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THE DESIGN OF AXIAL INDUCERS FOR TURBOPUMPS

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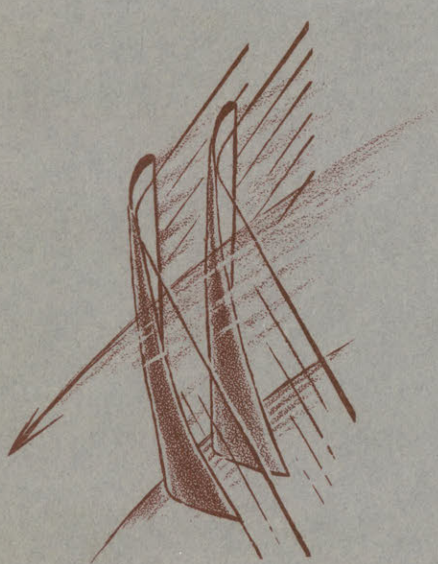

GAS TURBINE LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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THE DESIGN OF
AXIAL INDUCERS FOR TURBOPUMPS

by

Alan H. Stenning

Under the sponsorship
of
General Electric Company
Westinghouse Electric Corporation
Allison Division of the General Motors Corporation

Gas Turbine Laboratory

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1 INTRODUCTION

In many turbopump applications it is desirable to run the pump at the highest possible speed to minimise the size and weight of the unit and facilitate matching with a drive turbine. Frequently, a limit on rotational speed is imposed by pump cavitation with its associated deterioration in performance and structural damage. For conventional single sided centrifugal pumps cavitation occurs when the suction specific speed (defined as $\frac{N\sqrt{Q}}{(H_s)^{3/4}}$ *) exceeds 8-10,000 (1) so that for such pumps the maximum rotational speed without cavitation is determined by the flow and the suction head available. To permit operation at higher speeds an inducer or boost pump may be mounted in front of the main pump (fig. 1). A typical axial inducer is simply a very lightly loaded axial pump which raises the pressure of the fluid sufficiently to avoid cavitation in the main pump. It has been found possible to operate inducers successfully at suction specific speeds up to 30,000, so that considerable reductions in total pump weight can be achieved when they are employed. Of course, since the inducer is simply a lightly loaded pump which is capable of handling a cavitating fluid, the functions of inducer and main pump may be combined by designing the main pump so that the inlet is lightly loaded. However, it is not always convenient to do this and in many applications conventional centrifugal pumps are still used, preceded by a separate inducer. The objective of this report is to put the design of such inducers on a rational basis by developing a method of calculating blade shapes for the optimum pressure distribution.

*See Glossary for symbols.

2 CAVITATION LIMITS

For practical purposes, cavitation may be considered to occur when the pressure falls to the vapor pressure corresponding to the inlet temperature to the inducer, that is when $p = p_{v1}$. It is desirable to design the inducer so that p does not fall below p_{v1} at any point although the inducer will operate satisfactorily with some cavitation. On the other hand, to make the inducer as short as possible, we should have suction surface pressures on the blading as low as efficient operation permits. A reasonable compromise between these two requirements is to design for constant pressure on the suction surface, equal to the vapor pressure p_{v1} at the most unfavorable operating condition with respect to cavitation (Fig. 2). Since the inducer will perform efficiently with some cavitation a reasonable factor of safety is then included in the design.

Thus, the objective is to find the blade profile of the shortest inducer which, for a given set of inlet conditions, will produce the required rise in pressure without severe cavitation.

To compute the maximum allowable relative velocity in the inducer we assume that losses may be neglected so that the relative stagnation pressure is constant.

Then, at any point in the inducer

$$p/\rho + \frac{w^2}{2g_0} = p_1/\rho + \frac{w_1^2}{2g_0}$$

$$= p_2/\rho + \frac{w_2^2}{2g_0}$$

where station 1 is just outside the inducer and station 2 is at the mid-passage just inside the inducer (Fig. 3). At the point of maximum allowable relative velocity (and minimum static pressure) $p = p_{v1}$ and

$$\frac{p_{v1}}{\rho} + \frac{w_{max}^2}{2g_0} = \frac{p_1}{\rho} + \frac{w_1^2}{2g_0}$$

$$= \frac{p_1}{\rho} + \frac{c_{x1}^2}{2g_0} + \frac{u_1^2}{2g_0}$$

$$= \frac{p_{01}}{\rho} + \frac{u^2}{2g_0}$$

$$\frac{w_{max}^2}{2g_0} = \left(\frac{p_{01}}{\rho} - \frac{p_{v1}}{\rho}\right) + \frac{u^2}{2g_0} = H_{s1} + \frac{u^2}{2g_0} \tag{1}$$

An inducer design can be carried out only if w_{max} is greater than w_2 (the mid-passage velocity just inside the blading) and H_{s1} must be sufficiently large to ensure that this is so.

