Mechanics of Entrapment with Applications to Design of Industrial Parts Feeders

by

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B.S., Indian Institute of Technology (1989)
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Abstract

Parts feeders are devices used in automated assembly lines to present component parts to the assembly machinery in a consistent and predetermined orientation. The process of converting a randomly oriented set of parts (as obtained from a vendor) into an ordered array is called parts feeding. One method of orientation is to convey the parts over an oscillating pallet that has reliefs carved on it which trap the parts in the desired orientation. The mechanics of how a part that approaches a relief gets trapped in it is not well understood. As a result design parameters such as chamfer shapes and dimensions, oscillation pattern, oscillation frequency and amplitude are chosen empirically.

The objectives of this thesis are to understand the mechanics of entrapment for a one-part-relief system and to apply it to the design of entrapment reliefs and choice of oscillation parameters.

The thesis discusses conditions for a part to "get trapped" and "stay trapped" in a relief. The tendency for a part to be thrown out is shown to depend upon the kinetic energy of the part, pallet acceleration and chamfer geometry. Allowable limits on design parameters for the part to "stay trapped" are derived for simple part shapes and oscillation patterns. These are also experimentally verified. The event of "getting trapped" is shown to be probabilistic and entrapment is shown to depend upon the conditions at entry. The dependence of the probability of entrapment on the design parameters is explained. Possible design options to improve the likelihood of entrapment and the drawbacks of each option are discussed.

Thesis Supervisor: Mark J. Jakiela
Title: Associate Professor

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I am grateful to my thesis advisor, Professor Mark Jakiela for his guidance through the course of this work. His patience and trust helped me negotiate the many difficult turns this work took. I am indebted to him for this opportunity to work with him.

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This work would not have been possible without financial support from The MIT Leaders for Manufacturing Program. High-speed video recording equiment was provided by the MIT Edgerton Center and Polaroid Corporation. The APOS feeder used for the experiments was provided by Polaroid Corporation. I am grateful to Mr. Paul Moncevicz and Mr. Jack Duggin of Polaroid Corporation for their patience in answering all the questions I had about the design of entrapment reliefs and operation of the APOS Feeder. I thank Mr. Tony Callogero of the MIT Edgerton Center for his help in using the high-speed video camera.

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I thank my friends and family for their continuous support and encouragement.

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Introduction

This thesis constitutes an investigation into the mechanics of entrapment of a part in a relief and pertinent design issues. It was motivated by the need to improve performance of an industrial parts feeder. This chapter provides an introduction to the phenomenon of entrapmment, describes the industrial parts feeder that motivated this thesis and lays out the objectives and contributions of the thesis.

1.1 What is entrapment?

Entrapment is best described with the following example. Consider a golf ball putted towards a hole. The hole is usually only marginally larger than the ball. As the ball encounters the hole and interacts with the walls of the hole, it may either fall into the hole or get deflected off the walls away from the hole. The act of the ball falling into the hole is called entrapment. The likelihood of the ball getting trapped depends upon its velocity as it encounters the hole and the direction of its approach. Higher the velocity of the approaching ball, lower is its probability of getting trapped. Prior to its approach to the hole, the motion of the golf ball is characterized by rolling motion. Since rolling involves low energy dissipation, an initial "push" on the ball is sufficient to carry it to the vicinity of the hole. The interaction of the ball with the walls of the hole is characterized by either rolling (which is continuous contact) or collisions (intermittent contacts) with the walls of the hole.

Another example of entrapment is a basketball falling through the hoop. Depending upon its direction of approach and its velocity the motion of the ball is characterized by either rolling or bouncing. The energy that the ball loses as it falls through the hoop is much less than the energy that a golf ball loses.

This thesis deals with an instance of entrapment that arises in an industrial parts feeder. Entrapment is used as a means to orient and present component parts to the assembly robot. Unlike a golf ball, parts in a parts feeder usually cannot roll and need to be conveyed by vibration of the conveying surface. The likelihood of entrapment would depend upon the pallet oscillation conditions and shape of the relief in addition to the velocity of the part as it aproaches the relief. This thesis investigates the effect of each of these factors on entrapment.

1.2 Motivation: industrial parts feeding

This thesis was motivated by a need to improve the performance of an industrial parts feeder. Parts feeders are devices used in automated assembly lines to present parts to the assembly robot in the precise location and orientation. The need for parts feeders arises because typically injection molded plastic parts and stamped or rolled metallic parts are obtained in randomly oriented bulk from the vendor. They need to be separated and presented to the assembly robot in a consistent and predetermined orientation. Figure 1-1 shows some typical industrial parts that are oriented with the help of parts feeders. These parts are usually small gears, levers, screws or springs and have a range of sizes and shapes.

There are many different types of parts feeders used in the industry. Bowl feeders are one of the early types of feeders. This thesis, however, is concerned with a more recent type of feeder called Sony's Automated Parts Orienting System (APOS)([14, 19]).

1.2.1 Sony's APOS

Figure 1-2 shows the schematic of an APOS. The parts, initially stored in the hopper, are made to flow over the surface of a vibrating and slightly inclined pallet. As they flow over the pallet they get trapped (in the desired orientation) in reliefs carved on the pallet surface. Of the number of parts that flow over the pallet only a small fraction gets trapped.

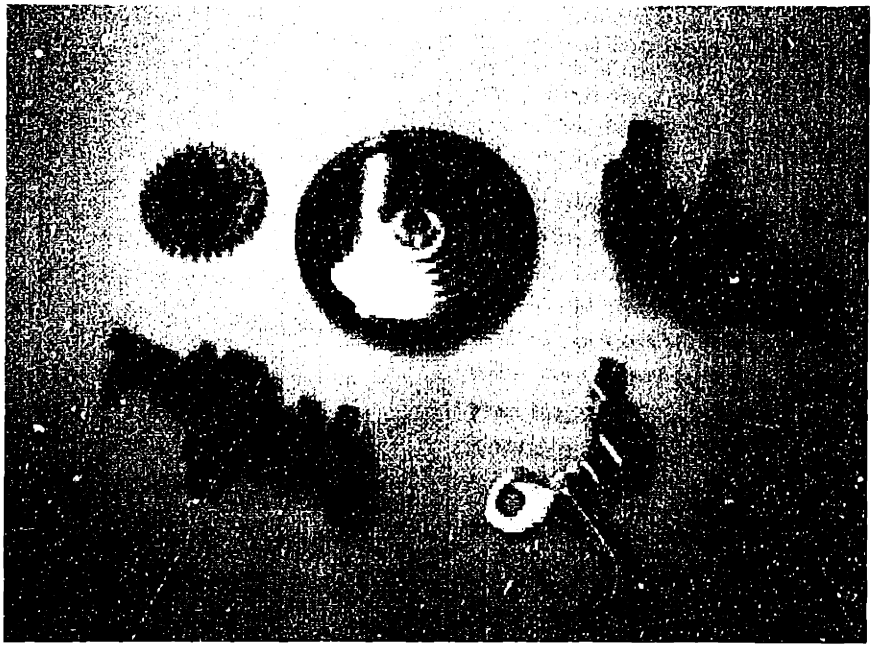

Figure 1-1: Typical Parts

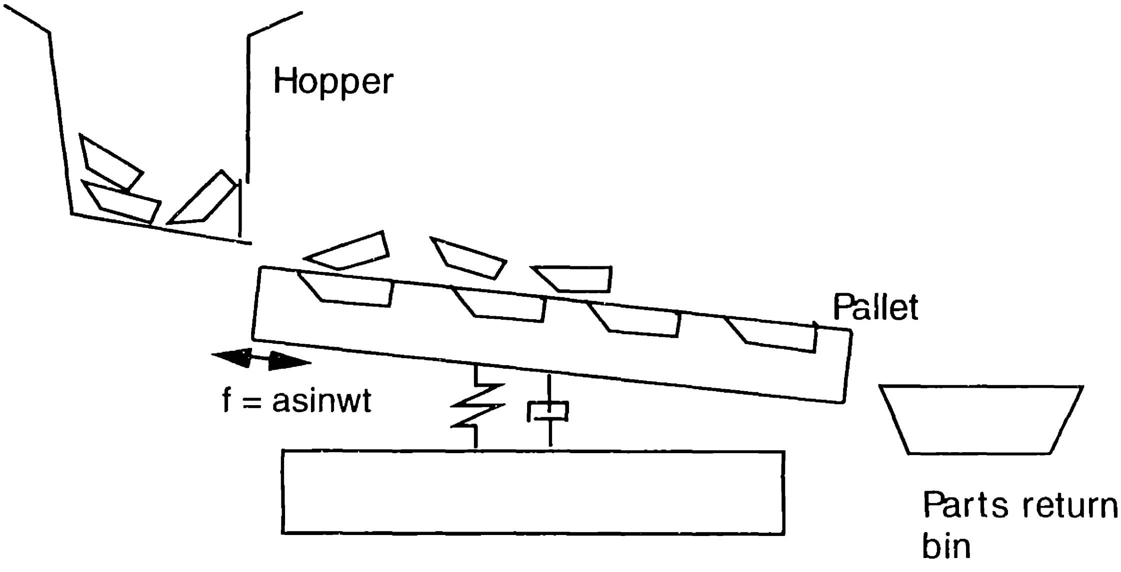

Figure 1-2: Schematic of APOS

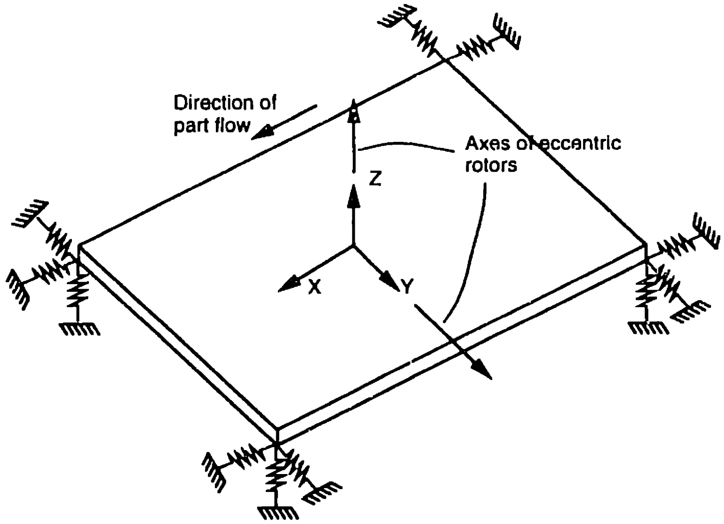

Figure 1-3: Schematic of pallet mount

Most of the parts flow over to the parts return bin and are recirculated back to the hopper. A continuous flow of parts is maintained for about two minutes. After this the pallet is detached from the shaker and transported to the assembly robot.

The shaker comprises of a pallet mount that has two eccentric rotors in two orthogonal directions shown as Y and Z directions in figure 1-3. The axes of both these rotors pass through the center of mass of the pallet-mount system. These rotors are driven, through flexible couplings, by induction motors whose speeds and braking torques can be continuously varied to get a range of oscillation frequencies and amplitudes. The pallet-mount system is suspended on a set of springs as shown in figure 1-3. The X and Y directional stiffnesses are approximately equal. The natural frequency of the pallet-suspension system is approximately 5 cycles per second.

The motors can be operated individually or in combination to get a range of oscillation patterns. When the Z-directional motor alone is operated, the pallet undergoes pure translation along a circular path parallel to its own plane (XY plane). When the Y-directional motor alone is operated, the pallet undergoes pure translation along a circular path parallel to the X-Z plane. Typical operating range corresponds to motor speeds in the range of 15-25 cycles/second. Since this is above the resonant frequency of the pallet-

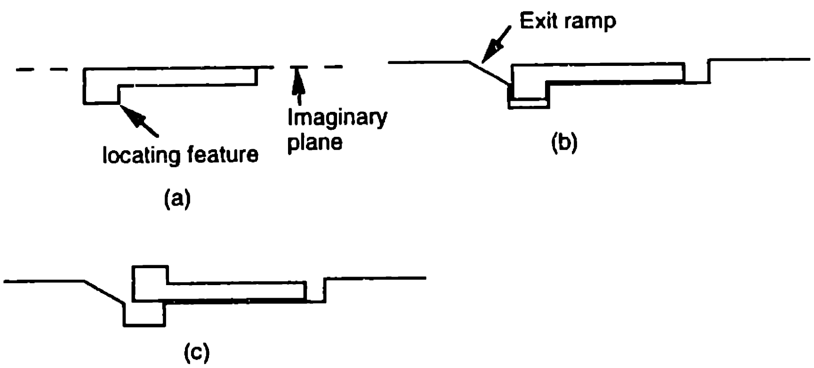

Figure 1-4: Basic design procedure

suspension system, amplitude of oscillation decreases gradually with increase in oscillation frequency if the braking torque is kept constant. Typical amplitude of oscillation is of the order of a millimeter. Typical dimension of the pallet is 240mm by 120mm.

1.2.2 Design parameters

A measure of efficiency of palletization is the fill-rate which is the number of correctly oriented parts that gets trapped in reliefs after a certain duration of parts flow over the pallet. Only a small fraction of the parts that flow over the pallet get trapped. To maximize fill-rate an important requirement is that the parts do not get trapped in any orientation but the desired orientation. Also parts that get conveyed down a pallet must not get tangled with parts that are already trapped. The fill-rate is a function of a number of design parameters. The principal parameters are pallet vibration pattern, vibration frequency and amplitude, pallet inclination and shapes of reliefs and guide vanes on the pallet. To palletize a given part these parameters have to be chosen so as to maximize the fill-rate. While the vibration pattern, frequency and amplitude can be adjusted easily by varying the speeds and braking torques of the two eccentric rotors, altering the chamfer geometry is an arduous process as it involves incremental material removal.

1.2.3 Current design procedure

The effect of each of the design parameters on fill-rate is not well understood and the parameter values are chosen by trial and error. The design procedure involves the following steps.

1. *Identification of a desired orientation and locating features:* The first step involves identifying an imaginary plane through the part that (when the part is palletized) would be flush with the pallet surface and one or many features on the part that can be used to locate it precisely on the pallet. Figure 1-4a illustrates one possible pair of an imaginary plane and a locating feature for a part.

2. *Carving out a basic relief shape:* The basic relief shape (figure 1-4b) is merely the negative of the locating feature(s) with a uniform clearance all around and chamfer along the edges. This basic relief ensures the stability of the trapped part in the desired orientation. The maximum allowable clearance is usually specified a priori based on assembly considerations. The chamfer is usually conical and uniform all around.

3. *Inlet and exit ramps:* Inlet ramps are usually added to facilitate entry into the relief. Besides, exit ramps are added to destabilize undesired orientations. Figure 1-4c shows an example of how an undesired orientation of the L-shaped part can be made less stable by adding an exit ramp.

4. *Tuning vibration pattern:* The vibration pattern, frequency and amplitude are chosen so as to destabilize all but the desired orientation. It takes experience and intuition on the part of the designer to arrive at the optimal vibration pattern. The vibration parameters must be chosen such that the part is stable only in the desired orientation and unstable in all other orientations.

The design procedure described above is largely empirical. It may take several weeks, depending upon the complexity of the part, to arrive at the optimal design. There is a need for a scientific basis for the selection of design parameters. Herein lies the motivation for the thesis.

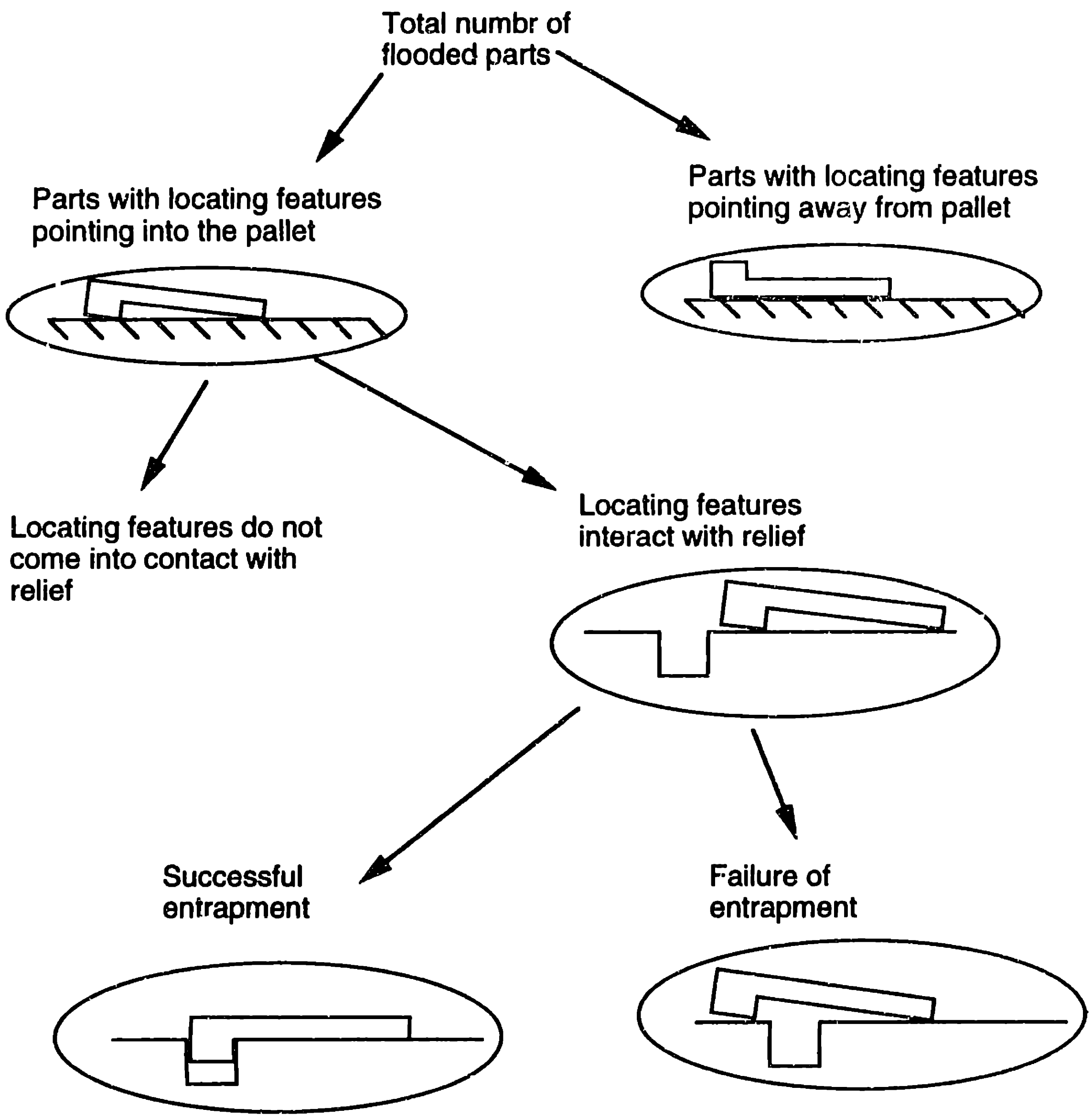

Figure 1-5: Stages of palletization

1.3 Thesis Objectives

Only a small fraction of the parts that are flooded over the pallet gets trapped. Most of the parts flow over to the bin and are recycled back to the hopper. The final fill-rate depends upon a multitude of factors. Figure 1-5 illustrates the various stages of the palletization process that influence fill-rate. Fill-rate is directly proportional to the number of parts that are being flooded over the pallet in the duration of interest. Of the parts that flow over the pallet only those parts that have their locating features pointing into the pallet surface (the remainder have their locating features pointing sideways or upwards) stand a chance of getting trapped. Of these parts only a fraction are conveyed in a way that their locating features actually interact with the relief (the remainder either bypass the relief or don't approach the relief in the right orientation). Of those parts that approach the relief with the right side down, in the right position and in the right orientation, only a fraction completely descend into the relief and get trapped. The remainder have their descent aborted.

The fill-rate can be improved by increasing the fraction of the parts that get through each stage of the palletization process. It may be noted that a design that is ideal for a particular stage of the palletization process may not be favorable as regards other stages of the palletization process. For example, high frequencies and amplitudes of oscillation result in high part conveyance rates but low rates of entrapment. It is thus important to explore all available design options and choose the option that is favorable to all stages of entrapment.

This thesis focuses on the last stage of the palletization process namely that of entrapment. Consider a part (shown in figure 1-6) being conveyed down an oscillating pallet and poised to descend into the relief. Because of its interaction with the relief it could either descend completely into the relief and get trapped or be thrown away from the relief. The goals of this thesis are

1. To understand the conditions under which a part that is poised to descend into the relief successfully completes its descent and gets trapped and stays trapped despite the motion of the pallet.

2. To identify ways and means to improve the rate of entrapment. It includes identifying

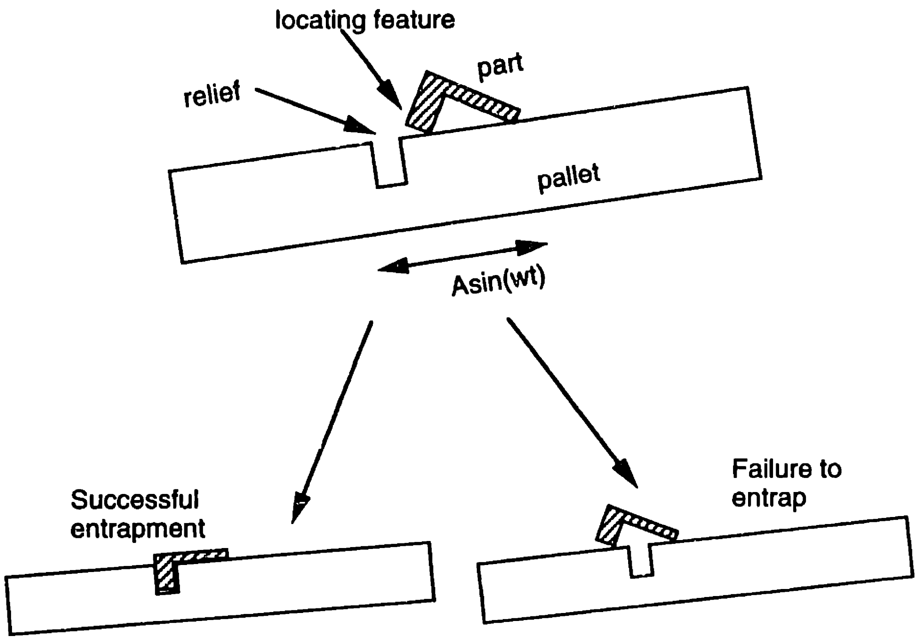

Figure 1-6: Phenomenon of entrapment

all of the options available to the designer and the pros and cons of each of those options.

