Optimizing Cycling Power

by

Alexander D. Springer

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Signature of Author: _____

Department of Mechanical Engineering
May 6, 2016

Certified by: _____

Anette Hosoi
Professor of Mechanical Engineering
Thesis Supervisor

Accepted by: _____

Anette Hosoi
Professor of Mechanical Engineering
Undergraduate Officer

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ABSTRACT

In this study we determine a viable bioenergetic model for power allocation during a cycling race. Various models have been proposed to address power allocation in races with two models rising above others: the Morton-Margaria Three Tank model and the Skiba Energy Balance model. The energy balance model was implemented in MATLAB and compared against the gold standard implementation in Golden Cheetah to model the depletion of an athlete's energy over the course a ride. The implementation of the model was successful as verified by ride data from a cyclist in the 2014 Tour de France. Additionally, the model was further tested with sample power profiles in order to understand the depletion of energy over the course of a ride.

Two key findings emerged from the investigation. First, we require a better account of exhaustion in the energy balance model which can be achieved by weighting the time spent below critical power over the time spent above critical power. This is because a cyclist becomes more exhausted by efforts at higher power outputs compared to the recovery at an effort below critical power.

Second, energy balance models should use a variable time constant as rides and races have highly variable recovery periods below critical power which affects the ability of an athlete to reconstitute their energy. Use of a variable time constant could address the weighting of efforts below critical power identified in the first finding as well.

Thesis Supervisor: Anette Hosoi
Tile: Professor of Mechanical Engineering

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1. INTRODUCTION

Athletes are increasingly seeking diminishing margins to optimize in order to distinguish themselves in their discipline. When podium places are separated by a few seconds, or even less, on races that vary from a few hundred meters to a few hundred miles, anything which can help an athlete cross the line a little faster can provide immense benefits. In the sport of cycling, performance optimizations can make a huge difference among athletes. In the 2009 Tour de France, 31 seconds separated the Levi Leipheimer, 3rd place, from Alberto Contador who won the grueling stage race in 91 hours and 26 seconds (McGann Publishing LLC). The time difference between the podium finishers represents 0.009% of the overall race time. Indeed, every little bit counts when optimizing race performance.

Cyclists have a singular goal while racing; to get across the finish line first. While the goal is simple, achieving it requires much more nuance. Like many modern sports, cycling is governed by numbers and metrics. For most metrics, the major unit of interest is watts, or energy over time. Watts are a measure of the power a cyclist puts into the bike and is dependent on how a cyclist allocates their energy over the duration of the race. Cyclists can choose to generate a lot of power for a short amount of time, or use less power over a long period of time. How a cyclist chooses to use their energy to generate power on the bike is a key differentiating factor in separating a first place finish from a mid-pack placing.

The goal of this thesis is to determine a mathematical model and set of parameters which optimizes the power allocation of a cyclist in a competitive cycling event. This model will take into account factors like the individual power output of a cyclist, the geography of the course, the recovery time of the athlete, and other parameters to provide an individual recommendation for the optimal power output for a cyclist. A few theoretical models such as the 3-Tank, Exertion

Curve, and Skiba model have been proposed to allocate power but none have yet been verified with experimental data. Using these models, predicted cycling power output will be compared to actual power output and recovery times using data from elite cyclists' training rides and professional cycling races in the Tour de France. A recommendation will be made for an optimal mathematical power model validated with real race data and with specific physiological parameters accounted for. This recommendation will be used to develop a model for cyclists to gauge real-time power output in a race with optimal power output. This model for cyclists would vastly improve the current real-time output of watts and turn data into actionable data, providing information on when to attack, when to ease up, and how much energy is remaining in an athlete's gas tank for a final sprint.

2. BUILDING THE OPTIMAL PACING STRATEGY

2.1 Critical Power Curve

Critical power (CP) is the maximum power level, in watts, that a cyclist can sustain for a given length of time. The maximum length of time a cyclist can hold a given power beginning with full energy constitutes a single data point on the critical power curve. For every watt value, there is a finite time period which a cyclist can hold a certain power. Each of these data points forms the critical power curve (CPC). Often, the shape of the graph follows a hyperbolic curve where time is along the x-axis and power is along the y-axis. This suggests that cyclists can hold extraordinarily high levels of power for a short duration (ex: in a sprint) while they can hold comparatively low levels of power for a much longer duration (ex: in a time trial). Fortunately, modern software suites such as Golden Cheetah and Strava, commonly used by cyclists, automatically calculate the critical power curve based on aggregated ride data from power

