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Nonlinear Interval Parameter Programming Combined with Cooperative Games: a Tool for Addressing Uncertainty in Water Allocation Using Water Diplomacy Framework

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Abstract This paper shows the utility of a new interval cooperative game theory as an effective water diplomacy tool to resolve competing and conflicting needs of water users from different sectors including agriculture, domestic, industry and environment. Interval parameter programming is applied in combination with cooperative game theoretic concepts such as Shapley values and the Nucleolus to provide mutually beneficial solutions for water allocation problems under uncertainty. The allocation problem consists of two steps: water resources are initially allocated to water users based on the Nash bargaining model and the achieved nonlinear interval parameter model is solved by transforming it into a problem with a deterministic weighted objective function. Water amounts and net benefits are reallocated to achieve efficient water usage through net benefit transfers. The net benefit reallocation is done by the application of different cooperative game theoretical methods. Then, the optimization problem is solved by linear interval programming and by converting it into a problem with two deterministic objective functions. The suggested model is then applied to the Zarrinehrud sub-basin, within Urmia Lake basin in Northwestern Iran. Findings suggest that a reframing of the

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problem using cooperative strategies within the context of water diplomacy framework - creating flexibility in water allocation using mutual gains approach - provides better outcomes for all competing users of water.

Keywords Water diplomacy · Cooperative games · Interval parameter · Uncertainty · Urmia Lake

1 Introduction

Allocation of and access to water is becoming increasingly complex because of population growth, uneven temporal and spatial distribution of water, rapid urbanization, ecosystem degradation, biodiversity losses, and global climate change. Many water-related problems are framed from a contested terrain in which many actors (individuals, communities, businesses, NGOs, states, and countries) compete to protect their own and often conflicting interests. Water-related problems are complex not only because they involve various stakeholders (e.g., farmers, industrial users, urban developers, environmental activists) who are competing for a limited and common resource but also because they cross multiple boundaries (e.g., physical, disciplinary, jurisdictional). As a result, there is rarely an acceptable solution for most water problems with multiple objectives and competing needs.

Water resources allocation - with competing and often conflicting needs of various users with different values, goals, and interests - is a complex wicked problem (Islam and Susskind 2015). The reason that water allocation remains a major problem is not the lack of sufficient scientific information to determine the likely impacts of climate change or certainty about hydrological variations in the future. Rather, water allocation will continue to be a problem because of population growth, current allocation practices, unchecked demand for water, and underinvestment in infrastructure and technology to increase efficiency of water use. An effective way to address complex water allocation problems is to reframe them as joint decision-making problems- from identifying and defining the problem to innovating and implementing mutual gains options for resolutions - tasks that can generate politically legitimate policies and projects based on science with active participation of all involved parties. The Water Diplomacy Framework (WDF) is emerging as an alternative to traditional techno- or values-focused approach to water management (Islam and Susskind 2013). The WDF diagnoses water problems, identifies intervention points, and proposes sustainable resolutions that are sensitive to diverse viewpoints and uncertainty as well as changing and competing demands.

In solving these types of multi-criteria multi-decision maker water problems, game theory provides an attractive alternative to traditional optimization methods. Game theory is an elegant mathematical formulation of competition and cooperation. Games can be classified as cooperative or non-cooperative. In cooperative games, players cooperate in bargaining or by forming coalitions and coordinating strategies to increase their benefits. In non-cooperative games, players make decisions independently when they cannot or do not want to coordinate their strategies or bargaining plans (Tisdell and Harrison 1992). The cooperative game approach can define efficient and fair solutions that provide the appropriate incentives among the parties involved (Sechi et al. 2013).

In the past decades, many studies have been carried out for developing deterministic cooperative game models for water resources allocation problems. Wang et al. (2003) proposed a cooperative game theoretic framework for obtaining equitable, efficient and sustainable short term water allocation schemes among competing water users in a river basin. Kucukmehmetoglu and Guldmann (2004) presented a linear programming model using cooperative game-theoretic concepts in the three riparian countries (Turkey, Syria and Iraq). Dinar et al. (2006) applied a role-playing game to a water allocation problem in the Kat watershed in South Africa by combining the companion modeling approach and a negotiation procedure as a cooperative game that can be mirrored as a mediated mechanism.

Madani and Dinar (2011) applied several commonly used cooperative game theoretic solution concepts through a numerical groundwater example. Mahjouri and Ardestani (2011) developed two cooperative and noncooperative methodologies for a large-scale water allocation problem in Southern Iran. Abed-Elmdoust and Kerachian (2012) utilized two fuzzy cooperative games for modeling equitable and efficient water allocation among water users in both inter-basin and intra-basin water allocation problems. Jafarzadegan et al. (2013) developed a new solution concept, called Fuzzy Variable Least Core, to model the fuzzy cooperative games. This solution concept is used for water and benefit allocation, considering the uncertainties associated with their benefit coefficients. Safari et al. (2014) applied a Stackelberg model to maximize the net benefit with the Iran Water Resources Management Company as the leader and agricultural, domestic, and industrial users as followers subject to the system's constraints. The Stackelberg (leader-follower) game is the hierarchical relationship between two autonomous, and possibly conflictual, decision makers (Colson et al. 2007). The suggested method is then applied to a case study in Iran, showing how cooperation in a water negotiation could create more value. However this paper does not consider the coalition among the followers and also does not model the uncertainty in the input variables. Roozbahani et al. (2014) proposes a multi-objective optimization model for sharing water among stakeholders of a transboundary river, assuming that the stakeholders cooperate.

In water resources management problems many parameters are often uncertain. Uncertainties might come from the conflicting water division, the randomness of rainfall events in spatial and temporal, the instability of water demand in different periods and the incongruity of strategies in the social and economic development (Zhang and Li 2014). In this case the cooperation is difficult to attain because of uncertainty in parameters and variables. Therefore, several inexact optimization techniques were developed to deal with the complexities in water resources systems, such as fuzzy and stochastic methods (Li and Huang 2008). In most cases, no sufficient data are available to estimate the probability density functions (PDFs) and/or membership functions, and only upper and lower bounds of the uncertain inputs are known. In this case, interval-parameter programming is effective in tackling uncertainties expressed as intervals with known lower and upper bounds but unknown distribution functions (Wang et al. 2012).

