Synthesis of Domain Specific CNF Encoders for Bit-Vector Solvers

by

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B.S., Massachusetts Institute of Technology (2016)

Submitted to the Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of

Master of Engineering in Electrical Engineering and Computer Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2016

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Abstract

SMT solvers are at the heart of a number of software engineering tools. These SMT solvers use a SAT solver as the back-end and convert the high-level constraints given by the user down to low-level boolean formulas that can be efficiently mapped to CNF clauses and fed into a SAT solver. Current SMT solvers are designed to be general purpose solvers that are suited to a wide range of problems. However, SAT solvers are very non-deterministic and hence, it is difficult to optimize a general purpose solver across all different problems. In this thesis, we propose a system that can automatically generate parts of SMT solvers in a way that is tailored to particular problem domains. In particular, we target the translation from high-level constraints to CNF clauses which is one of the crucial parts of all SMT solvers.

We achieve this goal by using a combination of program synthesis and machine learning techniques. We use a program synthesis tool called Sketch to generate optimal encoding rules for this translation and then use auto-tuning to only select the subset of these encodings that actually improve the performance for a particular class of problems.

Using this technique, the thesis shows that we can improve upon the basic encoding strategy used by CVC4 (a state of the art SMT solver). We can automatically generate variants of the solver tailored to different domains of problems represented in the bit-vector benchmark suite from the SMT competition 2015.

Thesis Supervisor: Armando Solar-Lezama
Title: Associate Professor

Acknowledgments

I would like to thank my advisor Prof. Armando Solar-Lezama for all his support in shaping this thesis the way it is. During the last few years, I learnt a lot from him–from doing research and identifying interesting problems to writing papers and communicating the results to others. He has great insights into this problem and was always there to help me look at the problem from a different perspective when I am stuck. I greatly appreciate all the countless hours he spent with me on developing this project.

This thesis is based on a paper in SAT 2016 [40]. I would like to thank my other collaborator, Rohit Singh, for helping me with the machine learning component of this project. I should especially thank him for being up late night to help me run the experiments and write the paper for our SAT submission. I would also like to thank the Computer Aided Programming group members for their support and for everything I learnt about research from the group meetings.

Last, I owe this thesis to my parents and my brother without whose constant support this thesis would not be the same.

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Chapter 1

Introduction

SMT solvers are at the heart of a number of software engineering tools such as automatic test generators for Java, JavaScript, Windows and Enterprise applications [52, 46, 12, 28], deterministic replay tools for long running database and web-server applications [17], and undefined behavior detection in C/C++ programs [56] where the goal of these tools is to make software robust and bug-free. Current SMT solvers (CVC4 [7], Z3 [21], Yices [23], Boolector [15], etc.) are designed to be general purpose solvers that are suited to a wide range of problems. For example, CVC4 solver is used as a backend in both [52] and [46]. But, since SAT solvers are very non-deterministic, it is difficult to optimize a general purpose solver across all different problems. On the other hand, specialized solvers that are tailored for particular classes of problems can potentially have a significant impact on the performance compared to a general solver. However, currently, this has to be done manually which is a very time consuming process given the vast number of problems that use SMT solvers.

In this thesis, we show how program synthesis and machine learning techniques can be used to automatically generate parts of SMT solvers in a way that is specialized to particular problem domains and thus, achieving better performance and at the same time, reducing the burden off solver writers. In particular, we target the translation step from high-level constraints to SAT in bit-vector solvers. Bit-vector solvers are widely used in [18, 29, 45, 50] because bit-vectors can be used to faithfully represent the full range of machine arithmetic and the translation from the input high-level

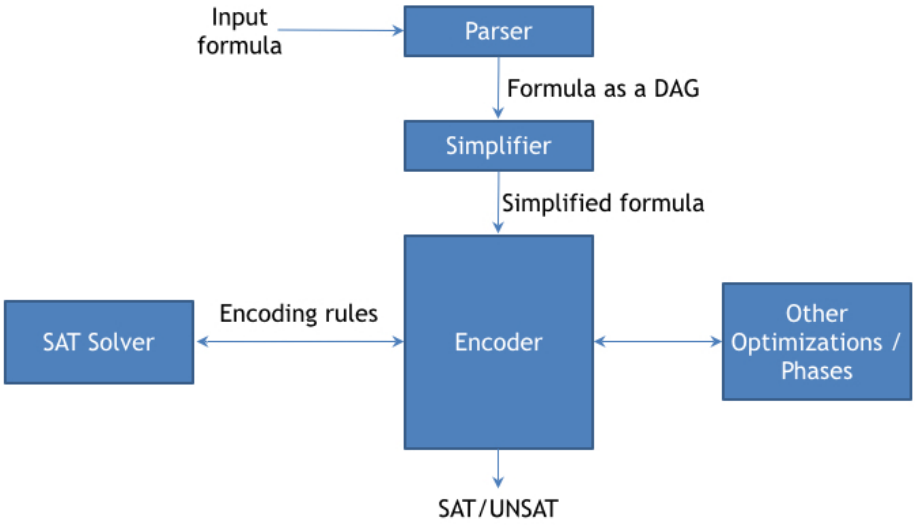


Figure 1-1: Transformation of a program to SAT in SMT solvers

constraints to SAT is a very crucial step in these solvers.

1.1 Translation from high-level constraints to SAT

Figure 1-1 shows different components of a typical bit-vector SMT solver. First, there is a parser that parses the input constraints that are either provided by a user or a program synthesis or program analysis tool that uses the SMT solver as the backend. Internally, these high-level constraints are stored as directed acyclic graphs (DAG) which we call formulas. Then, the solvers use "simplification rules" to aggressively optimize this formula. Finally, these high-level bit-vector formulas are mapped down to low-level CNF clauses that can be fed to a SAT solver—a process often referred to as *bit-blasting*. One approach to bit-blasting is to use the known efficient encodings for simpler boolean terms (such as AND or XOR) and compose them to generate CNF clauses for complex terms [53]. This approach can have a huge impact on the performance of the solver [43, 42], but generally, it relies on having optimal encodings for the simpler terms, and even then it does not guarantee any kind of optimality of the overall encoding.

In this thesis, we propose OptCNF, a new approach to automatically generate the code that converts high-level bit-vector terms into low-level CNF clauses. In addition

to the obvious benefits of having the code automatically generated instead of having to write it by hand, OptCNF has three novel aspects that together significantly improve the quality of the overall encoding: (a) OptCNF uses synthesis technology to automatically generate efficient encodings from high-level formulas to CNF (b) OptCNF relies on auto-tuning to choose encodings that produce the best results for problems from a given domain. (c) OptCNF identifies commonly occurring clusters of terms in a given domain and focuses on finding optimal encodings for such clusters.

1.2 Optimality of Encodings

The synthesis of encodings balances optimality among three criteria: number of clauses, number of variables and propagation completeness. The propagation completeness requirement has been proposed as an important criterion in order for the encoded constraints to solve efficiently in the SAT solver [11]. Modern SAT solvers rely heavily on unit propagation to infer the values of variables without having to search for them (see Section 2.1). Propagation completeness means that if a given partial assignment implies that another unassigned variable should have a particular value, then the solver should be able to discover this value through unit propagation alone. Prior work has demonstrated the synthesis of propagation complete encodings for terms involving a small number of variables [13]. OptCNF, however, is more flexible and is able to produce propagation complete encodings even for relatively large bit-vector terms by taking advantage of high-level hypothesis about the structure of the encoding (See Section 3.1).

In practice, however, propagation completeness does not *always* improve the performance of an encoding. For certain classes of problems, for example, the additional unit propagations caused by a propagation complete encoding can actually slow the solver down. Similarly, there is often a trade-off between the number of auxiliary variables and the number of clauses used by an encoding; for some problems having more variables but fewer clauses can be better, but for other problems, having fewer variables at the expense of more clauses can be better. In order to cope with this

variability, OptCNF uses auto-tuning to make choices about which encodings are best for problems from a particular domain. Prior work has demonstrated the value of tuning solver parameters in order to achieve optimal performance for problems from particular domains [35], but ours is the first work we know of where auto-tuning is used to make high-level decisions about how to encode particular terms (see Section 6).

Finally, OptCNF is able to better leverage its ability to synthesize optimal encodings by focusing on larger clusters of terms, as opposed to focusing on individual bit-vector operations independently. Given a corpus of sample problems from a domain, OptCNF is able to identify common recurring patterns in the formulas from those problems and then generate specialized encodings for those patterns.

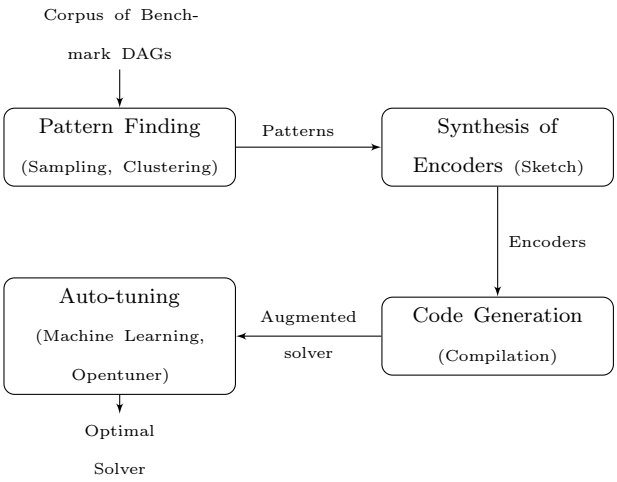


Figure 1-2: OptCNF: System Overview

1.3 Overview of OptCNF

Figure 1-2 shows how these ideas come together as OptCNF. The input to OptCNF is a collection of formulas represented as DAGs extracted from a set of benchmarks from a given problem domain. OptCNF samples these DAGs to extract representative clusters of terms—what the figure refers to as patterns. OptCNF then leverages Sketch synthesis system [48] to synthesize "optimal" encodings for those patterns and generates C++ code for the encodings that can be linked with a modified version

of CVC4 solver [7]. The auto-generated code contains a set of switches to turn different encodings on or off. Finally, the auto-tuner searches for the optimal configuration of those switches in order to produce the best performing domain-specific version of CVC4.