meters[1]. The programs identify parts of rides that correspond to high power outputs and low power outputs along with the duration of time at which that power was maintained and form the critical power curve accordingly. Figure 2-1 shows an example critical power curve using data compiled by Golden Cheetah software.
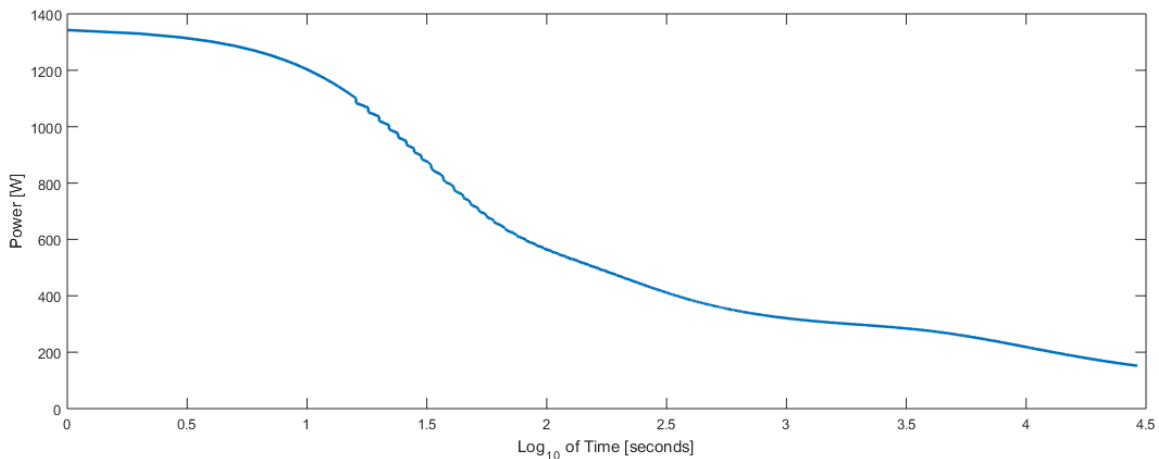

Figure 2-1: Example critical power curve plotted on a log time scale showing the decay of power as time increases. $Log_{10}(0.5)$ is equal to 3.2 seconds and $Log_{10}(4)$ is equal to 2.8 hours.

In conjunction with the critical power curve, another common term cyclists use is the functional threshold power (FTP). Functional threshold power is the average maximal power output a cyclist can sustain for one hour. When cyclists speak about their critical power, FTP is often used to describe their critical power with one value instead of the curve commonly used to give the full picture.

Testing for FTP is difficult for many athletes since pacing a maximal effort for one hour is both challening to do and costly in terms of time required to recover from such an effort. As a result, many athletes complete a 20 minute average maximal power test in a time trial type of fashion on a straight flat course to get a critical power reading for the twenty minute interval.

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[1] A power meter is a common tool used by cyclists which measures their power in real time either through the crank arms, the cassette, or pedal spindle depending on the brand of the power system. Common systems are the CycleOps PowerTap, Stages Power, and Quarq systems.

Then, FTP can be calculated by taking 95% of the average power from the 20 minute test (Wattbike). Together with the critical power curve, FTP provides a partial picture of a cyclist's fitness level through their ability to generate maximal power on the bike. Furthermore, these tools also provide a point of comparison to objectively compare cyclists in terms of raw power output. However, a fairer comparison of power between cyclists would normalize by weight because heavier cyclists must generate more power to travel the same distance and speed as lighter cyclists. Thus, normalized power metrics like FTP and the CP curve in units of watts per kilogram are also common in the sport of cycling.

Due to the fact that the critical power curve constitutes an athlete's maximum average power output for a given length of time, an athlete may exceed or fall below this power value during the course of race. Because the critical power value is an average, it would imply that any deviation above the value is equally matched by a deviation below the critical power value. However, race data shows that this is not the case (Skiba, Chidnok and Vahatalo). Exceeding critical power for a given time is physiologically costly and requires much more time below critical power to recover than simply the amount of time to average out to the critical power value. As a result, it is advisable when optimizing pacing to stay as close to the optimal critical power as possible without exceeding it. Furthermore, research has also shown that the greater the deviation above the critical power threshold; the faster an athlete becomes exhausted or must take exceedingly longer to recover (de Lucas, de Souza and Costa). In other words, exceeding critical power by small amount for a moderate amount of time is often less physiologically costly than exceeding critical power by a great amount for a short amount of time. Athletes will often give anecdotal evidence of the increase in perceived effort and recovery times for sprints in short interval workouts versus longer above-threshold intervals. From these results, a linear averaging of

efforts above and below critical power would not be physiologically relevant. Rather, any efforts above critical power must be weighted much more heavily as they are physiologically more costly and the athlete must take longer to recover. In determining an optimal pacing strategy, a cyclist must consider this phenomenon when considering exceeding their critical power to stay with the pack up a hill, or in deciding whether to launch a sprint attack.

2.2 Human Bioenergetic Models

Once a cyclist determines their critical power curve, there is a simple first order calculation that can be performed to find the optimal power allocation for a given race. This can be done by calculating the total energy needed to complete the race, and dividing by a critical power value to determine the time it would take to complete the race at that given power. If the time to complete is equal to the time a cyclist can maintain that power output, then that is the optimal power allocation for that ride. However, this method fails in real races due to variable pacing and power requirements.

To begin with, calculating the total energy needed to complete a race is difficult because of the variable forces that act upon a cyclist. There is a friction force which varies according to the road surface. The gravitational force would be relatively straightforward to calculate by taking the topology of the course into account and considering all elevation changes over the length of the course. There is also a drag force which varies according to a cyclist's speed, wind direction and speed, humidity, elevation, whether a cyclist is drafting behind another rider, and many more factors. Accounting for all of these factors reliably and predicting them ahead of time to calculate the anticipated energy expenditure in a race is not a reliable way of determining the fastest way to win a bike race.

