Essays on Frictions in Financial Institutions

by

Ameya Muley

B. Tech., Indian Institute of Technology Bombay (2009)
M. Sc., London School of Economics (2011)
Submitted to the Department of Economics
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in Economics
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
June 2016
© Ameya Muley, MMXVI. All rights reserved.
The author hereby grants to MIT permission to reproduce and to
distribute publicly paper and electronic copies of this thesis document
in whole or in part in any medium now known or hereafter created.

Signature redacted

Author ……

Signature redacted

Department of Economics
May 15, 2016

Certified by …

Iván Werning
Robert M. Solow Professor of Economics
Thesis Supervisor

Certified by …

Robert Townsend
Elizabeth & James Killian Professor of Economics
Thesis Supervisor

Certified by …

Alp Simsek
Rudi Dornbusch Career Development Associate Professor of Economics
Thesis Supervisor

Accepted by …

Ricardo Caballero
Ford International Professor of Economics
Chairman, Department Committee on Graduate Theses
DISCLAIMER NOTICE

Due to the condition of the original material, there are unavoidable flaws in this reproduction. We have made every effort possible to provide you with the best copy available.

Thank you.

The images contained in this document are of the best quality available.
Essays on Frictions in Financial Institutions

by

Ameya Muley

Submitted to the Department of Economics
on May 15, 2016, in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy in Economics

Abstract

In this thesis, I explore the consequences of frictions in financial intermediation. I theoretically analyse two financial contracts commonly found in the modern shadow banking system—rehypothecation and securitisation. Rehypothecation is the direct repledging of the collateral received in a debt contract by the intermediate lender, while securitisation is the use of the debt contract itself as collateral. I show that rehypothecation enables more efficient reuse of the collateral by the intermediate lender. I emphasise the role of the limited pledgeability of the intermediary in differentiating between the two contracts. In what has significant implications for monetary policy, I also show that open market operations undertaken with the intention of increasing liquidity and investment will take away collateral from the rehypothecation chain and be counterproductive to investment down the chain. I also examine the possibility of distortions created by large global financial institutions on emerging financial markets. In the context of India, I find that prices of firms that receive foreign institutional investor flows are not differentially affected relative to the firms that don’t.

Thesis Supervisor: Iván Werning
Title: Robert M. Solow Professor of Economics

Thesis Supervisor: Robert Townsend
Title: Elizabeth & James Killian Professor of Economics

Thesis Supervisor: Alp Simsek
Title: Rudi Dornbusch Career Development Associate Professor of Economics
Acknowledgments

I am immensely grateful to advisors Iván Werning, Robert Townsend and Alp Simsek, without whose constant guidance and support this thesis would have been impossible. I could not have asked for a better team of advisors, each of them experts on the topic. Every meeting with them was an enjoyable and illuminating experience, and their remarkable insights in response to my primitive arguments often left me feeling somewhat embarrassed. Their individual influences may be seen in parts of the thesis: Iván’s keenness to understand the economic intuition underlying the mathematical equations and the connections to well-known models; Rob’s urging for rigour and formalism and to describe in detail all the steps of the calculations; and Alp’s penchant for simplicity of the models and clarity of the arguments. I would especially like to thank them for giving me the freedom to pursue topics of my interest, rather than setting an agenda for me.

I would also like to thank Daron Acemoglu, Ricardo Caballero, Douglas Diamond, Leonid Kogan at MIT, and especially my classmates Rodrigo Adao, Nicolas Caramp and Dejanir Silva for valuable feedback.

The first two chapters of the thesis have benefited immensely from my visit to the Federal Reserve Bank of New York in 2015. Discussions with Antoine Martin, Tobias Adrian, Fernando Duarte and Thomas Eisenbach were extremely helpful in understanding the nuts and bolts of institutional finance in the United States and provided the foundations for the theoretical model. I would also like to thank Nina Boyarchenko, Nicola Cetorelli, Adam Copeland, Michael Fleming, James McAndrews, Asani Sarkar, James Vickery and other seminar participants at the New York Fed.

Special thanks to Manmohan Singh at the IMF whose papers on collateral reuse provided the motivation to delve deeper into the subject. Discussions with him also helped me to avoid making any unrealistic assumptions in the model, and to clarify the interpretations of the results.

The third chapter of the thesis is co-authored with Professor Ajay Shah of the National Institute of Public Finance and Policy, New Delhi. Work on this project was undertaken during my visits to the NIPFP in 2013 and 2014, and I thank Ajay for the opportunity. We thank Vikram Bahure for assistance with the data and valuable feedback. We also thank Chotibhak Jotikasthira, Christian Lundblad and Tarun Ramadorai for kindly providing data from their paper on the likely capital flows into emerging markets due to mutual funds’ capital shocks.

Many thanks, also, to Gary King, Esther Duflo, Thomas Dattilo, Eva Konomi, Lauren Fahey, Deborah Jamiol and Linda Woodbury for making the journey through the PhD programme and the job market as smooth as possible.
Contents

Introduction

1 Reuse of Collateral: Rehypothecation and Securitisation
  1.1 Introduction ................................................. 11
  1.1.1 Overview of Markets and Practices ...................... 16
  1.2 Model .................................................. 21
  1.3 Optimal Contracts .......................................... 25
    1.3.1 Borrowers’ Problem .................................... 26
    1.3.2 Lenders’ Problem: Securitisation vs Rehypothecation .. 27
    1.3.3 Equilibrium ........................................... 33
  1.4 Securitisation vs Rehypothecation ........................ 33
    1.4.1 Repo vs Mortgage Markets ............................ 38
  1.5 Comparative Statics ........................................ 39
  1.6 Conclusion ............................................... 41

Appendices .................................................. 43

Bibliography ................................................ 55

2 Rehypothecation and Monetary Policy ........................ 58
  2.1 Introduction .............................................. 58
  2.2 Model .................................................. 62
  2.3 Optimal Contracts .......................................... 65
  2.4 Central Bank Intervention .................................. 73
    2.4.1 Empirical Evidence .................................. 77
    2.4.2 Policy Implications .................................. 82
  2.5 Conclusion ............................................... 83

Appendices .................................................. 85
3 Identifying the Push Impact of Foreign Institutional Investor Flows on Indian Equity Prices

3.1 Introduction

3.2 Estimation Approach
  3.2.1 Previous Work
  3.2.2 Our Approach
  3.2.3 The Instrument

3.3 Data

3.4 Results
  3.4.1 Baseline Regressions
  3.4.2 Difference Regressions
  3.4.3 Discussion

3.5 Robustness Checks
  3.5.1 Periods with Non-zero Flows
  3.5.2 Additional Pricing Factors

3.6 Conclusion

Appendices

Bibliography
Introduction

The global financial crisis of 2007-08 has brought to the forefront the strong connections between the financial sector, its inefficiencies, and the real economy. The shadow banking sector, which has grown in size and importance over the last couple of decades, played a crucial role in the development of the crisis. It is, therefore, essential to closely analyse the financing channel of this sector and how it may be affected by policy. In this thesis, I attempt to understand financial institutions and the frictions such as limited borrowing capacity inherent in the intermediation setup. I explore the consequences of these frictions on aggregate investment, and highlight the unintended consequences of standard policy actions. In addition to discussing financial institutions in the context of large and advanced economies like the United States, I also explore the possibility of distortions created by large global financial institutions on smaller emerging markets.

In the first chapter, I examine the nature of two of the most common collateralised borrowing contracts used in the shadow banking sector. In this sector, which is not supported by explicit guarantees such as deposit insurance, collateral plays a vital role in enabling short term finance. The limited amount of high quality collateral means that its reuse is desirable. Rehypothecation and securitisation are two ways in which received collateral is reused by lenders. Rehypothecation is the direct repledging of the collateral received in a debt contract by the lender. This is a common occurrence in prime-brokerage and repo contracts. For example, a hedge fund borrows cash from its prime broker using treasury securities as collateral, and the prime broker then uses the same treasury securities to borrow from a money market fund for its short term liquidity needs. The practice is extremely common—about $3 trillion of received collateral was rehypothecated by US broker-dealers in 2007, roughly a third of their total assets. Securitisation (pyramiding), on the other hand, is the use of the debt contract itself as collateral. For example, in the mortgage market, a small bank owning mortgages borrows from a large investment bank against them. The investment bank then packages
this loan into an MBS or a covered bond and uses it as collateral to borrow in the repo market.

Debt contracts are typically over-collateralised to safeguard against the risk of the value of the collateral over the duration of the contract. I show that rehypothecation is a more efficient way of reusing the collateral in the short-run as it enables the lender to borrow more than the face value of the original debt, which is what the lender can borrow with securitisation. When a lender rehypothecates received collateral, the borrower is exposed to the risk of the lender defaulting and not being able to return the collateral. Thus, the lender effectively borrows the collateral haircut from the borrower. This leads to a two way loan, and the borrower becomes an unsecured creditor of the lender. The extent of this effective borrowing by the lender is determined by the pledgeability of the lender to the borrower. Pledgeability is defined as the upper bound on borrowing that does not trigger incentive problems in the borrower. I show that rehypothecation is observed in equilibrium if pledgeability is sufficiently high, and securitisation is observed otherwise. I argue that rehypothecation endogenously obtains in the prime-brokerage and bilateral repo markets due to high pledgeability among the sophisticated agents in those markets.

In the second chapter, building on the optimal rehypothecation contracts, I examine the optimality of the empirically observed segmented market arrangement in the shadow banking sector. In this arrangement, the borrower (such as a hedge fund) borrows from an intermediary (such as a broker-dealer), who then rehypothecates the borrower’s collateral to an ultimate lender (such as a money market fund). In this segmented market arrangement, the collateral is routed through an intermediary who uses it to fund his valuable intermediation functions. In a centralised market where the borrowers can borrow directly from the ultimate lender, some of the collateral goes directly to the lender without passing through the intermediary, who can no longer use it to fund his valuable projects. I show that, to the extent that the intermediary’s projects are socially more valuable than the borrower’s, the segmented market leads to better aggregate outcomes than the centralised market. The reason for this is the limited pledgeability, and consequently the limited capacity of the intermediary to borrow and rehypothecate the collateral. When the ultimate lender is available to lend to the borrower, the intermediary cannot outbid the ultimate lender to get hold of the borrower’s collateral. When the intermediary’s pledgeability is sufficiently high, this friction disappears, and the centralised market leads to the same outcome as the segmented market, with the intermediary being able to borrow all of the borrower’s collateral to rehypothecate.
This idea also raises a red flag for unconventional monetary policies like large scale asset purchases by the central bank. When a central bank purchases collateral in an open market operation, it provides an additional source of liquidity to the borrower. Again, the intermediary is unable to outbid the central bank due to his limited pledge-ability. The collateral sits idle with the central bank when it could have been used gainfully down the chain by the intermediary. This reduces the effectiveness of the policy action, the stated purpose of which is to increase liquidity and investment. I find strong empirical evidence for this channel—the repo rates with treasury collateral change significantly in the predicted direction when the supply of treasury collateral changes. This has important policy implications, such as using tools like interest on reserves (which do not involve removal of collateral) whenever possible, and restricting open market operations to ultimate cash lenders who do not further lend out collateral.

In the third chapter co-authored with Ajay Shah, we examine the possibility of distortions in emerging financial markets created by foreign financial institutions, known locally as foreign institutional investors (FII’s). Foreign portfolio flows and equity prices are strongly correlated. Policymakers, especially in emerging markets, are concerned that these flows could act as vectors for the transmission of global shocks to the emerging markets, or create distortions in the prices of certain asset classes. They often advocate global policy coordination which takes into account these spillover effects. To examine whether this concern is justified, we estimate the causal impact of the flows on equity prices in the context of an important emerging market—India. We note that the correlation could be due to news about Indian firms, or due to the price impact of the flows themselves. We try to isolate the latter. We regress the equity returns of firms on the contemporaneous foreign flows into them, controlling for aggregate market factors. We instrument for the endogenous flows by the exogenous variation in them produced by fire sales and purchases by US mutual funds caused by shocks to the capital of these funds. We find that the flows do not differentially and abnormally affect the prices of the firms which see these flows. Our result is consistent with FII’s internalising the potential price impact of their transactions and investing only in highly liquid firms. Our identification strategy, however, cannot identify the effect of the flows on the aggregate market factor—we only show that, controlling for the market factor, the flows do not differentially affect the prices of FII-active firms other than through their different loadings on the market factor.

A vast body of literature exists on financial intermediation, and this thesis by no means provides a complete picture of domestic and international financial interme-
diation. The first two chapters theoretically analyse certain under-explored financial contracts which have become ubiquitous in the last couple of decades, while the third chapter contributes a new identification strategy for a well known empirical question in international finance. Financial institutions are extremely important as the specialists that enable the channelling of savings to investment. The spirit of this thesis is to understand the frictions inherent in them so that they can be overcome or circumvented and the efficiency of the institutions can be maximised.
Chapter 1

Reuse of Collateral: 
Rehypothecation and Securitisation

1.1 Introduction

The last couple of decades have seen a spectacular growth in the shadow banking sector. As defined by Pozsar et al. (2012), this is the network of financial intermediaries other than deposit taking banks that has no explicit access to central bank credit or public sector guarantees. Pozsar et al. (2012), Cetorelli, Mandel and Mollineaux (2012), among others, describe this complex network of credit and maturity transformation that includes the traditional banking sector as just one node packaging and selling various types of retail loans into this network. Financial intermediation has grown beyond plain-vanilla banking, in which banks take deposits and lend them out to firms. Gorton (2009) suggests that this growth was the result of decreased regulation from the mid-1980s onwards and financial innovation such as securitisation.

The growth of shadow banking was naturally accompanied by a growth in short-term financing needs, which led to a rapid growth in the market for repurchase agreements, or “repos”, analogous to deposits at banks. The lack of FDIC and other guarantees meant that this type of financing would have to be collateralised. Figure 1-1 shows the multi-fold growth in the repo market since 1990. Growth in collateralised financing led to an increase in the demand for collateral, and in the reuse of collateral. Rehypothecation and securitisation are the two ways in which this is achieved.

Rehypothecation is the practice of the direct reuse by lenders of the collateral posted with them to borrow on their own account. This is different from securitising the cash
flows from the debt and using them as collateral, or “pyramiding” as it is called by Geanakoplos and Zame (2009), Kilenthong and Townsend (2011) etc. In rehypothecation, the received collateral itself is directly repledged. In securitisation, the entire debt contract is posted as collateral, which includes the cash flows from the repayment secured by the underlying collateral. Rehypothecation of collateral by lenders to a third party creates the lenders’ bankruptcy risk—if the lender defaults on his obligation to the third party, the collateral is confiscated by the third party and the original borrower does not get it back even if he is willing and able to fulfill his obligations. The possibility of rehypothecation is common knowledge at the time of contracting, and in most jurisdictions and types of borrowing contracts, a clause can be inserted limiting the amount of rehypothecable collateral or preventing rehypothecation altogether by keeping the collateral in a segregated account. Rehypothecation is common in prime brokerage and bilateral repo markets. Prime-brokers rehypothecate the collateral they receive from their hedge fund clients, and repo lenders reuse the collateral received in a repo contract. A number of recent IMF working papers by Singh and Aitken (2009, 2010) and Singh (2011, 2012) have pointed out its scale, and its implications during the financial crisis. Increased apprehension about the solvency of the lender can make the borrower hold the collateral in a segregated account. In the aftermath of the bankruptcy of Lehman Brothers, panic in the financial markets saw the availability of rehypothecable collateral collapse precipitously. Data from 10-Q forms filed with the
SEC indicates that the amount of collateral that was allowed to be repledged declined from about $4.5 trillion in November 2007 to about $2.1 trillion in December 2009 (Singh and Aitken, 2010).

Securitisation is another way to use received collateral. Small banks and savings and loans associations borrow against their mortgages by issuing debt contracts called mortgage backed securities (MBS) or covered bonds. These are purchased by large investment banks who may use them as collateral in the repo market. Alternatively, the investment banks may pool together MBS’s and borrow against a portfolio of them by issuing collateralised debt obligations (CDO). As is commonly understood, securitisation involves two steps. The first step is pyramiding, or converting a loan secured by the mortgage into a marketable instrument. The second is tranching, or pooling together several mortgages and creating instruments of different risk profiles. In this paper, I focus on the pyramiding part of securitisation.

In this paper, I study a unified model of rehypothecation and securitisation with the prime brokerage, repo and mortgage markets in mind. In my setup, both the borrower and the lender have variable scale investment projects. The lender can either securitise the debt or rehypothecate the collateral to borrow from a third party to invest in his project. With securitisation, the amount that the lender can borrow is restricted by the face value of the securitised debt, no matter how much of the collateral asset underlies the debt. Even if the lender wishes to borrow for the short term, the amount that he can borrow depends on the face value of the original debt. With rehypothecation, however, the amount that the lender can borrow is typically greater than the face value of the debt. There can be two ways in which this is achieved. One, if the lender wishes to borrow for the short term, he can borrow more by getting lower haircuts\(^1\) since the risk of the collateral is lower in the short term. Two, if the degree of overcollateralisation in the original debt contract is high (higher than that needed to account for the risk of the collateral), the lender can borrow much more than the face value of the original debt by rehypothecating the collateral. I show that rehypothecation achieves the constrained efficient outcome of maximising the investments of both the borrower and the lender, by allowing the lender to make the most efficient use of the collateral and squeeze a large amount of liquidity from it. Rehypothecation comes at a cost to the original provider of the collateral. The lender may fail to deliver the collateral on the settlement date, following which the borrower may have trouble recovering the collateral haircut from

\(^{1}\)Haircut = 1 – \(\frac{\text{Amount of Borrowing}}{\text{Value of Collateral}}\).
the bankrupt or credit-constrained lender. The lender compensates the borrower for this risk by offering him a lower interest rate.

I consider the role of the borrower-lender relationship in making rehypothecation feasible. When a lender rehypothecates the collateral, he effectively borrows from the borrower the difference between the value of the collateral and the face value of the debt. If the lender’s income is sufficiently pledgeable to the borrower, for example, in a close relationship characterised by effective monitoring, rehypothecation is optimal. In an arm’s length relationship, the borrower’s collateral constraint and the lender’s pledgeability constraint cannot be simultaneously satisfied and rehypothecation becomes infeasible. Securitisation, then, obtains as the equilibrium outcome.

The long term and multi-contractual nature of prime brokerage relationships between hedge-funds and their prime-brokers, and the hedge fund managers’ familiarity with the practices of their brokers, allows them to effectively monitor and discipline them. Similarly, the borrowers in the bilateral repo market are sophisticated hedge funds or other large dealers. The high pledgeability of the lenders in these markets makes rehypothecation feasible and optimal. On the other hand, in the mortgage market, the borrowing is done by relatively unsophisticated agents—a household borrows from a small bank by pledging the house, and a small bank borrows from a large investment bank by pledging the mortgage loan. The unsophisticated nature of the borrowers and their limited contractual arrangements with their lenders renders them ineffective at disciplining their lenders. Hence, rehypothecation becomes infeasible and securitisation obtains. The small bank uses the mortgage loan (secured by the house) as collateral to borrow from a large investment bank by issuing an MBS. The large investment bank uses the MBS (secured by the mortgage) as collateral to borrow in the repo market. Thus, I propose an explanation for the prevalence of securitisation in mortgage markets, and of rehypothecation in prime brokerage markets.

Rehypothecation is a key feature of the bilateral repo market. A number of papers, most notably Gorton and Metrick (2013), document the run-like situation in the bilateral repo market during the crisis, characterised by high rates and haircuts. In view of the documented facts, I study the comparative statics of a rehypothecation contract with respect to the lender’s risk and pledgeability, and the collateral risk. I find that an increase in the risk of the collateral increases the haircut, a channel which was at work during the crisis in the repo market. An increase in the risk of the lender’s project (keeping the return constant) lowers the haircut and the interest rate. If the pledgeability of the cash flows of the lender falls, the haircut falls and the interest rate rises.
If the net present value of the lenders’ investment becomes negative or if the lenders’ pledgeability is too low, rehypothecation is prevented by borrowers, as was the case during the crisis.

Related Literature

There has been very little work on modelling rehypothecation which is so widespread in the financial system. There has been considerable empirical work on the repos markets. Krishnamurthy, Nagel and Orlov (2014) and Copeland, Martin and Walker (2014) study the tri-party repo market and find little evidence of a run there during the crisis. Gorton and Metrick (2013) look at the bilateral repo market and find significant increases in haircuts and interest rates during the crisis. Martin, Skeie and Von Thadden (2014) model repo markets and show that repo runs can occur due to self-fulfilling expectations. They also note that the tri-party repo market may be fragile due to some of its peculiar mechanisms such as the morning unwind. However, they do not model rehypothecation.

Bottazzi, Luque, and Páscoa (2012) study the existence of equilibria with limited and unlimited rehypothecation, but they abstract from considerations of the lender’s default risk. In their recent papers, Eren (2014) and Infante (2014) specifically model rehypothecation and view it as a way for the intermediate broker to obtain liquidity by offering differential haircuts: higher haircuts to the cash borrower and lower haircuts to the lender on rehypothecation. Maurin (2014) considers a general equilibrium model with collateral constraints and rehypothecation in a frictionless setting, and finds that rehypothecation can at best be a substitute for complete markets. Andolfatto, Martin and Zhang (2015) focus on the liquidity creating role of rehypothecation, and argue that limiting it may be desirable in increasing the demand for cash balances in economies away from the Friedman rule. Lee (2015) studies the effect of collateral reuse on repo spreads. My paper is most closely related to Eren (2014), Infante (2014) and Maurin (2014). It differs from the literature by looking at the optimality of rehypothecation and showing that rehypothecation can be a major improvement over securitisation in the presence of market incompleteness arising out of moral hazard. This paper also studies the effects of monetary policy actions in the presence of rehypothecation chains.

The model of this paper also relates to the models of money which is valuable as a medium of exchange by different individuals at different points in time. Clearly, rehypothecation increases liquidity in the financial system since the same collateral can be used to finance multiple investment opportunities at different financial institutions at
Figure 1-2: Flow of cash and collateral.

different points of time. Collateral that is allowed to be repledged acquires some features of money reminiscent of Townsend (1980), in which intrinsically useless money acquires value as a medium of exchange between spatially separated agents who have good and bad endowments alternately as they move across space. Similarly with rehypothecation, a given amount of collateral can be passed from agent to agent to finance multiple investments at multiple points in time. Analogous to the velocity of money, collateral also acquires velocity, which is the number of times a piece of collateral is pledged and repledged.

Outline:

In Subsection 1.1.1, I provide an overview of the mechanics of the prime brokerage and mortgage markets. This subsection motivates the model by identifying the common features in these markets, and can be skipped without losing the essentials of the model and the results. Section 2.2 describes the model, the contracts and the timing. Section 2.3 analyses the securitisation and rehypothecation contracts individually. Section 1.4 examines in detail the optimality of rehypothecation over securitisation. Section 1.5 looks at a crisis scenario and discusses the effects of lender health and collateral quality on haircuts and interest rates. Section 2.5 concludes.

1.1.1 Overview of Markets and Practices

In this section, I describe the market practices which motivate my model. The reader may skip this section without losing the crux of the model and the results. The key mechanics of the model are described by a simple borrowing chain as in Figure 1-2. Borrower A borrows from lender B, who in turn securitises that debt or rehypothecated the collateral to borrow from C.
Prime brokerage and securities lending

The practice of rehypothecation is quite common in the shadow banking system in the United States, and especially in the United Kingdom. Broker-dealers routinely rehypothecate the collateral received from their hedge fund clients in a repo or a derivative contract. Rehypothecation is also a feature of repo contracts—in a repo contract, the permission to rehypothecate is implicit as the collateral is effectively sold on the origination date to the lender who can do with it what he wishes. Monnet (2011) discusses the legal minutiae of the ownership of the collateral in a repo transaction in different jurisdictions. In tri-party repos, the collateral sits with the clearing bank and is not rehypothecated. The total size of the US repo market is estimated to be about $3 trillion in 2012, with the tri-party segment making up for about half of it. A survey by Financial Services Authority (2012) indicates that repos make up 40-60% of the borrowing by hedge funds.

The basic steps that result in the creation of the smallest rehypothecation chain of length 2 are as follows. A hedge fund (or a bank) “A” needs to borrow cash from its prime broker (or another bank) “B” for a certain duration. The hedge fund receives the cash and posts a security as collateral. The collateral is usually of high quality, such as treasury bonds, Agency MBS’s or highly liquid shares. The lender usually takes a haircut to protect himself against a fall in the value of the collateral at the precise time when the borrower defaults. Often, the borrowing is in the form of margin buying of securities from the prime broker, in which case the purchased security itself serves as the collateral. If the hedge fund does not permit the prime-broker to rehypothecate the collateral, it is placed in a segregated client account and the borrower retains ownership of it. The collateral is not attacked by the creditors in the event of the prime broker’s bankruptcy. If the prime-broker is permitted to rehypothecate the collateral (for example, in a repo), it may use it to further borrow from a money market fund “C” through a tri-party repo. This second borrowing transaction may be of a duration shorter than the first, and when the settlement date of the first transaction (between the hedge fund and the prime broker) arrives, the prime broker will have to purchase the collateral of equivalent value to return to the hedge fund.

\(^2\)http://libertystreeteconomics.newyorkfed.org/2012/06/mapping-and-sizing-the-us-repo-market.html

\(^3\)Money market funds enter into repo contracts with broker-dealers in an arrangement intermediated by JP Morgan or Bank of New York Mellon, which match the cash lenders (MMF’s) with the cash borrowers and also hold the collateral on behalf of the MMF’s. This is called a tri-party repo.
Collateral

Figure 1-3: Payoff dependencies with Rehypothecation. An arrow from x to y indicates that the performance of y affects the payoff of x.

On the other hand, if the prime broker has rehypothecated the collateral it may be, on the settlement date of the original borrowing transaction, unable to produce the collateral to return to the borrower either due to a liquidity crunch or bankruptcy. If the value of the collateral to be returned is higher than the value of the debt, which it usually is due to a haircut, the borrower then faces the risk of losing that haircut. Thus, with rehypothecation, both the lender and the borrower are exposed to each other’s credit and liquidity risks. The crucial point about rehypothecation is that once the collateral is reused, the borrower no longer has first or exclusive access to it, becomes an unsecured creditor and must stand in the queue of creditors in the event of the lender’s bankruptcy if he wishes to recover the haircut. This lack of full security for the borrower may lead it to prevent the lender from rehypothecating the collateral if he is pessimistic about the solvency of the lender. This, however, comes at a cost which is reflected in the higher prime brokerage fees.

The payoff dependencies in a rehypothecation chain are shown in Figure 1-3. The payoffs of all three agents depends on the value of the collateral, since it underlies both debt contracts and is liquidated in the event of a bankruptcy. The payoffs of C and B also depend on the performance of the investments of their borrowers B and A respectively. Finally, the risk of losing the collateral haircut due to B’s default means that the payoff of A also depends on the performance of his lender B.

Singh and Aitken (2009, 2010) document the regulatory regimes regarding rehypothecation in the United Kingdom and the United States. Here, I summarise the key points. In the United States, broker-dealers are limited to rehypothecating margin securities amounting up to 140% of the debit balance by Rule 15c3-3 of the Securities

\[4\text{ An example from Singh and Aitken (2009): if$500 \text{ of securities are deposited as margin with the broker-dealer and the debit balance (the outstanding amount owed to the broker-dealer) is$200, then} \]

18
Investor Protection Act (1970). Non broker-dealers (such as banks) are not covered by this regulation, so that theoretically, there can be unrestricted rehypothecation in the United States. United Kingdom law neither distinguishes between broker-dealers and banks, nor provides for any cap on rehypothecation or customer protection in case of bankruptcy of the rehypothecating party. The regime in the European Union is less clear, but it appears that there is no cap provided for rehypothecation and contracting parties are free to bargain on how much collateral might be permitted to be reused.