Our evaluation shows that the resulting domain-specific encodings are able to significantly improve the performance of CVC4. Using OptCNF, we generated a separate solver for 7 different domains represented in the quantifier-free bit-vector benchmarks from the SMT-COMP 15 benchmark suite [8]. Using these specialized solvers on their respective domains, we were able to solve 83 problems from the test set (see Section 7) that CVC4 could not solve.

Chapter 2

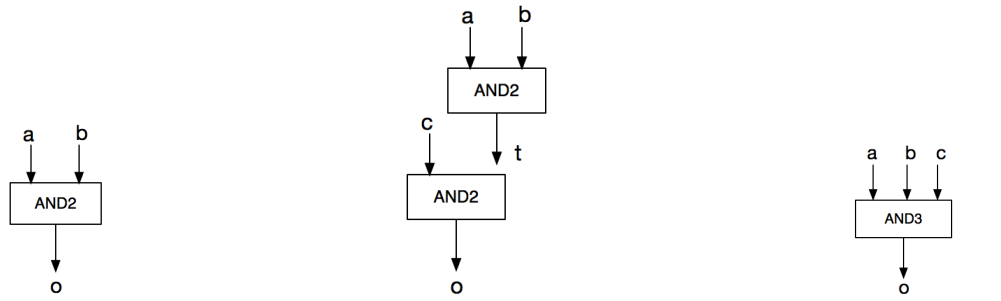
Background

In the following sections, we describe some background information on SAT solvers and the program synthesis tool we use (SKETCH).

2.1 SAT solvers

The goal of SAT solvers is to decide whether it is possible to satisfy a set of constraints. This boolean satisfiability problem is NP-complete, but many practical problems can be solved efficiently using certain heuristics. The input format for most of the SAT solvers is the conjunctive normal form (CNF). A CNF formula consists of a set of constraints called clauses that needs to be satisfied. Each clause is a disjunction of literals where a literal is a variable or its negation. If in a clause all but one literal are set to false, then that clause is known as a unit clause. The only way this clause can be satisfied is by making the unassigned literal true.

Modern SAT solvers such as MiniSAT [26], Chaff [44], Glucose [5] are based on the DPLL algorithm [20]. The DPLL algorithm uses depth-first search with backtracking to find a satisfying assignment for the CNF clauses. The algorithm starts by randomly guessing a variable to be true or false. Then the algorithm identifies all clauses that become unit as a result of this guess and sets the unassigned literals in these clauses to true. This process is called unit propagation. During propagation, it is possible that a clause cannot be satisfied. At this point, the algorithm learns a conflict clause



(a) AND2 DAG (b) Composing of two AND2 DAGs (c) AND3 DAG

Figure 2-1: DAGs for different AND circuits

and adds it to the set of CNF clauses so that it does not have to re-discover this conflict. The topmost guess that caused the conflict is then reversed. In the end, either a satisfiable assignment is found, or the entire search tree is exhausted. The latter case means that there is no satisfiable assignment for the given CNF clauses.

Unit propagation is very important for SAT solvers because it allows them to guess only a fraction of the variables and infer all the other variables based on this guess. Without unit propagation, the SAT solver has to find a satisfying assignment by just guessing and the probability of getting the correct assignment by guessing over all variables is exponentially low. Hence, it is very important for the encoding of the high level constraints down to CNF to capture as many unit propagations as possible.

2.2 Example encodings

Let us now look at some examples sub-formulas and their encodings to SAT to illustrate what we mean by "optimal" encodings.

Example 2.2.1. *Consider the* AND2 *term:* $o = a \wedge b$. *The DAG for this circuit is shown in Figure 2-1a. An efficient encoding for this circuit is shown below (each line*

$$a \vee \neg t$$
$$b \vee \neg t$$
$$\neg a \vee \neg b \vee t$$
$$t \vee \neg o$$
$$c \vee \neg o$$
$$\neg t \vee \neg c \vee o$$

(a) Encoding 1

$$a \vee \neg o$$
$$b \vee \neg o$$
$$c \vee \neg o$$
$$\neg a \vee \neg b \vee \neg c \vee o$$

(b) Encoding 2

Figure 2-2: Two different encodings for and3

represents a clause):

$$a \vee \neg o$$
$$b \vee \neg o$$
$$\neg a \vee \neg b \vee o$$

Now suppose that we want to encode this term : $o = a \wedge b \wedge c$. One can do this by decomposing it into two AND2 circuits and by introducing an extra auxiliary variable i.e. $t = a \wedge b$; $o = t \wedge c$ (Figure 2-1b). The corresponding encoding is shown in Figure 2-2a. This encoding has 6 clauses and 5 variables. However, if we represent the entire circuit using a single AND3 node as shown in Figure 2-1c and then, we can encode this circuit using only 4 clauses and 4 variables (Figure 2-2b) and yet, both encodings propagate the same amount of information. Extending this to a ANDn circuit, a naive DAG that is obtained by composing AND2 nodes will require an encoding with $3(n-1)$ clauses and $2n-1$ variables but the optimal encoding only requires $n+1$ clauses and $n+1$ variables. Thus, the optimal encoding has a 3X fewer clauses and 2X fewer variables.

Example 2.2.2. *The following example demonstrates the difference in the amount of propagation in two different encodings for the same sub-formula. Consider the following term: $o = ITE(a, b, c)$ where ite stands for if-then-else i.e. $o = b$ if a is true and $o = c$ otherwise. Consider the two encodings in Figure 2-3. Both of these encodings are correct, but the encoding on the right propagates more information than the one on*

$$
\begin{array}{c}
a \vee c \vee \neg o \\
a \vee \neg c \vee o \\
\neg a \vee b \vee \neg o \\
\neg a \vee \neg b \vee o
\end{array}
\qquad\qquad
\begin{array}{c}
a \vee c \vee \neg o \\
a \vee \neg c \vee o \\
\neg a \vee b \vee \neg o \\
\neg a \vee \neg b \vee o \\
b \vee c \vee \neg o \\
\neg b \vee \neg c \vee o
\end{array}
$$

Figure 2-3: Two different encodings for ite

the left. In particular, the encoding on the right captures that when $b = c$, o's value can be determined irrespective of a. Even though the left encoding has two fewer clauses, the right encoding is preferred in this case.

Example 2.2.3. *Unlike the above two examples which are usually encoded in an optimal manner in the existing solvers, this example illustrates a case where the encoding in most solvers is far from optimal. Consider the term that computes the max of two numbers: $o = MAX(a, b)$. This is usually encoded as shown in Figure 2-4 where the integers a, b, o are represented as 4 bit bit-vectors. Now, assume that in the middle of SAT solving, the SAT solver has chosen to set the most significant bit of a to 1. This partial assignment is enough to deduce that the most significant bit of the output o is also 1. However, since the current solvers encode node by node, the overall encoding for MAX is not propagation complete. Even if the encodings for the nodes $>$ and ITE are propagation complete, it is impossible to know the value of t just by knowing one bit of a and hence, it is impossible to predict any thing about o. This requires the SAT solver to try out all possible combinations of values for the other bits in a and b to figure out that the most significant bit of o should be 1. In our system, we can identify patterns like these, treat them as a single node, and generate encodings that take care of these corner cases.*

2.3 SKETCH

This section introduces SKETCH, a state of the art program synthesis tool, that OPTCNF uses to generate optimal encoding for a given sub-formula. SKETCH allows

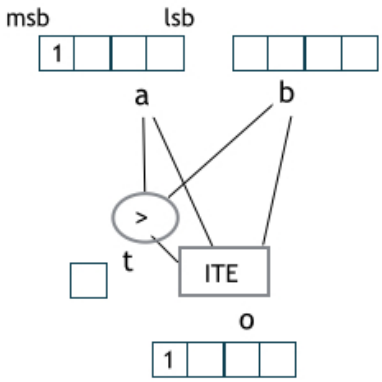


Figure 2-4: Max of two integers

users to write a template (a partial program) containing holes (represented as "??") to represent unknown integer or boolean values along with a specification. SKETCH uses a constraint-based approach to instantiate these holes with correct values so that the specification is satisfied. Figure 2-5 shows a toy SKETCH program and the corresponding synthesized solution. Here, SKETCH was able to figure out that the only possible value for the `??` is 2 that satisfies the statement `assert(t == x + x)`

```
int double(int x) {
    int t = x * ??;
    assert(t == x + x);
    return t;
}
```

(a) Input template

```
int double(int x) {
    int t = x * 2;
    assert(t == x + x);
    return t;
}
```

(b) Synthesized solution

Figure 2-5: Example SKETCH program

The synthesis problem reduces to solving a doubly quantified constraint of the form

$$\exists \phi.\ \forall \sigma.\ Q\left(\phi,\ \sigma\right)$$

where ϕ is a *control vector* describing the values of all the holes, σ is the input state of the program, and $Q\left(\phi,\ \sigma\right)$ is a predicate that is true if the program satisfies its correctness

specification under input σ and control vector ϕ. SKETCH uses counterexample guided inductive synthesis (CEGIS) to search for the control vector [49]. At every stage of the CEGIS algorithm, SKETCH uses the MiniSAT solver [26] to check if the predicate Q is satisfied.

In OPTCNF, we develop templates similar to Figure 2-5a (but more complex with many holes) that allow SKETCH to generate CNF clauses (see Section 3.1) and the specification for these templates captures the notion of "optimality" of these CNF clauses (see Section 3.2). Using this, OPTCNF is able to generate thousands of encodings for various sub-formulas.

Chapter 3

Synthesis of Encoders

Previous work [13] has attacked the problem of generating optimal propagation complete encodings for a given term by starting with an initial encoding and then exhaustively checking for violations of propagation completeness and incrementally adding more clauses to fix these violations. The resulting propagation complete encoding is then minimized to produce an equivalent but smaller encoding. Our approach to generating encodings is quite different because it relies on program synthesis technology, allowing us to symbolically search for an encoding based on a formal specification. An important advantage of our approach is flexibility. In particular, it allows us to generate *encoders* that generate encodings at solver run-time from terms that have parameters that will only be known at run-time (for example, the bit-width for a bit-vector operation).