Because of this problem, a model is needed which accounts for real-time output of power and considers the physiological response of the athlete in a race. Power data from a typical ride is plotted in Figure 2-2 along with the critical power line for a ride of the specific duration for the athlete. The figure demonstrates the high variability both above and below critical power for a given ride. Accounting for the recovery time below critical power as well as the exertion above critical power requires an advanced power allocation model which accounts for physiological factors along with the physical geography and course conditions. Various models have been proposed to account for the physiological factors, each with slightly different methods of implementation. Two of the leading models include the Morton-Margaria 3-Tank model and the Skiba Energy Store model.
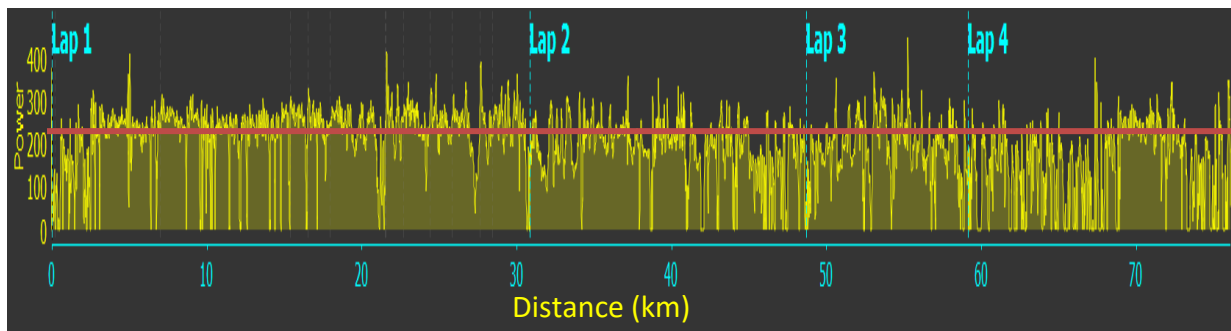

Figure 2-2: Power data from a typical ride plotted along with the cyclists' critical power line of 223 W for this ride length. The power data is highly variable with varying time intervals spent above and below the critical power threshold. Developing a model to account for recovery time below critical power and exertion above critical power is necessary to optimally allocate power in a cycling race.

2.2.1   Morton-Margaria Three Tank Model

The Morton-Margaria Three Tank model conceptualizes the human body as a series of vessels which follow the laws of fluid flow in determining work output (Morton). In accordance with the name of the model, there are three tanks which constitute the bioenergetics model and correspond to the body's three ways of producing energy. The three tank model is depicted in Figure 2-3. Tank P represents the phosphagen system and is connected to the work output W

through tap T which regulates the flow of all net energy expenditure above rest. The phosphagen system contributes to adenosine triphosphate (ATP) production in the fastest manner by using stored creatine phosphate molecules and can operate anaerobically, which is to say, without oxygen (Karp). While the phosphagen system is the most efficient at energy production, it depletes quickly because the body does not store a lot of creatine phosphate and is really only used for extreme efforts for periods of up to 10 seconds (Karp).

The O tank is only partially shown due to its infinite capacity because it represents the oxidative, also known as the aerobic, source. The O tank is connected to the P tank through the R_1 tube. The aerobic system, in contrast to the phosphagen system, has a very low rate of ATP production, but its capacity is infinite (Karp). The infinite capacity allows people to do aerobic activities like walking for seemingly endless amounts of time.

The last tank, the L tank, represents the glycolysis anaerobic energy production system. The upper level of the L tank is set at some height above the bottom level of the O tank but below the top level which means that the L tank only begins to fill the P tank when the O tank has partially emptied. Physiologically, this means that an athlete enters glycolysis anaerobic production at some point above rest but below the anaerobic threshold. Once an athlete exceeds the glycolysis rate of production, all they have remaining in terms of energy production is a small amount of creatine phosphate which the human body utilizes rapidly. Once depleted, the athlete must rest and recover to let their energy tanks refill. The L tank is connected to P through a one-way tube R_2 and P is connected back to L through another one-way tube R_3. R_2 is a much thicker tube than R_3 which means that L can empty into P much faster than it can be refilled by P. Incidentally, when an athlete has depleted all tanks and must rest, the reflow of energy comes from tank O as oxygen is used to produce more ATP, which is used as an energy source in the body to produce

the other molecules that are used in the phosphagen and glycolysis production systems. Effectively, this means that refill goes from tank O to tank P to tank L through tubes R_1 and R_3, respectively.

