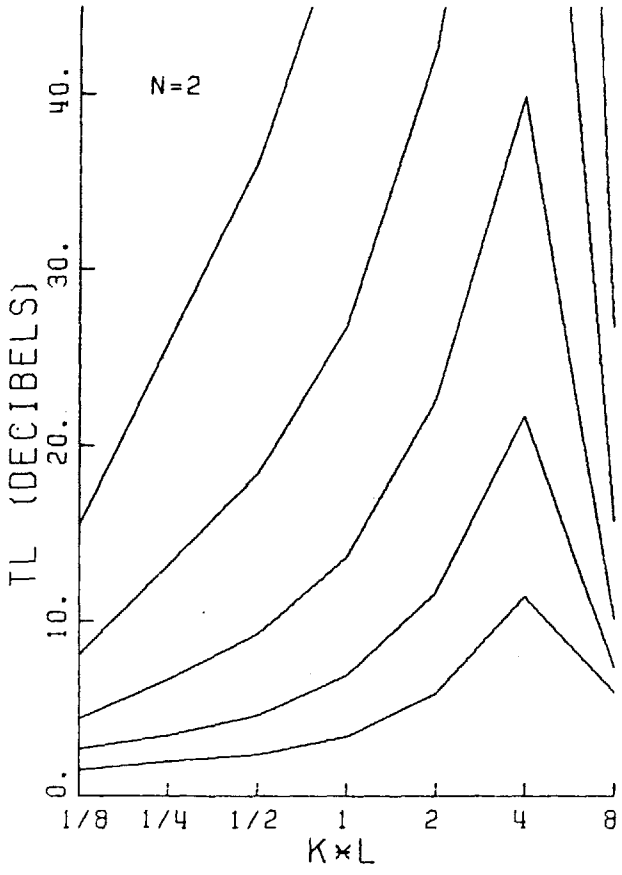


Figure A3.87

THETA=12.
D/L=2.000
AREA RATIO=1

S/D=16
8
4
2
1





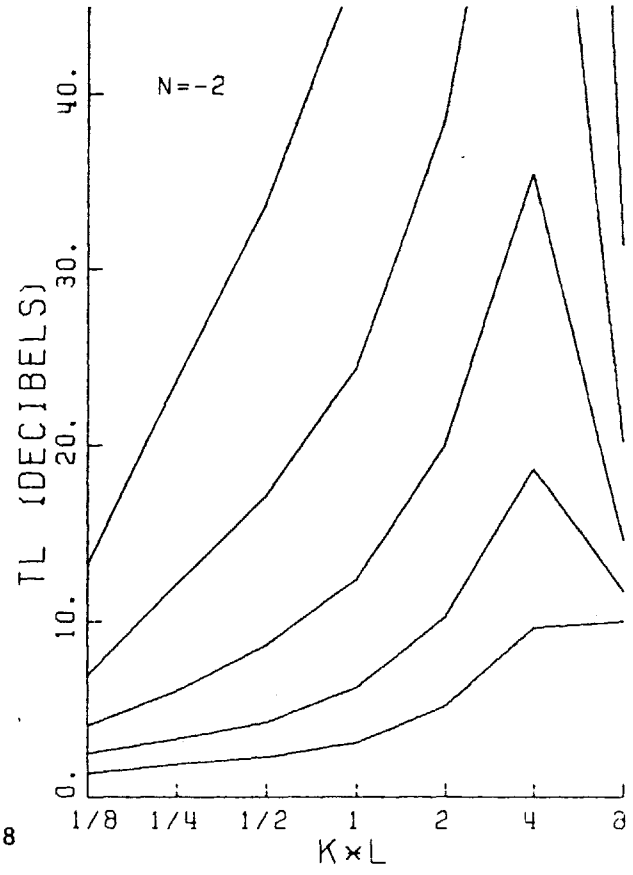
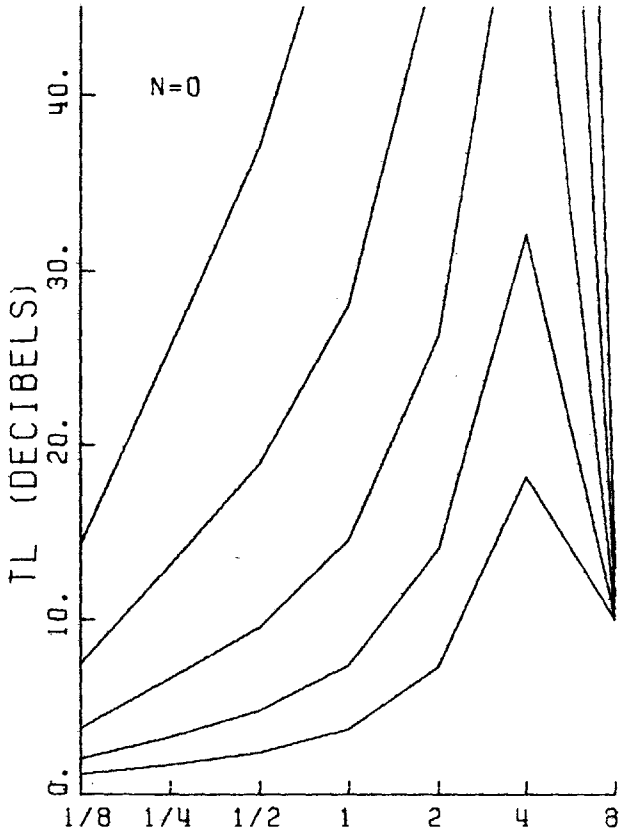
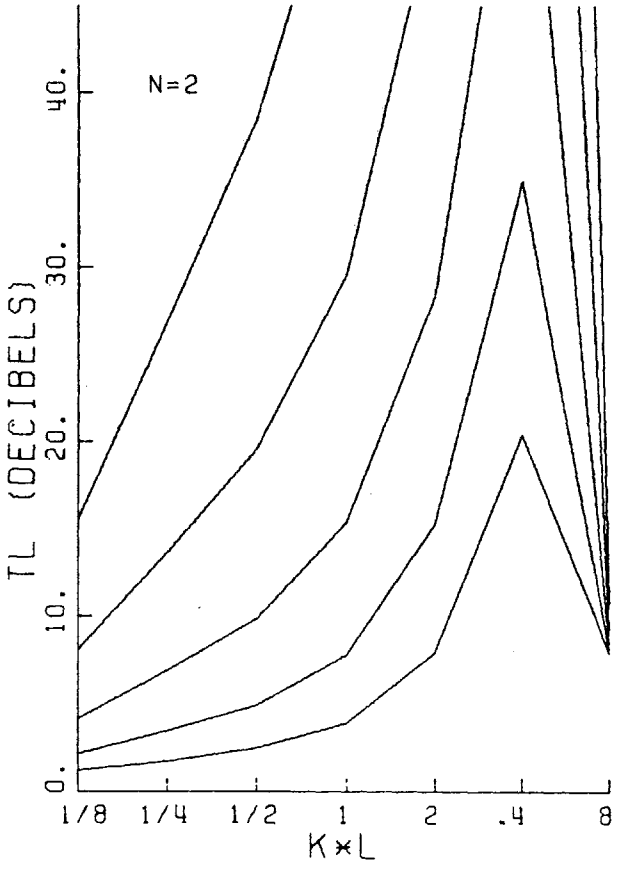


Figure A3.88

THETA=12.
D/L=4.828
AREA RATIO=1

S/D=16
8
4
2
1





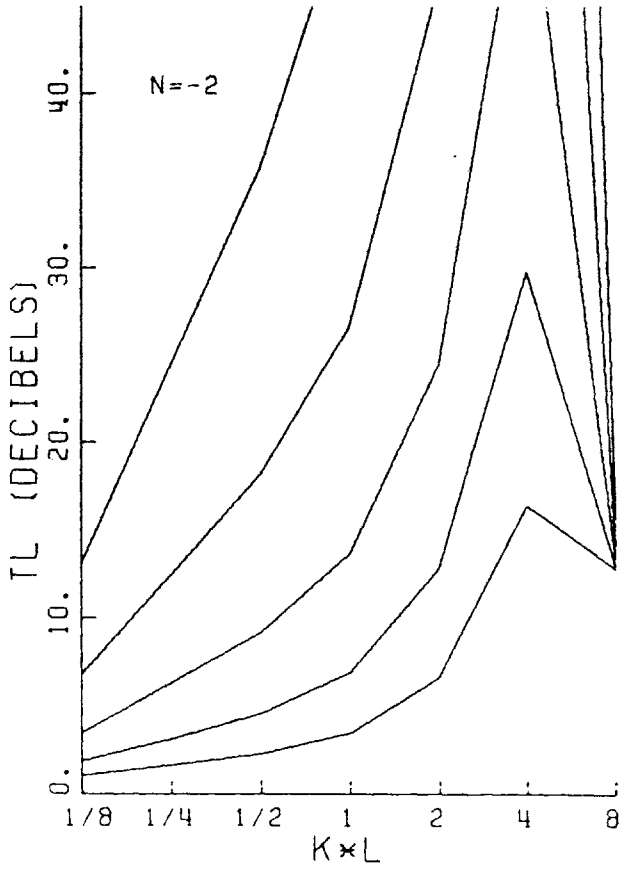
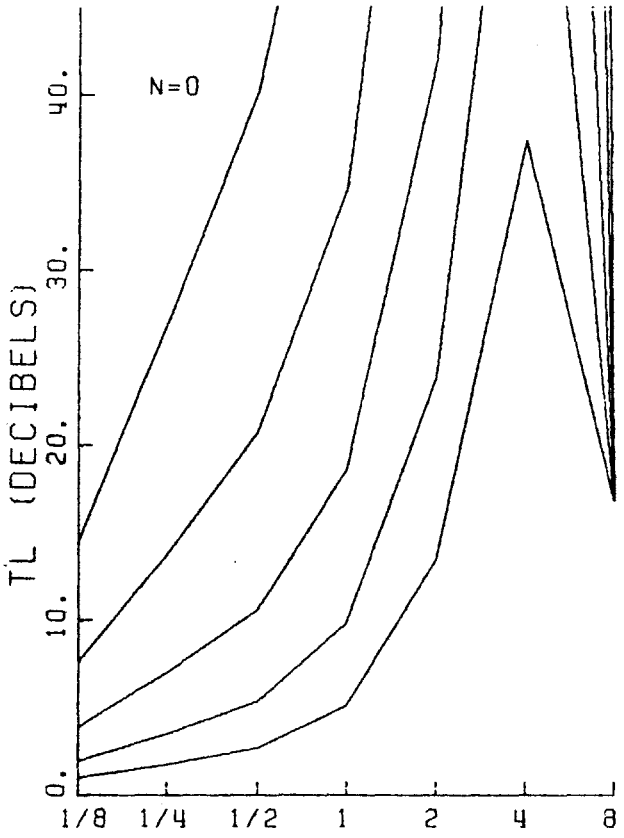
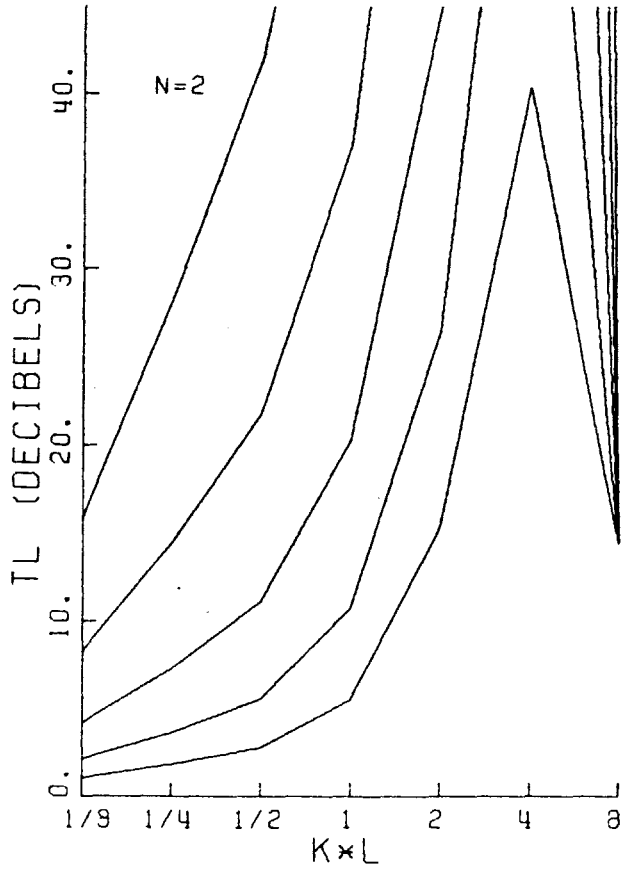


Figure A3.89

THETA=12.
D/L=12.928
AREA RATIO=1

S/D=16
8
4
2
1





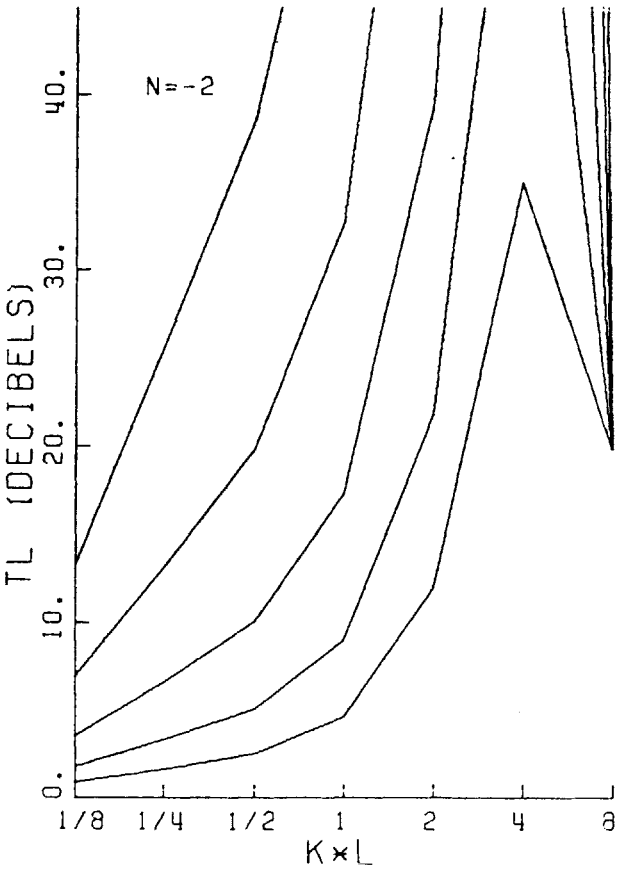
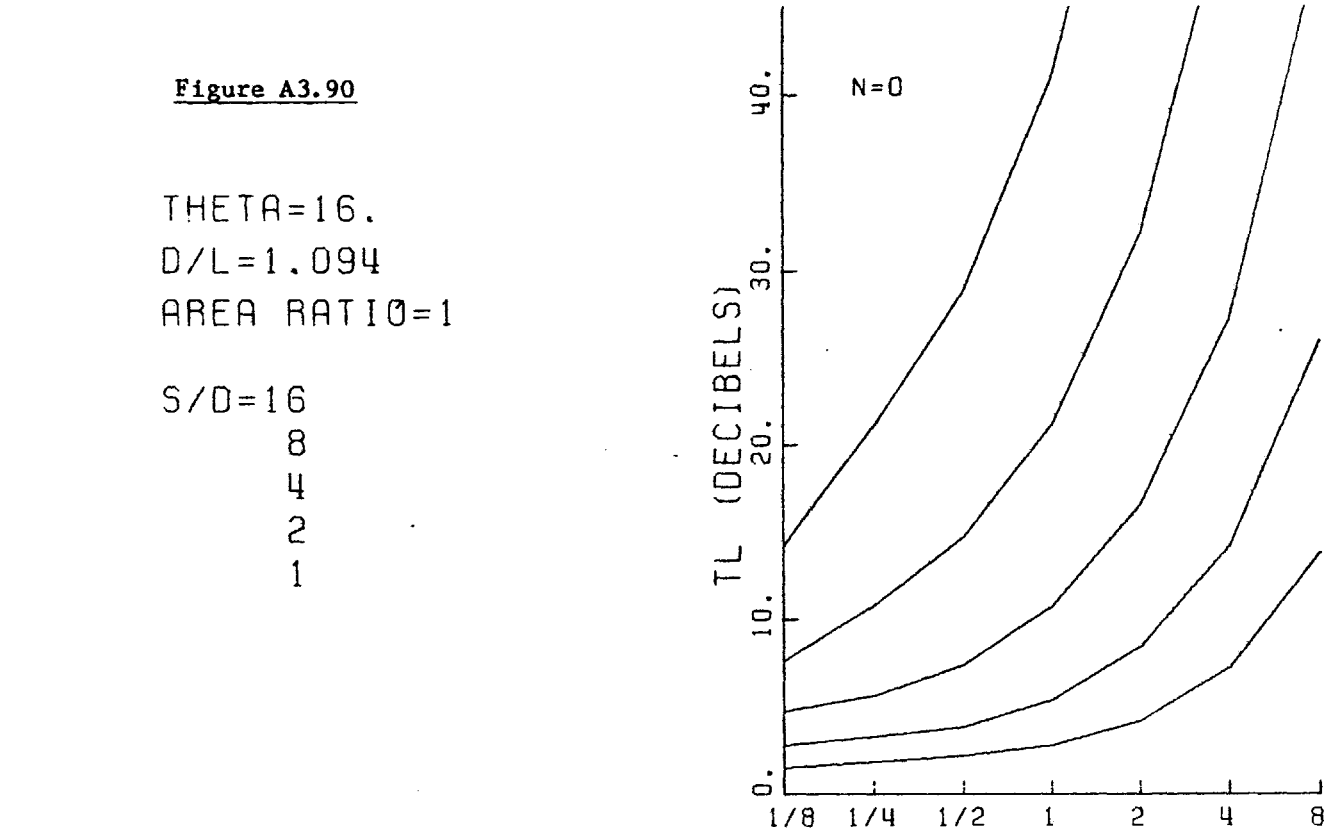
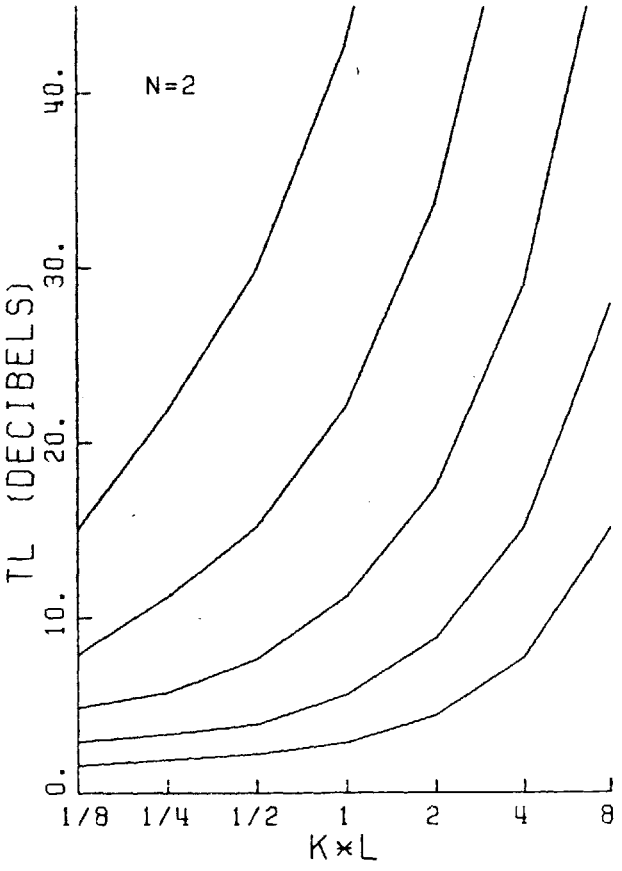


Figure A3.90

THETA=16.
D/L=1.094
AREA RATIO=1

S/D=16
8
4
2
1





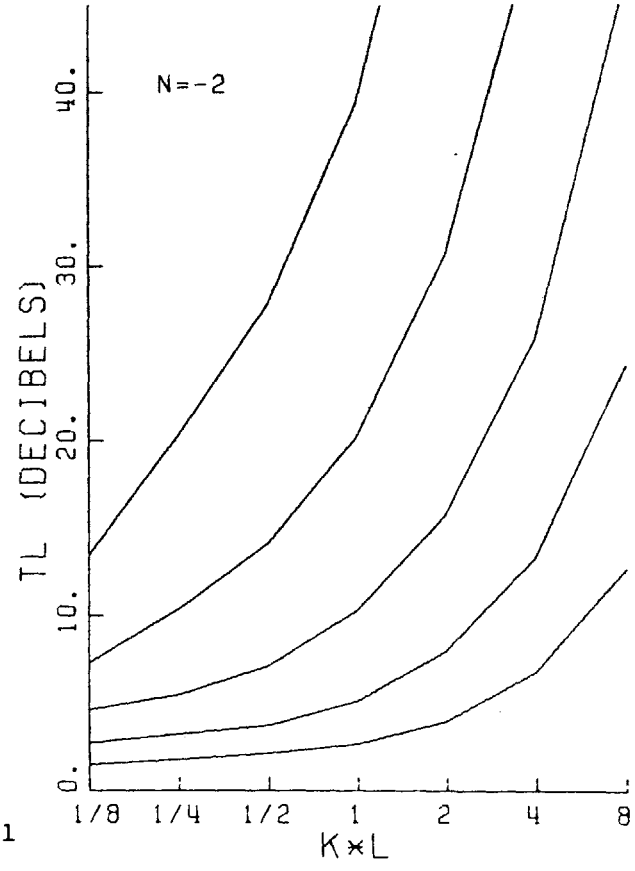
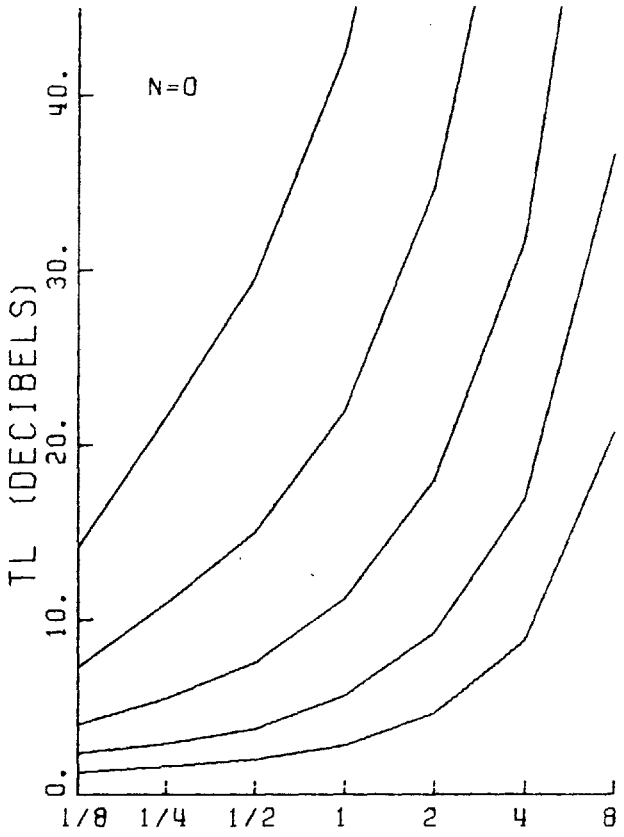
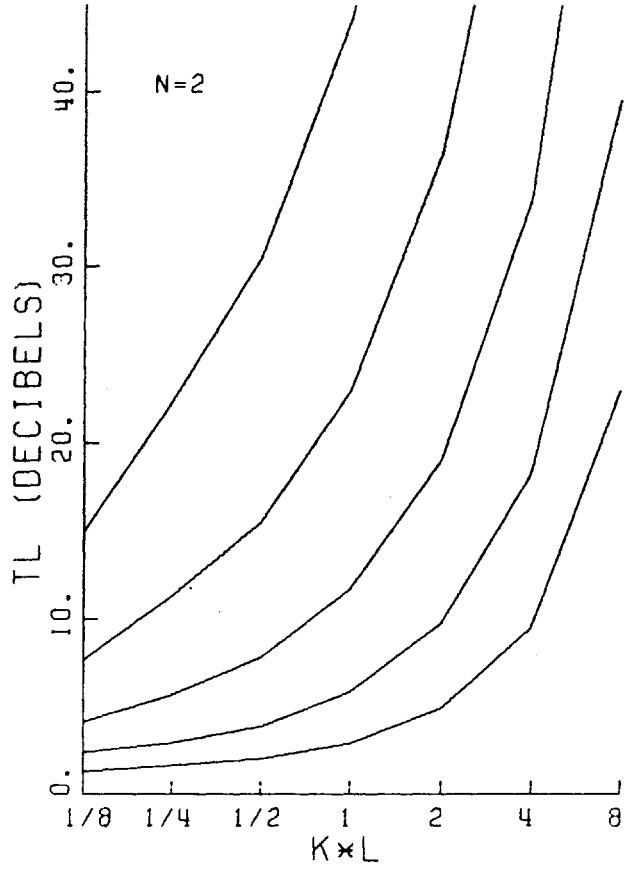