1.4 Thesis outline

This thesis is organized as follows. Chapter 2 discusses previous work on certain related issues. Chapter 3 describes experimental and numerical simulation studies that provided useful insight into the mechanics of entrapment. Chapter 4 describes the mechanics of entrapment. The mechanics is introduced for the example of a point mass in a cup and extended to actual parts and reliefs. Chapter 5 discusses available design options and the pros and cons of each of the options in light of the design constraints. Chapter 6 summarizes the contributions of this thesis and discusses topics for future research.

2

Background and Related Work

Useful insight into the problem of vibration-assisted entrapment was obtained from previous work on a number of closely related problems. This chapter describes results presented in previous work that served as "starting points" for this work. It also addresses, wherever appropriate, the short-comings of previous work as regards their applicability to the problem of vibration-assisted entrapment.

2.1 Vibration assisted manipulation

The APOS belongs to a class of parts feeders that requires vibrations for its operation. The bowl feeder is another example of a vibratory feeder that is common in industrial assembly lines. Vibration can be used to convey, orient and assemble parts. Vibration of the supporting surface has the following two effects: altering the contact force between the part and the vibrating surface and providing (or removing) energy to (or from) the parts.

The acceleration of the vibrating support helps increase or decrease the normal reaction force and hence the maximum possible friction force. It can also cause the part to intermittently lose contact with the supporting surface. This feature is made use of to enable part conveyance. Boothroyd [3] explains the mechanics of part conveyance in a bowl feeder track and brings out the relationship between direction of vibration, maximum track acceleration and rate of part conveyance. In a bowl feeder the parts move up the spiral feeder track and acquire a net increase in energy for each oscillation cycle.

Vibration can also result in a net loss in energy of the part causing it to reach locations and orientations of minimum potential energy. This feature can be made use of to orient parts. Bohringer et al [2] demonstrate orientation of flat parts on a transversely vibrating plate. The parts align themselves along lines of zero displacement.

The fact that support vibration also provides energy to the part can be made use of to destabilize parts in undesired orientations that also correspond to potential energy minima. A common application is the use of base vibration to separate tangled parts. Cohn et al [10] demonstrate the assembly of one thousand millimeter sized hexagonal pieces of silicon into a closely packed honeycomb structure in a vibrating concave membrane. The final assembled state corresponds to the global potential energy minimum. Vibration parameters are chosen so as to destabilize all configurations (which may be local potential energy minima) except for the closely packed honeycomb.

In an APOS, oscillation of the pallet serves the following three purposes: to overcome friction; to convey parts; to prevent parts from getting stuck in the reliefs in undesired orientations, and to prevent parts from getting tangled. The final trapped state of the part corresponds to minimum potential energy. Figure 2-1 shows an approximate potential energy distribution of the part as a function of its location and orientation. It should be noted that since the pallet is inclined the trapped state is not a global energy minimum but a local minimum and is represented by the potential energy trough of depth ΔE in the figure. The objective of this thesis is to understand the conditions for the part to successfully descend into this potential energy trough and stay inside the trough despite the oscillation of the pallet.

Stability of a part that is trapped in an oscillating relief is guaranteed if the maximum energy imparted to the part due to the oscillations does not exceed the "potential energy barrier ΔE." Caine([9]) proposed that the design parameters in the APOS have to be chosen so as to satisfy this condition. While this approach is generally applicable to all part shapes and oscillation patterns it does not yield practically useful results in the context of the APOS. This is because under typical operating conditions, the maximum possible energy that the part can acquire can exceed the potential energy barrier corresponding to

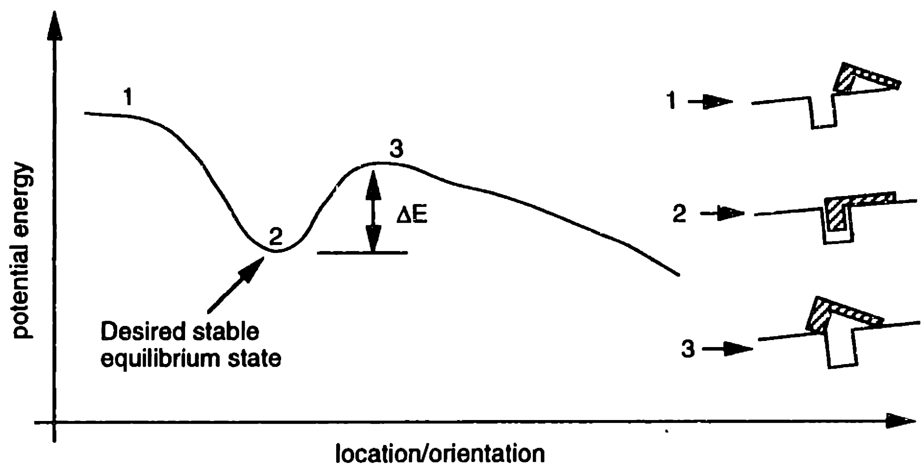

Figure 2-1: Variation of potential energy of contact states

the trapped state. The approach taken in this thesis is to take into account the momentum of the part in addition to its energy. It will be shown that this approach provides useful stability conditions, albeit, for a small subset of oscillation conditions and part shapes.

Stability in the desired configuration is not sufficient for entrapment. The part has to find a way to descend into the minimum potential energy configuration. Of interest to the designer is the set of initial configurations that are guaranteed to cause the part to reach the desired stable equilibrium state under the given forcing conditions. The ideal relief is one that maximizes this set of initial configurations. Lozano-perez [24] showed that a configuration space representation would be a useful tool for designing reliefs that facilitate entrapment. The configuration space representation was initially developed as a tool for robot path planning. The problem of designing entrapment reliefs can be formulated as an inverse of the robot path planning problem in which the motion is specified and the shapes that provide the constraints are determined. Caine [9] describes an interactive design tool in which the geometry of the configuration surface can be adjusted to achieve the desired motion constraints.

Limitations of configuration space based aproach: The configuration space based representation allows for design based on static or quasi-static analysis in which all forces are strictly functions of the relative position and orientation of the part. Since it does not allow for a representation of velocity and acceleration of the part it is inadequate for analysis of problems in which dynamic effects are significant. For example, the configuration space respresentation cannot account for the effect of a large initial velocity the part may have.

2.2 Work on other parts feeders

The APOS is just one of the many different parts feeders used in industry. Empirical studies have been performed on the various types of feeders, most notably the bowl feeder. Boothroyd and colleagues [3] provide general design guidelines relating to vibratory feeders as well as non-vibratory feeders. In [28] Murch and Poli provide experimental data on the design of an edge riser to reorient a rectangular block that is conveyed along a bowl feeder track. In [29] Poli and Murch provide experimental data to aid the design of a non-vibratory feeder. den Hamer [12] describes design guidelines for a variety of gravity parts feeders. Moncevicz et al [27] describe an experimental investigation into the use of the APOS for automated assembly of components. Han et al [18] describe the use of numerical simulations to iteratively arrive at optimal vane angles in a gravity feeder.

2.3 Behavior of trapped parts

An important requirement in the design of an entrapment relief is the stability of the trapped part in the presence of vibrations. A quantity of significant interest to the designer is the amount of energy imparted to the part from the oscillating relief. Useful insight into the mechanics of behavior of trapped parts was obtained from published results on topics such as behavior of impact dampers. Masri and Caughey [26] and Sadek [32] describe the mechanics of a one-dimensional impact damper. An impact damper consists of a mass moving freely inside an enclosed container. The container is attached to the structure whose vibration is to be damped. Energy is absorbed through impacts between the mass and the walls and the amount of energy absorbed varies with the operating frequency as well as the length of the impact damper. The energy of the mass is a maximum if successive impacts are energy giving thereby resulting in maximum damping.

2.4 Behavior of parts in bulk

Of significance to the analysis of the APOS is the behavior of parts in bulk. Of specific interest are issues such as the effect of vibration parameters on parts conveyance, tangling and the distribution of orientation of the parts as they get conveyed down a free surface of an oscillating APOS pallet. Researchers [21, 30] in the area of granular materials have reported on the behavior of materials such as sand, pharmacetical chemicals and food grains. Mathematical models have been developed to explain and predict the peculiarities in the behavior of such materials. These models however, have limited applications to parts feeding as they are based on the assumption that the individual particles are spherical or ellipsoidal.

3

Experimental and Numerical Analysis

An understanding of the mechanics of vibration assisted entrapment was acquired through experiments as well as numerical simulations. Experimental analysis involved filming and subsequent observation of high speed video images of a few simple parts getting trapped in reliefs. Numerical simulations involved simulation of the dynamics of interaction of a few simple parts with the reliefs. This chapter describes the experimental setup and the computer simulation tool used for the analysis.

3.1 Description of experimental setup and analysis

Experimental analysis involved high speed filming and subsequent viewing of the part-relief interaction. The experiments were performed with the APOS feeder described in chapter 1. This section describes the relevant details of the experimental setup. Figure 3-1 shows the typical experimental setup. The experimental part was placed on the free surface of the oscillating pallet and allowed to approach the relief several times and its interaction with the relief was filmed each time.

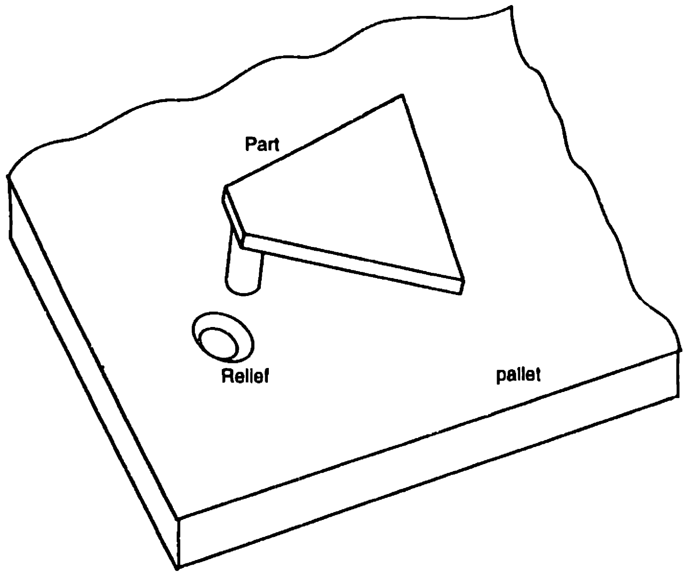

Figure 3-1: Experimental setup

High-speed video equipment: Entrapment of a part in a relief occurs over a time duration which is of the order of a few hundredths of a second. In order to observe such an event it was required to film it at very high speeds. Filming was carried out at the rate of 1000 frames per second using a Kodak EctaPro digital video camera. It allowed a maximum length of 1000 frames to be recorded and re-viewed.

Part shapes: Two simple part types shown in figure 3-2a were used for the study. The parts have only one locating feature each. Figure 3-3a shows the typical part dimensions. The locating feature is cylindrical and approximately 0.235 inches in diameter. One of the parts is an axisymmetric pin while the other part has a triangular plate attached to a cylindrical locating feature. The axisymmetric pin is representative of industrial parts such as screws and gears. The part with a triangular body is representative of the class of industrial parts that have a cylindrical locating feature and rest on three points.

Relief shapes: Two types of reliefs were considered. One was an axisymmetric relief. The basic relief shape is a cylinder of diameter exactly equal to 0.25 inches. Chamfers of different shapes and sizes were used. The other relief is a prismatic slot of the same cross-section as the axisymmetric relief. Figure 3-2b and 3-3b show the shapes and dimensions of the relief.

Some significant experimental observations: The following are some of the significant experimental observations.

1. Impacts with stationary surfaces resulted in a significant loss in the energy of the part.
2. Contacts with an *advancing wall* resulted in an increase in energy of the part.
3. Trapped axisymmetric pins were unstable at high oscillation frequencies even when the oscillations did not involve motion perpendicular to the plane of the pallet.
4. Whether a part that approached a relief got trapped in the relief depended upon the phase of the oscillation cycle of the pallet at which it approached the relief.

Figure 3-4 shows a sequence of images of an axisymmetric pin that is initially trapped but thrown out of the relief due to a disturbance input. Figure 3-5 shows the part with the triangular body approaching the oscillating relief but being thrown out due to the oscillation of the relief. In both the examples the frequency of oscillation was in the range of 20-25 cycles/second.

3.2 Numerical analysis of part-relief interaction

Observations from high speed video images were supplemented by results obtained from numerical simulations of the dynamics of the part-relief interaction. Numerical simulations made it possibe to record properties such as linear and angular velocities which are impossible to measure in an experimental setup. Besides, simulations also provided a "view" of the interaction of the part and the relief beneath the pallet surface. Simulations also allowed for ease of variation of part and relief shapes.

In order to facilitate the development of the simulation capability the model of the part relief interaction dynamics was simplified in the following respects.

- Only planar part-relief systems were considered for simulations.

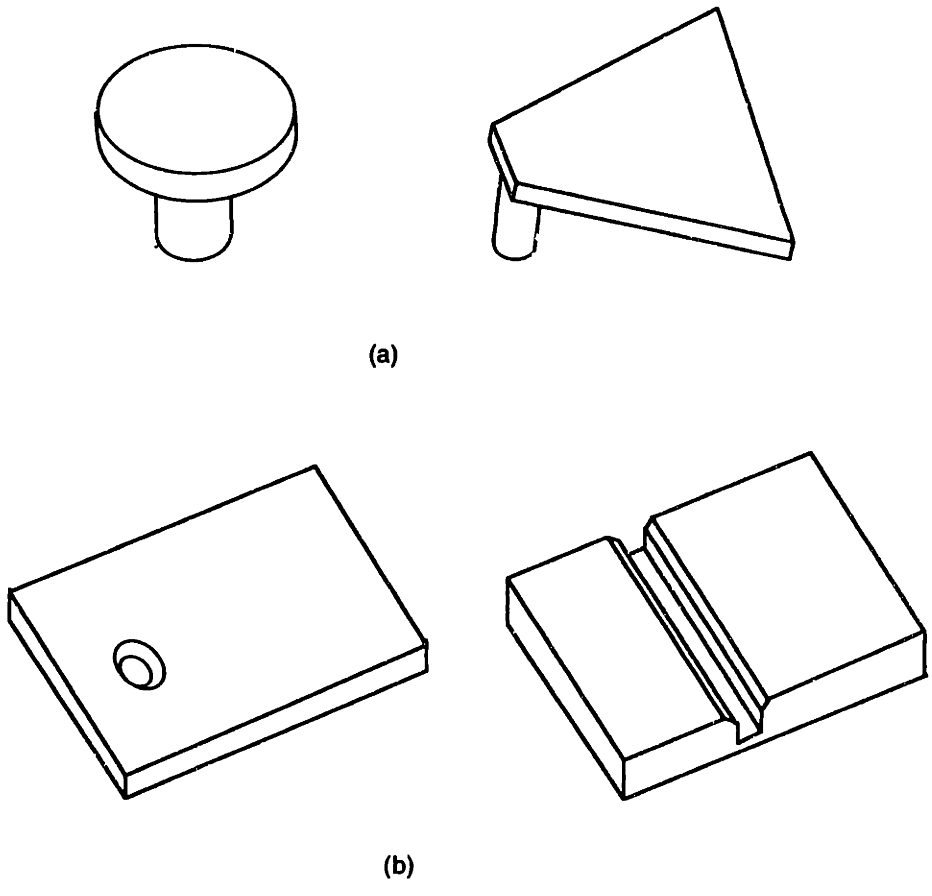

Figure 3-2: Experimental part and relief shapes

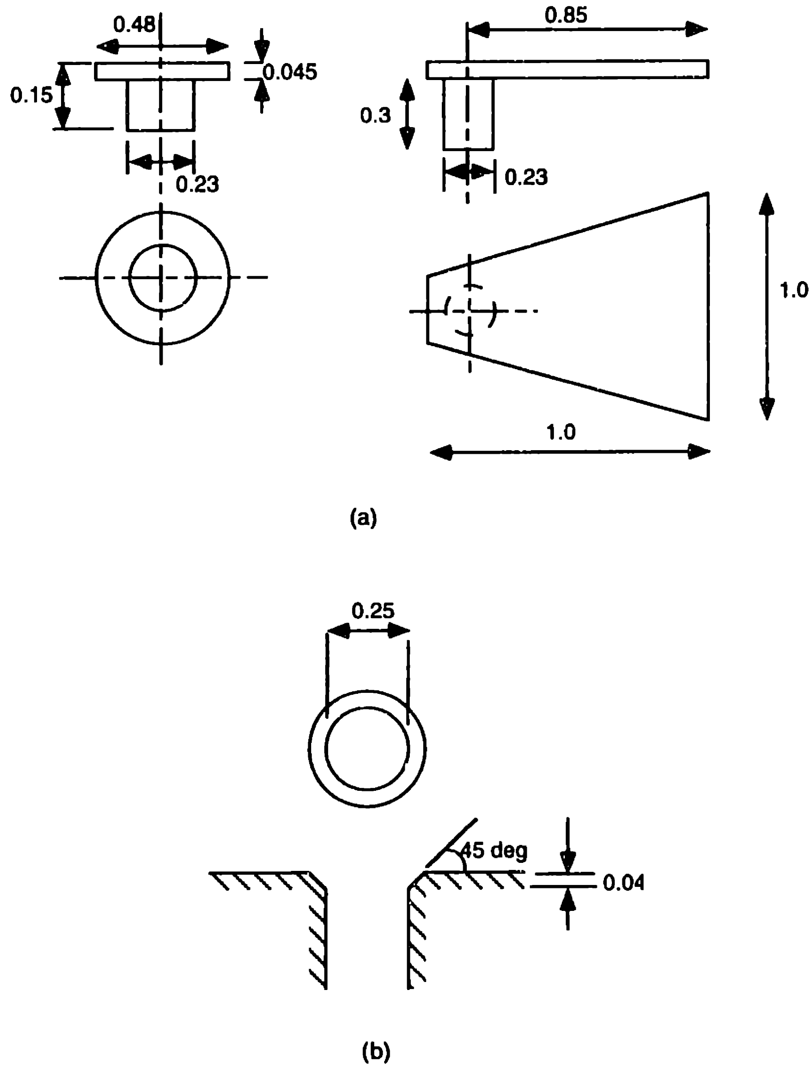

Figure 3-3: Experimental part and relief dimensions (in inches)

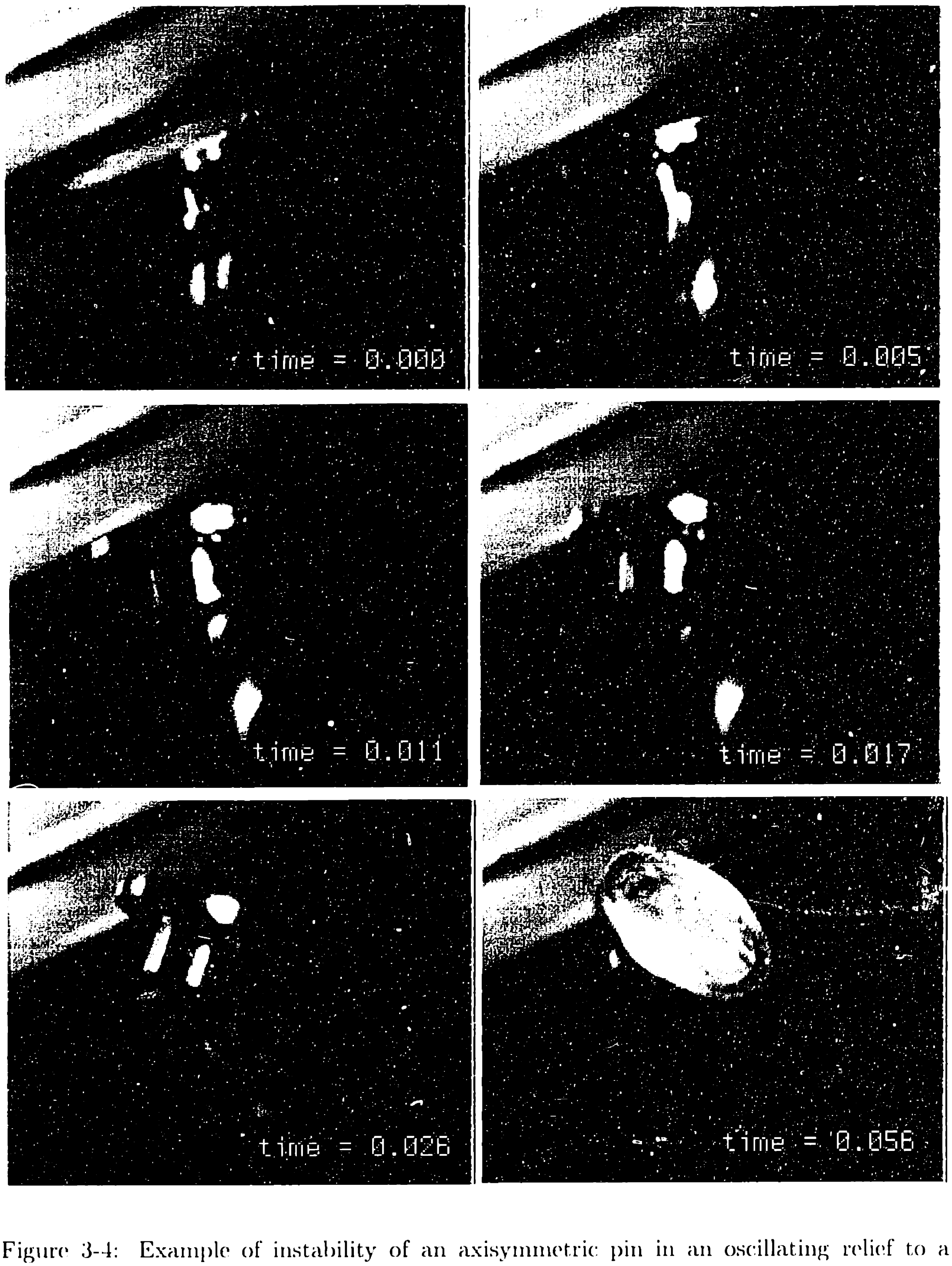

Figure 3-4: Example of instability of an axisymmetric pin in an oscillating relief to a disturbance (time in seconds)