If the inducer is operating with finite leading edge loading, then $c_{\theta 2} = c_{\theta 1} = 0$ since no change in c_θ can then occur in zero length.

$$w_2^2 = c_{x2}^2 + w_{\theta 2}^2$$

$$= c_{x2}^2 + w_{\theta 1}^2$$

$$w_2^2 = c_{x2}^2 + u^2 \tag{2}$$

From continuity $\qquad c_{x1} A_{x1} = c_{x2} A_{x2}$

$$c_{x2} = \frac{A_{x1}}{A_{x2}} c_{x1}$$

$$c_{x2} = f \cdot c_{x1} \tag{3}$$

where $f = \frac{A_{x1}}{A_{x2}}$ is the blockage factor of the blading. Then

$$W_2^{\,2} = u^2 + f^2 C_{x1}^{\,2}$$

$$\left(\frac{W_{max}}{W_2}\right)^2 = \frac{2g_0 H_{s1}}{W_2^{\,2}} + \frac{u^2}{W_2^{\,2}}$$

$$\left(\frac{W_{max}}{W_2}\right)^2 = \frac{2g_0 H_{s1} + u^2}{u^2 + f^2 C_{x1}^{\,2}}$$

$$\frac{W_{max}}{W_2} = \sqrt{\frac{\dfrac{H_{s1}}{C_{x1}^2/2g_0} + \dfrac{u^2}{C_{x1}^{\,2}}}{\dfrac{u^2}{C_{x1}^{\,2}} + f^2}} \qquad (4)$$

For given inlet conditions and blockage factor, the allowable acceleration may be calculated. For example, with $\frac{H_{s1}}{C_{x1}^2/2g_0} = 1.25$, $f = 1.03$, and $\frac{u}{C_{x1}} = 4.0$, $\frac{W_{max}}{W_2} = 1.011$. In this case, only 1.1% increase in velocity is allowable after the fluid enters the blading if cavitation is to be avoided.

3 INDUCER WORK

From the inlet conditions and the cavitation characteristics of the main pump, the increase in stagnation pressure to be supplied by the inducer may be calculated as follows.

Let the suction specific speed of the main pump for safe operation be $S = \frac{N\sqrt{Q}}{(H_{s3})^{3/4}}$. Then, from N, Q and S the necessary suction head at the inlet to the main pump can be found. The increase in head to be supplied by the inducer is equal to $H_{s3} - H_{s1}$ if the temperature (and hence the vapor pressure) does not change significantly in the inducer. For most fuels (including cryogenic fluids) the rate of change of vapor pressure with temperature is sufficiently small to permit this approximation. Thus,

$$\frac{p_{o3} - p_{o1}}{\rho} = H_{s3} - H_{s1}$$

$$\frac{u}{g_o}\left(c_{\theta 3} - c_{\theta 1}\right) = \frac{1}{\eta_I}\left(H_{s3} - H_{s1}\right)$$

from Euler's equation, where η_I is the inducer efficiency, usually of the order of 60 - 70%. With no preswirl, $c_{\theta 1} = 0$ and

$$\frac{u\, c_{\theta 3}}{g_o} = \frac{1}{\eta_I}\left(H_{s3} - H_{s1}\right) \tag{5}$$

from which $c_{\theta 3}$ may be found

$$c_{\theta 3} = u - w_{\theta 3}$$

$$c_{\theta 3} = u - c_{x3} \tan\beta_3 \tag{6}$$

For a design with constant mid-passage axial velocity (such as a free vortex design), $c_{x3} = c_{x2}$ and β_3 is calculated directly from equation (6). For designs other than free-vortex, it is necessary to compute the axial velocity distribution before β_3 is found. The inlet fluid angle β_2 is obtained from the relation $\tan\beta_2 = \frac{u}{c_{x2}}$.

4 BLADE SHAPE

Inducers are sufficiently lightly loaded so that the difference between fluid angles and blade angles can be neglected for a first approximation, and the blade loading can be computed by the simple one-dimensional approach described below. Consider the elementary control volume between two blades shown in figure 4.