3.1 Synthesis as a SyGus problem

OptCNF frames the task of generating these encoders as a Syntax Guided Synthesis problem (SyGuS) [2]. A SyGuS problem is a combination of a template or a grammar that represents the space of the candidate solutions and a specification that constrains the solution. The goal of a SyGuS solver such as Sketch is to find a candidate in the template that satisfies the specification. The two components, template and specification, are very crucial in determining the scalability of the problem. Here, we first

describe the templates that represent the space of CNF encoders for booleans and bit-vector terms. Then, we formalize the correctness and the optimality specification that constraints the template. Finally, we describe an efficient but equivalent specification that makes the SyGus synthesis problem more scalable.

3.1.1 CNF Encoders and Templates

The encoders generated by OptCNF work in two passes. Given a formula to be encoded into SAT, OptCNF first identifies terms for which it has learned to generate CNF constraints and replaces them by special placeholder operators N_i. Then, the pass that would normally have generated low-level constraints from the bit-vector terms is extended to recognize these placeholder operators and generate the specialized constraints for them.

The pass that identifies the known terms, and the scaffolding that iterates through the different operators in a DAG representation of the formula and identifies the placeholder nodes are all produced using relatively straightforward code-generation techniques. The synthesis problem focuses on the code that executes when one of these placeholder nodes is found. This is the encoder code that generates the CNF encoding for a previously identified term T.

The term T for which OptCNF is generating an encoding is known at synthesis time, so OptCNF can choose a template or a set of templates for this code depending on the properties of T. Figure 3-1 illustrates the three different kind of terms and the encodings that represent the terms. If T is not parametric—for example if it is just a collection of boolean operators—then the encoder just needs to generate a fixed set of clauses corresponding to the constraint represented by T, and the template will reflect that. On the other hand, many terms will be parameterized by bit-widths, so the encoder will have to produce clauses in one or more loops.

For bit-vector terms, which are parametric on the bit-width of their different operators, we differentiate between two different kinds – bit-parallel and non bit-parallel. Bit-parallel terms are those that are composed entirely of operations, such as bitwise AND, OR or XOR, where there is no dependency from one column of the bit-

$t \equiv and(x, or(y, z))$

```
clause({x, ~t})
clause({~x, ~y, t})
clause({~x, ~z, t})
clause({y, z, ~t})
```

$t \equiv bvAND_N(x, bvOR_N(y, z))$

```
for i from 1 to N:
  clause({x[i], ~t[i]})
  clause({~x[i], ~y[i], t[i]})
  clause({~x[i], ~z[i], t[i]})
  clause({y[i], z[i], ~t[i]})
```

$t \equiv bvEQ_N(x, y)$

```
t1 = true
for i from 1 to N:
  t2 = i == N ? t : newVar
  clause({x[i], y[i], ~t1, t2})
  clause({x[i], ~y[i], ~t2})
  clause({~x[i], y[i], ~t2})
  clause({~x[i], ~y[i], ~t1, t2})
  clause({t1, ~t2})
  t1 = t2
```

Figure 3-1: Encoders for three different kinds of terms

vector to another. For these kinds of terms, generating the encoding for a single column and then enumerating them over all columns will still preserve optimality. Hence, it is sufficient to just synthesize the encoding for the boolean term that represents operations in a single column. This is, however, not the case for all bit-vector terms.

Terms involving bitwise PLUS, for example, cannot be dealt in the same way because there are dependencies that flow from one column to another. These operations can still be represented as a loop of encodings, but there will be auxiliary variables that are threaded from one iteration of the loop to another. Figure 3-2 shows one such template for a bit-vector formula involving two bit-vector inputs of size N (taken as a parameter) and outputs another bit-vector of size N. For each column in the bit-vectors, the template calls encode_column which is another template for explicit encodings, but this template can be instantiated with variables specific to loop iteration. This template has one auxiliary variable per column. Every column has an incoming auxiliary variable (a constant for the first column) which carries information from the previous columns and

an outgoing auxiliary variable that carries information forward. This same template represents multiple formulas depending on how the encode_column template is instantiated. For example, this same template is used to generate encodings for both bitwise PLUS and bitwise MINUS operations.

The templates in OptCNF are all written in the Sketch language, which allows us to leverage the Sketch synthesis engine for the synthesis problem. A template in Sketch is a piece of code with integer and boolean holes to represent the set of candidate solutions that the synthesizer should consider. The standard template for an encoding is a list of clauses with holes representing the number of clauses, and the length and the literals present in each clause. We significantly reduce the size of the search space by enforcing an order among the literals in each clause and among clauses themselves and thus, eliminating symmetries. This canonical representation captures any general CNF encoding, but it does not impose any structure on the clauses. We found that this model is scalable enough for boolean formulas that expand into a small number of CNF clauses (about 20 to 30). But, in order to deal with bigger formulas like bit-vector operations, we need to represent the search space using loops to capture the structure.

OptCNF has a library of templates for different kinds of input types, output types and number of auxiliary variables per column. When running Sketch on a term, OptCNF runs different instances of Sketch with a different template and chooses the one that provides the best encoding (based on heuristics like number of clauses and number of auxiliary variables). Due to the scalability limits of Sketch, OptCNF can currently only synthesize encodings for non bit-parallel terms that have at most two input bit-vectors, at most two auxiliary variables per column and no nested loops in the template.

3.2 Problem formulation

In addition to the template, the other important component of a SyGus problem is the specification. Unlike the templates, which are very different for parameterized

```
Lit[N] encode(Lit[N] mval, Lit[N] fval) {
    Lit[N] out = newVar(N)   /* creates an array of out literals */
    Lit[N] aux = newVar(N)   /* creates an array of auxiliary literals */
    /* Specialize the first column */
    encode_column(mval[1], fval[1], out[1], const?, aux[1])
    for i from 2 to N:
      encode_column(mval[i], fval[i], out[i] aux[i−1], aux[i])
    return out
}
```

Figure 3-2: Template for a bitwise operation on two bit-vectors (with one auxiliary variable per column)

and non-parameterized terms, the specifications for both are actually very similar; the only difference is that for parameterized terms, the parameters must be threaded through to all the relevant predicates. Therefore, the rest of the section will omit these bit-width parameters in the interest of clarity.

A term $T(in)$ can be represented by a predicate $P(in, out)$ defined as $P(in, out) \Leftrightarrow out = T(in)$. For notational convenience, we will just write $P(x)$, where x is understood to be a vector containing both the input and the output variables. The goal is to generate an alternative representation of the predicate in terms of CNF clauses $C(x)$.

Definition 1 (Correctness Specification)**.** *A set of CNF clauses "represents" a boolean predicate P iff $P(x) \Leftrightarrow C(x)$.*

In addition to the correctness specification, however, we want to ensure propagation completeness which needs to be defined in terms of the behavior of the encoding under *partial* assignments. A partial assignment σ maps every variable to one of $\{true, false, \top\}$ where $\top$ indicates that the value has not been assigned by the solver and could be *true* or *false*. A partial assignment can be understood as the set of all complete assignments that are consistent with the partial assignment. Therefore, it is standard to define a partial order among partial assignments as:

$$\sigma \sqsupseteq \sigma' \iff \forall i.\ \sigma(x_i) \neq \top \Rightarrow \sigma'(x_i) = \sigma(x_i)$$

We generalize the predicate to be a function from partial assignments to the set

$\{true,\ false,\ \top\}$, and define $P(\sigma) = \top$ for any partial assignment where some variable x_i is set to $\top$.

Definition 2. *We define the following predicates on partial assignments:*

$$\begin{aligned}
complete(\sigma) &\equiv \forall i.\ \sigma(x_i) \neq \top \\
satisfiable(\sigma, P) &\equiv \exists \sigma' \sqsubseteq \sigma.\ P(\sigma') = true \\
unsatisfiable(\sigma, P) &\equiv \forall \sigma' \sqsubseteq \sigma.\ P(\sigma') \neq true \\
forces(\sigma, P, x_i, b) &\equiv (\sigma' = extend(\sigma, x_i, \neg b)) \Rightarrow unsatisfiable(\sigma', P) \\
maypropagate(\sigma, P) &\equiv \exists i, b.\ forces(\sigma, P, x_i, b)
\end{aligned}$$

Where $extend(\sigma, x_i, b)$ is defined as extending an assignment with $\sigma(x_i) = \top$ to one where variable x_i has value b. The predicate $maypropagate(\sigma, P)$ indicates that the partial assignment σ forces the value of some currently unassigned variable.

Lemma 3.2.1. *The forces() predicate has the following property.*

$$\begin{aligned}
&forces(\sigma, P, \hat{x_i}, \hat{b}) \ \wedge\ \sigma' = extend(\sigma, \hat{x_i}, \hat{b}) \\
&\quad \Rightarrow \forall_{(x_i, b) \neq (\hat{x_i}, \hat{b})}\ \ forces(\sigma, P, x_i, b) \Rightarrow forces(\sigma', P, x_i, b)
\end{aligned}$$

This means that if P and a partial assignment σ force $\hat{x_i}$ to take a particular value $\hat{b}$, then any other variable that was also forced by σ and P will also be forced after extending the assignment with $\sigma(\hat{x_i}) = \hat{b}$.

Lemma 3.2.2. *Another important property of forces() is the following.*

$$\begin{aligned}
&satisfiable(\sigma, P) \ \wedge\ forces(\sigma, P, \hat{x_i}, \hat{b}) \ \wedge\ \sigma' = extend(\sigma, \hat{x_i}, \hat{b}) \\
&\qquad\qquad \Rightarrow satisfiable(\sigma', P)
\end{aligned}$$

This means that if P and a partial assignment σ force $\hat{x_i}$ to take a particular value, then after extending the assignment with $\sigma(\hat{x_i}) = \hat{b}$, the new assignment is still satisfiable.

A clause c can be applied to a partial assignment as well, resulting in a value $c(\sigma) \in \{true, false, \mu, \top\}$. A clause is unit ($\mu$) if one of the literals in the clause has an unknown value and all others are $false$. A CNF encoding is a collection of clauses C.