Recently, interval parameter method was introduced for handling uncertainty efficiently. Details of linear interval parameter can be found in Huang (1996). This paper provides an interval parameter water quality management model. The model allows uncertain information, presented as interval numbers, to be effectively imbedded into the optimization process and the resulting solutions. Moreover, several interval parameter models have been introduced in the literature of water resources decision making (Li et al. 2006; Lu et al. 2010; Fan et al. 2012; Wang and Huang 2012, 2013, 2014). The results indicated that interval solutions can be used in modeling the objective function and decision variables. In the context of nonlinear interval parameter models, very few studies have been conducted. Li and Huang (2008) developed an interval parameter two-stage stochastic nonlinear programming method for supporting water-resources allocation within a multi-reservoir system. This model can deal with nonlinearities in the objective function such as the economies-of-scale effects. Nikoo et al. (2012) proposed a new methodology for simultaneous allocation of water and waste load in river basins. A nonlinear interval number optimization model is used to incorporate the uncertainties of model inputs and parameters.

This paper explores a new interval cooperative game model as an effective tool for WDF among competing users under uncertainty. This study combines key features of interval parameter methods and cooperative games to extend previously developed models for water allocation under uncertainty. In reality, there are nonlinear relationships among water resources systems and also noncooperative models cannot guarantee stability of allocation over time, thus this newly developed model considers both linear and nonlinear uncertainties in cooperative water allocation. Considering both linear and nonlinear relationships and using the Nash bargaining solution to find initial allocation are the main novelties of this paper. The total net benefit is then reallocated to the water users using cooperative game theoretic concepts. There are many cooperative solution methodologies; the Shapley values appear to be the most frequently used methodology in applications. In this study, in addition to Shapley values two variants of the nucleolus methodology are used. The Zarrinehrud sub-basin including its water resources system is modeled as a real case study to demonstrate the effectiveness of the model and solution methodology.

In the next section the fundamentals of interval parameter models and different cooperative game theoretic approaches will be described. Then in Section 3 the case study will be presented in details. Section 4 discusses the results from this study and concluding remarks are provided in Section 5.

2 Methodology

Water issues are complex because they cross multiple boundaries and involve various stakeholders with competing needs. The origin of many water issues is a dynamic consequence of competition, interconnections, and feedback among variables and processes in multiple sectors. When viewed as a limited resource, water lends itself to destructive conflicts over its division; knowledge of water use and allocation, however, can transform a finite water quantity into a flexible resource. To generate such a transformative knowledge base for water, we need a framework to synthesize explicit (scientific) and tacit (contextual) water knowledge. Such a framework must build on scientific objectivity and be cognizant of contextual differences inherent to water issues - Water Diplomacy Framework (WDF) proposed in Islam and Susskind (2013) is a step in that direction. The WDF is based on the following four propositions:

- water is not a fixed resource; knowledge of water can be shared across scales to make it a flexible resource;
- water issues are contextual and continuously changing in terms of their couplings with natural, societal and political forces;
- uncertainty, variability, nonlinearity, and feedback are not exogenous but rather linked in real world situations and must be accounted for;
- cooperative rather than competitive approaches to decision-making about water can provide an alternative to zero-sum thinking necessary for resolving water conflicts.

Key features of the WDF are presented in Fig. 1. To tackle the uncertainty and non-linearity issues related to water allocation for competing uses, the interval parameter programming approach will be used in this paper. To model the effect of cooperation, three different tools of cooperative game theory will be used to show how mutual gains can initiate trust building among involved parties compared to non-cooperative and zero-sum games.

2.1 Interval Parameter Programming

Because of the uncertainties in water resources management, interval programming model is considered to be a suitable approach to address cooperative water allocation problems. Let x denote a closed and bounded set of real numbers. An interval number $x^{\pm}$ is defined as an interval with known upper and lower bounds but unknown distribution membership information (Huang 1996):

$$x^{\pm} = [x^{-}, x^{+}] = \{t \in x | x^{-} \leq t \leq x^{+}\} \tag{1}$$

where x^{-} and x^{+} are the lower and upper bounds of $x^{\pm}$, respectively. When $x^{-}=x^{+}$ then $x^{\pm}$ becomes a deterministic number.

2.1.1 Linear Interval Parameter Programming

Let $R^{\pm}$ denote a set of interval numbers. A linear interval number programming model can be defined as follow (Huang 1996):

$$\text{Maximize } f^{\pm} = C^{\pm} X^{\pm} \tag{2}$$

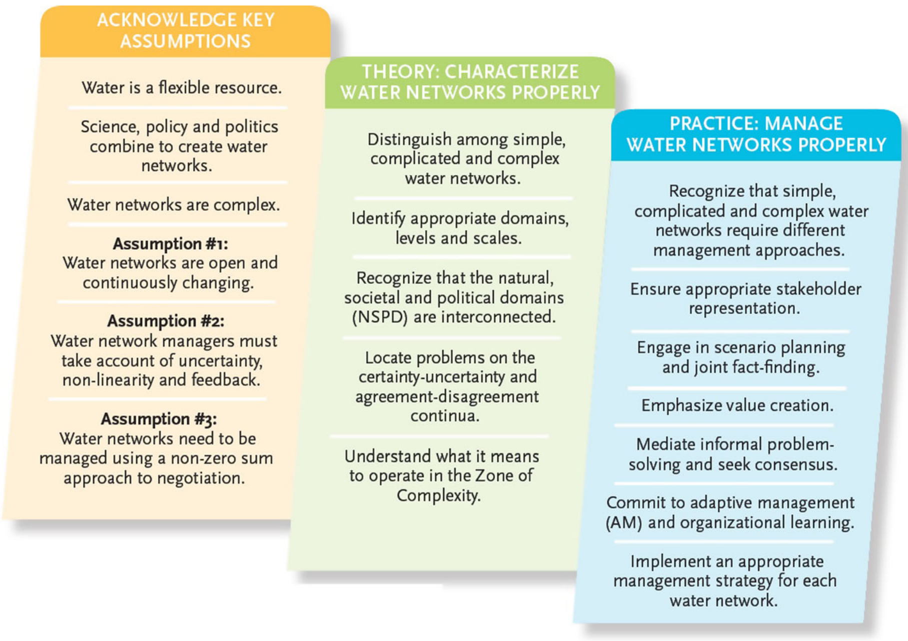


Fig. 1 The main ideas in the water diplomacy framework (Islam and Susskind 2013)

subject to:

$$A^{\pm}X^{\pm}\leq B^{\pm} \tag{3}$$

$$X^{\pm}\geq 0 \tag{4}$$

A robust two-step method was developed to solve this problem (Fan and Huang 2012). In this method a conservative submodel will be solved first; then an optimistic submodel will be derived based on solutions from the conservative submodel.