The glycolysis production system is used for high intensity exercise lasting from about 30 seconds to 2 minutes and produces ATP at a faster rate than the phosphagen system (Karp). The anaerobic energy production in the glycolysis system is responsible for the buildup of lactic acid in muscles (Karp). The buildup of lactic acid does a number of things in the body including inhibiting muscle contraction and interfering with electrical charges sent to the muscles (Karp). Thus, the body literally has an emergency stop mechanism built into this production system which forces the muscles to stop contracting and the athlete to rest when an athlete has emptied their tank and exceeded their ability to produce energy anaerobically.
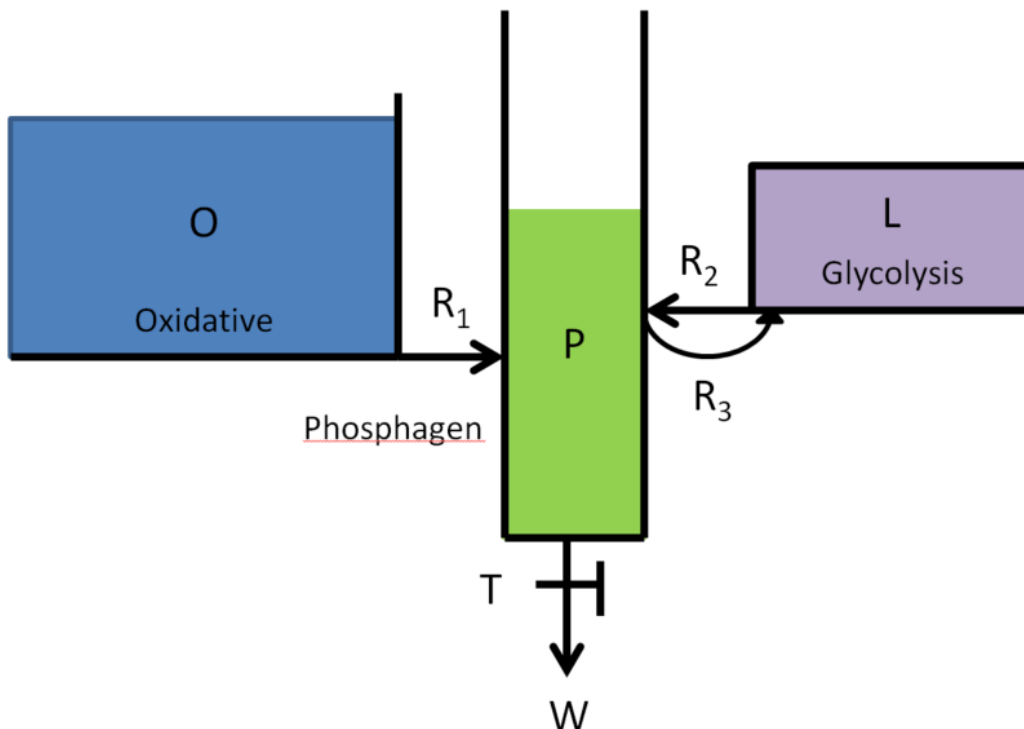

Figure 2-3: The Morton-Margaria Three Tank bioenergetics model. The model represents the three energy producing systems of the human body; oxidative, phosphagen, and glycolysis; as hydraulic tanks feeding into a work output, W for any energy expenditure.

While the Three Tank model provides an accurate conceptualization of energy flow through the body, Morton and Margaria did not quantify the flow rates which determine the model. It is known that the phosphagen and glycolysis anaerobic energy production is time limited and each system has less capacity than the oxidative system. An athlete's recovery rate is also generally known as evidenced by oxygen uptake and heart rate. From this information, the refill time of the tanks could theoretically be determined. However, to find out the relative flow rates of R_1, R_2, and R_3, one would have to come up with a novel testing scheme which may stretch the limits of the applicability of the hydraulic system metaphor. Alternatively, one could program the hydraulic fluid flow equations into a computer and match the parameters to work output data to try and fit the work curve as closely as possible. This last approach seems like the best possibility of quantifying this model until it is realized that this model does not factor in a criteria for exhaustion. In other words, an athlete exercising will eventually become exhausted with the emptying of the three tanks and will not perform according to the flow rates put forth by a curve matching algorithm. The Three Tank model, while appealing in its intuitiveness, does not lend itself well towards a quantitative model of work output and pacing optimization for cyclists.

2.2.2   Skiba Energy Store Model

The Skiba Energy Store model, like the Three Tank model, conceptualizes the human body in an intuitive way, this time by considering the body as a gas tank of energy storage. In the Skiba model, the primary parameter of interest is *W'*, which is the finite work capacity above critical power (Skiba, Chidnok and Vahatalo). For any ride of a given amount of time, a cyclist can maintain a work output less than or equal to their critical power for the entire duration of the ride, but as soon as a cyclist exceeds that critical power threshold, they start to use the finite amount of gas in their tank, *W'*. A parameter like *W'* is much more suitable than the Three Tank

theory for modeling optimal pacing strategies in a race because it allows consideration of supra-CP efforts like sprints or attacks which happen often in a cycling race to be factored into consideration. Furthermore, with modern cycling power meters, power output can be measured during the course of a ride thereby allowing *W'* to be calculated in real time to accurately respond to changes and deviations from CP during a race.

The calculation of *W'* relies on the hyperbolic relationship between power, CP, and time. It was first mathematically formulated in the current accepted state in 2010 in a paper examining exercise tolerance and VO_2 max (Jones, Vanhatalo and Burnley). *P* is equal to power output, and *t* is equal to the time to exhaustion at that power output (Jones, Vanhatalo and Burnley).

$$P = \left(\frac{W'}{t} \right) + CP \qquad [1]$$

The energy store model relies on three assumptions for calculating the balance of energy during a period of exercise. These assumptions are: 1) the expenditure of *W'* begins when the athlete exceeds CP, 2) the energy balance begins to increase again when the athlete falls below CP, and 3) the reconstitution of *W'* follows an exponential time recovery path which weights recent efforts more heavily than efforts further back in time (Skiba, Chidnok and Vahatalo). Given these assumptions, an equation can be formulated describing the balance of *W'* (*W'$_{bal}$*) where some amount of *W'* was expended (*W'$_{exp}$*):

$$W'_{bal} = W' - \int_0^t W'_{exp}(t)\, e^{-\frac{(t-u)}{\tau'_W}}\, dt \qquad [2]$$

where *W'* is calculated from equation [1], ($t - u$) is the time in seconds between exercise segments above CP, and $\tau_{W'}$ is the time constant for reconstituting *W'*. This formulation takes

into account the fading memory of *W'* during recovery below CP in which recent efforts are weighted more heavily than efforts which occurred in the distant past of the particular workout.