Figure A3.91

THETA=16.
D/L=2.000
AREA RATIO=1

S/D=16
8
4
2
1





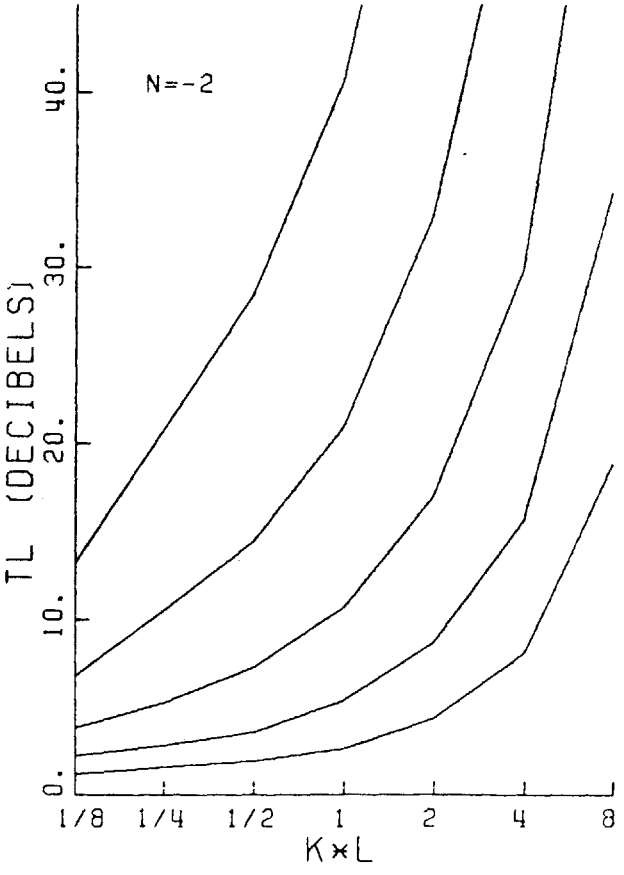
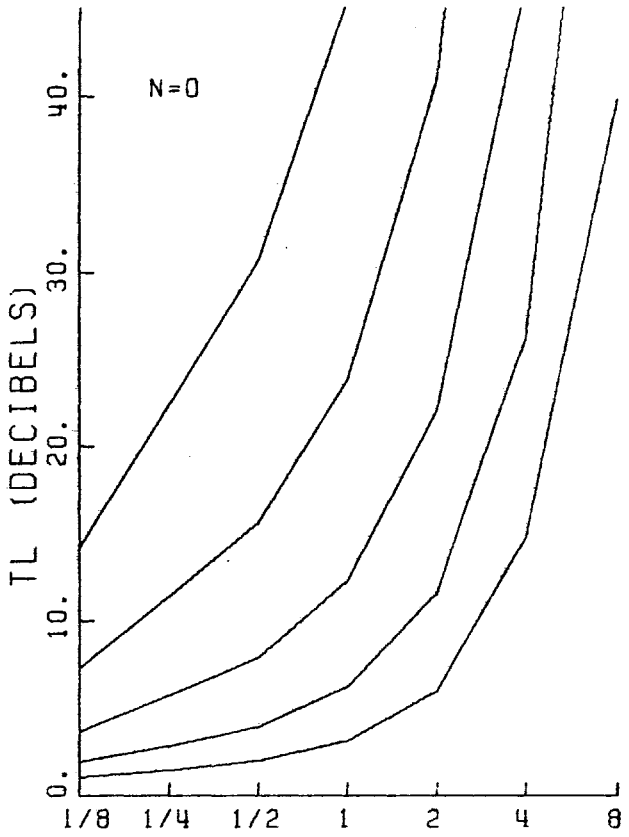
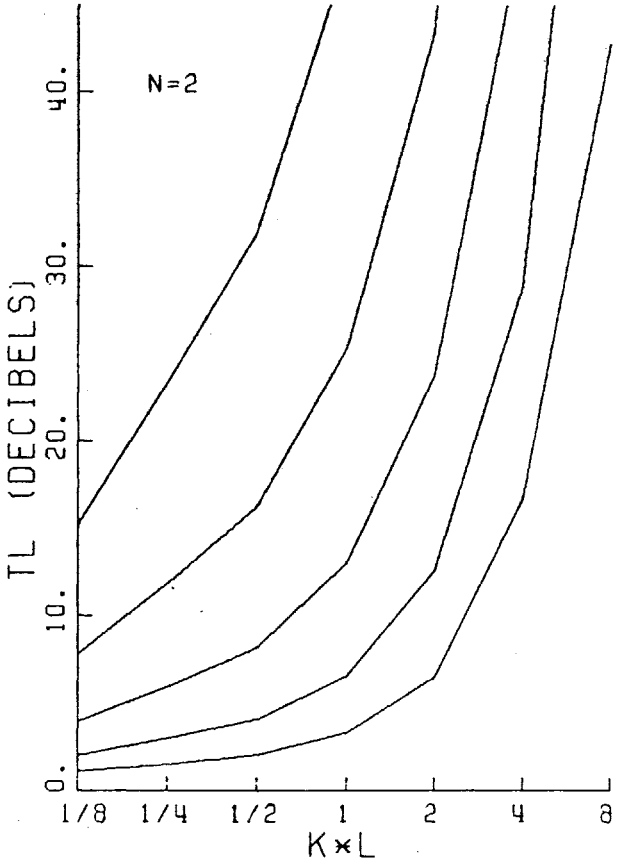


Figure A3.92

THETA=16.
D/L=4.828
AREA RATIO=1

S/D=16
8
4
2
1





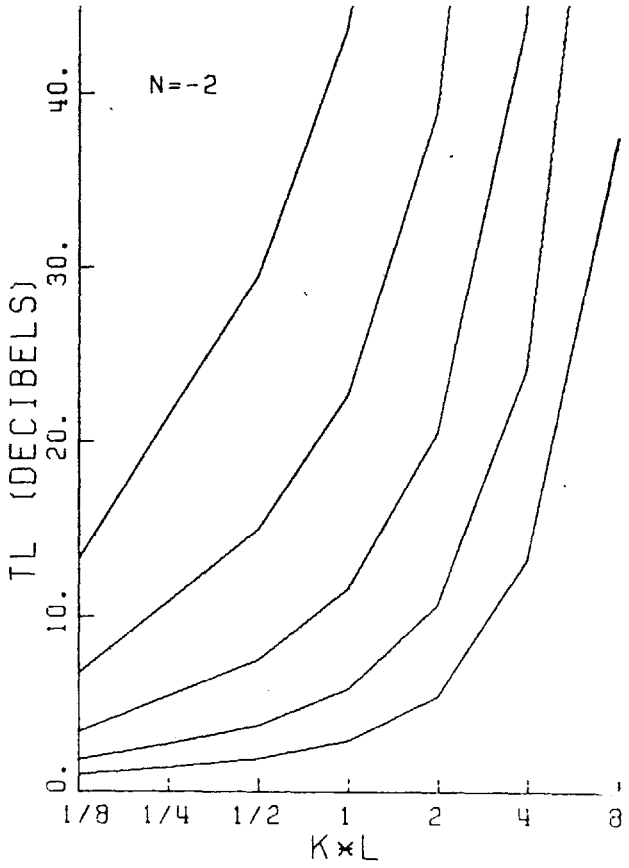
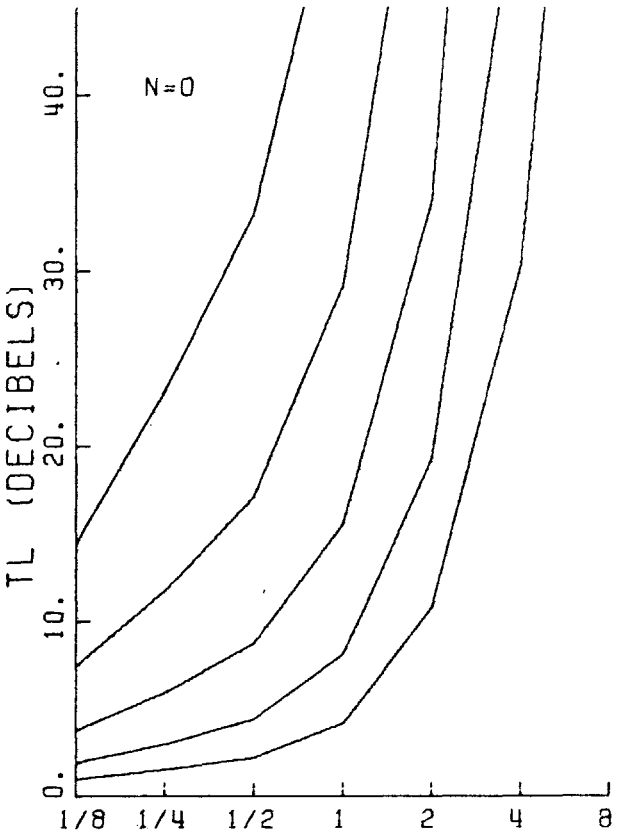
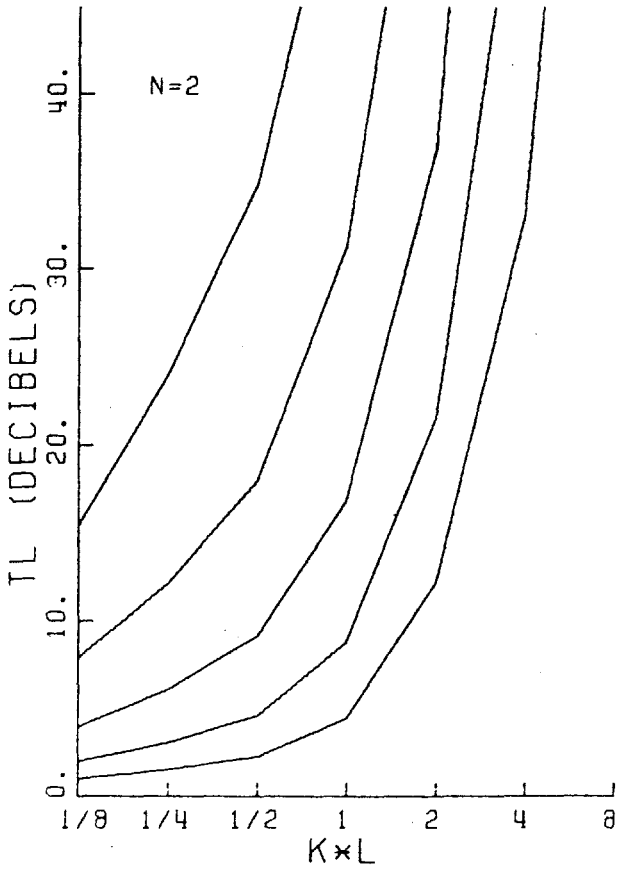


Figure A3.93

THETA=16.
D/L=12.928
AREA RATIO=1

S/D=16
8
4
2
1





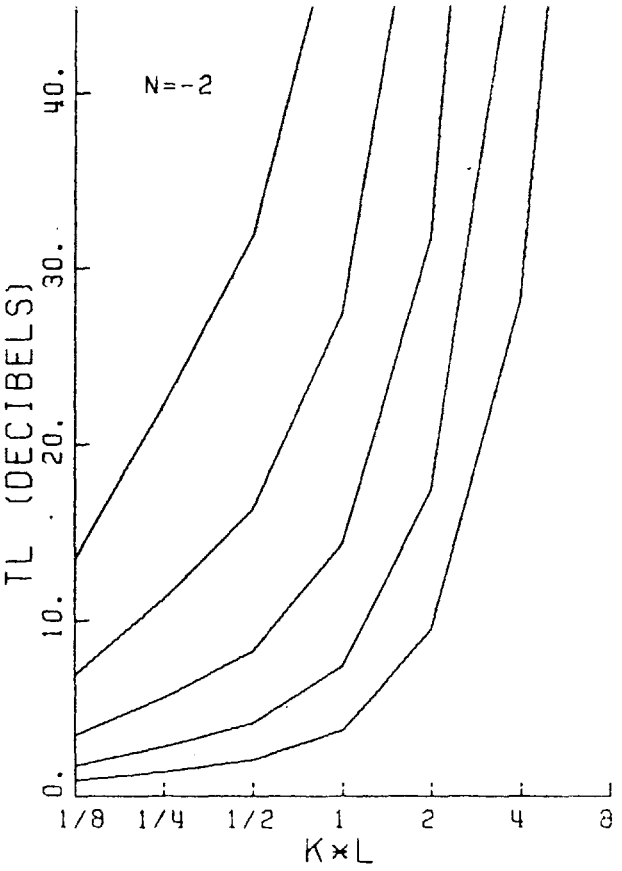
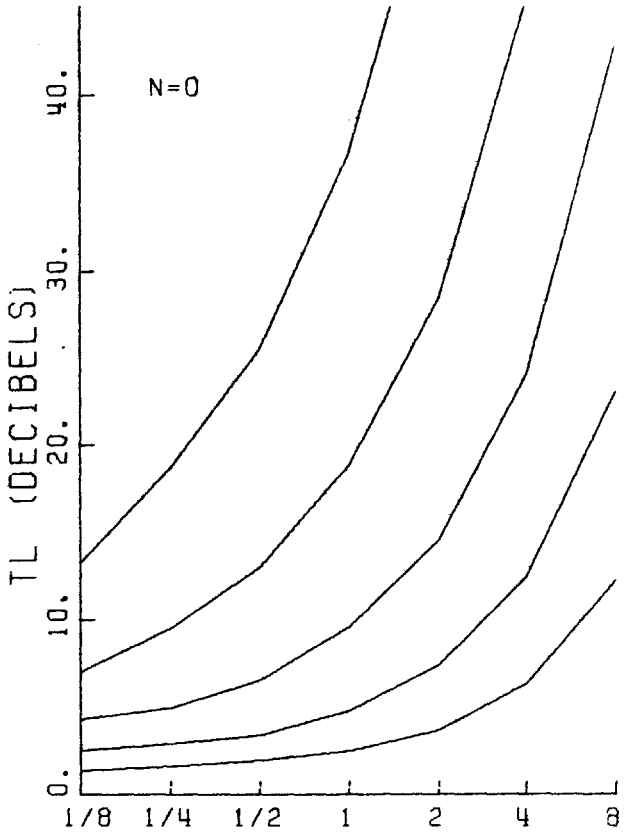
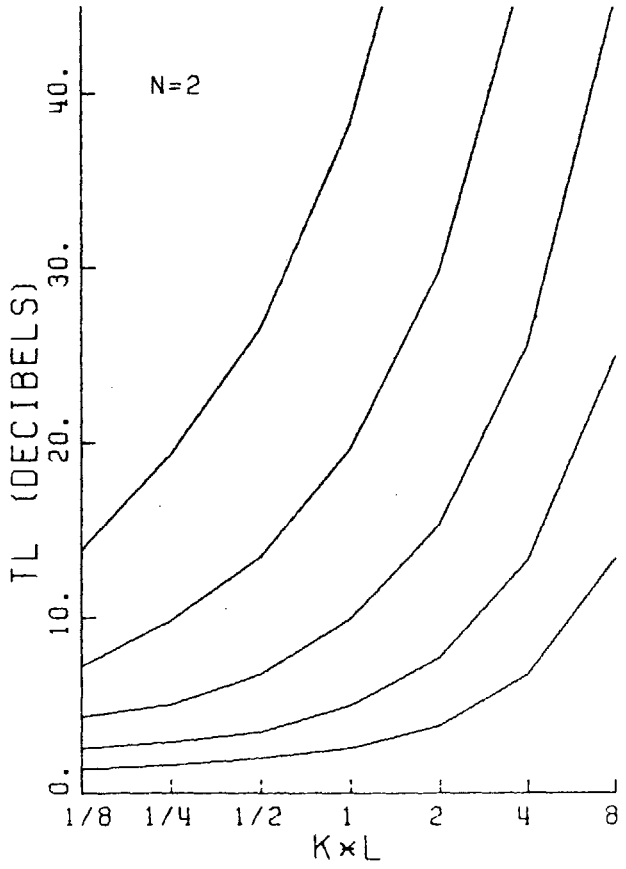


Figure A3.94

THETA=20.
D/L=1.094
AREA RATIO=1

S/D=16
8
4
2
1





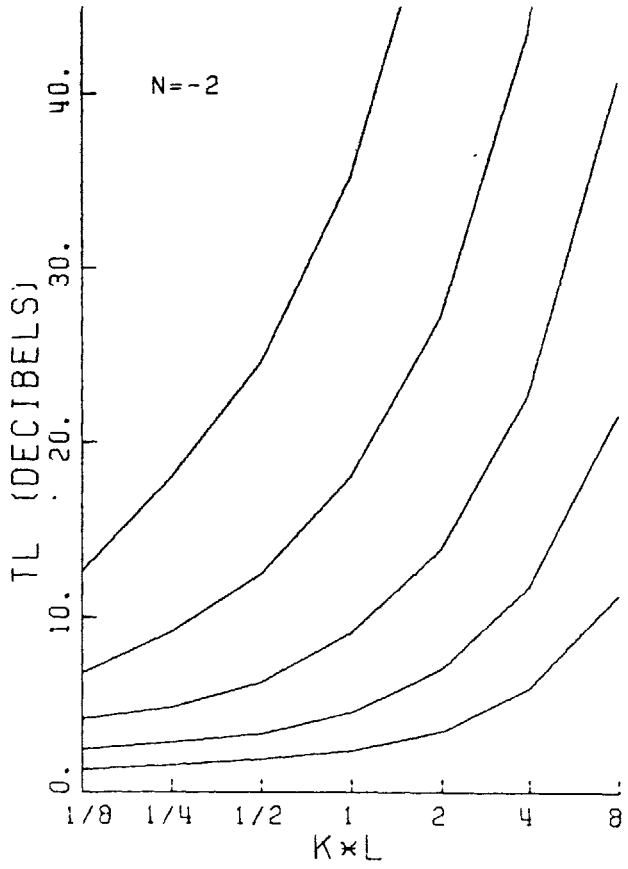
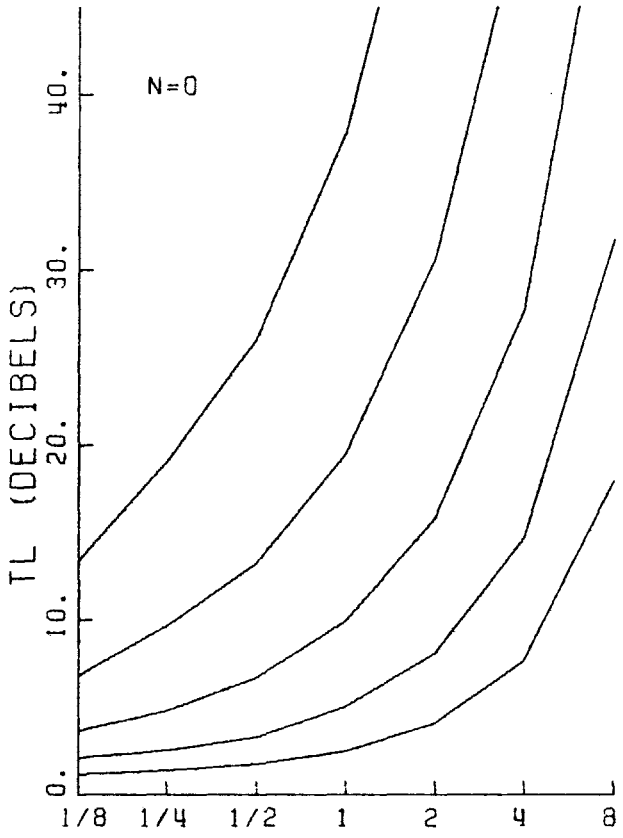
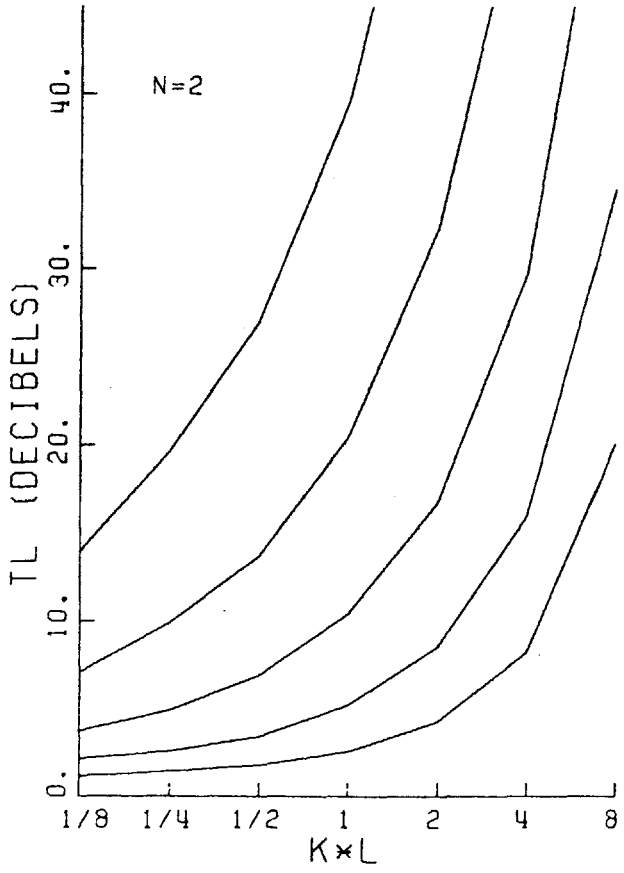


