

COLLAPSE OF THE INTER-ELECTRODE BREAKDOWN ARC
IN THE MAGNETIC FIELD DIRECTION

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ABSTRACT

A two-dimensional inter-electrode breakdown arc which is uniform in the magnetic field direction is shown to be unstable. The growth time for the instability is of the order of the arc development time. It is unlikely that a fully steady two-dimensional arc is ever established. The relationship of the instability to recent experiments revealing an inherently three-dimensional breakdown is discussed.

I. INTRODUCTION

In previous studies [Refs. (1), (2), (3), (4)] a two dimensional model of inter-electrode breakdown has proven adequate in predicting some features of the breakdown process. In particular, excess Joule heating of the plasma near the insulator surface due to the Hall effect has been identified as the driving mechanism for breakdown. Detailed MHD boundary layer calculations have predicted Hall voltage saturation and two-dimensional arc formation with average current densities of many tens of amp/cm². These predictions of Hall voltage saturation appear to match those measured in Ref. (5). The question of three-dimensionality, particularly collapse and pinching of the arc in the magnetic field direction, still remains however.

In the present report this question is examined. We shall show that the fully developed two-dimensional sheet arc overlaying the insulator surface is unstable to disturbances in the magnetic field direction for typical MHD generator breakdown conditions. Further, this instability has a growth time of the order of the turbulent heat diffusion time through the breadth of the arc. For typical MHD generator operating conditions this time is of the order of 10^{-3} seconds which is of the order of the growth time for the two-dimensional arc itself.

II. MODEL OF THE TWO DIMENSIONAL ARC

We shall model the two dimensional arc as a single zone of concentrated current density and temperature. A typical fully developed two-dimensional breakdown arc is shown in Fig. 1. It can be seen that the bulk of the breakdown current is spatially concentrated in a narrow region of breadth δ_a . As a simplified model of this continuum profile, we shall use a discrete profile as shown in Fig. 2. We shall assume the current vanishes outside this zone and the temperature is essentially the free stream temperature outside the arc zone. Inside the arc zone of breadth δ_a the temperature and current are uniform and have the values T and J_x respectively.

The energy equation for the arc of breadth δ_a governing the temperature T is

$$\begin{aligned} \rho C_p \frac{DT}{Dt} = & K_\infty (T_\infty - T) / (\delta_a^2 / 2) + K_w (T_w - T) / (\delta_a^2 / 2) \\ & + K_\perp \nabla_\perp^2 T + J^2 / \sigma(T) \end{aligned} \quad (2.1)$$

The convective derivative is $\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x}$. The effective turbulent heat conduction coefficient on the outer side of the arc is K_∞ , while the coefficient on the wall side is denoted K_w .

The conduction coefficient in the transverse direction is K_\perp with

$$\nabla_\perp^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$$

We assume that the transverse direction (y) and magnetic field direction (z) currents vanish in the steady state so that the only currents and fields in the steady state are J_{x0} , E_{x0} , E_{y0} . These are given in terms of the inter-electrode voltage V_x as

$$J_{x0} = \sigma(T_0)E_{x0} = -\sigma(T_0)V_x/l_i \quad (2.2)$$

$$E_{y0} = -\beta E_{x0}$$

where σ and β are the electrical conductivity and Hall parameter respectively. The Hall parameter is assumed constant in what follows.

The corresponding steady state temperature in the arc is T_0 .

The steady temperature T_0 is determined by the steady and uniform ($\nabla_{\perp}^2 T_0 \equiv 0$) solution of the energy equation (2.1) in terms of the applied voltage V_x and the outer (T_{∞}) and inner (T_w) temperatures:

$$T_0 = T_g + \frac{\sigma(T_0)\delta_a^2}{(K_{\infty} + K_w)/2} \left(\frac{V_x}{l_i}\right)^2 \quad (2.3)$$

The temperature T_g is the temperature the gas would achieve in the breakdown layer in the absence of Joule heating:

$$T_g \equiv (K_{\infty}T_{\infty} + K_wT_w)/(K_{\infty} + K_w) \quad (2.4)$$

Once the conductivity is specified as a function of the temperature, the non-linear algebraic Eq. (2.3) may be solved for the steady two-dimensional arc temperature T_0 .