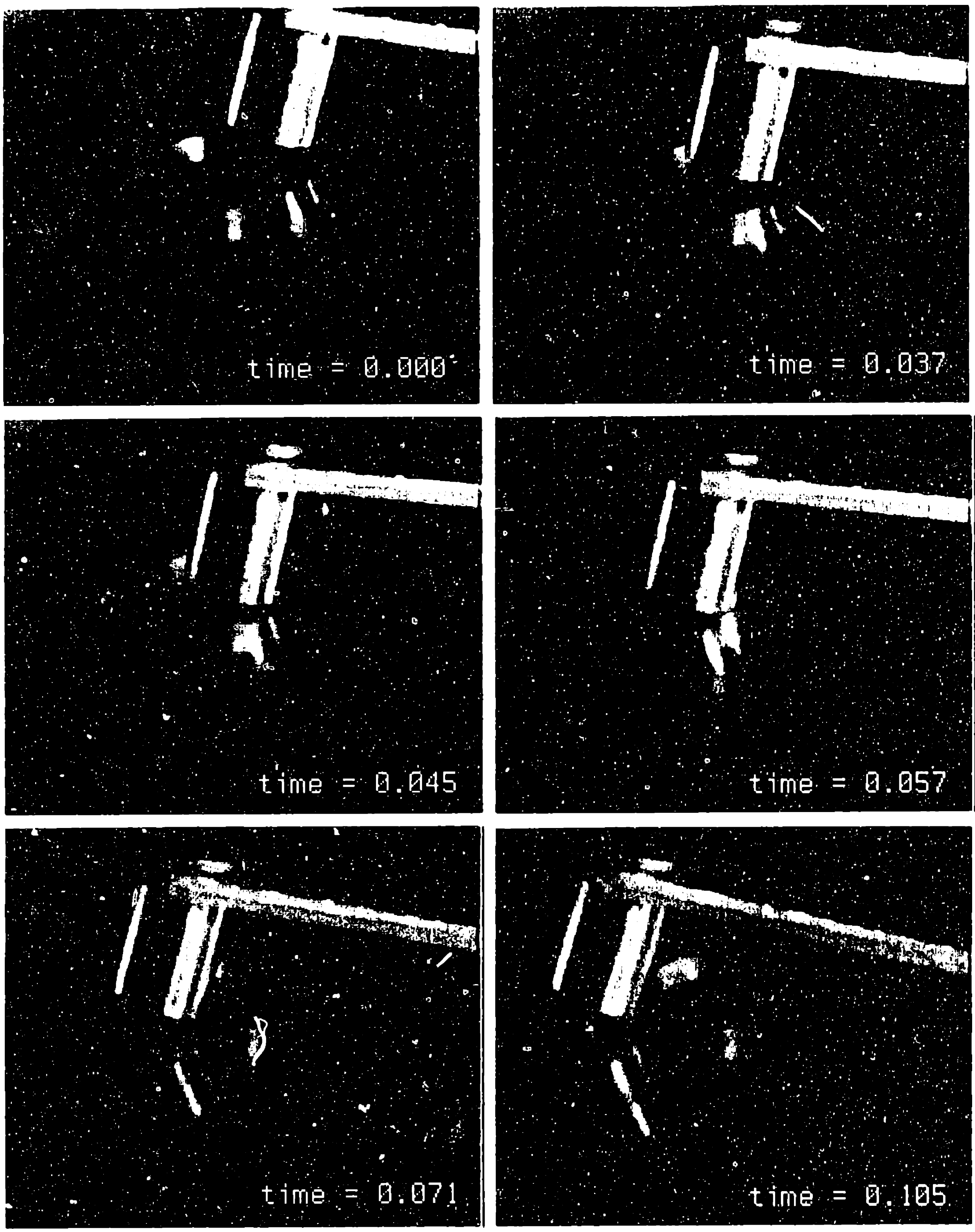

Figure 3-5: Example of failure of entrapment of a part that approaches an oscillating relief (time in seconds)

- Part and relief shapes were polygonal.
- Collisions between the part and the relief were assumed to be plastic (coefficient of restitution = 0.). The forces during a collision are impulsive.

The simulation is based on algorithms described by Gilmore and Cipra [15] and Krishnasamy and Jakiela [23].

The results of numerical simulations were represented in the following manner.

- Animation of the motion of the part and the relief on the computer screen.
- Plots of the variation of properties such as kinetic energy, total energy pallet velocity or other quantities of interest.

4

Mechanics of entrapment

This chapter addresses the principal objective of this thesis, namely, understanding the mechanics of entrapment of a part in a relief. When a part that is being conveyed on the surface of an oscillating pallet encounters a relief it begins to descend into the relief due to gravity. As it descends it comes into contact with the walls of the relief. The interaction between the part and the relief can result in two possible outcomes. As shown in figure 1-6, it could either descend into the relief and get trapped or get thrown out of the relief. This chapter discusses the mechanics of the part-relief interaction and explains the conditions that result in successful entrapment and those that result in failure of entrapment. The mechanics of entrapment is introduced with the help of some simple examples such as a point mass in a cup and then extended to some real parts and reliefs. Wherever possible the conditions for entrapment have been derived as a set of sufficient conditions expressed in terms of known design parameters such as vibration frequency, amplitude, chamfer shape and surface properties.

4.1 Introduction

The approach to determining the set of sufficient conditions for entrapment is as follows. The set of sufficient conditions for entrapment is the complement of the set of necessary conditions for a part to be thrown out of the relief during various stages of its descent into

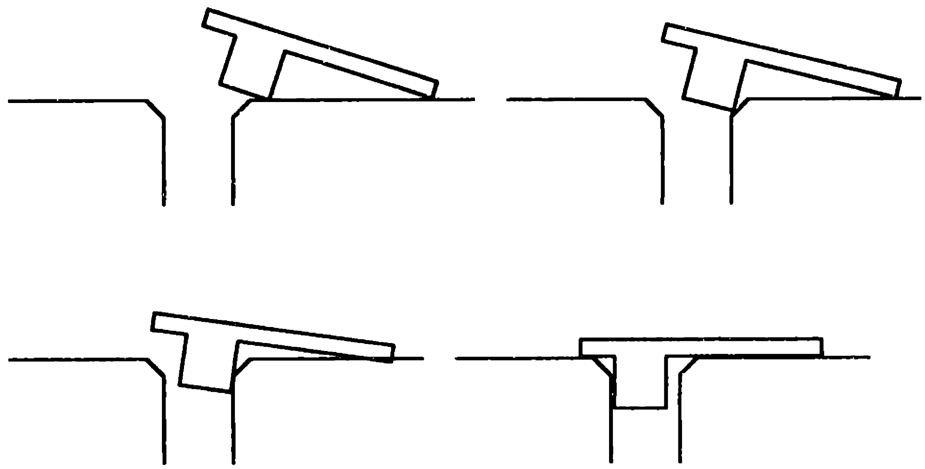

Figure 4-1: Various stages of descent

the relief. In other words, if any of the necessary conditions for a descending part to be thrown out of a relief is not satisfied then the part has to descend completely into the relief.[1] Thus the first step in determining sufficient conditions for entrapment is to understand the conditions for a part to be thrown off the relief at various stages of its descent.

Figure 4-1 shows an L-shaped part at various stages of its descent into the relief. A descending part has a momentum whose vertical component points downwards. For the part to be thrown out of the relief[2], the following condition has to be satisfied: the part has to acquire a momentum whose vertical component points upwards (along the positive Y direction). More specifically, the vertical component of the net impulse on the part (computed from the time when the part begins its descent) has to be pointing upwards. The net impulse on the part is the time integral of all the forces acting on the part. The forces acting on the part include the force of gravity and the constraining reaction forces at the contact points. Thus understanding the mechanics of entrapment involves being able to estimate the reaction forces, both direction and magnitude, that arise due to the part-relief

[1]This statement is based on the assumption that it is not possible for the part to get stuck at an intermediate position due to wedging, jamming or frictional sticking. This assumption is not always valid and will be discussed briefly in the next chapter.

[2]For the sake of this discussion, being thrown out can be assumed to mean that the lower most point on the part lies above the pallet free surface

interaction.

The reaction forces depend upon the geometry of the part-relief contact, friction at the interface, velocity and acceleration of the part and the relief. These quantities, in turn, are functions of the initial velocity of the part, oscillation pattern, oscillation frequency and amplitude. It will be shown later in this chapter that for some simple part and relief geometries and some simple vibration patterns, closed form expressions can be obtained relating the reaction forces to known design parameters such as vibration frequency, vibration amplitude, clearance, chamfer depth and part geometry.

The remainder of this chapter is organized as follows. The motion of a typical part in a relief is similar in nature to the motion of a point mass in a funnel. Useful insight into the mechanics can be obtained by analyzing the simpler problem of an imaginary point mass in a funnel. Section 4.2 describes this analysis. The motion of a point mass that is descending into a funnel can be of the following two types.

1. continuous contact characterized by sliding along the walls of the funnel.
2. intermittent contact characterized by bouncing between the walls of the funnel.

In either case the upward momentum the mass can gain due to interactions with the funnel is a function of the initial velocity of the mass, geometry of the funnel (specifically the inclination of the walls of the funnel) the damping properties of the walls of the funnel and the velocity and acceleration of the funnel as a function of time. Sections 4.2.1 and 4.2.2 explain the effect of initial velocity of the mass and geometry of the funnel on the mass getting trapped inside the funnel. Section 4.2.1 discusses the motion with intermittent contact with example of a point mass in a planar cup. Section 4.2.2 discusses the motion with continuous contact with the example of a point mass in a spatial cup. For the case of continuous contact the upward reaction force on the mass can be determined precisely in terms of the velocity of the mass and the cup geometry. For both of the examples, it is shown that the tendency of the mass to get thrown out of the cup increases with increase in its initial kinetic energy. For a mass that is initially inside the cup there is a minimum required initial kinetic energy for it to be thrown out of the cup. This minimum required kinetic energy is computed for the case when the initial velocity of the mass is parallel to the ground and there is no loss in energy due to damping. For very small inclinations of the walls of the cup the minimum required kinetic energy is much larger than the potential energy barrier (shown as ΔE in figure 2-1) that must be overcome.

Section 4.2.3 discusses the effect of damping on the minimum required initial kinetic energy described above. The results obtained for stationary cups are extended to oscillating cups in sections 4.2.4 and 4.2.5. In an oscillating cup, in addition to its initial energy the mass can acquire energy or lose energy through work interactions with the oscillating cup. A condition called resonance is described in which the mass continuously acquires energy from the cup and gets thrown out. For the point mass in an oscillating planar cup approximate conditions for resonance are derived in terms of the oscillation frequency. For the example of a point mass in continuous contact with an oscillating spatial cup where the oscillation is along a circular path parrallel to the base of the cup, the conditions for resonance can be precisely derived. The conditions for resonance in turn are used to derive conditions for stability of the point mass that is initially at the bottom of the oscillating cup.

The results on a point mass in a funnel are extended to derive design conditions for some simple parts and reliefs. Section 4.3 discusses motion of an axisymmetric pin in an axisymmetric relief for pallet oscillations along a circular path parallel to its own plane. Conditions for stability of the trapped pin are derived and shown to be of the same form as conditions for initiation of resonance for a point mass at the bottom of an oscillating spatial cup. Stability of the trapped pin to disturbances is also discussed. Conditions for successful descent of an approaching pin are also discussed in a qualitative manner. Exact expressions cannot be derived for these conditions because the early stages of descent are characterized by impacts and the response to impacts is not well defined. Section 4.4 describes motion of the asymmetric part in the axisymmetric relief. Failure of entrapment due to resonance is discussed.

4.2 Motion of a point mass in a funnel

The forces acting on a trapped part are the reaction forces from the walls of the relief and gravitational force. For the part to acquire a momentum pointing in the positive Y direction (shown in figure 4-2), the reaction forces must have components in the positive Y direction that are large enough to annul the effect of gravity. The direction of the reaction forces at

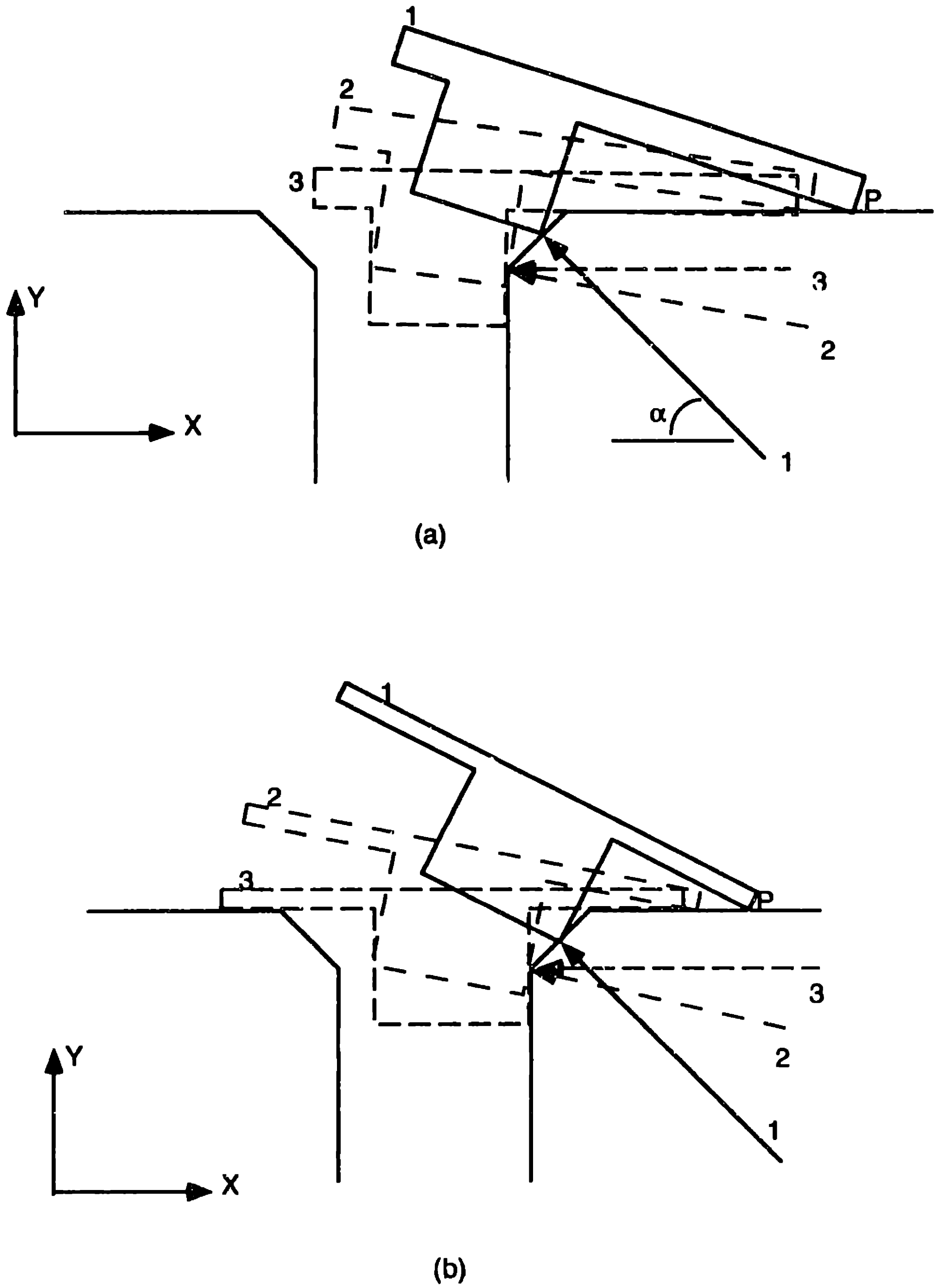

Figure 4-2: Direction of contact normals

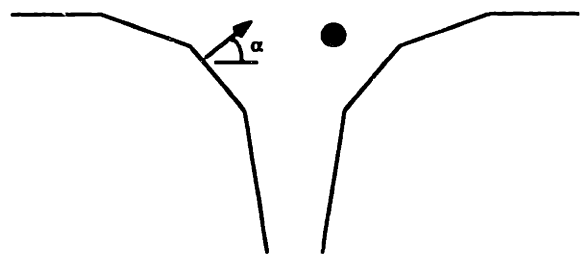

Figure 4-3: Point mass in a funnel

the contact points in the absence of significant friction is the same as the direction of the contact normals. The direction of the contact normal depends upon the orientation of the part. Figure 4-2a shows the contact normal at different stages of descent of the asymmetric part. The normals are drawn with the assumption that point P on the part remains in contact with the pallet surface. From figure 4-2 it is clear that the angle α made by the contact normal with the horizontal increases with elevation (or decreases with depth of descent). Figure 4-2b shows the variation of the contact normal of an axisymmetric pin-shaped part at various stages of its descent into the relief. Similar to the asymmetric part the inclination α increases with elevation.

The fact that the inclination α of the contact normal increases with increase in elevation suggests that useful insight into the motion of a part in a relief can be obtained by analyzing the simpler example of an imaginary "point mass" in a funnel (as shown in figure 4-3). This section discusses different aspects of the motion of a point mass in a funnel. The width of the funnel is representative of the clearance between the part and the relief in the fully descended state. Hence, in the discussion in the remainder of the chapter the funnel will also be referred to as a "clearance relief" for the part-relief system.

Of specific interest in the mechanics of entrapment are the conditions for stability of a point mass at various stages of its descent into an oscillating funnel. It is of interest to determine the conditions under which a mass that descends past a certain depth will remain below that depth notwithstanding the oscillation of the funnel. Since the inclination of the contact normal decreases with depth, it is intuitive that the stability of the mass increases with descent.

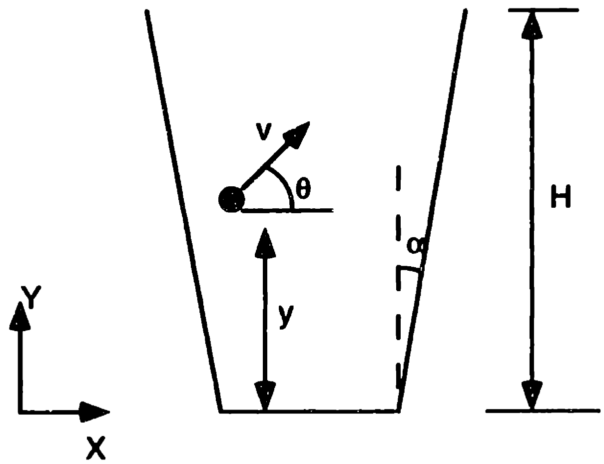

Figure 4-4: Point mass in a planar stationary cup

The discussion of the motion of a point mass in a funnel is preceded by a discussion of the motion of a point mass in a cup of constant inclination. A funnel can be thought of as an array of cups stacked on top of each other with the bottom of the cups (with the exception of the lowermost cup) removed.

4.2.1 Point mass in a planar stationary cup

The motion of a point mass in a cup can be of two types: intermittent contacts characterized by collisions with the walls of the funnel or continuous contacts characterized by sliding along the walls of the funnel. This subsection provides an analysis of the motion characterized by intermittent contacts with the example of a point mass in a planar cup shown in figure 4-4. It is of interest to determine the set of conditions that would cause an initially descending mass to leave the cup. For the sake of this discussion "leaving the cup" refers to the part rising above the top surface of the cup and not returning to the cup (the cup is of height H). In a stationary cup, there is no work interaction between the mass and the cup. As a result the energy of the mass undergoes a monotonic decrease as there is energy loss due to damping. Hence it is convenient to express the necessary conditions for a mass that is initially inside the cup to be thrown out as a minimum required initial kinetic energy. This minimum required kinetic energy is greater than or equal to the potential energy barrier that must be overcome and depends upon the direction of the initial velocity of the mass and the angle of inclination α of the walls of the cup.

In order to rise above the top surface of the cup, the mass has to acquire a momentum whose vertical component is positive. In other words the net impulse on the mass has to be positive until it leaves the cup. The forces that contribute to the vertical component of momentum are gravity and reaction forces that have positive vertical components. The motion of a point mass in a planar stationary cup is characterised by repeated collisions with the wall of the cup. The magnitude of the reaction forces during these collisions is proportional to the magnitude of the normal component of velocity of the mass at impact. Thus, for a given magnitude v of the incoming velocity, if γ is the angle of incidence, the magnitude of the reaction force decreases with increase in γ.