After a trial of seven subjects undergoing three different exercise tests to determine recovery rate, the recovery constant equation was determined as follows:

$$\tau'_W = 546e^{-0.01D_{CP}} + 316 \qquad [3]$$

where D_{CP} is the difference between recovery power and CP in the tests performed (Skiba, Chidnok and Vahatalo). To determine this equation, the data were best fit to an exponential regression with a close correlation of $r^2 = 0.77$, especially given the low number of test subjects (Skiba, Chidnok and Vahatalo). The equation suggests that the minimum recovery time for a complete reconstitution of *W'* upon emptying for an athlete is 316 seconds, a little over five minutes, with the recovery time increasing afterwards depending on the work output during the rest interval (D_{CP}).

The equations above, taken together, constitute a simple and practical application of real-time energy monitoring of athletes in competitive races. Modern cycling power meters can integrate data into these equations and provide actionable data in the form of *W'*, informing an athlete when it is necessary to recover or when it is permissible to attack given the history of work output in the race by the athlete.

3. MODELING THE OPTIMAL PACING STRATEGY

With the foundational equations in place, the optimal pacing strategy can be implemented in computer software. The ultimate goal is to create a real-time energy level indication for a cyclist while competing in a race. This can be done using equations [2] and [3] combined with data from a cyclist's power meter. Alternatively, for running, swimming, and other sports, power can be

replaced by speed, critical power can be replaced by critical speed, and time can be replaced by distance. Using the equations above, a similar energy tank model could be constructed.

The model was constructed using data posted online from various riders during the 2014 Tour de France (TrainingPeaks). From this data, time and power were collected over the course of the race. The implementation of the energy storage equations was done in MATLAB, chosen for its ease of use and ability to handle large datasets. With one power data point for every second of a ride, considering that rides last several hours long there can be many thousands of data points for a single ride which must each be integrated over separately, a computationally expensive process. MATLAB is a well-suited program to handle such a task.

Figure 3-1 below shows the calculation of *W'* for a cyclist plotted over the same time period as the corresponding power for that ride. When the power exceeds the critical power threshold, indicated by the dashed blue line, the W'_{bal} begins to decrease rapidly as a rider is drawing from their energy storage. However, when the cyclist falls below the critical power line, the W'_{bal} begins to increase again, albeit slowly. The difference in steepness of the slopes for recovery and depletion of W'_{bal} make intuitive sense; a cyclist must take a longer amount of time to recover than the time they spent above critical power. Furthermore, the closer to critical power that an athlete recovers at, the longer it will take to recover. Again, this makes intuitive sense as recovery at a very easy, low wattage output after an interval can be done much faster than recovery closer to critical power during a race. These two findings indicate that the change in W'_{bal} is dependent both on whether an athlete is above or below critical power as well as the magnitude of deviation from critical power.
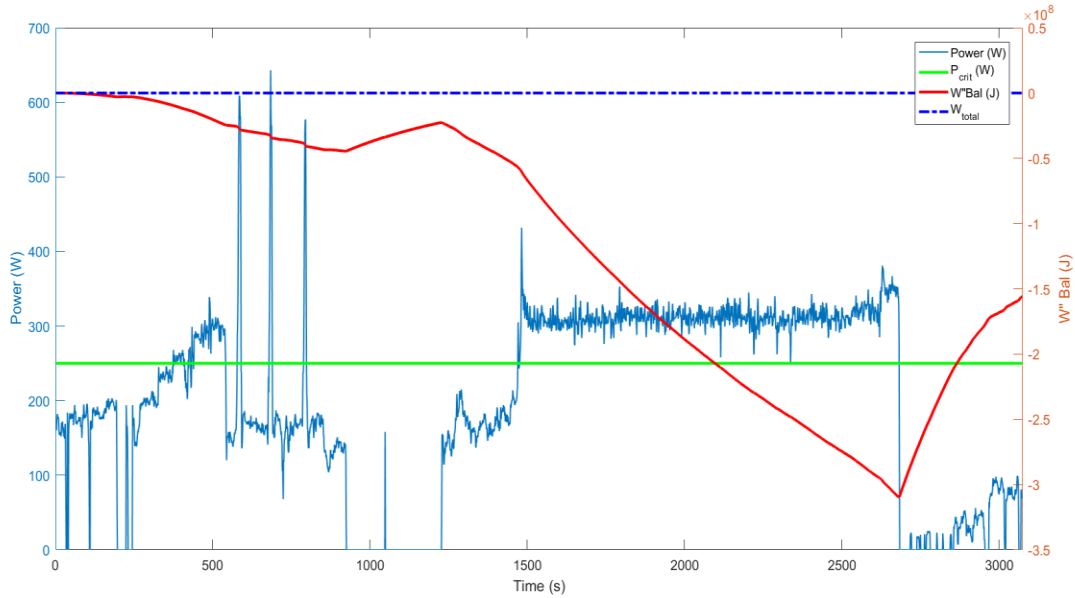
Figure 3-1: Modeled W'_{bal} of a Tour de France cyclist during a race plotted against power output. When the cyclist exceeds their critical power, indicated by the dashed blue line, the W'_{bal} falls sharply. When the cyclist enters recovery by cycling below their critical power, W'_{bal} rises slowly until full recovery.