Financial institutions like hedge funds have been taking advantage of the regulatory structure concerning rehypothecation by broker-dealers in the UK to borrow from UK-based or UK subsidiaries of US broker-dealers. Singh (2011) documents anecdotal evidence that prime-broker fees in the UK can be as low as LIBOR+50 bps when the broker-dealer is allowed to rehypothecate the collateral, and as high as LIBOR+250 bps otherwise. On the flip side, there is little protection afforded to customers in the event of the broker-dealer's bankruptcy. At the time of Lehman Brothers’ filing for bankruptcy in September 2008, its London branch, Lehman Brothers International Europe, had rehypothecated about $22 billion of its clients' assets, most of which were not recovered when British administrators took charge of it (Singh and Aitken (2009)).

**Mortgage market**

The mortgage market offers an alternative way to reuse collateral. Instead of the lender directly reusing the received collateral for his own borrowing, the lender packages the debt contract it holds into a marketable security and uses that as collateral.

The collateral chain starts with a small retail bank or savings or loan association “A” owning a mortgage on a house. To raise funds for additional investments, the bank sells the mortgage or a pool of mortgages to an investment bank “B” in return for a promise to service the mortgages and channel interest and principal payments from them to the buyer. These are called Mortgage-Backed Securities (MBS) or Mortgage Pass-through Securities. In most cases, this is done through an additional intermediary, which is a government-sponsored enterprise (GSE) such as the Fannie Mae, the Freddie Mac or the Ginnie Mae. The GSE securitises the pool of mortgages from the small banks and sells it on to an investment bank, applying its backing to it in the process. Alternatively, instead of selling the mortgages to the investment bank, the mortgage
originator borrows against them from the investment bank by issuing a security called a covered bond. In 2014, there were €2.1 trillion of covered bonds outstanding worldwide, most of this volume being in Europe (Adrian et al., 2013).

Once the investment bank receives these MBS’s or covered bonds, it can use them as collateral in a repo market to borrow from another bank or a money market fund “C”. Notice that the investment bank may use the MBS’s or covered bonds as collateral, and does not have the right to use the underlying mortgages as collateral, which may well be residing on the balance sheets of the originators of those mortgages. In each case, the mortgage originator is not affected by the investment bank’s default, as it pays the new holder of the MBS or covered bond under the same terms. Figure 1-4 shows the payoff dependencies with this type of (Type-I) securitisation. A’s payoffs only depend on the collateral. B’s payoffs depend on the performance of A, and of the collateral. C’s payoffs depend on the performance of B, and if B defaults, on the performance of A and the original collateral.

Often, instead of using the MBS’s as collateral, the investment bank ring-fences the pool of these MBS’s by creating a special purpose vehicle which issues bonds called Collateralised Debt Obligations (CDO). Using the proceeds from the CDO issue, the special purpose vehicle purchases these MBS’s from the investment bank. The CDO is purely a debt owed by the special purpose vehicle, backed by the assets of the vehicle. The investment bank thus isolates itself from the MBS’s, and the CDO investors from the balance sheet of the investment bank. The special purpose vehicle typically has no assets other than the MBS’s, and if one abstracts from the tranching of the payoffs from these MBS’s to suit the needs of different investors, the arrangement resembles a sale of the MBS’s to them. Figure 1-5 shows the payoff dependencies with these off-balance...
Figure 1-5: Payoff dependencies with **Type-II securitisation**. An arrow from $x$ to $y$ indicates that the performance of $y$ affects the payoff of $x$.

sheet arrangements (Type-II) securitisation. A’s payoffs only depend on the collateral. B has moved the borrowing contract off its balance sheet into a special purpose vehicle B’, and is isolated from the contract now. C’s payoffs depend on the performance of B’, and if B’ defaults, on the performance of A and the original collateral.

In the model of this paper, both Type-I and Type-II securitisation are equivalent in terms of payoffs and outcomes. I focus on analysing Type-I securitisation.

### 1.2 Model

I describe here a unified model of collateralised borrowing and the reuse of collateral. There are three dates, $t = 1, 2, 3$, and three types of agents, A, B and C. There is a continuum of unit mass of each type. I interpret an agent of type A to be a small bank or a hedge fund that needs to borrow, and an agent of type B to be an investment bank or prime-broker that can lend to the bank/hedge-fund A against collateral, and that can later rehypothecate that collateral. The small bank or hedge fund has a relatively longer term investment (e.g. a mortgage for the bank that can last for years, or an investment strategy for the hedge fund that can last for weeks or months), and the large bank or broker-dealer has a relatively short-term investment (e.g. an overnight liquidity need). Agent C is interpreted as a money market fund that lends to banks B against collateral.
There is one good called cash and an asset called collateral. The cash good is the numeraire. The collateral pays cash at $t = 3$.

**Preferences and Belief Disagreements**

Agents of type $A$, $B$ and $C$ are risk neutral, and consume cash at $t = 3$. There is no discounting. $A$, $B$ and $C$ have different beliefs about the collateral. At $t = 1$, $B$ believes that expected value of the payoff of the collateral is $1$. At every date, the expectation goes up by $u > 1$ with probability $q$ and down by $d < 1$ with probability $1-q$, such that

$$qu + (1-q)d = 1. \quad (1.1)$$

At $t = 1$, $A$ is more optimistic than $B$ about the payoff and believes that the expected value is $u$, whereas $C$ is less optimistic than $C$ and believes that the expected value is $d$. The agents’ beliefs are common knowledge. The beliefs are summarised in Figure 1-6.

**Endowments and Investments**

At $t = 1.1$, $A$ has $\Omega$ units of the collateral asset, and $B$ has 1 unit of cash. The first period is further divided into two: $t = 1.1$ (beginning of the period) and $t = 1.2$ (end of the period). $B$ cannot store the cash between $t = 1.1$ and $t = 1.2$. $B$ has access to a storage technology between $t = 1.1$ and $t = 3$, and between $t = 2$ and $t = 3$. $I$

---

$^5$This assumption is not essential, and only serves to fix the outside option of $B$. 

---
normalise this interest rate to 0. C has a large quantity of cash at each period that can be stored at an interest rate of 0 between any two periods.

At $t = 1.1$, A has access to a variable scale investment opportunity that yields $R_A > 1$ units of cash with certainty at $t = 3$ per unit investment of cash. At $t = 1.2$, B has access to an investment opportunity that yields $R_B > 1$ units of cash at $t = 2$ per unit investment of cash with probability $p$, and 0 with probability $1 - p$. I assume that the returns to the projects of all agents of type B are independent of each other, and of the value of the collateral.

**Borrowing Contracts**

I assume that the market is segmented: A and C cannot deal with each other, and must deal with the intermediary B.\(^6\) To finance their investments, A must borrow from B, and B must borrow from C.

**Assumption 1.** The income from A's project is not pledgeable to B or C. The income from B's project is not pledgeable to C. A fraction $\kappa$ of the income from B's project is pledgeable to A.

If A is an unsophisticated small bank or mortgage originator, I assume that $\kappa$ is low and B's income is not pledgeable to A as it isn't to C. If A is a sophisticated agent such as hedge fund, I assume that $\kappa$ is large and B's income is partly pledgeable to A. The partial pledgeability of B's income is motivated by its large size and strong reputation and the desire to protect it. I describe this in detail in a moral hazard setup in Appendix 2.A.

Without loss of generality, I consider contracts offered by agents B to A and C. All agents behave competitively. In particular, any agent B offering a contract takes as given the contracts offered by the other agents B. I first consider the contracts $C_{A,B} = (r, h, \theta)$ offered by B to A at $t = 1.1$. Here, $r$ is the net interest rate, $h$ is the haircut\(^7\), and $\theta \in \{0, 1\}$ is the rehypothecation permission: B can rehypothecate the collateral if $\theta = 1$ and not otherwise.\(^8\) I define one unit of this contract as lending one

\(^{6}\) I examine the consequences of relaxing this constraint in Chapter 2.

\(^{7}\) Haircut = $1 - \frac{\text{Units of cash borrowed}}{\text{Units of collateral pledged}}$.

\(^{8}\) The binary permission is without loss of generality. The framework does not preclude B offering multiple contracts to A. Some of these contracts may have $\theta = 1$, and others may have $\theta = 0$, so that in aggregate, any fraction between 0 and 1 of the total collateral may be permitted to be rehypothecated. The linear and risk-neutral nature of the framework will, however, ensure a corner solution with only one contract chosen between A and B, and with the aggregate $\theta \in \{0, 1\}$.
unit of cash by B to A under the given terms. I will frequently find it easier to refer to the contracts *per unit of borrowing* in terms of the face value of the debt $1 + r$, and the number of units of collateral posted $\frac{1}{1-h}$.

At $t = 1.2$, B may borrow from C by offering him a contract $C_{B,C} = (\tilde{r}, \tilde{h})$, depending on what contracts he offered at $t = 1.1$. Here, $\tilde{r}$ is the interest rate, and $\tilde{h}$ is the haircut. Again, I define one unit of this contract as lending one unit of cash by C to B. If $\theta = 1$, B can either directly rehypothecate the received collateral asset, or he can securitise the collateralised debt $C_{A,B}$ and post that as collateral. If $\theta = 0$, B can only securitise the collateralised debt.

Let $D_{A,B}(s_t)$ be the value of one unit of the underlying collateral in $C_{A,B}$, and $D_{B,C}(s_t)$ be the value of one unit of the underlying collateral in $C_{B,C}$ (the asset with rehypothecation, and the securitised debt with securitisation) in state $s_t$ at the time $t$ of maturity. Due to no-recourse, the payoff of $C_{A,B}$ is given by

$$\min \left\{ 1 + r, \frac{1}{1-h} D_{A,B}(s_t) \right\},$$

and that $C_{B,C}$ is given by

$$\min \left\{ 1 + \tilde{r}, \frac{1}{1-\tilde{h}} D_{B,C}(s_t) \right\}.$$

**Theorem 1.** For any fixed $d \leq 1$, there exists $\bar{u} > 1$ such that for $u > \bar{u}$ and $q$ satisfying Equation (1.1), the optimal contracts are such that

- $C_{A,B} = (r, h, \theta)$ is a long-term contract between $t = 1.1$ and $t = 3$ with

$$1 + r \leq \frac{1}{1-h} \min_{s_3} D_{A,B}(s_3).$$

- $C_{B,C} = (\tilde{r}, \tilde{h})$ is a short-term contract between $t = 1.2$ and $t = 2$ with

$$1 + \tilde{r} \leq \frac{1}{1-\tilde{h}} \min_{s_2} D_{B,C}(s_2).$$

Moreover, the collateralised borrowing contracts $C_{A,B}$ and $C_{B,C}$ are optimal compared to sales of the collateral.

This is similar to the optimal contracts in Geanakoplos (2009). If the debt contract is not safe, there will be default in states to which the borrower attaches a low probability.
Then, the borrower could increase the degree of collateralisation at little cost since the
default state had a low probability anyway according to him. But this makes the lender,
who was attaching a higher probability to the default state, feel more secure and reduce
the interest rate charged to the borrower. Similarly, a sale is suboptimal because the
more optimistic borrower (seller) would like to retain possession of the collateral.

Remark 1. The literature has dealt with the problem of repurchase agreements versus
asset sales. For example, Dang, Gorton and Holmström (2013) show that repurchase
agreements will be preferred to asset sales since the lender (or buyer) will not be re-
quired to acquire information at a cost. Monnet and Narajabad (2012) reach the same
conclusion by considering a hold-up problem at the repayment date when the lender
(or reseller) can extract all the surplus if the borrower has no predetermined right to
the collateral which is valuable to him. The focus of this paper is not to understand
the trade-off between asset sales and repurchase agreement, but to understand the con-
sequences of rehypothecation of collateral received in a debt contract. I therefore make
the following assumption.

Assumption 2. \( u > \bar{u} \).

Remark 2. The assumption of A’s extreme optimism in the short run, together with
Assumption 6, is to focus on safe debt contracts. Similarly, the assumption of C’s
extreme pessimism in the short run is to provide an upper bound on the haircut between
B and C in the short run. As I analyse in the following two sections, rehypothecation is
efficient because it enables more short-run borrowing for B from C than securitisation.
This is achieved because through rehypothecation, B can borrow at a lower haircut
from C in the short run, even at this upper bound defined by C’s pessimism.

At \( t = 2 \), the returns to B’s investment are realised, and then \( C_{B,C} \) is settled. At
\( t = 3 \), the returns to A’s investment are realised, and finally \( C_{A,B} \) is settled.

1.3 Optimal Contracts

I derive below the optimal contracts \( C_{A,B} \) and \( C_{B,C} \). I derive these contracts under the
assumption that all agents act competitively, taking the actions of other agents as given.
In particular, I assume that while offering the contracts, each agent of type B takes the
actions of other agents of type B as given. The optimal contracts determine the supply
of collateral from A and the demand for it from B, as a function of a “price”. This
"price" is the net return to A from "lending out" one unit of collateral to B. The market clearing for collateral pins down this equilibrium net return. I look for a symmetric equilibrium where all agents of type B offer the same contract.

Denote by \((x_i, z_i), i \in \{A, B\}\) the holdings of cash and collateral respectively at \(t = 1.1\), and by \((\tilde{x}_B, \tilde{z}_B)\) the holdings of cash and collateral of B at \(t = 1.2\).

## 1.3.1 Borrowers' Problem

A faces the offer \((r, h, \theta)\) from B. Suppose A holds \(z_A\) units of collateral and commits \(\Omega - z_A\) units from his endowment to this contract, and invests the proceeds from the borrowing into his opportunity. Let \(1 + \bar{r}\) be the gross cost of borrowing, inclusive of expected losses due to B’s default after rehypothecation. (If B does not rehypothecate, \(1 + \bar{r} = 1 + r\), the gross interest rate.) A solves

\[
\max_{z_A \leq \Omega} \frac{(R_A - (1 + \bar{r}))}{(1 - h)(\Omega - z_A)}. \tag{1.2}
\]

**Definition 1.** \(\bar{R} = (R_A - (1 + \bar{r}))(1 - h)\).

\(\bar{R}\) can be interpreted as the net return earned by A by lending collateral to B, or as the cost faced by B of borrowing collateral from A. Every agent of type B will have to provide at least an excess return of \(\bar{R}\) provided by the other competing agents of type B, for the contract to be accepted. Thus, \(\bar{R}\) also represents the outside option of A when considering a contract offered by agent of type B. An agent of type B will take \(\bar{R}\) as given and offer an optimal contract \((r(\bar{R}), h(\bar{R}), \theta(\bar{R}))\).

A will maximise his payoff by committing all of his endowment of collateral to this contract if \(\bar{R} > 0\), and by holding on to the collateral if \(\bar{R} < 0\). This gives the optimum holding of collateral

\[
z_A(\bar{R}) = \begin{cases} 0, & \text{if } \bar{R} > 0, \\ \in [0, \Omega], & \text{if } \bar{R} = 0, \\ \Omega, & \text{if } \bar{R} < 0. \end{cases} \tag{1.3}
\]

and the holding of cash\(^9\)

\[
x_A(\bar{R}) = (1 - h(\bar{R}))(\Omega - z_A(\bar{R})). \tag{1.4}
\]

\(^9\)From the budget constraint \(z_A + \frac{z_A}{1 - h(\bar{R})} = \Omega\).
1.3.2 Lenders' Problem: Securitisation vs Rehypothecation

At $t = 1.1$, B can choose from four actions: (i) do nothing and save the endowment of cash into the long-term storage technology and consume it at $t = 3$, (ii) offer a debt contract to A and hold on to it till $t = 3$, (iii) offer a debt contract to A and securitise the debt to borrow from C at $t = 1.2$, or (iv) offer a debt contract to A and rehypothecate the collateral to borrow from C at $t = 1.2$. If B has lent to A at $t = 1.1$, then B must decide whether and how to borrow for the short term to invest in his opportunity at $t = 1.2$. If the return to B from borrowing from C and investing is positive, the linearly scalable nature of his investment opportunity means that he will try to maximise the amount of investment. B's problem at $t = 1.2$ can, therefore, be split into two sub-problems: (i) deciding whether or not to invest, and (ii) maximising the amount of investment. I first look at the problem of maximising the investment, and then check whether the investment is profitable.

While offering a debt contract $C_{A,B}$ to A, B takes as given the net return $\bar{R}$ per unit of collateral that A gets from other agents B. The contract will, therefore, be a function of $\bar{R}$. Suppose that B has offered a contract $C_{A,B}(\bar{R}) = (r(\bar{R}), h(\bar{R}), 0(\bar{R}))$ to A at $t = 1.1$. Suppose also that A has accepted 1 unit of the contract, i.e. B has lent 1 unit of cash to A. Per unit of $C_{A,B}$, the value of the debt held by B is $1 + r$, and the value of the collateral received is $\frac{1}{1-\bar{R}}$. Now B has a choice of two assets to provide as collateral to borrow from C: the securitised debt or the underlying collateral asset itself. The securitised debt has a safe value $1 + r$, given the assumption that it is fully collateralised. The value of the underlying collateral is risky, and is $\frac{1}{1-\bar{R}}d$ in the worst case when the short-term contract with C expires at $t = 2$. The following lemma summarises the optimal contract $C_{B,C}$ that maximises the amount of borrowing from C.

**Proposition 1.** Given the contract $C_{A,B} = (r, h, \theta)$, the optimal contract $C_{B,C}$ is given by

$$(\tilde{r}, \tilde{h}) = \begin{cases} (0, 0), & \text{if the securitised debt is used as collateral, and} \\ (0, 1 - d), & \text{if } \theta = 1 \text{ and the underlying collateral is rehypothecated.} \end{cases}$$

The interest rate in each case is zero since C is risk neutral, has a large cash endowment, and access to an outside savings technology with a zero interest rate. The haircut that needs to be provided to C is zero when the collateral is the securitised debt.
\( C_{A,B} \) with a safe value, and is positive when the collateral is the rehypothecated asset with a risky value. Note that the degree of collateralisation of \( C_{A,B} \) is irrelevant to the amount that \( B \) can borrow by securitising it, as long as it is over-collateralised—the borrowing of \( B \) is only determined by the face value of the debt. Proposition 1 implies that per unit of the contract \( C_{A,B} \), \( B \) can borrow from \( C \) the amount

\[
\tilde{x}_B = \begin{cases} 
(1 + r), & \text{if the securitised debt is used as collateral, and} \\
\frac{1 - h}{1 - h_d}, & \text{if } \theta = 1 \text{ and the underlying collateral is rehypothecated.}
\end{cases} \tag{1.5}
\]

I now consider \( B \)'s decision to invest at \( t = 1.2 \), and whether to do it by securitising the debt or rehypothecating the collateral. Any contract offered to \( A \) must satisfy the assumed full-collateralisation constraint

\[
1 + r \leq \frac{1}{1 - h} d^2. \quad \tag{1.6}
\]

It must also satisfy \( A \)'s participation constraint in order for it to be accepted by \( A \).

**Consuming Endowment**

If \( B \) does not lend to \( A \), then at \( t = 1.1 \) he stores his endowment of 1 unit of cash into his long term storage technology and consumes it at \( t = 3 \) to get a utility of 1.

**Holding on to Debt**

When \( B \) offers a contract which he intends to hold on to till maturity, without borrowing against it and investing at \( t = 1.2 \), there is no risk to \( A \) of \( B \)'s default and loss of collateral. The cost of borrowing to \( A \) is, therefore, the interest rate \( r \). The contract offered by \( B \) must satisfy \( A \)'s participation constraint

\[
(R_A - (1 + r))(1 - h) \geq \bar{R}. \tag{1.7}
\]

The expected payoff of \( B \) is simply \( 1 + r \) and \( B \) solves the problem

\[
\max_{r, h} 1 + r, \tag{1.8}
\]

subject to the collateral constraint (2.6) and the participation constraint (2.5). Here, a haircut greater than the minimum required serves no purpose, and wastes useful
collateral. The solution is to make the collateral constraint bind, and raise the interest rate to the maximum possible so that A’s participation constraint binds.

**Securitisation**

When B securitises the debt contract $C_{A,B}$ and posts it as collateral to C, A is unaffected by B’s performance. With securitisation, the underlying collateral is not mixed with B’s balance sheet, and is kept in a segregated account, possibly with a third party or with A himself, and A has the first rights to it when he repays the debt. The underlying collateral does not pass through B’s bankruptcy proceedings. At $t = 2$, if B is solvent, B recovers $C_{A,B}$ from C. The collateral asset can then be recovered by A at $t = 3$ by paying off his debt. At $t = 2$, if B is insolvent, C impounds $C_{A,B}$. In this case, A pays the interest to C at $t = 3$ and recovers the underlying collateral asset. Thus, the cost of borrowing for A is again simply the interest rate $r$, and A’s participation constraint again takes the form of (2.5).

Now from Proposition 1, B is able to borrow a total of $1 + r$ cash from C which he invests into his opportunity. I explain in detail the state-by-state cash positions of A, B and C at $t = 2$ and $t = 3$ in Table 1.1 of Appendix 1.A. Intuitively, if the project succeeds, he gets the cash flow from the project, pays off the debt to C, and receives repayment from A. If the project fails, B gets nothing since $C_{A,B}$ is impounded by C.

Thus, B solves

$$
\max_{r,h} p \left( \frac{\text{Cash flow from project}}{R_B(1 + r)} - \frac{\text{Repayment to C}}{(1 + r)} + \frac{\text{Repayment from A}}{(1 + r)} \right) = pR_B(1 + r), \quad (1.9)
$$

subject to the collateral constraint (2.6) and the participation constraint (2.5). The optimal contract offered is the same as in that case of holding on to the debt.

**Rehypothecation**

From Lemma 1, B is able to borrow a total of $\frac{1}{1-h}d$ from C by rehypothecating the collateral received from A. On borrowing from C, B invests in his opportunity. The effective state-by-state cash positions of B and C after the settlement of $C_{B,C}$ and $C_{A,B}$ are explained in detail in Table 1.2 in Appendix 1.A. Intuitively, whether or not B’s project succeeds, C breaks even, either by getting repaid in full by B, or by liquidating the collateral. (C keeps the amount $\frac{1}{1-h}d$ and returns to B the excess amount as per
Assumption 4 in Appendix 1.A.) Thus, when B fails, he defaults on the returning of the collateral to A, A does not repay the debt to B, and instead loses the collateral value $\frac{1}{1-h}d$ to C.

The cost of borrowing to A is the interest rate when B is solvent with probability $p$, and the lost collateral to C when B is insolvent with probability $1-p$. A’s participation constraint, thus, takes the form

$$\left( \begin{array}{c} \text{Normal repayment} \\ R_A - p(1+r) \end{array} \right) - \left( 1 - p \right) \frac{1}{1-h}d (1-h) \geq \bar{R}. \quad (1.10)$$

A knows that C is pessimistic about the collateral and only lends an amount $\frac{1}{1-h}d$. Consequently, when B defaults, A knows that C will only keep the part of the collateral with value $\frac{1}{1-h}d$, and return the remainder. The above expression makes it clear that when B is insolvent, A loses an amount greater than the face value of the debt owed to B. Thus, per unit of borrowing, I can think of $1 + r$ as the riskless debt owed to B, and the amount

$$\frac{1}{1-h}d - (1+r),$$

as A’s exposure to B’s insolvency. This is the amount that B effectively borrows from A.\footnote{Alternatively, the situation can be interpreted as the following. Upon lending to A, B has net worth $1 + r$ receivable from A. After borrowing from C, B has assets $\frac{1}{1-h}d$ (cash) and liabilities $\frac{1}{1-h}d - (1+r)$ owed to A, leading to a net worth of $1 + r$. As described by Kirk et al. (2014), rehypothecation thus enables B to leverage to invest.}

I assume that B cannot “borrow” indiscriminately from A. Following Holmström and Tirole (1997), I limit the amount owed by B to A as

$$\frac{1}{1-h}d - (1+r) \leq \kappa \left( Pledgeable income \right) \leq \kappa \left( Income from investment \right) \leq R_B \frac{1}{1-h}d,$$

$$\quad (1.11)$$

where the right hand side is the income pledgeable by B to A—a fraction $\kappa$ of the gross cash flow from the investment. I motivate the limited pledgeability of B by the private benefit received by B by shirking on managing the risks of its investment and reducing the probability of success. If the amount owed by B to A is too large, B is highly leveraged and has little stake in the success of his investment, since it is A
that stands to lose a lot from B’s insolvency. He may then take undue risks leading to inefficient outcomes. As in Holmström and Tirole (1997), I focus on efficient contracts that disincentivise shirking. I discuss this in detail in Appendix 2.A.

When B succeeds, he earns the cash flow from the project, and repays C’s debt at \( t = 2 \) and recovers the collateral. He then gets repaid by A at \( t = 3 \), who then recovers the collateral. When B fails, he gets nothing since the collateral is impounded by C, and since A does not repay his debt to B due to B’s inability to return the collateral in full. Thus, B solves

\[
\max_{r,h} \left( \text{Cash flow from project} \left( \frac{1}{1 - h} d \right) \text{ Repayment to C} \left( \frac{1}{1 - h} d \right) \text{ Repayment from A} \left( 1 + r \right) \right),
\]

subject to the collateral constraint (2.6), the participation constraint (1.10) and the pledgeability constraint (2.7).

Combining the collateral constraint (2.6) and B’s incentive compatibility constraint (2.7), the following two-sided borrowing constraint must be satisfied,

\[
\frac{(1 - \kappa R_B)(1 - h)}{1 - h} d \leq 1 + r \leq \frac{1}{1 - h} d^2,
\]

which requires that A should put up sufficient collateral to cover the debt, and that B should not rehypothecate too much collateral. The two constraints will both be satisfied and rehypothecation will be feasible if

\[
\kappa \geq \tilde{\kappa} \equiv \frac{1 - d}{R_B}.
\]

Inequality (2.9) thus captures the two-sided risks inherent in rehypothecation—the risk of A not repaying the debt and the risk of B being irresponsible with the collateral and not being able to return it in full. If both of these risks are not taken care of, then rehypothecation is infeasible.

The following proposition summarises the optimal action of B and the contract \( C_{A,B} \).

**Proposition 2.** There exist cut-offs \( \tilde{R}^* < \tilde{R}^{**} < \tilde{R}^{***} \), such that

1. If \( pR_B \geq 1 \), and

   (a) if \( \kappa < \tilde{\kappa} \), B consumes the endowment if \( \tilde{R} > \tilde{R}^{**} \), and lends to A and
securitises the debt if $R \leq \bar{R}^{**}$, and

(b) if $\kappa \geq \bar{\kappa}$, B consumes the endowment if $R > \bar{R}^{***}$, and lends to A and rehypothecates the collateral if $\bar{R} \leq \bar{R}^{***}$. When $(pR_B - 1)d < \bar{R} \leq \bar{R}^{***}$, A’s collateral constraint binds. When $\bar{R} < (pR_B - 1)d$, B’s pledgeability constraint binds.

2. If $pR_B < 1$, B consumes the endowment if $\bar{R} > \bar{R}^{*}$, and lends to A and holds on to the debt if $\bar{R} \leq \bar{R}^{*}$.

The expressions for $\bar{R}^{*}$, $\bar{R}^{**}$ and $\bar{R}^{***}$ and for the optimal contracts $(r(\bar{R}), h(\bar{R}), \theta(\bar{R}))$ in each case are given in Appendix 2.B.