$C(\sigma)$ can either be *true* if $c(\sigma) = true$ for all the clauses, *false* if $c(\sigma) = false$ for at least one of the clauses, μ if σ makes at least one clause unit (and σ does not falsify any others), or $\top$ if none of the above. Thus, the result of applying C to a partial assignment helps identify the case when at least one of the clauses is a unit clause, and it is, therefore, possible to propagate further assignments. This is useful in describing unit propagation.

Definition 3 (UP)**.** *The function UP captures the unit propagation in SAT solvers. We say that C propagates σ to $UP(C, \sigma)$ under unit propagation according to the following rules:*

1. *if $C(\sigma) \neq \mu$, then $UP(C, \sigma) = \sigma$.*
2. *else, $C(\sigma)$ has a unit clause. If the unit clause forces $\sigma(x_i) = b$, then $UP(C, \sigma) = UP(C, \sigma')$ where $\sigma' = extend(\sigma, x_i, b)$.*

The definitions above give rise to an important lemma.

Lemma 3.2.3. *A set of CNF clauses C "represents" a boolean predicate P iff it satisfies the following two conditions:*

$$\begin{aligned} &1.\ satisfiable(\sigma, P) \ \Rightarrow\ C(UP(C,\ \sigma)) \ \neq\ false \\ &2.\ unsatisfiable(\sigma, P) \ \Rightarrow\ C(UP(C,\ \sigma)) \ \neq\ true \end{aligned} \tag{3.2.1}$$

i.e. if an assignment can be extended to a satisfiable assignment for P, then unit propagation should not lead to a contradiction. And similarly, if an assignment (possibly partial) already contradicts P, then unit propagation should not lead to a satisfiable assignment for the CNF clauses.

With the definitions above, we can now state the requirement for propagation completeness.

Definition 4 (Propagation Completeness)**.** *C is a set of propagation complete CNF clauses representing P if C "represents" P and*

$$\begin{aligned} &\forall \sigma.\ satisfiable(\sigma, P) \\ &\Rightarrow \forall\ x_i, b_i.\ (forces(\sigma, P, x_i, b_i) \Rightarrow\ \ UP(C, \sigma) \sqsubseteq extend(\sigma, x_i, b_i)) \end{aligned} \tag{3.2.2}$$

In other words, if a partial assignment can be completed into a satisfying assignment, and if there are unassigned variables x_i that if set to $\neg b_i$ would make the partial assignment unsatisfiable, then unit propagation must set all such x_i to b_i.

3.3 Synthesis-friendly propagation completeness

The above definition captures the notion of propagation complete encodings, but it is unsuitable as a specification for synthesis because the recursive definition of *UP* essentially defines a small SAT solver, making it too complex for a state of the art synthesizer. Instead, OPTCNF relies on an equivalent but simpler specification that does not require implementing a SAT solver. The idea is that instead of thinking in terms of full unit propagation, we now verify propagation only one step at a time. Specifically, the claim is that the following three rules guarantee propagation completeness.

$$\begin{aligned} &1.\forall\sigma.\ \mathit{satisfiable}(\sigma, P) \ \Rightarrow\ C(\sigma) \neq \mathit{false} \\ &2.\forall\sigma.\ \mathit{maypropagate}(\sigma, P) \ \Rightarrow C(\sigma) = \mu \\ &3.\forall\sigma.\ \mathit{unsatisfiable}(\sigma, P) \ \Rightarrow\ C(\sigma) = \mathit{false} \ \vee\ C(\sigma) = \mu \end{aligned} \tag{3.3.1}$$

Theorem 3.3.1. *Formula* (3.3.1) $\iff$ *Correctness* $\wedge$ *Formula* (3.2.2)

Proof: Formula (3.3.1) $\Rightarrow$ Correctness
This follows directly from Formula (3.3.1), because when σ is complete, *satisfiable*(σ, P) implies $P(\sigma) = \mathit{true}$ and similarly, *unsatisfiable*(σ, P) implies $P(\sigma) = \mathit{false}$.

Proof: Formula (3.3.1) $\Rightarrow$ Formula (3.2.2)
This can be proved by induction on the number of times σ can be extended before it fails *maypropagate*(σ, P). For the base case, $\neg$*maypropagate*(σ, P), (3.2.2) is vacuously satisfied because *forces*() fails for all variables. For the inductive case, *maypropagate*(σ, P), $C(\sigma) = \mu$ (by 3.3.1-2). Let $\sigma' = \mathit{extend}(\sigma, \hat{x}_i, \hat{b})$ which is obtained by propagating the unit clause in $C(\sigma)$. Note that $\mathit{UP}(C, \sigma) = \mathit{UP}(C, \sigma')$ by Definition 3. Applying

Lemma 3.2.2 tells us that *satisfiable*(σ', P), so applying the inductive hypothesis together with Lemma 3.2.1, we can prove the inductive case.

Proof: Correctness $\wedge$ Formula (3.2.2) $\Rightarrow$ Formula (3.3.1)
First, we use the fact that correctness is equivalent to Formula (3.2.1). If *satisfiable*$(\sigma$, $P)$, then $C(UP(C, \sigma)) \neq false$ and this implies $C(\sigma) \neq false$.
If σ can be propagated, then $\exists x_i, b.\ forces(\sigma, P, x_i, b)$. And hence, $UP(C, \sigma) \neq \sigma$ and this implies $C(\sigma) = \mu$.

If *unsatisfiable*(σ, P), then let $\sigma' \sqsupset \sigma$ be the maximal satisfying subset of σ i.e. σ' is satisfiable and $\forall \sigma' \sqsupset \sigma'' \sqsupset \sigma.\ \sigma''$ is unsatisfiable. Then, $C(\sigma') = \mu$ and since σ' is maximal subset, $C(\sigma) = false\ \vee\ C(\sigma) = \mu$.

3.4 Introducing Auxiliary Variables

In some cases, the encoding C will involve auxiliary variables t_i in addition to the variables x_i, in such cases, we write $C((x, t))$. In that case, the correctness specification must be generalized to

$$\forall x. \quad P(x) \iff \exists t.\ C((x, t))$$

Similarly, the conditions in Formula (3.3.1) generalize to the conditions below.

$$\begin{aligned}
&1.\forall \sigma.\ satisfiable(\sigma, P) \ \Rightarrow\ \exists \sigma_t.\ C((\sigma,\ \sigma_t)) \neq false \\
&2.\forall \sigma, \sigma_t.\ maypropagate(\sigma, P)\ \wedge\ C((\sigma,\ \sigma_t)) \neq false\ \Rightarrow\ C((\sigma,\ \sigma_t)) = \mu \\
&3.\forall \sigma, \sigma_t.\ unsatisfiable(\sigma, P)\ \Rightarrow\ C((\sigma,\ \sigma_t)) = false\ \vee\ C((\sigma,\ \sigma_t)) = \mu
\end{aligned} \tag{3.4.1}$$

The proof for this has a similar structure to the previous proof. Basically, once we establish the first rule above, auxiliary variables can be treated just as the other variables in P. It should be noted that this specification is more complex than Formula (3.3.1) because of the existential quantifier in the R.H.S of rule 1. The CEGIS algorithm employed by solvers like SKETCH is designed to deal with the outer universal

quantifiers, but cannot handle inner existential quantifiers. Hence, this existential quantifier should be translated into an explicit loop over all auxiliary assignments, which makes the synthesis problem hard. In practice, we found that this overhead is not significant when the number of auxiliaries used in the encodings is low.

3.5 Clause Minimization

Another important optimality criterion for the encodings is the clause minimization. If there are two propagation complete encodings having different number of clauses representing the same predicate, then the encoding with the lower number of clauses is preferred. OptCNF relies on binary search to find an encoding with an optimal number of clauses. This requires solving a logarithmic number of synthesis problems to generate a single encoding, which has proven to be reasonably efficient in practice. The number of CNF clauses in an encoding have an upward monotonicity property i.e. if there is an encoding for a function with n clauses, then there definitely exists an encoding with n' clauses where $n' \geq n$ and the converse. We leverage this property to design a random binary search algorithm to find the minimal encoding. The algorithm starts by randomly choosing n and queries the synthesizer to find an encoding with n clauses. If the synthesizer comes up with a solution, the algorithm refines the upper bound of the search to $n - 1$ and tries to find a solution with fewer number of clauses. If the synthesizer does not come with a solution, then the algorithm refines the lower bound of the search to $n + 1$ and continues the search. This process continues until the minimum solution is found. This algorithm can also run in parallel and this is very useful for cases where the synthesizer is solving a hard problem for every query.

3.6 Guarantees of the synthesized solution

When the formula is a boolean term or a bit-parallel term, Sketch performs full verification and hence, the output is guaranteed to be correct and propagation complete. When the input formula is a non bit-parallel bit-vector term, OptCNF does bounded

verification on the size of the bit-width parameters. The correctness specification is easier to verify than the propagation completeness requirement, so OptCNF allows the user to separately specify the checking bounds for both specifications. In our experiments, we check correctness for all inputs up to 6-bits and propagation completeness for up to 3-bits. Beyond these bounds, OptCNF relies on verifying the output (sat/unsat) of the solver on all the benchmarks used in our experiments to provide confidence on the correctness of the synthesized encodings. We did not encounter a single instance where OptCNF resulted in an incorrect output.

Chapter 4

Pattern Finding

In this phase, we identify commonly occurring patterns in the formulas arising from a given domain. For this, we build on prior work on representative sampling from DAG-based representations of formulas ([47]). The original sampling work on which we build takes as input a size k and produces a representative sample of all sub-terms of size k that appear in the corpus. When $k = 1$, for example, the process will return a sample of all the operations that appear in the corpus; the frequency with which a given operation appears in the sample will be approximately the same as the frequency with which it appears in the corpus. When sampling with higher values of k, the sampling process takes into account the fact that some operations are commutative, but not others.

Given a corpus, OptCNF collects representative samples for values of $k \leq 5$ for bit-parallel formulas and $k \leq 3$ for non bit-parallel formulas. The upper bounds are determined by the capabilities of our encoding synthesis algorithm, which is unable to generate encodings for larger terms.