III. STABILITY OF THE TWO-DIMENSIONAL ARC

We now consider fluctuations in the z direction about the steady state described in Part II. For this purpose the temperature, current, and field are represented in terms of steady and fluctuating parts as

$$\begin{aligned}
 T(z,t) &= T_0 + T'(z,t) \\
 \vec{J}(z,t) &= \vec{J}_0 + \vec{J}'(z,t) \\
 \vec{E}(z,t) &= \vec{E}_0 + \vec{E}'(z,t)
 \end{aligned}
 \tag{3.1}$$

where $\vec{J}_0 = (J_{x0}, 0, 0)$, $\vec{E}_0 = (E_{x0}, E_{y0}, 0)$.

The Maxwell Equations require that

$$\begin{aligned}
 J'_z &\equiv 0 \\
 E'_x &\equiv E'_y \equiv 0
 \end{aligned}
 \tag{3.2}$$

The Ohm's Law for the fluctuations is

$$\begin{aligned}
 J'_x + \beta J'_y &= \sigma_0 E'_x + \sigma' E_{x0} \\
 -\beta J'_x + J'_y &= \sigma E'_y + \sigma' E_{y0} \\
 J'_z &= \sigma_0 E'_z + \sigma' E_{z0}
 \end{aligned}
 \tag{3.3}$$

It follows that

$$J'_x = E_{x0} \sigma'$$

$$J'_y = J'_z = E'_z = 0$$

The fluctuating Joule heat is then (to first order in J'_x , σ')

$$\left(\frac{J^2}{\sigma}\right)' = 2 \frac{J_{x0}}{\sigma_0} J'_x - \frac{J_{x0}^2}{\sigma_0} \left(\frac{\sigma'}{\sigma_0}\right) = \frac{J_{x0}}{\sigma_0} \left(\frac{\sigma'}{\sigma_0}\right) \quad (3.4)$$

The fluctuating first order part of the energy equation (2.1) may now be expressed as

$$\tau \frac{DT'}{Dt} = \left[\frac{\left(\frac{J_{x0}^2}{\sigma_0 T_0}\right) \delta_a^2 \left(\frac{\partial \ln \sigma}{\partial \ln T}\right)_0}{(K_\infty + K_w)/2} - 1 \right] T' + \tau \alpha_1 \frac{\partial^2 T'}{\partial z^2} \quad (3.5)$$

The conductivity fluctuation is represented in terms of the temperature fluctuation as

$$\frac{\sigma'}{\sigma_0} = \left(\frac{\partial \ln \sigma}{\partial \ln T}\right)_0 \left(\frac{T'}{T_0}\right)$$

The time τ is the average effective diffusion time through the layer δ_a :

$$\tau \equiv \frac{\rho C_p (\delta_a^2/2)}{(K_\infty + K_w)}$$

while α_1 is the turbulent diffusivity in the z direction.

In the absence of z direction heat conduction ($\alpha_1 = 0$), the stability condition for growth of fluctuations is readily established from Eq. (3.5)

as

$$2 \left(\frac{J_{x0}^2}{\sigma_0 T_0}\right) \delta_a^2 / (K_\infty + K_w) \left(\frac{\partial \ln \sigma}{\partial \ln T}\right)_0 \geq 1 \quad (3.6)$$

This condition expressed in terms of the breakdown voltage V_x is

$$2 \sigma_o \left(\frac{V_x}{L_i}\right)^2 \delta_a^2 / (K_\infty + K_w) \left(\frac{\partial \ln \sigma}{\partial \ln T}\right)_o \geq 1 \quad (3.7)$$

The stability condition can also be expressed in terms of the steady state temperature elevation as

$$\left(\frac{T_o - T_g}{T_o}\right) \left(\frac{\partial \ln \sigma}{\partial \ln T}\right)_o \geq 1 \quad (3.8)$$

For typical two-dimensional breakdown we may have $T_g = 1700^\circ\text{K}$, $T_o = 3000^\circ\text{K}$, $(\partial \ln \sigma / \partial \ln T)_o \approx 10$ so that

$$\left(\frac{T_o - T_g}{T_o}\right) \left(\frac{\partial \ln \sigma}{\partial \ln T}\right)_o \approx 4.3$$

The stability condition is therefore readily violated.