Equating the tangential pressure torque on the control volume to the net efflux of angular momentum from the control volume we have

$$\Delta p . dx . r = -\rho \frac{W_{xm} \tau}{g_0} d(W_{\theta m}) . r$$

where Δ p is the difference in pressure between the two sides of the control volume and w_{xm}, $w_{\theta m}$ are the mass-weighted mean values between the blades which to a first approximation may be taken as the values at the middle of the passage w_x, w_θ. τ is the blade-spacing.

$$\frac{\Delta p}{\rho} = \frac{W_s^2 - W_p^2}{2g_0} = \frac{(W_s - W_p)(W_s + W_p)}{2g_0}$$

$$\approx \frac{2(W_s - W)(2W)}{2g_0}$$

where w is the relative velocity at mid-passage. Therefore,

$$W_s - W = -\frac{\tau}{2} \frac{W_x}{W} \frac{\partial W_\theta}{\partial x}$$

$$W_s - W = -\frac{\tau}{2} \cos\beta \frac{\partial W_\theta}{\partial x} \qquad (7)$$

If the blade loading is specified as a function of x, the allowable rate of change of w_θ may be calculated from equation (7).

For an inducer which is to be designed with constant suction surface velocity equal to $K w_2$, where $K = \frac{w_{max}}{w_2}$ and is found from the inlet conditions, the equation becomes

$$K w_2 - w = -\frac{\tau}{2} \cos\beta \frac{\partial w_\theta}{\partial x}$$

or

$$K \frac{c_{x2}}{\cos\beta_2} - \frac{c_x}{\cos\beta} = -\frac{\tau}{2} \cos\beta \frac{d(c_x \tan\beta)}{dx} \qquad (8)$$

If c_x is specified as a function of x, then equation (8) may be integrated to give β as a function of x. Unfortunately, in general c_x will not be known as a function of x along a mid-passage streamline, since the radial distribution of c_x at a given x depends on the radial distribution of β, as will be discussed later, yielding an exceedingly complicated differential equation for β. For one simple case only (free vortex design) c_x will be constant and the equation can easily be integrated. It appears that there actually are some merits to a free vortex design for axial inducers, so that this solution may be useful. The critical part of the inducer is the tip section and it is the blade shape at the tip which is to be calculated. At other radial stations the suction surface pressure is higher than at the tip and therefore the danger of cavitation is not as great.

With c_x constant = c_{x2} the equation reduces to

$$\frac{K}{\cos\beta_2} - \frac{1}{\cos\beta} = -\frac{\tau}{2}\cos\beta\,\frac{d(\tan\beta)}{dx}$$

$$= -\frac{\tau}{2\cos\beta}\,\frac{d\beta}{dx}$$

Therefore $$\frac{dx}{\tau} = \frac{\cos\beta_2\, d\beta}{2\,(K\cos\beta - \cos\beta_2)}$$

This equation integrates to

$$\frac{x}{\tau} = \frac{\cos\beta_2}{2\sqrt{K^2-\cos^2\beta_2}}\left[\arctan\left(\frac{\sin\beta_2\sqrt{K^2-\cos^2\beta_2}}{K-\cos^2\beta_2}\right) - \arctan\left(\frac{\sin\beta\sqrt{K^2-\cos^2\beta_2}}{K-\cos\beta\cos\beta_2}\right)\right] \tag{9}$$

and requires 7 figure accuracy in the individual components to give 3 figure accuracy in the final answer for the type of machine with which we are dealing. However, if β is greater than 65°, the assumption that $\cos\beta \approx \left(\frac{\pi}{2}-\beta\right) \approx \frac{1}{\tan\beta}$ can be made (β in radians) and the integral is simplified to

$$\frac{x}{\tau} = \frac{1}{2K\tan\beta_1}\log\left[\frac{K\frac{\tan\beta_2}{\tan\beta} - 1}{K-1}\right] \tag{10}$$

with negligible loss in accuracy.

For example, if $K = 1.011$, $\frac{u}{c_{x1}} = 4.0$, $\rho = 1.03$ then $c_{x2}/c_{x1} = 1.03$, $u/c_{x2} = 3.88$, $\beta_2 = 75.5^\circ$ and if $c_{\theta 2}/u = 0.25$ (a typical value)

$\beta_3 = 71.1^\circ$ and $x_3/\tau = .44$.