In the optimal contract when rehypothecation is feasible, exactly one of the collateral constraint (2.6) and the pledgeability constraint (2.7) will bind. As I show in the Appendix, when $(pR_B - 1)d > \bar{R}$, the net benefit from raising one unit of collateral from A is greater than the cost of raising that collateral. In this case, B’s incentive constraint binds. Intuitively, since the net return earned by A on the market is low, the cost of borrowing collateral from A and rehypothecating it is low. The lender B can then demand more collateral than is necessary to satisfy A’s collateral constraint, and rehypothecate this large amount of collateral to invest in the profitable opportunity. The amount of overcollateralisation, however, will be limited by B’s pledgeability constraint, which binds. When $(pR_B - 1)d \leq \bar{R}$, the reverse holds and the net benefit from raising one unit of collateral from A is less than the cost of raising that collateral. Intuitively, the high cost of borrowing collateral means that B is less willing to overcollateralise the debt with A. He demands only as much collateral as is needed to satisfy A’s collateral constraint, and B’s pledgeability constraint is slack.

Proposition 9 defines the optimal holding of collateral,

$$z_B(\bar{R}) = \begin{cases} 0, & \text{if } \bar{R} > \bar{R}^{cutoff}, \\ \in \left[0, \frac{1}{1-h(\bar{R})}\right], & \text{if } \bar{R} = \bar{R}^{cutoff}, \\ = \frac{1}{1-h(\bar{R})}, & \text{if } \bar{R} < \bar{R}^{cutoff}, \end{cases}$$

(1.15)

where $\bar{R}^{cutoff}$ is the cut-off above which there is no lending in the respective parametric
cases of Proposition 9. The optimal holding of cash is then given by\footnote{From the budget constraint $x_B + z_B(1 - h(\bar{R})) = 1$.}

$$x_B(\bar{R}) = 1 - (1 - h(\bar{R}))z_B(\bar{R}).$$ \hfill (1.16)

### 1.3.3 Equilibrium

With the optimal contracts of each type at hand, each agent of type B chooses between the four strategies taking $\bar{R}$ as given, and offers contracts $C_{A,B}(\bar{R}) = (r(\bar{R}), h(\bar{R}), \theta(\bar{R}))$ and $C_{B,C}(C_{A,B}(\bar{R})) = (\tilde{r}(\bar{R}), \tilde{h}(\bar{R}))$.

**Definition 2.** A symmetric competitive equilibrium is the collection of the net return $\bar{R}$ per unit collateral committed by A, the contracts $C_{A,B}(\bar{R})$ offered by B to A, the contracts $C_{B,C}(C_{A,B}(\bar{R}))$ offered by B to C, the allocations $(x_i(\bar{R}), z_i(\bar{R})), i \in \{A, B\}$ and $(\tilde{x}_B(\bar{R}), \tilde{z}_B(\bar{R}))$ such that

1. **A’s optimisation:** Agents A solve the problem (2.1) at $t = 1.1$ to obtain the demands for cash and collateral $(x_A(\bar{R}), z_A(\bar{R}))$ given by (2.3) and (1.4).

2. **B’s optimisation:** Agents B choose between the optima of the problems (2.4), (1.9) and (2.8) at $t = 1.1$ to offer optimal contracts $C_{A,B}(\bar{R}) = (r(\bar{R}), h(\bar{R}), \theta(\bar{R}))$ given by Proposition 9 and to obtain the demands for cash and collateral $(x_B(\bar{R}), z_B(\bar{R}))$ given by (2.11) and (1.16). Agents B maximise their borrowing at $t = 1.2$ from agents C by offering optimal contracts $C_{B,C}(C_{A,B}(\bar{R})) = (\tilde{r}(\bar{R}), \tilde{h}(\bar{R}))$ given by Proposition 1 to obtain the demands for cash and collateral $(\tilde{x}_B(\bar{R}), \tilde{z}_B(\bar{R}))$ given by (1.5).

3. **Market clearing:** The collateral market between A and B at $t = 1.1$ clears (the market for cash between A and B at $t = 1.1$ then clears by Walras’ law):

$$z_A(\bar{R}) + z_B(\bar{R}) = \Omega.$$ \hfill (1.17)

### 1.4 Securitisation vs Rehypothecation

I describe the equilibrium allocations with rehypothecation and securitisation when $pR_B \geq 1$. Figure 1-7 shows the collateral market clearing between A and B at $t = 1.1$. The demand for collateral $z_B(\bar{R})$ is downward sloping. The following proposition
summarises the equilibrium allocations of cash and the investments in the projects of A and B.

**Proposition 3 (Allocations).** If $p_{RB} \geq 1$ and $\kappa \geq \bar{\kappa}$, then in equilibrium

1. $x_A = 1$, $\bar{x}_B = \frac{R_A}{1 - \kappa p_{RB}}$, if $\Omega \in \left[ \frac{R_A}{d} \frac{1}{1 - \kappa p_{RB}}, \infty \right),$

2. $x_A = 1$, $\bar{x}_B = \Omega d$, if $\Omega \in \left[ \frac{1}{d p_{RG}(1 - \bar{\kappa})}, \frac{R_A}{d(1 - \kappa p_{RB})} \right],$

3. $x_A = \Omega d p_{RB}(1 - \bar{\kappa})$, $\bar{x}_B = \Omega d$, if $\Omega \in \left[ 0, \frac{1}{d p_{RG}(1 - \bar{\kappa})} \right].$

If $p_{RB} \geq 1$ and $\kappa < \bar{\kappa}$, then in equilibrium,

1. $x_A = 1$, $\bar{x}_B = R_A$, if $\Omega \in \left[ \frac{R_B}{d^2}, \infty \right),$

2. $x_A = 1$, $\bar{x}_B = \Omega d^2$, if $\Omega \in \left[ \frac{1}{p_{RB}d^2}, \frac{R_A}{d^2} \right],$

3. $x_A = p_{RB} \Omega d^2$, $\bar{x}_B = \Omega d^2$, if $\Omega \in \left[ 0, \frac{1}{p_{RB}d^2} \right].$

When collateral is abundant, A is always able to secure all of B’s cash endowment to invest. With rehypothecation, if collateral is too abundant, B is unable to charge very high haircuts and rehypothecate since A is not willing to take on the risk of losing a large
quantity of collateral. With securitisation, if collateral is too abundant, B is unable to charge very high interest rates to increase the value of the repledgeable debt since A will not participate. In this case, some collateral sits idle with A. When collateral is scarce, B reduces credit to A. The more useful the collateral is to B (\(pR_B\) is high), the more willing B is to provide credit to A as even smaller rates and haircuts enable B to generate revenue to break even.

The total expected output, or the sum of the consumptions of A, B and C at \(t = 3\) under their respective beliefs, is given by

\[
Y(\theta|d, \kappa) = (R_A - 1)x_A + (pR_B - 1)x_B,
\]

modulo a constant. Here \(\theta = 1\) indicates that the planner is permitted to rehypothecate, and \(\theta = 0\) indicates that he is not. I focus on the parameters \(d\) and \(\kappa\) which provide the two channels by which rehypothecation dominates securitisation.

**Definition 3.** Define the partial ordering “\(\succeq\)" on \(Y\) (defined in (2.14)) such that (i) \(Y(\theta|d, \kappa) \succeq Y(\theta'|d', \kappa')\) if and only if \(\forall \Omega \ Y(\theta|d, \kappa) \geq Y(\theta'|d', \kappa')\) and \(\exists \Omega\) such that \(Y(\theta|d, \kappa) > Y(\theta'|d', \kappa')\); (ii) \(Y(\theta|d, \kappa) \succ Y(\theta'|d', \kappa')\) if and only if \(\forall \Omega \ Y(\theta|d, \kappa) > Y(\theta'|d', \kappa')\); and (iii) \(Y(\theta|d, \kappa) \sim Y(\theta'|d', \kappa')\) if and only if \(\forall \Omega \ Y(\theta|d, \kappa) = Y(\theta'|d', \kappa')\).

The following result compares the optimal allocations with rehypothecation and securitisation.

**Proposition 4 (Optimality of Rehypothecation).** If \(pR_B \geq 1\),

1. \(Y(\theta = 1|d < 1, \kappa = \bar{\kappa}) \succ Y(\theta = 0|d < 1, \kappa = \bar{\kappa})\),
2. \(Y(\theta = 1|d = 1, \kappa > \bar{\kappa}) \succeq Y(\theta = 0|d = 1, \kappa > \bar{\kappa}), \text{ and}\)
3. \(Y(\theta = 1|d = 1, \kappa = \bar{\kappa}) \sim Y(\theta = 0|d = 1, \kappa = \bar{\kappa})\).

The result establishes the significance of both \(d\) and \(\kappa\) in making rehypothecation better than securitisation. The investments in the projects of A and B are shown in Figure 1-8.

The crux of this result and the comparison between securitisation and rehypothecation is seen with an example in a partial equilibrium setting, where B is offering a contract to A, taking \(\bar{R}\) as given. To fix ideas, suppose that rehypothecation is feasible (\(\kappa \geq \bar{\kappa}\)) and desirable (\(pR_B \geq 1\)). Suppose that \(p = 1\), so that A faces no risk of B’s
Rehypothecation is better than securitisation. The top two panels plot $x_A$ and $\tilde{x}_B$ for $d < 1$ and $\kappa = \bar{\kappa}$. The bottom two panels plot $x_A$ and $\tilde{x}_B$ for $d = 1$ and $\kappa > \bar{\kappa}$.

default. In this situation, it is easy to see that the binding participation constraints (2.5) and (1.10) are the same with securitisation and rehypothecation. If in addition, $\bar{R}$ is large and $(R_B - 1) d \leq \bar{R}$ holds, the collateral constraints also bind with both securitisation and rehypothecation from Proposition 9. This implies that the optimal haircuts and interest rates with both securitisation and rehypothecation are exactly the same. However, with securitisation, B can borrow from C an amount equal to $1 + r$, while with rehypothecation, B can borrow an amount

$$\text{Amt. with Rehypothecation} \quad \frac{1}{1 - h} d > \frac{1}{1 - h} d^2 = 1 + r.\]$$

Thus, rehypothecation disconnects the face value of the debt from the amount that can be borrowed by repledging. If B's borrowing requirement is short term and $d < 1$, it is better to rehypothecate since the low risk of the collateral in the short run enables B to borrow more with attractive haircuts. With securitisation on the other hand, the low risk of the collateral in the short term is irrelevant because what is being pledged as collateral is not the underlying collateral asset, but the cash flows from the original
long-term debt contract, and the value of these cash flows is limited by the large risk of the collateral in the long run. When \( p < 1 \), the borrower \( A \) faces the risk of losses due to \( B \)’s insolvency, and the interest rate and haircut are reduced appropriately to compensate \( A \) for those expected losses. However, even in this case, the ability to borrow more makes rehypothecation more attractive compared to securitisation.

When \((R_B - 1)d > \tilde{R}\), \( B \)’s pledgeability constraint binds, \( A \)’s collateral constraint is slack, and the collateral posted is strictly greater than what is required if \( \kappa > \tilde{\kappa} \). Assume that \( \tilde{R} = 0 \), so that the interest rate and the face value of the debt is the maximum possible without violating \( A \)’s participation constraint. In this case, the higher pledgeability of the lender \( B \) makes it possible for him demand larger than necessary haircuts to invest in his very profitable opportunity. Hence, the optimal haircut with rehypothecation is strictly greater than the optimal haircut with securitisation, and \( B \) can further borrow a lot more with rehypothecation than with securitisation due to lower collateral risk in the short term. In particular, with securitisation, \( B \) can borrow from \( C \) an amount equal to \( 1 + r \), while with rehypothecation, \( B \) can borrow an amount

\[
\frac{1}{1-h}d > \frac{1}{1-h}d^2 > 1 + r
\]

Again, these arguments must be tempered when \( A \) faces the risk of \( B \)’s insolvency, but the attractiveness of rehypothecation relative to securitisation remains.

We can summarise the two channels which make rehypothecation better than securitisation as

\[d < 1, \text{ and} \]

\[\kappa > \tilde{\kappa}.\]

When \( \tilde{R} \) is high, \( A \)’s collateral constraint binds. In this case, when \( d = 1 \), the amount of cash that \( B \) can borrow from \( C \) is the same with securitisation and rehypothecation, and they are equivalent. However, when \( d < 1 \), it is possible for \( B \) to borrow from \( C \) strictly more with rehypothecation than with securitisation. When \( \tilde{R} \) is low, the large pledgeability \( \kappa \) of \( B \) allows him to overcollateralise the debt with \( A \) and borrow more cash from \( C \) by rehypothecating the large amount of collateral thus obtained. This channel operates for any \( d \leq 1 \). The two channels allow the intermediate lender \( B \) to squeeze as much liquidity as possible from the collateral. \( A \) does not have access to \( C \)’s cash. \( B \)’s special position in the middle of the chain allows it access to \( A \)’s collateral and

37
C's cash, and rehypothecation allows B to make the most efficient use of the collateral to obtain the borrowing from C.

1.4.1 Repo vs Mortgage Markets

I differentiate between the repo and mortgage markets by the different pledgeabilities of the intermediate lender B's income to the borrower A. I propose that the difference between the prime brokerage and the mortgage markets is due to the costs of monitoring the lender. The original borrowers in the prime brokerage market, the hedge funds, are sophisticated investors. The fund managers are well versed with the practices of their brokers, the types of investments they make and how they manage the risks. Quite often, the fund managers themselves have had a stint in the brokerage, investment banking or risk management areas in the past, and tend to have close relationships with many market participants. This gives them a dual advantage—they understand the practices and investment tendencies of their brokers well, and are well informed about the general trends in the market, if not about the specific investments of their brokers. Furthermore, as mentioned above, the funds have long-term contracts with their brokers that go beyond these individual debt contracts, and the brokers compete for their services. If the fund manager feels that his broker is taking undue risks with his collateral, he can threaten to renegotiate or terminate the brokerage contract and move to another competing broker. This acts as a disciplining device for the brokers who would then be more conscientious about their risk taking. I interpret these factors in the prime brokerage market as a high pledgeability $\kappa$ of the lenders' incomes to the borrowers. This makes rehypothecation feasible and optimal.

In the mortgage market, on the other hand, the original borrowers are small banks and savings and loan associations. Their repertoire is generally limited to the plain-vanilla market of home loans. They are not generally informed about the ways and practices of their lenders, the investment banks. They also do not generally have relationships with the investment banks outside of the individual mortgage pass-through securities. Such narrow contractual arrangements prevents these mortgage originators from being able to influence their lenders to be more scrupulous about their risk taking. I interpret these factors in the mortgage market as a low pledgeability $\kappa$ of the lenders' incomes to the small banks. The low pledgeability arising from the large cost of monitoring makes rehypothecation infeasible. This market, therefore, sees a securitisation of mortgages into MBS's or covered bonds. A similar argument shows us why the un-
sophisticated homeowners do not allow their lenders (small banks) to rehypothecate the house collateral, which leads the lenders to use the mortgage loans as collateral against which to borrow from the large investment banks.

1.5 Comparative Statics

I study the behaviour of the repo and prime-brokerage markets, which are characterised by rehypothecation, during a crisis. The collateralised lending market experienced falling rehypothecation, as documented in Singh (2011, 2012) and Singh and Aitken (2009, 2010), and rising haircuts and borrowing rates, as documented in Gorton and Metrick (2013). The seminal work by Gorton and Metrick (2013) was the first to document the effects of the crisis on the bilateral repo market. Rehypothecation is a key feature of this market, and any theoretical analysis of it must incorporate the two sides—supply and demand—of the market for collateral, and how they determine the “price” of collateral. Below, I look at the comparative statics with respect to the variables which were affected by during crisis.

In view of the documented facts, I look at what happens when (i) the lenders’ projects become more risky (a fall in $p$ to $p' < p$) with the expected return remaining constant,12 (ii) the lenders’ pledgeability falls (a fall in $k$ to $k' < k$), and (iii) the risk of the collateral increases (a fall in $d$ to $d' < d$). I restrict attention to parameter values where rehypothecation is still feasible and optimal, i.e. $p'R_B >_1 \kappa$ and $K > 1-d'R_B$. This will enable me to relate to the facts documented by Gorton and Metrick (2013) who look at rates and haircuts in the bilateral repo market.

When the intermediate lender $B$ becomes more risky, $A$ needs a larger compensation for taking on a higher risk of $B$’s default, leading to a fall in the interest rate. Lower haircuts provide an additional benefit to $A$ by reducing the quantum of loss due to $B$’s default.

**Proposition 5 (Lender Risk).** Let $(r, h)$ and $(r', h')$ be the equilibrium rates and haircuts when the probability of success of $B$’s project is $p$ and $p' < p$ respectively. If $R_B$ rises proportionately to $R'_B > R_B$ to keep $pR_B = p'R'_B$ constant, then $h' \leq h$ and $r' \leq r$.

When the pledgeability of the lenders falls, possibly because of a more severe moral

---

12 As described in Appendix 2.A, $\kappa = 1 - b / R_B \Delta p$, where $b$ is the private benefit to $B$ from shirking and $\Delta p$ is the increase in the probability of success of the project due to not shirking. I assume that the when $p = p_H$ falls, so does $p_L$ by an equal amount such that $\Delta p$ and $\kappa$ do not change.
hazard problem, lenders are unable to demand a high haircut and rehypothecate too much collateral. The limited ability of the lenders to invest in their profitable opportunity means that they have less surplus to share with the borrowers, and the interest rate increases.

Proposition 6 (Moral Hazard). Let \((r, h)\) and \((r', h')\) be the equilibrium rates and haircuts when the pledgeability of B’s project is \(\kappa\) and \(\kappa' < \kappa\) respectively. Then \(h' \leq h\) and \(r' \geq r\).

It is important to distinguish between the risk of the project given by \(p\) and the relationship between A and B given by \(\kappa\). When the risk of the project changes, keeping the expected net present value constant, only the distribution of the surplus between A and B is affected. For a given expected net present value of the project, no matter what the risk of the project, the investments in A’s and B’s projects and the total surplus are the same, as is evident from Proposition 3. A fall in \(p\) simply moves the surplus from B to A as the riskier lenders compete to lend, rehypothecate and invest in equally profitable projects. However, when the relationship between A and B is impaired, possibly due to a worse moral hazard problem, the ability of B to borrow and rehypothecate A’s collateral reduces when the pledgeability constraint is binding. This reduces the total surplus through a reduction in the investment in B’s project, as is clear from Proposition 3.
When the risk of the collateral increases, the haircut increases as more collateral is needed to secure one unit of cash lending by A. The increased demand for the collateral means that borrowers need to be compensated with a lower interest rate.

**Proposition 7** (Collateral Risk). Let \((r, h)\) and \((r', h')\) be the equilibrium rates and haircuts when the lowest value of the collateral at \(t = 2\) is \(d\) and \(d' < d\) respectively. Then \(h' \geq h\), and if \(d' < \bar{g}(d)\) where \(\bar{g}'(\cdot) > 0\), \(r' \leq r\).

Gorton and Metrick (2013) document a rise in repo rates and haircuts across a wide variety of collateral classes during the crisis. The model provides three channels for the changes in haircuts and rates. According to the model, haircuts face a downward pressure as borrowers do not want to hand over too much rehypothecable collateral to risky or unreliable lenders, and face an upward pressure as the collateral becomes riskier. The second channel appears to have dominated during the crisis in the case of highly risky collateral such as corporate bonds. Interest rates face a downward pressure as lenders and the collateral become riskier, and an upward pressure as the limited pledgeability of the lenders also limits their ability to share the surplus. The first channel does not seem to have been dominant during the crisis. Gorton and Metrick (2013) suggest that the rise in the repo rates across a wide variety of collateral classes was to compensate for the increased default risk of the borrowers—a channel not modelled here.

When the expected net present value of B’s investment becomes negative \((pRB < 1)\), rehypothecation becomes inefficient. Borrowers then prevent rehypothecation and lenders simply hold on to the debt. Similarly, when the pledgeability falls below the cutoff \((\kappa < \bar{\kappa})\), rehypothecation becomes infeasible.\(^{13}\) This is consistent with the observed fall in the repledgeable collateral as a fraction of assets (Figure 1-9).

### 1.6 Conclusion

I construct a unified model of rehypothecation and securitisation with the prime brokerage, repo and mortgage markets in mind. I find that rehypothecation allows the lender to borrow for the short term against the full value of the collateral received by him,

\(^{13}\)Although rehypothecation becomes infeasible when the lenders’ pledgeability falls a lot, securitisation may still be desirable if the lenders’ projects are profitable. One reason why the switch to securitisation of repo contracts was not observed during the crisis could be the large fixed costs involved in developing a market and the associated legal framework for it.
which is typically larger than the face value of the debt contract. I consider the borrowers' ability to monitor and discipline their lenders into not taking too many risks with their collateral. As long as the costs of monitoring are small and the health of the lender is good, rehypothecation will be preferred to securitisation. I argue that the nature of the prime brokerage market and its participants makes it easier for the borrowers to monitor their lenders, and rehypothecation obtains in these markets. On the other hand, the unsophisticated nature of mortgage originators and their limited contractual arrangements with their lenders renders them ineffective at disciplining their lenders, and securitisation obtains in these markets.

I also study the effects of the collateral risk and lender's pledgeability on bilateral repo contracts which are characterised by rehypothecation. I find that an increase in the collateral risk raises haircuts, and a decrease in the lender's pledgeability raises repo rates, in line with the facts documented by Gorton and Metrick (2013).

A direction for future work would be to explore longer rehypothecation chains. Such chains will feature network externalities as in Acemoglu, Ozdaglar and Tahbaz-Salehi (2013), as the parties at one end of the rehypothecation chain will not internalise the effect of their contract on the parties at the other end of the chain. The network externalities will impair financial stability by allowing inefficient investment. A full analysis of the general chain will allow the formalisation of the concept of collateral velocity and provide additional insights into monetary and macro-prudential policy.
Appendices

1.A State-by-state Cash Positions with Securitisation and Rehypothecation

This appendix describes the mechanism of the settlement of contracts between A, B and C by listing out the state-by-state cash flows and positions. At \( t = 2 \), the returns to B’s investment are realised, and then \( C_{B,C} \) is settled. At \( t = 3 \), the returns to A’s investment are realised, and finally \( C_{A,B} \) is settled.

When \( C_{B,C} \) is being settled and B defaults, C is entitled to the collateral only up to the value of the debt, and returns any value from the liquidation in excess to the value of the debt back to B. This is in accordance with the legal practice for OTC derivatives and repos.\(^{14,15}\)

Assumption 3. During the settlement of \( C_{B,C} \), if B defaults, C liquidates a fraction \( \frac{(1+r)\bar{b}}{\bar{v}} \) of the available collateral, where \( \bar{v} \) is the total value of the collateral and \( \bar{b} \) is the amount borrowed. C gives the excess value from the liquidation \( \bar{v} - (1+r)\bar{b} \geq 0 \) back to B.

When \( C_{A,B} \) is settled, I assume that the loan is repaid only if the collateral is returned in full. I motivate this by the delivery versus payment systems.\(^ {16} \) I emphasise that even in the place of such systems, the borrower may be unsecured because the size of the loan may be smaller than the value of the collateral due to a high haircut. Since the lender is always fully secured, there will be default only when B is unable to return the collateral. In such an eventuality, I assume that A becomes an unsecured creditor of B, and receives from B the excess of the value of the collateral over the face value of the debt.

Assumption 4. During the settlement of \( C_{A,B} \), if B has value \( c \) on its balance sheet less than the value \( v \) of the collateral owed back to A, then B returns to A the amount \( \min\{c, v - (1+r)b\} \), where \( b \) is the amount borrowed.

\(^{14}\) http://www.sec.gov/Archives/edgar/data/1065696/000119312511118050/dex101.htm


\(^{16}\) E.g. Fedwire. Neither party faces settlement risk (also known as principal risk)
Table 1.1: Effective positions of A, B and C after the settlement of $C_{B,C}$ at $t = 2$ and $C_{A,B}$ at $t = 3$. The position of A is per unit of borrowing from B.

<table>
<thead>
<tr>
<th>State of the World</th>
<th>Position of A</th>
<th>Position of B</th>
<th>Position of C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(R_B, u^2)$</td>
<td>$R_A - (1+r) + \frac{1}{1-h} u^2$</td>
<td>$R_B(1+r) - (1+r) + (1+r)$</td>
<td>$1+r$</td>
</tr>
<tr>
<td>$(R_B, ud)$</td>
<td>$R_A - (1+r) + \frac{1}{1-h} ud$</td>
<td>$R_B(1+r) - (1+r) + (1+r)$</td>
<td>$1+r$</td>
</tr>
<tr>
<td>$(R_B, d^2)$</td>
<td>$R_A - (1+r) + \frac{1}{1-h} d^2$</td>
<td>$R_B(1+r) - (1+r) + (1+r)$</td>
<td>$1+r$</td>
</tr>
<tr>
<td>$(0, u^2)$</td>
<td>$R_A - (1+r) + \frac{1}{1-h} u^2$</td>
<td>$0$</td>
<td>$1+r$</td>
</tr>
<tr>
<td>$(0, ud)$</td>
<td>$R_A - (1+r) + \frac{1}{1-h} ud$</td>
<td>$0$</td>
<td>$1+r$</td>
</tr>
<tr>
<td>$(0, d^2)$</td>
<td>$R_A - (1+r) + \frac{1}{1-h} d^2$</td>
<td>$0$</td>
<td>$1+r$</td>
</tr>
</tbody>
</table>

Table 1.1 lists out the positions of A, B and C with securitisation at at $t = 2$ and $t = 3$. At $t = 2$, if B is solvent, he repays the total debt to C amounting to $\bar{x}_B(1+r) = 1+r$ and recovers the securitised contract $C_{A,B}$. If B is insolvent at $t = 2$, C impounds the contract $C_{A,B}$ and holds it up to $t = 3$ when he gets the safe payment $1+r$. (C may also sell it at $t = 2$ at the price $1+r$ to get the same utility.) When $C_{A,B}$ is being settled at $t = 3$, regardless of who is holding the contract, A must pay $1+r$ to the holder and recover all of the collateral. If B was insolvent at $t = 2$, he gets nothing at either $t = 2$ or $t = 3$. If B was solvent at $t = 2$, he would earn a total of $R_B(1+r)$ from his investment, would have to repay B’s debt amounting to $1+r$, and would be paid $1+r$ by A at $t = 3$. Thus, net of the collateral position, A’s expected payoff per unit of collateral committed is

$$(R_A - (1+r))(1-h),$$

and B’s expected payoff is

$$p R_B(1+r).$$
Table 1.2 lists out the positions of A and B with rehypothecation at \( t = 3 \). Let \( v_2 \) and \( v_3 \) be the expected payoffs of the collateral at \( t = 2 \) and \( t = 3 \) respectively. When B is solvent, he recovers all of the underlying collateral asset from C by repaying his debt amounting to \( \frac{1}{1-h}d \), and can pass it on to A when A repays the debt. On the other hand, when B fails, C liquidates a fraction \( \frac{d}{v_2} \) of the asset and returns the remainder to B. At \( t = 3 \) B owes to A a net amount \( \frac{1}{1-h}v_2 - (1 + r) \), which it would have effectively transferred to A after the repayment of A’s debt and the return of the collateral to A. But B now has only \( (1 - \frac{d}{v_2}) \frac{1}{1-h}v_3 \). It is easy to see that since \( 1 + r \leq \frac{1}{1-h}d^2 \), what B owes to A is always more than what B has. By Assumption 4, B returns to A all of the remaining collateral. Thus, given A’s optimistic beliefs, his expected payoff per unit of collateral committed, net of the collateral position is

\[
\left( R_A - p(1 + r) - (1 - p) \frac{1}{1-h}d \right) (1 - h).
\]

Similarly, B’s expected payoff is

\[
p \left( (R_B - 1) \frac{1}{1-h}d + 1 + r \right).
\]

1.B Moral Hazard

Suppose that B can either manage his risks well so that the probability of success of
his investment is \( p_H \), or he can shirk and get a private benefit \( b \) per unit of investment. In the latter case, the probability of success of the project becomes \( p_L < p_H \). A small value \( b \) can also be thought of as indicating a significant reputational cost to B in the event of a default. I assume that shirking is always socially suboptimal, i.e. \( p_H R_B > p_L R_B + b \), and wish to implement only the efficient contracts where B is sufficiently incentivised to manage his risks well. From (2.8), this requires

\[
p_H \left( (R_B - 1) \frac{1}{1-h} d + 1 + r \right) \geq p_L \left( (R_B - 1) \frac{1}{1-h} d + 1 + r \right) + b \frac{1}{1-h} d.
\]

Rearranging, I get

\[
\frac{1}{1-h} d - (1 + r) \leq \left( 1 - \frac{b}{R_B \Delta p} \right) R_B \frac{1}{1-h} d.
\]

Denoting \( \kappa = \left( 1 - \frac{b}{R_B \Delta p} \right) \) gives (2.7).