Figure 3-2 shows the same ride plotted using the Golden Cheetah software. The Golden Cheetah implementation of W'_{bal} is considered the gold standard in terms of fidelity to the original conception of the equation by Skiba (Mantica). For this reason, a close match to the Golden Cheetah W'bal equation will be considered to be an accurate implementation of the energy balance model.

As shown in the figure, the Golden Cheetah software shows a similar energy balance profile for the ride compared to the MATLAB implementation. The same parameters; $\tau_{W'}$, critical power, and W' were used in each example which further validates the implementation of the W'_{bal} equation in MATLAB.
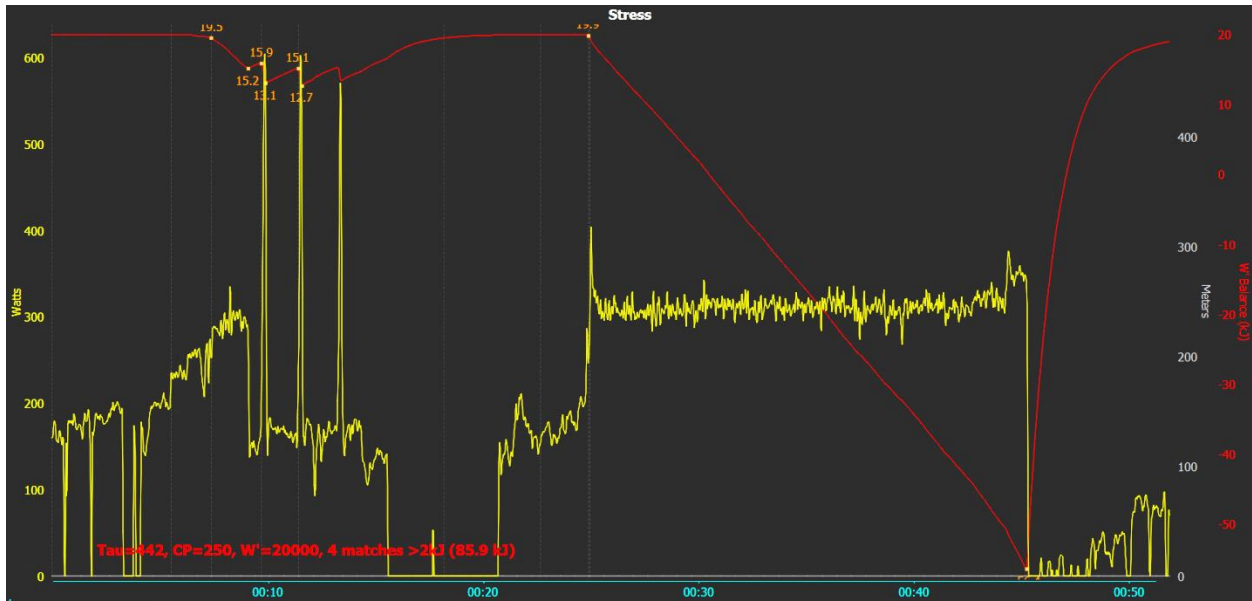
Figure 3-2: Golden Cheetah produced plot of W'_{bal} using the same $\tau_{W'}$, critical power, and W' parameters in the MATLAB implementation highlighting the similarities and repeatability of the energy balance model created.

To further test and refine the model, example power profiles were created to determine if the model behaved as expected. These test cases included intervals above and below critical power varying in length of time and in magnitude of deviation above and below critical power. Figure 3-3 shows four separate graphs consisting of different power profiles and the corresponding W'_{bal} profile. The first profile shows power as a step function going from below critical power to above critical power. The second profile shows power as a linear function increasing with time. While this is not a realistic model while riding, it nonetheless accurately shows the response that the W'_{bal} function decreases as the critical power threshold is approached and exceeded and furthermore, decreases more rapidly as critical power is increasingly exceeded. The third profile consists of a constant power function above critical power. This model is also outside of the realm of physiological possibility as W'_{bal} has a minimum value of 0 which represents complete exhaustion. However, if an athlete were to continue above critical power indefinitely, their W'_{bal} function would continue to decrease and become negative, as shown in the third graph. Lastly,