To obtain the tip coordinates of the blade, an x - θ plot is needed, whereas equation (10) gives only x versus β, an inconvenient form.

$$d\theta = \frac{dx \tan\beta}{r}$$

where r is the tip radius of the inducer. Therefore from equation (8)

$$d\theta = -\frac{\tau}{2r} \cdot \frac{\tan\beta \cos\beta_2 \, d\beta}{(K\cos\beta - \cos\beta_2)} \tag{11}$$

With the approximation that $\cos\beta \approx \frac{1}{\tan\beta} \approx (\frac{\pi}{2} - \beta)$ valid for large β equation (11) integrates to

$$\theta = \frac{\tau}{2r} \log\left[\frac{K - \frac{\tan\beta}{\tan\beta_2}}{K - 1}\right] \tag{12}$$

It is essential that the approximate expressions for x, θ should be consistent i.e. $r\frac{d\theta}{dx} = \tan\beta$ to ensure that the inducer does in fact have the desired inlet and exit angles. Equations (10) and (12) satisfy this requirement. In Figure (5) $\frac{x}{r}$ versus θ is shown for the tip section of a 3-bladed inducer with the specifications given on page (8). Now (at least for a free-vortex machine) it is possible to compute the blade coordinates for the inducer. Manufacturing considerations may dictate the use of blading other than free-vortex, in which case radial equilibrium considerations must be included.

Even for a very lightly loaded inducer, the fluid angle β will not be exactly equal to the blade angle β' but will in fact be greater than β'. The error will be on the safe side as far as blade loading is concerned, but if the inducer is designed with β_3' equal to the desired value of β_3, the design point work will not be attained. To remedy this, the inducer blading should be designed for $\beta_3' = \beta_3 - \delta$, where δ is the deviation and can be fairly accurately predicted from Carter's rule (Ref. 2), and the computation for blade shape should be continued on to β_3'. The resulting inducer should deliver the correct work at the design point flow.

5 RADIAL EQUILIBRIUM AND THE RADIAL BLADE ELEMENT INDUCER

The example shown in Figure 5 has been worked through with the assumption that c_x is constant at the tip through the inducer. This assumption is valid only if the design is free-vortex. A free-vortex design has the disadvantage of non-radial blade elements at each station, which entails some manufacturing difficulties, but the peripheral speeds are so low that no stress problems are introduced. It has the possible advantage that for a given amount of work at the tip, the static pressure leaving the inducer is greater than for a forced vortex design with increasing work from root to tip, so that for the same amount of tip work we might expect better cavitation characteristics from a free vortex inducer than from a forced vortex inducer. For a free vortex design rc_θ is invariant with r at a fixed x. Therefore, at any x,

$$r(u - w_\theta) = r_t(u_t - w_{\theta t})$$

$$r(u - c_x \tan\beta) = r_t(u_t - c_{xt}\tan\beta_t)$$

$$r(\tan\beta_2 - \tan\beta) = r_t(\tan\beta_{2t} - \tan\beta_t)$$

since $c_{xt} = c_x = c_{x2}$

At any (r,x), $\tan\beta = \tan\beta_2 - \frac{r_t}{r}(\tan\beta_{2t} - \tan\beta_t)$, where β_t is the tip inducer angle at that x, and this is the equation for the blading in conjunction with the tip shape.

The most attractive forced vortex inducer from the point of view of the machinist is one with radial blade elements i.e. one with $\tan\beta = r/r_t \tan\beta_t$ at any x. Neglecting meridional streamline curvature, let us use the simple radial equilibrium equation (Ref. 3) to study the radial c_x distribution in an inducer of this type.

$$\frac{d(c_x^2)}{dr} = 2g_oJ\frac{dh_o}{dr} - \frac{1}{r^2}\frac{d}{dr}(rc_\theta)^2 \tag{13}$$

If $\tan\beta = Ar$ where $A = \frac{\tan\beta_t}{r_t}$

then
$$\begin{aligned} h_o &= h_{o1} + \frac{uc_\theta}{g_oJ} \\ &= h_{o1} + \frac{u}{g_oJ}(u - w_\theta) \\ &= h_{o1} + \frac{\Omega r}{g_oJ}(\Omega r - Arc_x) \end{aligned}$$