Even within this bound, it is easier to generate encodings for some terms than for others. In particular, it is much easier to synthesize encodings for terms composed entirely of *bit-parallel* operations, compared with terms that involve non bit-parallel operations. As part of the sampling process, we, therefore, use simple heuristics to filter out patterns that contain too many operations that are not bit-parallel and for which synthesis is unlikely to scale.

Chapter 5

Code Generation

After getting the commonly occurring terms and their optimal encoders, the next step is to generate the code that augments the encoding phase in the target solver. OptCNF uses CVC4 as the target solver and generates the code for implementing the synthesized encoders in two phases: (1) Pattern matching in the decreasing order of the pattern size and (2) Extending the existing encoding phase in CVC4. OptCNF generates code for a straight-forward pattern matching phase while handling symmetries by enumerating all equivalent permutations of patterns with commutative operations. The generated code for augmenting CVC4 implements the synthesized encoder for each matched pattern and provides a command-line interface for switching them on or off individually.

However, there is scope for optimizing this code by implementing: (1) fast pattern matching that reuses common terms in the matched patterns (2) caching and reusing newly generated literals in the encoding phase (3) reduction in number of function calls in the generated code and (4) simplifying the encodings for patterns with constant inputs. Even without these optimizations, we are able to show significant improvement in CVC4's performance on certain domains (Section 7).

Chapter 6

Auto-tuning

For each domain, we use OpenTuner [3] to auto-tune the set of encoders (one for each pattern) obtained from the synthesis phase according to a performance metric based on the number of benchmark problems solved and the time taken to solve them. The evaluation function (**fopt**) to be optimized takes as input a set of encoders to be used and returns a real number. The number is the sum of all the times taken by the benchmarks to solve; for any benchmarks that time out, their time is counted as the timeout bound times two. The auto-tuner tries to minimize this value by trying out various subsets of encoders provided to it as input while learning a model of the dependence of **fopt** on the selection of encoders.

6.1 Optimization Problem Setup

We specify the set of all encodings generated for a particular domain to the tuner and create the following two configuration parameters: (1) p : a permutation parameter that permutes the list of all encodings. (2) n : total number of encodings to be used. We normalize the configuration parameters by truncating the permuted list p to have only first n elements and sorting the truncated permutation.Note that we chose this configuration instead of a bit-vector representing the set of choices to allow the tuner to apply techniques that can potentially hill-climb based on the number of encodings being used.

Chapter 7

Evaluation

We extend CVC4 solver (ranked 2 in the bit-vector category of SMT-COMP 2015 [9]) with synthesized encoders for each domain and evaluate the impact on its performance. Each generated solver is evaluated on the non-incremental quantifier free bit-vector (QF_BV) benchmark suite from SMT-COMP 2015. This benchmark suite consists of 26320 benchmarks that are grouped into 36 sub-categories. In most cases, these sub-categories represent a particular domain of problems–for example, the log – slicing category represents benchmarks that verify bit-vector translation from operations like addition and multiplication to a set of base operations and the mcm category represents multiple constant multiplication problems that commonly occur in digital audio and video processing and wireless communications. Some other sub-categories like asp are themselves a collection of benchmarks from multiple different domains from the Answer Set Programming community that includes combinatorial problems, planning and verification.

7.1 Experimental Setup

OptCNF generates a domain specific solver in four stages:

1. Randomly sampling 10% of the benchmarks from the domain and running CVC4 to collect all the formulas just before they are encoded to SAT.

2. Pattern finding (Section 4) on these formulas and filtering the terms based on capabilities of the synthesis phase of OptCNF.

3. Translation of each term to multiple SyGus problems one for each possible template that is suitable for the type and the size of the term. For problems involving non bit-parallel terms, OptCNF uses Sketch with 4 cores to parallelize the clause minimization algorithm (Section 3.5). All other problems use a single core. Each problem is also given a timeout of 3 hours.

4. Augmenting CVC4 code with the generated encoders (Section 5) and auto-tuning to find a subset of encoders that improve the performance (Section 6).

Different parts of OptCNF system were run on different machines. Pattern finding and synthesis of encoders were run on a machine with forty 2.4 GHz Intel Xeon processors and 96 GB RAM. For auto-tuning, we used a private cluster running OpenStack with parallelism of 150 on 75 virtual machines each with 4 cores and 8GB RAM of processing power. Finally, the performance experiment evaluating the solvers on QF_BV benchmarks was run on the StarExec [51] cluster infrastructure with a timeout of 900 seconds and a memory limit of 200 GB (similar to the resources used for the SMT competition).

7.2 Domains and Benchmarks

We generate a total of 7 domain-specific solvers and a general solver which is obtained by using the entire QF_BV benchmark suite for pattern finding and synthesis. For the general solver, we enable all the generated encoders and do not auto-tune them. The 7 domains are chosen from the 36 categories in QF_BV. We chose these categories based on the criteria that the number of benchmarks in the domain is at least 20 and the average run-time is significant enough to see an improvement. The solvers for these domains are referred by their category name.

7.3 Experiments

We focus on the following questions: (1) Can OptCNF generate domain-specific solvers in reasonable amount of time? (2) How does the performance of the domain-specific optimal solvers generated by OptCNF compare to CVC4? (3) How domain-specific are the encoders generated by OptCNF?

7.3.1 Time taken to generate optimal encoders

Table 7.1 shows the number of generated (gen) and selected (sel) encoders (selected after auto-tuning, differentiated by the type of patterns), and, the total time taken to synthesize these encoders (both cpu time and clock time). In addition to this, Pattern Finding was run for an hour per domain and Auto-tuning was run for 7.5 hours per domain. In total, OptCNF was able to generate domain-specific encoders in $10 - 22$ hours per domain which is a reasonable amount of time as compared to a software engineer implementing and debugging encoders in a solver.

Table 7.1: Encoder statistics and Sketch running times

Domain	# boolean		# bit parallel		# non bit parallel		Total patterns		Synthesis time	
	gen	sel	gen	sel	gen	sel	gen	sel	(cpu hrs)	(clock hrs)
general	336	336	334	334	12	12	682	682	497	17
asp	29	22	0	0	4	3	33	25	8	2
brummayerbiere2	66	0	12	7	2	2	80	9	16	2
brummayerbiere3	35	0	13	3	5	3	53	6	15	3
bruttomesso	21	4	1	0	1	0	23	4	5	2
float	272	17	294	18	3	0	569	35	360	13
log-slicing	19	0	86	60	5	5	110	65	49	4
mcm	13	3	2	1	4	1	19	5	7	2

7.3.2 Impact of domain-specific solvers

With the exception of the general solver, all the other solvers are auto-tuned to select a subset of the generated encodings that improves the performance. For all domains except asp and bruttomesso, the training set for auto-tuning contains 50% benchmarks chosen randomly from the domain. For these domains, we perform 2-fold cross-validation i.e. we swap training/test sets and run auto-tuning again. For asp and bruttomesso, the training set contains only 20% benchmarks due to resource constraints for auto-tuning resulting from them having a large number of benchmarks. For these two domains, we run auto-tuning again for approximating cross-validation with another disjoint training set that contains 20% benchmarks form the domain.

Table 7.2: Performance comparison: Domain-specific, general and CVC4 solvers on 7 categories of QF_BV benchmark suite (first training set)

	CVC4		general		Domain-Specific		Boolector	
Benchmark category	solved	time (s)	solved	time (s)	solved	time (s)	solved	time (s)
asp (365)	240	32652.8	238	33291.8	**288**	**34971.5**	308	29821.6
brummayerbiere2 (33)	28	1202.8	24	1653.2	**29**	**1691.0**	33	1371.2
brummayerbiere3 (40)	23	1165.2	23	2239.4	**24**	**1272.1**	32	1760.7
bruttomesso (676)	623	32880.8	604	35808.6	**623**	**32840.2**	774	8461.1
float (62)	59	4015.9	55	3599.6	**60**	**4395.5**	58	6152.9
log-slicing (79)	33	12636.1	57	17290.6	**62**	**21115.4**	53	9534.8
mcm (61)	40	3933.9	38	3355.0	**43**	**4193.0**	39	8333.1
	1046	88487.5	1039	97238.2	**1129**	**100479.8**	1297	65435.4

We compare the performance of the domain-specific solvers (auto-tuned on the first training set) with the general solver and CVC4 in Table 7.2. Only the benchmarks from the first test set are considered for evaluation in the table. The best-performing solver for every domain is marked as bold. The auto-tuned solver solves 83 benchmarks more than CVC4 in total. For all domains, the domain-specific solvers outperform CVC4. The domain-specific solvers auto-tuned on the second training set for each domain also outperform CVC4 and solve 73 more benchmarks on their corresponding test sets

(the details are in the appendix).

Table 7.2 also presents the performance of the Boolector solver (the best bit-vector solver in SMT-COMP'15) on the same test set benchmarks for reference. CVC4 is already better than Boolector on two domains (mcm, float) and OptCNF improves it slightly further. On one domain ($\log-$slicing), CVC4 is notably worse than Boolector, but OptCNF makes it outperform Boolector. In addition, OptCNF significantly bridges the gap between CVC4 and Boolector on the mcm domain.

Table 7.3: Performance comparison between general optimal solver and CVC4 on the other domains of QF_BV benchmarks

	CVC4		general		Boolector	
Benchmark category	solved	time (s)	solved	time (s)	solved	time (s)
VS3 (10)	**2**	**742.2**	0	0.0	3	434.9
uclid (416)	**416**	**1625.3**	416	1981.6	416	450.9
tacas07 (5)	5	1257.0	**5**	**831.1**	5	251.3
stp_samples (426)	**424**	**72.4**	424	182.3	426	9.9
spear (15)	**12**	**251.8**	12	786.1	12	128.0
sage (22390)	**22390**	**6225.7**	22390	7683.3	22390	3690.3
brummayerbiere (52)	39	2611.4	**39**	**1714.7**	41	448.4
bmc-bv (135)	135	520.2	**135**	**473.9**	135	51.9
fft (16)	**8**	**886.2**	7	41.0	9	597.1
calypto (16)	9	2.99	**11**	**985.1**	15	1447.8
	23440	14195.2	**23339**	**14679.1**	23452	7510.5

The run-times for benchmarks from domains where we did not perform auto-tuning can be found in the Table 7.3. The general solver performs better on some domains but not the others, and, slightly worse than CVC4 overall. In all cases where we performed auto-tuning, the domain-specific solvers beat the general solver (Table 7.2). Two scatter plots showing the performance of CVC4 versus general and the domain-specific solvers on these 7 domains can be found in Figure 7-1. It is evident from the graphs that the domain-specific solvers reduce the number of negative points (in the upper left triangle) thereby improving the performance when compared to CVC4 overall.