Suppose that \( b \) is too large and B’s pledgeable income is too small to effect a re-hypothecation contract. B’s lender A can then choose to monitor his operations so that the private benefit from shirking reduces. Monitoring can take the form of closely following the investment activity and threatening costly renegotiation if it is perceived that risk is not being managed well. Suppose that A incurs a cost \( c \) of monitoring per unit of B’s investment. In order that it is incentive compatible for A to monitor B, I must have from (1.10)

\[
R_A - p_H (1 + r) - (1 - p_H) \frac{1}{1-h} d - c \frac{1}{1-h} d \geq R_A - p_L (1 + r) - (1 - p_L) \frac{1}{1-h} d.
\]

Rearranging, I get

\[
\frac{1}{1-h} d - (1 + r) \geq \frac{c}{\Delta p} \frac{1}{1-h} d.
\]

In order that monitoring is useful, conditions (2.20) and (1.20) together with A’s collateral constraint must define a feasible region. Hence, I must have

\[
\frac{c + b}{\Delta p} \leq R_B, \text{ and,}
\]

\[
\frac{c}{\Delta p} \leq 1 - d.
\]

Thus, if the cost of monitoring \( c \leq \min \{ \Delta p R_B - b, \Delta p (1 - d) \} \) is sufficiently small, the
1.C Proof of Proposition 9

Consuming the endowment gives B a utility

\[ U^{CE}(\bar{R}) = 1. \]

Lending to A may give him a higher utility. There are two cases.

\( \kappa < \bar{\kappa} \):

I first consider the case when \( \kappa < \bar{\kappa} \) and rehypothecation is infeasible. Suppose that B offers one unit of the contract \((r, h, \theta = 0)\) to A, and securitises a fraction \( w \) and holds on to a fraction \( 1 - w \) of the contract. Write \( 1 + r = X \) and \( \frac{1}{1 - h} = Z \). B thus solves a combination of the two problems (2.4) and (1.9):

\[
\max_{X,Z} p R_B X w + X (1 - w), \text{ s.t.}
\]

Collateral const.: \( X \leq Z d^2 \),

Participation const.: \( R_A - X \geq \bar{R} Z \),

The collateral and participation constraints occurs in both problems and remain unchanged.

Since the participation constraint will always bind, I can rewrite the objective function as

\[
[1 + (p R_B - 1) w] (R_A - \bar{R} Z).
\]

If \( p R_B < 1 \), we immediately get \( w = 0 \) and B will hold on to the debt. Since any additional collateral over the minimum required by the collateral constraint is wasteful and costs A a utility of \( \bar{R} \) per unit, B will try to minimise the collateral demanded and the collateral constraint will bind in the optimum. This gives the optimal contract

\[
1 + r = \frac{R_A}{1 + \frac{R}{d^2}}, \quad \frac{1}{1 - h} = \frac{1}{d^2} \frac{R_A}{1 + \frac{R}{d^2}}, \quad \theta = 0.
\]

This gives B a utility of

\[
U^{HO}(\bar{R}) = \frac{R_A}{1 + \frac{R}{d^2}}.
\]
B will stop lending and consume the endowment when $\bar{R} > \bar{R}^*$ such that $U^{HO}(\bar{R}^*) = 1$. This gives

$$\bar{R}^* = (R_A - 1)d^2.$$  

This proves part (1) of the proposition for $\kappa < \bar{\kappa}$.

If $pR_B \geq 1$, $w = 1$ and B will securitise the debt. Again, since collateral is costly, B will try to minimise it and the collateral constraint will bind. This gives the optimal contract

$$1 + r = \frac{R_A}{1 + \frac{R}{d^2}}, \quad \frac{1}{1 - h} = \frac{R_A}{d^2} \frac{1}{1 + \frac{R}{d^2}}, \quad \theta = 0.$$  

This gives B a utility of

$$U^{SE}(\bar{R}) = \frac{pR_B R_A}{1 + \frac{R}{d^2}}.$$  

B will stop lending and consume the endowment when $\bar{R} > \bar{R}^*$ such that $U^{SE}(\bar{R}^{**}) = 1$. This gives

$$\bar{R}^{**} = (pR_B R_A - 1)d^2.$$  

This proves part (2a) of the proposition.

$\kappa \geq \bar{\kappa}$:

Now I consider the case when $\kappa \geq \bar{\kappa}$ and rehypothecation is feasible. Suppose that B offers one unit of the contract $(r, h, \theta = 1)$ to A, and rehypothecates a fraction $w_1$, securitises a fraction $w_2$ and holds on to a fraction $1 - w_1 - w_2$ of the contract. Write $1 + r = X$ and $\frac{1}{1 - h} = Z$. B thus solves a combination of the three problems (2.4), (1.9) and (2.8):

$$\max_{X,Z} p ((R_B - 1)Zd + X) w_1 + pR_B X w_2 + X(1 - w_1 - w_2), \text{ s.t.}$$

> Collateral const.: $X \leq Zd^2$,

> Participation const.: $R_A - (pX + \frac{1 - p}{Zd})w_1 - Xw_2 - X(1 - w_1 - w_2) \geq \bar{R}Z$,

> Pledgeability const.: $Zd - X \leq \kappa R_B Zd$.

The collateral constraint occurs in all three problems and remains unchanged. The pledgeability constraint also remains unchanged as B's project is linearly scalable and the condition on the proportion of the face value $X$ and the amount of collateral $Z$
to overcome moral hazard with rehypothecation is unchanged. The participation constraint changes to reflect that a smaller fraction of the collateral is rehypothecated and the expected loss of collateral is proportionately smaller.

Since the participation constraint will always bind, I can rewrite the objective function as

\[ [(pR_B - 1)dw_1 - \bar{R}] Z + (pR_B - 1)Xw_2 + R_A. \]

If \( pR_B < 1 \), we immediately get \( w_1 = w_2 = 0 \) and B will hold on to the debt. This proves part (1) of the proposition for \( \kappa \geq \bar{\kappa} \).

When \( pR_B \geq 1 \), \( 1 - w_1 - w_2 = 0 \) as there is always a non-negative utility from securitisation. The derivative w.r.t \( w_1 \) is

\[(pR_B - 1)dZ,\]

and that w.r.t \( w_2 \) is

\[(pR_B - 1)X.\]

Now by the collateral constraint, \( X \leq Zd^2 < Zd \), the derivative w.r.t \( w_1 \) is always larger. B can borrow more from C by rehypothecation at a given \( \bar{R} \), and rehypothecation is always preferred to securitisation. Thus, we have \( w_1 = 1 \). Now there are two cases:

Case I: \( 0 \leq (pR_B - 1)d \leq \bar{R} \). In this case, the first term in the modified objective function is negative. The benefit from rehypothecating collateral is less than the cost of raising that collateral. B will minimise \( Z \) so that the collateral constraint binds. The optimal contract is

\[ 1 + r = \frac{R_A}{1 + \frac{R}{d^2} \frac{(1-p)(1-d)}{d^2}}, \quad \frac{1}{1 - h} = \frac{1}{d^2} \frac{R_A}{1 + \frac{R}{d^2} + \frac{(1-p)(1-d)}{d^2}}, \quad \theta = 1, \]

and the utility is

\[ U^{RE}(\bar{R}) = \frac{p \left( 1 + \frac{R - 1}{d} \right) R_A}{1 + \frac{R}{d} + \frac{(1-p)(1-d)}{d}}. \]

Case II: \( \bar{R} < (pR_B - 1)d \). In this case, the first term in the modified objective function is positive. The benefit from rehypothecating collateral is greater than the cost of raising that collateral. B will maximise \( Z \) so that the pledgeability constraint
binds. The optimal contract is
\[ 1 + r = \frac{R_A(1 - \kappa R_B)}{1 + \frac{\bar{R}}{d} - \kappa p R_B}, \quad \frac{1}{1 - h} = \frac{R_A}{d(1 + \frac{\bar{R}}{d} - \kappa p R_B)}, \quad \theta = 1, \]
and the utility is
\[ U^{RE}(\bar{R}) = \frac{p R_B(1 - \kappa)R_A}{1 + \frac{\bar{R}}{d} - \kappa p R_B}. \]

I guess and verify an \( \bar{R}^{**} \) such that \( \bar{R}^{**} > (p R_B - 1)d \) and \( U^{RE}(\bar{R}^{**}) = 1 \). From the expression for the utility in the guessed range of \( \bar{R}^{**} \), I have
\[ \bar{R}^{**} = p(R_B - 1 + d)(R_A - 1)d + (p R_B - 1)d. \]

This proves part (2b) of the proposition.

1. D Belief Disagreements and Proof of Theorem 1

Lemma 1. With securitisation and rehypothecation, there exists a \( \bar{u} > 1 \) such that for \( u > \bar{u} \), the long-term optimal contract \( C_{A,B} \) takes the form \( 1 + r \leq \frac{1}{1-h} d^2 \).

Proof. Without loss of generality, let B offer the contract to A and C. Write \( 1 + r = X \) and \( \frac{1}{1-h} = Z \).

**Securitisation:** I first look at the optimal contracts with securitisation. B is able to optimally raise \( E_C[\min\{X, Zu\}] \) from C by selling the contract \( C_{A,B} \). B solves
\[ \max_{X,Z} p R_B E_C[\min\{X, Zu\}], \]
\[ \text{s.t. } R_A - E_A[\min\{X, Zu\}] \geq \bar{R} Z, \]
where \( v \) is the payoff of one unit of the collateral at \( t = 3 \), and the expectations \( E_A[\cdot] \) and \( E_C[\cdot] \) are taken according to the beliefs of A and C respectively.

It is clear that the constraint will always bind since we can increase \( X \) otherwise. Due to the linearity of the problem, it is sufficient to compare the values of the maximand at \( X \in (-\infty, Z d^2] \cup \{Z u d\} \cup [Z u^2, \infty) \).

\( X \in [Z u^2, \infty) \): Here, there is always default, and we have \( R_A - Zu = \bar{R} Z \). This
gives \( Z = \frac{R_A}{u + R} \), and B's payoff is

\[
U(X \geq Zu^2) = pR_B \frac{R_A}{u + R} d.
\]

\( X = Zud \): Here, there is default when \( v \leq ud \). We have \( R_A - X = \tilde{R}Z \) since A does not believe he will ever default. This gives \( Z = \frac{R_A}{ud + \tilde{R}} \). C believes that A will always default. Then B's payoff is

\[
U(X = Zud) = pR_B \frac{R_A d}{ud + \tilde{R}}.
\]

\( X \in (-\infty, Zd^2] \): Here, there is no default. The optimal contract is given by Proposition 9 and B's payoff does not feature \( u \).

It is clear that as \( u \to \infty \), \( U(X \geq Zu^2) \to 0 \) and \( U(X = Zud) \to 0 \) for any given \( \tilde{R} \). The safe debt contract is optimal because otherwise, there is default and A loses collateral that he values more than B. Moreover, the selling of the safe debt contract by B to C is equivalent to borrowing against it at a zero interest rate.

**Rehypothecation**: Next, I look at the optimal contracts with rehypothecation. B is able to raise \( Ec[Zv] = Zd \) by borrowing against it at a zero interest rate from C for the short term. (Borrowing for the short term from C dominates selling the collateral to C as B is able to raise the same amount \( Zd \) in both cases but loses valuable collateral to C by selling.) B solves

\[
\max_{X,Z} \left( R_B Zd - Zd + E_B[\min\{X, Zv\}] \right),
\]

s.t. \( R_A - pE_A[\min\{X, Zv\}] - (1 - p)Zd \geq \tilde{R}Z \), and

\[
Zd - E_B[\min\{X, Zv\}] \leq \kappa R_B Zd.
\]

where \( v \) is the payoff of one unit of the collateral at \( t = 3 \), and the expectations \( E_A[\cdot] \) and \( E_B[\cdot] \) are taken according to the beliefs of A and B respectively.

A's participation constraint always binds. Again, due to linearity, it is sufficient to restrict attention to \( X \in (-\infty, Zd^2] \cup \{Zud\} \cup [Zu^2, \infty) \).

\( X \in [Zu^2, \infty) \): Here, there is always default. B's pledgeability constraint does not bind. We have \( R_A - pZu - (1 - p)Zd = \tilde{R}Z \). This gives \( Z = \frac{R_A}{pu + (1 - p)d + \tilde{R}} \), and B's
payoff is
\[ U(X \geq Zu^2) = \frac{p(R_Bd - d + 1)R_A}{pu + (1 - p)d + R}. \]

\[ X = Zu^2 : \text{Here, there is default when } v \leq ud. \text{ Again, it can be shown that B's pledgeability constraint does not bind for } \kappa \geq \bar{\kappa}. \text{ Since A believes there will be no default, we have } R_A - pZud - (1 - p)Zd = \bar{R}Z. \text{ This gives } Z = \frac{R_A}{pud + (1 - p)d + \bar{R}}, \text{ and B's payoff is} \]
\[ U(X = Zu^2) = \frac{p \left( (R_B - 1)d + \left( \frac{u - d}{u - d} \right)^2 (u - 1)ud + \left( \frac{u - 1}{u - d} \right) ud \right) R_A}{pud + (1 - p)d + \bar{R}}. \]

\[ X \leq Zu^2 : \text{Here, there is no default. The optimal contract is given by Proposition 9 and B's payoff does not feature } u. \]

Again, as } u \to \infty, U(X \geq Zu^2) \to 0 \text{ and } U(X = Zu^2) \to 0 \text{ for any given } \bar{R}, \text{ and the safe debt contract is optimal as it does not involve default by A and loss of valuable collateral.} \]

The proof of Lemma 1 also nests the case of a sale of the collateral to B, with \( 1 + r > \frac{1}{1 - h} u^2 \), so that there is always default.

**Corollary 1.** Under the parameter restrictions of Lemma 1, the collateralised borrowing contract between A and B is optimal compared to a sale of the collateral.

If A were to borrow short term from B, A would have to default in all states of the world since A has no cash flows at } t = 2. \text{ This case is also nested above.}

**Corollary 2.** The optimal borrowing contract between A and B is long-term.

This section implicitly assumes that B does not wilfully default, and A has recourse to B's balance sheet if B does not return the collateral. If we assume that no-recourse holds for both A and B, then any borrowing contract will essentially be a sale, and for a sufficiently high } u, \text{ will be suboptimal to A simply holding on to the collateral without borrowing/selling and investing.}
1.E Comparative Statics

Define

\[ \Omega_1 = \frac{1}{d p R_B (1 - \kappa)} \]
\[ \Omega_2 = \frac{1}{d p R_B (1 - \kappa)} \]
\[ \Omega_3 = \frac{1}{d p R_B (1 - \kappa)} \]
\[ \Omega_4 = \frac{1}{d 1 - \kappa p R_B} \]

\( \Omega_1 \) is the cutoff below which the supply of collateral is too low and \( \bar{R} \) is too high for B to be willing to supply all of the cash to A. \( \Omega_4 \) is the cutoff above which the supply of collateral is too high and \( \bar{R} = 0 \), and A does not use all of the collateral to borrow from B. Between \( \Omega_1 \) and \( \Omega_2 \), A’s collateral constraint binds, and between \( \Omega_3 \) and \( \Omega_4 \), B’s pledgeability constraint binds. Between \( \Omega_2 \) and \( \Omega_3 \), \( \bar{R} = (p R_B - 1)d \) and neither constraint binds.

I provide a graphical proof for the propositions in Section 1.5. From Propositions 9 and 3 and the proof of Proposition 9, the equilibrium haircuts and interest rates are given by

\[ \frac{1}{1 - h} = \begin{cases} 
\Omega_1, & \text{if } \Omega \in [0, \Omega_4], \\
\Omega, & \text{if } \Omega \in [\Omega_1, \Omega_4], \\
\Omega_4, & \text{if } \Omega \in [\Omega_4, \infty), 
\end{cases} \]

\[ 1 + r = \begin{cases} 
\Omega_1 d^2 & \text{if } \Omega \in [0, \Omega_1], \\
\Omega d^2 & \text{if } \Omega \in [\Omega_1, \Omega_2], \\
\Omega \left[ \frac{\Omega_3 - \Omega_2}{\Omega_3 - \Omega_2} d^2 + \frac{\Omega_3 - \Omega_2}{\Omega_3 - \Omega_2} (1 - \kappa R_B) d \right] & \text{if } \Omega \in [\Omega_2, \Omega_3], \\
\Omega (1 - \kappa R_B) d & \text{if } \Omega \in [\Omega_3, \Omega_4], \\
\Omega_4 (1 - \kappa R_B) d & \text{if } \Omega \in [\Omega_4, \infty). 
\end{cases} \]

Figure 1-10 shows the change in the haircuts and interest rates for various values of \( \Omega \) when (i) the lender’s risk increases (keeping the project NPV constant), (ii) the lender’s pledgeability decreases, and (iii) the collateral risk increases. It is clear that the haircuts and interest rates are affected as in Section 1.5. The effect of a decrease in
\[ d \text{ on the interest rate is ambiguous. However, for a large enough fall in } d, \text{ the interest rate decreases for all values of } \Omega. \text{ The decrease has to be such that the value of } \Omega_2 \text{ with } d' \text{ is greater than the value of } \Omega_3 \text{ with } d, \text{ i.e.} \]

\[\frac{1}{d' p(R_B - 1 + d')} > \frac{1}{d pR_B (1 - \kappa)} ,\]

\[d'^2 + (R_B - 1)d' - dR_B (1 - \kappa) < 0,\]

\[d' < \frac{-(R_B - 1) + \sqrt{(R_B - 1)^2 + 4R_B(R_B - 1)(1 - \kappa)d}}{2} \equiv g(d).\]
Bibliography


56


Chapter 2

Rehypothecation and Monetary Policy

2.1 Introduction

The spectacular growth in shadow banking over the past couple of decades (Cetorelli, Mandel and Mollineaux, 2012; Pozsar et al., 2012) has led to a significant increase in the demand for and utilisation of financial securities as collateral in short-term repurchase agreements. The limited supply of “safe” collateral has also led to an increase in the practice of reusing available collateral. Rehypothecation is the practice of the direct reuse by lenders of the collateral posted with them to borrow on their own account. This is commonly observed in prime-brokerage arrangements, where prime brokers rehypothecate their hedge fund clients’ collateral to borrow from money market funds, and in repo markets, where cash lenders reuse the securities they receive in a repurchase agreement. Data from 10-Q filings with the SEC shows that in 2013, approximately $2 trillion dollars of collateral was rehypothecated by the US broker-dealers, or about 30% of their total assets. The rapid expansion in shadow banking has also made it necessary to understand the transmission of monetary policy through these funding channels characterised by extensive collateral reuse.

In this paper, I study a model of intermediation through rehypothecation with the repo and mortgage markets in mind. In my setup, both the borrower and the lender have variable scale investment projects. The lender is an intermediary and can rehypothecate the collateral to borrow from a third party to invest in his project. I show that rehypothecation allows the intermediate lender to invest in his profitable...
project by lending cash at a high haircut, and then rehypothecating the collateral at a lower haircut. The intermediate lender, thus, effectively borrows the haircut from the original borrower. The extent of this borrowing is restricted by the pledgeability of the intermediate lender. Pledgeability is the upper bound on borrowing, as a fraction of an investor’s expected cash flow, that does not trigger incentive problems. The higher the pledgeability, the stronger is the ability of the intermediate lender to effectively borrow from the borrower at high haircuts.

The model of rehypothecation is motivated by the commonly observed funding chain in the prime-brokerage market, where hedge funds (the original borrowers) borrow from their prime brokers (the intermediate lenders) who rehypothecate the collateral to money market funds (the ultimate lenders). This market exhibits segmentation, and the original borrowers—the hedge funds—cannot directly borrow from the ultimate lenders—the MMF’s. I examine and compare the equilibrium aggregate output in the presence and absence of this segmentation. I find that if the intermediate lender’s project is more valuable than the borrower’s project, then the aggregate output with market segmentation is higher than that in the case when the borrowers can directly borrow from the ultimate lenders. The reason for this is a pecuniary externality. When there is no segmentation, the presence of other lenders (who have no productive projects of their own and who do not rehypothecate) increases the cost of borrowing and rehypothecating the collateral. This moves investment away from the intermediary’s project towards relatively less valuable projects, leading to a fall in the aggregate output. This suggests that the observed segmentation in the market might reflect an efficient institutional arrangement, to the extent that the social value of the broker-dealers’ intermediation operations is higher than the social value of their hedge fund borrowers’ projects.

Next, I consider the central bank’s activities as another way of providing an alternative source of funding to the hedge funds in this segmented market. Following the Quantitative Easing actions of the Federal Reserve, concerns have been raised by market participants about the “illiquidity” in the treasury market. Singh (2013) has also documented a fall in the “velocity” (number of times a piece of collateral is pledged and repledged) of treasury collateral after QE-2 and QE-3. Motivated by this, I use the optimal rehypothecation contract to study the intervention of a central bank in the repo market and show that the central bank’s actions may be ineffective. During an expansionary open market operation to stimulate the economy, the central bank...

---

removes high quality collateral from the system, tightens collateral constraints down the rehypothecation chain and crowds out private investment. The counterparty to the central bank ends up providing too much collateral to the central bank, and raises the cost of borrowing collateral in the private market to inefficiently high levels. The collateral removed from the system sits with the central bank and does no work. As a result, the effectiveness of the expansionary operation is reduced. This effect holds when collateral is scarce and the cost of borrowing it in the private market is high. When collateral is abundant, however, some of it is idle and is not being used for borrowing and rehypothecation. In this case, the central bank’s intervention simply takes away the idle collateral without affecting rehypothecation and investment down the funding chain. Conversely, a contractionary operation has the side effect of stimulating private lending. This is because the collateral which was hitherto locked up with the central bank, is released and loosens borrowing constraints.

I suggest the bilateral repo spread as a measure of collateral scarcity. When collateral is idle, the cost of borrowing it in the private market is zero, and the repo spread is high. When collateral is scarce, the cost of borrowing it is high and the repo spread is low. I find evidence for this channel in the general collateral bilateral repo spreads for US treasury securities. When the repo spread is high (as it was before 2009), an increase in the supply of treasuries has little effect on the bilateral repo spread, indicating that there is little effect on the cost of rehypothecation down the chain. When the repo spread is low (as it has been since 2009), an increase in the supply of treasuries significantly increases the repo spread, indicating a fall in the cost of rehypothecation.

This has a number of policy implications. The result shows that open market operations must be conducted when collateral is abundant and idle. In view of the imminent reversal of many of the world’s central banks’ quantitative easing policies, too, it has important implications. First, it suggests that tools such as interest on reserves, which do not affect the supply of collateral, would be more effective than open market operations when collateral is scarce. Indeed, an intention in this direction was signalled in the FOMC meeting of July 2014 (Federal Reserve, 2014). Furthermore, high quality collateral such as treasuries tends to be relatively scarce in times of crises. Conducting expansionary open market operations using this safe collateral at precisely these times is more likely to backfire. Second, the result indicates that when the central bank conducts expansionary open market operations, it should restrict the purchases of assets from dealers and money funds to directly provide liquidity to where it is most valuable. Also, when the central bank, with a contractionary intent, borrows cash
against collateral through facilities such as the Overnight Reverse Repo Program (ON-RRP), it should deny rehypothecation permissions to the counterparties receiving the collateral, as rehypothecating the collateral may generate additional financing activity.

Related Literature

There has been a host of recent papers focussing specifically on rehypothecation. Bottazzi, Luque, and Páscoa (2012) study the existence of equilibria with limited and unlimited rehypothecation, but they abstract from considerations of the lender’s default risk. In their recent papers, Eren (2014) and Infante (2014) specifically model rehypothecation and view it as a way for the intermediate broker to obtain liquidity by offering differential haircuts: higher haircuts to the cash borrower and lower haircuts to the lender on rehypothecation. Maurin (2014) considers a general equilibrium model with collateral constraints and rehypothecation in a frictionless setting, and finds that rehypothecation can at best be a substitute for complete markets. Andolfatto, Martin and Zhang (2015) focus on the liquidity creating role of rehypothecation, and argue that limiting it may be desirable in increasing the demand for cash balances in economies away from the Friedman rule. Lee (2015) studies the effect of collateral reuse on repo spreads. My paper is most closely related to Eren (2014), Infante (2014) and Maurin (2014). It differs from the literature by looking at the optimality of the funding chains commonly observed in prime brokerage and repo markets which are characterised by segmentation and rehypothecation. This paper also studies the effects of monetary policy actions in the presence of rehypothecation chains.

The results on the impact of central bank intervention on repo markets relate to several papers that compare the different tools of monetary policy. Goodfriend (2002) suggests that the market interest rate and the level of reserves can be set independently, while Keister, Martin and McAndrews (2008) describe a floor system to divorce the two. Martin et al. (2013) analyse the effectiveness of the various tools of the Fed in maintaining the floor on rates. Stein (2012) prescribes the use of interest on reserves to avoid excessive short-term debt creation, while Kashyap and Stein (2012) advocate the use of a combination of open market operations, reserve requirements and open market operations depending on the nature of the financial system. Cochrane (2014) suggests that a regime with a large central bank balance sheet and interest paying reserves creates financial stability as the interest paying reserves reduce the incentives to create runsusceptible “inside money”. Ewerhart and Tapking (2008) study the optimal choice of
collateral in repurchase agreements, and examine the welfare implications of the central bank’s purchases of different types of collateral. Araújo, Schommer and Woodford (2013) study the effect of open market operations on borrowing constraints. While in their paper, asset purchases may tighten or relax collateral constraints depending on the risk of the asset, in my model, asset purchases tighten collateral constraints due to an externality.

The effect of treasury supply on their prices is investigated by Krishnamurthy and Vissing-Jorgensen (2012) who find that treasury securities enjoy a scarcity premium. Krishnamurthy and Vissing-Jorgensen (2013) argue that depriving the economy of liquid treasury bonds may have adverse welfare consequences. D’Amico, Fan and Kitsul (2013) find that purchases of specific CUSIP’s by the Fed leads to an increase in the corresponding special repo spread as the supply decreases. Fleming, Hrung and Keane (2010) similarly find that the repo spread between treasury and non-treasury collateral narrowed due to the TSLF which lent treasuries against other assets. Caballero (2006) and Caballero, Farhi and Gourinchas (2006) discuss the macroeconomic and international implications of collateral shortages, and Caballero and Farhi (2013) highlight the benefits of supplying safe assets to the system.

Outline:

Section 2.2 describes the model, the contracts and the timing. Section 2.3 analyses the the optimal rehypothecation contracts with and without market segmentation. Section 2.4 looks at a central bank intervention in a market characterised by rehypothecation, and looks at the empirical evidence and policy implications. Section 2.5 concludes.