Assume n interval coefficients $C_j^{\pm}$ (j=1, 2,…, n) in the objective function of the model. If the first k of them are positive, the rest are negative, then, the following expressions can be developed for the conservative submodel;

$$\text{Max } f^{-}=\sum\nolimits_{j=1}^{k}c_j^{-}x_j^{-}+\sum\nolimits_{j=k+1}^{n}c_j^{-}x_j^{+}, \tag{5}$$

subject to:

$$\sum\nolimits_{j=1}^{k}\left|a_{ij}\right|^{+}Sign\left(a_{ij}^{+}\right)x_j^{-}+\sum\nolimits_{j=k+1}^{n}\left|a_{ij}\right|Sign\left(a_{ij}\right)x_j^{+}\leq b_i^{-}\quad\forall i \tag{6}$$

$$x_j^{-}\geq 0,\quad j=1,2,\ldots,k \tag{7}$$

$$x_j^{+}\geq 0,\qquad j=k+1,k+2,\ldots,n \tag{8}$$

x_{jopt}^{-}(j=1,2,…,k) and x_{jopt}^{+}(j=k+1,k+2,…,n) will be obtained from conservative submodel. The optimistic submodel can be formulated as follows (assume$b_j^{\pm}$>0, $f^{\pm}$>0):

$$\text{Max } f^{+}=\sum\nolimits_{j=1}^{k}c_j^{+}x_j^{+}+\sum\nolimits_{j=k+1}^{n}c_j^{+}x_j^{-} \tag{9}$$

subject to:

$$\sum\nolimits_{j=1}^{k}\left|a_{ij}\right|Sign\left(a_{ij}^{-}\right)x_j^{+}+\sum\nolimits_{j=k+1}^{n}\left|a_{ij}\right|^{+}Sign\left(a_{ij}^{+}\right)x_j^{-}\leq b_i^{+}\quad i=1,2,\ldots,m. \tag{10}$$

$$\sum\nolimits_{j=1}^{l_{i1}}a_{ij}^{-}x_j^{+}+\sum\nolimits_{j=l_{i1}+1}^{k}a_{ij}^{-}x_{jopt}^{-}+\sum\nolimits_{j=k+1}^{l_{i2}}a_{ij}^{-}x_j^{-}+\sum\nolimits_{j=l_{i2}+1}^{n}a_{ij}^{-}x_{jopt}^{+}\leq b_i^{+} \tag{11}$$

$$x_j^{+}\geq x_{jopt}^{-}\quad j=1,2,\ldots,k \tag{12}$$

$$x_j^{-}\leq x_{jopt}^{+}\quad j=k+1,k+2,\ldots,n \tag{13}$$

$$x_j^{+}\geq 0,\quad j=1,2,\ldots,k \tag{14}$$

$$x_j^{-}\geq 0,\quad j=k+1,k+2,\ldots,n \tag{15}$$

$$\left|a_{ij}\right|=\min\left\{a_{ij}^{-},a_{ij}^{+}\right\},\quad\left|a_{ij}\right|^{+}=\max\left\{a_{ij}^{-},a_{ij}^{+}\right\}\quad\forall i \tag{16}$$

where $a_{ij}^{\pm}\geq 0 \quad (j=1,2,\ldots,l_{i1};j=l_{i2}+1,\ldots,n)$, $a_{ij}^{\pm}\leq 0 \quad (j=l_{i1}+1,l_{i1}+2,\ldots,l_{i2})$ and $l_{i1}\leq k$, and $l_{i2}\geq k$.

Therefore, solutions $x_{j\ opt}^{\pm}=[x_{j\ opt}^{-},x_{j\ opt}^{+}]$ and $f_{j\ opt}^{\pm}=[f_{j\ opt}^{-},f_{j\ opt}^{+}]$ are the ultimate solutions for model (2).

2.1.2 Nonlinear Interval Parameter Programming

A general nonlinear interval number programming problem with uncertain interval coefficients both in the objective function and in the constraints is defined as follows:

$$\text{Minimize } f(X,U) \tag{17}$$

Subject to:

$$g_i(X,U)\geq(=\leq)\left[\nu_i^L,\nu_i^R\right],\quad i=1,\ldots,l, \tag{18}$$

$$X\in\Omega^n,\quad U=\left[U^L,U^R\right],\quad U_i=\left[U_i^L,U_i^R\right],\quad i=1,2,\ldots,q, \tag{19}$$

where f is objective function of X (decision vector) and U (uncertain vector). g_i is ith constraint. ν_i is the allowable interval number of ith constraint. To solve this nonlinear interval number programming problem the method of Jiang et al. (2008) can be applied. Based on an order relation of interval numbers, the uncertain objective function is transformed into a model with two deterministic objective functions. By this transformation, the uncertainty will be decreased and this objective function provides an accurate estimation of the sensitivity of outputs to the fluctuation of the uncertain input coefficients, thus the robustness of the decisions is also considered.