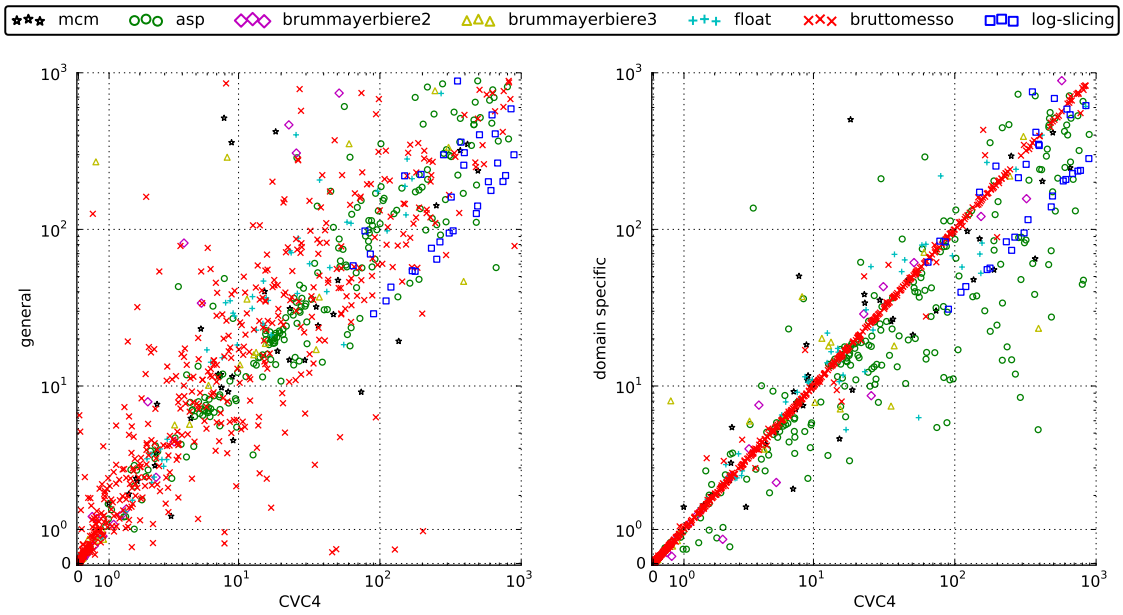


Figure 7-1: Scatter plots showing run-times (log scale) for different solvers on the 7 domains

7.3.3 Domain specificity

We ran each domain-specific solver (obtained from the first training set) on all the other domains and the results are summarized in Table 7.4. The best performing result for each domain is marked as bold and the results that are worse than CVC4 are underlined. 5 out of 7 of the domains are very domain-specific; the solvers that are tuned specially for them perform significantly better than all the other solvers. In some cases, using one solver on another domain makes it worse than CVC4. However, mcm domain has a solver optimal for other two domains performing almost identical to their respective solvers. These results substantiate our argument for domain-specific solvers.

Table 7.4: Cross-domain performance

solver →	asp		brummayerbiere2		brummayerbiere3		bruttomesso		float		log-slicing		mcm	
domain ↓	solved	time (s)	solved	time (s)	solved	time (s)	solved	time (s)	solved	time (s)	solved	time (s)	solved	time (s)
asp	**288**	**34971.5**	227	28173.5	253	34061.6	240	33118.9	236	29491.3	227	28230.5	255	35159.5
brummayerbiere2	28	786.9	**29**	**1691.0**	29	2363.0	29	2174.8	29	1804.2	29	1705.3	28	1706.6
brummayerbiere3	22	1206.7	22	1149.3	24	1272.1	23	1169.2	23	1410.1	22	911.3	**25**	**1945.6**
bruttomesso	606	37216.1	609	38744.1	623	32809.8	623	32840.1	623	32867.5	607	37164.7	**623**	**32683.5**
float	57	1650.8	57	2179.3	60	4853.1	59	3599.5	**60**	**4395.5**	57	1832.4	59	4100.9
log-slicing	58	20816.6	59	20125.7	35	12955.7	35	14640.7	32	11796.1	**62**	**21115.4**	36	14021.6
mcm	38	4301.6	40	3413.1	39	3411.2	41	3940.7	39	3759.5	39	5313.0	**43**	**4193.0**

Chapter 8

Related Work

8.1 Generating Propagation Complete Encodings

A recent paper [13] on automatically generating propagation complete encodings is the closest to this work. Encodings generated through OptCNF are propagation complete and OptCNF also minimizes the number of clauses across the template being used for the encoder similar to [13]. But, OptCNF is different in two important ways: (1) Instead of encodings, OptCNF generates *encoders* which produce encodings at run-time (enabled by program synthesis) (2) The generated encoders are specialized for a particular domain (enabled by pattern finding and auto-tuning).

8.2 Notions of strength of Encodings

Different notions of propagation strength of encodings have been considered in both Knowledge Compilation [19] (e.g. unit-refutation completeness [22] and its generalizations [30, 31, 32]) and Constraint Programming [6, 14] communities. Propagation complete encodings (PCEs) have been established [11] to be "well-posed" for a SAT solver's deduction mechanism, which provides a tractable reasoning on the constraints. [11] reduces the problem of generating PCEs to iteratively solving QBF formulas whereas OptCNF relies on CEGIS based program synthesis [48] to generate encoders producing PCEs at run-time. There has also been some recent work on using SAT

solvers for enumeration of prime implicants in the Knowledge Compilation community [30, 31]. In Constraint Programming, Generalized Arc-Consistency (GAC) [6] is connected to propagation completeness and has been adopted in SAT [27] but is usually only enforced on input/output variables and not on auxiliary variables which provides a weaker notion of propagation strength as compared to PCEs. [10] shows that certain global constraints can require exponential sized formulas for PCEs. In our work, we do not encounter this issue since we consider only small patterns as constraints.

8.3 Reducing Encodings size

Reducing the size of the CNF encodings derived from SAT formulas has been shown to be an effective way of optimizing SAT solvers [25, 16, 33, 24, 44, 55]. There has been a lot of work on optimal encodings for specific kinds of constraints like cardinality constraints [1], sequence constraints [14], verification of microprocessors [55]. There is also some work on logic minimization techniques like Beaver [41]. But, to our knowledge, we are the first ones to generate domain specific encodings that are propagation complete and minimal for multiple challenging domains using program synthesis technology.

8.4 Other SMT Solvers

OptCNF can be extended to other SMT solvers besides CVC4 such as Z3 [21], Beaver [41], Boolector [15] and Yices [23]. In Beaver and Boolector, intermediate data structures like And-Inverter graphs (AIGs) are employed and are later on transformed to CNF efficiently. Consequently, they have numerous optimizations on the AIG representation before translating it to CNF. Applying OptCNF directly to such solvers can override these optimizations and hence, requires more work. These solvers can also use lazy bit-blasting strategy as opposed to eager bit-blasting that we use in our experiments. OptCNF can be extended to solvers employing lazy bit-blasting by

using the generated encodings at the time of bit-blasting.

8.5 Algorithm Configuration

Finally, algorithm configuration [37, 4, 36], an active area of research in artificial intelligence, has been used in generation of encodings for Planning Domain Models [54] and improving CSP solving by searching for optimal solver choices and the different encodings for the CSP constraints [34]. It has also been shown to be successful for tuning parameters for SAT solvers [39]. Unlike OpenTuner [3], where the optimization function is a black-box, algorithm configuration can use the structure of certain types of functions and employ additional heuristics [38, 39] to optimize them.

Chapter 9

Conclusion and Future Work

OptCNF is a new technique to generate CNF encodings for bit-vector terms that are optimal with respect to propagation completeness. We combined it with machine learning based techniques namely pattern finding and auto-tuning to generate domain-specific SMT solvers. Our evaluation showed that this technique can noticeably improve CVC4, a state of the art SMT solver. We were able to automatically generate solvers that are specialized for the domains in the benchmark suite from SMT-COMP 2015 and these solvers perform significantly better than CVC4. Moreover, we have also showed that these solvers are very domain specific and hence, validating the argument for domain specific solvers.

There are many possible future directions based on OptCNF.

1. This thesis only applies OptCNF to a particular SMT solver, CVC4. However, we believe that OptCNF can be extended to solvers such as Z3, Boolector with little effort.

2. The techniques in this thesis can be extended to theories beyond bit-vectors in SMT solvers. For example, it will be useful to extend this approach to unary representation of integers. Unary representation is another way to deal with integer arithmetic in solvers and is used by several solvers including Sketch. One can then envision choosing between bit-vector representation or unary representation based on the target domain to achieve even better performance.

3. They are other parts of SAT/SMT solvers that can be similarly made domain specific using a combination of synthesis and machine learning techniques. One possible target for future work is to allow SAT solvers to operate on richer types of clauses (not being restricted to CNF representation), but this requires implementing the propagation logic for these rich clauses that do the same thing as unit propagation for CNF clauses. We can use program synthesis to automatically generate this propagation logic for each type of clause.

Appendix A

Additional results

Table A.1 shows the performance comparison between domain specific solvers that is auto-tuned on the second training set and CVC4. These are evaluated on the corresponding second test sets and hence, are not directly comparable to Table 7.2.