2.2 Model

I describe here a model of rehypothecation of collateral. There are two dates, \( t = 1, 2 \). The first period is further divided into two: \( t = 1.1 \) (beginning of the period) and \( t = 1.2 \) (end of the period). There are three types of agents, A, B and C. There is a continuum of unit mass of each type. I interpret an agent of type A to be a hedge fund that needs to borrow, and an agent of type B to be an investment bank or prime-broker that can lend to the bank/hedge-fund A against collateral, and that can later rehypothecate that collateral. Agent C is interpreted as a money market fund that lends to banks B against collateral.
There is one good called cash and an asset called collateral. Cash is the numeraire. Agents of type A, B and C are risk neutral, and consume cash at \( t = 2 \). There is no discounting. There are belief disagreements between the agents about the payoff of the collateral. Agents B and C believe that one unit of the collateral pays off 1 unit of cash at \( t = 2 \) with probability 1. Agents A are more optimistic and believe that one unit of the collateral pays off \( u > 1 \) units of cash at \( t = 2 \) with probability 1.

At \( t = 1.1 \), A has \( Q \) units of the collateral asset, and B has 1 unit of cash. C has a large quantity of cash at each period that can be stored at an interest rate of 0 between any two periods. At \( t = 1.1 \), A has access to a variable scale investment opportunity that yields \( R_A > 1 \) units of cash with certainty at \( t = 2 \) per unit investment of cash. At \( t = 1.2 \), B has access to an investment opportunity that yields \( R_B > R_A \) units of cash at \( t = 2 \) with certainty. B cannot store the cash between \( t = 1.1 \) and \( t = 1.2 \). B has access to a storage technology between \( t = 1.1 \) and \( t = 3 \). I normalise this interest rate to 0.

**Borrowing Contracts**

To finance their investments, A must borrow from B and/or C, and B must borrow from C.

**Assumption 5.** The income from A’s project is not pledgeable to B or C. The income from B’s project is not pledgeable to C. A fraction \( \kappa > 0 \) of the income from B’s project is pledgeable to A.

The contract between A and B can be thought of as a prime-brokerage contract or a bilateral repo contract, whereas the contracts with the money market fund C can be thought of as a tri-party repo contract. The partial pledgeability of B’s income is motivated by its large size and strong reputation and the desire to protect it. I describe this in detail in a moral hazard setup in Appendix 2.A.

I first consider the contracts \( C_{A,B} = (r, h, \theta) \) between A and B traded at \( t = 1.1 \). Here, \( r \) is the net interest rate, \( h \) is the haircut\(^4\), and \( \theta \in \{0, 1\} \) is the rehypothecation

---

\(^2\)This assumption is not essential, and only serves to fix the outside option of B.

\(^3\)Bilateral repos are traded over the counter between hedge funds and broker-dealers. Money market funds enter into repo contracts with broker-dealers in an arrangement intermediated by JP Morgan or Bank of New York Mellon, which match the cash lenders (MMF’s) with the cash borrowers and also hold the collateral on behalf of the MMF’s. This is called a tri-party repo.

\(^4\)Haircut = \( 1 - \frac{\text{Units of cash borrowed}}{\text{Units of collateral pledged}} \).
Figure 2-1: Borrowing contracts. The arrows indicate the flow of cash at the time of lending.

permission: B can rehypothecate the collateral if $\theta = 1$ and not otherwise.\textsuperscript{5} I define one unit of this contract as lending one unit of cash by B to A under the given terms. I will frequently find it easier to refer to the contracts \textit{per unit of borrowing} in terms of the face value of the debt $1 + r$, and the number of units of collateral posted $\frac{1}{1 - h}$.

At $t = 1.2$, B may borrow from C under the contract $C_{B,C} = (\tilde{r}, \tilde{h})$, depending on what contracts were traded between A and B at $t = 1.1$. Here, $\tilde{r}$ is the interest rate, and $\tilde{h}$ is the haircut. Again, I define one unit of this contract as lending one unit of cash by C to B. If $\theta = 1$, B can directly rehypothecate the received collateral asset. If $\theta = 0$, B can only hold on to the debt.

At $t = 1.1$, A may also borrow from C under the contract $C_{B,C} = (\tilde{r}, \tilde{h})$. All agents behave competitively. Without loss of generality, I assume $C_{A,B}$ is offered by B to A, while $C_{A,C}$ and $C_{B,C}$ are offered by C to A and B respectively. Due to no recourse, the payoff of each debt contract in any state is the minimum of the face value of the debt and the value of the underlying collateral.

**Theorem 2.** There exists $\bar{u} > 1$ such that for $u > \bar{u}$, the optimal contracts are such that

$$1 + r \leq \frac{1}{1 - \bar{h}}, \quad 1 + \tilde{r} \leq \frac{1}{1 - \bar{h}}, \quad \text{and} \quad 1 + \tilde{r} \leq \frac{1}{1 - \bar{h}}.$$  

Moreover, the collateralised borrowing contracts $C_{A,B}$ and $C_{B,C}$ are optimal compared to sales of the collateral.

This is similar to the optimal contracts in Geanakoplos (2009). If the debt contract is not safe, there will be default in states to which the borrower attaches a low probability. Then, the borrower could increase the degree of collateralisation at little cost since the

\textsuperscript{5}The binary permission is without loss of generality. The framework does not preclude B offering multiple contracts to A. Some of these contracts may have $\theta = 1$, and others may have $\theta = 0$, so that in aggregate, any fraction between 0 and 1 of the total collateral may be permitted to be rehypothecated. The linear and risk-neutral nature of the framework will, however, ensure a corner solution with only one contract chosen between A and B, and with the aggregate $\theta \in \{0, 1\}$. 

64
default state had a low probability anyway according to him. But this makes the lender, who was attaching a higher probability to the default state, feel more secure and reduce the interest rate charged to the borrower. Similarly, a sale is suboptimal because the more optimistic borrower (seller) would like to retain possession of the collateral.

Remark 3. The literature has dealt with the problem of repurchase agreements versus asset sales. For example, Dang, Gorton and Holmström (2013) show that repurchase agreements will be preferred to asset sales since the lender (or buyer) will not be required to acquire information at a cost. Monnet and Narajabad (2012) reach the same conclusion by considering a hold-up problem at the repayment date when the lender (or reseller) can extract all the surplus if the borrower has no predetermined right to the collateral which is valuable to him. The focus of this paper is not to understand the trade-off between asset sales and repurchase agreements (collateralised borrowing contracts), but between securitisation and rehypothecation. I therefore make the following assumption.

Assumption 6. $u > \bar{u}$.

At $t = 2$, the returns to A’s and B’s investments are realised, $C_{B,C}$ is settled first, and finally $C_{A,B}$ is settled.

### 2.3 Optimal Contracts

In this section, I describe the optimal contracts and equilibrium outcomes when trade between A and C is restricted and when it is allowed. Segmentation in typically observed in these markets as hedge funds do not seem to have direct access to the sources of cash such as money market funds, but have to go through broker-dealers who act as intermediaries. I compare the aggregate equilibrium outcomes with and without segmentation in this market where rehypothecation of collateral plays a central role. An exploration of the reasons behind this segmentation is beyond the scope of this work.

I start by describing the optimal contracts $C_{A,B}$, $C_{B,C}$ and $C_{A,C}$, in the no-market-segmentation case when A and C are allowed to trade contracts $C_{A,C}$. I derive these contracts under the assumption that all agents act competitively, taking the actions of other agents as given. I look for a symmetric equilibrium where all agents of type B offer the same contracts $C_{A,B}$ and $C_{B,C}$, and all agents of type C offer the same contract $C_{A,C}$.  

65
C’s Problem

C has a large endowment of cash at \( t = 1.1 \) and \( t = 1.2 \) which it can store at a zero interest rate. It is easy to see that in any equilibrium, C would be willing to lend to A or B at a zero interest rate and a zero haircut. The interest rate cannot be lower than C’s outside option of zero. If the interest rate is higher than zero, then an agent C can attract all lending by lowering it slightly. I summarise this in the following proposition.

**Proposition 8.** The optimal contracts \( C_{B,C} \) and \( C_{A,C} \) are given by \( (\bar{r}, \tilde{h}) = (\tilde{r}, \tilde{h}) = (0, 0) \).

A’s Problem

A faces the offers \( (r, h, \theta) \) from B and \( (\tilde{r}, \tilde{h}) = (0, 0) \) from C. Let \( z_{A,B}^A \) and \( z_{A,C}^A \) be the units of collateral provided by A to B and C under the contracts \( C_{A,B} \) and \( C_{A,C} \) respectively. The amount of cash borrowed from A from B and C is therefore \( (1-h)z_{A,B}^A \) and \( (1-h)z_{A,C}^A = z_{A,C}^A \) respectively. A solves the following problem:

\[
\begin{align*}
\max_{z_{A,B}^A, z_{A,C}^A} & \quad \text{Net return per unit borrowing} \\
& \quad \left( R_A - (1 + r) \right) \cdot (1-h)z_{A,B}^A + \left( R_A - 1 \right) \cdot z_{A,C}^A, \\
\text{s.t.} & \quad z_{A,B}^A + z_{A,C}^A \leq \Omega.
\end{align*}
\] (2.1)

Even though B rehypothecates A’s collateral, B’s project never fails as assumed and A always gets back his collateral, making A’s cost of borrowing from B equal to the interest rate. Now A will allocate all its collateral to the contract that gives it the maximum net return per unit of collateral.

**Definition 4.** \( \bar{R} = (R_A - (1 + r))(1 - h) \).

\( \bar{R} \) can be interpreted as the net return earned by A by lending its collateral, or as the cost faced by B of borrowing collateral from A. The solution of the problem gives the supply of collateral in the contract \( C_{A,B} \):

\[
z_{A,B}^A(\bar{R}) \begin{cases} 
= 0, & \text{if } \bar{R} < R_A - 1, \\
\in [0, \Omega], & \text{if } \bar{R} = R_A - 1, \\
= \Omega, & \text{if } \bar{R} > R_A - 1.
\end{cases}
\] (2.3)
The remainder of the collateral $\Omega - z_{A,B}^A = z_{A,C}^A$ is allocated to the contract $C_{A,C}$.

**B’s Problem**

At $t = 1.1$, B can choose from three actions: (i) do nothing and save the endowment of cash into the storage technology and consume it at $t = 2$, (ii) offer a debt contract to A and hold on to it till $t = 2$, or (iii) offer a debt contract to A and rehypothecate the collateral to borrow from C at $t = 1.2$. The terms $C_{B,C}$ under which B is able to borrow from C by rehypothecation are given by Proposition 8. I derive below the optimal contract $C_{A,B} = (r(\bar{R}), h(\bar{R}), \theta(\bar{R}))$ that gives A a net return of at least $\bar{R}$ per unit of collateral.

If B offers a contract $(r, h, \theta = 0)$ to A, intending to hold it till maturity, B solves

$$\max_{r,h} 1 + r,$$  

subject to

$$(R_A - (1 + r))(1 - h) \geq \bar{R}. \tag{2.5}$$

The expected payoff of B is simply $1 + r$ and B solves the problem subject to A’s participation constraint (2.5). By Theorem 2, it is sufficient to consider the safe debt contracts given by the collateral constraint

$$1 + r \leq \frac{1}{1 - h}. \tag{2.6}$$

In this problem, a haircut greater than the minimum required serves no purpose, and wastes useful collateral. The solution is to make the collateral constraint bind, and raise the interest rate to the maximum possible so that A’s participation constraint binds.

Suppose now that B offers a contract $(r, h, \theta = 1)$ to A, intending to rehypothecate the collateral. B then holds $\frac{1}{1-h}$ units of the collateral. Then at $t = 1.2$ and from Proposition 8, B is able to borrow a total of $\frac{1}{1-h}$ from C at a zero interest rate by rehypothecating the collateral. On borrowing from C, B invests in his opportunity. Upon lending to A, B has receivables $1 + r$ from A, and owes $\frac{1}{1-h}$ to A. Thus, B owes to A a net amount

$$\frac{1}{1-h} - (1 + r).$$

This can be thought of as A’s exposure to B’s default. This is the amount that B effectively borrows from A.

I assume that B cannot “borrow” indiscriminately from A. Following Holmström
I limit the amount owed by B to A as

\[
\frac{1}{1-h} - (1 + r) \leq \kappa \cdot R_B \cdot \frac{1}{1-h},
\]

where the right hand side is the income pledgeable by B to A—a fraction \( \kappa \) of the gross cash flow from the investment. I motivate the limited pledgeability of B by the private benefit received by B by shirking on managing the risks of its investment and reducing the probability of success. If the amount owed by B to A is too large, B is highly leveraged and has little stake in the success of his investment, since it is A that stands to lose a lot from B's insolvency. He may then take undue risks leading to inefficient outcomes. As in Holmström and Tirole (1997), I focus on efficient contracts that disincentivise shirking. I discuss this in detail in Appendix 2.A.

B earns the cash flow from the project at \( t = 2 \), and repays C's debt and recovers the collateral. He then gets repaid by A, who then recovers the collateral. Thus, B solves

\[
\max_{r,h} \left( R_B \frac{1}{1-h} - \frac{1}{1-h} + 1 + r \right),
\]

subject to the collateral constraint (2.6), the participation constraint (2.5) and the pledgeability constraint (2.7).

Combining the collateral constraint (2.6) and B's incentive compatibility constraint (2.7), the following two-sided borrowing constraint must be satisfied,

\[
\frac{(1 - \kappa R_B)}{1-h} \leq 1 + r \leq \frac{1}{1-h},
\]

which requires that A should put up sufficient collateral to cover the debt, and that B should not rehypothecate too much collateral. The two constraints will both be satisfied and rehypothecation will be feasible if

\[
\kappa \geq 0.
\]

Inequality (2.9) thus captures the two-sided risks inherent in rehypothecation—the risk of A not repaying the debt and the risk of B being irresponsible with the collateral and not being able to return it in full. If both of these risks are not taken care of, then...
rehypothecation is infeasible.\footnote{I restrict attention to $\kappa < \frac{1}{R_B}$, since otherwise the pledgeability constraint is trivially satisfied.}

The following proposition summarises the optimal action of B and the contract $C_{A,B}$.

**Proposition 9.** There exists a cut-off $\tilde{R}^* > 0$, such that B consumes the endowment if $\tilde{R} > \tilde{R}^*$, and lends to A and rehypothecates the collateral if $\tilde{R} \leq \tilde{R}^*$. When $R_B - 1 < \tilde{R} \leq \tilde{R}^*$, A’s collateral constraint binds. When $\tilde{R} < R_B - 1$, B’s pledgeability constraint binds. The optimal contracts $(r(\tilde{R}), h(\tilde{R}), \theta(\tilde{R}))$ are given in Appendix 2.B

Since $R_B > 1$ and investing in B’s project is profitable, B always finds it optimal to rehypothecate than simply holding on to the debt. When $\tilde{R}$ is too high, B finds the market interest rate too low and simply consumes his endowment. In the optimal contract, exactly one of the collateral constraint (2.6) and the pledgeability constraint (2.7) will bind. As I show in the Appendix, when $R_B - 1 > \tilde{R}$, the net benefit from raising one unit of collateral from A is greater than the cost of raising that collateral. In this case, B’s incentive constraint binds. Intuitively, since the net return earned by A on the market is low, the cost of borrowing collateral from A and rehypothecating it is low. The lender B can then demand more collateral than is necessary to satisfy A’s collateral constraint, and rehypothecate this large amount of collateral to invest in the profitable opportunity. The amount of overcollateralisation, however, will be limited by B’s pledgeability constraint, which binds. When $R_B - 1 \leq \tilde{R}$, the reverse holds and the net benefit from raising one unit of collateral from A is less than the cost of raising that collateral. Intuitively, the high cost of borrowing collateral means that B is less willing to overcollateralise the debt with A. He demands only as much collateral as is needed to satisfy A’s collateral constraint, and B’s pledgeability constraint is slack.

Given the optimal rehypothecation contract, B’s demand for collateral in the contract $C_{A,B}$ is given by

$$z_{A,B}^B(\tilde{R}) = \begin{cases} \frac{1}{1 - h(\tilde{R})}, & \text{if } \tilde{R} < \tilde{R}^*, \\ \left[0, \frac{1}{1 - h(\tilde{R})}\right], & \text{if } \tilde{R} = \tilde{R}^*. \end{cases}$$

(2.11)

**Definition 5.** A symmetric competitive equilibrium is the collection of the net return $\tilde{R}$ per unit collateral committed by A, and the contracts $C_{A,B}, C_{B,C}$ and $C_{A,C}$, such that

1. C’s optimisation: Agents C offer contracts $C_{B,C}$ and $C_{A,C}$ given by Proposition 8.
2. *A's optimisation:* Agents A solve the problem (2.1) at $t = 1.1$ to obtain the supply of collateral $z_{A,B}^A(\tilde{R})$ given by (2.3).

3. *B's optimisation:* Agents B choose between the optima of the problems (2.4) and (2.8) at $t = 1.1$ to offer optimal contracts $C_{A,B}(\tilde{R}) = (r(\tilde{R}), h(\tilde{R}), \theta(\tilde{R}))$ given by Proposition 9 and to obtain the demand for collateral $z_{A,B}^B(\tilde{R})$ given by (2.11).

4. *Market clearing:* The collateral market between A and B at $t = 1.1$ clears:

$$z_{A,B}^A(\tilde{R}) = z_{A,B}^B(\tilde{R})$$

(2.12)

The supply of collateral (2.3) defines the collateral that A is willing to allocate to the contract $C_{A,B}$ which gives A a net return of $\tilde{R}$ per unit of collateral. The demand for collateral (2.11) defines the amount of collateral that B asks for in the optimal contract $C_{A,B}$ that gives A a net return of $\tilde{R}$ per unit of collateral. The market clearing condition for the collateral in the contract $C_{A,B}$ pins down the equilibrium net return $\tilde{R}$ per unit of collateral lent by A.

The equilibrium is described in Figure 2-2a. When $R_A$ is small and B's project is very valuable relative to A's, B is able to borrow all of A's collateral at a high interest rate and a high haircut (low $\tilde{R}$) to rehypothecate and invest. When $R_A$ is large, this is no longer possible and B is not able to command all of A's collateral, when the collateral endowment of A is large. A uses a part of its collateral to borrow directly from C and invest in its relatively valuable project.
Market Segmentation

When A and C cannot trade with each other, i.e. the contract $C_{A,C}$ is not tradable. A only faces the offers $(r, h, \theta)$ from B, and $z^{A}_{A,C} = 0$. Here, the equilibrium is again given by Definition 5, with A’s supply of collateral (2.3) modified to

\[
\begin{align*}
    z^{A}_{A,B}(\bar{R}) &= 0, \quad \text{if } \bar{R} < 0, \\
    &\in [0, \Omega], \quad \text{if } \bar{R} = 0, \\
    &= \Omega, \quad \text{if } \bar{R} > 0.
\end{align*}
\] (2.13)

The equilibrium is described in Figure 2-2b. Here, A is restricted to borrow from B, and B is able to borrow all of A’s collateral to rehypothecate if its pledgeability is high enough. If A’s collateral endowment is very high, then B’s pledgeability is not high enough to borrow all of A’s collateral, and some collateral lies idle with A.

Aggregate Output

The total investment in A’s project is $I_A = z^{A}_{A,B}(1 - h) + z^{A}_{A,C}$, while that in B’s project is $I_B = z^{B}_{A,B}$. The total expected output, or the sum of the consumptions of A, B and C at $t = 2$, is given by

\[Y = (R_A - 1)I_A + (R_B - 1)I_B,\] (2.14)

Let $Y^{AB,AC}$ and $Y^{AB}$ be the expected output at $t = 2$ without market segmentation and with it, respectively.

**Theorem 3.** Suppose $R_B > R_A$. Then

1. If $\kappa \geq \frac{R_A}{R_B}$, then $Y^{AB,AC} = Y^{AB} \forall \Omega$;
2. If $\frac{1}{R_B} \leq \kappa < \frac{R_A}{R_B}$, then $Y^{AB,AC} \leq Y^{AB} \forall \Omega$;
3. If $\kappa < \frac{1}{R_B}$, then $\exists \Omega^*$ such that $Y^{AB,AC} \leq Y^{AB}$ for $\Omega \leq \Omega^*$ and $Y^{AB,AC} > Y^{AB}$ for $\Omega > \Omega^*$;

This theorem establishes the importance of B’s pledgeability in determining aggregate outcomes. Due to the fact that B’s project is more profitable than A’s, it is desirable to maximise the investment in B’s project. When the pledgeability is very high, then even without market segmentation, B is always able to borrow and rehypothecate all of...
A's collateral endowment to invest in his profitable project. The allocations with and without market segmentation are exactly the same.

When the pledgeability is small, B is no longer able to rehypothecate all of A's collateral endowment, especially when the endowment is high. As a result, A ends up using part of its collateral to secure additional borrowing from C. This moves investment towards A's relatively less profitable project from B's project, when there is no market segmentation and A is allowed to borrow from C. As a result, the aggregate expected output with market segmentation is higher.

When the pledgeability is very small, B is unable to rehypothecate all of A’s collateral even in the absence of competition from C with market segmentation. Consequently, some collateral lies idle with A with market segmentation when the collateral endowment is high. Allowing borrowing from C then frees up the use of the collateral to invest in A’s project when investment in B’s project is severely restricted by low pledgeability.

The intuition behind this result is that when A can contract with C as well as B, A gives away some of the collateral to C who demands it to guard against A’s default. But the collateral does not produce additional surplus in the hands of C. If instead A borrows only from B, B is able to make good use of it by rehypothecating it and investing in his profitable project. The inefficiency of the equilibrium is the result of a pecuniary externality. The presence of an alternative but unproductive cash lender C bids up the net return $\bar{R}$ to A from lending out collateral. The intermediate lender B, due to limited pledgeability, cannot out-bid C even though his project is more profitable.\footnote{I have described the inefficiency in terms of aggregate output. In the absence of market segmentation, A is better off and B is worse off compared to the case with market segmentation. It is straightforward to show that in an equilibrium with market segmentation, an ex-post non-pledgeable transfer from B to A achieves a Pareto improvement over the outcome without segmentation.}

The central role of pledgeability in causing aggregate inefficiency in the absence of market segmentation can be seen from the fact that when there is full pledgeability, the outcomes with and without market segmentation are the same.

**Corollary 3.** When $R_B > R_A$ and $\kappa = 1$, $Y^{AB,AC} = Y^{AB}$.

The result indicates that when collateral is scarce and the intermediate lenders' activities are socially valuable ($R_B$ is high), it is desirable to have segmentation and so that collateral is restricted to flow through these productive lenders. The intermediate lenders' projects can be thought of as provision of intermediation services to search and match clients, which require liquidity in the short term in the event of demand
and supply imbalances (Adrian et al., 2013). These services are likely to be especially valuable in times of crises and high market volatility, compared to the activities of the speculative hedge funds. The observed segmentation in the shadow banking financing channel, therefore, provides an efficient way to transmit funding at such times.

2.4 Central Bank Intervention

I now look at what happens when a central bank intervenes in an existing chain to exchange cash for collateral. In addition to the model in Section 2.2 with the observed market segmentation, I introduce a central bank which can lend directly to agents A, thus providing an alternative to the contract \( C_{A,C} \). I show that a monetary policy action of the central bank may lose bite as the taking out or putting in of collateral in exchange for cash will affect collateral constraints down the chain in a counterproductive way. While the central bank is not a profit maximising agent, a mechanism similar to that in Section 2.3 will produce an adverse effect due to the provision of an alternative lending source to A.

The channel I highlight here works through changing repo rates and quantities of treasury collateral available with borrowers to pledge to private lenders, and is independent of the monetary policy channels that work through changing bond prices and interest rates. Although this channel will be operating in normal situations, a useful way to think of the model is in a situation in which large scale purchases or sales of treasury collateral are carried out by the central bank. Such purchases were undertaken by central banks in advanced economies in response to the global financial crisis. There is also a possibility of large scale sales of collateral being carried out in the near future as these economies emerge out of the recession. The large quantities involved make this channel quantitatively important.

I consider the observationally relevant case with market segmentation.\(^8\) I assume a repo-style intervention by the central bank—the central bank borrows or lends cash against collateral, rather than buying or selling it. This is in keeping with the tenor of the model, and does not lose generality. Purchases and sales of collateral will have the same effect on collateral constraints down a rehypothecation chain as the collateral

\(^8\)I also restrict attention to the realistic case \( \kappa R_B < 1 \). If \( \kappa R_B \geq 1 \), then \( 1 + \tau \leq 0 \), and B pays A to be able to lend cash to A against collateral. More significantly in this case as \( \tilde{R} \to \kappa R_B - 1 \to 0 \), B is able to borrow an indefinite amount of collateral to A by promising to pay indefinitely large amounts together with returning the collateral.
Figure 2-3: Expansionary operation—central bank lending against collateral.

enters or leaves a central bank. I also assume that the central bank lends cash to or borrows cash from A against collateral. To recap, A can be thought of as a hedge fund or a dealer, B as another dealer, and C as a money fund. Figure 2-3 shows the flow of cash and collateral as the central bank lends cash to A against collateral. The choice of A as the point of entry has two reasons. First, the unintended effects on collateral constrains arise when the entry is at A, and not when it is at B or C. Second, this is quantitatively relevant: Carpenter et al. (2015) show that hedge funds tend to be the largest buyers and sellers of treasury securities, far larger than broker dealers. Their analysis estimates that of the $600 billion of treasuries purchased by the Fed during LSAP-2, about 60% were sold by hedge funds.\footnote{While hedge funds are not counterparties to the Fed’s open market operations, they can sell securities to the Fed through their broker-dealers, who are.} They propose a preferred habitat explanation for this evidence.

The central bank targets a supply of cash $m$ to be infused into the system. (I abstract from the reasons why the bank would target a particular $m$.) This is without loss of generality. In the model, targeting a money supply is equivalent to targeting the interest rate between A and B. Before the market between A and B opens at $t = 1.1$, the central bank at $t = 0$ lends $m \in \mathbb{R}$ units of cash to A until $t = 3$, at the terms $(r^{CB}, h^{CB})$ set by the market in equilibrium. $m > 0$ indicates an expansionary operation and $m < 0$ indicates a contractionary operation. I assume that the central bank makes the loan fully secured with $1 + r^{CB} = \frac{1}{1 - h^{CB}}$, so that only $r^{CB}$ is left to be determined by the market equilibrium. The central bank rebates lump-sum all profits to A, B or C at $t = 2$. As A borrows from the central bank, it moves collateral away from B by an amount

$$\Delta \Omega = m \frac{1}{1 - h^{CB}}. \quad (2.15)$$
The changed supply of collateral affects the equilibrium between A and B. If \( m > 0 \), the central bank takes away collateral and increases \( \bar{R} \) and reduces the interest rate between A and B. If \( m < 0 \), the central bank supplies collateral, decreases \( \bar{R} \) and increases the interest rate. In the new equilibrium, A is indifferent between borrowing from B and borrowing from the central bank, so that

\[
(R_A - (1 + r^{CB})) (1 - h^{CB}) = \bar{R}.
\]

**(Definition 6)** An equilibrium with central bank intervention of \( m \in \mathbb{R} \) units of cash is the collection of the net return \( \bar{R} \) per unit collateral committed by A, the terms \((r^{CB}, h^{CB})\) of borrowing from (or lending to) the central bank, and the contracts \( C_{A,B} \), \( C_{B,C} \) and \( C_{A,C} \), such that

1. **Optimisation at \( t > 0 \):** Conditions (1)-(3) in Definition 5 are satisfied for the optimisation of A, B and C at \( t > 0 \), with A’s modified supply of collateral given by

\[
z^A_{A,B}(\bar{R}) = \begin{cases} 
0, & \text{if } \bar{R} < 0, \\
[0, \Omega - \Delta \Omega], & \text{if } \bar{R} = 0, \\
\Omega - \Delta \Omega, & \text{if } \bar{R} > 0,
\end{cases}
\]

where \( \Delta \Omega \) is given by Equation (2.15).