Consider the simplest example of a mass at an elevation y from the bottom of the cup having a velocity that is exactly vertical. If the velocity is pointing vertically upwards (angle $\theta = 90\,\text{deg}$ in figure 4-4) then the minimum required kinetic energy for the mass to leave the cup is given by

$$KE_{min} = mg(H - y) \Rightarrow v = \sqrt{2g(H - y)} \tag{4.1}$$

where v is the magnitude of the initial velocity. The minimum required kinetic energy is the potential energy barrier that must be overcome. If the velocity points downwards instead of upwards, then the mass will acquire an upward pointing momentum upon impinging on the bottom of the cup. However, it also loses some energy due to the impact. The loss in energy can be expressed as $v(1 - e)$ where v is the velocity before impact and e is the coefficient of restitution. Energy loss is a maximum when $e = 0$. The minimum required kinetic energy is given by

$$KE_{min} = \frac{mg(H - ye^2)}{e^2} \tag{4.2}$$

If the velocity of the mass is not exactly vertical but points at an angle θ to the horizontal then the mass can gain a vertical component of momentum through reaction forces from the walls of the cup, in addition to forces from the bottom of the cup. The effect of the angle of inclination of the walls, α, on the vertical component of impulse needs to be understood. These effects will be discussed first for small angles of inclination α and then extended to larger angles of inclination.

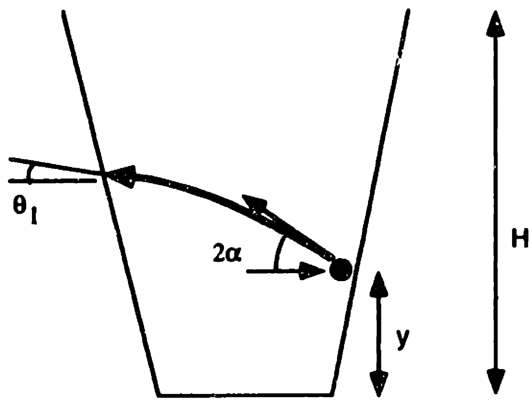

Figure 4-5: Point mass in a cup with steep slopes

Motion in a cup with steep slopes

Consider a mass at an elevation y from the bottom of the cup with a velocity pointing parallel to the base of the cup ($\theta = 0$). The cup has walls of very steep slopes, (small inclinations α such that $\sin\alpha << 1$) as shown in figure 4-5. Such a cup constitutes the lowermost part of the funnel shown in figure 4-3. It is of interest to determine the minimum kinetic energy the mass should have for it to be thrown out of the relief. On impinging the walls of the cup, in the absence of any damping, the mass acquires a velocity pointing at an angle 2α to the horizontal. The trajectory of the free flying mass is parabolic as shown in figure 4-5. In order to sustain its flight upwards, the mass has to reach the opposite wall of the cup with an angle θ_l that is greater than $(-\alpha)$. For convenience in computing the minimum required kinetic energy, it can be stipulated that θ_l be positive (measured clockwise from the horizontal) that is, the direction of the trajectory when the mass reaches the opposite wall is atleast horizontal, if not pointing upwards. Then a lower limit on the magnitude of velocity v can be computed as follows.

$$v^2 > \frac{2gl_{av}}{\sin 4\alpha} \tag{4.3}$$

where l_{ave} is the average width of the cup over the flight. The use of l_{ave} is justified for a cup with very steep walls. Although the above expression was derived for a specific initial condition, it provides useful insight into the effect of various parameters on the event of the mass being thrown out. For example, the minimum required kinetic energy increases with increase in the width l of the cup. This also means that for the mass to make successive "climbs" during each flight the actual kinetic energy required is much more than that provided by equation 4.3 which is based on the width of the cup at the initial elevation. The equation also reveals that the energy required increases with decrease in inclination α of the walls of the cup.

The minimum required kinetic energy for the mass to be thrown out of the cup is given by

$$KE_{min} = \frac{1}{2}mv_{min}^2 = max(\frac{1}{2}m(\frac{2gl_{max}}{\sin 4\alpha}), mg(H - y)) \quad (4.4)$$

While the second term is the potential energy barrier given by equation 4.1 , the first term is the kinetic energy required by the part to acquire sufficient upward pointing momentum from the walls of the relief. Clearly for small inclinations α, the first term is much larger than the second term. The above expression is derived for a cup with steep slopes in which the mass has to go through several collisions with the walls of the cup before it gets thrown out. It was also assumed that the mass has an initial horizontal velocity and is just about to collide with the wall of the cup.

Motion in a cup with intermediate or shallow slopes

Equation 4.4 was derived with the assumption that the slope of the walls of the cup, α is small. A mass that has an initial velocity pointing parallel to the ground has to go through several flights between the walls inorder to leave the cup. The upper portion of the funnel can be represented as a cup of intermediate or shallow slopes. In a cup with slope $\alpha = 45$ deg (in figure 4-6), the upward components of the reaction forces are much larger. A mass with an initial horizontally pointing velocity can acquire a vertically pointing velocity after just one collision. The minimum required kinetic energy for the mass to leave the cup reduces to the potential energy barrier that needs to be overcome. It is given by

$$KE_{min} = \frac{1}{2}mv_{min}^2 = mg(H - y) \quad (4.5)$$

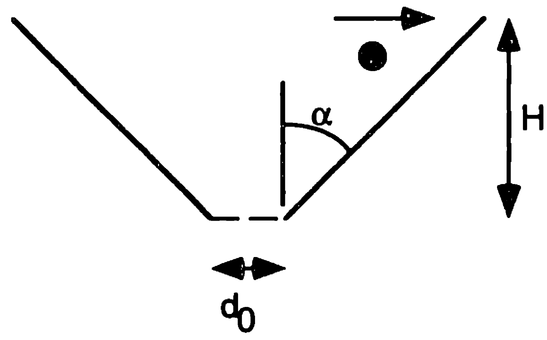

Figure 4-6: Point mass in a cup with intermediate inclinations

4.2.2 Point mass in a stationary spatial cup

The motion of a point mass inside a cup is characterized by collisions with the walls of the cup (intermittent contact) or sliding along the walls of the cup (continuous contact). The previous example of a point mass in a stationary planar cup provided an analysis of the intermittent motion in a cup. The minimum required kinetic energy was derived for the example of a point mass with an initial horizontally pointing velocity and just about to collide with a wall of the cup. This subsection provides an analysis of the motion of a point mass characterized by continuous contact with the wall of the cup. This is done with the example of a point mass inside a spatial cup as shown in figure 4-7. Unlike the example of the mass in a planar cup, precise analytical expressions can be derived to describe the motion characterized by continuous contact. This in turn leads to precise conditions on the minimum required kinetic energy for the mass to get thrown off the relief.

The equations of motion can be derived in cylindrical coordinates (r,y and θ). If the mass is constrained to remain in contact with the surface of the cup, r and y are related as follows.

$$r = r_0 + y \tan(\alpha) \tag{4.6}$$

At any instant the acceleration **a** of the point mass can be written in terms of the unit

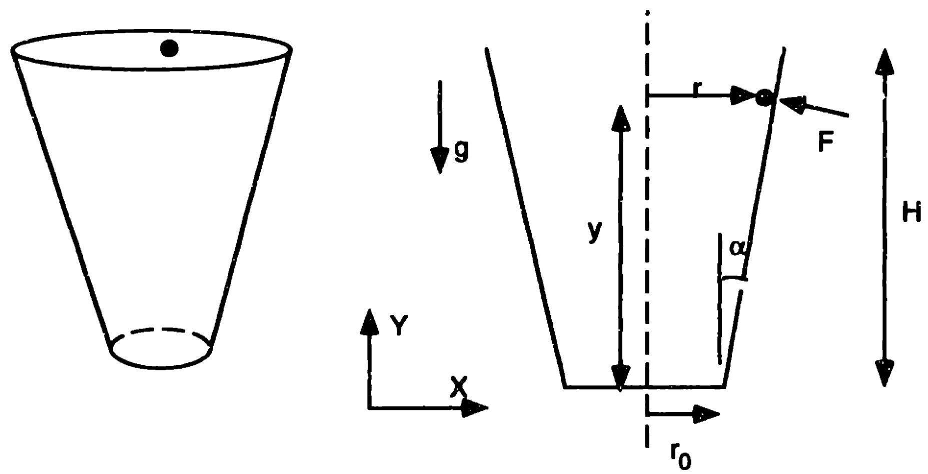

Figure 4-7: Mass in a stationary spatial cup

vectors $\mathbf{i}_r$, $\mathbf{i}_\theta$ and $\mathbf{i}_y$ as follows.

$$\mathbf{a} = \mathbf{i}_r(\ddot{r} - r\dot{\theta}^2) + \mathbf{i}_\theta(2\dot{r}\dot{\theta} + r\ddot{\theta}) + \mathbf{i}_y\ddot{y} \tag{4.7}$$

The external forces acting on the mass are gravitational force and the normal reaction force F from the walls of the cup. Assuming frictionless contacts, the equations of motion can be written as follows.

$$m\ddot{y} = F\sin\alpha - mg \tag{4.8}$$

$$m(\ddot{r} - r\dot{\theta}^2) = -F\cos\alpha \tag{4.9}$$

$$2\dot{r}\dot{\theta} + r\ddot{\theta} = 0 \tag{4.10}$$

Since friction is assumed to be zero, the angular momentum about the axis of the cone is conserved. This is indicated by the third equation which can be simplified as follows.

$$(\dot{\theta}r^2) = constant \tag{4.11}$$

The above three equations and equation 4.6 can be combined to obtain the following equation in terms of a single variable z.

$$\ddot{z}(1 + \tan^2(\alpha)) + g - \frac{c^2\tan(\alpha)}{z^3} = 0. \tag{4.12}$$

where $z = r_0 + y\tan(\alpha)$ and c is a constant and equal to $\dot{\theta}r^2$. The above equation is of the form,

$$\ddot{z} - \frac{a}{z^3} + b = 0 \tag{4.13}$$

where $a, b > 0$. This means that there is a stable equilibrium elevation, y_{eq} at which $\ddot{y}$ and $\dot{y}$ are zero and the mass has a steady motion at a constant elevation. In the absence of damping however, if the mass starts at an elevation different from the equilibrium elevation, the mass will never reach this steady motion, but will oscillate about it. From equation 4.12 it can be shown that at equilibrium the following condition has to be satisfied.

$$\tan(\alpha) = \frac{g}{\dot{\theta}_{st}^2(r_0 + y_{st}\tan(\alpha))} \tag{4.14}$$

This equilibrium occurs at an elevation where the vertical component of the reaction force exactly balances the weight of the mass and the horizontal component provides the required centripetal force. Therefore, in order to have a steady motion at a height y above the bottom of the cup the mass has to have a velocity, v_{st} given by

$$v_{st} = \dot{\theta}_{st}(r_0 + y\tan(\alpha)) \tag{4.15}$$

Also, at any given elevation, if the mass has a velocity parallel to the ground and greater than v_{st}, it undergoes an increase in elevation. If the velocity is less than v_{st} the mass undergoes a decrease in elevation.

Motion in a cup with steep slopes

Similar to the example of a point mass in a planar cup, for a mass that has an initial horizontally pointing velocity, the minimum required kinetic energy for it to leave the cup can be determined. Starting from an initial elevation y, to undergo an increase in elevation the mass has to have a velocity of atleast v_{st} which is given by equation 4.15. From equation 4.14 the minimum required kinetic energy for the mass to acquire sufficient upward momentum is given by

$$v^2 > \frac{g(r_0 + y\tan\alpha)}{\tan\alpha} \tag{4.16}$$

and the minimum required kinetic energy for the mass to leave the cup is given by

$$\frac{1}{2}mv^2 > max(\frac{1}{2}m\frac{g(r_0 + y\tan\alpha)}{\tan\alpha}, mg(H - y)) \tag{4.17}$$

The first term is the energy required for the mass to acquire sufficient upward pointing momentum and the second term is the potential energy barrier. Clearly, the first term is much larger than the second, if the inclination α is very small.

Motion in a cup with intermediate or shallow slopes

Minimum required kinetic energy given by equation 4.17 is also valid for cups with large wall inclinations. Close to the bottom of the cup, if $r_0 >> y$ the minimum required kinetic energy is very small. At higher elevations, as y becomes much larger than r_0, the minimum required kinetic energy increases due to the increasing radius of the cup.

4.2.3 Effect of damping

Estimates of the minimum required kinetic energy for a point mass to leave a cup were derived based on the assumption that there is no energy loss due to damping. If the energy loss due to damping is significant the minimum required kinetic energy for the part to be thrown out of the relief is larger. Energy loss can occur due to one of two possible reasons. There is energy loss associated with collisions between the mass and the wall (as in a planar cup). The total amount of energy lost is a function of the number of collisions, velocity of the mass prior to each collision and the surface properties. Energy is also lost due to frictional sliding (as in a spatial cup). The total amount of energy lost is a function of the duration of the sliding contact, the frictional properties and the sliding velocity of the mass. In general, if the cup is stationary a mass that has a sequence of collisions with the walls of the cup loses a lot more energy than a mass that slides along the walls of the cup.

For the example of a point mass in a cup, energy loss due to damping is significant if the angle α is small as well as if the angle α is very large. If the angle α is small in a planar cup, the mass has to make several flights between the walls of the cup before it gets thrown out of the cup. The energy loss is proportionately larger. Similarly for the example of a mass in sliding contact with a spatial cup, for very small α, the mass will have to make several spirals inside the cup before it reaches the top of the cup. The energy loss is proportionately larger. The energy loss is significant if the angle α is very large. This is because at large α (α close to 90), the number of collisions (for a planar cup) and the duration of contact (for a spatial cup) are larger.

4.2.4 Point mass in a planar oscillating cup

This subsection describes the motion of a point mass in an oscillating planar cup and the conditions under which the mass gets thrown out of the cup. In a stationary cup, the conditions were derived as lower limits on the initial kinetic energy of the mass. It was shown that the upward momentum the part might acquire is proportional to the kinetic energy. For a cup with very steep slopes, the required kinetic energy is much larger than the potential energy barrier.

In an oscillating cup the mass can gain energy due to work interactions with the walls of the cup and this gain in energy must be taken into account in determining conditions for entrapment. The energy the part may gain is a function of the vibration pattern, vibration frequency and amplitude, the width of the cup and the slope of the walls of the cup. For the results in the previous sections to be of utility in design, the kinetic energy of the mass must be expressible in terms of the above design parameters. It will be shown in this section that this is possible for simple oscillation patterns. It will also be shown that under certain conditions, called resonance, the part can get thrown out of the relief. The conditions for resonance are derived in terms of the design parameters.

Energy providing and absorbing contacts:

A mass in an oscillating cup can undergo an increase or decrease in energy due to work interactions. Consider a mass in contact with the left wall of the cup that is moving to the right as shown in figure 4-8. The change in energy of the mass, ΔE, due to the contact is given by

$$\Delta E = \int_0^{\Delta t} \mathbf{f}_c \bullet \mathbf{v}_c dt \qquad (4.18)$$

where, $\mathbf{f}_c$ is the contact force, $\mathbf{v}_c$ is the velocity of the contact point on the part and Δt is the duration of contact. Contacts in which $\mathbf{f}_c$ and $\mathbf{v}_c$ point in the same direction are energy

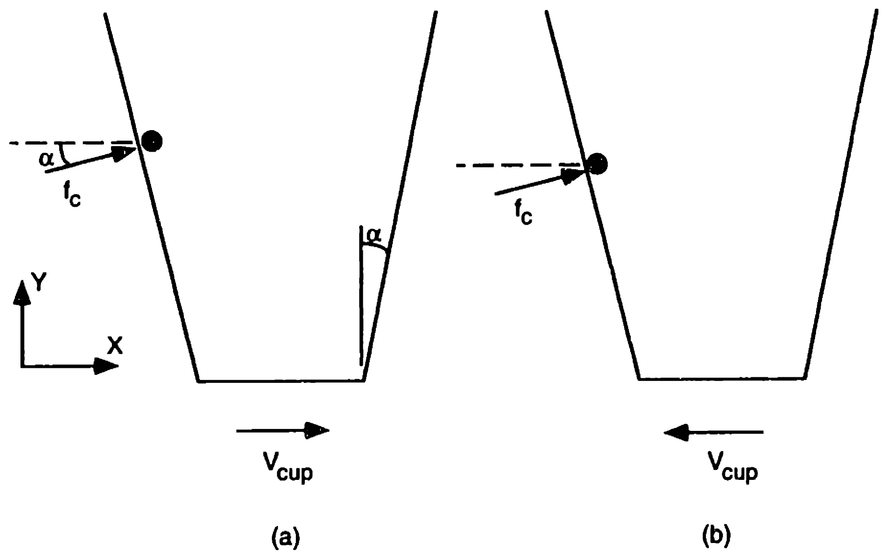

Figure 4-8: Work interactions with an oscillating cup

providing (ΔE is positive) and contacts in which $\mathbf{f}_c$ and $\mathbf{v}_c$ point in opposite directions are energy absorbing (ΔE is negative). Thus, if the cup has a velocity v_{cup} in the positive x-direction, the contact is energy ***providing*** and if the cup has a velocity pointing in the negative x-direction, the contact is energy ***absorbing.*** In general, contacts with an advancing wall are energy providing and contacts with a receding wall are energy absorbing.

The magnitude of the change in energy of the mass, ΔE depends upon the duration of contact, contact force, velocity of the mass as well as the angle between the force vector and the velocity vector. In contacts that arise due to collisions, the contact force is very large, but the duration of contact is very small. ΔE increases with increase in velocity of the cup. If the contact is frictionless, and if the cup has a velocity parallel to the x-axis, ΔE decreases with increase in the the angle of inclination α. This is because, as shown in figure 4-8, the angle between vectors $\mathbf{v}_c$ and $\mathbf{f}_c$ increases with an increase in α.

Conditions for a mass to be thrown out

It was shown in subsection 4.2.1 that for a mass with an initial horizontally pointing velocity to be thrown out of a stationary cup with small inclinations α, there is a minimum required kinetic energy that is given by equation 4.3. This is the minimum kinetic energy required

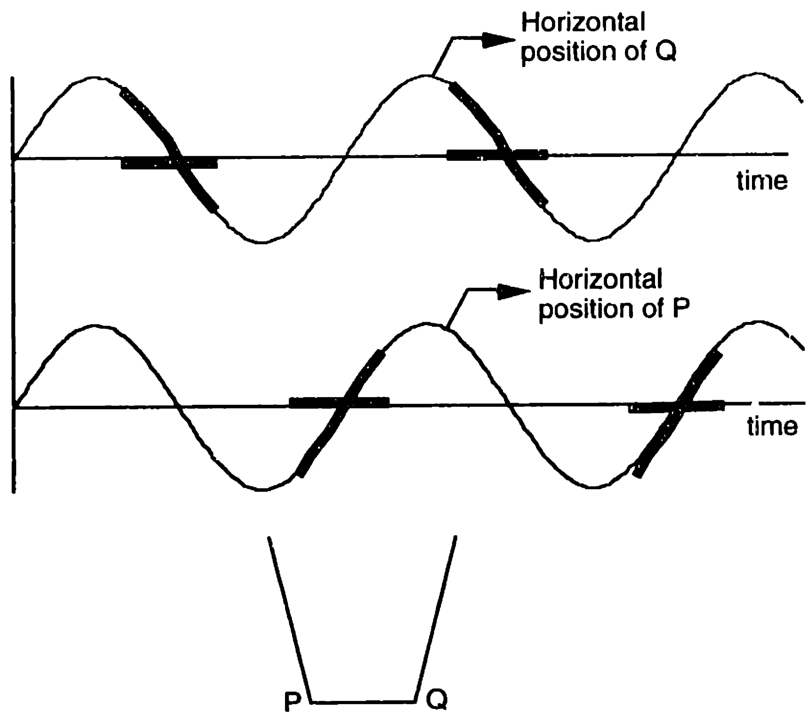

Figure 4-9: Energy providing intervals

for a mass to travel from one wall to the opposite wall without a loss in elevation. If the cup were oscillating, it is possible for the mass to acquire this energy through energy providing contacts with the oscillating cup. Since the mass has to make several flights back and forth before it leaves the cup, its kinetic energy has to exceed that described by equation 4.3 at the onset of each flight. With each flight the required kinetic energy increases due to an increase in the width of the cup. The potential energy also increases for each flight due to an increase in elevation. The mass can gain this energy before each flight if successive contacts with the walls are energy providing. This phenomenon is referred to as *resonance* because it is similar to the resonance in the forced oscillation of a simple spring-mass-damper system in which there is continuous positive energy input and the energy of the system increases *monotonically* until the rate of energy input equals the rate of energy dissipation due to damping. Resonance is a worst-case scenario in the sense that there is no loss in energy due to negative work interactions. The only loss in energy is due to damping.