Through a modified possibility degree, the uncertain inequality and equality constraints are changed to deterministic inequality constraints. The possibility degree of interval number represents certain degree for comparing interval numbers. Zhang et al. (1999) introduced an approach using the possibility degree for comparing any two intervals. Thus the uncertain objective function can be transformed into a two-objective optimization problem as follows:

$$\text{Minimize } [m(f(X,U)),w(f(X,U))], \tag{20}$$

where

$$m(f(X,U))=\frac{1}{2}\left(f^L(X)+f^R(X)\right) \tag{21}$$

$$w(f(X,U))=\frac{1}{2}\left(f^L(X)-f^R(X)\right) \tag{22}$$

With a certain value of X, $f(X,U)$ is an interval number, when its bounds $f^L(X)$, $f^R(X)$ can be obtained (Ma 2002) as

$$f^L(X)=\min_{U\in\Gamma}f(X,U),\quad f^R(X)=\max_{U\in\Gamma}f(X,U), \tag{23}$$

$$U\in\Gamma=\left\{U|U^L\leq U\leq U^R\right\}$$

We require that the inequality constraint $g_i(X,U)\geq[\nu_i^L,\nu_i^R]$ in Eq. (18) is satisfied with a possibility degree level, and reformulate it as a deterministic inequality:

$$P_{C\geq D}\geq\lambda_i, \qquad C=\left[g_i^L(X),g_i^R(X)\right], \qquad D=\left[\nu_i^L,\nu_i^L\right] \quad , \tag{24}$$

where $P_{C\geq D}$ is the possibility degree of the ith constraint. $0\leq\lambda_i\leq1$ is a predetermined possibility degree level. C is the interval of the constraint function at X and its bounds can be obtained through two deterministic optimization processes:

$$g^L(X)=\min_{U\in\Gamma}g(X,U), \qquad g^R(X)=\max_{U\in\Gamma}g(X,U) \tag{25}$$

Thus Eq. (12) can be transformed into an unconstraint and single-objective optimization problem in terms of a penalty function $\tilde{f}$;

$$\text{Minimize } \tilde{f}=\frac{(1-\beta)(m(f(X,U))+\xi)}{\phi}+\frac{\beta(w(f(X,U))+\xi)}{\varphi} \\ +\sigma\sum_{i=1}^{k}\varphi(P_{M_i\geq N_i}-\lambda_i), \tag{26}$$

where

$$\phi=\min_{X\in\Omega^n}(m(f(X,U))+\xi), \quad \varphi=\min_{X\in\Omega^n}(w(f(X,U))+\xi), \tag{27}$$

$0\leq\beta\leq1$ is a weighting factor of the two objective functions (Zarghami and Szidarovszky 2011), furthermore ξ is a number which makes $m(f(X,U))+\xi$ and $w(f(X,U))+\xi$ non-negative. In addition, ϕ and φ are two normalization factors, σ is the penalty factor which is usually specified as a large value and φ is a function with the following form (Jiang et al. 2008):

$$\varphi(P_{M_i\geq N_i}-\lambda_i)=(\max(0,-(P_{M_i\geq N_i}-\lambda_i)))^2 \tag{28}$$

The water resource allocation mechanism is designed in two stages. In Fig. 2, the proposed methodology is explained in details. The users could select the noncooperative solution such as Nash equilibrium or might choose a cooperative solution such as Shapley value or Nucleolus. In the first step of allocation, an economic objective function is used to find an initial water allocation scheme to the stakeholders. The initial allocation scheme is subject to the systems constraints. Since the players make their decisions simultaneously, a logical assumption is that the players select the Nash bargaining solution (Nash 1953). Nash proposed his bargaining solution for two-person games, when the players maximize the product of their gains over what each would receive without agreement. This is the only solution satisfying certain properties such as efficiency, symmetry, independence of unit changes and independence of irrelevant alternatives (Nash 1953). In the second stage, cooperative game theoretical concepts are applied to this model. In cooperative games, the actors are the coalitions. They may range from non-cooperative coalitions where the players act to maximize only their own benefits, to full cooperative coalitions where all players act cooperatively to maximize the coalition's benefit. Partial coalitions, with certain subsets of players, may also be formed. In applying cooperative games, we have to determine the characteristic function value for each possible coalition. The discrete function given by the costs or benefits of every coalition is called the characteristic function and represents a key element of the cooperative game solution (Sechi and Zucca 2015).

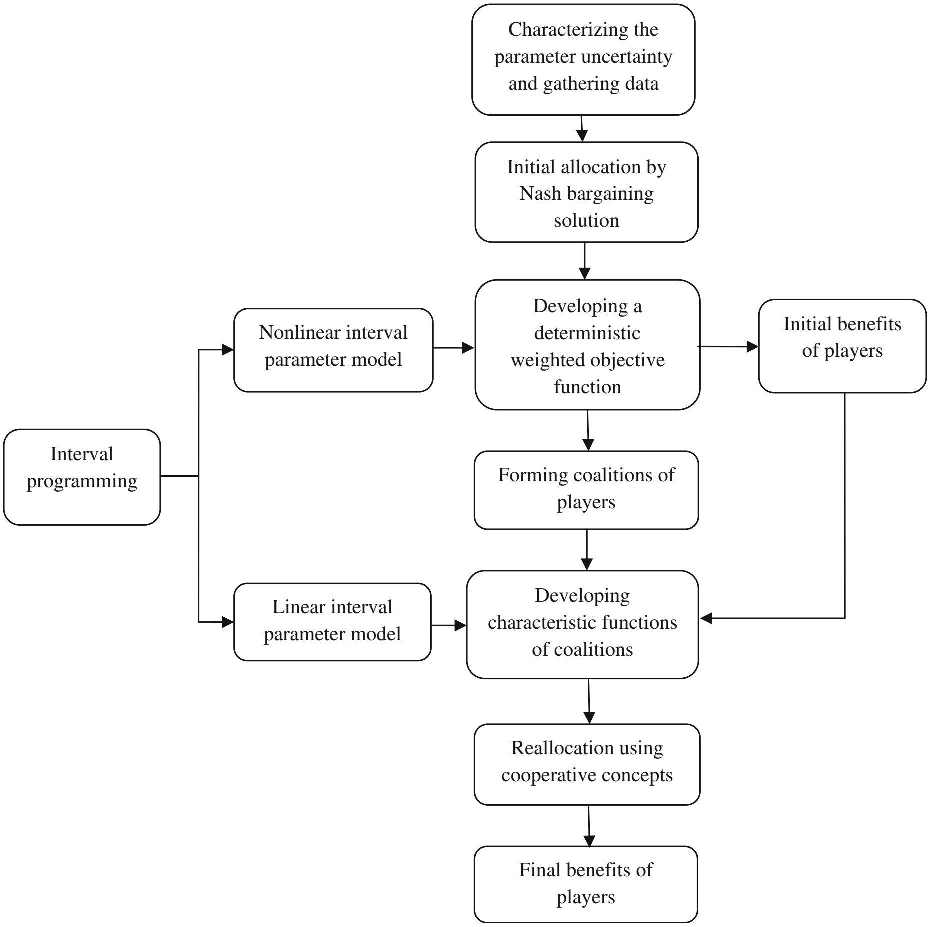


Fig. 2 Framework of the proposed methodology

In the first stage, the water allocation model maximizes the net benefits derived for the various water users, including agricultural, domestic and industrial consumption. In the second stage the characteristic function values of the various coalitions are first calculated, and then the concepts of Shapley values or Nucleolus are used to obtain optimal cost allocations. There are other alternative methods and concepts to solve cooperative games, they can be used similarly to the cases presented in this paper.