Figure A3.95

THETA=20.
D/L=2.000
AREA RATIO=1

S/D=16
8
4
2
1





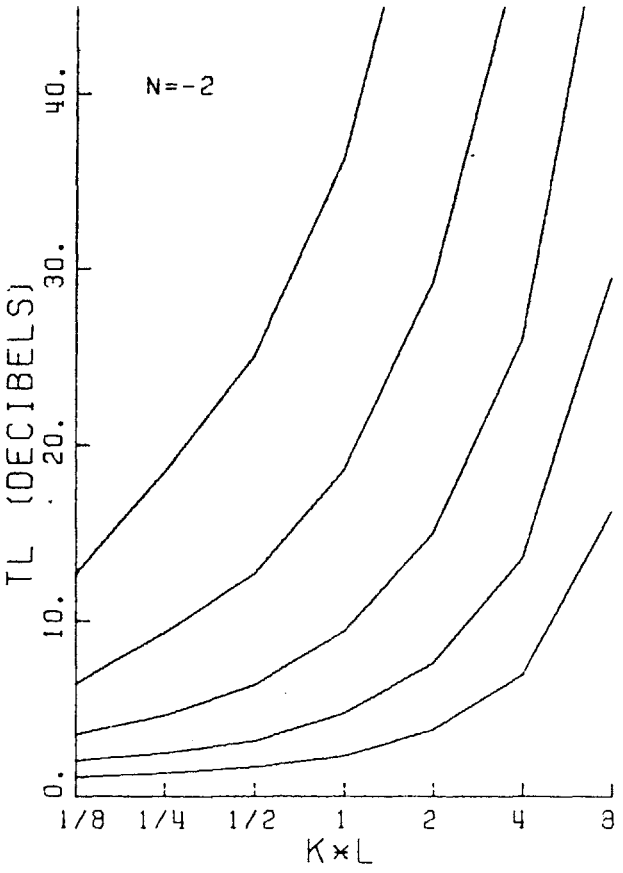
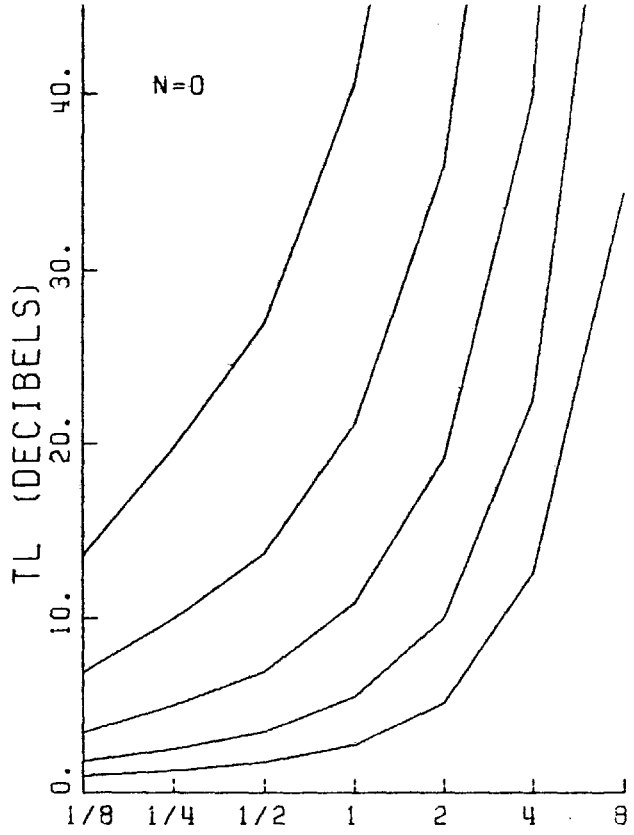
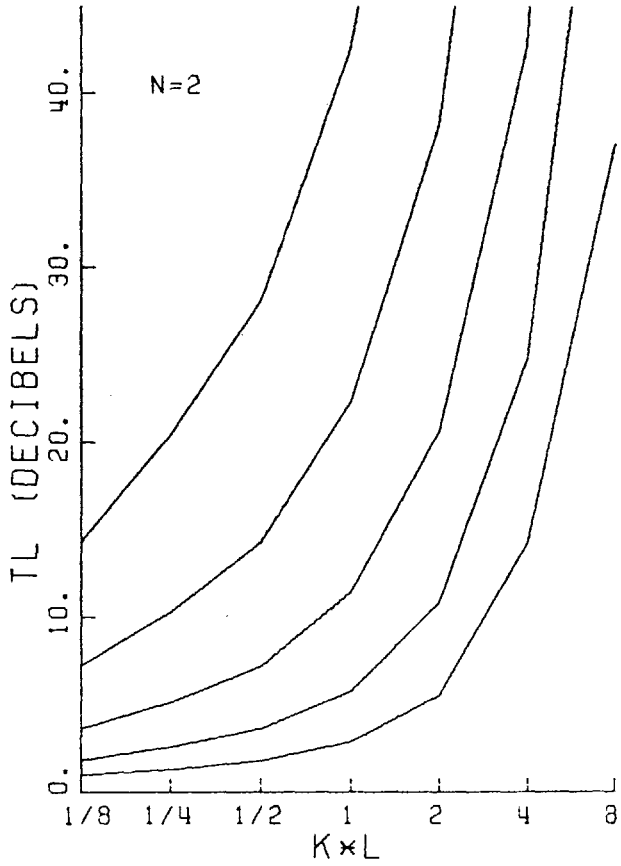


Figure A3.96

THETA=20.
D/L=4.828
AREA RATIO=1

S/D=16
8
4
2
1





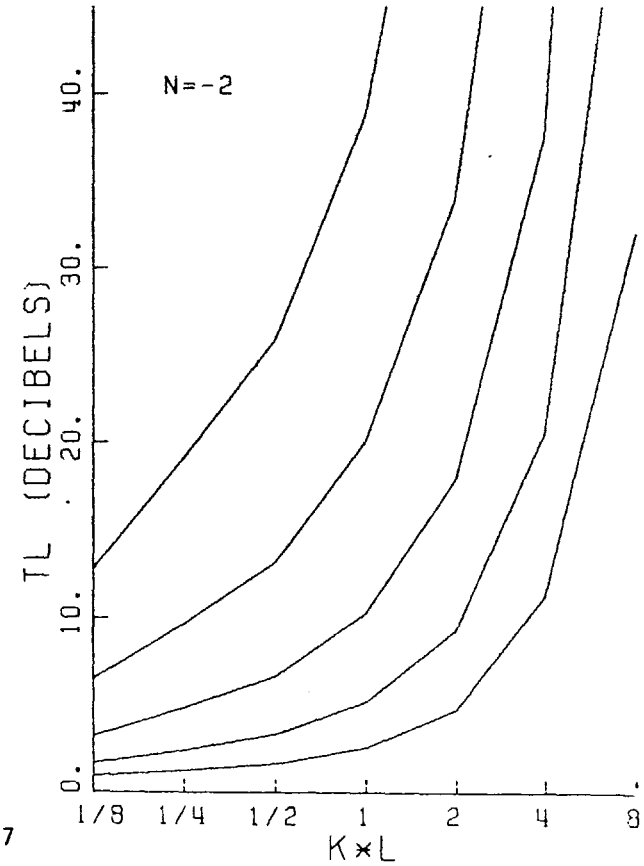
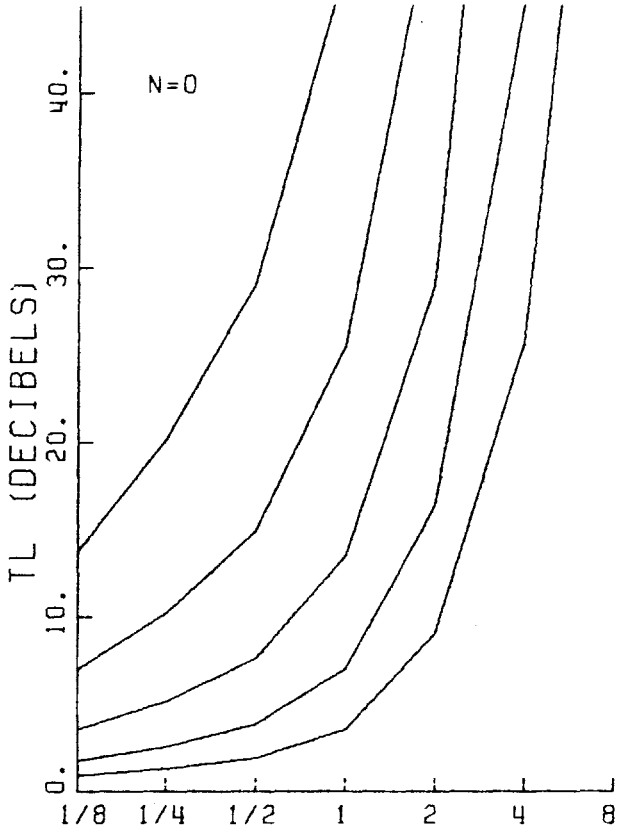
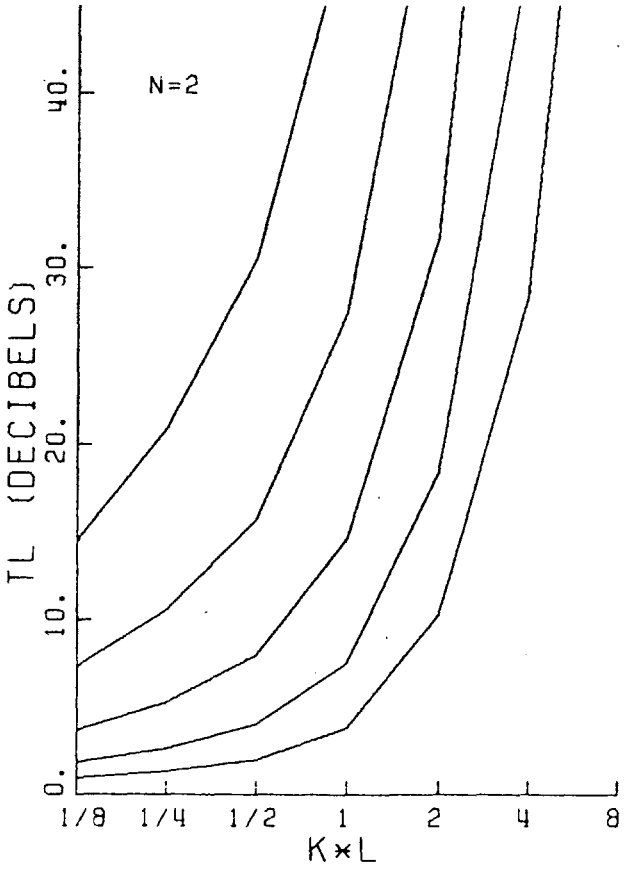


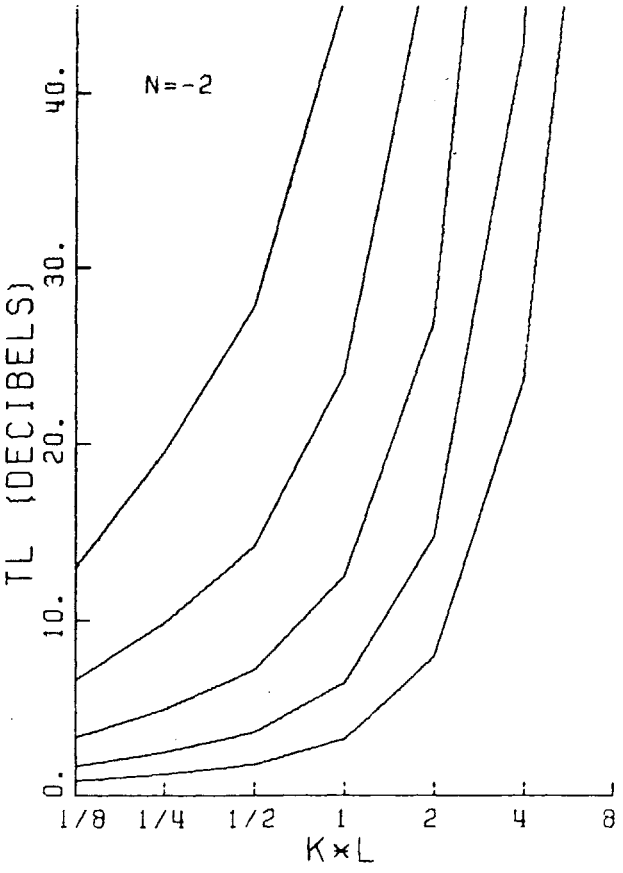
Figure A3.97

THETA=20.
D/L=12.928
AREA RATIO=1

S/D=16
8
4
2
1







Figures A3.110 - A3.113: Octave band TL vs S/D for a circular duct lined with a resistive screen type resonator liner. Each figure corresponds to a different value of D/L. In each figure four frames are shown, corresponding to θ = 0.5, 1, 2, and 4. In each frame four curves are plotted, corresponding to four different values of kL as indicated by symbols.