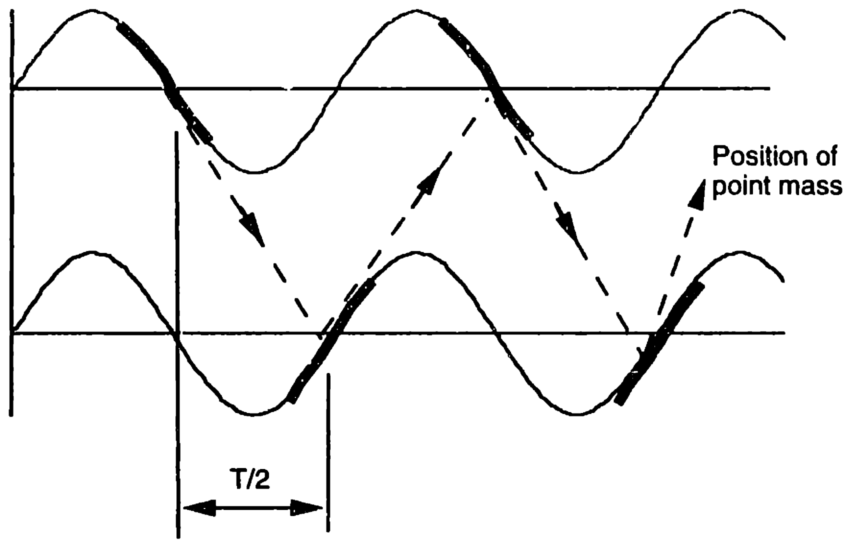

Figure 4-10: A sequence of energy providing contacts

Approximate conditions for resonance can be easily derived for the example of a point mass in a cup with small inclinations. Let the cup have a sinusoidal oscillation in a direction parallel to the base of the cup. The positions of the two points P and Q on the cup as a function of time are represented as sinusoids shown in figure 4-9. As discussed earlier the mass gains energy through contacts with an advancing wall and loses energy through contacts with a receding wall. The darkened regions on each sinusoid represent approximate intervals that correspond to energy providing contacts for each wall. Contacts with the left wall are energy providing when the cup is moving to the right and contacts with the right wall are energy providing when the cup is moving to the left. Contacts with the walls outside their respective energy providing intervals result in an energy loss. Resonance occurs when the mass makes contact with the two walls of the funnel during their respective energy providing intervals. Figure 4-10 shows schematically an example of a sequence of contacts that are energy providing. The mass travels from one wall to the other in approximately half the period of oscillation. Therefore a necessary condition for resonance that velocity v should satisfy is given by equation 4.19.

$$v \approx v_{res} = \frac{d\omega}{\pi} \tag{4.19}$$

where d is the horizontal distance covered which is approximately the width, l of the cup. To sustain resonance, the increase in energy of the mass must be accompanied by an increase in the width, d and hence an increase in elevation. To have an increase in elevation for each flight, the minimum required velocity, v_{climb} is given by

$$v_{climb} = \sqrt{\frac{2gd}{\sin 4\alpha}} \tag{4.20}$$

Equation 4.20 is the same as equation 4.3. For the mass to monotonically gain energy and elevation with each contact, $v \approx v_{res} > v_{climb}$. From the above two equations it follows that for a given width, d there is a minimum frequency $\omega_{min,res}$ below which resonant escape cannot occur. $\omega_{min,res}$ is given by equation 4.21.

$$\omega_{min,res} \approx \pi\sqrt{\frac{2g}{d \sin 4\alpha}} \tag{4.21}$$

There are two conditions that need to be satisfied for resonance. One is the frequency condition specified by equation 4.21 and the other is the velocity (or kinetic energy) condition given by equation 4.20. In addition, for the mass to have successive energy providing contacts, the amplitude has to be smaller than clearance.

If amplitude is much larger than clearance, the mass cannot have successive energy providing contacts. This is evident from figure 4-11 which shows the position of the points P and Q and the approximate position of the point mass. The mass cannot have a monotonic increase in energy because on gaining energy from one wall the mass reaches the opposite wall before the onset of energy providing interval corresponding to the opposite wall. Hence, energy providing contacts are interspersed by contacts that result in a loss in energy. Still, the mass can leave the cup if it has a *net* gain in energy and elevation for each oscillation cycle. This occurs at large amplitudes when the gain in elevation from an energy providing contact exceeds the loss in elevation following energy dissipating contacts. Approximate conditions for the mass to be thrown out of the cup can be derived as follows. To travel from one wall of the cup to the opposite wall, the effective horizontal distance the mass has to cover is approximately equal to the amplitude of oscillation. Since there is energy loss at the end of each flight the kinetic energy the mass has at the beginning of each flight is approximately given by the maximum velocity of the cup which is $a\omega$. The minimum

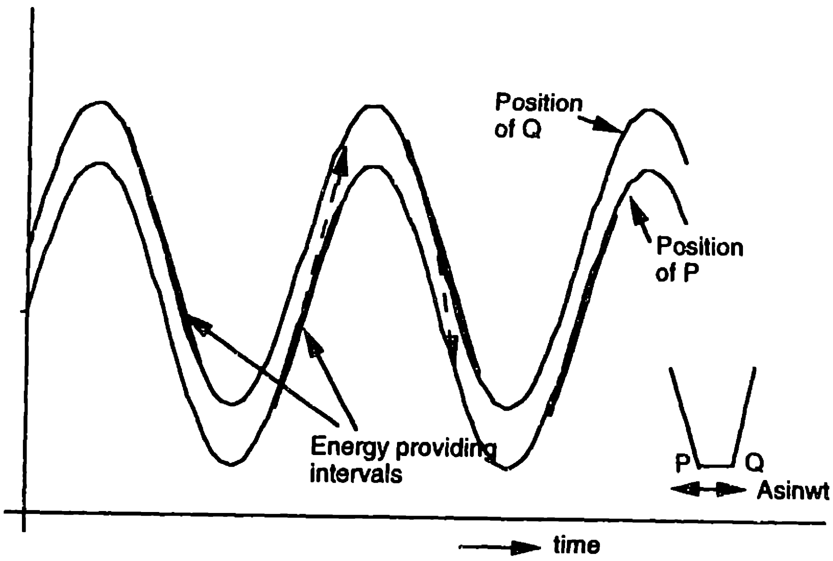

Figure 4-11: Large amplitude oscillation

frequency and amplitude required for the mass to be thrown out is given by

$$a\omega > v_{climb} = \sqrt{\frac{2ga}{\sin 4\alpha}} \tag{4.22}$$

The right hand side is obtained by substituting $d = a$ in equation 4.3. The above equation reduces to

$$a\omega^2 > \frac{g}{\sin 4\alpha} \tag{4.23}$$

For a mass at the bottom of the cup to be thrown out the minimum frequency is given by equation 4.21 or equation 4.23. The mass also needs a minimum velocity given by equation 4.20. It can gain this velocity from the cup if the amplitude of oscillation of the cup is large enough. For a mass that is initially at rest the maximum velocity it can gain from the oscillating cup is approximately given by $a\omega$ where a is the amplitude of oscillation and ω is the frequency of oscillation. So the velocity condition can be expressed as

$$a\omega > v_{climb} \tag{4.24}$$

Thus in addition to a minimum frequency there is a minimum required amplitude for the mass to be thrown out of the cup. However, if the mass has initial kinetic energy that satisfies the condition in equation 4.20 then a smaller amplitude may suffice.

Effect of changes in inclination of the cup

In deriving expressions for minimum required frequency and amplitude for the mass to be thrown out of the cup, it was assumed that the inclination α of the walls of the cup are small. In an oscillating planar cup in which the inclination α is not small, the complexity of the motion does not allow for derivation of expressions for minimum frequency and amplitude. It can however be shown qualitatively that the tendency for resonance decreases as the inclination α increases for the following reasons.

1. With increase in α, the upward component of the reaction forces increases. As a result the elevation of the mass increases at a higher rate.

2. For larger α, the width d of the cup increases faster with elevation. This in turn results in an increase in the rate at which v_{climb} increases with elevation. Thus the velocity required to sustain resonance increases with increase in α.

3. Owing to the above two effects the required rate of energy input to sustain resonance increases.

4. In addition, in accordance with equation 4.18, with increase in α, the energy input/output from work interactions decreases. Hence, at very large α the oscillating cup cannot provide energy at the rate at which it is required to sustain resonance.

In summary, with increase in α, the rate of energy input required to sustain resonance increases and this requirement cannot be met at very large α.

A note on maximum possible kinetic energy:

In the discussion on resonance it was shown that it is possible for the energy of the mass to increase monotonically due to consecutive energy providing contacts. There is however an upper limit on the maximum attainable kinetic energy [17]. This is because of energy loss due to damping which is generally proportional to the velocity (or kinetic energy) of the mass. Consider for example a mass with a velocity v_i before the ith impact and colliding with a wall of the cup which is moving at a velocity V. The energy loss due to the impact can be quantified with the coefficient of restitution e. By the simplest definition of e [31], the velocity of the mass after the ith impact is given by

$$v_{i+1} = V(1+e) + ev_i \tag{4.25}$$

The maximum value of V is $a\omega$. If every impact of the mass is energy providing the energy of the mass increases until the energy input is balanced by the energy loss due to damping. The maximum possible velocity can be determined by equating v_i to v_{i+1}. This corresponds to

$$v_{max} = a\omega\frac{1+e}{1-e} \tag{4.26}$$

This maximum kinetic energy is attainable only when successive contacts are energy giving. This inturn requires that the conditions for resonance be satisfied. For resonance, increase in kinetic energy has to be accompanied by an increase in the width of the cup and hence in the elevation. For example if the walls of the cup were perfectly vertical ($\alpha = 0$) the mass cannot gain elevation and the maximum energy it can attain is limited by the width d of the cup.

4.2.5 Point mass in an oscillating spatial cup

This subsection describes the motion of a point mass in a cup that describes a periodic motion along a circular path in a plane parallel to the base of the cup as shown in figure 4-12. The cup is oscillating at a constant frequency Ω and radius (or amplitude) R. Similar to the motion in a stationary cup, with the assumption that the mass is in continuous contact with the walls of the cup the equations of motion can be derived precisely. Conditions for the mass to be trapped in the cup or be thrown out of the cup can be derived from these equations.

Equations of motion

Consider a point mass in a cup as shown in figure 4-12. The exact position of the mass can be specified by the following set of coordinates shown in figure 4-12: ϕ, θ, and y. The coordinate system $(\mathbf{e}_r, \mathbf{e}_\theta)$ is rigidly fixed to the cup and rotates at an angular velocity Ω the frequency of oscillation of the cup. At an elevation y the radius of the cup r is given by

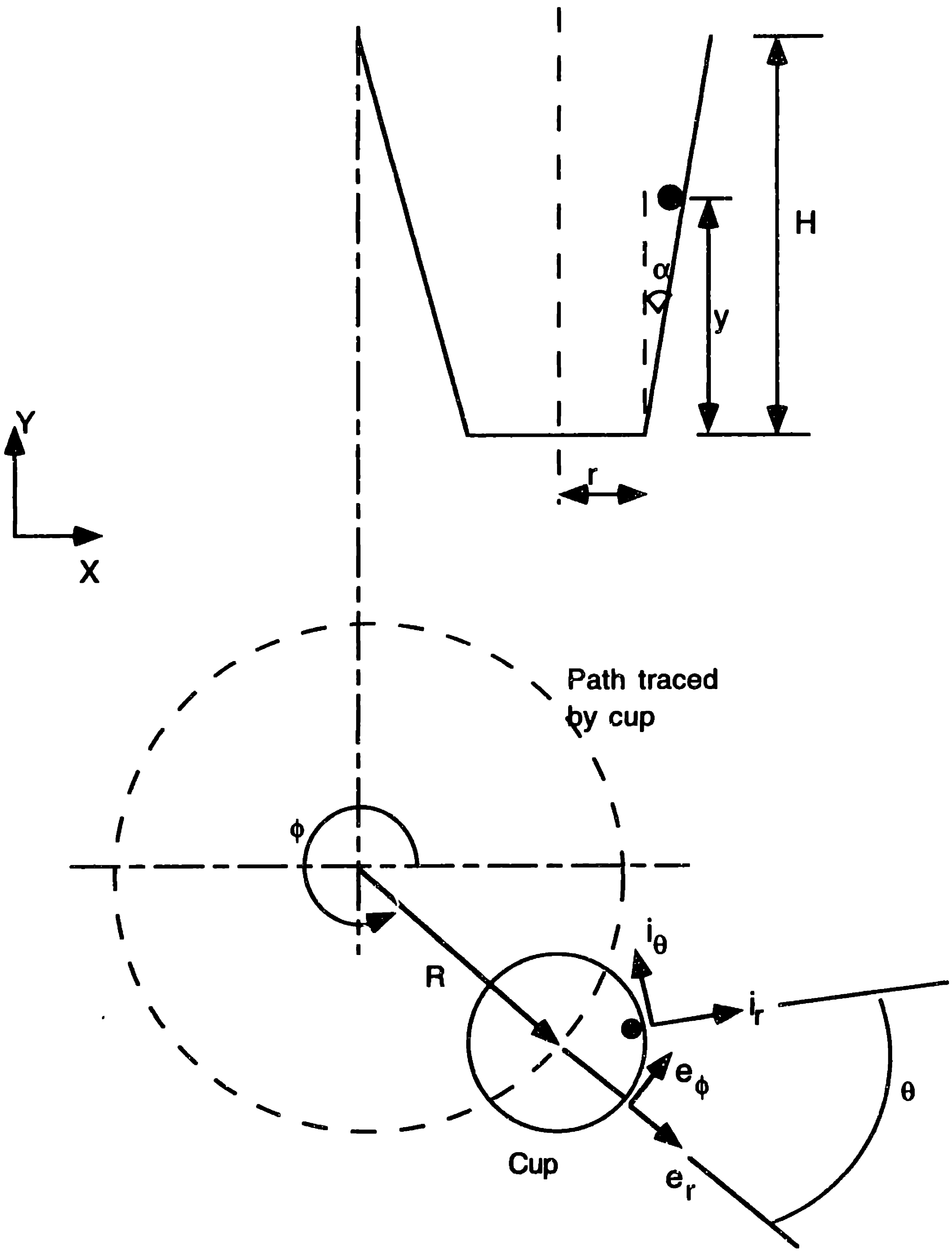

Figure 4-12: Point mass in an oscillating spatial cup

equation 4.6. The acceleration $\mathbf{a}$ in terms of the unit vectors $\mathbf{i}_r$, $\mathbf{i}_\theta$ and $\mathbf{i}_y$ is given by

$$\mathbf{a} = \mathbf{i}_r[\ddot{r} - r(\Omega + \dot{\theta})^2 - a\Omega^2 \cos\theta] + \mathbf{i}_\theta[r\ddot{\theta} + 2\dot{r}(\Omega + \dot{\theta}) + a\Omega^2 \sin\theta] + \mathbf{i}_y \ddot{y} \tag{4.27}$$

Let F be the reaction force from the walls of the cup. In the absence of friction, F points normal to the wall of the cup. Therefore the equations of motion can be written as follows.

$$m(-\ddot{r} + (\Omega + \dot{\theta})^2 r + R\Omega^2 \cos\theta) = F \cos\alpha \tag{4.28}$$

$$2\dot{r}(\Omega + \dot{\theta}) + r\ddot{\theta} + R\Omega^2 \sin\theta = 0 \tag{4.29}$$

$$m\ddot{y} + mg = F \sin\alpha \tag{4.30}$$

For the mass to remain in contact, $F \geq 0$. Eliminating F and expressing r in terms of y using equation 4.6, we have the following.

$$2\dot{y}\tan\alpha(\Omega + \dot{\theta}) + (r_0 + y\tan\alpha)\ddot{\theta} + R\Omega^2 \sin\theta = 0 \tag{4.31}$$

$$\ddot{y}(1 + \tan^2\alpha) = (\Omega + \dot{\theta})^2 (r_0 + y\tan\alpha)\tan\alpha + R\Omega^2 \cos\theta \tan\alpha - g \tag{4.32}$$

Steady motion

Although the equations of motion are coupled and nonlinear they provide useful insight into the motion of the mass inside the oscillating cup. For example it can be shown that for a given frequency of oscillation, Ω there is a height y_{st} at which the mass will have a steady motion. This height y_{st} can be determined by substituting $\dot{y}, \ddot{y}, \dot{\theta}, \ddot{\theta} = 0$ in the above equation. Equation 4.32 reduces to

$$\Omega^2(r_0 + y_{st}\tan\alpha) + R\Omega^2 \tan\alpha - g = 0 \tag{4.33}$$

$$\Omega^2 = \frac{g}{\tan\alpha[r_0 + y_{st}\tan\alpha + R]} \tag{4.34}$$

Thus, for a given Ω there is a y_{st} at which the mass will exhibit steady motion such that $\dot{y}$, $\ddot{y}$, $\dot{\theta}$, $\ddot{\theta} = 0$. It should also be noted that Ω decreases with increase in y_{st} as well as an increase in α.

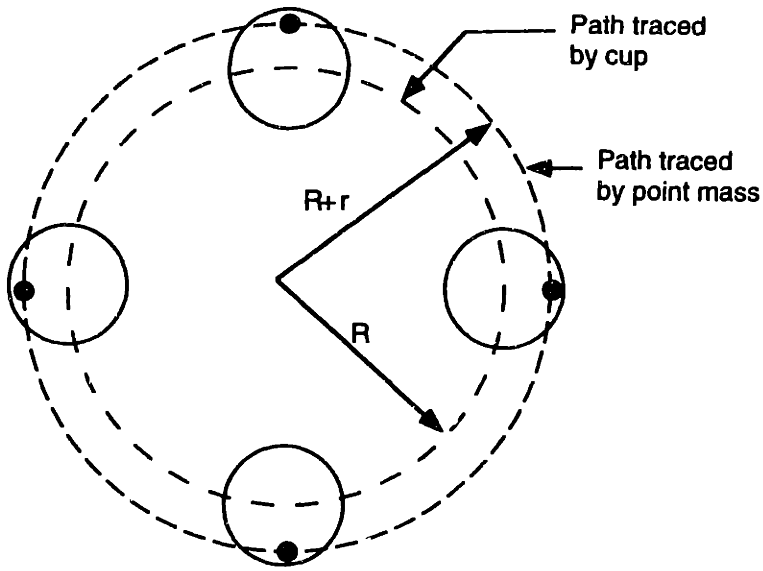

Figure 4-13: Position of mass with respect to cup during steady motion

Figure 4-13 describes the position of the mass with respect to the cup when it is in steady motion. The mass describes a circular path with respect to the cup as the cup oscillates along a circular path. This steady motion occurs at a height when the horizontal component of the reaction force provides the centripetal force and the vertical component exactly balances the weight of the mass. At steady motion the mass has a constant velocity equal to $\Omega(R + r)$.

Unlike the example of the mass in a stationary cup, the steady motion in an oscillating cup is not stable to perturbations in y or θ. This instability can be illustrated by linearizing the equations of motion for small perturbations of y and θ about the steady motion. If δ is the perturbation of y about y_{st} and ψ is the perturbation of θ about $\theta = 0$, the linearized equations are as shown below.

$$\ddot{\psi}(r_0 + y_{st} \tan\alpha) + R\omega^2\psi = 0 \tag{4.35}$$

$$\ddot{\delta}(1 + \tan^2\alpha) - \dot{\psi}[2\Omega \tan\alpha(r_0 + y_{st} \tan\alpha)] - \delta[\Omega^2 \tan^2\alpha] = 0 \tag{4.36}$$

The instability is due to the fact that increase in elevation keeping θ constant results in

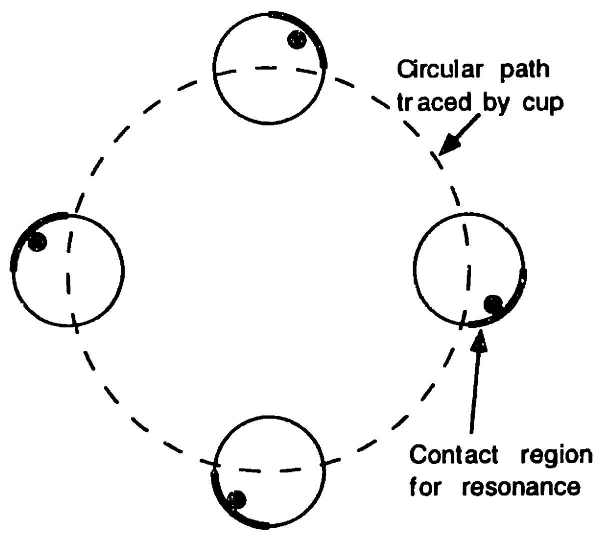

Figure 4-14: Contact region for resonance

an increase in the centripetal force and hence an increase in the upward component of the reaction force. This in turn results in a further increase in the elevation of the mass. Similarly, decrease in the elevation results in a decrease in centripetal force and hence a further decrease in the elevation of the mass.

Conditions for the mass to be thrown out

In the case of a stationary spatial cup it was shown that there is a minimum required kinetic energy for a descending mass to be thrown out. For a cup with small inclinations α it was shown that this energy is much larger than the potential energy barrier. The minimum kinetic energy was found from the requirement that the upward component of the reaction force exceeds the weight of the point mass. If the cup is oscillating, the mass can acquire this energy through energy providing contacts with the cup. As it gains elevation, it requires continuous energy input to compensate for the loss in kinetic energy due to increase in elevation and also to provide for the increase in the required kinetic energy due to the increase in diameter of the cup. This continuous energy input is possible through continuous energy providing contacts with the oscillating cup. Such a condition is called resonance as it involves a monotonic increase in energy. The only loss in energy is due to damping.