$$rc_\theta = r(\Omega r - c_x A r)$$

The equation reduces to

$$(1+r^2A^2)\frac{dc_x}{dr} + 2rA^2c_x - 2\Omega A r = 0$$

or $$\frac{d}{dr}\left[(1+r^2A^2)c_x\right] = 2\Omega A r$$

Therefore

$$\left[(1+r^2A^2)\,c_x\right] = A\Omega r^2 + \text{Constant} \tag{14}$$

and

$$\left[(1+r_q^2A^2)c_{x2}\right] = A\Omega r_q^2 + \text{constant} \tag{15}$$

where r_q is the reference radius where $c_x = c_{x2}$. But $Ar = \tan\beta$. Therefore, subtracting (15) from (14)

$$c_x(1+\tan^2\beta) - c_{x2}(1+\tan^2\beta_q) = u\tan\beta - u_q\tan\beta_q$$

$$u = c_{x2}\tan\beta_2 \qquad u_q = c_{x2}\tan\beta_{2q}$$

Solving for c_x/c_{x2}

$$\frac{c_x}{c_{x2}} = \frac{1+\tan^2\beta_q + \tan\beta_2\tan\beta - \tan\beta_{2q}\tan\beta_q}{1+\tan^2\beta}$$

But c_x will equal the mean value c_{x2} approximately halfway from root to tip, so that $\tan\beta_q \simeq \left(\frac{\lambda+1}{2}\right)\tan\beta$ where λ is the ratio of hub diameter to tip diameter. Therefore

$$\frac{c_x}{c_{x2}} = \frac{1+\left(\frac{\lambda+1}{2}\right)^2\tan^2\beta + \tan\beta_2\tan\beta\left[1-\left(\frac{\lambda+1}{2}\right)^2\right]}{1+\tan^2\beta} \tag{16}$$

For an inducer with $\beta_{2t} = 75.5°$, $\beta_{3t} = 71.1°$ and $\lambda = 0.5$ we find that $\frac{c_{x3t}}{c_{x2}} = 1.13$, representing a substantial acceleration at the tip.

Returning to equation (8), in order to find the inducer blade shape it is necessary to substitute (16) into (8) and integrate. With (16) in its exact form, the equation cannot be integrated analytically. However, a good approximation to (16) for the machines we are considering is

$$\frac{c_x}{c_{x2}} = \left(\frac{\lambda+1}{2}\right)^2 + \frac{\cos\beta}{\cos\beta_2}\left[1 - \left(\frac{\lambda+1}{2}\right)^2\right] \tag{17}$$

This expression deviates from (16) by less than 1% over the range of interest.

Eliminating c_x between (17) and (8) and integrating with the approximation $\cos\beta \simeq \left(\frac{\pi}{2}-\beta\right) \simeq \frac{1}{\tan\beta}$ the equations for the shape of the inducer are

$$\frac{x}{\tau} = \frac{P}{2\tan\beta_2[K-(1-P)]}\log\left[\frac{\frac{\tan\beta_2}{\tan\beta}[K-(1-P)] - P}{K-1}\right] \tag{18}$$

$$\theta = \frac{\tau}{2r}\log\left[\frac{[K-(1-P)] - \frac{\tan\beta}{\tan\beta_2}P}{K-1}\right] \tag{19}$$

which reduce to (10) and (12) (as they should) when the hub-tip ratio tends to unity.

In a design problem, $\frac{\Delta C_\theta}{u}$ at the tip is known and we wish to find the outlet angle and the blade shape.

$$\Delta C_{\theta t} = C_{\theta 3t} = u_t - C_{x3t}\tan\beta_{3t}$$

$$\frac{\Delta C_{\theta t}}{u_t} = 1 - \frac{C_{x3t}}{u_t}\tan\beta_{3t} = 1 - \frac{C_{x3t}}{C_{x2}}\cdot\frac{C_{x2}}{u_t}\cdot\tan\beta_{3t}$$

$$\frac{C_{x3t}}{C_{x2}} = \frac{1-\frac{\Delta C_{\theta t}}{u_t}}{\frac{C_{x2}}{u_t}\tan\beta_{3t}} = \frac{\tan\beta_{2t}}{\tan\beta_{3t}}\left(1 - \frac{\Delta C_{\theta t}}{u_t}\right) \tag{20}$$

Eliminating $\frac{c_{x3t}}{c_{x2}}$ between (17) and (20)

$$\left(1 - \frac{\Delta C_{\theta t}}{U_t}\right) = \frac{\tan\beta_{3t}}{\tan\beta_{2t}}\left[P + \frac{\cos\beta_{3t}}{\cos\beta_{2t}}(1-P)\right] \tag{21}$$

β_3 is found by a trial and error solution of (21).

x/τ and θ are then found as functions of β from (18) and (19). For the example under consideration (page 8) with $\lambda = 0.5$, $\frac{\Delta C_\theta}{U} = .25$, $\beta_{2t} = 75.5^\circ$ it is found that β_{3t} is 66.2° for the radial blade element inducer (as opposed to 71.1° for the free vortex inducer). From equation (18) $\frac{x_3}{\tau} = .46$ for the radial blade element inducer -- slightly longer than the free vortex inducer. $c_{x3t}/c_2 = 1.27$ for the radial element inducer.