2.2 The Shapley Values

The Shapley Value is a solution concept that prescribes a single payoff for each player. It is the average of all marginal contributions of that player to each coalition he or she can be a member of (Shapley 1953). The Shapley Value is a unique solution concept in cooperative game theory that allocates the total surplus generated by the grand coalition to each member based on each player's average contribution to all coalitions in that game (Dinar and Nigatu 2013). The Shapley Value for player i is defined as

$$\varphi_i(\nu) = \sum_{S \subseteq N} \frac{(s-1)!(N-s)!}{N!} [\nu(S) - \nu(S-i)] \tag{29}$$

where S runs through all coalitions containing player i, $S-\{i\}$ is the coalition obtained by excluding i, s is the number of players in coalition S, N is the total number of players, $\nu(S)$ is the characteristic function value of coalition S. In (29), the first part gives the probability of a particular player joining coalition S and the bracketed expression gives the contribution that the particular player can make to the coalition by joining.

2.3 The Nucleolus

The Nucleolus concept is based on the idea of excesses. The Nucleolus is the reward vector for which the excesses for all coalitions are as small as possible uniformly. That is, this concept makes all players less unhappy uniformly, so it can serve as a mutually acceptable solution concept. The Nucleolus allocation is a single solution that is always in the Core, if the Core is non-empty and provides a fair and efficient allocation of benefits (Madani and Dinar 2011). An excess is the amount by which the worth of a coalition exceeds the aggregate payoff to its members in isolation. The excess of coalition S with respect to payoff vector x is defined as

$$e(S,x) = \nu(S) - \sum_S x_i \tag{30}$$

The Nucleolus minimizes lexicographically the maximum excess of the coalitions by sequentially solving the following problem (Schmeidler 1969):

$$\text{Minimize } \ e \tag{31}$$

subject to:

$$\sum_{i \in S} x_i \geq \nu(S) - e \quad (\text{for remaining coalitions}) \tag{32}$$

$$\sum_{i \in N} x_i = \nu(N) \tag{33}$$

This optimization problem is first solved involving all coalitions. The coalition for which $e(S, x)$ equals the critical value is eliminated from constraint (32) in the next step and this reduced model is then solved. Continuation in this manner reduces the number of constraints by one at each step. The procedure terminates if either a unique optimal solution is obtained or all coalitions are eliminated.

2.4 The Normalized Nucleolus

In applying Normalized Nucleolus (Lejano and Davos 1995) the excess is replaced by the ratio of the excess and the total payoff of the coalition: $e_n = \frac{\nu(S) - \sum_S x_i}{\sum_S x_i}$. Then the Normalized Nucleolus is found by replacing problem (31)–(33) by the following:

$$\text{Minimize } \ e_n \tag{34}$$

subject to:

$$\sum_{i \in S} x_i \geq \nu(S)/(1+e_n) \quad (\text{for remaining coalitions}) \tag{35}$$

$$\sum_{i \in N} x_i = \nu(N) \tag{36}$$

An important property of the normalized nucleolus is that it is monotonic in the aggregate in a proportional manner.

3 Case Study

The Zarrinehrud river basin with an area of about 11,578 km^2 is in Northwestern Iran. It is located among four provinces of West Azerbaijan, East Azerbaijan, Kurdistan and Zanjan but its largest share belongs to West Azerbaijan (Fig. 3a). The motivation to select this basin is due to its importance in Northwestern part of Iran. This basin is the major water supplier to the Urmia Lake, the second largest salt lake on earth (Encyclopædia Iranica 2014). The lake is now under critical condition because of over-using its fresh recharging water resources by agriculture, domestic and industrial firms. In addition to higher number of dams on the rivers inside the basin, climate change has also a significant effect.

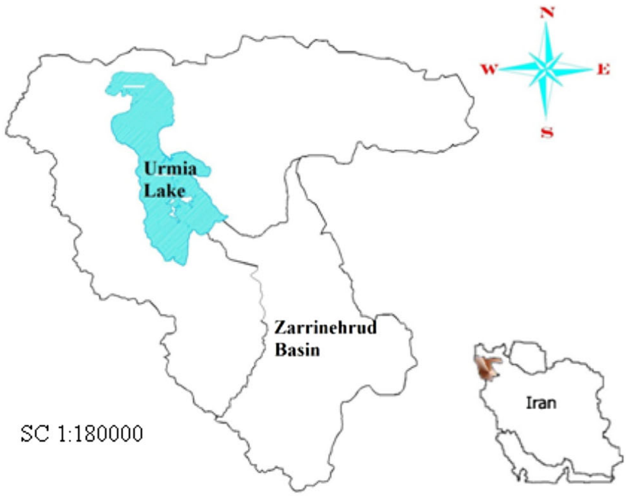
The length of the Zarrinehrud River is 240 km and the average montly discharge of the river is about 139.5 MCM (West Azerbaijan Regional Water Authority 2006). The Martyr Kazemi dam was built on this river in 1970, which also called the Zarrinehrud dam. This dam provides the required water amount to the Mianduab plain, a part of Bonab and the Malekan plains. Gross agriculture area of this basin is 64,640 and 58,171 ha from this area is irrigable.

In addition to agriculture consumption, the Zarrinehrud River supplies more than 40 % of the domestic water demand of Tabriz city by a pipeline. Maximum annual water transmission value according to the design is 157 MCM for domestic consumption and industrial uses. Figure 3b presents a systematic sketch of the Zarrinehrud Sub-Basin.