For resonance, the contact between the mass and the cup has to be energy providing. In addition, the mass describes a circle of radius approximately $R + r$ as the cup describes a circle of radius R. In addition to doing work, the reaction force must provide the required centripetal force to sustain the circular motion of the point mass. In order for these two conditions to be satisfied, the point of contact between the mass and the cup lies somewhere on the darkened quarter circle shown in figure 4-14 for different stages of the oscillation cycle. There is a minimum frequency for resonance and this is the frequency beyond which the mass gains a net upward momentum. This minimum required frequency can be computed as follows. At the minimum required frequency, the upward component of the reaction force equals the weight of the mass. The horizontal component of the reaction force is the centripetal force and is a maximum for $\theta = 0$. Hence the minimum required frequency for resonance can be derived as follows.

$$T \sin\alpha > mg \tag{4.37}$$

$$T_{max} = \frac{m(r_0 + y\tan\alpha)\Omega^2}{\cos\alpha} \tag{4.38}$$

$$\Omega^2 > \frac{g}{\tan\alpha(r_0 + y\tan\alpha)} \tag{4.39}$$

Comparing with equation 4.34, it also follows that for a given Ω the condition for resonance can be written as

$$y > y_{st} \tag{4.40}$$

In addition to the frequency condition for resonance, there is a velocity (or kinetic energy) requirement for resonance. The velocity condition can be written as,

$$v_{min} \approx \Omega(r + R) \tag{4.41}$$

This is the velocity required for the mass to describe a circle of radius $R + r$ as the cup describes a circle of radius R. If the mass does not have this initial kinetic energy, it will undergo a drop in elevation until it acquires the kinetic energy, either due to gravity or due to energy input from the oscillating cup. The mass can easily gain this energy from the oscillating cup if the amplitude of oscillation R of the cup is much larger than the clearance

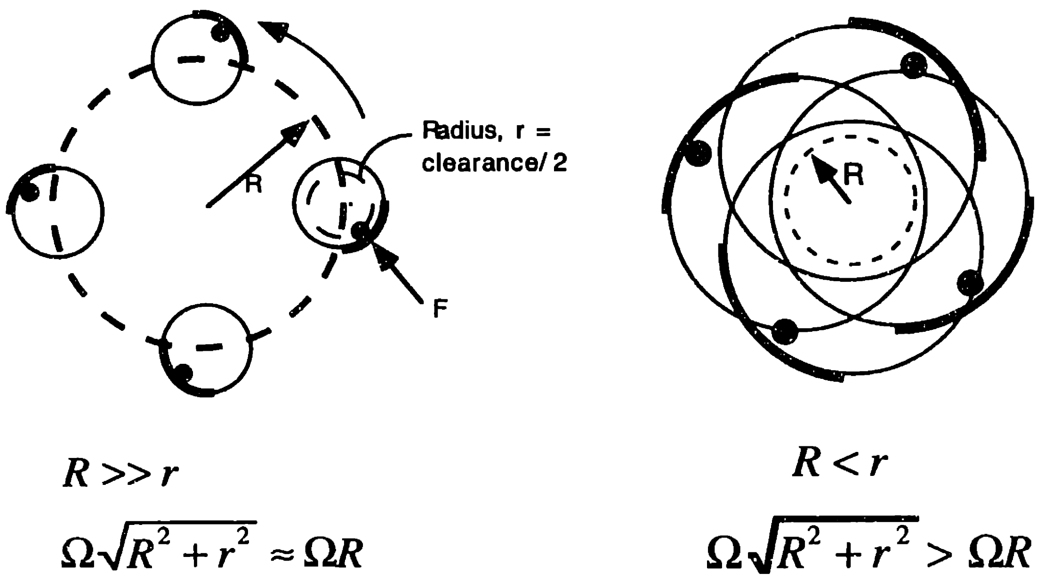

Figure 4-15: (a)Amplitude much larger than clearance; (b)Amplitude much smaller than clearance

r.

Conditions for a mass at the bottom of a cup to be thrown out:

The conditions for steady motion and resonance derived above can be used to derive useful design conditions for the motion of a point mass inside an oscillating cup. Consider a mass that is initially at the bottom of the cup. For the mass to be thrown out of the cup the steady state oscillation height y_{st} must be below the bottom of the cup. that is, $y_{st} \leq 0$. Hence the minimum required frequency of oscillation, Ω_{min} is given by

$$\Omega^2_{min} = \frac{g}{\tan\alpha[r_0 + R]} \tag{4.42}$$

Since the minimum required frequency decreases with increase in elevation, at frequencies larger than Ω_{min} the mass monotonically gains energy and elevation and leaves the cup. It should however be noted that there is a minimum initial kinetic energy requirement for the onset of resonance. Since the mass has to describe a circle of radius $R + r$, it has to have a minimum kinetic energy equal to $\frac{1}{2}m\Omega^2(r+R)^2$ given by equation 4.41. The mass should either have this initial energy or acquire this energy from the cup. It can acquire this energy from the cup if $R >> r$, as shown in figure 4-15a, because the minimum required velocity is almost equal to the velocity of the cup. However if $R << r$, as shown in figure 4-15b the mass cannot gain this energy from the cup and will remain at the bottom of the cup even if the frequency exceeds the minimum required frequency.

Time delay in onset of resonance:

Even if frequency conditions for resonance are satisfied, there may be a time delay in the onset of resonance. This delay can occur due to the following reasons.

1. Insufficient initial energy, $v < \Omega(r+R)$: If the initial kinetic energy of the mass is less than the required energy given by equation 4.41, then $\dot{\theta} < 0$ (the mass tends to lag behind the cup) and from equations 4.32 and 4.33, $\ddot{y} < 0$. The mass drops in elevation until it acquires the required kinetic energy, either due to gravity or due to energy input from the oscillating cup.

2. Excess initial energy, $v > \Omega(r+R)$: If the initial energy of the mass is greater than the required energy given by equation 4.41, then $\dot{\theta} > 0$ and from equations 4.32 and 4.33, $\ddot{y} > 0$. The mass has an increase elevation until it loses energy, either due to gravity or due to the energy absorbing contact with the oscillating cup.

3. Initial energy absorbing contact: when the initial point of contact is not an energy providing contact, that is, it lies outside the quarter circle shown darkened in figure 4-14, it takes a finite amount of time (upto a period of oscillation of the cup) before the onset of resonance.

Conditions for a point mass entering the cup to stay inside

From the point of view of entrapment, it is required to determine the conditions for a mass that enters an oscillating cup to remain inside the cup. Consider the case where the mass enters the cup (which is oscillating at a frequency Ω) and begins to make continuous contact at an elevation y_{in} and velocity v_{in} such that $\dot{y} < 0$ where $\dot{y}$ is the vertical component of its velocity. It can be shown that the mass will remain inside the cup if

$$y_{in} < y_{st} \tag{4.43}$$

and

$$v_{in} < v_{min} = \Omega(R + r_0 + y_{in} \tan\alpha) \tag{4.44}$$

The inequality $y_{in} < y_{st}$ ensures that the upward component of the reaction force does not exceed the weight and as a result $\ddot{y} < 0$. The second inequality ensures that there is no increase in elevation after the onset of contact. This is because if $v > v_{min}$, $\dot{\theta} > 0$ which in turn might result in $\ddot{y} > 0$ (from equations 4.32 and 4.33). Hence the condition for a mass descending into a cup of height H to remain inside the cup is obtained by substituting $y_{in} = H$ in the previous two equations,

$$y_{st} > H \tag{4.45}$$

which is the same as

$$\Omega^2 < \frac{g}{\tan\alpha[r_0 + H\tan\alpha + R]} \tag{4.46}$$

and the velocity condition is given by

$$v < \Omega(R + r_0 + H\tan\alpha) \tag{4.47}$$

While the condition given by equation 4.43 is in terms of known quantities, the condition given by equation 4.44 involves the velocity of the point mass when it begins to slide. This velocity in turn depends upon the initial entry velocity of the mass and the phase of oscillation of the cup at which it enters the cup and the nature of the motion in the initial stages of its descent into the cup. The motion in the initial stages of its descent is characterized by collisions and as a result the velocity at the onset of sliding cannot be precisely determined.

Effect of inclinations of the walls of the cup

The above conditions for resonance were derived for small values of α. As α increases, the following effects have to be taken into account.

- With increase in α the upward normal component of the reaction force increases. This in turn increases the value of $\ddot{y}$ and the rate at which the elevation of the mass increases.

- With increase in α the radius of the cup increases faster with an increase in elevation. Hence the minimum required kinetic energy for resonance also increases.

- As a result of the above two reasons, at large inclinations, the required rate of energy input to sustain resonance increases.

- With increase in α, however, the maximum possible rate of energy input decreases. This places an upper limit on the duration for which resonance can be sustained. With increase in α the duration of resonance (and hence a sustained increase in elevation) decreases.

The implications of the above observations can be illustrated through simulation results of the motion of a point mass in a cup for three different inclinations of the walls of the cup. Let the base radius of the cup, r_0 be $2cm$. The cup is oscillating along a circular path of radius $R = 6cm$ in a plane parallel to the base of the cup. The mass is initially at the bottom of the cup. The frequency of oscillation of the cup Ω is above the minimum required oscillation frequency for resonance for a wall inclination, $\alpha = 5\,\text{deg}$. Figure 4-16 shows simulation results of the elevation of the mass y as a function of the number of oscillations of the cup until the time when the mass ceases to have an energy providing contact with the cup (that is, when $\theta < -90\,\text{deg}$). Figure 4-17 shows the plot of angle θ as a function of number of oscillations of the cup for different inclinations of the walls of the cup.

At low wall inclinations, the net increase in elevation before the loss of energy providing contact is higher even though the rate of increase in elevation is lower. At higher wall inclinations, the rate of increase in elevation is higher, but the total increase in elevation before loss of contact is lower. This is because the rate at which energy can be provided decreases with increase in inclination. An important implication of the above observation is that for cups with large inclinations, the mass cannot get thrown out of the cup if the depth of the cup is very large. Resonance that is initiated at the bottom of the cup cannot be sustained until the mass reaches the top of the cup. This effect is even more pronounced

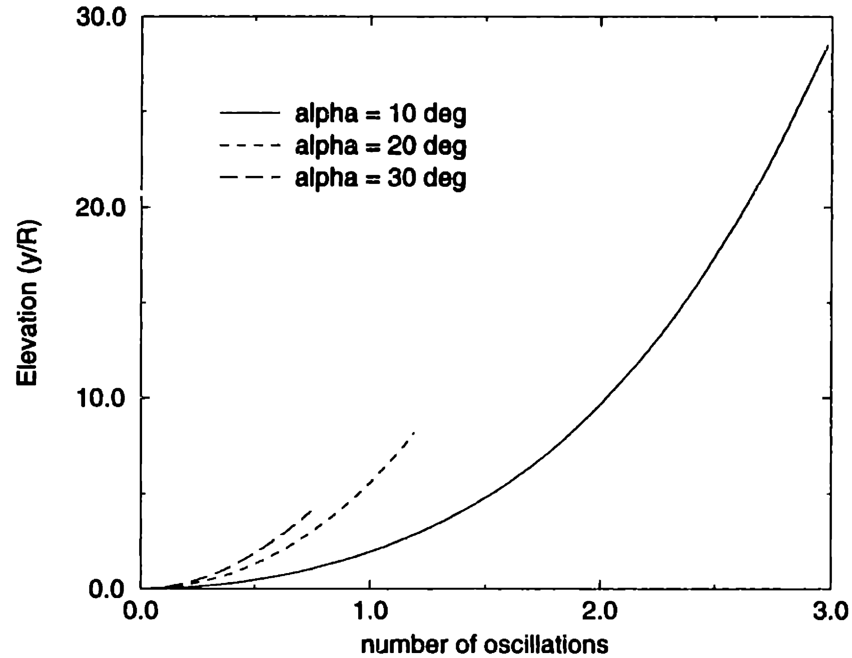

Figure 4-16: Increase in elevation of a point mass in a cup due to resonance

if the energy loss due to damping is significant.

4.2.6 Point mass in a spatial oscillating funnel

The discussion on the mechanics of a point mass in a clearance relief will be concluded with the final example of a point mass in an oscillating funnel. The interest in a funnel arises out of the fact that a typical clearance relief (such as that shown in figure 4-18) is shaped like a funnel. The angle of inclination α increases with increase in elevation y. Of specific interest are the possible outcomes on the motion of a point mass that is approaching the funnel as shown in figure 4-18.

The mass first makes contact with the funnel in the initial shallow regions characterized by very large α. At large α, work interactions are minimal and the conditions derived for a stationary funnel can be applied. Also, there is significant energy loss due to damping. The mass can get thrown out of the funnel if it has a certain minimum kinetic energy as it enters the funnel. The minimum required kinetic energy depends upon the initial velocity,

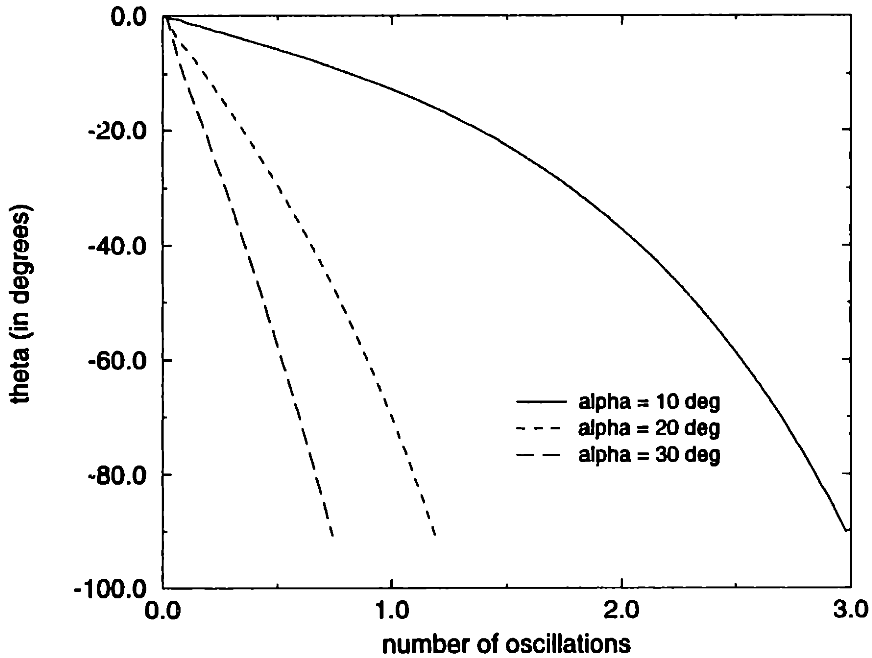

Figure 4-17: Variation of contact angle θ of a point mass in a cup due to resonance

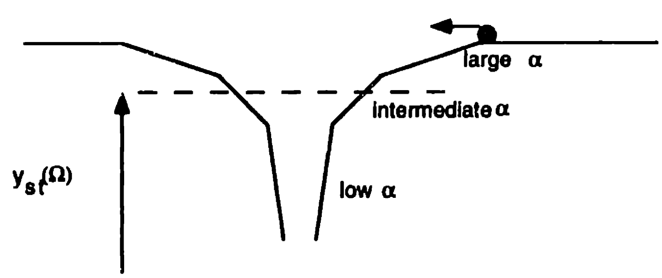

Figure 4-18: point mass in a spatial oscillating funnel

both direction and magnitude. Equation 4.1 provides the minimum required initial kinetic energy for the case when the mass has an initial horizontally pointing velocity. If the mass has insufficient initial kinetic energy it descends below the initial shallow part of the funnel.

Below the shallow part of the funnel the slope of the wall is characterized by low or intermediate values of α. Work interactions between the funnel and the wall become significant. It is possible for the mass to acquire sufficient energy and get thrown out of the funnel. The mass successfully descends into the funnel if conditions described by equations 4.43 and 4.44 are satisfied. If the condition in equation 4.44 is satisfied and not the condition in equation 4.43, the mass may still descend successfully if the time delay for the onset of resonance is large enough that the mass descends past y_{st} before the onset of resonance.

It is important to understand the conditions under which equation 4.44 will be satisfied when equation 4.43 is already satisfied. The velocity or kinetic energy of a descending mass depends upon a number of factors. One of the factors is the height through which the mass has fallen at the time of interest. More the height through which it has fallen, greater is its kinetic energy. Another important factor, is the work interaction it has had with the oscillating funnel. If the mass is in contact with a receding wall its energy is lower. This means that the phase of oscillation at which the mass enters the funnel is important. Finally, the energy of the mass depends upon its initial energy at the time of its entry into the funnel. Clearly, increasing y_{st} and increasing v_{min} increase the chances for entrapment. From equation 4.34 it is clear that y_{st} can be raised by increasing the clearance or decreasing oscillation frequency. From equation 4.41, v_{min} also increases with increase in clearance. In addition, decrease in amplitude decreases the rate of work input from the oscillating funnel and therefore increases the time delay for the onset of resonance. Increasing time delay in turn improves the chances for the part to get trapped. In summary, increasing clearance and decreasing frequency and amplitude help in entrapment. If clearance cannot be increased, another available option is make the inclination above $y = y_{st}$ large. Since shallow slopes preclude resonance and generally result in energy loss due to damping, they increase the likelihood of the mass descending below $y = y_{st}$ satisfying the energy condition in equation 4.44.

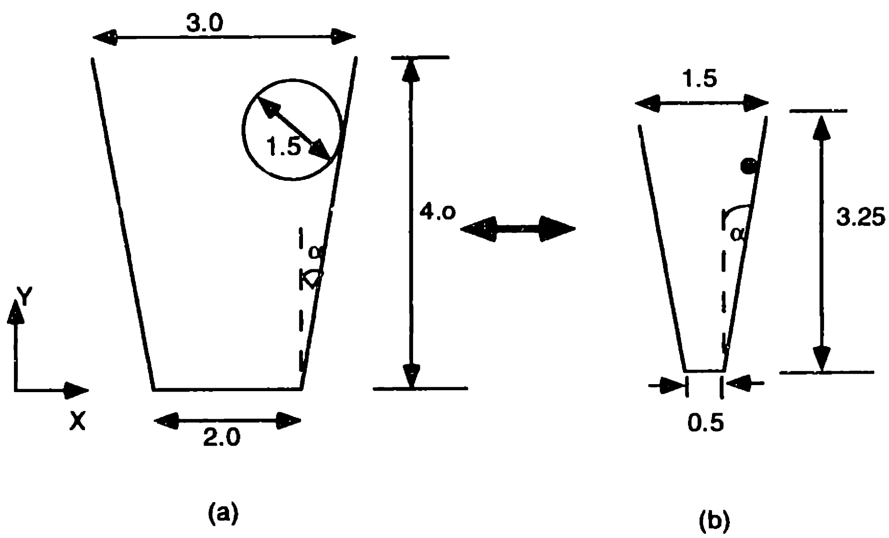

Figure 4-19: Pingpong ball in a cup (dimensions in inches)

4.2.7 Motion of a ping-pong ball in an oscillating cup

The results derived for the motion of a point mass in a cup can be easily verified with a ping-pong ball in a coffee cup as shown in figure 4-19a. If the ball can be assumed to be a point mass (this is so if the ball slides without rolling) the point mass equivalent of the cup is shown in figure 4-19b. The radius of this equivalent cup is the difference in the radius of the actual cup and the radius of the ping-pong ball. Typical dimensions of the ball and the cup are also shown (in inches). For a ball that is initially at the bottom of the cup, the minimum frequency of oscillation for which the ball gets thrown out is given by

$$\Omega_{min}{}^2 = \frac{g}{(r_0 + R)\tan\alpha} \tag{4.48}$$

where R is the radius of the circle described by the cup and r_0 is the effective radius of the base of the cup. The inclination α is 8.75 deg. For the cup shown, the minimum frequency is approximately, 5 cycles/second if amplitude R is $50mm$.

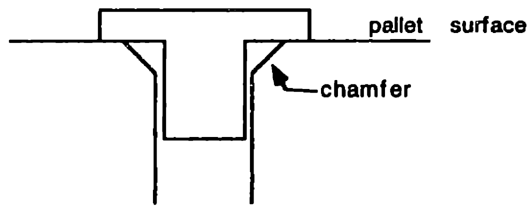

Figure 4-20: Axisymmetric pin in a relief - cross sectional view

4.3 Axisymmetric pin in an axisymmetric relief

The mechanics of interaction of a point mass in a relief can be extended to explain the behavior of real parts. This section discusses the motion of an axisymmetric pin in an axisymmetric relief. Dimensions of a typical pin and relief used in the analysis are shown in figure 3-3. The relief is carved out on a pallet that is mounted on an APOS. The oscillation of the pallet is a translation along a circular path of radius approximately $1mm$ in its own plane. Typical APOS operating frequencies range from 15 to 25 cycles/second.

The motion of the axisymmetric pin in the relief under the given oscillation conditions is similar in nature to the motion of a point mass in a cup that is describing a circular path in a plane parallel to its base. Figure 4-20 shows the cross-sectional view of the pin inside the cup. The pin gains energy and momentum through contacts with the walls of the relief. Similar to the case of the point mass in the cup, the pin can have energy providing and absorbing contacts with the oscillating cup. The pin can get thrown out of the cup if it acquires sufficient momentum in the positive y direction. It acquires this momentum if the upward components of the reaction forces from the walls of the relief exceed the weight of the pin. It needs to be understood how these upward reaction forces arise. The upward reaction forces arise due to the inclination of the pin with the vertical. This inclination, in turn, is due to the non-zero clearance between the pin and the relief and the tilting moments due to reaction forces.

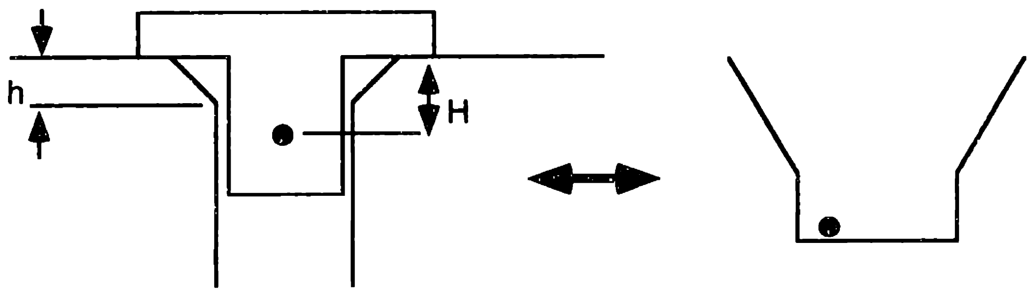

Figure 4-21: Pin whose center of mass is below bottom of chamfer

4.3.1 Conditions for stability

This subsection addresses issues of stability of the pin which is already inside the relief. Of specific interest is the minimum frequency and amplitude of oscillation of the relief that would throw the pin out of its trapped state. Conditions for stability can be derived in a manner similar to the conditions for stability of a point mass inside a cup. Consider figure 4-21 which shows a trapped pin whose center of mass is below the bottom of the chamfer. Under such conditions, the motion of the pin inside the relief is similar to the motion of a point mass at the bottom of a cup with vertical walls. There is no vertical component for the reaction forces from the walls of the relief. The weight of the pin is balanced by the reaction forces from the pallet surface. If the radius of oscillation is sufficiently large the pin has a steady motion along a circle of radius $(R + r)$. Therefore if the center of mass of the pin is below the bottom of the chamfer the trapped pin is stable for any frequency of oscillation. There is however, one difference between the motion of a point mass and that of a pin. While the mass slides along the walls of the cup, the pin tends to roll along the walls of the cup. Hence, in addition to its rotation along a circle of radius $(R + r)$, the pin has a rotation about its own axis.