Table A.1: Performance comparison: Domain-specific, general and CVC4 solvers on 7 categories of QF_BV benchmark suite (second training set)

	CVC4		general		Domain-Specific		Boolector	
Benchmark category	solved	time (s)	solved	time (s)	solved	time (s)	solved	time (s)
asp (365)	237	36330.9	228	33828.7	**273**	**37212.5**	300	31661.7
brummayerbiere2 (32)	26	1030.6	21	1885.6	**29**	**3390.7**	31	999.5
brummayerbiere3 (39)	17	1653.4	16	2333.7	**18**	**2465.4**	27	668.4
bruttomesso (676)	**621**	**30642.0**	610	35967.9	620	31492.5	774	7832.0
float (62)	53	3829.2	53	7670.0	**54**	**4086.5**	49	6462.2
log-slicing (79)	29	9340.1	57	17955.2	**58**	**17465.5**	60	11098.1
mcm (61)	39	3159.6	39	6382.2	**43**	**4274.5**	40	9379.3
	1022	85985.8	1024	106023.3	**1095**	**100387.6**	1281	68101.2

Figure A-1: Scatter plots for performance comparison between domain specific solvers and CVC4 for each domain

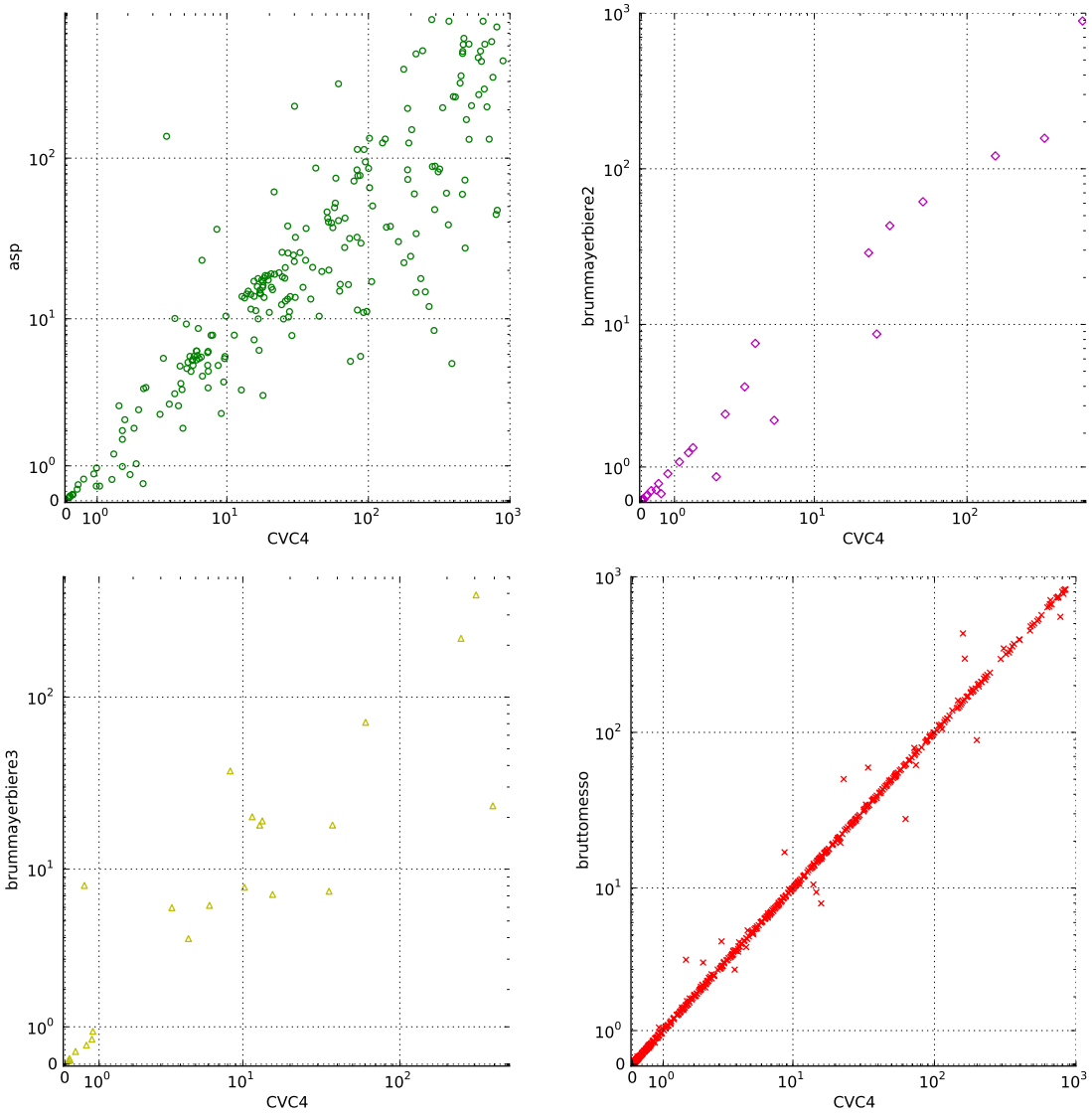
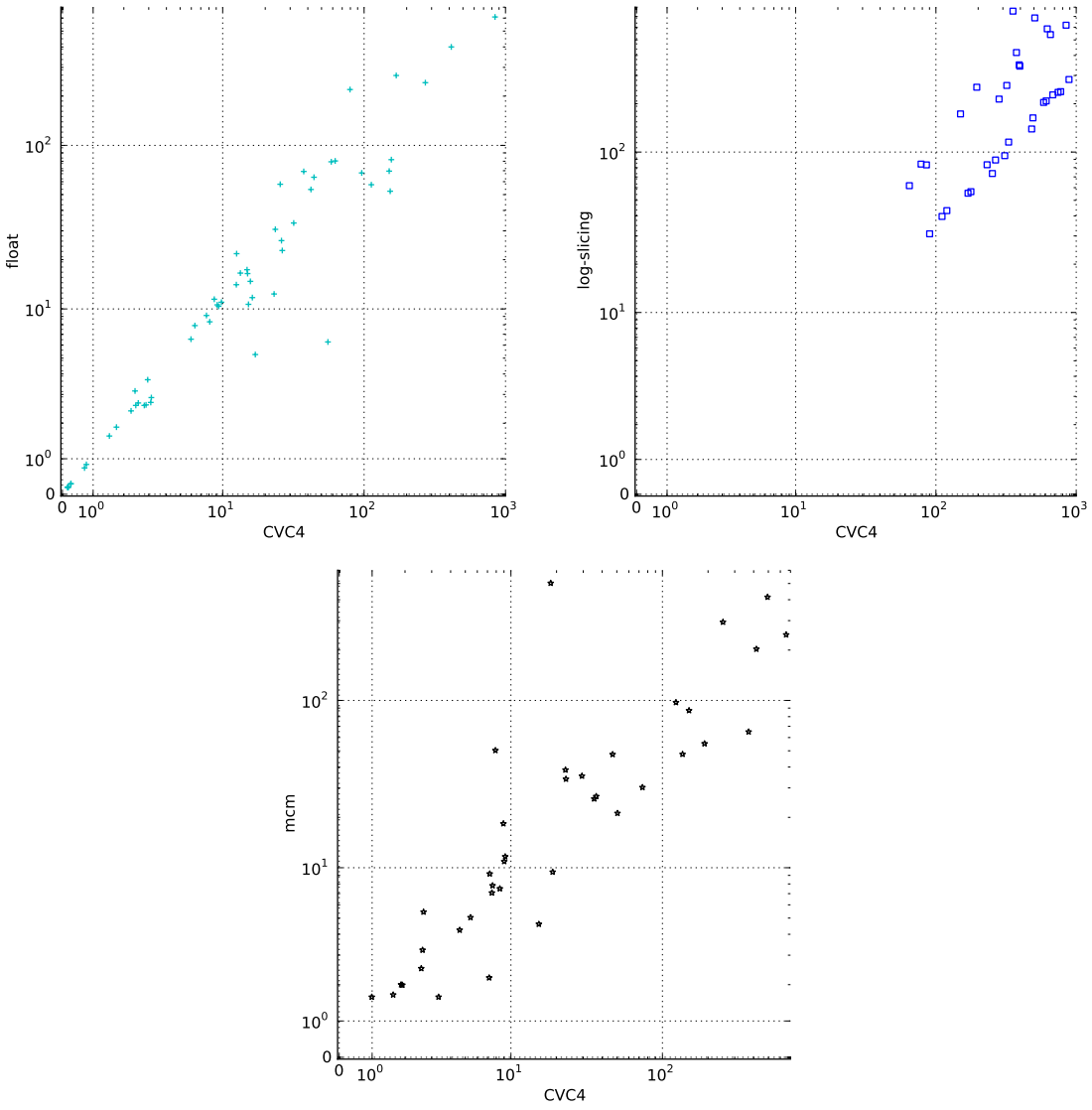


Figure A-2: Scatter plots for performance comparison between domain specific solvers and CVC4 for each domain (cont.)



Bibliography

[1] Ignasi Abío, Robert Nieuwenhuis, Albert Oliveras, and Enric Rodríguez-Carbonell. *Principles and Practice of Constraint Programming: 19th International Conference, CP 2013, Uppsala, Sweden, September 16-20, 2013. Proceedings*, chapter A Parametric Approach for Smaller and Better Encodings of Cardinality Constraints, pages 80–96. Springer Berlin Heidelberg, Berlin, Heidelberg, 2013.

[2] Rajeev Alur, Rastislav Bodik, Garvit Juniwal, Milo MK Martin, Mukund Raghothaman, Sanjit A Seshia, Rishabh Singh, Armando Solar-Lezama, Emina Torlak, and Abhishek Udupa. Syntax-guided synthesis. *Dependable Software Systems Engineering*, 40:1–25, 2015.

[3] Jason Ansel, Shoaib Kamil, Kalyan Veeramachaneni, Jonathan Ragan-Kelley, Jeffrey Bosboom, Una-May O'Reilly, and Saman P. Amarasinghe. Opentuner: an extensible framework for program autotuning. In José Nelson Amaral and Josep Torrellas, editors, *International Conference on Parallel Architectures and Compilation, PACT '14, Edmonton, AB, Canada, August 24-27, 2014*, pages 303–316. ACM, 2014.

[4] Carlos Ansótegui, Meinolf Sellmann, and Kevin Tierney. A gender-based genetic algorithm for the automatic configuration of algorithms. In Ian P. Gent, editor, *Principles and Practice of Constraint Programming - CP 2009, 15th International Conference, CP 2009, Lisbon, Portugal, September 20-24, 2009, Proceedings*, volume 5732 of *Lecture Notes in Computer Science*, pages 142–157. Springer, 2009.