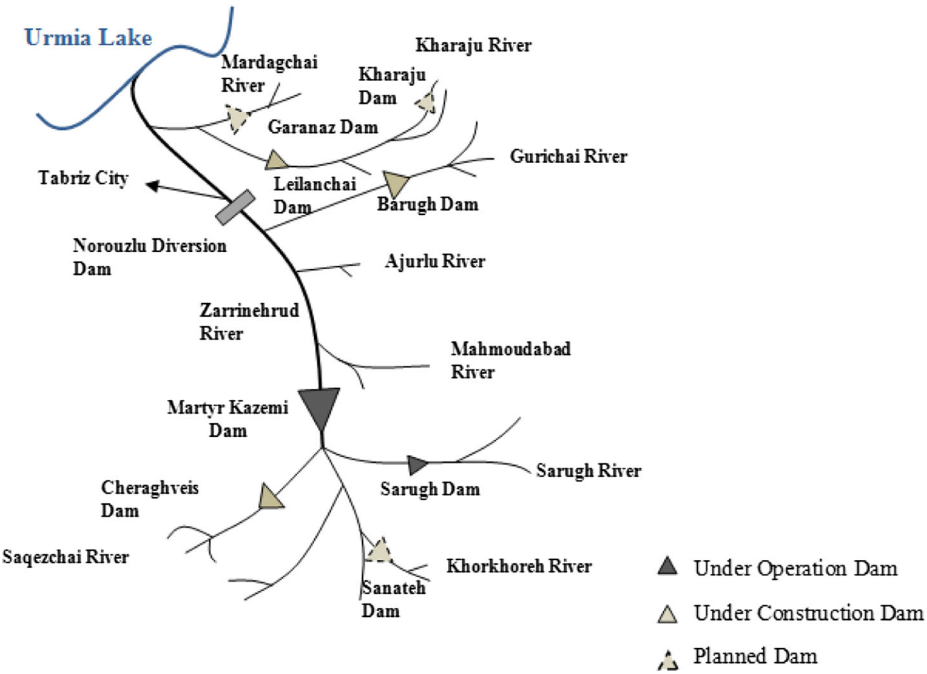
In the first step for initial allocation, we used the Nash bargaining solution. Since the users make decisions simultaneously without knowing those of the other users and compete for limited resources, a logical assumption is that the users select a bargaining solution such as the Nash solution which satisfies Nash fairness axioms. In this game, three players are considered: Player 1 is the agricultural section, Player 2 is the domestic section and Player 3 is the industrial stakeholder. A crucial issue for the water allocation is the protection of the environmental needs of the Zarrinehrud River and Urmia Lake. The lake's surface area has been estimated to have been as large as 6100 km^2 but since 1995 it has generally been declining and was estimated from Landsat satellite data to be only 2366 km^2 in August of 2011 (UNEP and GEAS 2012) and just about 1400 km2 in July 2015.

Therefore, the model should include the minimum environmental requirements of the Urmia Lake and Zarrinehrud River as constraints. The environmental constraints were used in the optimization models for finding minimum values of the objectives so they were not explicitly used in determining the disagreement point, they were implicitly taken into account as constraints.

We assume that the stakeholders select the Nash bargaining solution. In the initial water allocation model, the constraints include the water continuity equation in the Shahid Kazemi



(a)



(b)

Fig. 3 Location of the Zarrinehrud river sub-basin within the Urmia lake basin, Iran (**a**) The sketch of the Zarrinehrud main streams and hydro-structures (**b**)

reservoir, the constraints related to the environmental flow of the Zarrinehrud River and other physical constraints. There is a significant uncertainty in the inflow to the reservoir because of its high dependency on climate variability. Moreover, cost data are seldom available as deterministic values or reliable probability distributions. In this case, inflow and uncertain economic inputs can hardly specified as with a PDF and/or fuzzy membership function however they can be expressed as intervals. Therefore interval parameter programming is the effective method to handle these uncertain parameters. The variations for the ranges of uncertain parameters are determined by combining historical data and engineering judgment. The lower and upper bounds of long-term average monthly inflows to the reservoir are presented in Table 1. The problem is modeled for a 12 months planning periods. Therefore, this leads to the following nonlinear interval parameter optimization model:

$$MaximizeZ^{\pm} = \prod\left(NB^{\pm}_{1,m} - d_{1,m}\right)\left(NB^{\pm}_{2,m} - d_{2,m}\right)\left(NB^{\pm}_{3,m} - d_{3,m}\right) \tag{37}$$

subject to:

$$R_m = \sum\nolimits_{i=1}^{n} w_{i,m} + d_{env,m}, \tag{38}$$

$$S^{\pm}_{m+1} = S^{\pm}_m + I^{\pm}_m - R_m - L_m \quad , \tag{39}$$

$$S_{\min} \le S^{\pm}_m \le S_{\max}, \tag{40}$$

$$w_{i,m,\min} \le w_{i,m} \le w_{i,m,\max}, \tag{41}$$

where $NB^{\pm}_{i,m}(i=1,2,3)$,$I^{\pm}_m$, $S^{\pm}_m$ and also $Z^{\pm}$ are interval numbers. Here $NB^{\pm}_{1,m}$, $NB^{\pm}_{2,m}$, $NB^{\pm}_{3,m}$ are the net benefit functions of agricultural, domestic and industrial demands in month m, $d_{1,m}$, $d_{2,m}$, $d_{3,m}$ are the corresponding disagreement values. It is assumed that the disagreement points are calculated using the following equation when the decision makers are unable to reach an agreement (Kerachian and Karamouz 2007):

$$d_{i,m} = NB_i\left(x_{i,\min}\right) \tag{42}$$

Table 1 The lower and upper bounds of the monthly long-term inflow to the reservoir (MCM)

Month	Lower bound	Upper bound
January	16.61	19.50
February	30.21	35.46
March	194.35	228.14
April	193.34	226.96
May	54.26	63.70
June	17.66	20.74
July	14.65	17.19
August	13.6	15.96
September	10.71	12.57
October	7.86	9.22
November	16.59	19.47
December	21.52	25.26

where $x_{i,\min}$ is the minimum acceptable value of x_i for decision maker i. By this assumption, in disagreement point, each decision maker achieves the minimum value of available resources. The minimum values of monthly consumption data of previous years are estimated as $x_{i,\min}$. Furthermore $d_{env,m}$ is the minimum environmental requirement of the Urmia Lake and Zarrinehrud River in month m, $w_{i,m}$ is the allocated water volume to section i in month m and $w_{i,m,\min}$, $w_{i,m,\max}$ are its minimum and maximum values, R_m is the reservoir release in month m, $I_m^{\pm}$ is the inflow to the reservoir in month m, L_m is the amount of reservoir evaporation, $S_m^{\pm}$, $S_{m+1}^{\pm}$ are the reservoir storage amounts in months m, $m+1$ and $S_{\min}$, $S_{\max}$ are the common minimum and maximum values of $S_m^{\pm}$.