In figure (5), the tip blade coordinates of the radial element inducer are shown for purposes of comparison with the free vortex inducer doing the same amount of work at the tip. The difference between fluid angle and blade angle has been neglected. With the deviation included, both inducers would be slightly longer.

6 CONCLUSIONS

An inducer design system has been developed which permits the designer to find the blading coordinates for the shortest inducer which will do the required task without serious cavitation. To test the validity of this design system, three inducers are presently being made at the Gas Turbine Laboratory and will be tested in a pump test stand which is also under construction. These inducers have been designed for $\beta_2 = 78^\circ$,

$\Delta c_{\theta t}/u_t = 0.20$, $\int = 1.05$, $\lambda = 0.4$ and values of K of 1.005, 1.012 and 1.016. The corresponding values of suction specific speed at incipient cavitation are 30,000, 25,000 and 23,000, respectively. One of these inducers is shown in figure (6).

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2) Howell, A. R., "Fluid Dynamics of Axial Compressors", The Institution of Mechanical Engineers, Proceedings 1945, Vol. 153.

3) Adams, H. T., "Internal Combustion Turbine Theory", Cambridge University Press.

GLOSSARY

A_x	cross-section area	ft^2
c_θ	tangential absolute velocity	ft/sec
c_x	axial velocity	ft/sec
f	blockage factor $\frac{A_{x1}}{A_{x2}}$	
g_o	constant in Newton's Law $F = \frac{ma}{g_o}$	$\frac{\text{lb.m. ft}}{\text{lb.f. sec}^2}$
H_s	suction head	$\frac{\text{ft lbs f.}}{\text{lb.m.}}$
K	ratio of $\frac{\text{maximum velocity}}{\text{inlet velocity}}$	
N	rotational speed	r.p.m.
p	static pressure	lbs f/ft^2
p_o	stagnation pressure	lbs f/ft^2
p_v	vapor pressure	lbs f/ft^2
P	$\left(\frac{\lambda+1}{2}\right)^2$	
Q	flow	g.p.m.
r	radius	ft
S	suction specific speed	
u	blade speed	ft/sec
w	relative velocity	ft/sec
x	axial distance	ft
β	fluid angle from axis	
β'	blade angle from axis	
λ	hub-tip ratio	
η_I	inducer efficiency	
θ	rotation about x axis	

GLOSSARY, cont.

SUBSCRIPTS

1	before inducer
2	inside entrance
3	leaving inducer
t	tip
q	radial station where $c_x = c_{x2}$