Figure A3.110

AREA RATIO=1.0 D/L=1.054

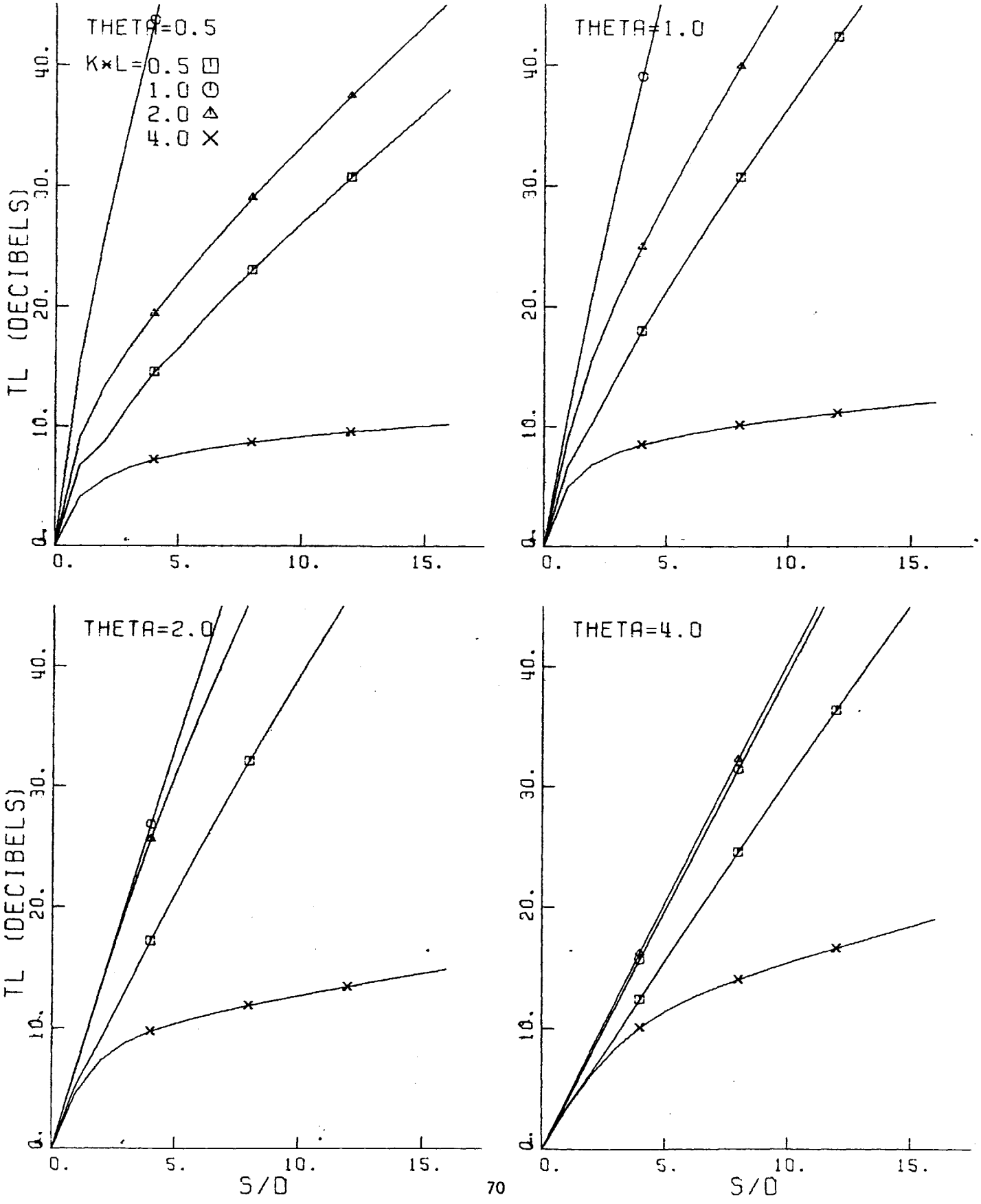


Figure A3.111

AREA RATIO=1.0 D/L=2.000

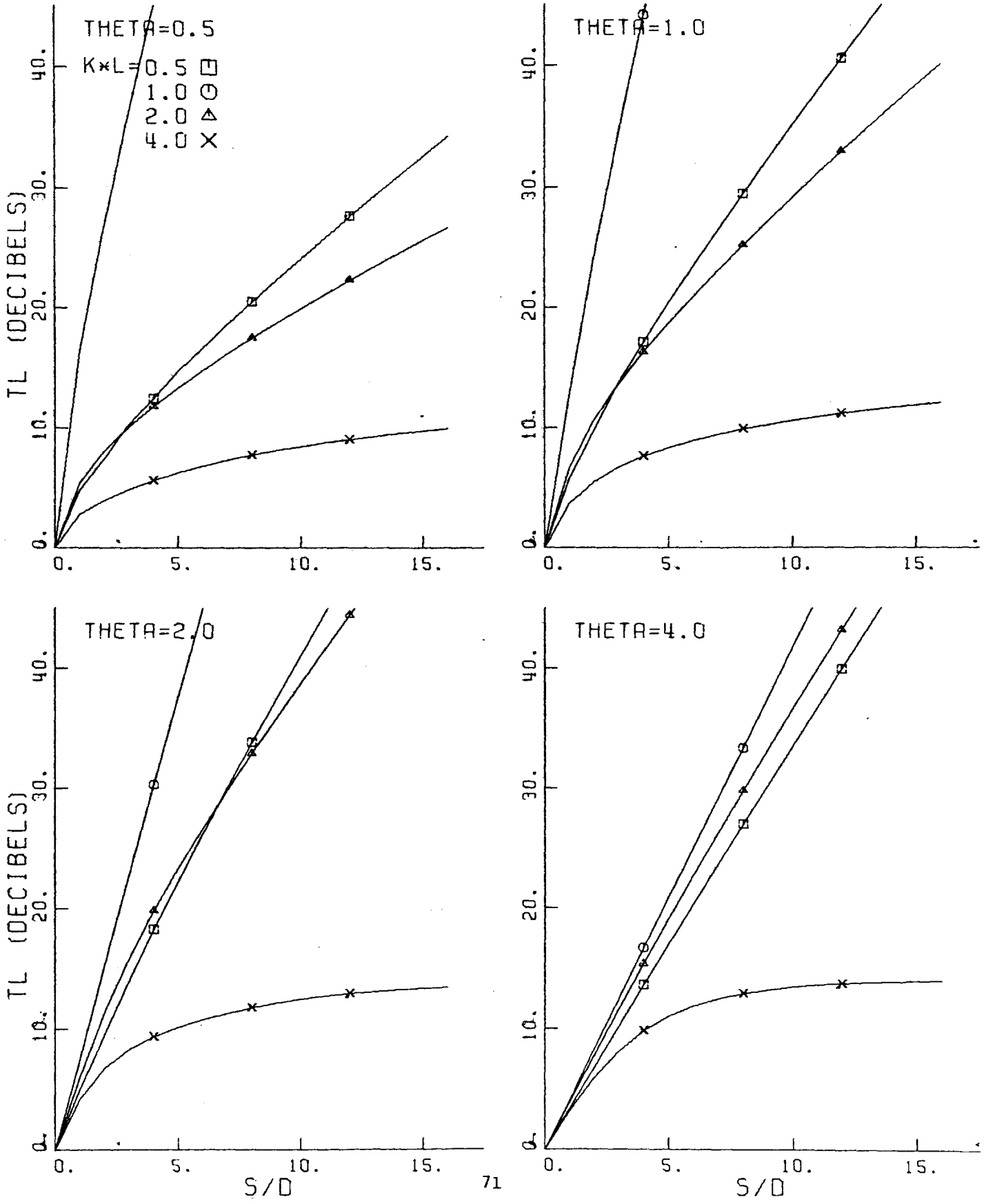


Figure A3.112

AREA RATIO=1.0 D/L=4.828

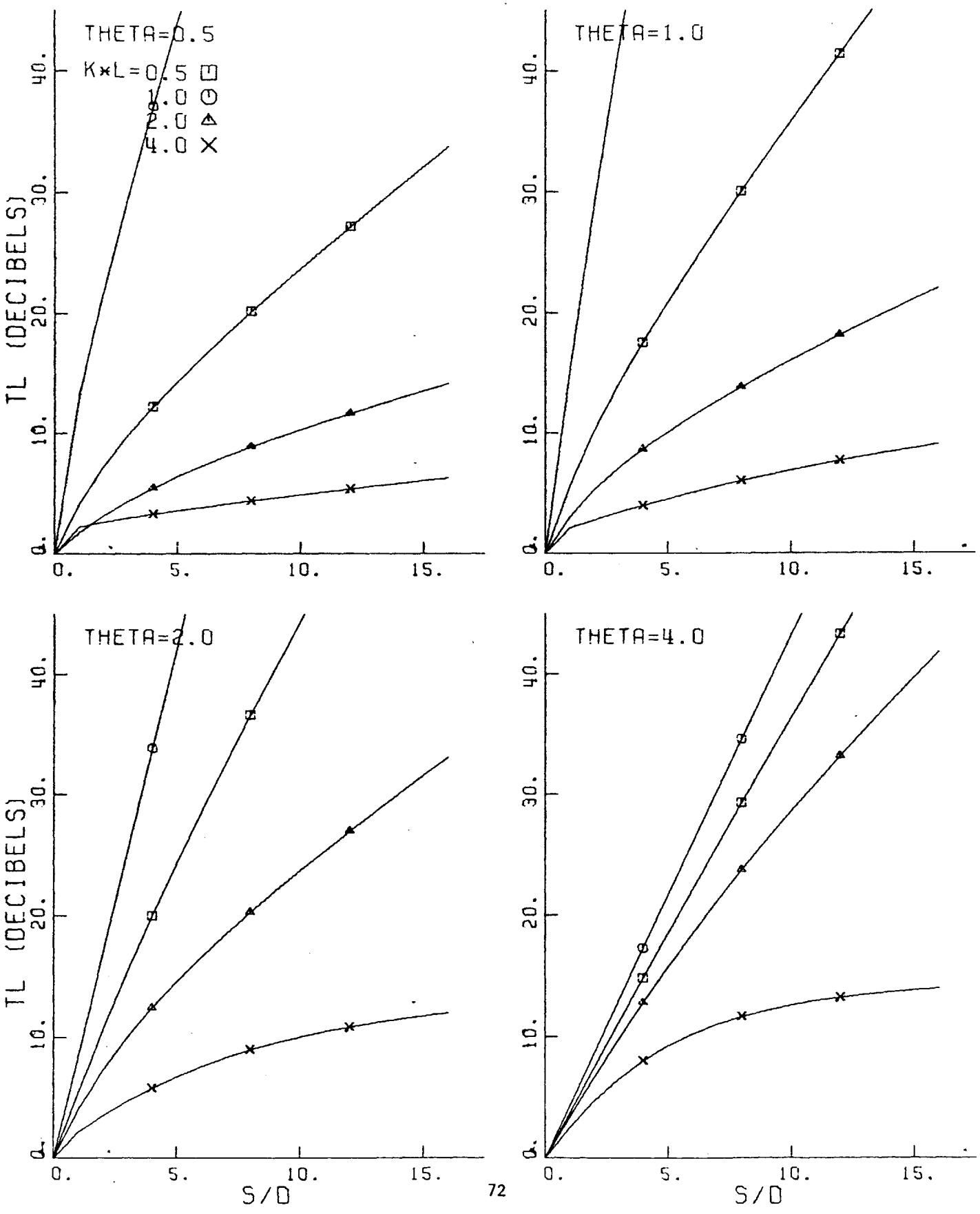
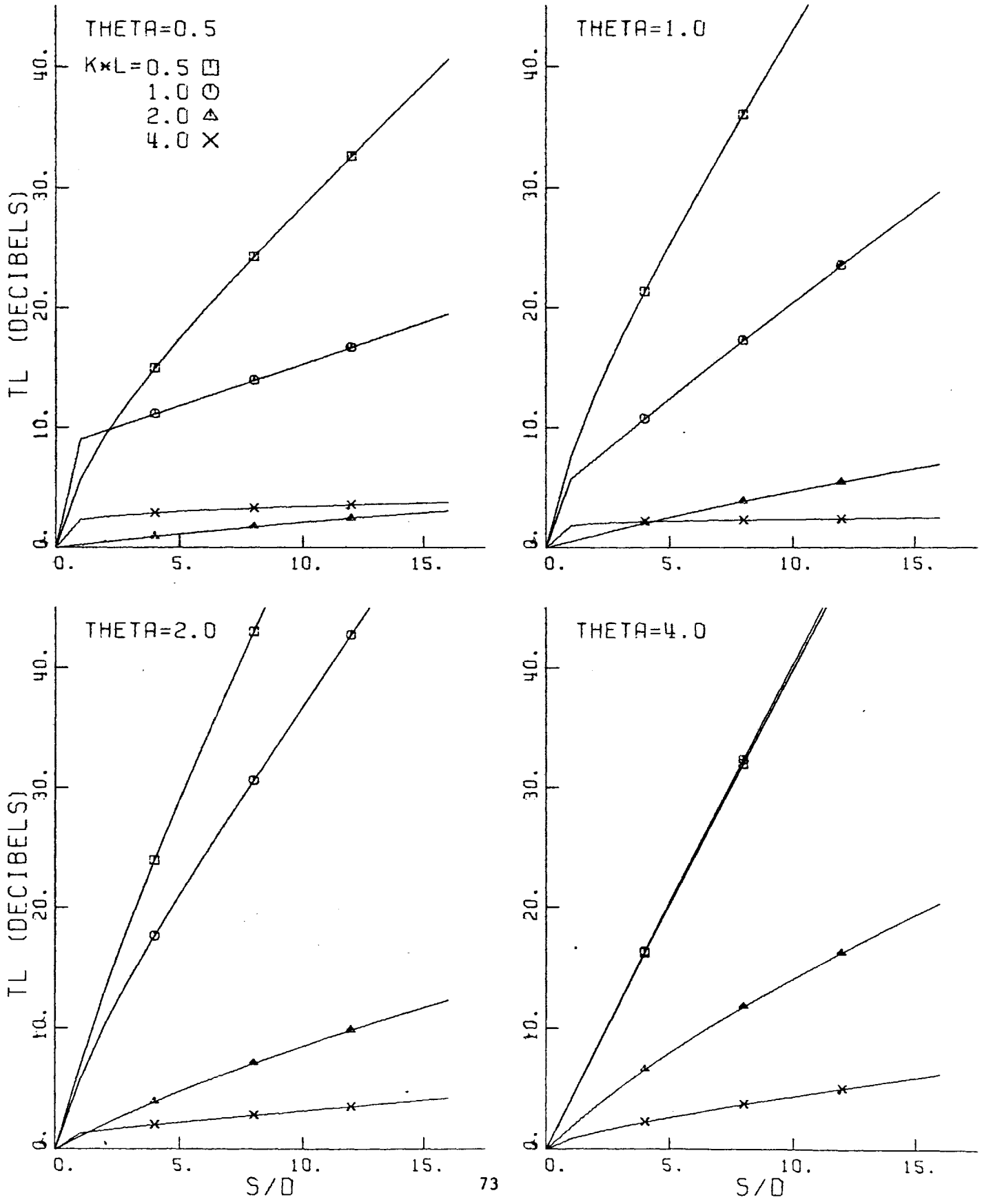


Figure A3.113

AREA RATIO=1.0 D/L=12.928



Figures A3.114 - A3.121: Octave band TL vs S/D for a circular duct lined with a resistive screen type resonator liner. The format is the same as in Figs. A3.110 - A3.113.