Now consider the pin in figure 4-22 for which the center of mass lies above the bottom of the chamfer. The reaction forces from the walls of the chamfer exert moments about the center of mass of the pin. If this moment exceeds the restoring moment due to the reaction force from the pallet surface, the pin acquires an inclination with the vertical.

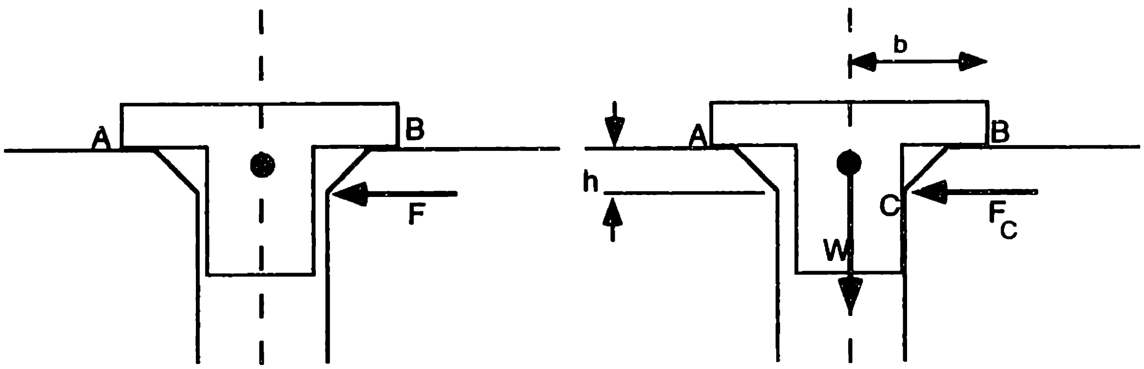

Figure 4-22: Pin whose center of mass is above bottom of chamfer

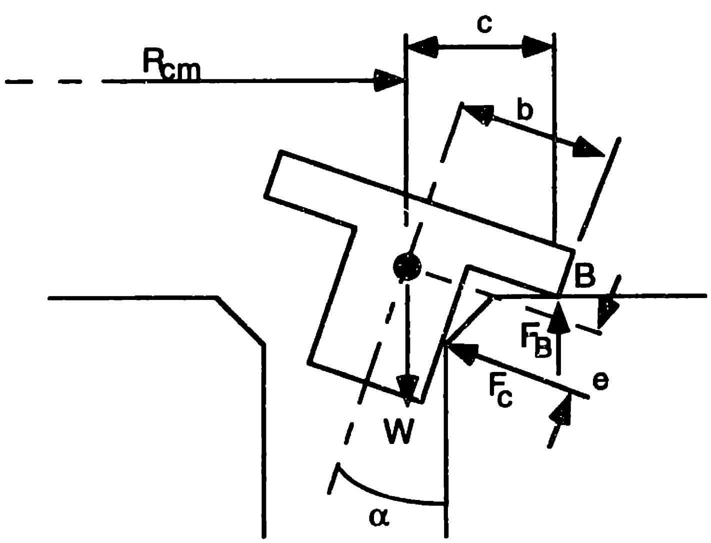

Figure 4-23: Pin inclined with the chamfer

This inclination inturn results in reaction forces that have components in the positive Y direction. It is required to determine the minimum frequency Ω above which the pin begins to tilt. Consider figure 4-22 which shows the forces acting on the pin when it is just about to tilt to the right. The reaction force at A, F_A vanishes when the pin tilts and the only contact with the pallet surface is at point B. The contact with the relief is at the bottom of the chamfer and the reaction force, F_C is given by

$$F_C = m(r + R)\Omega^2 \tag{4.49}$$

F_C is the centripetal force required for the pin to describe a circle of radius $(R+r)$ where R is the amplitude of oscillation and r is half of the clearance between the pin and the relief. Taking moments of the forces about point B, for the mass that is about to tilt,

$$m(r + R)\Omega^2 h = mgb \tag{4.50}$$

As shown in figure 4-22 h is the depth of chamfer. The minimum required frequency of oscillation of the relief for the pin to tilt is given by

$$\Omega_{min}^2 = \frac{gb}{(r + R)h} \tag{4.51}$$

If $\Omega > \Omega_{min}$, the pin begins to roll out of the relief. Figure 4-23 shows the direction of reaction forces for a small angle of inclination α. For a given frequency Ω, the overturning moment is greater for $\alpha > 0$ than for $\alpha = 0$. Hence, if the pin is not stable for $\alpha = 0$, it is not stable for any $\alpha > 0$. The strong similarity with the motion of a point mass at the bottom of a cup should be taken note of. There is a minimum frequency after which the mass leaves the bottom of the cup and monotonically gains energy and elevation. This is the condition of resonance described earlier.

4.3.2 Stability to disturbances

It is of interest to determine the extent of stability of the pin to small disturbance inputs. Specifically it is of interest to determine the the maximum disturbance tilt α_{max} upto which the pin would return to $\alpha = 0$ for a frequency of oscillation $\Omega < \Omega_{min}$ where Ω_{min} is defined by equation 4.51. In the example of the point mass in a cup it was shown that for a given oscillation frequency Ω, there was an elevation y_{st} at which the mass would have a steady motion. This steady motion was shown to be unstable. Similarly, for a pin, for a given oscillation frequency Ω there is an inclination α_{st} at which the pin would have a steady precession. This steady precession is unstable. Therefore, if $\alpha < \alpha_{st}$ the pin would be returned to the $\alpha = 0$ orientation. For $\alpha > \alpha_{st}$ the pin would have a monotonic increase in energy resulting in an increase in its energy and elevation. Hence for a given frequency of oscillation, α_{st} is the maximum allowable perturbation in the inclination from the fully trapped state.

The inclination, α_{st} can be determined from the equations of motion for steady precession. Figure 4-23 shows the reaction forces that act on the pin in steady precession. The center of mass of the pin describes a circle of radius R_{cm}. The centripetal force is provided by the horizontal component of the reaction force, F_C. The weight of the pin must be balanced by the upward vertical components of the reaction forces. These two conditions can be expressed as follows.

$$F_C \cos\alpha = mR_{cm}\Omega^2 \tag{4.52}$$

$$F_C \sin\alpha + F_B = mg \tag{4.53}$$

In addition, the moment of the reaction forces about the center of mass of the pin must equal the torque required to maintain the precession. This torque is equal to the rate of change of angular momentum of the pin. The pin describes an oscillation along a circular path with frequency Ω and in addition rotates about its own axis with an angular velocity ω.

If I_1 and I_2 are the moment of inertia of the pin about its principal axes, the angular momentum,**H** is given by

$$\mathbf{H} = I_1(\Omega\cos\alpha - \omega)\mathbf{e_1} + I_2\Omega\sin\alpha\mathbf{e_2} \tag{4.54}$$

where $\mathbf{e_1}$, $\mathbf{e_2}$ and $\mathbf{e_3}$ are unit vectors along the principal axes. The torque equation in terms of the reaction forces can be written as

$$F_B c - F_C e = \Omega^2\cos\alpha\sin\alpha(I_2 - I_1) + I_1\Omega\omega\sin\alpha \tag{4.55}$$

At very low values of α the right hand side of the above equation can be assumed to be zero. The three equations can be solved to obtain α_{st}, F_B and F_C. It is also apparent from figure 4-23 that the steady motion is unstable. A small perturbation in α results in a monotonic increase or decrease in α.

4.3.3 Conditions for a pin descending into a relief to get trapped

The discussion of stability of a fully trapped pin to small disturbances can be extended to explain the conditions for a pin that is descending into a relief to get trapped in the relief. The motion of a pin in the final stages of its descent into an axisymmetric relief is equivalent to the motion of a point mass in a cup whose inclination α decreases with depth as shown in figure 4-24. For a given oscillation frequency Ω there is an $\alpha_{st}(\Omega)$ and a $y_{st}(\Omega)$ at which the pin can have steady precession. At elevations below y_{st} the pin monotonically loses energy and gets fully trapped. Thus the frequency condition for successful entrapment can be expressed as follows.

$$y < y_{st}(\Omega) \tag{4.56}$$

As the oscillation frequency Ω increases the elevation for steady precession, y_{st} decreases. In addition to the frequency condition, there is a velocity condition for resonance. As the relief describes a circle of radius R the pin has to describe a circle of radius $R + r$ where r is one-half the clearance between the pin and the relief. This means that the pin has to have a velocity of atleast $\Omega(R + r)$. The pin can acquire this velocity from the relief if the amplitude of oscillation R is much larger than the clearance r.

For the specific part-relief pair under consideration and in general for typical parts that are routinely palletized in an APOS, the clearance for a fully trapped part which is of the order of a tenth of a millimeter is much smaller than the amplitude of oscillation which is of the order of a millimeter. On the other hand, for a part in its early stages of descent, the clearance is much larger than amplitude and as a result the part does not acquire the minimum required kinetic energy for resonance. Hence resonance cannot occur in the early stages of descent of the pin.

Resonance zone: The above observations have important implications. For a given frequency Ω, resonance cannot occur at depths below y_{st}. In addition, due to insufficient energy, resonance cannot occur in the initial stages of descent. This means that there is an intermediate zone called the "resonance zone" (shown in figure 4-25) where resonance can occur and throw the part out. This is the zone where the inclination α is large but the clearance r is still much smaller than the amplitude R. The resonance zone is sandwiched between a lower zone of very small inclination and an upper zone of very large clearance both of which preclude resonance. It does not however mean that resonance is always initiated when the pin is in the resonance zone. For resonance to be initiated, the pin has to have an energy providing contact with the relief. Depending upon the phase of the oscillation cycle at which the pin enters the resonance zone there is a time delay in the onset of the energy providing contact and hence in the onset of resonance. If this time delay exceeds the time taken by the pin to descend through the resonance zone, resonance does not occur and the pin gets trapped in the relief.

The time delay in the onset of resonance depends upon the phase of the oscillation cycle at which the pin descends into the resonance zone and the location of the point of contact between the pin and the relief. This in turn depends upon the motion of the pin prior to its descent into the resonance zone. Experimental observations show that the motion of the pin in the initial stages of its entry into the relief is characterized by intermittent contacts or collisions between the pin and the relief. As a result of this complexity in the motion of the pin, conditions at the time of entry into the resonance zone cannot be precisely determined. Hence the time delay and the conditions for entrapment cannot be exactly determined. It is useful, however, to compare approximate estimates of the time delay with those of the time taken for descent through the resonance zone for the specific part-relief pair under consideration. The length of the time delay can vary from 0 to half a period of oscillation. Hence for an oscillation frequency of 25 cycles/second, the upper limit on the time delay is approximately 0.04 seconds. An approximate estimate of the time for descent through the resonance zone can be obtained by assuming that the depth of the resonance zone is equal to the amplitude of oscillation which is approximately 1 mm. For a free falling object starting from rest, the time taken for descent through a depth of 1 mm is approximately 0.015 seconds. This approximate calculation shows that the time delay is of the same order of magnitude as the time for descent through the resonance zone. Hence depending upon the actual time delay the part can either successfully descend into the relief or get thrown out of the relief due to resonance. It is also useful to understand the effect of various design parameters on the occurence of resonance. Increasing the oscillation frequency decreases $y_{st}(\Omega)$ and hence increases the depth of the resonance zone and hence the likelihood of resonance. Decreasing clearance or increasing amplitude raises the upper boundary of the resonance zone and as a result lessens the likelihood of occurrence of resonance.

4.3.4 Failure of entrapment in the initial stages of descent

The discussion in the previous subsection dwelled on failure of entrapment due to resonance. This kind of failure occurs in the resonance zone that corresponds to clearance being much smaller than amplitude. In the case of the pin and the axisymmetric relief under consideration, it occurs when the pin is very close to its fully descended state. Failure due to resonance is similar to the phenomenon of a pingpong ball being thrown out of a cup that is oscillating along a circular path in a plane parallel to the ground. Failure due to resonance is just one cause for failure of entrapment. It was observed that failure of entrapment can also occur in the initial stages of descent, that is, even before the pin reaches the resonance zone. Figure 4-26 depicts the occurence of one such event of failure. Failure in the initial stages of descent occurs due to an energy providing contact very early in the descent. It depends upon the phase of oscillation of the pallet at the time of initial contact between the pin and the relief and the direction of approach of the pin. Determining conditions for failure involves understanding the various possible ways in which the pin can approach the relief and the resulting motion of the pin. This will be the subject of future research.

4.4 Asymmetric part with a cylindrical locating feature

Failure of entrapment due to resonance was also observed in parts that are not axisymmetric. This section provides a brief description of failure of entrapment of the asymmetric part shown in figure 3-2a in the initial stages of its descent into the relief. The part has a cylindrical locating feature attached to a flat triangular body. While being conveyed on the pallet surface, it rests on three points, two of them being the corner points on the triangular body and the third being a point on the cylindrical locating feature. The relief is a cylindrical hole with a conical chamfer as shown in figure 3-2b. The dimensions of the experimental part and the relief are as shown in figure 3-3. The oscillation of the pallet is a translation

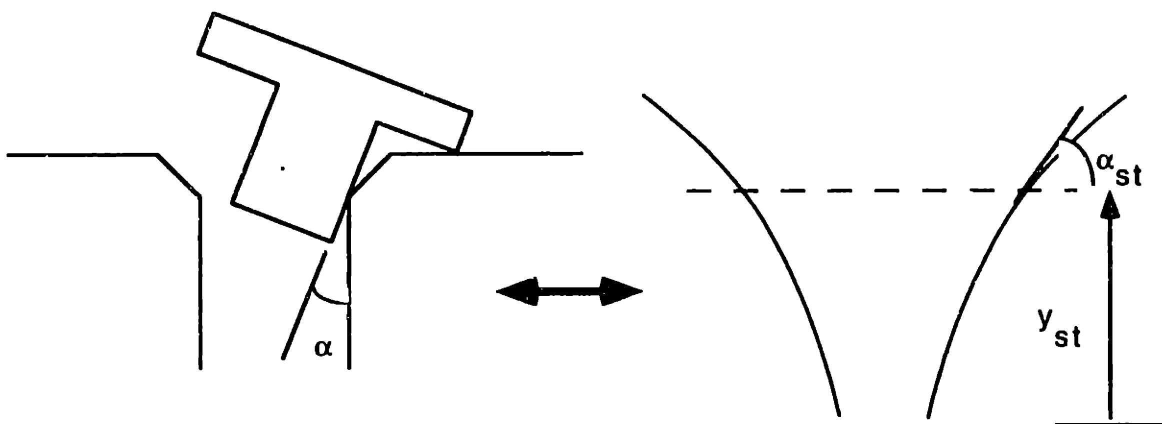

Figure 4-24: Equivalence between aximmetric pin and point mass in a funnel

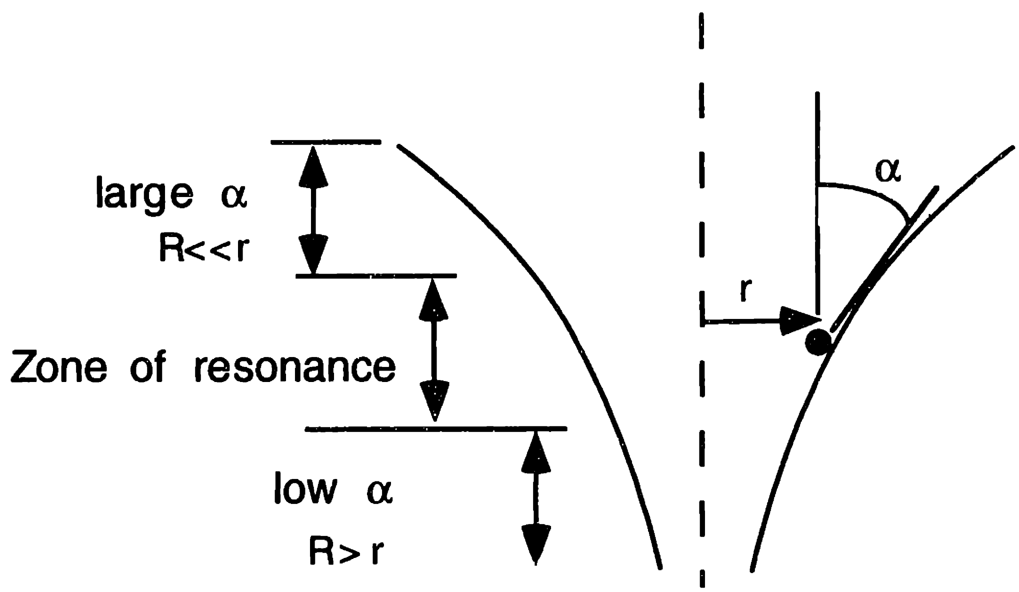

Figure 4-25: Zone of resonance

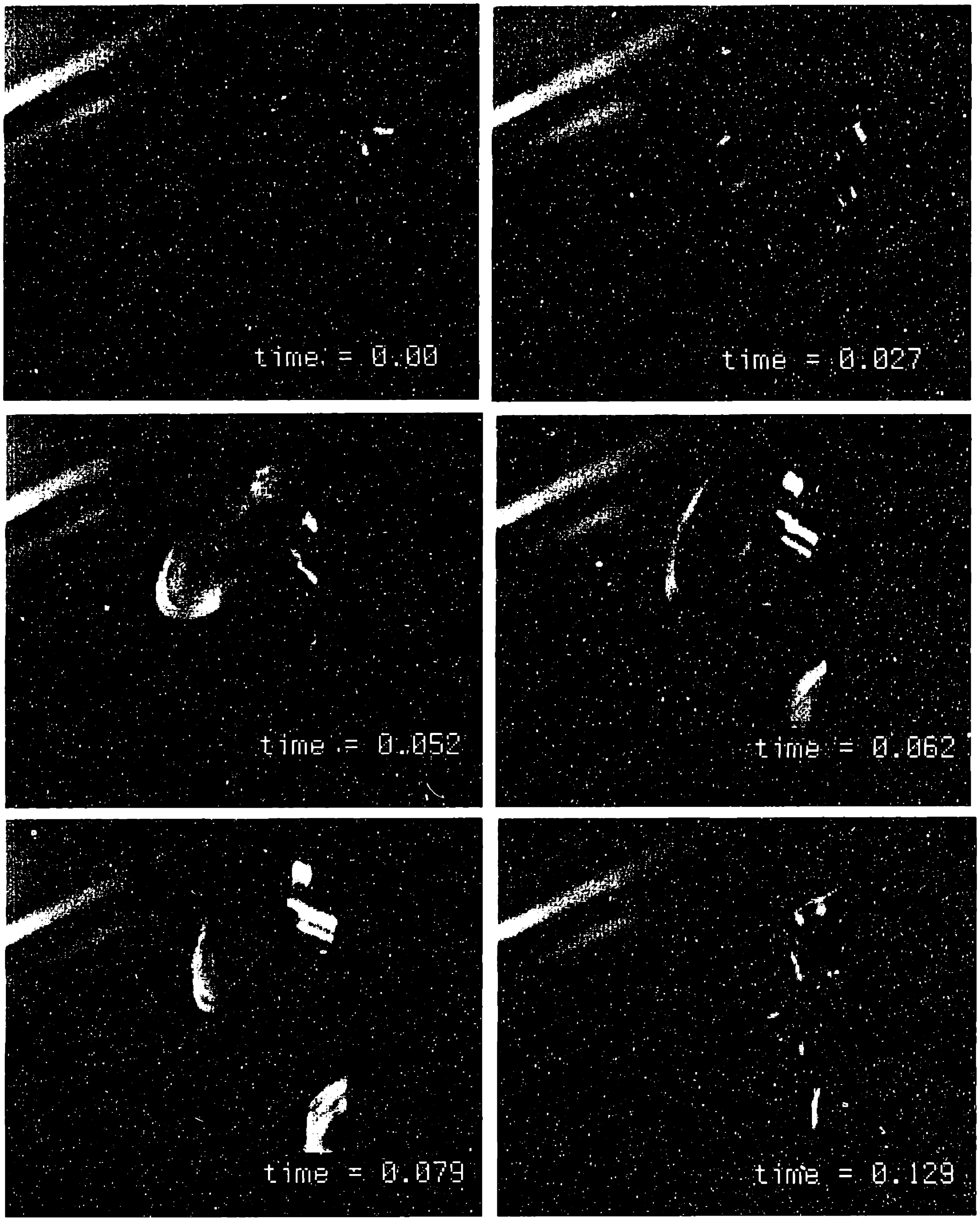

Figure 4-26: Example of failure of entrapment of a pin in the initial stages of its descent (time in seconds)

along a circular path parallel to the plane of the pallet with a radius approximately 1 mm. The clearance between the fully trapped part and the relief is approximately 0.1 mm.

The asymmetry in the geometry of the part makes its interaction with the relief qualitatively different from that of an axisymmetric pin. Unlike an axisymmetric pin, the asymmetric part cannot precess about the vertical axis. As a result, resonance leading to monotonic energy input does not occur in its fully descended state. On the other hand, due to its asymmetry, the part is vulnerable to be thrown out of the relief due to resonance in the early stages of its descent.