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the fourth profile shows a square function alternating between equal periods of above and below critical power output. Although the deviation from critical power is the same number of watts on either side, the W'_{bal} function progressively decreases. This is because W'_{bal} accounts for the increased recovery needed after activities above critical power as the body takes longer to recover after bouts of intense exercise than the duration of exercise performed. Because the W'_{bal} function closely follows the implementation in the Golden Cheetah program, it adequately handles the four described test cases, and is in agreement with physiological intuition.
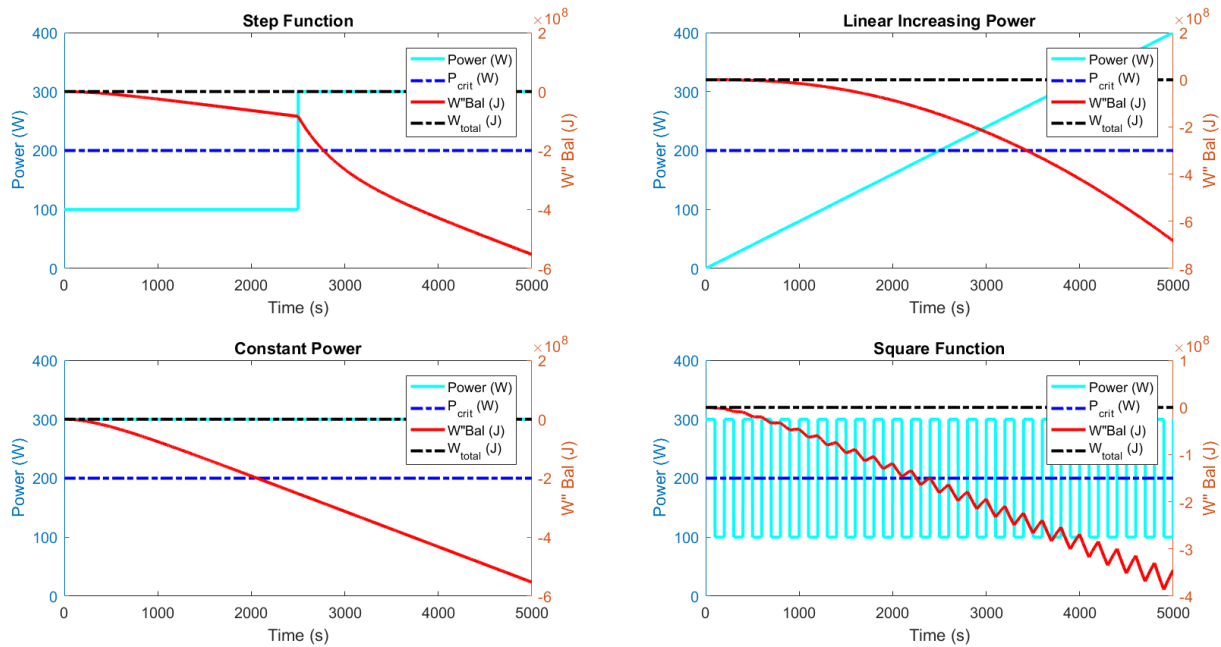

Figure 3-3: Four test cases of the W'_{bal} function plotted on the same axes as power profiles. (*clockwise from left*) 1) Step function from below critical power to above critical power beginning at time *t*. 2) Linear power output increasing monotonically throughout the sample time. 3) Constant power output above critical power threshold. 4) Square function alternating evenly above and below critical power and showing the difference in recovery and exhaustion rates that W'_{bal} predicts.

4. DISCUSSION

4.1 Weighting Recovery Time

After modeling the energy tank reserve of a cyclist, a key finding must be raised. That is, intuitively, recovery time for efforts above critical power should take much longer than the respective time interval above critical power. For example, a cyclist who sprints for five seconds should not be able to recover from the effort within five seconds. However, the current formulation of Skiba's model does not consider any kind of weighting factor for recovery intervals after critical power is exceeded. In the most simple implementation of the weighting, a recovery constant, $\alpha(W'_{exp}(t))$, could be inserted in equation [2] and be implemented only when current power is below critical power. Since the recovery time interval for below critical power efforts should be longer, α should be less than or equal to one.

$$W'_{bal} = W' - \alpha(W'_{\exp}(t)) \int_0^t W'_{exp}(t)\, e^{-\frac{(t-u)}{\tau'_W}}\, dt \qquad [4]$$

$$\text{where} \quad \alpha \leq 1 \quad if \quad W'_{exp} \leq CP$$
$$a = 1 \quad if \quad W'_{exp} \geq CP$$

The α value would be another rider parameter which could be derived from ride data or an equation could be fitted to it dependent on the rider similar to the variable $\tau_{W'}$ in equation [3].

4.2 Limitations of the Energy Balance Model

The first limitation of the model is the data from the Tour de France rides, and indeed a ride from any athlete, does not imply that a rider will empty their tank. Since W' is calculated based on previous rides, this is a value that will have to be recalculated and changed frequently depending on fitness and how hard a cyclist pushes themselves. This is because W'_{bal} should never be below zero, which would signal a negative energy balance. If an inaccurate W' is used

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and an athlete enters a particularly hard effort, it is feasible that their W'_{bal} could become negative if W' is not calibrated correctly. Fortunately, a negative W'_{bal} has no effect on the W'_{bal} overall profile, only the specific value of W'_{bal}. If W'_{bal} were displayed on a cycling computer, it could cause confusion for the user but not disrupt the actual data.

A good method of finding an upper limit of W'_{bal} for cyclists would be to look at the W'_{bal} of elite cyclists during monument races. Monument races are one day stage races which take place in Europe over the course of the spring and summer and are grueling six plus hour races in which riders push themselves as hard as they can to win. Monument races, as opposed to stage races or other forms of riding, are ideal because a rider is not consciously saving any energy over the course of the ride so the W' calculation from an all-out race like a monument would be an excellent indication of the total energy balance of an athlete.

Finally, a major assumption made in the calculation of W'_{bal} is a constant $\tau_{W'}$. As seen in equation [3], $\tau_{W'}$ is variable based on the difference in recovery below critical power. However, a rider is hardly ever consistently constant in power while recovering over the course of a ride and the power data is quite noisy with variance of around 5% in most cases. This would mean that $\tau_{W'}$ would have to be recalculated every second of the ride which is very computationally costly. Of course, $\tau_{W'}$ would vary relatively little during each of these calculations because even deviations of 5% in power would affect $\tau_{W'}$ in a relatively similar magnitude depending on the critical power and recovery power output of the rider.