Figure A3.114

AREA RATIO=1.0 D/L=1.054

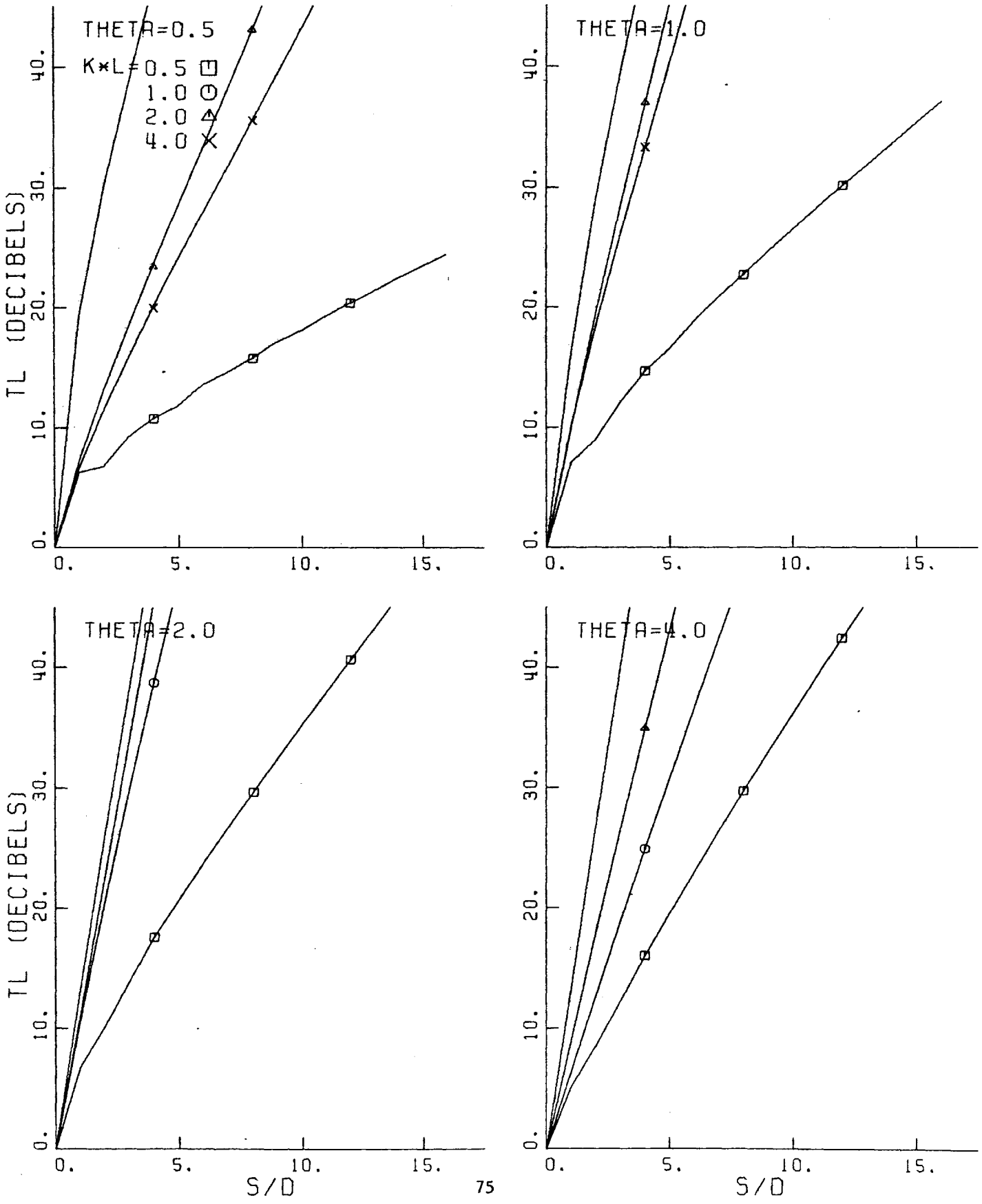


Figure A3.115

AREA RATIO=1.0 D/L=1.054

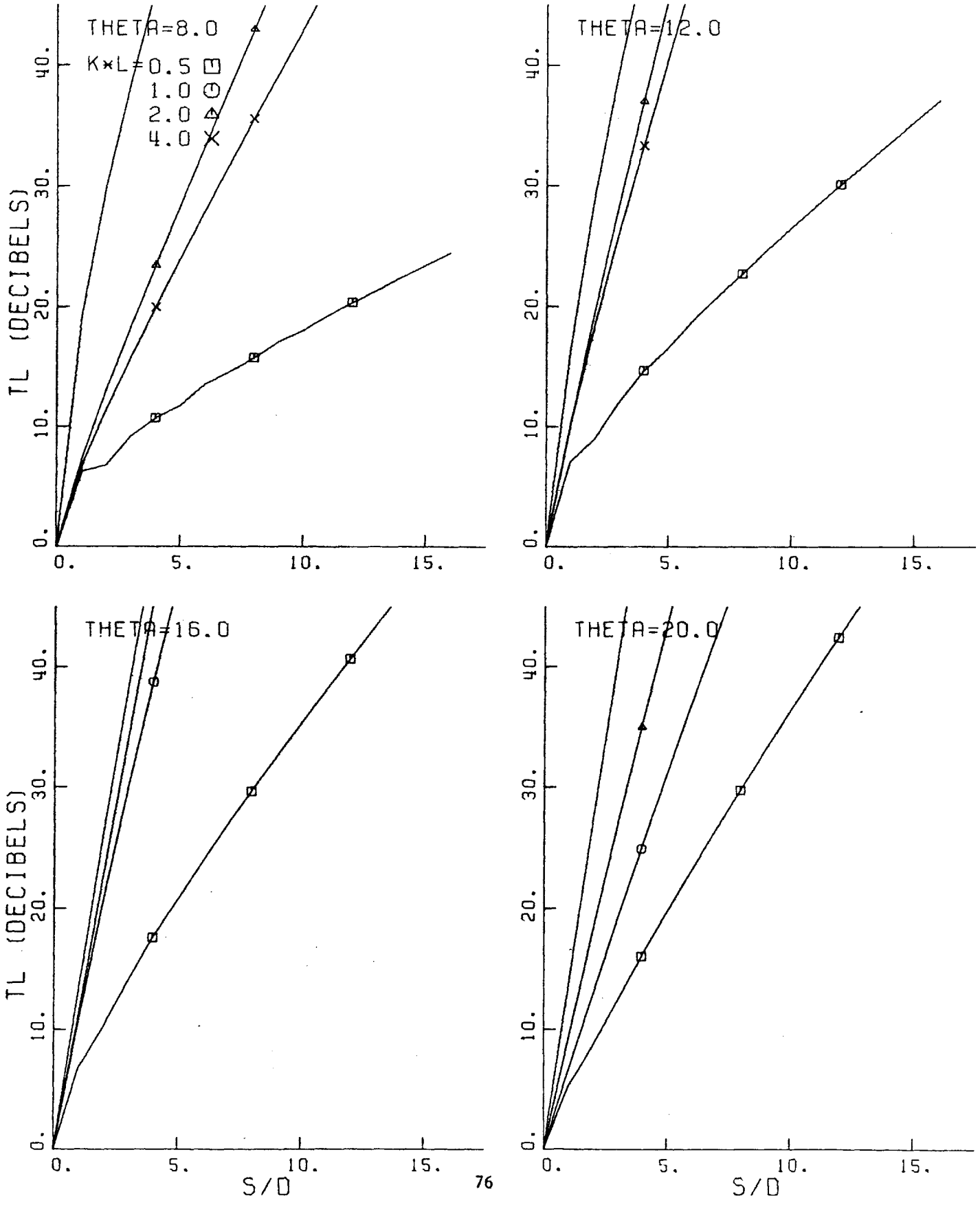


Figure A3.116

AREA RATIO=1.0 D/L=2.000

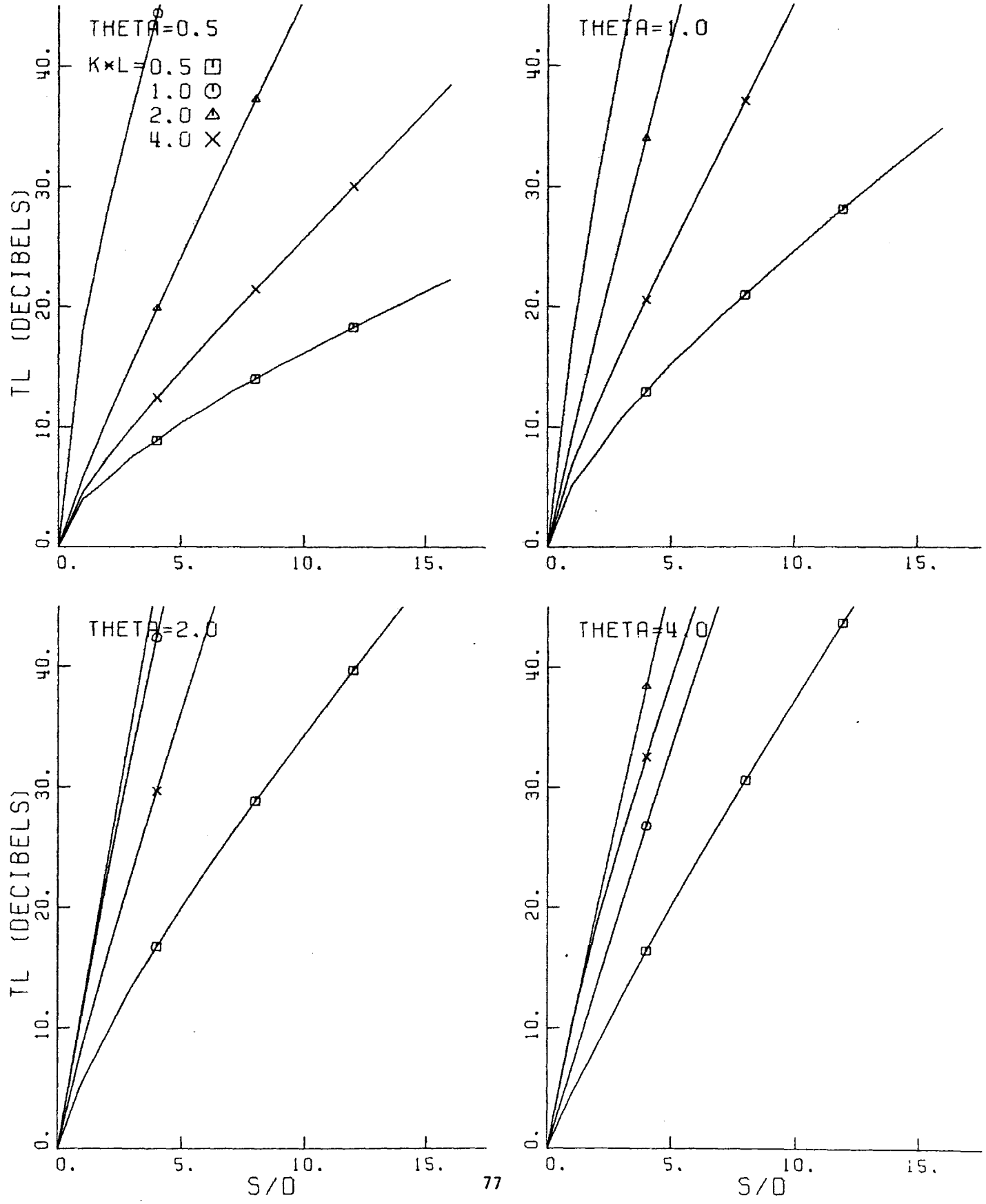


Figure A3.117

AREA RATIO=1.0 D/L=2.000

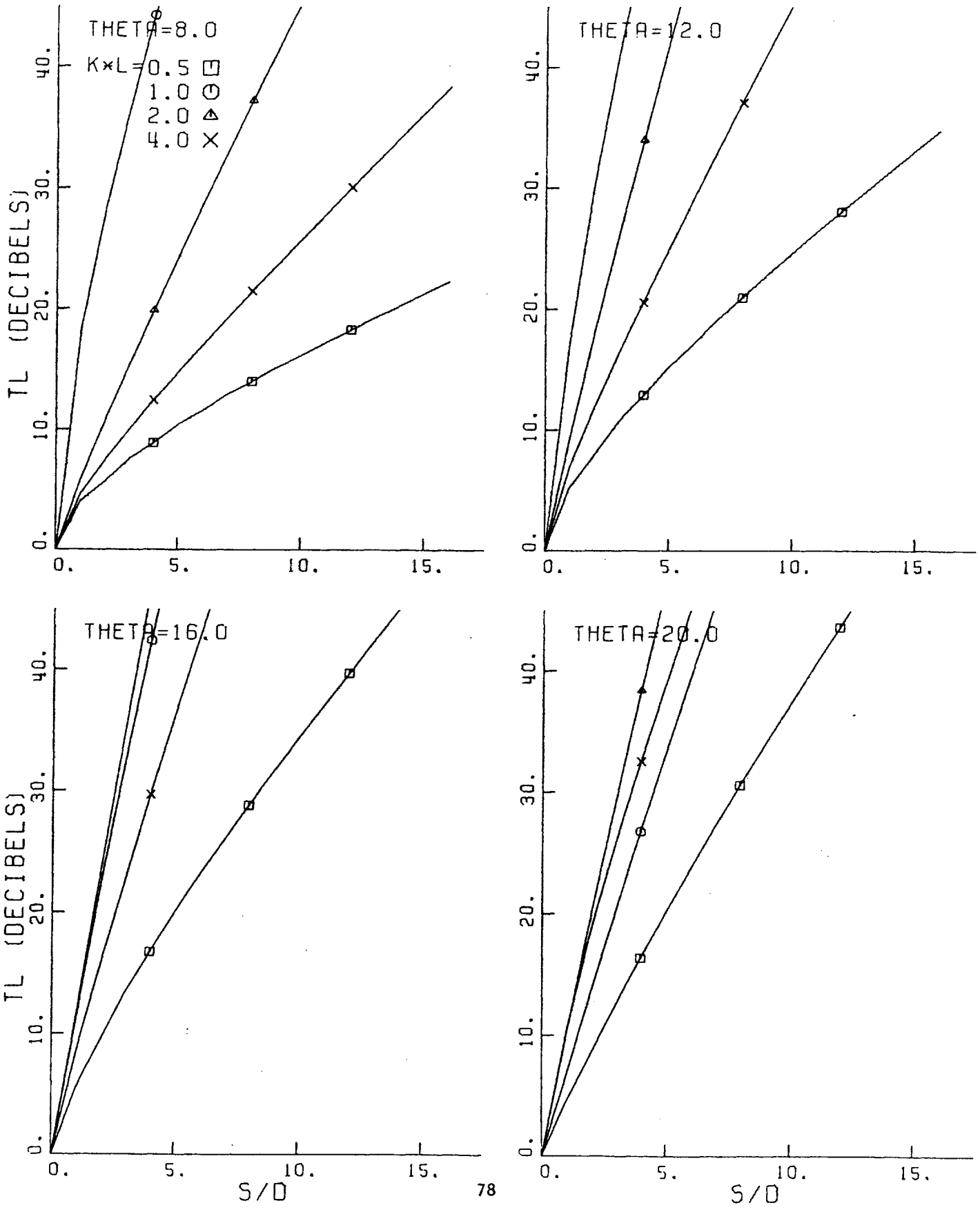


Figure A3.118

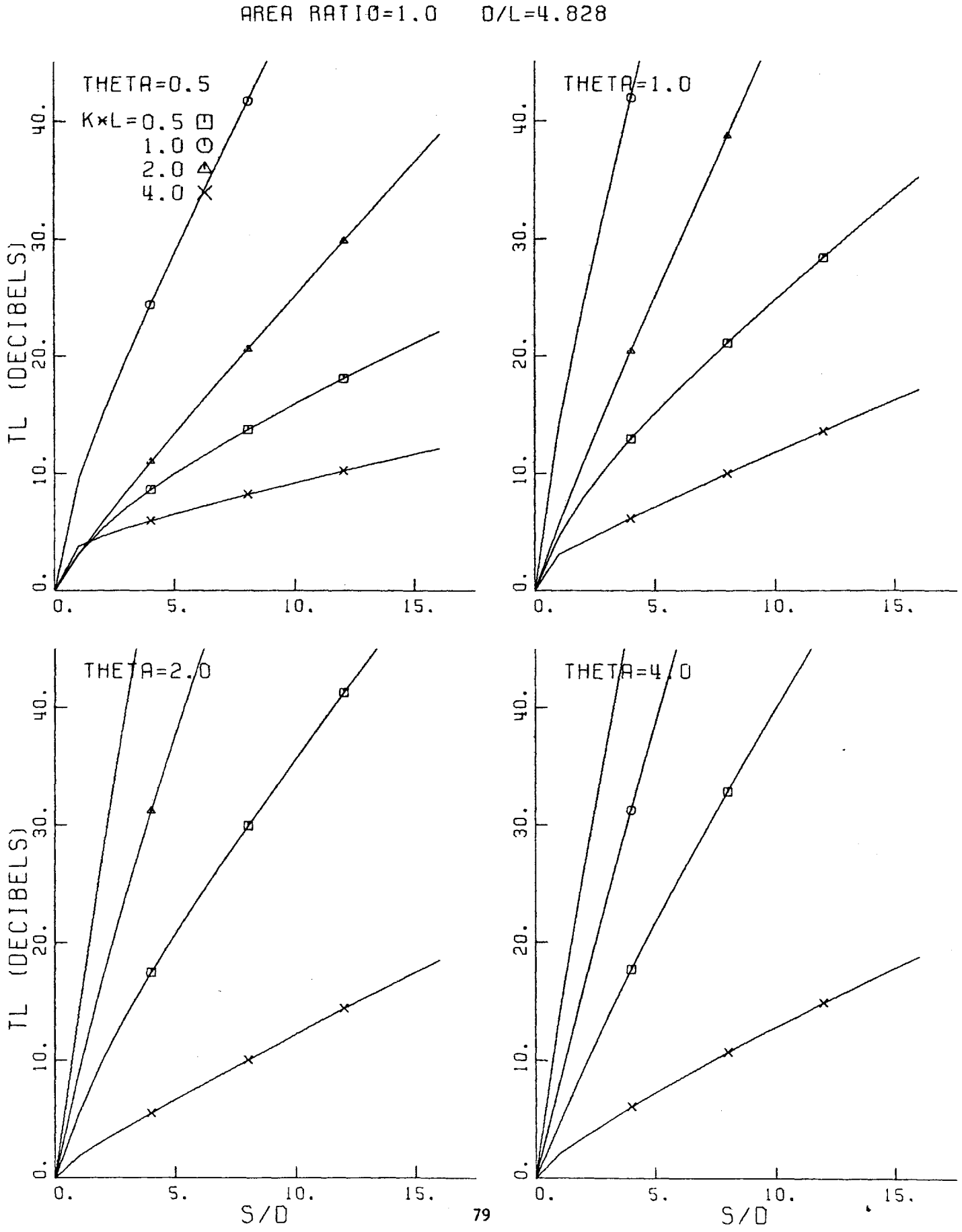


Figure A3.119

AREA RATIO=1.0 D/L=4.828

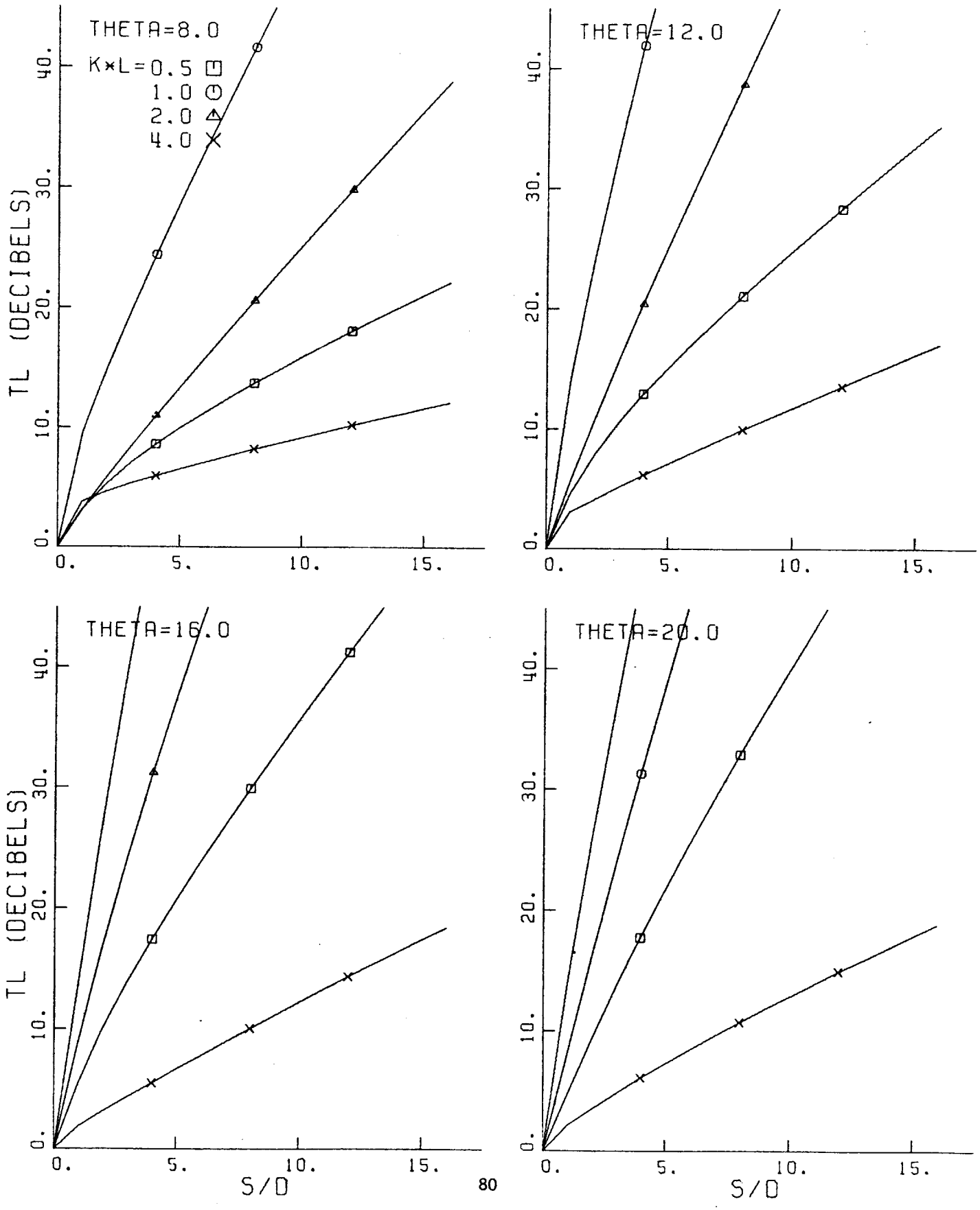


Figure A3.120

AREA RATIO=1.0 D/L=12.928

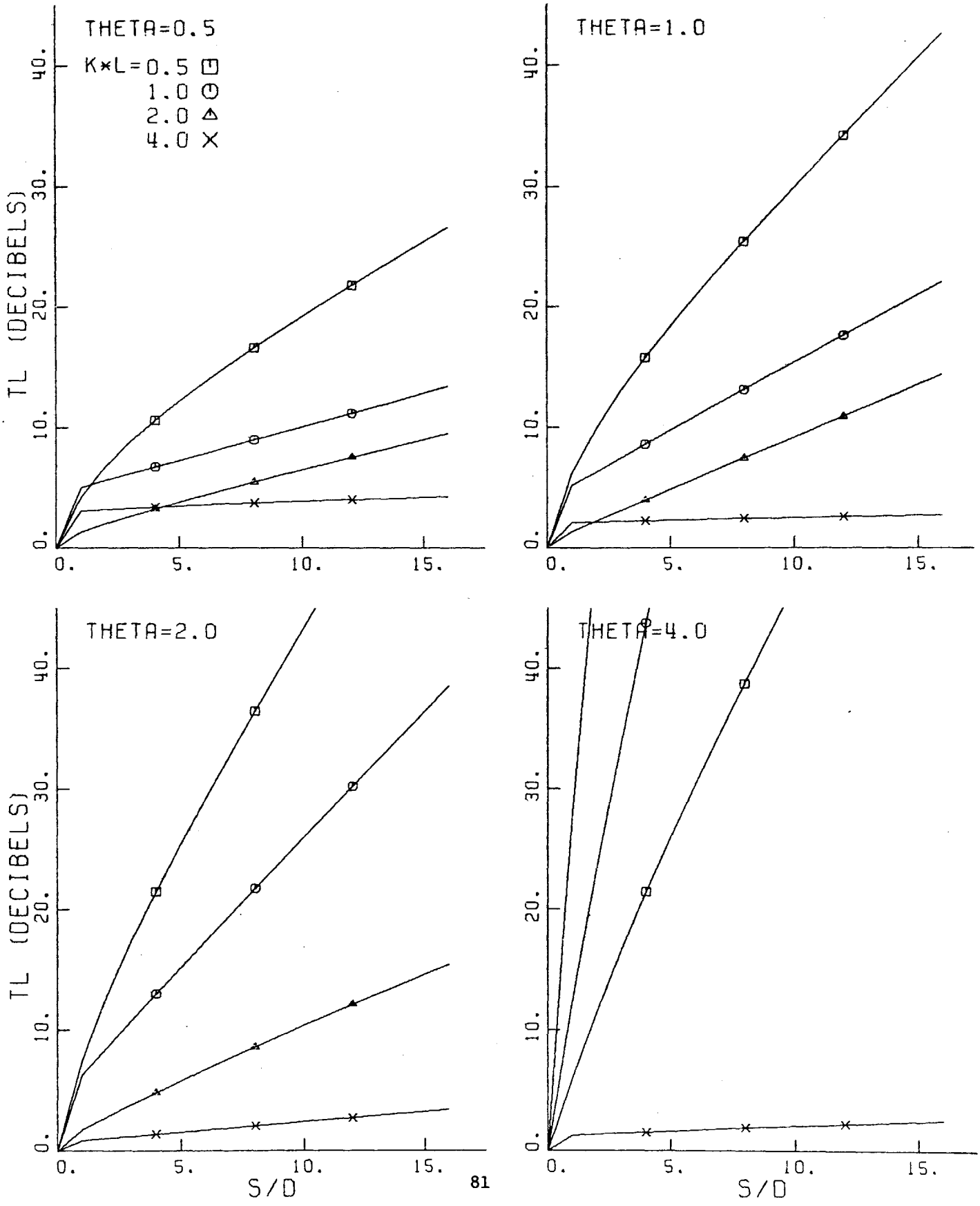
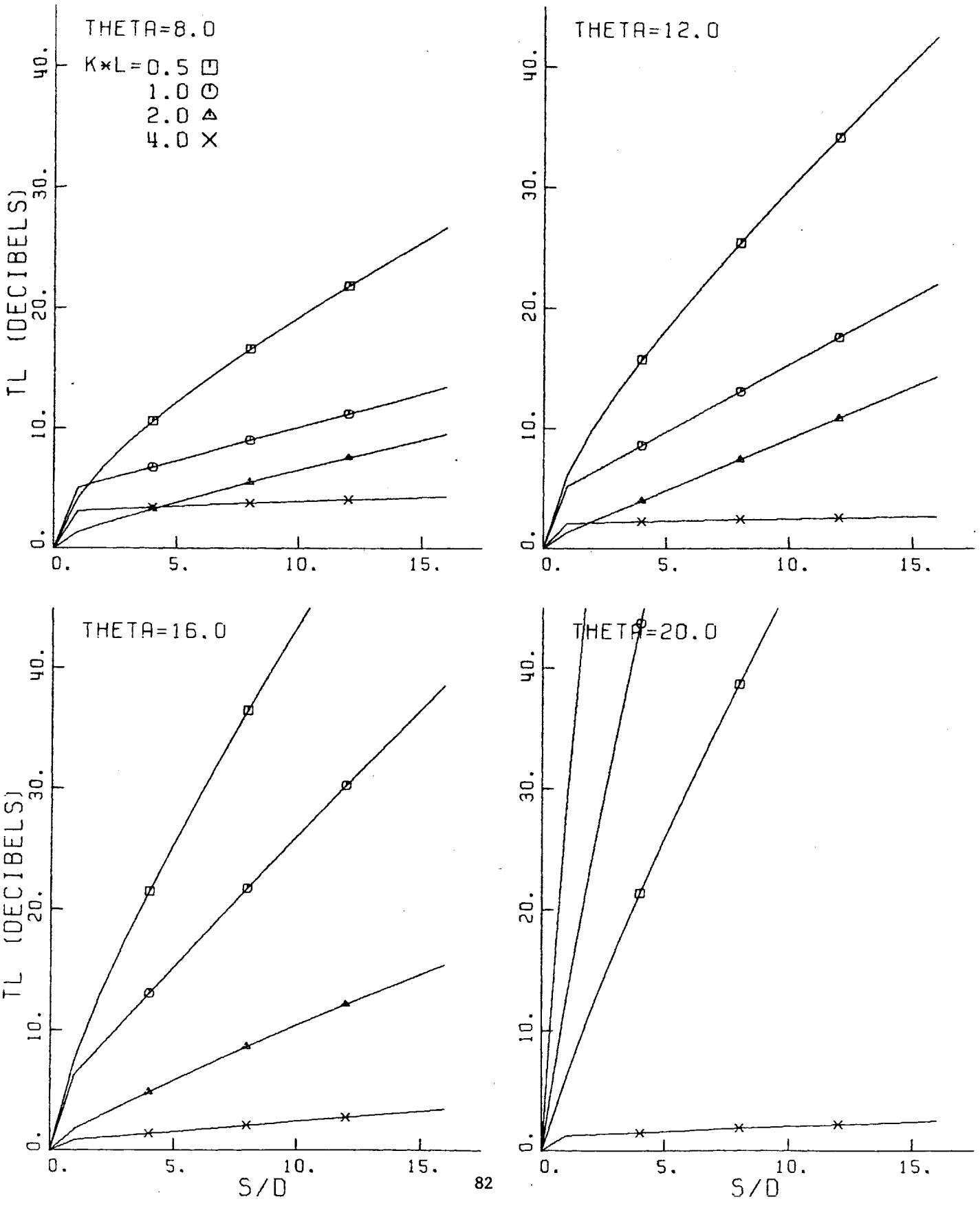


Figure A3.121

AREA RATIO=1.0 D/L=12.928



A2. COMPUTER PROGRAMS

The computer program CCHOC, which obtains complex wave numbers in lined circular ducts, has been revised. In the new program (YCHO) the Hankel functions with complex arguments are used to obtain the exact boundary impedances.

During the preparation of this addendum we have developed a new subroutine (CMJYB) to compute the complex J- and Y-Bessel functions. This subroutine is superior to the M.I.T. Math. Library subroutines for the complex Bessel functions in accuracy and speed.

To accommodate CMJYB the subroutines RTCHO, BECHO* and complex function FUN(Z)* have been revised. The other subroutines remain unchanged.