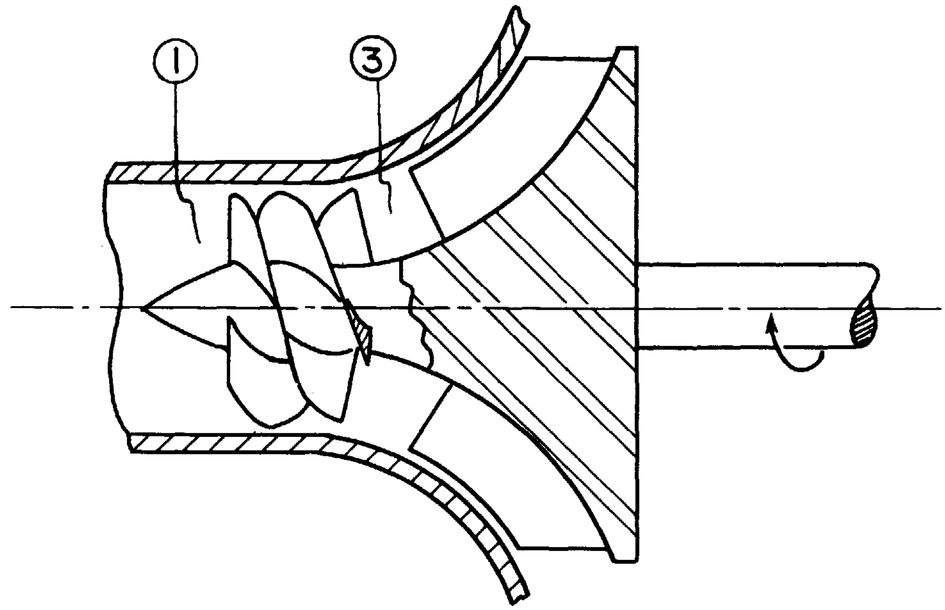

FIG. 1- PUMP WITH INDUCER

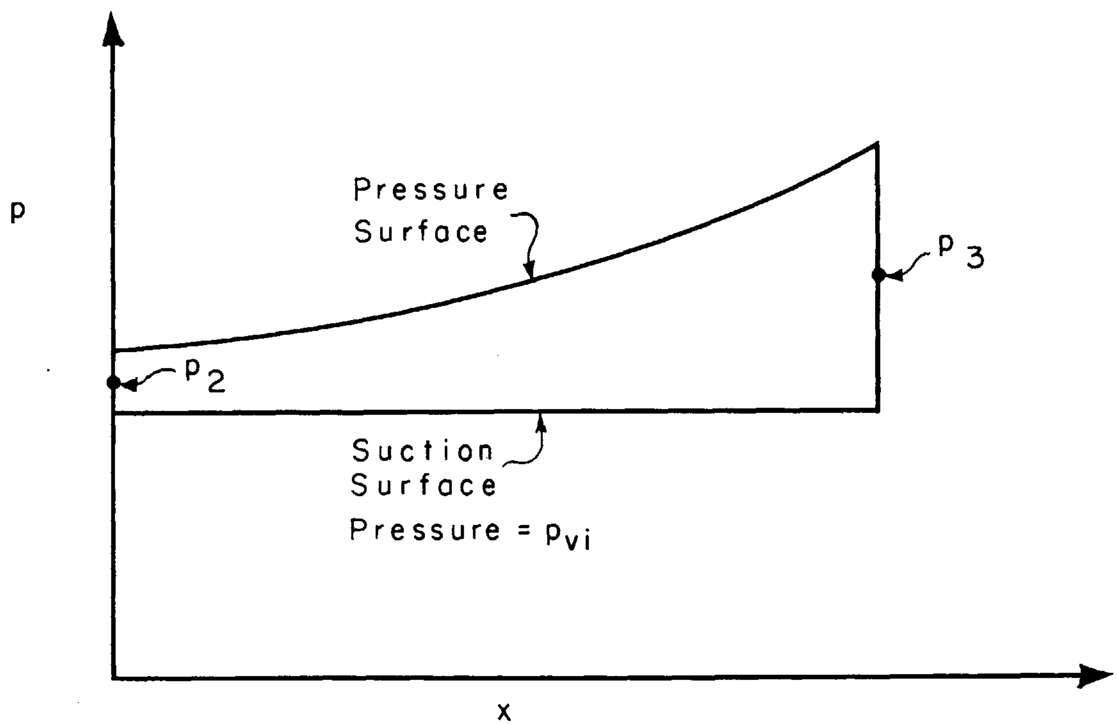

FIG. 2- DESIRED INDUCER BLADE LOADING

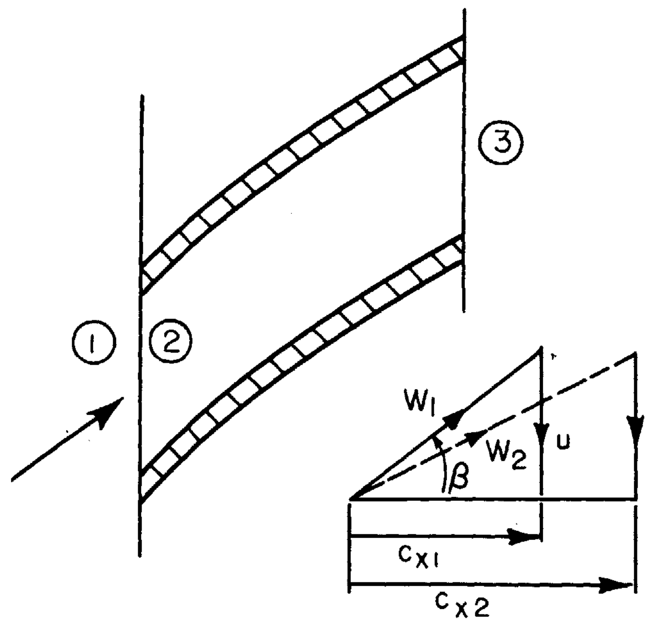

FIG. 3- ENTRANCE VELOCITY TRIANGLES