The growth time of an unstable z direction fluctuation is

$$\tau'_{\text{undamped}} = \frac{\tau}{\left(\frac{T_o - T_g}{T_o}\right) \left(\frac{\partial \ln \sigma}{\partial \ln T}\right)_o - 1}$$

The time, τ , may also be expressed as

$$\tau = \frac{\frac{1}{4} (\delta_a / \delta)^2}{N_{st} (u/D)}$$

where N_{st} is the Stanton number and on the definition

$$N_{st} \equiv \frac{(K_\infty + K_w)/2}{D \rho C_p u}$$

For $\delta_a / \delta \approx 1/4$, $u = 250$ m/sec, $\delta = 1$ cm, $D = 10$ cm, $N_{st} = 0.02$, the time τ is approximately 0.3 ms.

If axial conduction is included, the fluctuation may be represented in terms of its Fourier mode as

$$T' = \tilde{T} e^{(t/\tau' + ikz)}$$

where $k = 2\pi/\lambda$ is the wavenumber of the mode of wavelength λ . The growth time is then given in terms of k as

$$(\tau')^{-1} = \tau^{-1} \left[\left(\frac{T_o - T_g}{T_o} \right) \left(\frac{\partial \ln \sigma}{\partial \ln T_o} \right) - 1 \right] - \alpha_{\perp} k^2$$

$$(\tau')^{-1} = \tau_{\text{undamped}}^{-1} - \alpha_{\perp} k^2$$

Wavelengths shorter than λ_{crit} given by

$$\frac{4\pi^2}{\lambda_{\text{crit}}^2} \approx \frac{\left(\frac{T_o - T_g}{T_o} \right) \left(\frac{\partial \ln \sigma}{\partial \ln T_o} \right) - 1}{\tau \alpha_{\perp}}$$

will be damped by axial diffusion. Assuming $\alpha_{\perp} \approx \alpha$, the cut-off wavelength for typical MHD generator conditions is about 2 mm.

IV. CONCLUSION

We conclude that magnetic field direction collapse of a two dimensional inter-electrode arc should occur on a time scale of the order of the breakdown time scale itself. Hence it is unlikely that a uniform two-dimensional arc is ever established in an inter-electrode breakdown. Rather, a series of arc streamers distributed in the magnetic field direction is more likely. If these streamers are numerous and somewhat uniformly distributed then the aggregate begins to resemble a two-dimensional arc once again with 'fine structure' in the magnetic field direction. Two dimensional breakdown theory of Refs. (1), (4) should again be generally descriptive of such a situation. (Since the streamers are non-uniform in the magnetic field direction anomalous impedance effects due to the Hall effect are not possible.) It should be noted that all wave lengths save the shortest to be cut off by turbulent diffusion in the z direction are permitted to grow equally fast; hence many wave modes and numerous resulting streamers are possible. On the other hand if some selection mechanism (outside that purely operative in the gas) were to select only the longest wave lengths, then only one or two resulting streamers would be likely and the breakdown would become essentially three-dimensional. Preliminary evidence from Stanford [Ref. (6)] indicates that the formation of a single streamer is intimately associated with the heat-up of the insulation material itself. It is therefore possible that the thermal dynamics of the insulator provide a selection mechanism for a single dominant unstable mode. The manner in which this might occur remains a subject for further study.

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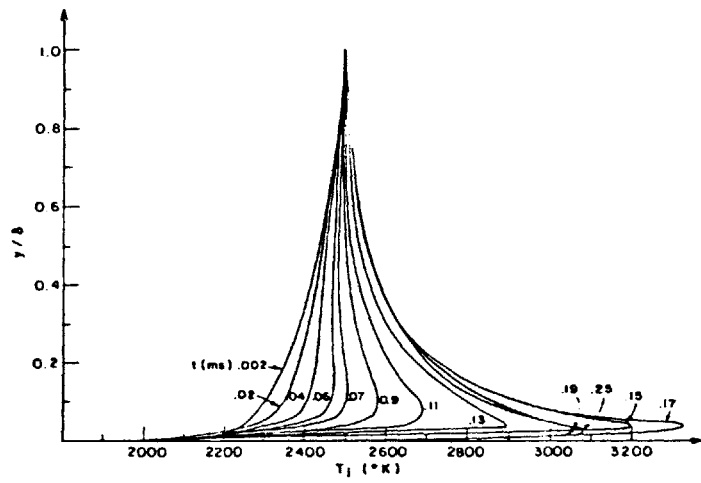