The net benefits are the differences of the gross benefits (B_i), derived from agricultural, domestic and industrial water uses in the basin and the costs (C_i) of water supply (Abrishamchi et al. 2011).

The total net benefit of water usage to agriculture is:

$$NB_{1,m}^{\pm} = \sum_{j=1}^{n} \left[Y_{j,m} p_{j,m} - C_{j,m}^{\pm} \right] A_{j,m}, \tag{43}$$

where

n	Number of crops
$NB_{1,m}^{\pm}$	Net benefit in month m (IR Rails)
j	Crop index
$A_{j,m}$	Area of crop j (ha) as a function of water amount allocated to its demand in month m
$p_{j,m}$	Price of crop j in month m (IR Rials/kg)
$Y_{j,m}$	Yield of crop j in month m (kg/ha)
$C_{j,m}^{\pm}$	Uncertain cost of crop j production in month m (IR Rials /ha).

The net benefit function to domestic water usage is defined as follows:

$$NB_{2,m}^{\pm} = \left(w_{2,m} \right) \times \left(s_{2,m} - r_{2,m}^{\pm} \right) \tag{44}$$

where

$NB_{2,m}^{\pm}$	Benefit received by domestic users in month m (IR Rials)
$w_{2,m}$	Amount of water consumed by domestic users in month m (MCM)
$s_{2,m}$	Domestic water charge (IR Rials/m^3)
$r_{2,m}^{\pm}$	Price of water for domestic demand presented by the Iran Ministry of Energy (IRMOE 2010)

$$NB_{3,m}^{\pm} = \left(w_{3,m} \right) \times \left(s_{3,m} - r_{3,m}^{\pm} \right) \tag{45}$$

where

$NB_{3,m}^{\pm}$	Benefit of water consumption to industrial users in month m (IR Rials)
$w_{3,m}$	Amount of water consumed by industrial demand in month m (MCM)
$s_{3,m}$	Industrial demand water charge (IR Rials/m^3)
$r_{3,m}^{\pm}$	Price of water delivery to the industrial section presented by the IRMOE (2010) (IR Rials/m^3).

In model (26) ξ is a number which makes $m(f(X,U))+\xi$ and $w(f(X,U))+\xi$ non-negative. In this model, $m(f(X,U))+\xi$ and $w(f(X,U))+\xi$ are non-negative and ξ selected as zero. The normalizing factors ϕ and ψ in Eq. (26) are specified as 5.63E+15 and 7.73E+14 by solving Eq. (27). The weighting factor β can be determined by any standard approach known for the literature of multiobjective programming. To simplify, in this work we assumed equal preferences for two objective functions and the weighting factor β is set to be 0.5. The penalty factor σ is assumed to be 1000 as a large number in comparison to $\sum_{i=1}^{k}\varphi(P_{M_i \geq N_i} - \lambda_i)$ and 0.80 is selected for λ that is a acceptable possibility degree level in this model. In this problem the reliability and security are the most important issues, therefore a relatively large satisfactory degree level is specified. The required data are provided by the West Azerbaijan Agriculture Jihad Organization and West Azerbaijan Regional Water Authority.

4 Results and Discussion

The general algebraic modeling system (GAMS), a widely used modeling language is employed to optimize the deterministic weighted objective function (Brooke et al. 1998). Fig. 4 shows the numerical results.

According to several meetings by the officials working in the region of case study, a cooperative water management system will be the strategic solution for the lake. Therefore cooperative game theoretic concepts are applied next to this allocation problem. The seven possible coalitions of the players are listed in Table 2.

In the next step we calculate the characteristic function values for all coalitions. The characteristic function value is the best possible outcome of a coalition without the cooperation of the other players. Each coalition solves the corresponding optimization problem to maximize its net benefit obtained from the use of the available water by all participating stakeholders in each coalition with the assumption that the players outside the coalition do not cooperate. The linear interval optimization model will be used to calculate the characteristic function values of these coalitions. This model is based on interactive algorithm resulting in two deterministic sub models, which correspond to the lower and upper bounds of the values

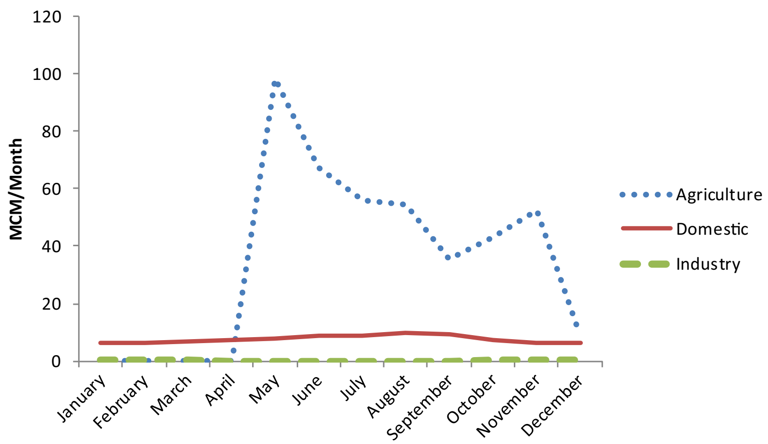


Fig. 4 Initial monthly water allocations based on the Nash bargaining solution (MCM)

Table 2 All possible coalitions after satisfying the environmental needs and the annual ranges of characteristic function values of the coalitions (million IR Rials/year)

No.	Players in coalitions	Coalition type	Characteristic function values
1	Agriculture	Non-cooperative	[201652, 222402]
2	Domestic	Non-cooperative	[142870, 151970]
3	Industry	Non-cooperative	[6600, 6800]
4	Agriculture + Domestic	Partial Cooperation	[391500, 437520]
5	Agriculture + Industry	Partial Cooperation	[259860, 322030]
6	Domestic + Industry	Partial Cooperation	[212620, 278180]
7	Agriculture + Domestic + Industry	Full Cooperation	[420420, 507190]

of the objective functions and the constraints. Table 2 summarizes the ranges of overall annual net benefits for all possible coalitions.