Following the practice set forth in the Skiba et al. paper, $\tau_{W'}$ was calculated by taking the average power below critical power over the course of the ride and using that value to calculate D_{CP}. For a first-order approximation, this assumption yields surprisingly accurate results, though consideration of a variable $\tau_{W'}$ would yield more accurate results. Furthermore, this assumption

works relatively well while doing interval training where the recovery periods are similar following the work periods; however it breaks down when considering data from races. Because the course elevation and speed due to attacks vary greatly over the course of a race, deviations below critical power could be significant such as when a rider is coasting down a descent or they could be minor such as when a rider is sitting in the draft of the peloton. In this case, the assumption of a uniform $\tau_{W'}$ for the whole ride fails due to the high variability of power exerted during sub-critical power periods.

Implementing a variable $\tau_{W'}$ could also account for the weighting factor necessary between efforts over and under critical power. For instance, an athlete recovering significantly below their critical power will regain their energy much faster than an athlete recovering near their critical power. As such, the D_{CP} of the athlete recovering at a lower power output will be much higher, which will cause the recovery time constant to be lower, leading to a faster recovery. In this way, a variable D_{CP} could allow consideration of recovery efforts at different power outputs to affect the speed at which a cyclist regains their energy after efforts above critical power.

5. CONCLUSION

To reiterate, the goal of this thesis is to build upon existing human bioenergetic models and provide recommendations for the optimal allocation of power for an athlete during a race. After evaluation of many models, the two most promising models considered were the Morton-Margaria Three-Tank model and the Skiba Energy Balance model. However, the Morton-Margaria model was discarded due to a lack of consideration for an exhaustion factor as well as a lack of data for determining flow rate parameters between an athlete's energy tanks. Accordingly, the Energy Balance model was modeled in MATLAB using data from a cyclist

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who rode in the 2014 Tour de France in order to test the validity of the model in predicting energy capacity and exhaustion in athletes. The derived model performed very well against the gold standard implementation of the model in the Golden Cheetah software suite. Additionally, characteristic power profiles were also tested on the model to see how it would account for variations in power and were validated against intuition about physiological responses to exertion.

Two novel findings arose as a result of this investigation. First, the energy balance model must consider a weighting for recovery below critical power versus time spent above it. The present model uses an equal weighting for time spent above and below critical power when reconstituting energy. However, consideration of exhaustion by the athlete must be made as athletes become more exhausted exercising above critical power than a comparable amount of time spent riding below their critical power. Secondly, the assumption of a constant recovery rate $\tau_{W'}$ used in the Skiba et al. paper is invalid due to the highly variable recovery outputs from cyclists over the course of a ride and especially in a race. Moving forward, implementations of W'_{bal} should include a variable recovery rate dependent on the deviation below critical power for the recovery interval. Considering this, implementing the variable recovery rate could address the issue raised by the first finding by evaluating recovery intervals further below critical power with lower time constants signifying that athletes will regain their energy faster than recovery periods closer to the critical power threshold. With this in mind, implementing a variable recovery rate should be a top priority for future iterations of performance optimization models.

Apart from the variable recovery rate, future work in the area of human bioenergetics models could examine modeling in other sports such as running and swimming. Although the implementation would switch critical power for critical speed and time for distance in both cases,

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it is likely that the individual athlete parameters and equations are sport specific and would require reformulation using additional tests from athletes. Building future bioenergetics models more accurately could usher in a new level of performance optimization as athletes are able to determine how much energy their bodies have to guide their decisions during crucial moments in races. Formulating these models with data measured by various sport instruments turns data into more than simply a number; it becomes a guide for decision making. In short, bioenergetic models can turn data from numbers into actions.

6. REFERENCES

de Lucas, R.D, et al. "Time to exhaustion at and above critical power in trained cyclists: The relationshipo between heavy and severe intensity domains." *Science & Sports* (2013): e9-e14.

Jones, Andrew M, et al. "Critical Power: Implications for Determination of VO2 Max and Exercise Tolerance." *Med Sci Sports Exerc* (2010): 1876-1890.

Karp, Jason. *The Three Metabolic Energy Systems*. 1 February 2009. 19 April 2016. <http://www.ideafit.com/fitness-library/the-three-metabolic-energy-systems>.

Mantica, Stefano. "Human Performance Modellling on Wearable Devices." *Research Gate* (2015). Online. <https://www.researchgate.net/publication/281934699>.

McGann Publishing LLC. *Tour De France Winners, Podium, Times*. 2015. 18 April 2016. <http://bikeraceinfo.com/tdf/tdfindex.html>.

Morton, R. Hugh. "The critical power and related whole-body bioenergetic models." *European Journal of Applied Physiology* (2006): 339-354.

Skiba, Philip F, et al. "Modeling the Expenditure and Reconstitution of Work Capacity above Critical Power." *American College of Sports Medicine* (2012): 1526-1532.

TrainingPeaks. *2014 Tour de France Week 1 Power Analysis*. 5 July 2014. 20 April 2016. <http://home.trainingpeaks.com/blog/article/2014-tour-de-france-power-analysis-week-1>.

Wattbike. *Functional Threshold Power*. n.d. 19 April 2016. <https://wattbike.com/us/functional-threshold-power>.