Figure 1: Calculated Two-Dimensional Continuum Breakdown Arc Profile (Ref. 4)

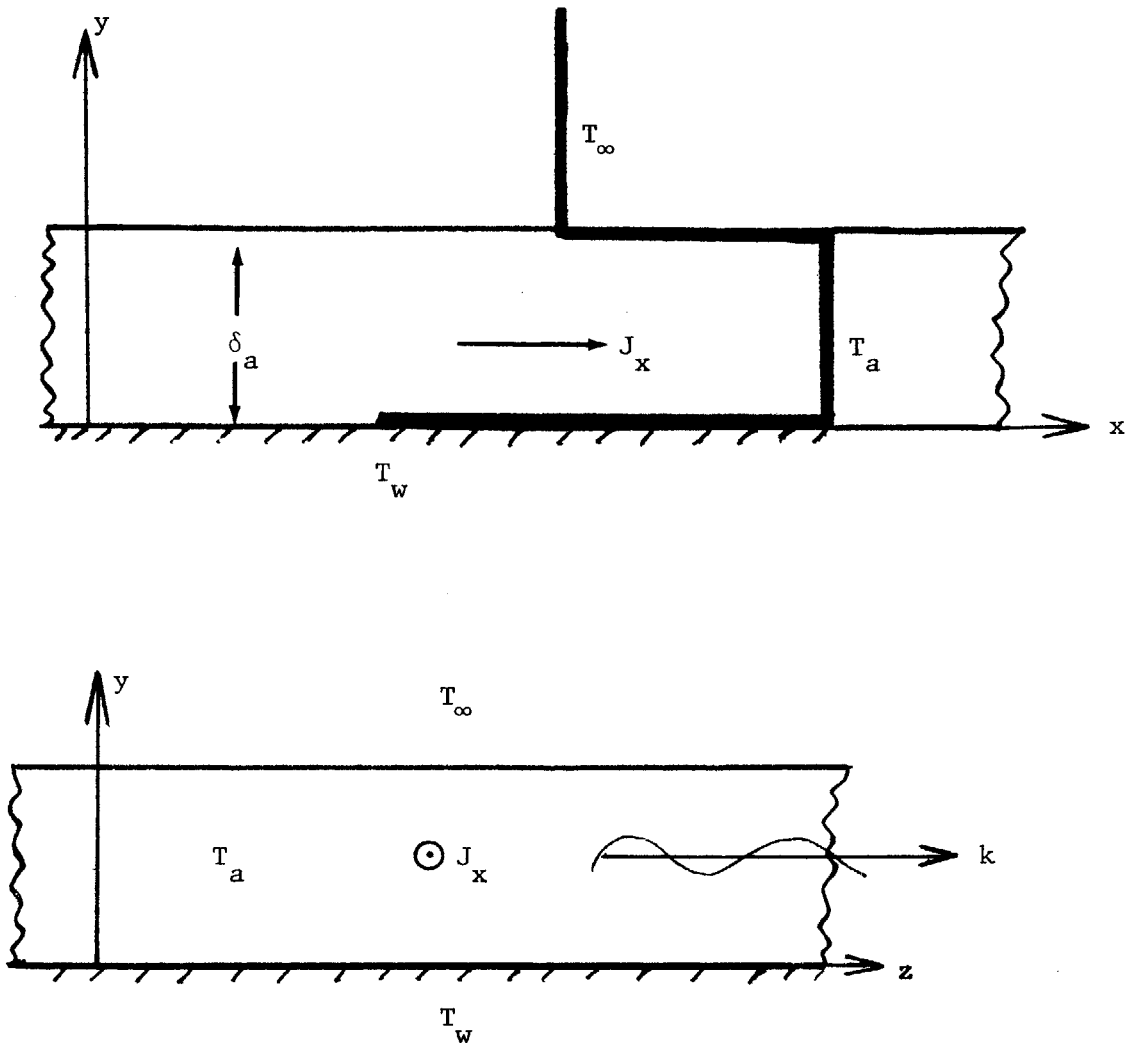


Figure 2: Discrete Model of Two-Dimensional Arc