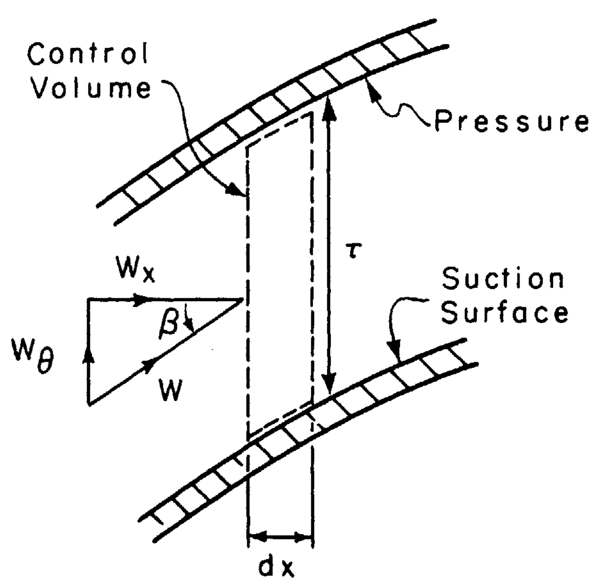

FIG. 4 – CONTROL VOLUME NOTATION

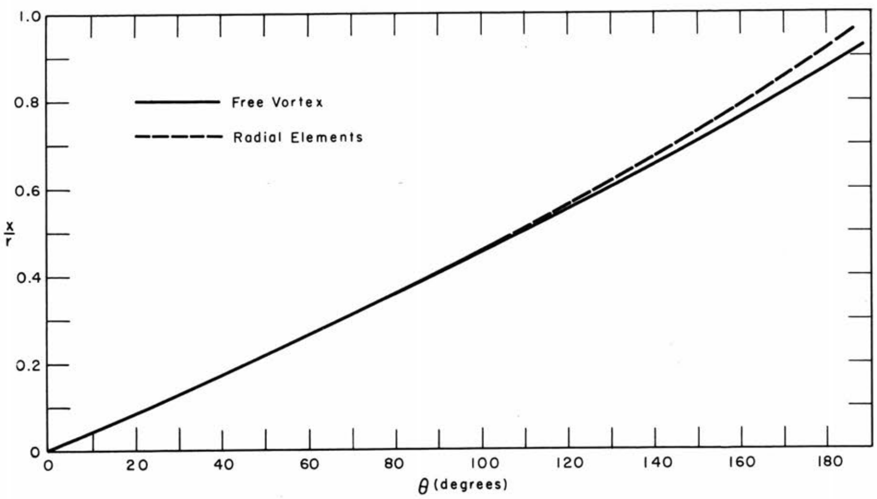

FIG. 5 - INDUCER TIP COORDINATES

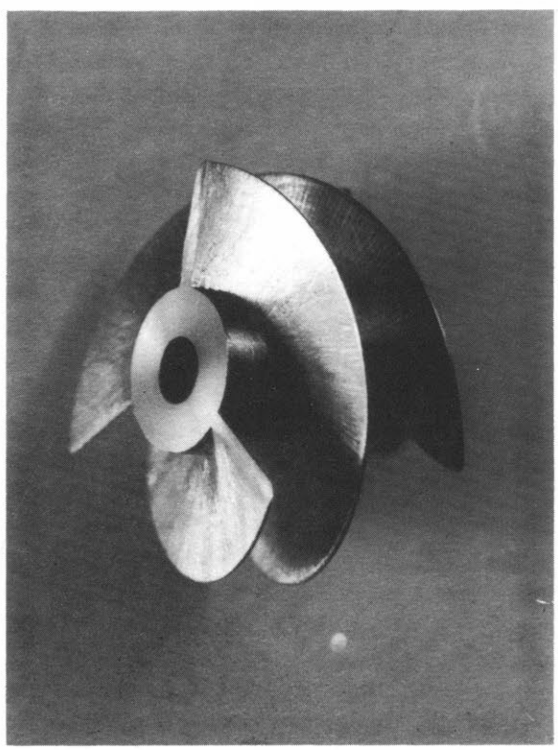

FIG. 6 - TEST INDUCER