Depending upon the phase of the oscillation cycle at which the part begins to descend into the relief the motion in the initial stages of descent is characterized by either intermittent contacts and collisions or continuous sliding contact. It was observed that the part can have continuous energy providing contact with the relief causing it to be thrown off the relief. Figure 3-5 shows an example of such an event. The part gets thrown off in about half an oscillation cycle. As the relief describes a circular path of radius R which is the amplitude of oscillation, to be in resonance, the locating feature has to describe a circle of radius $R + r$ where r is the clearance between the locating feature and the relief. This means that there is a minimum required kinetic energy for the initiation of resonance. The part can gain this energy from the oscillating relief if the clearance is much smaller than the amplitude of oscillation, that is, $R + r \approx R$. As a result, resonance can get initiated only in the lower part of the chamfer where the clearances are small. Approximate frequency conditions for resonance is given by the following condition.

$$\Omega_{min}^2 > \frac{g \tan \alpha}{R + r} \tag{4.57}$$

where Ω_{min} is the minimum required frequency for resonance and α is the inclination of the contact normal. The minimum frequency for resonance is not well defined because unlike an axisymmetric pin, the asymmetric part cannot have a steady precession at a constant inclination of the contact normal. The inclination of the contact normal varies along the circumference of the relief. This variation in the inclination of the contact normal is illustrated in figure 4-27 for the equivalent planar part-relief system[3]. The inclination of

[3]the planar part-relief system is chosen for the purpose of illustration because of the simplicity of the contact geometry.

the contact normal, α for a contact with the right wall of the relief is different from the inclination for a contact with the left wall of the relief. This is illustrated in the two columns in figure 4-27 for different levels of descent into the relief. Notice that in the early stages of descent the slope of the contact normal for contacts with the right wall of the relief is the slope of the chamfer β while for contacts with the left wall of the relief it is $90 - \theta$ where θ, as shown in figure, is the inclination of the part.

Effect of depth and width of chamfer: Despite the complexity of the motion of the asymmetric part in the relief, useful insight about the effect of design parameters on entrapment can be obtained. In the section on the motion of a point mass in an oscillating cup, it was shown that the depth of the cup is significant in determining whether a descending mass would get thrown off. For cups with very large depths, even if resonance is initiated in the lower levels of the cup, it cannot be sustained until the mass reaches the top of the cup. This effect is even more pronounced if there is significant energy loss due to sliding friction. A similar situation arises if the chamfer for the asymmetric part is deep and wide as shown in figure 4-28. For deep and wide chamfers, clearance is very large near the surface of the pallet. For resonance to be initiated near the surface of the pallet, the part must have a velocity of about $\Omega(R+r^*)$ where r^* is the clearance at the pallet surface. Since the typical velocity of a part conveying on a pallet surface is approximately the velocity of the pallet which is ΩR resonance cannot be initiated near the pallet surface. For resonance to be initiated, the part has to descend to the lower part of the chamfer where clearance becomes comparable to the amplitude. Even if resonance is initiated close to the bottom of the chamfer, if the chamfer is very deep it cannot be sustained until the part fully leaves the relief. The part will descend back again into the relief. Thus the part can make several attempts at getting into the relief. This inturn increases the overall likelihood of it getting trapped. Thus a deeper and wider chamfer (for the same inclination) will have a higher efficiency of entrapment.

From the point of view of design, it is useful to know how deep a chamfer should be in order to be considered deep. The chamfer can be considered deep if the potential energy difference between the top and bottom of the chamfer is much larger than the kinetic energy of the part when it is in resonance (approximately equal to the velocity of the relief). This corresponds to the case where the only way the part can leave the chamfer is through

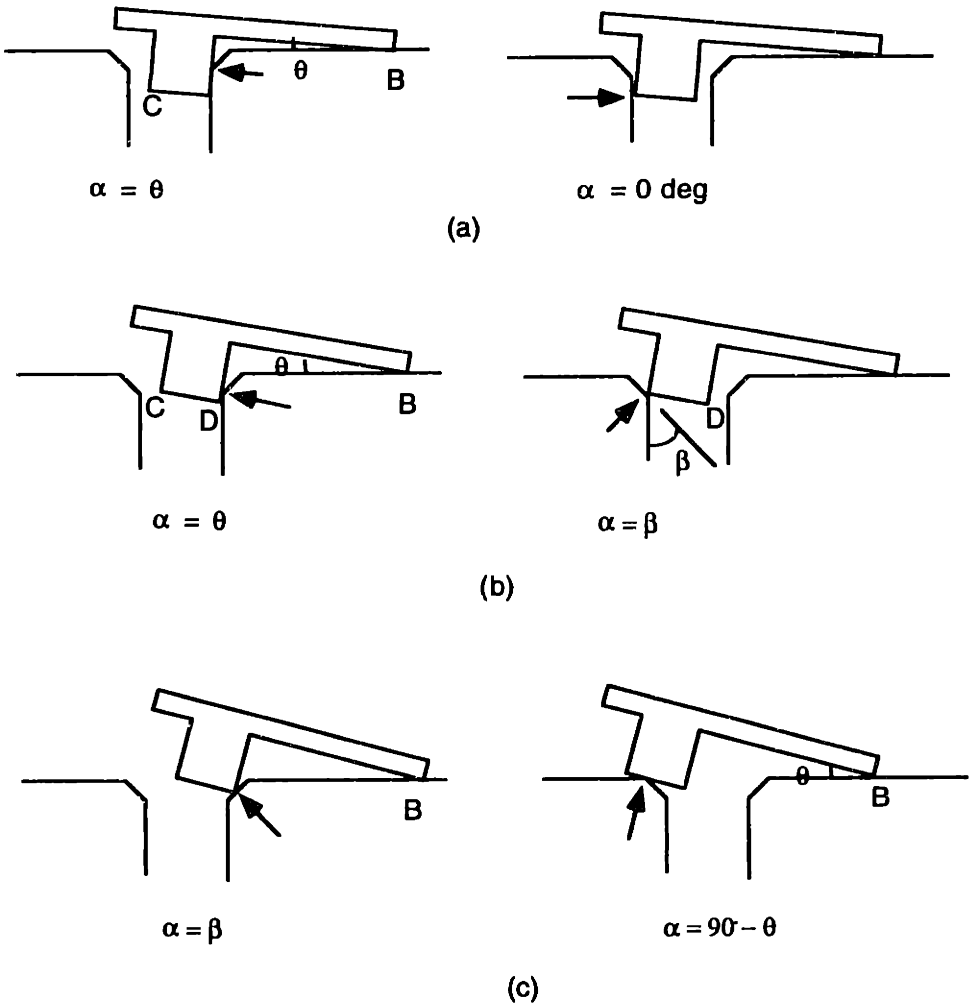

Figure 4-27: Various levels of disturbance inclination

resonance sustained over several oscillations. If h is the depth of the chamfer, R is the amplitude of oscillation, r is the clearance and Ω is the frequency of oscillation, the chamfer can be considered deep and wide if

$$h >> \frac{(R+r)^2\Omega^2}{2g} \tag{4.58}$$

For $R = 1mm$, $r = 0.1mm$, $\Omega = 150rad/s$, $h >> 1.2mm$.

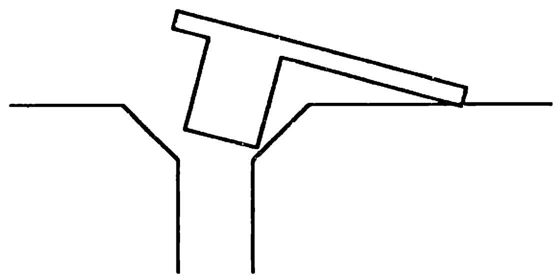

Figure 4-28: Deep and wide chamfer

4.5 Summary

This chapter discusses the mechanics of entrapment, conditions for a part to stay trapped and conditions for successful descent. The mechanics is introduced for the example of a point mass in a cup and then extended to some real parts and reliefs. The entrapment of a part in a relief is influenced by the initial velocity of the part, geometry of the relief and the relief oscillation parameters. The effect of intial velocity and relief geometry is explained with the example of a point mass in a stationary cup. It is shown that for a mass with an initial velocity that is initially parallel to the ground, there is a minimum required kinetic energy for the mass to be thrown out of the cup. This minimum required kinetic energy is a function of the inclination of the walls of the cup. At very small inclinations it is much larger than the potential energy barrier that needs to be overcome. The motion of the mass in a cup could be characterized by intermittent contacts (due to collisions) or continuous sliding contact with the walls of the cup. The example of a point mass in a planar cup was used to illustrate motion characterized by intermittent contacts and the example of the point mass in a spatial cup was used to illustrate motion characterized by continuous contact.

The analysis of motion in a stationary cup was extended to an oscillating cup where the oscillation is parallel to the base of the cup. It was shown that the mass can have energy providing or absorbing contacts with the surface of an oscillating cup. In the presence of oscillations it is possible for the mass to have successive energy providing contacts. Resonance was described as the condition under which the energy and elevation of the part increases monotonically until the part leaves the relief. Precise conditions for resonance are derived for the example of the point mass in a spatial cup where the oscillation is along a circular path parallel to the base of the cup. It is shown that there is a minimum frequency beyond which a mass that is initially at the bottom of the cup can monotonically gain energy and get thrown out of the cup. In addition to a minimum frequency requirement, there is also a minimum amplitude requirement for resonance. The amplitude has to be much larger than clearance. At large wall inclinations, the minimum frequency required for resonance decreases while the minimum amplitude required increases due to larger clearances. There is also a time delay in the onset of resonance. This delay is due to insufficient initial kinetic energy and delay in occurrence of an energy providing contact with the cup.

The motion of an axisymmetric pin is similar in nature to a point mass in a cup but for the fact that the inclination in the contact normals arise due to inclination of the pin rather than the walls of the relief. The inclination of the pin, in turn, arises due to tilting moments from the reaction forces. For a tilting moment to arise, the center of mass of the pin has to be above the bottom of the chamfer and the frequency of oscillation has to be above a certain minimum frequency. In addition to failure due to resonance, failure of entrapment can also occur in the early stages of descent of the part into the relief. This failure usually occurs due to an energy providing contact in the very early stages of descent.

The motion of an asymmetric part differs from an axisymmetric pin in that due to asymmetry the direction of the contact normal varies with the phase of oscillation. As a result, asymmetric parts are more stable in their fully trapped state. However, entrapment can fail due to resonance in the chamfer zone. The occurrence of the event of entrapment depends upon the time delay in the onset of resonance. This delay, in turn, depends upon the conditions at the time of approach to the relief.

5

Design Options and Tradeoffs

The discussion of the effect of shape and oscillation parameters in the previous chapter was confined to the last stage of the palletization process, namely the entrapment of a single part that approaches an oscillating relief. The design parameters that are ideal for this last stage of palletization are not necessarily ideal in the context of the entire palletization process. This chapter summarizes the options available to the designer to improve the efficiency of entrapment, that is the last stage of the palletization process, and discusses the pros and cons of each of the options from the point of view of efficiency of the overall palletization process. This chapter is organized in the following manner. Section 5.1 discusses the option of altering oscillation parameters. Section 5.2 discusses chamfer shapes and dimensions.

5.1 Oscillation Parameters

It was shown in the previous chapter that the magnitude of the reaction forces are generally proportional to the kinetic energy of the part as well as the acceleration of the relief. The larger the magnitude of the reaction force, the greater is the vertical component. A part in continuous contact gets thrown out of the relief if the sum of the upward component of the reaction forces exceeds the weight of the part. The kinetic energy of the part while it is conveying on the pallet surface and when it is interacting with the walls of the relief is proportional to the maximum velocity of the relief. Since the maximum velocity of the

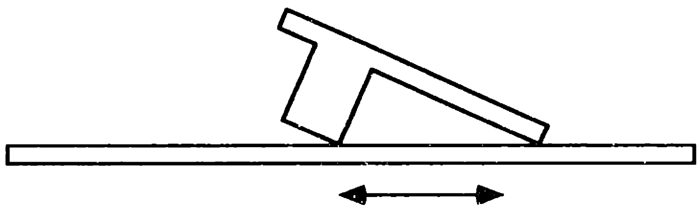

Figure 5-1: Frictional contact between part and pallet

relief is given by $R\Omega$ where R is the amplitude of oscillation and the maximum acceleration of the relief is $R\Omega^2$, it is obvious that decreasing frequency and amplitude will assist in the part getting trapped. In addition, decreasing frequency increases the tme delay in the occurrence of the first energy providing contact. Larger time delay means higher efficiency of entrapment.

At too low an amplitude or frequency the very purpose of having an oscillating pallet would be defeated. Oscillation has the following uses in part conveyance. Oscillation of the pallet helps break frictional contacts both on the pallet surface as well as inside the relief. For example, in order to break the frictional contacts between the part and the pallet shown in figure 5-1, the minimum required pallet acceleration is given by

$$A\Omega^2 > g\mu \tag{5.1}$$

Oscillation of the pallet also helps avoid clustering of parts. A part that is being conveyed on a pallet will encounter several obstructions to its flow either in the form of other parts or certain features on the pallet surface. These obstructions essentially constitute potential energy minima which are shallow in comparison to the minimum that corresponds to the fully trapped state. The part needs a certain minimum kinetic energy to overcome these obstructions or to get around them. This minimum kinetic energy is proportional to the physical dimension of the obstruction. Thus, while decreasing the frequency and amplitude assist the entrapment process, it is generally detrimental to the conveyance of the parts.

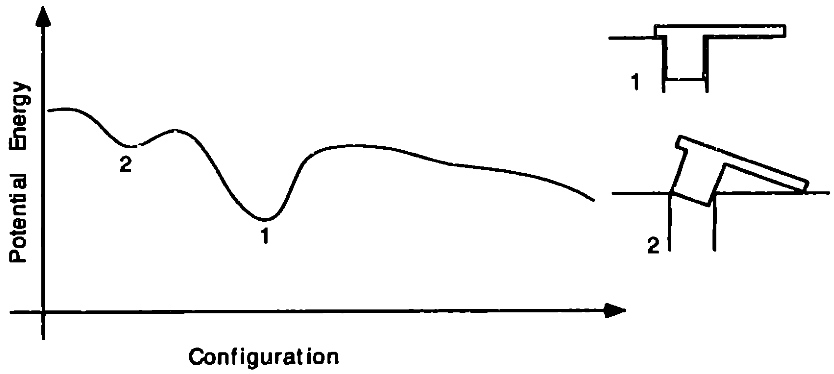

Figure 5-2: Undesired stable configurations due to inadequate chamfer

5.2 Shape of relief

The shape of the relief affects the entrapment process in the following two ways.

1. It influences the direction of the reaction force. It was shown that in the absence of friction the direction of the reaction force is along the contact normal.

2. It affects the inclining moment acting on the part. It was shown that there is an inclining moment on the part if the center of mass lies above the bottom of the chamfer and the moment increases with incease in the depth of the chamfer.

Altering the depth of chamfer has contrasting effects on the stability of a trapped part and the likelihood of it getting trapped. Decreasing the depth of chamfer stabilizes the trapped part because the inclining moment decreases. However, having a very small chamfer or having no chamfer would result in the part being stable in one or many undesired configurations. As shown schematically in figure 5-2, the effect of reducing chamfer is to add other local potential energy minima (such as configuration 2) in the neighborhood of the minimum that corresponds to the fully trapped state (shown as configuration 1).

Making the chamfer wider also helps in increasing the efficiency of the part getting trapped as discussed in section 4.5.4. A wide and deep chamfer shown in figure 4-28 increases the probability of entrapment. There is however a limit to the extent to which the chamfer width can be increased. This is because a very large chamfer increases the chances of a conveying part to get stuck in orientations other than the desired orientation.

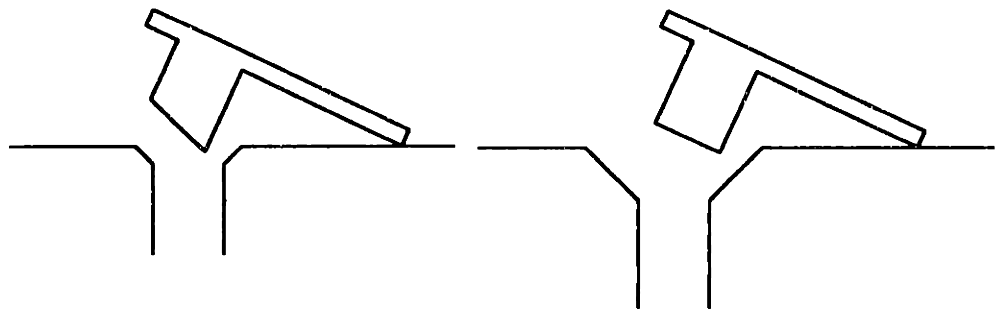

Figure 5-3: (a) Part with a taper added to the locating feature; (b)Effective chamfer shape

Increasing the clearance between the part and the relief assists entrapment. However, increasing the clearance is usually not an option available to the designer. Clearance is usually specified based on assembly requirements.

A useful option available to the designer is to modify the shape of the part if that is allowed. For example, increasing the major radius of the axisymmetric pin (shown as b in figure 4-22) makes it more stable. For the asymmetric part, increasing the L-shaped part makes it more stable. For the asymmetric part, a useful modification that significantly improved the entrapment efficiency was to add a taper to the locating feature on the part as shown in figure 5-3. Adding a taper effectively increases the clearance in the initial stages of descent as shown in figure 5-3. A deep and wide chamfer increases the efficiency of entrapment of the part.

6

Conclusion

This chapter summarizes the contributions of this thesis and suggests topics for future work.

6.1 Contributions

The thesis discusses mechanics of entrapment in the context of an industrial parts feeder, namely Sony's Automated Parts Orienting System (APOS). The orientation of the parts is accomplished by conveying them over an oscillating pallet that has reliefs carved on it to trap the parts in the desired orientation. The scope of this thesis is restricted to the entrapment of a single part in a relief. The effect of neighboring parts on the entrapment is ignored.

The thesis discusses the conditions for a part that is initially descending into a relief to get thrown out of the relief. The motion of the descending part is characterized by either continuous sliding contact or intermittent contact due to collisions with the walls of the relief. In either case the net upward momentum the part can acquire is a function of its kinetic energy, geometry of the relief, oscillation frequency and amplitude and oscillation pattern.

The mechanics of entrapment is introduced for the simple example of a point mass in a cup and extended to some real parts and reliefs. The effect of kinetic energy and relief geometry is illustrated with the example of a point mass in a stationary cup. It is shown that for a point mass that is initially inside a cup the minimum required kinetic energy for it to leave the cup increases with decrease in inclination of the walls of the cup. If the direction of its initial velocity is parallel to the ground, the minimum required initial kinetic energy is much larger than the potential energy barrier for small wall inclinations.

The effect of oscillation parameters on entrapment is illustrated with the example of a point mass in an oscillating cup. It is shown that the energy of the mass can increase due to a contact with an advancing wall (an energy providing contact) or decrease due to contact with a receding wall (energy absorbing contact). A condition called resonance is discussed in which the mass can have continuous energy providing contacts with the oscillating cup resulting in a monotonic increase in its energy (both potential energy and kinetic energy) causing it to be thrown out of the cup. Approximate conditions for resonance are derived for some simple oscillation patterns of the cup. It is shown that there is a minimum required frequency for resonance and this frequency increases with decrease in inclination of the walls of the cup with the vertical. In addition, there is a minimum required initial kinetic energy for the onset of resonance. This requirement is usually met if the amplitude of oscillation is much larger than the diameter of the cup.

The mechanics of the motion of a point mass in a cup is extended to the stability of an axisymmetric pin that is trapped in an axisymmetric relief that describes a circular path in a plane parallel to the plane of the pallet. It is shown that if the center of mass of the trapped pin is above the bottom of the chamfer the pin can monotonically acquire energy and get thrown out of the relief in the same manner that a point mass gets thrown out of an oscillating cup. Similar to the example of a point mass in a cup there is a minimum frequency and amplitude for the trapped pin to be unstable. The minimum frequency is expressed as a function of the part-relief geometry and amplitude of oscillation. The requirement on amplitude is that it be much larger than the clearance. The theoretical results have been verified through high-speed video images of the process.

The analysis on the stability of a fully trapped pin is extended to explain the stability of a pin that has an initial inclination with the axis of the relief. Conditions for stability of the pin to a disturbance inclination are derived for the worst case scenario that the pin in its disturbed state has an initial energy providing contact. The conditions for stability of a disturbed pin are used to explain conditions for successful descent of an approaching pin in a relief. The descent of a pin in an oscillating relief is equivalent to the descent of a point mass in a cup in which the inclination of the walls increases with elevation. For the specific pin-relief pair that is considered, failure of entrapment due to resonance does not occur in the initial stages of descent because of large clearances. It does not occur in the final stages of descent because of small inclinations. It is most likely to occur in the intermediate stages of descent referred to as the resonance zone. The actual ocurrence of resonance depends upon the phase of the oscillation cycle at which the part enters the resonance zone and cannot be precisely predicted because of the chaotic motion of the pin prior to the entry into the resonance zone. Failure of entrapment due to resonance was also observed in the case of an asymmetric part with a cylindrical locating feature interacting with an axisymmetric relief.

Failure due to resonance is not the only mode of failure. It was observed that the axisymmtric pin could fail to get trapped due to an energy providing contact in the initial stages of descent.

6.2 Future work

Future work must be directed at enhancing the scope of this thesis. The following are some of the issues that need to be addressed.

- The thesis primarily addresses failure of entrapment due to resonance when the part-relief interaction is characterized by continuous contact. Continuous contact allows for precise modeling of the mechanics of part-relief interaction. A majority of industrial parts, however, are essentially prismatic and the part-relief interaction is characterized by discontinuous contacts and collisions. A simple example is the interaction of the axisymmetric pin in a prismatic keyway shown in figure 3-2. The motion is characterized by back and forth motion between the walls of the relief. To precisely determine the conditions for failure of entrapment a complete understanding of the mechanics of collisions between the part and the relief is required.

- The thesis only briefly discusses failure of entrapment for reasons other than resonance. Experimental observations revealed that failure at the initial stages of descent, for reasons other than resonance, are a significant fraction of the total number of instances of failure. Causes and conditions for failure in the initial stages of descent must be investigated.

- The thesis considers entrapment of a single part. Effects of neighboring parts on entrapment are ignored. The conditions under which the one-part-oncrelief assumption is justified must be determined.

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