2. **Central Bank’s haircut choice assumption:** The central bank secures the debt fully and does not overcollateralise, so that \( 1 + r^{CB} = \frac{1}{1 - h^{CB}} \).

3. **A’s optimisation at \( t = 0 \):** Agents A are indifferent between using the collateral to borrow from B and accepting the central bank’s offer, so that Equation (2.16) holds.

4. **Market clearing:** The collateral market between A and B at \( t = 1.1 \) clears:

\[
z^A_{A,B}(\bar{R}) = z^B_{A,B}(\bar{R}).
\]

Define the total output \( Y \) as the sum of the expected consumptions of A, B and C at \( t = 2 \) and \( t = 3 \). I have the following proposition:

**(Proposition 10)** In the equilibrium with central bank intervention described in Definition 6, the marginal effect due to the intervention of \( m \in \mathbb{R} \) units of cash is given
The proposition says that the action by the central bank may have unintended consequences that work against the central bank’s objective. For example, in the case of an expansionary operation with \( m > 0 \), the decreased supply of collateral may tighten the collateral constraint between B and C, having an adverse effect on output. Figure 2-4 shows the equilibria in the three regions which react varyingly to an expansionary operation with \( m > 0 \).

In the first region, when collateral is abundant and \( \bar{R} = 0 \), the action of the central bank will have the desired effect of increasing output as A invests the extra cash. There will be no effect on the equilibrium between A and B as A will use the idle collateral to borrow from the central bank.

However, when collateral is scarce in the second region and \( \bar{R} > 0 \), useful collateral is taken away from B through a lower haircut, and B cannot borrow as before by rehypothecating the collateral to C. This effect is particular to a rehypothecation chain, and would be absent if the central bank had entered at B. It is clear that the tightening of B’s collateral constraint causes the monetary policy action to lose its bite. In fact, as detailed in Appendix 2.E, it could possibly have the opposite effect and cause aggregate expected output to decrease.

**Corollary 4.** If \( R_B > R_A \), there exists an \( \bar{R}^\# > 0 \) such that \( \frac{\partial Y}{\partial m} \bigg|_{m=0} < 0 \) for \( \bar{R} \in (0, \bar{R}^\#) \).

The reason why output may even fall due to the expansionary intervention is reminiscent of the pecuniary externality in Section 2.3. A may end up giving too much collateral to the central bank, without taking into account the adverse effects of doing so on the collateral constraint between B and C. The central bank is only contracting with A, and A cannot internalise the loss due to the tightening of B’s collateral constraint. The collateral sits idle with the central bank, when otherwise it could have been used profitably by B. Even though the collateral is more useful to B, B cannot outbid the central bank to borrow it due to limited pledgeability.\(^{10}\)

\(^{10}\)This assumes that the cash (reserves) received by A from the central bank cannot be pledged.
Figure 2-4: Equilibrium with an expansionary open market operation.

In the third region, $\bar{R} = \bar{R}^*$. B is not lending to A its full endowment of cash, and the central bank’s entry causes A to simply substitute to borrowing from the central bank instead of from B. A’s borrowing and investment will increase by the difference in the haircuts $h$ and $h^{CB}$ offered by B and the central bank respectively, and B will consume the idle cash. This effect is not particular to a rehypothecation chain, and would have existed even if the central bank had entered at B.

The case of contractionary operations is nearly symmetric, with an additional feature: the central bank must decide whether to allow A to rehypothecate the received collateral. With a sale of the collateral, the rehypothecation permission is implicit. With lending collateral, however, the permission needs to be made explicit.

**Proposition 11** (Contractionary Operation). The marginal effect due to a contractionary operation ($m < 0$) is given by Proposition 10 if rehypothecation by A is permitted, and by $\frac{\partial y}{\partial m}_{m=0} = R_A - 1$, $\forall \bar{R} \in [0, \bar{R}^*]$ if rehypothecation by A is not permitted.

If collateral is not permitted to be rehypothecated by A, it just sits with A and does not circulate to generate more borrowing and investment.

### 2.4.1 Empirical Evidence

The model predicts that an increase in the supply of collateral lowers the return $\bar{R}$ from lending collateral and increases the repo rate $r$ in the bilateral repo market between A to B. In Appendix 2.E.2, I relax this assumption and show that the result about the fall in expected output still holds. The intuition is that when B is rehypothecating a A’s collateral at bad terms for A (low $\bar{R}$), A chooses to invest the cash received from the central bank in its project instead of lending it to B. B’s limited pledgeability prevents it from borrowing more from A under the same terms.
and B.\textsuperscript{11} I consider the weekly regression

\[ \Delta s_t = \alpha + \rho_1 \Delta s_{t-1} + \rho_2 s_{t-1} + X_t \gamma + \beta_1 \Delta T_t + \beta_2 I_{t-1} \cdot \Delta T_t + \epsilon_t. \tag{2.18} \]

Here, \( s_t = r_t - y_t^{3mo} \) is the overnight borrowing rate with treasury collateral, normalised by the short term risk free rate, the three month treasury yield. \( X_t \) are various market controls including the lagged equity market, the VIX, short and long term treasury yields, and their changes. \( \Delta T_t \) are the weekly changes in outstanding treasury securities. The product \( I_{t-1} \cdot \Delta T_t \) of a measure of collateral idleness and the change in treasury supply is included to test for the hypothesis that the interest rate reacts less to changing collateral supply when collateral is idle.

The model also predicts that an increase in the supply of collateral increases financing activity between A and B and between B and C as the investment in B’s project increases, and more so when collateral is scarce. I consider the weekly regressions

\[ \Delta F_t = \alpha + \rho_1 \Delta F_{t-1} + \rho_2 F_{t-1} + D_t \delta + X_t \gamma + \beta_1 \Delta T_t + \beta_2 I_{t-1} \cdot \Delta T_t + \epsilon_t. \tag{2.19} \]

\( \Delta F_t \) is the weekly change in aggregate overnight borrowing or lending reported weekly by Primary Dealers. I run the regressions for \( \Delta F_t \) equal to the change in the aggregate borrowing, the aggregate lending, and the net borrowing (borrowing minus lending) using treasury collateral. \( D_t \) are weekly dummies at end-of-quarter weeks.\textsuperscript{12}

For the index of collateral idleness, I use \( I_t = s_t \) itself, since the model predicts that when collateral is abundant and idle, the interest rate is high. The model predicts that in both regressions, \( \beta_1 > 0 \) and \( \beta_2 < 0 \).

Data

The data for \( F_t \) are obtained from the aggregate FR 2004 reports published by the Federal Reserve Bank of New York. These data are for aggregate (repo and other) borrowing and lending by the set of primary dealers using treasury collateral. The series for \( T_t \) are obtained by subtracting the treasury holdings of the System Open

\textsuperscript{11}While in the simple model with homogeneous agents, \( \bar{R} \) does not change for small changes in the supply of collateral when \( \bar{R} \in \{\bar{R}^*, \bar{R}_B - 1\} \), it can be shown that for a continuum of agents B with project returns in some interval \( [\bar{R}_{B,L}, \bar{R}_{B,U}] \), \( \frac{\partial \bar{R}}{\partial m} < 0 \) and \( \frac{\partial \bar{R}}{\partial m} > 0 \) when collateral is scarce and \( \bar{R} > 0 \), and \( \frac{\partial \bar{R}}{\partial m} = \frac{\partial \bar{R}}{\partial m} = 0 \) when collateral is idle and \( \bar{R} = 0 \).

\textsuperscript{12}These are included because there is an observed drop in financing at the end of the quarter due to cyclical factors.
Market Account of the FRBNY from the federal debt held by the public\textsuperscript{13}. The series for $r_t$ is the index of overnight US treasury collateral repo rates USRG1T obtained from Bloomberg. The data are weekly from 2005Q2 to 2013Q1. I use high frequency weekly data to minimise lower frequency fluctuations and other secular trends caused by regulatory changes. The weekly frequency also eliminates the possibility of the left-hand side variables affecting the changes in the supply of treasuries, which are announced well in advance. I use overnight financing data to increase the power of the test, since it is overnight financing, which needs to be rolled over every night, that will be impacted most by the changing supply of collateral.

Figure 2-5 shows the spread between the overnight treasury collateral repo rates and the three month treasury yield. The fact that the spread was positive in the years leading up to the crisis, and has been close to zero since, seems to indicate that collateral has been less idle since the crisis.

\textsuperscript{13}The weekly data for the federal debt held by the public are obtained from www.treasutydirect.gov. These data are the face value of all US treasury securities. The ideal variable to look at would be the market value of all outstanding treasury securities, the data for which is not publicly available at a weekly frequency to the best of my knowledge. The use of the face value data will only underestimate the significance of my results. An increase is the supply of collateral will reduce the price of the collateral. The measurement error in the increase in the supply of collateral will bias the coefficient downwards.
<table>
<thead>
<tr>
<th></th>
<th>Whole period</th>
<th>Before 01-01-09</th>
<th>After 01-01-09</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta T_t$</td>
<td>0.37*</td>
<td>1.06**</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.40)</td>
<td>(0.65)</td>
</tr>
<tr>
<td>$s_{t-1} \cdot \Delta T_t$</td>
<td>-3.08***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.65)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Controls</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.28</td>
<td>0.32</td>
<td>0.31</td>
</tr>
<tr>
<td>$N$</td>
<td>383</td>
<td>383</td>
<td>179</td>
</tr>
</tbody>
</table>

Table 2.1: Regressing repo-treasury spread on treasury supply. HAC standard errors.

Figure 2-6: Conditional correlation between treasury supply and the repo-treasury spread, before and after 1 January 2009. Conditioning on the control variables.

Results

The identification assumption in the regressions is that the unobserved factors affecting the dependent variables are uncorrelated with the weekly changes in treasury supply. The issuance of bonds by the Treasury is according to a preset timetable, which does not interact with the unobserved factors at a weekly frequency.

The results for regression (2.18) are in table 2.1. The spread between the repo rate and the risk-free rate increases significantly as the treasury supply increases, but not so much when the spread is already high and collateral is idle. Since the time series for the spread indicates that collateral has been less idle since the crisis, I rerun the regression for before and after 1 January 2009. I find that the effect of treasury supply on the spread is significant only after that date, when collateral was scarce. Figure 2-6 shows this conditional correlation before and after that date.
Table 2.2: Regressing aggregate overnight borrowing, lending and net borrowing on treasury supply. HAC standard errors.

The results for regression (2.19) are in Table 2.2. I find that an increase in the supply of collateral significantly increases the amount of financing by broker-dealers. Most importantly, in the third column I find that net borrowing by primary dealers from outside their set also increases. However, this may be consistent with the absence of a rehypothecation chain, and the primary dealer simply borrowing from a money fund to purchase the treasury security from the central bank. This concern is mitigated by the fact that lending by primary dealers also goes up significantly, which indicates that the dealers receive treasury collateral to do with as they wish. The coefficient on the interaction term is not significant. The reason for this is that the quantities are not disaggregated between hedge fund-dealer/inter-dealer ($C_{A,B}$) financing and dealer-money fund ($C_{B,C}$) financing. Idle collateral (high $s_{t-1}$) in the bilateral market could mean cheaper funding from money funds for the dealers, and this would show up in their increased borrowing. The effect on the bilateral market and rehypothecation, especially when collateral is idle, can be better identified by looking at the bilateral repo rate regression (2.18). The bilateral market is heavily characterised by rehypothecation. As shown in Figure 2-7, 75-80% of the collateral that is eligible to be rehypothecated is actually rehypothecated by the top 5 U.S. broker dealers. A change in the terms of borrowing collateral is, therefore, likely to have a significant impact on the ability of the dealers to satisfy their short-term liquidity needs through rehypothecation.
2.4.2 Policy Implications

The trade-off between cash and collateral created by open market operations leads to several policy implications. The fact that the removal of collateral during an expansionary open market operation can contract financing and investment suggests that the ideal way to conduct such operations is for the central bank to not demand collateral for the cash it supplies. The potential losses to the central bank due to default may then be covered by increased taxes on the profits of the financial sector. Similarly, the effective way to conduct contractionary operations would be to not provide collateral in exchange for cash, and use instruments like reserve requirements and interest on reserves (IOR, IEOR). Indeed, good quality collateral like treasuries is most likely to be scarce, and the magnitude of the pecuniary externality in likely to be strongest, in times of crises. Conducting expansionary open market operations using treasuries at such times is more likely to backfire.

Other tools such as the Term Deposit Facility (TDF) could be expanded. So could the coverage of reserve requirements across institutions and liability types, in order for reserves to be effectively manipulated on a large scale using IOR without resorting to open market operations. This conclusion relates to Stein (2012) and Kashyap and Stein (2012), who advocate using a combination of interest on reserves and open

Figure 2-7: Collateral actually rehypothecated by top 5 U.S. broker-dealers, as a fraction of collateral allowed to be rehypothecated.
market operations to optimally conduct monetary policy. In their framework which focusses on financial stability, active adjustment of the level of reserves prevents excessive short-term debt creation. My model highlights the role of externalities down the rehypothecation chain when open market operations are used when collateral is scarce.

If open market operations are to be conducted, they must be conducted in a way that does not affect the supply of collateral that can be profitably rehypothecated. One potential variable to look at would be the spread between the collateralised borrowing rate and the short term risk free rate. A high value of this spread would mean that collateral is abundant and idle and that it is safe to conduct an open market operation. With contractionary open market operations, it is much easier to ensure that the collateral provided does not enter into circulation by denying rehypothecation rights. The Fed’s Overnight Reverse Repo Programme (ON-RRP) is conducted through a tri-party arrangement wherein the collateral stays with the tri-party bank and is not rehypothecated. It may be tempting to consider statutorily limiting rehypothecation to provide a greater control to the central bank over the outcomes of its monetary policy actions. However, this may not be ideal since it will prevent collateral from flowing to where it is needed the most.

Another way of minimising the unintended consequences of open market operations is to conduct them with counterparties who are at the lending end of the chain. Contractionary operations which lend collateral to money funds will be more effective since they are not natural rehypothecators and will hold on to it. The ON-RRP’s expanded counterparties include such entities. In fact, primary dealers rarely avail of this facility and most of the volumes are transacted with money market funds. Similarly, expansionary open market operations in crises can be restricted to purchase assets from the money funds and dealers (agents B and C in the model) to ensure that liquidity is provided directly to where it is most valuable for intermediation purposes, instead of to hedge funds (agents A) who engage in speculative activities.

2.5 Conclusion

I construct a model of rehypothecation with the prime brokerage and repo in mind. I show that rehypothecation allows the intermediate lender to invest in his profitable project by effectively borrowing from the borrower by charging a high haircut. Funding chains in the prime-brokerage market typically exhibit segmentation, with the hedge
fund borrowers being unable to directly borrow from the ultimate MMF lenders. I show that when the pledgeability of the intermediate lenders is limited, segmentation may achieve superior aggregate outcomes. The reason for this is a pecuniary externality generated by the presence of other competing but unproductive lenders who reduce the collateral available to the productive intermediate lenders to borrow and rehypothecate. I also show that open market operations of the central bank can backfire as high quality collateral removed from the system can tighten collateral constraints down the rehypothecation chain and impair financing and investment activity. I discuss a number of implications for policy.

A direction for future research could be to further explore the quantitative effects of open market operations on collateral constraints and financing. Special repo spreads provide a more accurate measure of the discrepancy between the supply and demand for collateral. Examining the impact of treasury supply on financing using treasury collateral when the spreads are high will clarify the quantitative importance of these effects.

A further direction would be to analyse the desirability of bilateral, over the counter markets, over anonymous, centralised markets. Over the counter markets enable the exploitation of pledgeability through relationships with known counterparties. The analysis in this paper shows that having the intermediate lender B in the middle of the chain is desirable as his cash flows are pledgeable to the borrower. Given a distribution of agents with varying pledgeabilities, the optimal network and institutional arrangement can be studied.
Appendices

2.A Moral Hazard

Suppose that B can either manage his risks well so that the probability of success of his investment is \( p_H = 1 \), or he can shirk and get a private benefit \( b \) per unit of investment. In the latter case, the probability of success of the project becomes \( p_L < p_H \). I assume that shirking is always socially suboptimal, i.e. \( p_H R_B > p_L R_B + b \), and wish to implement only the efficient contracts where B is sufficiently incentivised to manage his risks well. From (2.8), this requires

\[
p_H \left( (R_B - 1) \frac{1}{1 - h} + 1 + r \right) \geq p_L \left( (R_B - 1) \frac{1}{1 - h} + 1 + r \right) + b \frac{1}{1 - h}.
\]

Rearranging, I get

\[
\frac{1}{1 - h} - (1 + r) \leq \left( 1 - \frac{b}{R_B \Delta p} \right) R_B \frac{1}{1 - h}.
\]

Denoting \( \kappa = 1 - \frac{b}{R_B \Delta p} \) gives (2.7).

2.B Proof of Proposition 9

Consuming the endowment gives B a utility

\[
U^{CE}(\bar{R}) = 1.
\]

Lending to A may give him a higher utility. Suppose that B offers one unit of the contract \((r, h, \theta = 1)\) to A, and rehypothecates a fraction \(w\), and holds on to a fraction \(1 - w\) of the contract. Write \(1 + r = X\) and \(\frac{1}{1 - h} = Z\). B thus solves a combination of the problems (2.4) and (2.8):

\[
\max_{X,Z} \left( (R_B - 1)Z + X \right) w + X(1 - w), \text{ s.t.}
\]
Collateral const.: \( X \leq Z \),
Participation const.: \( R_A - Xw - X(1 - w) \geq \bar{R}Z \),
Pledgeability const.: \( Z - X \leq \kappa R_B Z \).

The collateral constraint occurs in all three problems and remains unchanged. The pledgeability constraint also remains unchanged as B’s project is linearly scalable and the condition on the proportion of the face value \( X \) and the amount of collateral \( Z \) to overcome moral hazard with rehypothecation is unchanged. The participation constraint changes to reflect that a smaller fraction of the collateral is rehypothecated and the expected loss of collateral is proportionately smaller.

Since the participation constraint will always bind, I can rewrite the objective function as
\[
[(R_B - 1)w - \bar{R}] Z + R_A.
\]
Since \( R_B > 1 \), \( w = 1 \) maximises the objective. Now there are two cases:

Case I: \( 0 \leq R_B - 1 \leq \bar{R} \). In this case, the first term in the objective function is negative. The benefit from rehypothecating collateral is less than the cost of raising that collateral. B will minimise \( Z \) so that the collateral constraint binds. The optimal contract is
\[
1 + r = \frac{R_A}{1 + \bar{R}}, \quad \frac{1}{1 - h} = \frac{R_A}{1 + \bar{R}}, \quad \theta = 1,
\]
and the utility is
\[
U^{RE}(\bar{R}) = \frac{R_B R_A}{1 + \bar{R}}.
\]

Case II: \( \bar{R} < R_B - 1 \). In this case, the first term in the objective function is positive. The benefit from rehypothecating collateral is greater than the cost of raising that collateral. B will maximise \( Z \) so that the pledgeability constraint binds. Plugging the binding participation constraint \( R_A - X = \bar{R}Z \) into the pledgeability constraint, I have
\[
(1 + \bar{R} - \kappa R_B)Z \leq R_A.
\]

If \( R_B - 1 > \bar{R} > \kappa R_B - 1 \), the optimal contract is
\[
1 + r = \frac{R_A(1 - \kappa R_B)}{1 + \bar{R} - \kappa R_B}, \quad \frac{1}{1 - h} = \frac{R_A}{1 + \bar{R} - \kappa R_B}, \quad \theta = 1,
\]
and the utility is

$$U^{RE}(\bar{R}) = \frac{R_B(1 - \kappa)R_A}{1 + \bar{R} - \kappa R_B}.$$ 

As $\bar{R} \to \kappa R_B - 1 > 0$, B is able to borrow an indefinite amount of collateral to A by promising to pay indefinitely large amounts together with returning the collateral. Thus, in any equilibrium, $\bar{R} > \kappa R_B - 1$.

I guess and verify an $\bar{R}^*$ such that $\bar{R}^* > R_B - 1$ and $U^{RE}(\bar{R}^*) = 1$. From the expression for the utility in the guessed range of $\bar{R}^*$, I have

$$\bar{R}^* = R_B(R_A - 1) + (R_B - 1).$$

2. C Proof of Theorem 3

As assumed, $R_B > R_A$. I consider the three cases in turn.

Case I: $\kappa \geq \frac{R_A}{R_B}$

![Diagram](image)

Figure 2-8: $\kappa \geq \frac{R_A}{R_B}$. The solid blue line is the supply curve without segmentation. The dashed blue line is the supply curve with segmentation.

The equilibrium in this case is described in Figure 2-8. Here, $\bar{R} \geq \kappa R_B - 1 > R_A - 1$, and even without market segmentation, B is able to borrow all of A's collateral. Thus, the outcomes with and without segmentation are exactly equal.
Case II: \( \frac{1}{R_B} \leq \kappa < \frac{R_A}{R_B} \).

The equilibrium in this case is described in Figure 2-9. Let \( \Omega_1 \) be such that \( \bar{R} = R_A - 1 \). The left panel of Figure 2-9 depicts the equilibrium for \( \Omega > \Omega_1 \), while the right panel depicts the equilibrium for \( \Omega \leq \Omega_1 \). When \( \Omega \leq \Omega_1 \), \( \bar{R} \geq R_A - 1 \), and again the equilibrium outcomes with or without segmentation are exactly equal.

When \( \Omega > \Omega_1 \) as in the left panel, the investments and output without segmentation are given by

\[
I_A = 1 + \Omega - \Omega_1, \quad I_B = \Omega_1, \quad Y^{AB,AC} = (R_A - 1)(1 + \Omega - \Omega_1) + (R_B - 1)\Omega_1.
\]

The investments and output with segmentation are given by

\[
I_A = 1, \quad I_B = \Omega, \quad Y^{AB} = (R_A - 1) + (R_B - 1)\Omega.
\]

Thus,

\[
Y^{AB,AC} - Y^{AB} = (R_A - R_B)(\Omega - \Omega_1) < 0.
\]
Case III: $\kappa < \frac{1}{R_B}$.

Figure 2-10: $\kappa < \frac{1}{R_B}$. The solid blue line is the supply curve without segmentation. The dashed blue line is the supply curve with segmentation.

The equilibrium in this case is described in Figure 2-10. Let $\Omega_2 > \Omega_1$ be such that $\bar{R} = 0$. The left panel of Figure 2-10 depicts the equilibrium for $\Omega > \Omega_2$, while the right panel depicts the equilibrium for $\Omega \leq \Omega_2$. When $\Omega \leq \Omega_2$, $\bar{R} \geq 0$ and we have $Y^{AB,AC} \leq Y^{AB}$ as in Case II.

When $\Omega > \Omega_2$ as in the left panel, the investments and output without segmentation are given by

$$I_A = 1 + \Omega - \Omega_1, \quad I_B = \Omega_1, \quad Y^{AB,AC} = (R_A - 1)(1 + \Omega - \Omega_1) + (R_B - 1)\Omega_1.$$ 

The investments and output with segmentation are given by

$$I_A = 1, \quad I_B = \Omega_2, \quad Y^{AB} = (R_A - 1) + (R_B - 1)\Omega_2.$$ 

Thus,

$$Y^{AB,AC} - Y^{AB} = (R_A - 1)(\Omega - \Omega_2) - (R_B - R_A)(\Omega_2 - \Omega_1) > 0$$

if and only if $\Omega > \Omega^* \equiv \Omega_2 + \frac{R_B - R_A}{R_A - 1}(\Omega_2 - \Omega_1)$. 

89
2.D  Belief Disagreements and Optimality of Safe Debt

Without loss of generality, let B offer the contract to A and C. Write \( 1 + r = X \) and \( \frac{1}{1-h} = Z \), and let \( v \) be the value of one unit of the collateral at \( t = 2 \). B is able to raise by borrowing against it at a zero interest rate from C. (Borrowing from C is equivalent to selling the collateral to C.) B solves

\[
\max_{X,Z} R_B Z - Z + E_B[\min\{X, Zv\}],
\]

s.t.

\[
R_A - E_A[\min\{X, Zv\}] \geq \tilde{R} Z, \quad \text{and}
\]

\[
Z - E_B[\min\{X, Zv\}] \leq \kappa R_B Z.
\]

where \( v \) is the payoff of one unit of the collateral at \( t = 3 \), and the expectations \( E_A[\cdot] \) and \( E_B[\cdot] \) are taken according to the beliefs of A and B respectively.

A’s participation constraint always binds. Again, due to linearity, it is sufficient to restrict attention to \( X \in (-\infty, Z] \cup [Zu, \infty) \).

\( X \in [Zu, \infty) \): Here, there is always default. B’s pledgeability constraint does not bind. We have \( R_A - Zu = \tilde{R} Z \). This gives \( Z = \frac{R_A}{u + \tilde{R}} \), and B’s payoff is

\[
U(X \geq Zu) = \frac{R_BR_A}{u + \tilde{R}}.
\]

\( X \leq Z \): Here, there is no default. The optimal contract is given by Proposition 9 and B’s payoff does not feature \( u \).

As \( u \to \infty \), \( U(X \geq Zu) \to 0 \) for any given \( \tilde{R} \), and the safe debt contract is optimal as it does not involve default by A and loss of valuable collateral.

2.E  Central Bank Intervention

2.E.1  Proof of Proposition 10

Fix an \( m \in \mathbb{R} \). \( m > 0 \) stands for cash supplied by the central bank in exchange for collateral, and \( m < 0 \) stands for cash withdrawn by the central bank.\(^{14}\) I prove the result for \( m > 0 \). The case for \( m < 0 \) is exactly symmetric. Write \( 1 + r = X, \frac{1}{1-h} = Z \),

\(^{14}\)Assume that A starts off with an endowment of cash which he invests in the project.
$1 + \tilde{r} = \tilde{X}$ and $\frac{1}{1-h} = \tilde{Z}$. The total expected output at $t = 2$ can be written as

$$Y = \frac{\text{A's cons.}}{\Omega + R_AI_A - Xz_{A,B}^A - (1 + r^{CB})m + T_A}
+ \frac{\text{B's cons.}}{R_BI_B - \tilde{X}I_B + Xz_{A,B}^A + 1 - z_{A,B}^A + T_B + \tilde{X}I_B - I_B + T_C},$$

where $T_A$, $T_B$ and $T_C$ are the lump sum rebates from the central bank, and $x_A = x_A^R + m$ is the sum of the amounts borrowed from B and the central bank respectively. Since the central bank rebates all profits lump sum,

$$T_A + T_B + T_C = r^{CB}m.$$

Thus, modulo a constant, the total expected output can be written as

$$Y = (R_A - 1)I_A + (R_B - 1)I_B.$$

Now in equilibrium, for small enough $|m|$, 

$$x_A = \begin{cases} 
1 + m & \text{if } \tilde{R} = 0, \\
1 + m & \text{if } (0, \tilde{R}^*), \\
(\Omega - \Delta\Omega)(1 - h) + m & \text{if } \tilde{R} = \tilde{R}^*,
\end{cases}$$

and

$$\bar{x}_B = \begin{cases} 
\frac{R_A}{1 - \kappa R_B} & \text{if } \tilde{R} = 0, \\
(\Omega - \Delta\Omega)d & \text{if } \tilde{R} \in (0, \tilde{R}^*), \\
(\Omega - \Delta\Omega)d & \text{if } \tilde{R} = \tilde{R}^*.
\end{cases}$$

An explanation for these values is in order. In the first region, when the initial endowment of collateral is large, $\tilde{R} = 0$ and A gives away idle collateral to the central bank. The investment into A's project increases by $m$. The investment into B's project is at its highest level possible given the limited pledgeability of B, and is unaffected. In the intermediate region, the investment into B's project increases by $m$, and the investment into B's project falls by $\Delta\Omega d$, since the amount collateral lent by A to B falls by $\Delta\Omega$. Finally, in the region where the initial endowment of collateral is too small, the investment into A's project increases by $m$, but decreases by $\Delta\Omega(1 - h)$ as A substitutes to
borrowing from the central bank. I now look at the three regions in turn.