* These two subroutines were previously included in the listing of RTCHO.

```
C   THIS PROGRAM COMPUTES COMPLEX WAVE VECTORS IN A LINED CIRCULAR          YCHO0001
C   DUCT USING COMPLEX EIGENVALUES OBTAINED BY THE SUBROUTINE RTCHO,        YCHO0002
C   THEN PERFORMS INTEGRATION TO OBTAIN OCTAVE BAND TRANSMISSION            YCHO0003
C   LOSS OF SOUND POWERS.                                                  YCHO0004
C   TO FIND EXACT WALL IMPEDANCE, COMPLEX HANKEL FUNCTIONS HAVE BEEN       YCHO0005
C   USED.                                                                  YCHO0006
C THIS PROGRAM HAS BEEN CREATED BY  Y.C.CHO  AT M.I.T. GAS TURBINE         YCHO0007
C LABORATORY IN 1974.                                                      YCHO0008
      IMPLICIT COMPLEX*8 (C)                                               YCHO0009
      COMMON /RIMIT/ZIN(64,4)                                              YCHO0010
      COMMON /QFNT/AAA,BBB                                                 YCHO0011
      COMMON/BYCF/DDY0,DDY1                                                YCHO0012
      COMPLEX ANSW(4),W(4),ZIN,CKIA(2,8,4,114)                             YCHO0013
      REAL TT(8)/.5,1.,2.,4.,8.,12.,16.,20./                               YCHO0014
      REAL ARATIO(4)/.125,.25,.5,.75/                                      YCHO0015
      REAL XBS(5)/1.,2.,4.,8.,16./,CABS,COS,COUNT(7)                       YCHO0016
      REAL V(7)/.0625,.125,.25,.5,1.,2.,4./                                YCHO0017
      REAL GN(113),GP(4),YO(17),YP(17),YN(17),ZO(17),ZP(17),ZN(17)         YCHO0018
      REAL ATTO(7),ATTP(7),ATTN(7)                                         YCHO0019
      REAL ATTD(2,2,8,4,5,3,7),YCOM(113),COTAN                             YCHO0020
      REAL RCTNC(4,113)                                                    YCHO0021
      REAL*4 DDY0(30),DDY1(30)                                             YCHO0022
      REAL*8 DY0(30),DY1(30),DK,DKN                                        YCHO0023
      COMPLEX*16 ZV1,ZV2,ZZ01,ZZ11,ZY01,ZY11,ZZA,ZYA                       YCHO0024
      DIMENSION CJP(2),CYP(2)                                              YCHO0025
  100 FORMAT(8F10.5)                                                       YCHO0026
  120 FORMAT(8F10.5)                                                       YCHO0027
  110 FORMAT(8I10)                                                         YCHO0028
  202 FORMAT(1X,3I1,1X,7E17.7/)                                            YCHO0029
  300 FORMAT('1  SOUND ATTENUATION IN  CIRCULAR SILENCER LINED WITH        YCHO0030
     1RESONATOR.'/)                                                        YCHO0031
  301 FORMAT('1  SOUND ATTENUATION IN  CIRCULAR SILENCER LINED WITH        YCHO0032
     1POROUS MATERIAL.'/)                                                  YCHO0033
  601 FORMAT('1  SOUND ATTENUATION IN CIRCULAR DUCT LINED WITH             YCHO0034
     1RESONATOR.'/)                                                        YCHO0035
  602 FORMAT('1  SOUND ATTENUATION IN CIRCULAR DUCT LINED WITH             YCHO0036
```

```
     1POROUS MATERIAL.'/)                                                    YCH00037
  410 FORMAT(1X,4I2,I4,2E12.4,' NO ROOT...'/)                                YCH00038
  401 FORMAT(1X,2I2,I4,3F10.5,4E15.7//)                                      YCH00039
  404 FORMAT(3X,3I3,I4,5E15.4/)                                              YCH00040
  407 FORMAT(6X,7F17.8/)                                                     YCH00041
  408 FORMAT(1X,' TRANSMISSION COEFFICIENT COMPUTED ON OCTAVE BANDS'/        YCH00042
     11X,' CENTER FREQUENCIES ARE EQUAL TO'/)                                YCH00043
C 409 FORMAT(2X,' GAMMA=',F4.2/)                                            YCH00044
  415 FORMAT(10X,' T=',F5.2/)                                               YCH00045
  411 FORMAT(20X,' AREA RATIO=',F5.3,'   OR D/L=',F6.3/)                    YCH00046
  412 FORMAT(30X,' LENGTH OF LINING/D=',F5.2/)                              YCH00047
  501 FORMAT(3I1,I3,2X,6E12.5)                                              YCH00048
  505 FORMAT(6I1,4X,7F10.4)                                                 YCH00049
 3031 FORMAT(1X,' NUMBER OF INCORRECT ROOTS =',I5/)                         YCH00050
      READ(5,100)  ((ZIN(L,I),I=1,4),L=1,64)                               YCH00051
      READ(5,120)  (GN(L),L=1,113)                                          YCH00052
      READ(5,110)  NK,NGK,NI,NGI,NM,NGM,IJK1,IJK2                           YCH00053
      GB(1)=1./(SQRT(8.)-1.)                                                YCH00054
      GB(2)=1.                                                              YCH00055
      GB(3)=1./(SQRT(2.)-1.)                                                YCH00056
      GB(4)=1./(2./SQRT(3.)-1.)                                             YCH00057
      CEI=(0.,1.)                                                           YCH00058
      ATMX=EXP(160.)                                                        YCH00059
      PI=3.14159                                                            YCH00060
      PV=2./PI                                                              YCH00061
      ATMIV=1./ATMX                                                         YCH00062
      ATDMX=10.*ALOG10(ATMX)                                                YCH00063
      DDY0(1)=1.D0                                                          YCH00064
      DDY1(1)=1.25D0                                                        YCH00065
      DY0(1)=1.D0                                                           YCH00066
      DY1(1)=1.25D0                                                         YCH00067
      DO 1010 K=2,30                                                        YCH00068
      DK=K+1                                                                YCH00069
      DKN=K                                                                 YCH00070
      DY0(K)=DY0(K-1)+1.D0/DKN                                              YCH00071
      DY1(K)=DY0(K)+5.D-1/DK                                                YCH00072
```

```
      DDY0(K)=DY0(K)/DY0(K-1)                                          YCH00073
      DDY1(K)=DY1(K)/DY1(K-1)                                          YCH00074
 1010 CONTINUE                                                         YCH00075
      DO 40 IJK=IJK1,IJK2                                              YCH00076
      NQQ=0                                                            YCH00077
      IF(IJK.EQ.2) NGK=NGK+4                                           YCH00078
      DO 40 IJL=2,2                                                    YCH00079
      IF(IJK.EQ.1.AND.IJL.EQ.1) WRITE(6,300)                           YCH00080
      IF(IJK.EQ.1.AND.IJL.EQ.2) WRITE(6,601)                           YCH00081
      IF(IJK.EQ.2.AND.IJL.EQ.1) WRITE(6,301)                           YCH00082
      IF(IJK.EQ.2.AND.IJL.EQ.2) WRITE(6,602)                           YCH00083
C     IF(IJK.EQ.1) NJ=1                                                YCH00084
C     IF(IJK.EQ.2) NJ=2                                                YCH00085
C     IF(IJK.EQ.1) NGJ=1                                               YCH00086
C     IF(IJK.EQ.2) NGJ=3                                               YCH00087
C     DO 50 J=NJ,NGJ                                                   YCH00088
C     GAMJ=GAM(J)                                                      YCH00089
      GAMJ=1.5                                                         YCH00090
      DO 50 K=NK,NGK                                                   YCH00091
      TTK=TT(K)                                                        YCH00092
      DO 50 I=NI,NGI                                                   YCH00093
      GBI=GB(I)                                                        YCH00094
      DO 50 M=NM,NGM                                                   YCH00095
      XRSM=XRS(M)                                                      YCH00096
      DO 55 LS=1,113                                                   YCH00097
      GNL=GN(LS)                                                       YCH00098
      GNB=GNL*GBI                                                      YCH00099
      GLB=GNL+GNB                                                      YCH00100
      IF(M.NE.1) GO TO 33                                              YCH00101
      IF(IJK.EQ.2) GO TO 15                                            YCH00102
      IF(K.NE.NK) GO TO 666                                            YCH00103
      ERB=.0001                                                        YCH00104
      CALL BFSJ(GNB,0,BJ01,ERB,IER)                                    YCH00105
      CALL BESJ(GNB,1,BJ11,ERB,IER)                                    YCH00106
      CALL BESJ(GLB,1,BJ12,ERB,IER)                                    YCH00107
      CALL BESY(GNB,0,BY01,IER)                                        YCH00108
```

```
      CALL BESY(GNB,1,BY11,IER)                                          YCHO0109
      CALL BESY(GLB,1,BY12,IER)                                          YCHO0110
      RJNOR=BJ01*BY12-BY01*BJ12                                          YCHO0111
      RJDOM=BJ11*BY12-BY11*BJ12                                          YCHO0112
      RCTNC(I,LS)=RJNOR/RJDOM                                            YCHO0113
  666 CTTK=RCTNC(I,LS)+(0.,1.)*TTK                                       YCHO0114
      CAB=GNB/CTTK                                                       YCHO0115
      GO TO 16                                                           YCHO0116
   15 RTG=SQRT(.5*GAMJ)                                                  YCHO0117
      SQRTN=SQRT(1.+(TTK/GNL)**2)                                        YCHO0118
      QPKR=RTG*SQRT(SQRTN+1.)                                            YCHO0119
      QPKI=RTG*SQRT(SQRTN-1.)                                            YCHO0120
      CQHR=CMPLX(QPKR,QPKI)                                              YCHO0121
      X1=GNB*QPKR                                                        YCHO0122
      Y1=GNB*QPKI                                                        YCHO0123
      X2=X1+QPKR*GNL                                                     YCHO0124
      Y2=Y1+QPKI*GNL                                                     YCHO0125
      RXY1=SQRT(X1*X1+Y1*Y1)                                             YCHO0126
      RXY2=SQRT(X2*X2+Y2*Y2)                                             YCHO0127
      CZ1=CMPLX(X1,Y1)                                                   YCHO0128
      CZ2=CMPLX(X2,Y2)                                                   YCHO0129
      CPL1=(1.,0.)                                                       YCHO0130
      IF(RXY1.GE.7.) GO TO 7301                                          YCHO0131
      CALL CMJYES(CZ1,RXY1,CJB,CYB)                                      YCHO0132
      CHK011=CJB(1)+CEI*CYB(1)                                           YCHO0133
      CHK021=CJB(1)-CEI*CYB(1)                                           YCHO0134
      CHK111=CJB(2)+CEI*CYB(2)                                           YCHO0135
      CHK121=CJB(2)-CEI*CYB(2)                                           YCHO0136
      IF(CABS(CHK011).LT.1.E-50) GO TO 7201                              YCHO0137
      CPU1=CHK111/CHK011                                                 YCHO0138
      CPU2=CHK121/CHK011                                                 YCHO0139
      CPL2=CHK021/CHK011                                                 YCHO0140
      GO TO 7202                                                         YCHO0141
 7201 CPU1=CHK111                                                        YCHO0142
      CPU2=CHK121                                                        YCHO0143
      CPL1=(0.,0.)                                                       YCHO0144
```

```
      CPL2=CHK021                                                       YCHO0145
7202  IF(RXY2.GE.7.) GO TO 7401                                         YCHO0146
      CALL CMJYFS(CZ2,RXY2,CJB,CYB)                                     YCHO0147
      CPUL=(CJB(2)+CEI*CYB(2))/(CJB(2)-CEI*CYB(2))                      YCHO0148
      GO TO 7501                                                        YCHO0149
7401  IF(Y2.GT.80.) GO TO 7402                                          YCHO0150
      KP2=RXY2+1.                                                       YCHO0151
      IF(KP2.GE.11) KP2=11                                              YCHO0152
      ZV2=CZ2                                                           YCHO0153
      CALL JYHSL(1,KP2,ZV2,ZZA,ZYA)                                     YCHO0154
      CFTAI=EXP(-2.*Y2)                                                 YCHO0155
      CFTAR=CEI*COS(2.*X2)-SIN(2.*X2)                                   YCHO0156
      CZTA=ZZA                                                          YCHO0157
      CYTA=ZYA                                                          YCHO0158
      CPUL=CFTAI*CFTAR*(CZTA+CEI*CYTA)/(CZTA-CEI*CYTA)                  YCHO0159
      GO TO 7501                                                        YCHO0160
7402  CPUL=(0.,0.)                                                      YCHO0161
      GO TO 7501                                                        YCHO0162
7301  ZV2=CZ2                                                           YCHO0163
      ZV1=CZ1                                                           YCHO0164
      KP1=RXY1+1                                                        YCHO0165
      KP2=RXY2+1                                                        YCHO0166
      IF(KP1.GE.11) KP1=11                                              YCHO0167
      IF(KP2.GE.11) KP2=11                                              YCHO0168
      CALL JYHSL(0,KP1,ZV1,ZZ01,ZY01)                                   YCHO0169
      CALL JYHSL(1,KP1,ZV1,ZZ11,ZY11)                                   YCHO0170
      CZZ01=ZZ01                                                        YCHO0171
      CZY01=ZY01                                                        YCHO0172
      CZZ11=ZZ11                                                        YCHO0173
      CZY11=ZY11                                                        YCHO0174
      CPPF=CZZ01+CEI*CZY01                                              YCHO0175
      CPU1=-CEI*(CZZ11+CEI*CZY11)/CPPF                                  YCHO0176
      CPU2=-CEI*(CZZ11-CEI*CZY11)/CPPF                                  YCHO0177
      CPL2=-(CZZ01-CEI*CZY01)/CPPF                                      YCHO0178
      Y21=Y2-Y1                                                         YCHO0179
      IF(Y21.GT.80.) GO TO 7302                                         YCHO0180
```

```
      X21=X2-X1                                                          YCHO0181
      CALL JYHSL(1,KP2,ZV2,ZZA,ZYA)                                      YCHO0182
      CZZ2=ZZA                                                           YCHO0183
      CZY2=ZYA                                                           YCHO0184
      CFTAI=EXP(-2.*Y21)                                                 YCHO0185
      CFTAR=COS(2.*X21)+CEI*SIN(2.*X21)                                  YCHO0186
      CPUL=CFTAI*CFTAR*(CZZ2+CEI*CZY2)/(CZZ2-CEI*CZY2)                   YCHO0187
      GO TO 7501                                                         YCHO0188
 7302 CPUL=(0.,0.)                                                       YCHO0189
 7501 CTANM=CPU1-CPUL*CPU2                                               YCHO0190
      CTAND=CPL1-CPUL*CPL2                                               YCHO0191
      IF(CABS(CTAND).LT.1.E-50) GO TO 7502                               YCHO0192
      CTAN=CTANM/CTAND                                                   YCHO0193
      CAB=GNB*CTAN/CQHR                                                  YCHO0194
      GO TO 16                                                           YCHO0195
 7502 IF(CABS(CTANM).GT.1.E-30) GO TO 7503                               YCHO0196
      CAB=1.E+20                                                         YCHO0197
      GO TO 16                                                           YCHO0198
 7503 WRITE(6,401) K,I,LS,TTK,GBI,GNL,CZ1,CZ2                            YCHO0199
      CAB=1.1111                                                         YCHO0200
   16 AAA=REAL(CAB)                                                      YCHO0201
      BBB=AIMAG(CAB)                                                     YCHO0202
      IF(BBB.EQ.0..AND.AAA.GT.1.E+4) GO TO 303                           YCHO0203
      GO TO 351                                                          YCHO0204
  303 AAA=10000.                                                         YCHO0205
      ANSW(1)=CMPLX(2.40483,0.)                                          YCHO0206
      W(1)=CMPLX(0.,0.)                                                  YCHO0207
      NQ7=1                                                              YCHO0208
      TERMQ=GNB**2-2.40483**2                                            YCHO0209
      IF(TERMQ) 304,305,305                                              YCHO0210
  304 REALK=0.                                                           YCHO0211
      AIMAK=SQRT(-TERMQ)                                                 YCHO0212
      GO TO 22                                                           YCHO0213
  305 REALK=SQRT(TERMQ)                                                  YCHO0214
      AIMAK=0.                                                           YCHO0215
      GO TO 22                                                           YCHO0216
```

```
351 IF(ABS(AAA).LE.1.E-5) AAA=0.                                                  YCHO0217
    IF(ABS(BBB).LE.1.E-5) BBB=0.                                                  YCHO0218
    CALL RTCHO(NQ7,ANSW,W)                                                        YCHO0219
    IF(NQ7.EQ.0) GO TO 60                                                         YCHO0220
    XIRE=REAL(ANSW(1))                                                            YCHO0221
    XIIM=AIMAG(ANSW(1))                                                           YCHO0222
    CALL WAVEK(GNB,XIRE,XIIM,REALK,AIMAK)                                         YCHO0223
 22 CKIA(IJK,K,I,LS)=CMPLX(REALK,AIMAK)                                           YCHO0224
    AW=CABS(W(1))                                                                 YCHO0225
    IF(AW.LT..01) GO TO 33                                                        YCHO0226
    WRITE(6,404) IJK,K,I,LS,AW,AAA,BBB,ANSW(1)                                    YCHO0227
 33 CYKE=CKIA(IJK,K,I,LS)*XRSM*2.                                                 YCHO0228
    IF(IJL.EQ.1) CKR=GBI*CKIA(IJK,K,I,LS)/GNL/(1.+GBI)**2                         YCHO0229
    IF(IJL.EQ.2) CKR=CKIA(IJK,K,I,LS)/GNB                                         YCHO0230
    CKRV=(0.,.5)*(CKR+1./CKR)                                                     YCHO0231
    AMCK=AIMAG(CYKE)                                                              YCHO0232
    IF(AMCK.GT.80.) GO TO 5                                                       YCHO0233
    CSUM=CCOS(CYKE)-CKRV*CSIN(CYKE)                                               YCHO0234
    ASUM=CABS(CSUM)                                                               YCHO0235
    YCOM(LS)=1./ASUM**2*ATMX                                                      YCHO0236
    GO TO 55                                                                      YCHO0237
  5 IF(AMCK.GT.160.) GO TO 6                                                      YCHO0238
    YCOM(LS)=4.*EXP(-2.*(AMCK-80.))/CABS(1.+CKRV)**2                              YCHO0239
    GO TO 55                                                                      YCHO0240
  6 YCOM(LS)=0.                                                                   YCHO0241
    GO TO 55                                                                      YCHO0242
 60 WRITE(6,410) NQ7,IJK,K,I,LS,AAA,BBB                                           YCHO0243
    LSSS=LS-1                                                                     YCHO0244
    CKIA(IJK,K,I,LS)=CKIA(IJK,K,I,LSSS)                                           YCHO0245
    YCOM(LS)=YCOM(LSSS)                                                           YCHO0246
    NQQ=NQQ+1                                                                     YCHO0247
    IF(NQQ.GT.20) GO TO 3030                                                      YCHO0248
 55 CONTINUE                                                                      YCHO0249
    CKIA(IJK,K,I,114)=(0.,0.)                                                     YCHO0250
    DO 20 L=1,7                                                                   YCHO0251
    HI=SQRT(2.)*V(L)                                                              YCHO0252
```

```
      H=.0625*HI                                                        YCH00253
      LM=16*(L-1)                                                       YCH00254
      DO 30 KL=1,17                                                     YCH00255
      KLM=KL+LM                                                         YCH00256
      YO(KL)=YCOM(KLM)/HI                                               YCH00257
      AKL=FLOAT(KL-1)                                                   YCH00258
      YP(KL)=(HI+H*AKL)**2*YO(KL)                                       YCH00259
      YN(KL)=YO(KL)/(HI+H*AKL)**2                                       YCH00260
   30 CONTINUE                                                          YCH00261
      CALL QSF(H,YO,ZO,17)                                              YCH00262
      IF(ZO(17).LT.ATMIV) GO TO 21                                      YCH00263
      ATTD(IJK,IJL,K,I,M,1,L)=-10.*ALOG10(ZO(17))+ATDMX                 YCH00264
      GO TO 26                                                          YCH00265
   21 ATTD(IJK,IJL,K,I,M,1,L)=2.*ATDMX                                  YCH00266
   26 CALL QSF(H,YP,ZP,17)                                              YCH00267
      ATTP(L)=3.*ZP(17)/HI**2/7.                                        YCH00268
      IF(ATTP(L).LT.ATMIV) GO TO 23                                     YCH00269
      ATTD(IJK,IJL,K,I,M,2,L)=-10.*ALOG10(ATTP(L))+ATDMX                YCH00270
      GO TO 24                                                          YCH00271
   23 ATTD(IJK,IJL,K,I,M,2,L)=2.*ATDMX                                  YCH00272
   24 CALL QSF(H,YN,ZN,17)                                              YCH00273
      ATTN(L)=2.*ZN(17)*HI**2                                           YCH00274
      IF(ATTN(L).LT.ATMIV) GO TO 25                                     YCH00275
      ATTD(IJK,IJL,K,I,M,3,L)=-10.*ALOG10(ATTN(L))+ATDMX                YCH00276
      GO TO 20                                                          YCH00277
   25 ATTD(IJK,IJL,K,I,M,3,L)=2.*ATDMX                                  YCH00278
   20 CONTINUE                                                          YCH00279
   50 CONTINUE                                                          YCH00280
      DO 54 LX=1,7                                                      YCH00281
      COUNT(LX)=2.*V(LX)                                                YCH00282
   54 CONTINUE                                                          YCH00283
      WRITE(6,408)                                                      YCH00284
      WRITE(6,407) (COUNT(LX),LX=1,7)                                   YCH00285
C     DO 56 JX=NJ,NGJ                                                   YCH00286
C     WRITE(6,409)  GAM(JX)                                             YCH00287
      DO 56 KX=NK,NGK                                                   YCH00288
```

```
      KP=KX                                                                  YCH00289
      WRITE(6,415) TT(KX)                                                    YCH00290
      DO 56 IX=NI,NGI                                                        YCH00291
      TGB=2.*GE(IX)                                                          YCH00292
      WRITE(6,411) ARATIO(IX),TGB                                            YCH00293
      DO 56 MX=NM,NGM                                                        YCH00294
      WRITE(6,412) XRS(MX)                                                   YCH00295
C     IF(IJK.EQ.1) JY=JX                                                     YCH00296
C     IF(IJK.EQ.2) JY=JX-1                                                   YCH00297
      WRITE(6,202) KP,IX,MX,(ATTD(IJK,IJL,KX,IX,MX,1,LX),LX=1,7)             YCH00298
      WRITE(6,202) KP,IX,MX,(ATTD(IJK,IJL,KX,IX,MX,2,LX),LX=1,7)             YCH00299
      WRITE(6,202) KP,IX,MX,(ATTD(IJK,IJL,KX,IX,MX,3,LX),LX=1,7)             YCH00300
   56 CONTINUE                                                               YCH00301
   40 CONTINUE                                                               YCH00302
      NGK=4                                                                  YCH00303
      DO 1001 IJKB=IJK1,IJK2                                                 YCH00304
      IF(IJKB.EQ.2) NGK=NGK+4                                                YCH00305
      DO 1001 KB=NK,NGK                                                      YCH00306
      KN=KB                                                                  YCH00307
      DO 1001 IB=1,4                                                         YCH00308
      DO 1002 LAB=1,38                                                       YCH00309
      LABI=3*(LAB-1)+1                                                       YCH00310
      LABT=LABI+2                                                            YCH00311
      WRITE(7,501) IJKB,KN,IB,LABI,(CKIA(IJKB,KB,IB,LSB),LSB=LABI,LABT)      YCH00312
 1002 CONTINUE                                                               YCH00313
 1001 CONTINUE                                                               YCH00314
      NGK=4                                                                  YCH00315
      DO 2001 IJKD=IJK1,IJK2                                                 YCH00316
      IF(IJKD.EQ.2) NGK=NGK+4                                                YCH00317
      DO 2001 IJLD=2,2                                                       YCH00318
      DO 2001 NB=1,3                                                         YCH00319
      DO 2001 KD=NK,NGK                                                      YCH00320
      KM=KD                                                                  YCH00321
      DO 2001 ID=1,4                                                         YCH00322
      DO 2001 MD=1,5                                                         YCH00323
      WRITE(7,505) IJKD,IJLD,NB,KM,ID,MD,(ATTD(IJKD,IJLD,KD,ID,MD,NB,        YCH00324
```

```
    1LXD),LXD=1,7)                                                  YCH00325
2001 CONTINUE                                                       YCH00326
3030 WRITE(6,3031) NQQ                                              YCH00327
     STOP                                                           YCH00328
     END                                                            YCH00329
```

```
      SUBROUTINE CMJYB(CZ,CJB,CYB)                                      CMJY0001
C  CMJYB COMPUTES THE FIRST AND SECOND COMPLEX J- AND Y-BESSEL.          CMJY0002
C THIS PROGRAM HAS BEEN CREATED BY  Y.C.CHO  AT M.I.T. GAS TURBINE      CMJY0003
C LABORATORY IN 1974.                                                   CMJY0004
      COMPLEX*8 CZ,CJB(2),CYB(2)                                        CMJY0005
      RXY=CABS(CZ)                                                      CMJY0006
      IF(RXY.GE.7.) GO TO 200                                           CMJY0007
      CALL CMJYBS(CZ,RXY,CJB,CYB)                                       CMJY0008
      GO TO 201                                                         CMJY0009
  200 CALL CMJYBG(CZ,CJB,CYB)                                           CMJY0010