Since the water shares are calculated based on achieving the highest total net revenue, the net benefit proportions to the different users is not necessarily equal. Hence, the reallocation of the net benefits among the water users is carried out in order to provide a relatively fair condition (Mahjouri and Ardestani 2011). Next the total net benefit is reallocated to the water users using cooperative game theoretical approaches such as the Shapley values, the Nucleolus or the Normalized Nucleolus. The calculation is based on the data of the years 2000–2010. Table 3 shows the upper and lower bounds of the reallocated total net benefits in a year for players based on the different cooperative solution concepts and the results for the Nash bargaining solution. For comparison, the result of using deterministic model is also calculated. These results are obtained for all four methods however to save the space, the results shown in the last column of Table 3 is for the Shapley value.

As shown in Table 3, the solutions under the four methods present different benefits intervals. In all patterns, the total benefit of allocation to the agriculture sector is higher than allocation to the domestic and industrial users. In certain cooperative games sum of the Shapley values equals the total benefit of the grand coalition (Shapley 1953). This property is not necessarily true in this interval parameter model as it is shown by the numerical results, however the interval of the total benefit in the Shapley values contains the interval obtained based both versions of the nucleolus. Therefore the Shapley values have larger uncertainty. This observation might show a difficulty in using Shapley values in certain cases. In this case, the results using the Nucleolus are very similar to those based on the Normalized Nucleolus but the Shapley values have very large intervals of the objective values and therefore they are

Table 3 Reallocation of annual benefits according to the Shapley value, the Nucleolus, the normalized nucleolus, the Nash bargaining solution and the deterministic Shapley value in the Zarrinehrud River (million IR Rials/year)

Player	Shapley value	Nucleolus	Normalized nucleolus	Nash bargaining solution	Deterministic Shapley value
Agriculture	[196729, 274004]	[215517, 236797]	[213495, 235270]	[201652, 222402]	215598
Domestic	[142906, 217674]	[168270, 192940]	[167520, 192410]	[142870, 151970]	201679
Industry	[12851, 83444]	[36633, 77453]	[39405, 79510]	[6600, 6800]	50398
Total	[352486, 575122]	[420420, 507190]	[420420, 507190]	[351122, 381172]	467675

not applicable due to large uncertainty in the results. Table 3 also describes the initial net benefits for each stakeholder based on the nonlinear interval parameter model. The upper bounds in the three cooperative solutions are always higher than the upper bound obtained by the Nash bargaining solution. The same holds for the lower bounds as well, except for the Shapley values.

The industrial water user has the lowest total benefit among the sectors. It is necessary to mention that this benefit is for the sector and not benefit per water unit. Therefore, the participation of the industrial sector in any coalition, results in a considerable increase in its benefit and therefore this sector should join the coalition.

The characteristic function value for the grand coalition is $f^{\pm}$=[420420, 507190] $\times 10^6$ IR Rials which means that the system benefit would change between f^- and f^+ as the decision variables take different values within their lower and upper bounds. In fact, the decision maker can acquire a higher system benefit in the upper values, however it will lead to a higher risk for the optimization model and make the water resource management system more unstable. Conversely, lower values of the objective function and the constraints would decrease the risk and enhance the system stability, however they would generate lower system benefits and also may lead to less use of the available water resources. Consequently this solution can generate appropriate decision alternatives for the decision makers. Therefore, the decision makers could adjust the level of the uncertainty within their lower and upper bounds to reach a tradeoff between risk and system benefit, and then generate appropriate decision alternatives depending on the optimism degree of the decision makers.

5 Conclusions

This paper introduces the concept of cooperative game theory in combination with interval parameter programming for a water allocation problem in the Zarrinehrud river basin, Iran. Since many of the factors have uncertain features, and may be known only as interval numbers, a new interval parameter cooperative game theoretic model is formulated for this problem. In this game, the agricultural, domestic and industrial users are competing for their equitable share of water while satisfying the environmental constraint. The paper offers a methodology based on game theory that can encourage innovative problem-solving where the players join in forming a coalition for mutual gains in the midst of uncertainties. The Shapley values, the Nucleolus and the Normalized Nucleolus approach lead to efficient water usage and improved efficiency through water transfers to achieve maximum benefits for the basin as a whole.

The results show that water allocation agreement can improve the efficiency of water allocation if the benefits of cooperation are distributed properly. This is an incentive to reach an agreement and it decreases the player's probability to leave the coalition. Therefore, the new model provides stable and more sustainable solutions to the problem. However, to ensure the stability and sustainability of the cooperation, suitable laws or authorizations should be introduced by the governmental organization based on the findings of this research. Overall, the results presented above indicate that there is an increase in benefit to all players through cooperation in comparison to their gains under the Nash bargaining solution. The benefits interval according to the Nucleolus are [215517, 236797], [168270, 192940] and [36633, 77453], and based on the Nash bargaining solution are [201652, 222402], [142870, 151970] and [6600, 6800] for the agriculture, domestic and industrial users, respectively. Actually, the total net benefit obtained in the cooperative water allocation models (Shapley and Nucleolus) is higher than the total net benefit in the initial allocation model (Nash Bargaining). Findings suggest that use of cooperative game theory could objectively demonstrate creation of mutual gains to achieve an effective negotiated agreement for water allocation. In addition, these results indicate that, through the proposed modeling approach, uncertain information can be effectively incorporated by using the interval parameter programming into the cooperative water allocation processes. It is then clear that modeling the uncertainty within the cooperative game theoretic approaches encourage players to join coalitions to achieve overall maximum and reliable benefits in the basin. The paper also does not provide a comparison between the two cooperative game solutions of Shapley and the Nucleolus. Another limitation of the paper is that it does not consider recent decision made by the Urmia Lake Restoration Program, for example, effects of restoring the lake by temporarily taking some of the farmlands out of agricultural use – to enhance water flow to the lake – by providing rent to the owner. We plan to explore this creative option in our future study.

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Compliance with Ethical Standards This paper is original manuscript and it is not submitted elsewhere. Research does not involve Human Participants and/or Animals.

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