Case I: $\tilde{R} = 0$.

I can write

$$Y = (R_A - 1) + (R_B - 1) \frac{R_A}{1 - \kappa R_B} + (R_A - 1)m,$$

which gives

$$\frac{\partial Y}{\partial m} \bigg|_{m=0} = R_A - 1.$$

Case II: $\tilde{R} \in (0, \tilde{R}^*)$.

I can write

$$Y = (R_A - 1) + (R_B - 1)\Omega + (R_A - 1)m - (R_B - 1)\Delta\Omega.$$

Differentiating w.r.t. $m$,

$$\frac{\partial Y}{\partial m} = (R_A - 1) - (R_B - 1)\frac{\partial \Delta\Omega}{\partial m}.$$

Now from Equation (2.15), $\Delta\Omega = m\frac{1}{1-hCB}$, which gives

$$\frac{\partial \Delta\Omega}{\partial m} = \frac{1}{1-hCB} + m\frac{\partial}{\partial m} \frac{1}{1-hCB}.$$

Evaluating the result at $m = 0$,

$$\frac{\partial Y}{\partial m} \bigg|_{m=0} = (R_A - 1) - (R_B - 1)\frac{1}{1-hCB}.$$

Since $\frac{1}{1-hCB} = \frac{R_A}{1+\tilde{R}}$, we can have $\frac{\partial Y}{\partial m} \bigg|_{m=0} < 0$ for $\tilde{R} \in (0, \tilde{R}^*)$, where $\tilde{R}^* = \frac{R_B-1}{R_A-1}R_A - 1$.

Case III: $\tilde{R} = \tilde{R}^*$.

I can write

$$Y = (R_A - 1)\Omega(1-h) - (R_A - 1)\Delta\Omega(1-h) + (R_A - 1)m + (R_B - 1)\Omega - (R_B - 1)\Delta\Omega.$$
In this region, \( h \) is constant. This gives

\[
\frac{\partial Y}{\partial m}
\bigg|_{m=0} = (R_A - 1) - (R_B - 1) \frac{1}{1 - h^{CB}} - (R_A - 1)(1 - h) \frac{1}{1 - h^{CB}}.
\]

### 2.E.2 Allowing Reserves to be Pledged

The discussion so far has assumed that \( A \) invests the cash received from the central bank. It has implicitly assumed that the cash (reserves) received from the Central Bank is not pledgeable to \( B \). Pledging of the reserves as collateral in order to borrow cash effectively involves lending the haircut to \( B \). The assumption that this is not allowed is reasonable since the mandate of hedge funds generally does not allow this type of direct unsecured lending to intermediaries like prime brokers, even though it is effectively achieved indirectly through rehypothecation. However, I show that even after relaxing this assumption, the result about the possible fall in expected output still holds.

The cash (reserves) and the collateral are equivalent for \( B \). \( B \)'s demand curve for "collateral", which could either be the original collateral asset or the reserves from the central bank, remains unchanged. However, for \( A \), the cash and the collateral are not equivalent. \( A \) is forced to go to \( B \) and borrow cash against collateral to invest in his project. On receiving the cash from the central bank, \( A \) can either invest it in his project, or borrow against it from \( B \). The net return from investing is \( R_A - 1 \), and the net return from using it as collateral to borrow from \( B \) is \( \bar{R} \). Thus, when \( \bar{R} < R_A - 1 \), \( A \) will invest the cash in his project, and when \( \bar{R} \geq R_A - 1 \), \( A \) will borrow against it from \( B \). In view of this, the indifference Equation (2.16) gets modified to

\[
(R_A - (1 + r^{CB}))(1 - h^{CB}) = \bar{R}, \text{ if } \bar{R} < R_A - 1,
\]

\[
(1 + \bar{R} - (1 + r^{CB}))(1 - h^{CB}) = \bar{R}, \text{ if } \bar{R} \geq R_A - 1.
\]

In each case, the right hand side represents the net return to \( A \) from borrowing from \( B \) and investing the proceeds. In the first case, the left hand side represents the net return to \( A \) from borrowing from the central bank and investing the proceeds. In the second case, the left hand side represents the net return to \( A \) from borrowing from the central bank and then borrowing against the reserves from \( B \).

Thus, when \( \bar{R} \geq R_A - 1 \), \( r^{CB} = h^{CB} = 0 \). The central bank exchanges the collateral for cash one for one, and \( A \) extends the cash to \( B \), leaving the equilibrium unchanged.
The central bank’s intervention is neutral when $\bar{R} \geq R_A - 1$.

But when $\bar{R} < R_A - 1$, A finds it more profitable to invest the cash in the project. The situation in Section 2.4 holds, and so do Proposition 10 and Corollary 4. $\frac{\partial Y}{\partial m} \bigg|_{m=0} < 0$ for $\bar{R} < \bar{R}^\#$. Since, $\bar{R}^\# = \frac{R_B}{R_A - 1} R_A - 1 > R_A - 1$ for $R_B > R_A$, this is true for all $\bar{R} < R_A - 1$.

When $\bar{R}$ is sufficiently small, A is already lending collateral to B to the binding pledgeability limit. Then A does not get a good enough return from lending a marginal amount to B, and instead chooses to invest in the project.
Bibliography


Chapter 3

Identifying the Push Impact of Foreign Institutional Investor Flows on Indian Equity Prices

with Ajay Shah

3.1 Introduction

"...even a measured pace of exit may cause severe market turbulence and collateral damage."

In the field of international financial flows with emerging markets, an important question concerns the causal impact of foreign order flow on stock prices in an emerging market. As is evident from the quote above, policy makers have long expressed concerns about the extent to which foreign investors might withdraw capital on a large scale: this could either worsen a domestic crisis, or act as a vector for crisis transmission. A related set of concerns arises about “big fish in a small pond”, where portfolio rebalancing by global investors can kick off large changes in asset prices in emerging markets. These concerns are implicitly grounded in the belief that foreign order flow has an economically significant causal impact upon stock prices.

There are two possibilities. On the one hand, foreign investors might recklessly send large orders into relatively illiquid emerging markets, and kick off contemporar-
eous price distortions. Alternatively, foreign investors might be mindful of the cost of transacting, and be careful in order placement. As an example, Patnaik et al. (2015) suggest that foreign investors are mindful of the cross-sectional variation in liquidity: changes in global risk conditions are largely expressed through orders in the most liquid securities. To the extent that foreign investors are mindful about market illiquidity, and place orders which the market is able to absorb with low price impact, the causal impact of order flow upon prices would be small.

It is not easy to assess whether foreign order flow has a causal impact upon local stock prices, as good news impacts upon foreign orders and upon local stock prices. This gives an apparent correlation between foreign orders and stock market returns. One class of papers has analysed reduced form effects, e.g. Patnaik et al. (2013); Stigler et al. (2010). A recent strand of the literature has identified causal effects Ülkü and Weber (2013b,a) by utilising identification through heteroskedasticity in a VAR setting. In this paper, we propose an instrumental variables approach to this question.

Traditional theories of frictionless asset pricing, such as the CAPM and other factor pricing theories, as well as consumption-based asset pricing theories, posit that that the price of an asset is determined by (i) the expectations of future payoffs, and (ii) a pricing factor or a stochastic discount factor. Keeping the stochastic discount factor constant, the quantity of an asset does not affect its price since the optimising consumers’ marginal rate of substitution between current and future consumption is equalised by the equilibrium price. The demand curve for an asset is, therefore, horizontal. The presence of transaction costs (“illiquidity”) can make the demand curve downward sloping, so that a decrease in the supply will increase the price, ceteris paribus. From a theoretical perspective, our exercise will effectively estimate the slope of the demand curve for Indian equities in which FIIs are active.

We assume a CAPM-style asset pricing equation with net FII inflows as an additional term. The coefficient on this term will give us the required impact of flows on the price over and above the change due to the aggregate market factor. Since the foreign flows into the stock of a firm will also be correlated with its idiosyncratic information, we will need to instrument for the flows with a foreign factor that generates exogenous variation in the flows. Our candidate instruments are the S&P 500 VIX index and the variable FIFA from Jotikasthira, Lundblad and Ramadorai (2012). FIFA is a generated variable that is a measure of the likely flows into the country when U.S. mutual funds rescale their portfolios after infusion or withdrawal of capital by their investors. Both of these represent exogenous shocks to the U.S. economy and are unlikely to be caused by
Indian conditions. We restrict attention to Indian firms with low internationalisation so that these variables do not directly convey idiosyncratic information about them.

Our results show that flows are very strongly correlated with returns, and OLS estimates give a misleading picture of a large impact of flows on prices. When we instrument for the flows with our exogenous variables, however, we find that flows have no impact on prices, over and above the impact due to the aggregate market factor. We also find that there is no differential impact on the prices of the firms which see the flows, compared to the prices of the firms which do not. The results are robust to the way the “FII-active” firms are selected for analysis, and to the inclusion of the Fama-French portfolios as factors in addition to the market risk premium.

The results indicate that the correlation between prices and flows is driven by aggregate or idiosyncratic information about Indian firms. Our identification strategy, however, cannot identify the causal price effects of the flows that work through the aggregate stochastic discount factor. Indeed, the foreign shocks showing up in the VIX or FIFA may be highly correlated with the returns on the aggregate Indian equity market, and may indirectly impact the stock prices of all firms. But the crux of our results is that they will not disproportionately impact the prices of those firms which actually see the flows.

The results are consistent with a frictionless view of the market. They may also indicate that FII’s are usually active in widely held and liquid stocks with low transaction costs. This interpretation points to a certain degree of perspicacity on the part of the foreign investors, who may not a priori buy into stocks which will be costly to exit from.

The paper is organised as follows. Section 3.2 describes the previous work on the relationship between flows and prices and contrasts it with our approach. It also discusses the instruments we use. Section 3.3 describes the sources of our data. Section 3.4 describes our main results. Section 3.5 performs some robustness checks on the main analysis. Section 3.6 concludes.

### 3.2 Estimation Approach

#### 3.2.1 Previous Work

Previous empirical work on the relationship between foreign portfolio flows and stock prices has mainly focussed on the correlations between them. Patnaik et al. (2013)
conduct an event study on days with extreme movements in the Indian NIFTY index or extreme net flows into India, and find a strong correlation between flows and stock returns. The correlation is consistent with both the price-pressure and the information theories. No causal inference can be drawn from the results. Gordon and Gupta (2003) find that FII flows are negatively related to lagged stock market returns, an evidence of negative feedback trading. Acharya et al. (2014) use firm-level FII flow data and look at the differential returns to stocks facing inflows. They find significant cumulative abnormal differential returns to stocks facing high FII flows. However, again, their results are consistent with both the information and the price-pressure stories, and no causal inference can be drawn.

A few papers look for causality from flows to prices, but the work suffers from the key issue that certain global shocks can directly convey information about the more internationalised firms, and the causal inference is diluted. Chakrabarti (2001) tests for Granger causality using a VAR and finds causality from flows to prices during the Asian financial crisis, and a possible reverse causality after the crisis. Mukherjee et al. (2002) find that only sales by foreign portfolio investors affect prices. Anshuman et al. (2010) find that the FII activity dampens the volatility of the aggregate stock market, which is evidence for liquidity provision by FII's. Stigler et al. (2010) test for Granger causality in a VAR which also includes exchange rates, and find causality from stock returns to flows. Froot and Ramadorai (2008) look at the forecasting power of portfolio flows for stock prices and find evidence of the information story, but not the price-pressure story. On the other hand, Choe et al. (1999) find that Korean stock market data during the Asian financial crisis is more consistent with the price-pressure story.

More recently, Ülkii and Weber (2013b) undertake a concerted exercise in identification of the push and pull effects by using the heteroskedasticity-based identification methodology of Rigobon (2003). They use daily data for the Korean stock exchange and find evidence of both the information and the price pressure stories. However, this exercise also suffers from the problem that certain global shocks could directly convey information about local firms, in which case the Rigobon (2003)-identification is inconsistent.
3.2.2 Our Approach

Our approach differs from the previous work on the relationship between flows and prices in that we use exogenous variation in the flows to identify the “push” impact of flows on prices. Shleifer (1986) conducts an event study in which he uses an exogenous event—the inclusion of a stock in an index—to identify the impact of the subsequent bulk purchases on prices, and finds evidence for the price-pressure story. Hau et al. (2010) perform a similar exercise for currencies and find strong price effects of sales and purchases. Lakonishok et al. (1992) try to identify the price impact of the exogenous herding policies of institutional investors, and find mixed evidence.

Our approach is similar to a demand estimation methodology. Theories of asset pricing provide pricing formulas that relate the price of an asset to the stochastic discount factor and the expected payoffs of the asset. With frictionless markets, the quantity of the asset purchased does not affect its price, and the demand curve for the asset is horizontal. The stochastic discount factor and the expected payoff of the asset serve as demand shifters. Frictions such as transaction costs may lead to the demand curve being downward sloping. Our methodology will effectively estimate the slope of the demand curve.

The supply of a domestic asset is the total stock of it available net of the holdings by foreign investors. The shifters of this supply are the foreign stochastic discount factor and the expected payoffs. Trying to directly estimate the slope of the demand curve of a stock of a firm by OLS will confound the movements of the demand and supply curves due to information about the firm. The OLS estimates, thus, will be biased and inconsistent. As in traditional demand estimation methods, our approach will be to instrument for the flows (quantities) with the exogenous variation in the supply curve which is provided by the foreign stochastic discount factor. We will rely heavily on the assumption that the foreign stochastic discount factor is orthogonal to idiosyncratic information about the firms, and we will select the set of firms for our estimation appropriately.

Formally, we assume the demand curve for the stock of a firm $i$ to be given by a CAPM-style factor pricing model

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_i (R_{m,t} - R_{f,t}) + \gamma F_{i,t} + \varepsilon_{i,t},$$

(3.1)

where $R_{i,t}$ is the net return on the price of the stock at time $t$ over $t - 1$, $R_{f,t}$ is the
risk-free rate between \( t \) and \( t - 1 \), \( R_{m,t} \) is the return on the market as a whole, the risk premium on which we take to be the stochastic discount factor, and \( \varepsilon_{i,t} \) is the idiosyncratic information received at \( t \) about the future payoff of the stock, determined by the profitability of the firm. \( F_{i,t} \) is the net purchases of shares of firm \( i \) by foreign institutional investors normalised by the total number of outstanding shares. Traditional asset pricing theories such as the Arbitrage Pricing Theory by Ross (1976) do not include this term since they assume away all frictions.

Our interpretation of the model (3.1) is as follows. The stock price of firm grows at a constant trend rate \( \alpha_i \) above the risk-free rate. For a 1% market return, the stock price increases by \( \beta_i \% \). This can be interpreted as the way the aggregate state of the economy—the economy-wide conditions and expectations of domestic aggregate demand—affect the profitability of firm \( i \). \( \varepsilon_{i,t} \) represents idiosyncratic information about the firm which is orthogonal to the aggregate innovations in the economy. Finally, frictions will impact the stock price by \( \gamma \) per unit sale or purchase. For simplicity, we assume \( \gamma \) to be the same for all stocks.

Our coefficient of interest is \( \gamma \). A finding of \( \gamma = 0 \) will mean that there is no “push” impact of FII flows on prices. This will be consistent with the frictionless view of markets with horizontal demand curves. On the other hand \( \gamma > 0 \) would mean the presence of frictions such as large transaction costs in the market. It could also mean that foreign investors are “big fish in a small pond”, and that the concerns of policymakers about “hot money” are justified.

3.2.3 The Instrument

To estimate the slope of the demand curve, we need a proxy for the supply shifter—the foreign stochastic discount factor. We consider two candidates: the S&P 500 VIX index and the variable FIFA from Jotikasthira, Lundblad and Ramadorai (2012) which represents forced flows into the Indian stock market due to fire sales and purchases by US mutual funds when their investors pull out or put in capital.

The S&P 500 VIX index is calculated as the weighted average of the implied volatilities of options on the S&P 500 index. It represents a measure of apprehension about the U.S. markets. When the VIX is high, investors in the U.S. follow a “risk-off” approach and desist from investing in what they perceive as risky emerging markets, instead moving their cash to safer assets like U.S. treasuries. When the VIX is low, U.S. investors are willing to take “risk-on” and invest in emerging markets.
The variable FIFA at week $t$ is calculated by Jotikasthira, Lundblad and Ramadorai (2012) as

$$\text{FIFA}_t = \sum_{\text{Funds investing globally}} \text{FundDollarExposureToIndia}_{t-1} \times \text{FundPercentCapitalChange}_t,$$

with the summation over funds domiciled in the US and Europe which invest in developed and emerging markets across the world. This variable represents the likely forced flows into India due to changes in funds’ capital if the funds simply scale their between-country portfolios up or down in response to actions by their investors. They have two series: a raw version which is computed as above and represents the dollar value of the likely flows, and another version which is normalised by the total value of the country’s stock market.

Each of these variables represents a shock to the US economy. We need to ensure that these variables are uncorrelated with idiosyncratic information about Indian firms. India forms a minuscule fraction of the international portfolio holdings of the United States. The figures in Treasury (2010) indicate that the weight of India in the foreign portfolio of the US is less than 1%. It is, therefore, unlikely that either aggregate information about India or idiosyncratic information about Indian firms will affect the U.S. economy significantly to produce a shock to either the VIX or the FIFA. For our purposes, it suffices that idiosyncratic shocks to Indian firms in which FIIs are active do not affect the VIX or the FIFA.

Conversely, for VIX or FIFA to be valid as instruments, they should also not generate idiosyncratic information about Indian firms. We, therefore, restrict attention to non-financial Indian firms with low internationalisation, i.e. low exports and imports. This will exclude, for example, large IT firms catering to the export market, as well as petroleum companies dependent on imports of oil. Restricting attention to firms with low internationalisation will ensure that the foreign shock will only generate information about them indirectly through aggregate effects on the Indian economy. A shock to VIX or FIFA will not generate idiosyncratic information about these firms.

3.3 Data

We make use of a unique dataset on daily firm-level FII flows maintained by the Securities and Exchanges Board of India (SEBI). FIIs are required to settle their trades
Figure 3-1: Weekly returns on the COSPI index and weekly net inflows by foreign institutional investors (FII’s) into COSPI firms.

through custodian banks, who are required to report them to SEBI (Shah and Patnaik, 2011). The raw data for the net FII flows is in rupees. We normalise it by the market capitalisation of the firm at the start of the day. We consider 2359 of the most important Indian firms from the CMIE COSPI index.

The daily stock price and market capitalisation data for these firms is obtained from the CMIE Prowess database. Figure 3-1 shows the weekly net FII inflows and the weekly market capitalisation weighted stock price returns for the COSPI firms. Figure 3-2 shows a scatter plot of the weekly flows and stock price returns of the COSPI index. The figure shows clear evidence of a strong correlation between flows and returns.

We obtain the weekly series for FIFA from Jotikasthira, Lundblad and Ramadorai (2012). We use the version of their series that is normalised by the country market capitalisation. Figure 3-3 shows the weekly series for FIFA. The weekly net FII inflows
are also shown for comparison. It is clear that the likely flows induced by the changing capital base of U.S. mutual funds are very strongly correlated with the actual flows.

We obtain daily data for the risk free rate, the realised market return, and the returns on the Fama and French (1993) portfolios SMB and HML for India from Agarwala et al. (2013). They compute market returns as the market-cap weighted average of the returns to all COSPI firms after excluding some illiquid firms. We also use publicly available data on the VIX index.

Our sample period for the daily regressions is 1 April 2003 through 31 March 2013. Since the series for FIFA ends in 2009, our sample period for the weekly regressions is 1 April 2003 through 31 March 2009.

Figure 3-2: Correlation between flows and returns. $\beta = 0.33^{***}(0.02)$
3.4 Results

3.4.1 Baseline Regressions

With firm-level FII flow data in hand, we would ideally have liked to estimate the coefficient $\gamma$ in Equation (3.1) with a panel of the firms, instrumenting for the endogenous flows. However, both of our instruments, FIFA and VIX, are aggregate variables, and are only weakly correlated with the flows for individual firms. This is because when a shock hits US markets, idiosyncratic conditions ensure that FII's do not simply scale their within-country portfolio up or down. They may buy or sell some stocks more than others.
We therefore focus on an aggregate of non-financial firms, and estimate

\[ R_t - R_{f,t} = \alpha + \beta (R_{m,t} - R_{f,t}) + \gamma F_t + \epsilon_t, \quad (3.2) \]

where \( R_t \) is the market-capitalisation weighted return on an aggregate set of firms, and \( F_t \) is the net FII inflow into them normalised by the market capitalisation. While US shocks may not be strongly correlated with flows into individual firms, they are strongly correlated with the average flows into an aggregate of firms. The reason for this is that many funds are mandated to hold close to a certain fixed portfolio weight in a country. As a result, a shock in the US translates roughly into a scaling up or down of the between-country portfolio by these funds. Jotikasthira, Lundblad and Ramadorai (2012) also provide empirical evidence for the general increase or decrease of country positions in response to capital inflows or outflows.

To make the estimation more relevant, we focus on an aggregate of firms which see high FII activity. We sort the firms by the volatility of the net FII flows into them over the sample period. We pick the firms with the standard deviation \( \sigma_t^{FII} \) of net FII flows in the top 100q%. From among these firms, we need to exclude firms with low internationalisation as described above. For each year, we construct the market-cap weighted \( R_t \) and \( F_t \) by including only those firms with gross trade (exports plus imports) less than \( \theta = 5\% \) of sales in that year. Formally, for each year \( \tau \), we construct a set

\[ Z_\tau = \left\{ i \mid \frac{X_{i,\tau} + M_{i,\tau}}{S_{i,\tau}} < \theta \right\} \cap \{ i \mid \sigma_t^{FII} \text{ is in the top 100q}\% \}, \]

where \( X_{i,\tau}, M_{i,\tau} \) and \( S_{i,\tau} \) are the exports, imports and sales of firm \( i \) in year \( \tau \). With such a set for each year, we construct

\[ R_t = \sum_{i \in Z_\tau} w_{i,t} R_{i,t}, \text{ when day or week } t \text{ is in year } \tau, \text{ and} \]

\[ F_t = \sum_{i \in Z_\tau} w_{i,t} F_{i,t}, \text{ when day or week } t \text{ is in year } \tau, \]

where \( w_{i,t} \) are the start of the day or week market-cap weights for firm \( i \). Since this aggregate of firms contains a small subset of the universe of Indian firms, although diversified, we are not concerned about the possible equality of the variables on the left and right sides of Equation (3.2).

While constructing the set of firms with low internationalisation, we bear a caveat
in mind. Our set may include firms which do not themselves export, but produce tradable goods which are strong substitutes for those that are actually exported. It may also include firms which do not import, but locally source tradable goods which are substitutes for imports. A foreign shock will be a direct signal of the idiosyncratic profitability of such firms. However, we believe the bias introduced by this to be small since we are considering an aggregate of firms. The positive impact of a favourable foreign shock on firms producing substitutes of exports will cancel out its negative impact on those sourcing substitutes of imports.

We estimate Equation (3.2) for these aggregates. We run two sets of regressions, one with weekly data and the weekly FIFA as the instrument, and the other with daily data and the change in the VIX over the previous business day as the instrument. We do not use the weekly changes in the VIX as an instrument in the weekly estimations since it turns out to be a very weak instrument. With time series data and the possibility of heteroskedastic and autocorrelated errors, we compute the Heteroskedasticity and Autocorrelation Consistent standard errors using the Bartlett kernel. The optimal lag length is obtained using the automatic lag selection algorithm of Newey and West (1994).

The top panel of Table 3.1 shows the OLS regression of Equation (3.2) for different values of \( q \) with weekly data. The average number of firms each year in the index is 44, 54, 69 and 83 for \( q = 0.15, 0.20, 0.25, 0.30 \) respectively, and are a diversified set of large firms with representations from all the (non-financial) NIC Sections. The sample period is 1 April 2003 to 31 March 2009. The strong correlation between flows and excess returns is clearly visible even after controlling for the market factor. The estimate of the coefficient is, however, inconsistent.

The bottom panel of Table 3.1 shows the just-identified IV regression of Equation (3.2) with the weekly FIFA as the instrument. The coefficient on the net FII flows is very small in magnitude and not significantly different from zero. Thus, flows seem to have no impact on stock prices. The table also lists the \( F \)-statistics from the first stage of the IV regressions, which show that the instrument is highly relevant. The full first-stage estimation results are in Table 3.5 in Appendix 3.6. A forced flow of 1 percent into India causes an actual flow of 1.5 to 1.8 basis points into the firms under consideration.

\(^{1}\)Indian firms are classified into 5-digit National Industrial Classification (NIC) codes. The first two digits are further classified into alphabetical Sections A-U. For example, Section C representing manufacturing encapsulates codes the first two digits from 10 to 33. (http://mospi.nic.in/Mospi_New/upload/nic_2008_17apr09.pdf)
### Baseline Weekly OLS:

<table>
<thead>
<tr>
<th>Dependent Variable: $R_t - R_{f,t}$</th>
<th>q = 0.15</th>
<th>q = 0.20</th>
<th>q = 0.25</th>
<th>q = 0.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>$-0.31$</td>
<td>$-0.40^*$</td>
<td>$-0.31^*$</td>
<td>$-0.25$</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.19)</td>
<td>(0.16)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>$R_{m,t} - R_{f,t}$</td>
<td>$1.05^{***}$</td>
<td>$1.17^{***}$</td>
<td>$1.21^{***}$</td>
<td>$1.16^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.06)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>$F_t$</td>
<td>$0.06^{***}$</td>
<td>$0.09^{***}$</td>
<td>$0.07^{***}$</td>
<td>$0.07^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.73</td>
<td>0.78</td>
<td>0.80</td>
<td>0.81</td>
</tr>
<tr>
<td>Num. obs.</td>
<td>313</td>
<td>313</td>
<td>313</td>
<td>313</td>
</tr>
</tbody>
</table>

### Baseline Weekly IV:

<table>
<thead>
<tr>
<th>Dependent Variable: $R_t - R_{f,t}$</th>
<th>q = 0.15</th>
<th>q = 0.20</th>
<th>q = 0.25</th>
<th>q = 0.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>$-0.14$</td>
<td>$0.03$</td>
<td>$-0.03$</td>
<td>$-0.03$</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(0.22)</td>
<td>(0.20)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>$R_{m,t} - R_{f,t}$</td>
<td>$1.11^{***}$</td>
<td>$1.33^{***}$</td>
<td>$1.32^{***}$</td>
<td>$1.25^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.15)</td>
<td>(0.12)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>$F_t$</td>
<td>$-0.04$</td>
<td>$-0.01$</td>
<td>$-0.02$</td>
<td>$-0.01$</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.05)</td>
<td>(0.04)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Num. obs.</td>
<td>313</td>
<td>313</td>
<td>313</td>
<td>313</td>
</tr>
<tr>
<td>First-stage $F$</td>
<td>$44.59^{***}$</td>
<td>$33.99^{***}$</td>
<td>$37.96^{***}$</td>
<td>$45.38^{***}$</td>
</tr>
<tr>
<td>Wu-Hausman $F$</td>
<td>$0.33$</td>
<td>$4.03^*$</td>
<td>$4.36^*$</td>
<td>$4.61^*$</td>
</tr>
</tbody>
</table>

|                                          | $***p < 0.001$, $**p < 0.01$, $*p < 0.05$ |

Table 3.1: Estimation of the basic model (3.2) with weekly data. IV estimation with FIFAt as instrument. Sample period: 1 April 2003 to 31 March 2009. HAC standard errors. All net returns in percent and flows in basis points.