  201 RETURN                                                            CMJY0011
      END                                                               CMJY0012
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```
      SUBROUTINE CMJYBS(CZ,RXY,CJB,CYB)                                 CJYS0001
      IMPLICIT COMPLEX*8 (C,Z)                                          CJYS0002
      COMMON/BYCF/DDY0,DDY1                                             CJYS0003
      DIMENSION ZX0(30),CJB(2),CYB(2),DDY0(30),DDY1(30)                 CJYS0004
      DPI=3.14159                                                       CJYS0005
      DPV=2./DPI                                                        CJYS0006
      DGM=.577216                                                       CJYS0007
      ZHF=CZ/2.                                                         CJYS0008
      ZLG=CLOG(ZHF)+DGM                                                 CJYS0009
      NL=30                                                             CJYS0010
      IF(RXY.LT.2.) NL=10                                               CJYS0011
      IF(RXY.LT..2) NL=5                                                CJYS0012
      IF(RXY.LT..01) NL=3                                               CJYS0013
      IF(RXY.LT.1.E-5) NL=2                                             CJYS0014
      IF(RXY.LT.1.E-20) GO TO 100                                       CJYS0015
      ZZ0=1.                                                            CJYS0016
      ZZ1=1.                                                            CJYS0017
      ZY1=1.                                                            CJYS0018
      ZY0=1.                                                            CJYS0019
      NLS=NL+1                                                          CJYS0020
      ZT=-.25*CZ*CZ                                                     CJYS0021
      DO 50 K=1,NL                                                      CJYS0022
      NS=NLS-K                                                          CJYS0023
      DK0=NS*NS                                                         CJYS0024
      DK1=NS*(NS+1)                                                     CJYS0025
      ZTD0=ZT/DK0                                                       CJYS0026
      ZZ0=1.00+ZZ0*ZTD0                                                 CJYS0027
      ZTD1=ZT/DK1                                                       CJYS0028
      ZZ1=1.00+ZZ1*ZTD1                                                 CJYS0029
      ZY0=1.00+ZY0*ZTD0*DDY0(NS)                                        CJYS0030
      ZY1=1.00+ZY1*ZTD1*DDY1(NS)                                        CJYS0031
   50 CONTINUE                                                          CJYS0032
      CJB(1)=ZZ0                                                        CJYS0033
      ZJB1=ZZ1*ZHF                                                      CJYS0034
      CJB(2)=ZJB1                                                       CJYS0035
      CYB(1)=DPV*(ZLG*ZZ0-ZY0+1.00)                                     CJYS0036
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      CYB(2)=DPV*(ZLG*ZJB1-ZHF*(ZY1-.5)-1./CZ)                    CJYS0037
      GO TO 2                                                     CJYS0038
  100 CJB(1)=(1.,0.)                                              CJYS0039
      CJB(2)=.5*CZ                                                CJYS0040
      CYB(1)=DPV*ZLG                                              CJYS0041
      CYB(2)=-DPV/CZ                                              CJYS0042
    2 RETURN                                                      CJYS0043
      END                                                         CJYS0044
```

```
      SUBROUTINE CMJYBG(CZ,CJB,CYB)                                     CJYG0001
C  IF ABS(DY) IS GREATER THAN 50.,EXP(-ABS(DY))*CJB AND CYB ARE RETURNED CJYG0002
      IMPLICIT REAL*8 (D),COMPLEX*8 (C),COMPLEX*16 (Z)                  CJYG0003
      REAL*8 DPHR(2),DCSR(2),DSNR(2),DOSR(2),DOSI(2),DINR(2),DINI(2)    CJYG0004
      REAL*8 DRZA(2),DRYA(2),DIZA(2),DIYA(2),DRW(2),DIW(2)              CJYG0005
      REAL*8 DJR(2),DJI(2)                                              CJYG0006
      REAL*8 DRV(2),DIV(2),DYR(2),DYI(2)                                CJYG0007
      COMPLEX*16 DCMPLX                                                 CJYG0008
      DIMENSION CJB(2),CYB(2)                                           CJYG0009
      DPI=3.141592653590                                                CJYG0010
      DX=REAL(CZ)                                                       CJYG0011
      DY=AIMAG(CZ)                                                      CJYG0012
      DXY=DSQRT(DX*DX+DY*DY)                                            CJYG0013
      ZV=DCMPLX(DX,DY)                                                  CJYG0014
      DXS=DX/DXY                                                        CJYG0015
      DQR=DSQRT(1.D0+DXS)                                               CJYG0016
      DQI=-DSQRT(1.D0-DXS)                                              CJYG0017
      DAPT=DSQRT(1.D0/DPI)/DSQRT(DXY)                                   CJYG0018
      IF(DY.GT.5.D+1) GO TO 20                                          CJYG0019
      IF(DY.LT.-5.D+1) GO TO 21                                         CJYG0020
      DXPP=DEXP(DY)                                                     CJYG0021
      DXPN=1./DXPP                                                      CJYG0022
      GO TO 30                                                          CJYG0023
   20 DXPP=1.D0                                                         CJYG0024
      DXPN=0.D0                                                         CJYG0025
      GO TO 30                                                          CJYG0026
   21 DXPP=0.D0                                                         CJYG0027
      DXPN=1.D0                                                         CJYG0028
   30 DCSH=5.D-1*(DXPP+DXPN)                                            CJYG0029
      DSNH=5.D-1*(DXPP-DXPN)                                            CJYG0030
      KP=DXY+1.                                                         CJYG0031
      IF(KP.GE.11) KP=11                                                CJYG0032
      DO 10 II=1,2                                                      CJYG0033
      DII=2.5D-1                                                        CJYG0034
      IF(II.EQ.2) DII=7.5D-1                                            CJYG0035
      DPHR(II)=DX-DII*DPI                                               CJYG0036
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      DCSR(II)=DCOS(DPHR(II))                                          CJYG0037
      DSNR(II)=DSIN(DPHR(II))                                          CJYG0038
      DOSR(II)=DCSR(II)*DCSH                                           CJYG0039
      DOSI(II)=-DSNR(II)*DSNH                                          CJYG0040
      DINR(II)=DSNR(II)*DCSH                                           CJYG0041
      DINI(II)=DCSR(II)*DSNH                                           CJYG0042
      NKU=II-1                                                         CJYG0043
      CALL JYHSL(NKU,KP,ZV,ZZA,ZYA)                                    CJYG0044
      DRZA(II)=DREAL(ZZA)                                              CJYG0045
      DIZA(II)=DIMAG(ZZA)                                              CJYG0046
      DRYA(II)=DREAL(ZYA)                                              CJYG0047
      DIYA(II)=DIMAG(ZYA)                                              CJYG0048
      DRW(II)=DRZA(II)*DOSR(II)-DIZA(II)*DOSI(II)-DRYA(II)*DINR(II)+   CJYG0049
     1 DIYA(II)*DINI(II)                                               CJYG0050
      DIW(II)=DRZA(II)*DOSI(II)+DIZA(II)*DOSR(II)-DRYA(II)*DINI(II)-   CJYG0051
     1 DIYA(II)*DINR(II)                                               CJYG0052
      DJR(II)=DAPT*(DQR*DRW(II)-DQI*DIW(II))                           CJYG0053
      DJI(II)=DAPT*(DQR*DIW(II)+DQI*DRW(II))                           CJYG0054
      DRV(II)=DRZA(II)*DINR(II)-DIZA(II)*DINI(II)+DRYA(II)*DOSR(II)-   CJYG0055
     1 DIYA(II)*DOSI(II)                                               CJYG0056
      DIV(II)=DRZA(II)*DINI(II)+DIZA(II)*DINR(II)+DRYA(II)*DOSI(II)+   CJYG0057
     1 DIYA(II)*DOSR(II)                                               CJYG0058
      DYR(II)=DAPT*(DQR*DRV(II)-DQI*DIV(II))                           CJYG0059
      DYI(II)=DAPT*(DQR*DIV(II)+DQI*DRV(II))                           CJYG0060
      CJB(II)=DCMPLX(DJR(II),DJI(II))                                  CJYG0061
      CYB(II)=DCMPLX(DYR(II),DYI(II))                                  CJYG0062
   10 CONTINUE                                                         CJYG0063
      RETURN                                                           CJYG0064
      END                                                              CJYG0065
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```
      SUBROUTINE JYHSL(N,KP,ZX,ZZTA,ZYTA)                                JYSL0001
C     JYHSL OBTAINS FACTOR OF A COMPLEX BESSEL FUNCTION FOR A LARGE ARG. JYSL0002
      IMPLICIT REAL*8 (D),COMPLEX*16 (Z)                                 JYSL0003
      ZT=1.5625D-2/ZX/ZX                                                 JYSL0004
      DN4=4*N*N                                                          JYSL0005
      ZZTM=1.D0                                                          JYSL0006
      ZYTM=1.D0                                                          JYSL0007
      IF(N.EQ.1) KP=KP+1                                                 JYSL0008
      IF(KP.LE.2) KP=2                                                   JYSL0009
      KPS=KP-1                                                           JYSL0010
      DO 10 J=1,KPS                                                      JYSL0011
      DJ4=4*(KP-J)                                                       JYSL0012
      DJ2=2*(KP-J)                                                       JYSL0013
      DNJ41=DN4-(DJ4-1.D0)*(DJ4-1.D0)                                    JYSL0014
      DNJ43=DN4-(DJ4-3.D0)*(DJ4-3.D0)                                    JYSL0015
      DJ21=DJ2*(DJ2-1.D0)                                                JYSL0016
      ZFTZ=DNJ41*DNJ43/DJ21                                              JYSL0017
      ZZTM=1.D0-ZZTM*ZFTZ*ZT                                             JYSL0018
      DNJ45=DN4-(DJ4+1.D0)*(DJ4+1.D0)                                    JYSL0019
      DJ22=DJ2*(DJ2+1.D0)                                                JYSL0020
      ZFTY=DNJ41*DNJ45/DJ22                                              JYSL0021
      ZYTM=1.D0-ZYTM*ZFTY*ZT                                             JYSL0022
   10 CONTINUE                                                           JYSL0023
      ZZTA=ZZTM                                                          JYSL0024
      ZYTA=(DN4-1.D0)*1.25D-1/ZX*ZYTM                                    JYSL0025
      RETURN                                                             JYSL0026
      END                                                                JYSL0027
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      SUBROUTINE RTCHO(NQ7,ANSW,W)                                                    RTCH0001
C     SUBROUTINE RCHO OBTAINS FUNDAMENTAL EIGENVALUES FOR WAVES IN A                  RTCH0002
C     LINED CIRCULAR DUCT.  IN OTHER WORD, IT OBTAINS FIRST ROOTS                     RTCH0003
C     OF THE EQUATION,  X*J1(X)/J0(X)=CMPLX(AAA,BBB), WHERE J0(X)                     RTCH0004
C     AND J1(X) ARE BESSEL FUNCTIONS OF THE ORDERS OF ZERO AND ONE                    RTCH0005
C     WITH COMPLEX ARGUMENTS.                                                         RTCH0006
C     RTCHO CALLS BECHO AND YSQNK9.  BECHO CALLS CMJYB AND SSP                        RTCH0007
C     SUBROUTINES FOR REAL BESSEL FUCTIONS.  YSQNK9 IS THE                            RTCH0008
C     SUBROUTINE REVISED FORM M.I.T. MATH. LIBRARY SUBROUTINE.                        RTCH0009
C     IN CALLING PROGRAM, COMMON STATEMENT SHOULD BE MADE TO SUPLY VALUES             RTCH0010
C     OF AAA AND BBB, AND ZIN(64,4).                                                  RTCH0011
      IMPLICIT COMPLEX*8 (C)                                          RCHO0020 RTCH0012
      COMPLEX*8 Z(4),ERROR,ANSW(4),RETERR,FUN,W(4),CMPLX              RCHO0030 RTCH0013
      COMPLEX*8 CJB(2),CYB(2)                                                  RTCH0014
      COMPLEX*8 ZIN,ZAB                                               RCHO0040 RTCH0015
      COMMON /RIMIT/ZIN(64,4)                                                  RTCH0016
      COMMON /QFNT/AAA,BBB                                                     RTCH0017
      REAL*4 JR0,JI0,JR1,JI1,CABS,COS                                          RTCH0018
      EXTERNAL FUN,FCT,FCTI                                           RCHO0070 RTCH0019
   91 FORMAT(1X,' ROOT IS NOT ACCURATE')                              RCHO0080 RTCH0020
      AAQ=AAA**2                                                      RCHO0090 RTCH0021
      BBQ=BBB**2                                                      RCHO0100 RTCH0022
      ABQ=AAQ+BBQ                                                     RCHO0110 RTCH0023
      IF(ABQ.GT.4.) GO TO 44                                          RCHO0120 RTCH0024
      CAB=CMPLX(AAA,BBB)                                              RCHO0130 RTCH0025
      CABDP=CAB+(2.,0.)                                               RCHO0140 RTCH0026
      CABBR=CAB+(4.,0.)                                               RCHO0150 RTCH0027
      CRDF=CSQRT(CABDP**2-CABBR*CAB)                                  RCHO0160 RTCH0028
      CX=(2.,0.)*(CABDP-CRDF)/CABBR                                   RCHO0170 RTCH0029
      IF(CABS(CX).GE.1.) GO TO 44                                     RCHO0180 RTCH0030
      CZT=(2.,0.)*CSQRT(CX)                                           RCHO0190 RTCH0031
      IF(AIMAG(CZT).GT.0.) CZT=-CZT                                   RCHO0200 RTCH0032
      DO 8601 LSD=1,20                                                RCHO0210 RTCH0033
      XPT=REAL(CZT)                                                   RCHO0220 RTCH0034
      YPT=AIMAG(CZT)                                                  RCHO0230 RTCH0035
      CALL BECHO(XPT,YPT,ART,BIT,GJR0,GJI0,GJR1,GJI1)                 RCHO0240 RTCH0036
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      WT1=ART-AAA                                                       RCH00250 RTCH0037
      WT2=BIT-BBB                                                       RCH00260 RTCH0038
      W(1)=CMPLX(WT1,WT2)                                               RCH00270 RTCH0039
      CANSP=CZT                                                         RCH00280 RTCH0040
      IF(CABS(W(1)).LT..0005) GO TO 8511                                RCH00290 RTCH0041
      CJ1=CMPLX(GJR1,GJI1)                                              RCH00300 RTCH0042
      CJ0=CMPLX(GJR0,GJI0)                                              RCH00310 RTCH0043
      CZJ0=CZT*CJ0                                                      RCH00320 RTCH0044
      CZJ1=CZT*CJ1                                                      RCH00330 RTCH0045
      CAJ0=CAF*CJ0                                                      RCH00340 RTCH0046
      CAJ1=CAB*CJ1                                                      RCH00350 RTCH0047
      CAZJ=CAB*CJ1/CZT                                                  RCH00360 RTCH0048
      C1=CZJ0+CAJ1                                                      RCH00370 RTCH0049
      C2=(2.,0.)*(CZJ1-CAJ0)                                            RCH00380 RTCH0050
      CJDB=CJ0-CZJ1+CAJ0-CAZJ                                           RCH00390 RTCH0051
      CROT=CSQRT(C1**2-C2*CJDB)                                         RCH00400 RTCH0052
      CZDLT=(CROT-C1)/CJDB                                              RCH00410 RTCH0053
      CZT=CZT+CZDLT                                                     RCH00420 RTCH0054
 8601 CONTINUE                                                          RCH00430 RTCH0055
      CANSP=CZT                                                         RCH00440 RTCH0056
      IF(CABS(W(1)).LT..005) GO TO 8511                                 RCH00450 RTCH0057
   44 AOS=.666667+.333333*AAA                                           RCH00460 RTCH0058
      IF(ABQ.GT.AOS) GO TO 31                                           RCH00470 RTCH0059
      IF(BBB.NE.0.) GO TO 80                                            RCH00480 RTCH0060
      IF(AAA.LT.0.) GO TO 45                                            RCH00490 RTCH0061
      XAB=SQRT(8.*AAA/(4.+AAA))                                         RCH00500 RTCH0062
      YAB=0.                                                            RCH00510 RTCH0063
      D=1.E-5                                                           RCH00520 RTCH0064
      NON=0                                                             RCH00530 RTCH0065
      CALL BESJ(XAB,NON,BJ,D,IER)                                       RCH00540 RTCH0066
      BJ0=BJ                                                            RCH00550 RTCH0067
      NON=1                                                             RCH00560 RTCH0068
      CALL BESJ(XAB,NON,BJ,D,IER)                                       RCH00570 RTCH0069
      BJ1=BJ                                                            RCH00580 RTCH0070
      ERE=XAB*BJ1-AAA*BJ0                                               RCH00590 RTCH0071
      ANSW(1)=CMPLX(XAB,0.)                                             RCH00600 RTCH0072
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      W(1)=CMPLX(FRE,0.)                                          RCH00610 RTCH0073
      NQ7=1                                                       RCH00620 RTCH0074
      GO TO 21                                                    RCH00630 RTCH0075
   45 XAB=0.                                                      RCH00640 RTCH0076
      YAB=SQRT(-8.*AAA/(4.+AAA))                                  RCH00650 RTCH0077
      CALL IO(YAB,RIO)                                            RCH00660 RTCH0078
      CALL IBI(YAB,RI)                                            RCH00670 RTCH0079
      FRE=AAA*RIO+YAB*RI                                          RCH00680 RTCH0080
      ANSW(1)=CMPLX(0.,-YAB)                                      RCH00690 RTCH0081
      W(1)=CMPLX(FRE,0.)                                          RCH00700 RTCH0082
      NQ7=1                                                       RCH00710 RTCH0083
      GO TO 21                                                    RCH00720 RTCH0084
   80 SQSD=4.*AAA+ABQ                                             RCH00730 RTCH0085
      SQSF=SQRT(SQSD**2+16.*BBQ)                                  RCH00740 RTCH0086
      SQDN=4.+2.*AAA+.25*ABQ                                      RCH00750 RTCH0087
      XAB=SQRT((SQSF+SQSD)/SQDN)                                  RCH00760 RTCH0088
      YAB=-SQRT((SQSF-SQSD)/SQDN)                                 RCH00770 RTCH0089
      ANSW(1)=CMPLX(XAB,YAB)                                      RCH00780 RTCH0090
      W(1)=FUN(ANSW(1))                                           RCH00790 RTCH0091
      NQ7=1                                                       RCH00800 RTCH0092
      GO TO 21                                                    RCH00810 RTCH0093
   31 IF(BBB.NE.0.) GO TO 81                                      RCH00820 RTCH0094
      IF(AAA) 82,83,83                                            RCH00830 RTCH0095
   82 IF(AAA.LT.-.7055) GO TO 251                                 RCH00840 RTCH0096
      XLI=-AAA+.57                                                RCH00850 RTCH0097
      XRI=-AAA+.6                                                 RCH00860 RTCH0098
      GO TO 90                                                    RCH00870 RTCH0099
  251 IF(AAA.LT.-1.705) GO TO 252                                 RCH00880 RTCH0100
      XLI=-AAA+.59                                                RCH00890 RTCH0101
      XRI=-AAA+.61                                                RCH00900 RTCH0102
      GO TO 90                                                    RCH00910 RTCH0103
  252 IF(AAA.LE.-2.) GO TO 8500                                   RCH00920 RTCH0104
      XLI=-AAA+.5                                                 RCH00930 RTCH0105
      XRI=-AAA+.6                                                 RCH00940 RTCH0106
      GO TO 90                                                    RCH00950 RTCH0107
   90 EPS=1.E-4                                                   RCH00960 RTCH0108
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      IEND=300                                                          RCHO0970 RTCH0109
      CALL RTMI(YN,FI,FCTI,XLI,XRI,EPS,IEND,IER)                        RCHO0980 RTCH0110
      ANSW(1)=CMPLX(0.,-YN)                                             RCHO0990 RTCH0111
      W(1)=CMPLX(FI,0.)                                                 RCHO1000 RTCH0112
      NQ7=1                                                             RCHO1010 RTCH0113
      IF(IER.NE.0) WRITE(6,91)                                          RCHO1020 RTCH0114
      GO TO 21                                                          RCHO1030 RTCH0115
   83 IF(AAA.GT.1.0944) GO TO 161                                       RCHO1040 RTCH0116
      XLI=1.25                                                          RCHO1050 RTCH0117
      XRI=1.31                                                          RCHO1060 RTCH0118
      GO TO 69                                                          RCHO1070 RTCH0119
  161 IF(AAA.GT.1.3385) GO TO 162                                       RCHO1080 RTCH0120
      XLI=1.29                                                          RCHO1090 RTCH0121
      XRI=1.41                                                          RCHO1100 RTCH0122
      GO TO 69                                                          RCHO1110 RTCH0123
  162 IF(AAA.GT.1.6351) GO TO 163                                       RCHO1120 RTCH0124
      XLI=1.39                                                          RCHO1130 RTCH0125
      XRI=1.51                                                          RCHO1140 RTCH0126
      GO TO 69                                                          RCHO1150 RTCH0127
  163 IF(AAA.GT.2.0023) GO TO 164                                       RCHO1160 RTCH0128
      XLI=1.49                                                          RCHO1170 RTCH0129
      XRI=1.61                                                          RCHO1180 RTCH0130
      GO TO 69                                                          RCHO1190 RTCH0131
  164 IF(AAA.GT.2.4679) GO TO 131                                       RCHO1200 RTCH0132
      XLI=1.59                                                          RCHO1210 RTCH0133
      XRI=1.71                                                          RCHO1220 RTCH0134
      GO TO 69                                                          RCHO1230 RTCH0135
  131 IF(AAA.GT.3.0788) GO TO 169                                       RCHO1240 RTCH0136
      XLI=1.69                                                          RCHO1250 RTCH0137
      XRI=1.81                                                          RCHO1260 RTCH0138
      GO TO 69                                                          RCHO1270 RTCH0139
  169 IF(AAA.GT.3.9181) GO TO 132                                       RCHO1280 RTCH0140
      XLI=1.79                                                          RCHO1290 RTCH0141
      XRI=1.91                                                          RCHO1300 RTCH0142
      GO TO 69                                                          RCHO1310 RTCH0143
  132 IF(AAA.GT.5.1519) GO TO 133                                       RCHO1320 RTCH0144
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      XLI=1.89                                                          RCHO1330 RTCH0145
      XRI=2.01                                                          RCHO1340 RTCH0146
      GO TO 69                                                          RCHO1350 RTCH0147
  133 IF(AAA.GT.7.1631) GO TO 134                                       RCHO1360 RTCH0148
      XLI=1.99                                                          RCHO1370 RTCH0149
      XRI=2.11                                                          RCHO1380 RTCH0150
      GO TO 69                                                          RCHO1390 RTCH0151
  134 IF(AAA.GT.10.) GO TO 8100                                         RCHO1400 RTCH0152
      XLI=2.09                                                          RCHO1410 RTCH0153
      XRI=2.21                                                          RCHO1420 RTCH0154
   69 IEND=300                                                          RCHO1430 RTCH0155
      EPS=1.E-4                                                         RCHO1440 RTCH0156
      CALL RTMI(X,F,FCT,XLI,XRI,EPS,IEND,IER)                           RCHO1450 RTCH0157
      ANSW(1)=CMPLX(X,0.)                                               RCHO1460 RTCH0158
      W(1)=CMPLX(F,0.)                                                  RCHO1470 RTCH0159
      NQ7=1                                                             RCHO1480 RTCH0160
      IF(IER.NE.0) WRITE(6,91)                                          RCHO1490 RTCH0161
      GO TO 21                                                          RCHO1500 RTCH0162
   81 IF(AAA.LT.0.) GO TO 2001                                          RCHO1510 RTCH0163
      IF(ABQ.GE.99.) GO TO 8100                                         RCHO1520 RTCH0164
       IF(AAA.GT.0.66) GO TO 1001                                       RCHO1530 RTCH0165
      IF(BBB.LT.-1.) GO TO 1101                                         RCHO1540 RTCH0166
      L=1                                                               RCHO1550 RTCH0167
      GO TO 1000                                                        RCHO1560 RTCH0168
 1101 IF(BBB.LT.-1.475) GO TO 1102                                      RCHO1570 RTCH0169
      L=2                                                               RCHO1580 RTCH0170
      GO TO 1000                                                        RCHO1590 RTCH0171
 1102 IF(BBB.LT.-2.1) GO TO 1103                                        RCHO1600 RTCH0172
      L=3                                                               RCHO1610 RTCH0173
      GO TO 1000                                                        RCHO1620 RTCH0174
 1103 IF(BBB.LT.-3.5) GO TO 1104                                        RCHO1630 RTCH0175
      L=4                                                               RCHO1640 RTCH0176
      GO TO 1000                                                        RCHO1650 RTCH0177
 1104 IF(BBB.LT.-5.) GO TO 1121                                         RCHO1660 RTCH0178
      L=5                                                               RCHO1670 RTCH0179
      GO TO 1000                                                        RCHO1680 RTCH0180
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1121 L=6                                                        RCHO1690 RTCH0181
     GO TO 1000                                                 RCHO1700 RTCH0182
1001 IF(AAA.GT.1.) GO TO 1002                                   RCHO1710 RTCH0183
     IF(BBB.LT.-1.5) GO TO 1201                                 RCHO1720 RTCH0184
     IF(BBB.LT.-.4) GO TO 1211                                  RCHO1730 RTCH0185
     L=7                                                        RCHO1740 RTCH0186
     GO TO 1000                                                 RCHO1750 RTCH0187
1211 IF(BBB.LT.-.7) GO TO 1213                                  RCHO1760 RTCH0188
     L=8                                                        RCHO1770 RTCH0189
     GO TO 1000                                                 RCHO1780 RTCH0190
1213 IF(BBB.LT.-1.) GO TO 1212                                  RCHO1790 RTCH0191
     L=60                                                       RCHO1800 RTCH0192
     GO TO 1000                                                 RCHO1810 RTCH0193
1212 L=9                                                        RCHO1820 RTCH0194
     GO TO 1000                                                 RCHO1830 RTCH0195
1201 IF(BBB.LT.-2.4) GO TO 1202                                 RCHO1840 RTCH0196
     L=10                                                       RCHO1850 RTCH0197
     GO TO 1000                                                 RCHO1860 RTCH0198
1202 IF(BBB.LT.-4.) GO TO 1203                                  RCHO1870 RTCH0199
     L=11                                                       RCHO1880 RTCH0200
     GO TO 1000                                                 RCHO1890 RTCH0201
1203 IF(BBB.LT.-5.) GO TO 1204                                  RCHO1900 RTCH0202
     L=12                                                       RCHO1910 RTCH0203
     GO TO 1000                                                 RCHO1920 RTCH0204
1204 IF(BBB.LT.-7.) GO TO 1205                                  RCHO1930 RTCH0205
     L=62                                                       RCHO1940 RTCH0206
     GO TO 1000                                                 RCHO1950 RTCH0207
1205 L=63                                                       RCHO1960 RTCH0208
     GO TO 1000                                                 RCHO1970 RTCH0209
1002 IF(AAA.GT.1.7) GO TO 1003                                  RCHO1980 RTCH0210
     IF(BBB.LT.-.85) GO TO 1301                                 RCHO1990 RTCH0211
     L=13                                                       RCHO2000 RTCH0212
     GO TO 1000                                                 RCHO2010 RTCH0213
1301 IF(BBB.LT.-1.9) GO TO 1302                                 RCHO2020 RTCH0214
     L=14                                                       RCHO2030 RTCH0215
     GO TO 1000                                                 RCHO2040 RTCH0216
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1302 IF(BBB.LT.-3.4) GO TO 1303                                    RCHO2050 RTCH0217
     L=15                                                          RCHO2060 RTCH0218
     GO TO 1000                                                    RCHO2070 RTCH0219
1303 IF(BBB.LT.-6.) GO TO 1304                                     RCHO2080 RTCH0220
     L=16                                                          RCHO2090 RTCH0221
     GO TO 1000                                                    RCHO2100 RTCH0222
1304 L=17                                                          RCHO2110 RTCH0223
     GO TO 1000                                                    RCHO2120 RTCH0224
1003 IF(AAA.GT.3.5) GO TO 1004                                     RCHO2130 RTCH0225
     IF(BBB.LT.-1.) GO TO 1401                                     RCHO2140 RTCH0226
     L=18                                                          RCHO2150 RTCH0227
     GO TO 1000                                                    RCHO2160 RTCH0228
1401 IF(BBB.LT.-2.7) GO TO 1402                                    RCHO2170 RTCH0229
     L=19                                                          RCHO2180 RTCH0230
     GO TO 1000                                                    RCHO2190 RTCH0231
1402 IF(BBB.LT.-6.) GO TO 1403                                     PCHO2200 RTCH0232
     L=20                                                          RCHO2210 RTCH0233
     GO TO 1000                                                    RCHO2220 RTCH0234
1403 L=21                                                          RCHO2230 RTCH0235
     GO TO 1000                                                    RCHO2240 RTCH0236
1004 IF(AAA.GT.8.) GO TO 1005                                      RCHO2250 RTCH0237
     IF(BBB.LT.-3.5) GO TO 1501                                    RCHO2260 RTCH0238
     L=22                                                          RCHO2270 RTCH0239
     GO TO 1000                                                    RCHO2280 RTCH0240
1501 L=23                                                          RCHO2290 RTCH0241
     GO TO 1000                                                    RCHO2300 RTCH0242
1005 L=24                                                          RCHO2310 RTCH0243
     GO TO 1000                                                    RCHO2320 RTCH0244
2001 PHI=ATAN(-AAA/BBB)*180./3.14159                               RCHO2330 RTCH0245
     IF(PHI.LT.-23.3) GO TO 3001                                   RCHO2340 RTCH0246
     IF(ABQ.GE.99.) GO TO 8100                                     RCHO2350 RTCH0247
     IF(BBB.LT.-.72) GO TO 2101                                    RCHO2360 RTCH0248
     L=25                                                          RCHO2370 RTCH0249
     GO TO 1000                                                    RCHO2380 RTCH0250
2101 IF(BBB.LT.-1.005) GO TO 2201                                  RCHO2390 RTCH0251
     L=26                                                          RCHO2400 RTCH0252
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      GO TO 1000                                                        RCHO2410 RTCH0253
2201  IF(BBB.LT.-1.486) GO TO 2301                                      RCHO2420 RTCH0254
      L=27                                                              RCHO2430 RTCH0255
      GO TO 1000                                                        RCHO2440 RTCH0256
2301  IF(BBB.LT.-1.9) GO TO 2401                                        RCHO2450 RTCH0257
      L=28                                                              RCHO2460 RTCH0258
      GO TO 1000                                                        RCHO2470 RTCH0259
2401  IF(BBB.LT.-2.4) GO TO 2501                                        RCHO2480 RTCH0260
      IF(AAA.LT.-.6) GO TO 2402                                         RCHO2490 RTCH0261
      L=29                                                              RCHO2500 RTCH0262
      GO TO 1000                                                        RCHO2510 RTCH0263
2402  L=30                                                              RCHO2520 RTCH0264
      GO TO 1000                                                        RCHO2530 RTCH0265
2501  IF(BBB.LT.-3.) GO TO 2601                                         RCHO2540 RTCH0266
      IF(AAA.LT.-.6) GO TO 2502                                         RCHO2550 RTCH0267
      L=31                                                              RCHO2560 RTCH0268
      GO TO 1000                                                        RCHO2570 RTCH0269
2502  IF(AAA.LT.-1.) GO TO 2503                                         RCHO2580 RTCH0270
      L=32                                                              RCHO2590 RTCH0271
      GO TO 1000                                                        RCHO2600 RTCH0272
2503  IF(AAA.LT.-1.2) GO TO 2504                                        RCHO2610 RTCH0273
      L=33                                                              RCHO2620 RTCH0274
      GO TO 1000                                                        RCHO2630 RTCH0275
2504  L=34                                                              RCHO2640 RTCH0276
      GO TO 1000                                                        RCHO2650 RTCH0277
2601  IF(BBB.LT.-3.5) GO TO 2701                                        RCHO2660 RTCH0278
      IF(AAA.LT.-.5) GO TO 2602                                         RCHO2670 RTCH0279
      L=35                                                              RCHO2680 RTCH0280
      GO TO 1000                                                        RCHO2690 RTCH0281
2602  IF(AAA.LT.-.8) GO TO 2603                                         RCHO2700 RTCH0282
      L=36                                                              RCHO2710 RTCH0283
      GO TO 1000                                                        RCHO2720 RTCH0284
2603  IF(AAA.LT.-1.) GO TO 2604                                         RCHO2730 RTCH0285
      L=37                                                              RCHO2740 RTCH0286
      GO TO 1000                                                        RCHO2750 RTCH0287
2604  L=38                                                              RCHO2760 RTCH0288
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      GO TO 1000                                                        RCHO2770 RTCH0289
2701  IF(BBB.LT.-5.) GO TO 2801                                         RCHO2780 RTCH0290
      IF(AAA.LT.-.8) GO TO 2702                                         RCHO2790 RTCH0291
      L=39                                                              RCHO2800 RTCH0292
      GO TO 1000                                                        RCHO2810 RTCH0293
2702  IF(BBB.LE.-4.) GO TO 2703                                         RCHO2820 RTCH0294
      L=40                                                              RCHO2830 RTCH0295
      GO TO 1000                                                        RCHO2840 RTCH0296
2703  L=61                                                              RCHO2850 RTCH0297
      GO TO 1000                                                        RCHO2860 RTCH0298
2801  L=41                                                              RCHO2870 RTCH0299
      GO TO 1000                                                        RCHO2880 RTCH0300
3001  IF(AAA.LE.-1.5) GO TO 8500                                        RCHO2890 RTCH0301
      IF(AAA.GT.-1.5) GO TO 7001                                        RCHO2900 RTCH0302
      IF(BBB.LT.-1.5) GO TO 6002                                        RCHO2910 RTCH0303
      L=42                                                              RCHO2920 RTCH0304
      GO TO 2000                                                        RCHO2930 RTCH0305
6002  IF(BBB.LT.-2.5) GO TO 6003                                        RCHO2940 RTCH0306
      L=43                                                              RCHO2950 RTCH0307
      GO TO 2000                                                        RCHO3980 RTCH0308
6003  IF(BBB.LT.-3.5) GO TO 6004                                        RCHO3990 RTCH0309
      L=44                                                              RCHO4000 RTCH0310
      GO TO 2000                                                        RCHO4010 RTCH0311
6004  L=45                                                              RCHO4020 RTCH0312
      GO TO 2000                                                        RCHO4030 RTCH0313
7001  IF(BBB.LT.-.5) GO TO 7101                                         RCHO4040 RTCH0314
      IF(AAA.GT.-1.1) GO TO 7002                                        RCHO4050 RTCH0315
      L=46                                                              RCHO4060 RTCH0316
      GO TO 2000                                                        RCHO4070 RTCH0317
7002  IF(AAA.GT.-.5) GO TO 7003                                         RCHO4080 RTCH0318
      L=47                                                              RCHO4090 RTCH0319
      GO TO 2000                                                        RCHO4100 RTCH0320
7003  L=48                                                              RCHO4110 RTCH0321
      GO TO 2000                                                        RCHO4120 RTCH0322
7101  IF(BBB.LT.-1.) GO TO 7201                                         RCHO4130 RTCH0323
      IF(AAA.GT.-1.1) GO TO 7102                                        RCHO4140 RTCH0324
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      L=49                                                          RCHO4150 RTCH0325
      GO TO 2000                                                    RCHO4160 RTCH0326
7102  IF(AAA.GT.-.5) GO TO 7103                                     RCHO4170 RTCH0327
      L=50                                                          RCHO4180 RTCH0328
      GO TO 2000                                                    RCHO4190 RTCH0329
7103  L=51                                                          RCHO4200 RTCH0330
      GO TO 2000                                                    RCHO4210 RTCH0331
7201  IF(BBB.LT.-1.5) GO TO 7301                                    RCHO4220 RTCH0332
      IF(AAA.GT.-1.1) GO TO 7202                                    RCHO4230 RTCH0333
      L=52                                                          RCHO4240 RTCH0334
      GO TO 2000                                                    RCHO4250 RTCH0335
7202  L=53                                                          RCHO4260 RTCH0336
      GO TO 2000                                                    RCHO4270 RTCH0337
7301  IF(BBB.LT.-2.) GO TO 7401                                     RCHO4280 RTCH0338
      L=54                                                          RCHO4290 RTCH0339
      GO TO 2000                                                    RCHO4300 RTCH0340
7401  IF(BBB.LT.-2.5) GO TO 7501                                    RCHO4310 RTCH0341
      L=55                                                          RCHO4320 RTCH0342
      GO TO 2000                                                    RCHO4330 RTCH0343
7501  IF(BBB.LT.-2.7) GO TO 7601                                    RCHO4340 RTCH0344
      IF(AAA.GT.-1.3) GO TO 7502                                    RCHO4350 RTCH0345
      L=56                                                          RCHO4360 RTCH0346
      GO TO 2000                                                    RCHO4370 RTCH0347
7502  L=57                                                          RCHO4380 RTCH0348
      GO TO 3000                                                    RCHO4390 RTCH0349
7601  IF(AAA.GT.-1.4) GO TO 7602                                    RCHO4400 RTCH0350
      IF(BBB.LT.-2.94) GO TO 7611                                   RCHO4410 RTCH0351
      L=58                                                          RCHO4420 RTCH0352
      GO TO 2000                                                    RCHO4430 RTCH0353
7611  L=64                                                          RCHO4440 RTCH0354
      GO TO 2000                                                    RCHO4450 RTCH0355
7602  L=59                                                          RCHO4460 RTCH0356
      GO TO 2000                                                    RCHO4470 RTCH0357
1000  Z(1)=ZIN(L,1)                                                 RCHO4480 RTCH0358
      Z(2)=ZIN(L,2)                                                 RCHO4490 RTCH0359
      Z(3)=ZIN(L,3)                                                 RCHO4500 RTCH0360
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      Z(4)=ZIN(L,4)                                                                    RCHO4510 RTCH0361
      GO TO 9000                                                                       RCHO4520 RTCH0362
 2000 ZAB=CMPLX(-BBB,AAA)                                                              RCHO4530 RTCH0363
      Z(1)=ZAB-ZIN(L,1)                                                                RCHO4540 RTCH0364
      Z(2)=ZAB-ZIN(L,2)                                                                RCHO4550 RTCH0365
      Z(3)=ZAB-ZIN(L,3)                                                                RCHO4560 RTCH0366
      Z(4)=ZAB-ZIN(L,4)                                                                RCHO4570 RTCH0367
      GO TO 9000                                                                       RCHO4580 RTCH0368
 3000 ZAB=CMPLX(-BBB,0.)                                                               RCHO4590 RTCH0369
      Z(1)=ZAB-ZIN(L,1)                                                                RCHO4600 RTCH0370
      Z(2)=ZAB-ZIN(L,2)                                                                RCHO4610 RTCH0371
      Z(3)=ZAB-ZIN(L,3)                                                                RCHO4620 RTCH0372
      Z(4)=ZAB-ZIN(L,4)                                                                RCHO4630 RTCH0373
 9000 M=2                                                                              RCHO4640 RTCH0374
      N=4                                                                              RCHO4650 RTCH0375
      NPRINT=0                                                                         RCHO4660 RTCH0376
      ERROR=(1.E-2,1.E-2)                                                              RCHO4670 RTCH0377
      CALL YSONK9(Z,N,ERROR,M,NPRINT,ANSW,W,RETERR,NO,FUN,NQ7)                         RCHO4680 RTCH0378
      GO TO 21                                                                         RCHO4690 RTCH0379
 8100 XO=2.40483                                                                       RCHO4700 RTCH0380
      CAB=CMPLX(AAA,BBB)                                                               RCHO4710 RTCH0381
      CAHF=CAB+(.5,0.)                                                                 RCHO4720 RTCH0382
      DQN=XO**2+.25                                                                    RCHO4730 RTCH0383
      CQRT=CSQRT((1.,0.)+1.33333*DQN/CAHF**2)                                          RCHO4740 RTCH0384
      CANSP=XO*((1.,0.)-1.5*CAHF*(CQRT-(1.,0.))/DQN)                                   RCHO4750 RTCH0385
      DO 8505 LPS=1,20                                                                 RCHO4760 RTCH0386
      XP=REAL(CANSP)                                                                   RCHO4770 RTCH0387
      YP=AIMAG(CANSP)                                                                  RCHO4780 RTCH0388
      CALL BECHO(XP,YP,ARE,BIM,JR0,JI0,JR1,JI1)                                        RCHO4790 RTCH0389
      W1=ARE-AAA                                                                       RCHO4800 RTCH0390
      W2=BIM-BBB                                                                       RCHO4810 RTCH0391
      W(1)=CMPLX(W1,W2)                                                                RCHO4820 RTCH0392
      IF(CABS(W(1)).LT..0005) GO TO 8511                                               RCHO4830 RTCH0393
      CIAP=CMPLX(JR0,JI0)/CMPLX(JR1,JI1)                                               RCHO4840 RTCH0394
      CZAJ=CANSP/CAB                                                                   RCHO4850 RTCH0395
      CMAPJ=CIAP*CZAJ                                                                  RCHO4860 RTCH0396
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      CADM=1./CANSP+CZAJ                                                        RCHO4870 RTCH0397
      CZMAP=1.+CMAPJ                                                            RCHO4880 RTCH0398
      CQI=2.*CADM*(CIAP-CZAJ)/CZMAP**2                                          RCHO4890 RTCH0399
      CDELZ=CZMAP*((1.,0.)-CSQRT((1.,0.)-CQI))/CADM                             RCHO4900 RTCH0400
      CANSP=CANSP+CDELZ                                                         RCHO4910 RTCH0401
 8505 CONTINUE                                                                  RCHO4920 RTCH0402
      GO TO 8511                                                                RCHO4930 RTCH0403
 8500 CDAB=(.5,0.)                                                              RCHO4940 RTCH0404
      CAB=CMPLX(AAA,BBB)                                                        RCHO4950 RTCH0405
      CEI=(0.,1.)                                                               RCHO4960 RTCH0406
      DO 8501 KPS=1,20                                                          RCHO4970 RTCH0407
      CZ=CEI*(CAB-CDAB)                                                         RCHO4980 RTCH0408
      XP=REAL(CZ)                                                               RCHO4990 RTCH0409
      YP=AIMAG(CZ)                                                              RCHO5000 RTCH0410
      IF(XP.NE.0..AND.YP.NE.0.) GO TO 8503                                      RCHO5010 RTCH0411
      CALL BECHO(YP,YP,ARE,BIM,JR0,JI0,JR1,JI1)                                 RCHO5020 RTCH0412
      CALP=CMPLX(JR1,JI1)/CMPLX(JR0,JI0)                                        RCHO5030 RTCH0413
      GO TO 8504                                                                RCHO5040 RTCH0414
 8503 CALL CMJYB(CZ,CJB,CYB)                                                    RTCH0415
      CALP=CJB(2)/CJB(1)                                                        RTCH0416
 8504 CALP2=CALP**2                                                             RCHO5100 RTCH0417
      CPLP=CALP2+(1.,0.)                                                        RCHO5110 RTCH0418
      CPLN=(1.,0.)-CALP2                                                        RCHO5120 RTCH0419
      CALPP=CPLP/CALP                                                           RCHO5130 RTCH0420
      CZAP=CZ*CALP                                                              RCHO5140 RTCH0421
      CPLZ=CPLN/CZAP                                                            RCHO5150 RTCH0422
      CQ=(CPLP+(.5,0.)*CPLZ)*((1.,0.)-CAB/CZAP)/CALPP**2                        RCHO5160 RTCH0423
      CQRP=(1.,0.)-CSQRT((1.,0.)-(4.,0.)*CQ)                                    RCHO5170 RTCH0424
      CDNM=(2.,0.)*CPLP+CPLZ                                                    RCHO5180 RTCH0425
      CANSP=CZ-CALPP*CQRP/CDNM                                                  RCHO5190 RTCH0426
      XP=REAL(CANSP)                                                            RCHO5200 RTCH0427
      YP=AIMAG(CANSP)                                                           RCHO5210 RTCH0428
      CALL BECHO(XP,YP,ARE,BIM,JR0,JI0,JR1,JI1)                                 RCHO5220 RTCH0429
      W1=ARE-AAA                                                                RCHO5230 RTCH0430
      W2=BIM-BBB                                                                RCHO5240 RTCH0431
      W(1)=CMPLX(W1,W2)                                                         RCHO5250 RTCH0432
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      IF(CABS(W(1)).LT..0005) GO TO 8511                        RCHO5260 RTCH0433
      DA=ARE-YP                                                 RCHO5270 RTCH0434
      DB=RIM+XP                                                 RCHO5280 RTCH0435
      CDAB=CMPLX(DA,DB)                                         RCHO5290 RTCH0436
 8501 CONTINUE                                                  RCHO5300 RTCH0437
 8511 ANSW(1)=CANSP                                             RCHO5310 RTCH0438
      NQ7=1                                                     RCHO5320 RTCH0439
   21 RETURN                                                    RCHO5330 RTCH0440
      END                                                       RCHO5340 RTCH0441
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      SUBROUTINE BECHO(XP,YP,ARE,BIM,JR0,JI0,JR1,JI1)                    BECH0001
      REAL*4 JR0,JI0,JR1,JI1                                             BECH0002
      COMPLEX*8 CJB(2),CYB(2),CZ,CMPLX                                   BECH0003
      IF(ABS(XP).LT.1.E-30) XP=0.                                        BECH0004
      IF(ABS(YP).LT.1.E-30) YP=0.                                        BECH0005
      IF(YP.NE.0.) GO TO 21                                              BECH0006
      D=1.E-6                                                            BECH0007
      NJ=0                                                               BECH0008
      CALL BESJ(XP,NJ,JR0,D,IER)                                         BECH0009
      IF(JR0.LT.1.E-10.AND.JR0.GE.0.) JR0=1.E-10                         BECH0010
      IF(JR0.LT.0..AND.JR0.GT.-1.E-10) JR0=-1.E-10                       BECH0011
      NJ=1                                                               BECH0012
      CALL BESJ(XP,NJ,JR1,D,IER)                                         BECH0013
      ARE=XP*JR1/JR0                                                     BECH0014
      BIM=0.                                                             BECH0015
      JI0=0.                                                             BECH0016
      JI1=0.                                                             BECH0017
      GO TO 50                                                           BECH0018
   21 IF(XP.NE.0.) GO TO 31                                              BECH0019
      YN=-YP                                                             BECH0020
      CALL I0(YN,JR0)                                                    BECH0021
      IF(JR0.LT.1.E-10.AND.JR0.GE.0.) JR0=1.E-10                         BECH0022
      IF(JR0.LT.0..AND.JR0.GT.-1.E-10) JR0=-1.E-10                       BECH0023
      CALL IBI(YN,JI1)                                                   BECH0024
      ARE=YP*JI1/JR0                                                     BECH0025
      JI0=0.                                                             BECH0026
      JR1=0.                                                             BECH0027
      BIM=0.                                                             BECH0028
      GO TO 50                                                           BECH0029
   31 CZ=CMPLX(XP,YP)                                                    BECH0030
      CALL CMJYB(CZ,CJB,CYB)                                             BECH0031
      JR0=REAL(CJB(1))                                                   BECH0032
      JR1=REAL(CJB(2))                                                   BECH0033
      JI0=AIMAG(CJB(1))                                                  BECH0034
      JI1=AIMAG(CJB(2))                                                  BECH0035
      IF(JR0.LT.1.E-20.AND.JR0.GE.0.) JR0=1.E-20                         BECH0036
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      IF(JI0.LT.1.E-20.AND.JI0.GE.0.)  JI0=1.E-20                    BECH0037
      IF(JR1.LT.1.E-20.AND.JR1.GE.0.)  JR1=1.E-20                    BECH0038
      IF(JI1.LT.1.E-20.AND.JI1.GE.0.)  JI1=1.E-20                    BECH0039
      IF(JR0.LT.0..AND.JR0.GT.-1.E-20)  JR0=-1.E-20                  BECH0040
      IF(JI0.LT.0..AND.JI0.GT.-1.E-20)  JI0=-1.E-20                  BECH0041
      IF(JR1.LT.0..AND.JR1.GT.-1.E-20)  JR1=-1.E-20                  BECH0042
      IF(JI1.LT.0..AND.JI1.GT.-1.E-20)  JI1=-1.E-20                  BECH0043
      RD=JR0*JR0+JI0*JI0                                             BECH0044
      IF(RD.LT.1.E-30)  RD=1.E-30                                    BECH0045
      RA=JR0*JR1+JI0*JI1                                             BECH0046
      RB=JR0*JI1-JR1*JI0                                             BECH0047
      ARE=(XP*RA-YP*RB)/RD                                           BECH0048
      BIM=(XP*RB+YP*RA)/RD                                           BECH0049
   50 RETURN                                                         BECH0050
      END                                                            BECH0051
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      COMPLEX FUNCTION FUN(Z)                                          FUNZ0001
      COMPLEX*8 Z,CMPLX,ACM,CJB(2),CYB(2)                              FUNZ0002
      DIMENSION BJRE(2),BJIM(2)                                        FUNZ0003
      COMMON /QFNT/AAA,BBB                                             FUNZ0004
      AC=AAA                                                           FUNZ0005
      BC=BBB                                                           FUNZ0006
      ACM=CMPLX(AC,BC)                                                 FUNZ0007
      CALL CMJYB(Z,CJB,CYB)                                            FUNZ0008
      FUN=Z*CJB(2)-ACM*CJB(1)                                          FUNZ0009
      RETURN                                                           FUNZ0010
      END                                                              FUNZ0011
```