The Wu-Hausman $F$-statistics for the test of endogeneity of flows (equivalently, for the test of equality of the OLS and IV estimates), also significantly reject exogeneity for $q = 0.20, 0.25, 0.30$. The coefficient on the market factor represents the average $\beta$ of the firms under consideration. A high $\beta$ is consistent with the observation in Patnaik and Shah (2013) about the FII's choice of stocks.

The top panel of Table 3.2 shows the OLS regressions of Equation (3.2) with daily data for different values of $q$. The average number of firms each year in the index is
Baseline Daily OLS:

<table>
<thead>
<tr>
<th>Dependent Variable: $R_t - R_{f,t}$</th>
<th>$q = 0.15$</th>
<th>$q = 0.20$</th>
<th>$q = 0.25$</th>
<th>$q = 0.30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>$-0.08^*$</td>
<td>$-0.08^*$</td>
<td>$-0.08^{**}$</td>
<td>$-0.09^{***}$</td>
</tr>
<tr>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td></td>
</tr>
<tr>
<td>$R_{m,t} - R_{f,t}$</td>
<td>$1.23^{***}$</td>
<td>$1.21^{***}$</td>
<td>$1.17^{***}$</td>
<td>$1.15^{***}$</td>
</tr>
<tr>
<td>(0.04)</td>
<td>(0.07)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td></td>
</tr>
<tr>
<td>$F_t$</td>
<td>$0.07^{***}$</td>
<td>$0.09^{***}$</td>
<td>$0.08^{***}$</td>
<td>$0.09^{***}$</td>
</tr>
<tr>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.73</td>
<td>0.77</td>
<td>0.78</td>
<td>0.80</td>
</tr>
<tr>
<td>Num. obs.</td>
<td>2407</td>
<td>2407</td>
<td>2407</td>
<td>2407</td>
</tr>
</tbody>
</table>

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$

Baseline Daily IV:

<table>
<thead>
<tr>
<th>Dependent Variable: $R_t - R_{f,t}$</th>
<th>$q = 0.15$</th>
<th>$q = 0.20$</th>
<th>$q = 0.25$</th>
<th>$q = 0.30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.06</td>
<td>0.05</td>
<td>0.05</td>
<td>$-0.01$</td>
</tr>
<tr>
<td>(0.07)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>$R_{m,t} - R_{f,t}$</td>
<td>$1.33^{***}$</td>
<td>$1.30^{***}$</td>
<td>$1.26^{***}$</td>
<td>$1.21^{***}$</td>
</tr>
<tr>
<td>(0.08)</td>
<td>(0.07)</td>
<td>(0.06)</td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>$F_t$</td>
<td>$-0.04$</td>
<td>$-0.06$</td>
<td>$-0.05$</td>
<td>$-0.03$</td>
</tr>
<tr>
<td>(0.06)</td>
<td>(0.07)</td>
<td>(0.06)</td>
<td>(0.05)</td>
<td></td>
</tr>
<tr>
<td>Num. obs.</td>
<td>2407</td>
<td>2407</td>
<td>2407</td>
<td>2407</td>
</tr>
<tr>
<td>First-stage $F$</td>
<td>43.69^{***}</td>
<td>47.84^{***}</td>
<td>47.54^{***}</td>
<td>39.73^{***}</td>
</tr>
<tr>
<td>Wu-Hausman $F$</td>
<td>7.24^{**}</td>
<td>8.56^{**}</td>
<td>8.42^{**}</td>
<td>7.46^{**}</td>
</tr>
</tbody>
</table>

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$

Table 3.2: Estimation of the basic model (3.2) with daily data. IV estimation with $\Delta VIX_{t-1}$ as instrument. Sample period: 1 April 2003 to 31 March 2013. HAC standard errors. All net returns in percent and flows in basis points.

39, 57, 77 and 89 for $q = 0.15, 0.20, 0.25, 0.30$ respectively, and are a diversified set of large firms with representations from all the (non-financial) NIC Sections. The sample period is 1 April 2003 to 31 March 2013. Again, we can see a strong correlation between flows and daily returns after correcting for the market factor.

The bottom panel of Table 3.2 shows the just-identified IV regressions of Equation (3.2) with the change in the US VIX index over the previous business day as the instrument. As in the weekly case, the coefficient on the FII flows is again small in
magnitude and insignificantly different from zero. The strong relevance of the change in VIX as the instrument is borne out by the high first-stage $F$-statistics. The full first-stage estimation results are in Table 3.6 in Appendix 3.6. A rise of 1 point in the VIX index causes an outflow of about 0.3 basis points. The equality of the OLS and the IV estimates is also strongly rejected by the Wu-Hausman test. The high market $\beta$'s are consistent with the evidence in Patnaik and Shah (2013).

3.4.2 Difference Regressions

In this subsection, we consider an alternative specification and see whether the prices of stocks in which FII's are active respond to flows into them any more than the prices of stocks without any FII activity. In particular, we take the difference of Equation (3.2):

$$\Delta R_t = \Delta \alpha + \Delta \beta (R_{m,t} - R_{f,t}) + \gamma \Delta F_t + \Delta \varepsilon_t, \tag{3.3}$$

where

$$\Delta R_t = R_t^h - R_t^l,$$
$$\Delta \alpha = \alpha^h - \alpha^l,$$
$$\Delta \beta = \beta^h - \beta^l,$$
$$\Delta F_t = F_t^h - F_t^l,$$ and
$$\Delta \varepsilon_t = \varepsilon_t^h - \varepsilon_t^l,$$

and the superscripts $h$ and $l$ refer to market-cap weighted averages across sets of firms with high and low FII activity respectively. We construct the high FII activity sets exactly as in the baseline exercise with $q = 0.15, 0.20, 0.25, 0.30$. For the low FII activity set, we simply take the low internationalisation firms which do not see any FII activity over the sample period, i.e. $\sigma_t^F = 0$.

The top panel of Table 3.3 shows the OLS regressions of the difference Equation (3.3) with daily data. The sample period is 1 April 2003 to 31 March 2009. Again, the coefficients on the differential flows seem to suggest that firms which face flows see larger changes in their prices. However, these coefficients are inconsistent.

The bottom panel of Table 3.3 shows the just-identified IV regression of Equation (3.3) with the weekly FIFA as the instrument for the endogenous differential flows. Again, the coefficient on the differential net FII flows is very small in magnitude and
### Difference Weekly OLS:

<table>
<thead>
<tr>
<th>Dependent Variable: $\Delta R_t$</th>
<th>$q = 0.15$</th>
<th>$q = 0.20$</th>
<th>$q = 0.25$</th>
<th>$q = 0.30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>$-0.56$</td>
<td>$-0.65^*$</td>
<td>$-0.54^*$</td>
<td>$-0.50$</td>
</tr>
<tr>
<td></td>
<td>$(0.33)$</td>
<td>$(0.32)$</td>
<td>$(0.26)$</td>
<td>$(0.26)$</td>
</tr>
<tr>
<td>$R_{m,t} - R_{f,t}$</td>
<td>$0.06$</td>
<td>$0.17^*$</td>
<td>$0.22^*$</td>
<td>$0.11$</td>
</tr>
<tr>
<td></td>
<td>$(0.10)$</td>
<td>$(0.08)$</td>
<td>$(0.08)$</td>
<td>$(0.10)$</td>
</tr>
<tr>
<td>$\Delta F_t$</td>
<td>$0.07^*$</td>
<td>$0.10^{***}$</td>
<td>$0.07^{**}$</td>
<td>$0.07^{**}$</td>
</tr>
<tr>
<td></td>
<td>$(0.03)$</td>
<td>$(0.03)$</td>
<td>$(0.02)$</td>
<td>$(0.02)$</td>
</tr>
<tr>
<td>$R^2$</td>
<td>$0.09$</td>
<td>$0.13$</td>
<td>$0.10$</td>
<td>$0.04$</td>
</tr>
<tr>
<td>Num. obs.</td>
<td>313</td>
<td>313</td>
<td>313</td>
<td>313</td>
</tr>
</tbody>
</table>

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$

### Difference Weekly IV:

<table>
<thead>
<tr>
<th>Dependent Variable: $\Delta R_t$</th>
<th>$q = 0.15$</th>
<th>$q = 0.20$</th>
<th>$q = 0.25$</th>
<th>$q = 0.30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>$-0.16$</td>
<td>$-0.09$</td>
<td>$-0.11$</td>
<td>$-0.18$</td>
</tr>
<tr>
<td></td>
<td>$(0.42)$</td>
<td>$(0.41)$</td>
<td>$(0.37)$</td>
<td>$(0.33)$</td>
</tr>
<tr>
<td>$R_{m,t} - R_{f,t}$</td>
<td>$0.18$</td>
<td>$0.40^*$</td>
<td>$0.39^{**}$</td>
<td>$0.23$</td>
</tr>
<tr>
<td></td>
<td>$(0.12)$</td>
<td>$(0.16)$</td>
<td>$(0.12)$</td>
<td>$(0.12)$</td>
</tr>
<tr>
<td>$\Delta F_t$</td>
<td>$0.00$</td>
<td>$-0.06$</td>
<td>$-0.06$</td>
<td>$-0.04$</td>
</tr>
<tr>
<td></td>
<td>$(0.05)$</td>
<td>$(0.09)$</td>
<td>$(0.08)$</td>
<td>$(0.07)$</td>
</tr>
<tr>
<td>Num. obs.</td>
<td>313</td>
<td>313</td>
<td>313</td>
<td>313</td>
</tr>
</tbody>
</table>

First-stage $F$ 44.59*** 33.99*** 37.96*** 45.40***

Wu-Hausman $F$ 1.28 4.45* 3.83* 2.20

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$

Table 3.3: Estimation of the difference model (3.3) with weekly data. IV estimation with FIFAt as instrument. Sample period: 1 April 2003 to 31 March 2009. HAC standard errors. All net returns in percent and flows in basis points.

not significantly different from zero. The $F$–statistics from the first stage of the IV regressions show that the instrument is highly relevant. The full first-stage estimation results are in Table 3.7 in Appendix 3.6. The Wu-Hausman $F$–statistics for the test of equality of the OLS and IV estimates also significantly reject the equality of the OLS and IV estimates for $q = 0.20, 0.25$. The coefficient on the market factor represents the difference in the average $\beta$ of the sets of firms under consideration. A positive and significant $\beta$ is consistent with the observation in Patnaik and Shah (2013) that FII's
Table 3.4: Estimation of the difference model (3.3) with daily data. IV estimation with ΔVIX_{t-1} as instrument. Sample period: 1 April 2003 to 31 March 2013. HAC standard errors. All net returns in percent and flows in basis points.

<table>
<thead>
<tr>
<th></th>
<th>Difference Daily OLS:</th>
<th></th>
<th>Difference Daily IV:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dependent Variable: ΔR_t</td>
<td></td>
<td>Dependent Variable: ΔR_t</td>
</tr>
<tr>
<td></td>
<td>q = 0.15</td>
<td>q = 0.20</td>
<td>q = 0.25</td>
</tr>
<tr>
<td>Constant</td>
<td>0.02</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.05)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>R_{m,t} - R_{f,t}</td>
<td>0.07</td>
<td>0.05</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.09)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>ΔF_t</td>
<td>0.07***</td>
<td>0.08***</td>
<td>0.09***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>R²</td>
<td>0.02</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>Num. obs.</td>
<td>2407</td>
<td>2407</td>
<td>2407</td>
</tr>
<tr>
<td>First-stage F</td>
<td>43.73***</td>
<td>46.79***</td>
<td>47.96***</td>
</tr>
<tr>
<td>Wu-Hausman F</td>
<td>4.88*</td>
<td>3.00</td>
<td>2.18</td>
</tr>
</tbody>
</table>

***p < 0.001, **p < 0.01, *p < 0.05

tend to pick riskier firms.

The top panel of Table 3.4 shows the OLS regressions of Equation (3.3) with daily data. The sample period is 1 April 2003 to 31 March 2013. Again, we can see a strong correlation between the differential flows and differential returns after correcting for the market factor.

The bottom panel of Table 3.4 shows the just-identified IV regressions of Equation (3.3) with the change in the US VIX index over the previous business day as the instru-
ment. The coefficient on the differential FII flows is negative, but is not significantly different from zero. The strong relevance of the change in VIX as the instrument is borne out by the high first-stage $F$–statistics. The full first-stage estimation results are in Table 3.8 in Appendix 3.6. Although the $F$–statistics for the Wu-Hausman test for equality of IV and OLS estimates are high (except for $q = 0.30$), the equality is rejected with 95% probability only for $q = 0.15$.

3.4.3 Discussion

The baseline regressions, thus, point strongly to the absence of any push impact of foreign portfolio flows on Indian equity prices. All the comovement in the prices and flows is explained by the aggregate pricing factor and idiosyncratic information about the firms. The difference regressions reinforce the results of the baseline regressions and show that the prices of the stocks in which FII’s are active do not react to flows any more than the prices of other stocks. The comovement between prices and flows is fully explained by information about firms. This is consistent with a frictionless view of the Indian equity market with low transaction costs. It also points to the fact that the stocks that FII’s are active in are widely held, and Indian investors do not face market segmentation with respect to them. An alternative interpretation could be that FII’s are shrewd investors, and do not a priori get exposed to stocks with frictions, as such illiquid or segmented stocks would be costly to get out of when needed. The results mitigate the concerns of policymakers about the adverse effects of FII flows.

Our empirical methodology is, however, unable to identify the aggregate macroeconomic impact of flows, which there would be undoubtedly. The global shocks showing up in the VIX or FIFA may be correlated with the returns on the aggregate Indian equity market, and may indirectly impact the stock prices of all firms. However, they will not impact the prices of the FII-active firms any more than those of FII-inactive firms.

3.5 Robustness Checks

3.5.1 Periods with Non-zero Flows

We check the robustness of our results when the firms for our regressions are selected in a different way. Rather than sorting the firms by the standard deviations of their
net flows, we sort them by the number of weeks or days with non-zero FII activity. As before, we pick firms with the top 100$q\%$ non-zero activity weeks or days and estimate the basic model (3.2) by IV. The results are in Tables 3.9 and 3.10 in Appendix 3.6.

We find that the crux of the results holds with this way of picking firms. In the weekly regressions, the coefficient on the flows is almost zero and not significant. The Wu-Hausman test also rejects the hypothesis of exogeneity of the flows. In the daily regressions, the coefficient on the flows is negative, but not significantly different from zero. The Wu-Hausman test strongly rejects the hypothesis of the exogeneity of flows.

### 3.5.2 Additional Pricing Factors

We also check the robustness of the results by including the $SMB$ (small minus big firms) and $HML$ (high minus low book-to-market firms) from Fama and French (1993). The firms are picked as in the baseline regression based on the standard deviations of their FII flows. We estimate the basic model (3.2) by IV. The results are in Tables 3.11 and 3.12 in Appendix 3.6.

Again, we see that the results are robust to the inclusion of additional factors. In the weekly regression, the the coefficient on the flows is almost zero and insignificant. The Wu-Hausman test for the exogeneity of the flows is rejected except for $p = 0.15$. In the daily regressions, too, the coefficient on the flows is small and insignificant, and the Wu-Hausman test strongly rejects the exogeneity of the flows.

### 3.6 Conclusion

We identify the push impact of foreign institutional investor flows on Indian stock prices. We instrument for the endogenous flows by exogenous shocks: the U.S. VIX index and the likely forced flows from U.S. mutual funds due to their changing capital bases. We find that flows have no causal impact on prices, over and above the impact due to changes in the stochastic discount factor. This is consistent with the frictionless view of the Indian equity market with low transaction costs and low market segmentation. It may also point to rational investing by FII's who refrain from buying into illiquid stocks out of the foresight that getting out of these stocks will be costly. In any case, our results alleviate policymakers' concern that foreign portfolio flows cause volatility in asset prices. Our methodology, however, is unable to identify the impact of flows on the aggregate stochastic discount factor and the macroeconomy.
Appendices

3.A IV First Stage

This section lists the first-stage of the IV regressions from Sections 3.4.1 and 3.4.2. Tables 3.5 and 3.6 show the first-stages of the IV regressions of the basic model (3.2). Tables 3.7 and 3.8 show the first-stages of the IV regressions of the difference model (3.3).

Baseline Weekly IV First Stage:

<table>
<thead>
<tr>
<th>Dependent Variable: ( F_t )</th>
<th>( q = 0.15 )</th>
<th>( q = 0.20 )</th>
<th>( q = 0.25 )</th>
<th>( q = 0.30 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>5.17***</td>
<td>3.19**</td>
<td>2.84***</td>
<td>2.50***</td>
</tr>
<tr>
<td></td>
<td>(1.3)</td>
<td>(0.96)</td>
<td>(0.86)</td>
<td>(0.70)</td>
</tr>
<tr>
<td>( R_{m,t} - R_{f,t} )</td>
<td>1.62***</td>
<td>1.34***</td>
<td>1.20***</td>
<td>1.04***</td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td>(0.26)</td>
<td>(0.21)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>FIFA_t</td>
<td>181.98***</td>
<td>146.94***</td>
<td>153.09***</td>
<td>150.37***</td>
</tr>
<tr>
<td></td>
<td>(35.39)</td>
<td>(34.40)</td>
<td>(32.55)</td>
<td>(27.80)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.19</td>
<td>0.21</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>( F )-statistic</td>
<td>44.59***</td>
<td>33.99***</td>
<td>37.96***</td>
<td>45.38***</td>
</tr>
<tr>
<td>Num. obs.</td>
<td>313</td>
<td>313</td>
<td>313</td>
<td>313</td>
</tr>
</tbody>
</table>

Table 3.5: First Stage IV estimation of the basic model (3.2) with weekly data with FIFA_t as instrument. Sample period: 1 April 2003 to 31 March 2009. HAC standard errors. All net returns in percent and flows in basis points.
### Baseline Daily IV First Stage:

Dependent Variable: $F_t$

<table>
<thead>
<tr>
<th>$q$</th>
<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
<th>0.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.02***</td>
<td>0.93***</td>
<td>0.93***</td>
<td>0.71***</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.19)</td>
<td>(0.17)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>$R_{m,t} - R_{f,t}$</td>
<td>0.66***</td>
<td>0.63***</td>
<td>0.59***</td>
<td>0.47***</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.10)</td>
<td>(0.09)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>$\Delta VIX_{t-1}$</td>
<td>-0.36***</td>
<td>-0.32***</td>
<td>-0.28***</td>
<td>-0.26***</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.08)</td>
<td>(0.07)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$F$-statistic</td>
<td>43.69***</td>
<td>47.84***</td>
<td>47.54***</td>
<td>39.73***</td>
</tr>
<tr>
<td>Num. obs.</td>
<td>2407</td>
<td>2407</td>
<td>2407</td>
<td>2407</td>
</tr>
</tbody>
</table>

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$

Table 3.6: First Stage IV estimation of the basic model (3.2) with daily data with $\Delta VIX_{t-1}$ as instrument. Sample period: 1 April 2003 to 31 March 2013. HAC standard errors. All net returns in percent and flows in basis points.

### Difference Weekly IV First Stage:

Dependent Variable: $\Delta F_t$

<table>
<thead>
<tr>
<th>$q$</th>
<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
<th>0.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>5.17***</td>
<td>3.19**</td>
<td>2.84***</td>
<td>2.50***</td>
</tr>
<tr>
<td></td>
<td>(1.30)</td>
<td>(0.96)</td>
<td>(0.86)</td>
<td>(0.70)</td>
</tr>
<tr>
<td>$R_{m,t} - R_{f,t}$</td>
<td>1.62***</td>
<td>1.35***</td>
<td>1.20***</td>
<td>1.04***</td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td>(0.26)</td>
<td>(0.21)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>$FIFA_t$</td>
<td>181.98***</td>
<td>146.94***</td>
<td>153.09***</td>
<td>150.36***</td>
</tr>
<tr>
<td></td>
<td>(35.39)</td>
<td>(34.40)</td>
<td>(32.55)</td>
<td>(27.80)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.19</td>
<td>0.21</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>$F$-statistic</td>
<td>44.59***</td>
<td>33.99***</td>
<td>37.96***</td>
<td>45.40***</td>
</tr>
<tr>
<td>Num. obs.</td>
<td>313</td>
<td>313</td>
<td>313</td>
<td>313</td>
</tr>
</tbody>
</table>

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$

Table 3.7: First Stage IV estimation of the difference model (3.3) with weekly data with $FIFA_t$ as instrument. Sample period: 1 April 2003 to 31 March 2009. HAC standard errors. All net returns in percent and flows in basis points.
Difference Daily IV First Stage:

<table>
<thead>
<tr>
<th>Dependent Variable: $\Delta F_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q = 0.15$</td>
</tr>
<tr>
<td>Constant</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$R_{m,t} - R_{f,t}$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$\Delta VIX_{t-1}$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
</tr>
<tr>
<td>$F$-statistic</td>
</tr>
<tr>
<td>Num. obs.</td>
</tr>
</tbody>
</table>

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$

Table 3.8: First Stage IV estimation of the difference model (3.3) with daily data with $\Delta VIX_{t-1}$ as instrument. Sample period: 1 April 2003 to 31 March 2013. HAC standard errors. All net returns in percent and flows in basis points.

3.B Robustness Checks

This appendix lists the results for the robustness checks described in 3.5. Tables 3.9 and 3.10 are the IV regressions for the basic model (3.2) when the firms are selected based on the number of weeks and days with non-zero FII flows. Tables 3.11 and 3.12 are the IV regressions for the basic model (3.2) including the Fama-French factors $SMB$ and $HML$. 

119
### Baseline Weekly IV: Non-zero weeks

<table>
<thead>
<tr>
<th>Dependent Variable: $R_t - R_{f,t}$</th>
<th>$q = 0.15$</th>
<th>$q = 0.20$</th>
<th>$q = 0.25$</th>
<th>$q = 0.30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.11</td>
<td>0.11</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.12)</td>
<td>(0.11)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>$R_{m,t} - R_{f,t}$</td>
<td>0.99***</td>
<td>0.99***</td>
<td>1.01***</td>
<td>1.02***</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>$F_t$</td>
<td>-0.01</td>
<td>0.00</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Num. obs.</td>
<td>313</td>
<td>313</td>
<td>313</td>
<td>313</td>
</tr>
<tr>
<td>First-stage $F$</td>
<td>40.82***</td>
<td>40.24***</td>
<td>46.98***</td>
<td>45.32***</td>
</tr>
<tr>
<td>Wu-Hausman $F$</td>
<td>8.10**</td>
<td>5.66*</td>
<td>5.25*</td>
<td>5.92*</td>
</tr>
</tbody>
</table>

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$

Table 3.9: IV estimation of the basic model (3.2) with weekly data with FIFA$_t$ as instrument. Firm selection based on number of weeks with non-zero flows. Sample period: 1 April 2003 to 31 March 2009. HAC standard errors. All net returns in percent and flows in basis points.

### Baseline Daily IV: Non-zero days

<table>
<thead>
<tr>
<th>Dependent Variable: $R_t - R_{f,t}$</th>
<th>$q = 0.15$</th>
<th>$q = 0.20$</th>
<th>$q = 0.25$</th>
<th>$q = 0.30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.10</td>
<td>0.09</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>$R_{m,t} - R_{f,t}$</td>
<td>1.15***</td>
<td>1.12***</td>
<td>1.11***</td>
<td>1.10***</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.05)</td>
<td>(0.04)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>$F_t$</td>
<td>-0.37</td>
<td>-0.32</td>
<td>-0.32</td>
<td>-0.32</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.17)</td>
<td>(0.16)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>Num. obs.</td>
<td>2407</td>
<td>2407</td>
<td>2407</td>
<td>2407</td>
</tr>
<tr>
<td>First-stage $F$</td>
<td>33.46***</td>
<td>25.06***</td>
<td>24.99***</td>
<td>24.41***</td>
</tr>
<tr>
<td>Wu-Hausman $F$</td>
<td>8.91**</td>
<td>7.64**</td>
<td>7.12**</td>
<td>7.11**</td>
</tr>
</tbody>
</table>

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$

Table 3.10: IV estimation of the basic model (3.2) with daily data with ΔVIX$_{t-1}$ as instrument. Firm selection based on number of days with non-zero flows. Sample period: 1 April 2003 to 31 March 2013. HAC standard errors. All net returns in percent and flows in basis points.
Baseline Weekly IV: Fama-French factors

<table>
<thead>
<tr>
<th>Dependent Variable: $R_t - R_{f,t}$</th>
<th>$q = 0.15$</th>
<th>$q = 0.20$</th>
<th>$q = 0.25$</th>
<th>$q = 0.30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.07</td>
<td>0.04</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(0.27)</td>
<td>(0.24)</td>
<td>(0.20)</td>
</tr>
<tr>
<td>$R_{m,t} - R_{f,t}$</td>
<td>1.17***</td>
<td>1.38***</td>
<td>1.37***</td>
<td>1.30***</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.16)</td>
<td>(0.12)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$SMB_t$</td>
<td>0.48**</td>
<td>0.42*</td>
<td>0.37**</td>
<td>0.37***</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.17)</td>
<td>(0.14)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>$HML_t$</td>
<td>-0.26**</td>
<td>-0.28</td>
<td>-0.22</td>
<td>-0.20*</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.16)</td>
<td>(0.13)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>$F_t$</td>
<td>0.03</td>
<td>-0.01</td>
<td>-0.02</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.05)</td>
</tr>
</tbody>
</table>

Num. obs. | 313 | 313 | 313 | 313 |

First-stage $F$ | 27.51*** | 23.79*** | 31.56*** | 34.72*** |
Wu-Hausman $F$ | 0.29 | 3.84* | 3.96* | 4.22* |

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$

Table 3.11: IV estimation of the basic model (3.2) with weekly data with $FIFA_t$ as instrument. Including the Fama-French factors $SMB$ and $HML$. Sample period: 1 April 2003 to 31 March 2009. HAC standard errors. All net returns in percent and flows in basis points.
<table>
<thead>
<tr>
<th></th>
<th>( q = 0.15 )</th>
<th>( q = 0.20 )</th>
<th>( q = 0.25 )</th>
<th>( q = 0.30 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.05</td>
<td>0.05</td>
<td>0.04</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.07)</td>
<td>(0.06)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>( R_{m,t} - R_{f,t} )</td>
<td>1.40***</td>
<td>1.36***</td>
<td>1.32***</td>
<td>1.26***</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.09)</td>
<td>(0.07)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>( SMB_t )</td>
<td>0.27***</td>
<td>0.23**</td>
<td>0.24***</td>
<td>0.19**</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.08)</td>
<td>(0.07)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>( HML_t )</td>
<td>0.08</td>
<td>0.06</td>
<td>0.08</td>
<td>0.12*</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>( F_t )</td>
<td>-0.05</td>
<td>-0.07</td>
<td>-0.06</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.06)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Num. obs.</td>
<td>2407</td>
<td>2407</td>
<td>2407</td>
<td>2407</td>
</tr>
<tr>
<td>First-stage ( F )</td>
<td>21.67***</td>
<td>22.86***</td>
<td>22.03***</td>
<td>19.97***</td>
</tr>
<tr>
<td>Wu-Hausman ( F )</td>
<td>7.91**</td>
<td>9.12**</td>
<td>10.24**</td>
<td>10.13**</td>
</tr>
</tbody>
</table>

### Table 3.12
IV estimation of the basic model (3.2) with daily data with \( \Delta VIX_{t-1} \) as instrument. Including the Fama-French factors \( SMB \) and \( HML \). Sample period: 1 April 2003 to 31 March 2013. HAC standard errors. All net returns in percent and flows in basis points.
Bibliography


124