[5] Gilles Audemard and Laurent Simon. Predicting learnt clauses quality in modern sat solvers. In *Proceedings of the 21st International Jont Conference on Artifical Intelligence*, IJCAI'09, pages 399–404, San Francisco, CA, USA, 2009. Morgan Kaufmann Publishers Inc.

[6] Fahiem Bacchus. Gac via unit propagation. In *Proceedings of the 13th International Conference on Principles and Practice of Constraint Programming*, CP'07, pages 133–147, Berlin, Heidelberg, 2007. Springer-Verlag.

[7] Clark Barrett, Christopher L. Conway, Morgan Deters, Liana Hadarean, Dejan Jovanović, Tim King, Andrew Reynolds, and Cesare Tinelli. Cvc4. In *Proceedings of the 23rd International Conference on Computer Aided Verification*, CAV'11, pages 171–177, Berlin, Heidelberg, 2011. Springer-Verlag.

[8] Clark Barrett, Morgan Deters, Leonardo Moura, Albert Oliveras, and Aaron Stump. 6 years of smt-comp. *Journal of Automated Reasoning*, 50(3):243–277, 2012.

[9] Clark W. Barrett, Leonardo de Moura, and Aaron Stump. Smt-comp: Satisfiability modulo theories competition. In *Computer Aided Verification, 17th International Conference, CAV 2005, Edinburgh, Scotland, UK, July 6-10, 2005, Proceedings*, volume 3576 of *Lecture Notes in Computer Science*, pages 20–23. Springer, 2005. `http://smtcomp.sourceforge.net/2016/`.

[10] Christian Bessiere, George Katsirelos, Nina Narodytska, and Toby Walsh. Circuit complexity and decompositions of global constraints. In Craig Boutilier, editor, *IJCAI 2009, Proceedings of the 21st International Joint Conference on Artificial Intelligence, Pasadena, California, USA, July 11-17, 2009*, pages 412–418, 2009.

[11] Lucas Bordeaux and Joao Marques-Silva. Knowledge compilation with empowerment. In *Proceedings of the 38th International Conference on Current Trends in Theory and Practice of Computer Science*, SOFSEM'12, pages 612–624, Berlin, Heidelberg, 2012. Springer-Verlag.

[12] Ella Bounimova, Patrice Godefroid, and David Molnar. Billions and billions of constraints: Whitebox fuzz testing in production. In *Proceedings of the 2013 International Conference on Software Engineering*, ICSE '13, pages 122–131, Piscataway, NJ, USA, 2013. IEEE Press.

[13] Martin Brain, Liana Hadarean, Daniel Kroening, and Ruben Martins. Automatic generation of propagation complete sat encodings. In *Verification, Model Checking, and Abstract Interpretation*, pages 536–556. Springer, 2016.

[14] Sebastian Brand, Nina Narodytska, Claude-Guy Quimper, Peter J. Stuckey, and Toby Walsh. Encodings of the sequence constraint. In Christian Bessiere, editor, *Principles and Practice of Constraint Programming - CP 2007, 13th International Conference, CP 2007, Providence, RI, USA, September 23-27, 2007, Proceedings*, volume 4741 of *Lecture Notes in Computer Science*, pages 210–224. Springer, 2007.

[15] Robert Brummayer and Armin Biere. Boolector: An efficient smt solver for bit-vectors and arrays. In *Proceedings of the 15th International Conference on Tools and Algorithms for the Construction and Analysis of Systems: Held As Part of the Joint European Conferences on Theory and Practice of Software, ETAPS 2009,*, TACAS '09, pages 174–177, Berlin, Heidelberg, 2009. Springer-Verlag.

[16] Benjamin Chambers, Panagiotis Manolios, and Daron Vroon. Faster sat solving with better cnf generation. In *Proceedings of the Conference on Design, Automation and Test in Europe*, DATE '09, pages 1590–1595, 3001 Leuven, Belgium, Belgium, 2009. European Design and Automation Association.

[17] Alvin Cheung, Armando Solar-Lezama, and Samuel Madden. Partial replay of long-running applications. In *Proceedings of the 19th ACM SIGSOFT symposium and the 13th European conference on Foundations of software engineering*, ESEC/FSE '11, pages 135–145, New York, NY, USA, 2011. ACM.

[18] Byron Cook, Daniel Kroening, Philipp Rümmer, and Christoph M. Wintersteiger. Ranking function synthesis for bit-vector relations. In *Proceedings of the 16th International Conference on Tools and Algorithms for the Construction and Analysis of Systems*, TACAS'10, pages 236–250, Berlin, Heidelberg, 2010. Springer-Verlag.

[19] Adnan Darwiche and Pierre Marquis. A knowledge compilation map. *J. Artif. Intell. Res. (JAIR)*, 17:229–264, 2002.

[20] Martin Davis and Hilary Putnam. A computing procedure for quantification theory. *J. ACM*, 7(3):201–215, July 1960.

[21] Leonardo De Moura and Nikolaj Bjørner. Z3: an efficient smt solver. In *Proceedings of the Theory and practice of software, 14th international conference on Tools and algorithms for the construction and analysis of systems*, TACAS'08/ETAPS'08, pages 337–340, Berlin, Heidelberg, 2008. Springer-Verlag.

[22] Alvaro del Val. Tractable databases: How to make propositional unit resolution complete through compilation. In Jon Doyle, Erik Sandewall, and Pietro Torasso, editors, *Proceedings of the 4th International Conference on Principles of Knowledge Representation and Reasoning (KR'94). Bonn, Germany, May 24-27, 1994.*, pages 551–561. Morgan Kaufmann, 1994.

[23] Bruno Dutertre. Yices 2.2. In *Computer Aided Verification*, pages 737–744. Springer, 2014.

[24] Niklas Eén and Armin Biere. Effective preprocessing in sat through variable and clause elimination. In *Proceedings of the 8th International Conference on Theory and Applications of Satisfiability Testing*, SAT'05, pages 61–75, Berlin, Heidelberg, 2005. Springer-Verlag.

[25] Niklas Een, Alan Mishchenko, and Niklas Sörensson. Applying logic synthesis for speeding up sat. In *Proceedings of the 10th International Conference on Theory and Applications of Satisfiability Testing*, SAT'07, pages 272–286, Berlin, Heidelberg, 2007. Springer-Verlag.

[26] Niklas Eén and Niklas Sörensson. An extensible SAT-solver. In *SAT*, pages 502–518, 2003.

[27] Ian P. Gent. Arc consistency in SAT. In Frank van Harmelen, editor, *Proceedings of the 15th European Conference on Artificial Intelligence, ECAI'2002, Lyon, France, July 2002*, pages 121–125. IOS Press, 2002.

[28] Patrice Godefroid. Test generation using symbolic execution. In Deepak D'Souza, Telikepalli Kavitha, and Jaikumar Radhakrishnan, editors, *IARCS Annual Conference on Foundations of Software Technology and Theoretical Computer Science, FSTTCS 2012, December 15-17, 2012, Hyderabad, India*, volume 18 of *LIPIcs*, pages 24–33. Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2012.

[29] Sumit Gulwani, Saurabh Srivastava, and Ramarathnam Venkatesan. Constraint-based invariant inference over predicate abstraction. In *Proceedings of the 10th International Conference on Verification, Model Checking, and Abstract Interpretation*, VMCAI '09, pages 120–135, Berlin, Heidelberg, 2009. Springer-Verlag.

[30] Matthew Gwynne and Oliver Kullmann. Generalising and unifying SLUR and unit-refutation completeness. In Peter van Emde Boas, Frans C. A. Groen, Giuseppe F. Italiano, Jerzy R. Nawrocki, and Harald Sack, editors, *SOFSEM 2013: Theory and Practice of Computer Science, 39th International Conference on Current Trends in Theory and Practice of Computer Science, Špindlerův Mlýn, Czech Republic, January 26-31, 2013. Proceedings*, volume 7741 of *Lecture Notes in Computer Science*, pages 220–232. Springer, 2013.

[31] Matthew Gwynne and Oliver Kullmann. Towards a theory of good SAT representations. *CoRR*, abs/1302.4421, 2013.

[32] Matthew Gwynne and Oliver Kullmann. Generalising unit-refutation completeness and SLUR via nested input resolution. *J. Autom. Reasoning*, 52(1):31–65, 2014.

[33] Marijn Heule, Matti Järvisalo, and Armin Biere. Clause elimination procedures for CNF formulas. In Christian G. Fermüller and Andrei Voronkov, editors, *Logic for Programming, Artificial Intelligence, and Reasoning - 17th International Conference, LPAR-17, Yogyakarta, Indonesia, October 10-15, 2010. Proceedings*, volume 6397 of *Lecture Notes in Computer Science*, pages 357–371. Springer, 2010.

[34] Barry Hurley, Lars Kotthoff, Yuri Malitsky, and Barry O'Sullivan. Proteus: A hierarchical portfolio of solvers and transformations. In Helmut Simonis, editor, *Integration of AI and OR Techniques in Constraint Programming - 11th International Conference, CPAIOR 2014, Cork, Ireland, May 19-23, 2014. Proceedings*, volume 8451 of *Lecture Notes in Computer Science*, pages 301–317. Springer, 2014.

[35] Frank Hutter, Domagoj Babic, Holger H. Hoos, and Alan J. Hu. Boosting verification by automatic tuning of decision procedures. In *Proceedings of the Formal Methods in Computer Aided Design*, FMCAD '07, pages 27–34, Washington, DC, USA, 2007. IEEE Computer Society.

[36] Frank Hutter, Holger H. Hoos, and Kevin Leyton-Brown. Sequential model-based optimization for general algorithm configuration. In Carlos A. Coello Coello, editor, *Learning and Intelligent Optimization - 5th International Conference,*

LION 5, Rome, Italy, January 17-21, 2011. Selected Papers, volume 6683 of *Lecture Notes in Computer Science*, pages 507–523. Springer, 2011.

[37] Frank Hutter, Holger H. Hoos, Kevin Leyton-Brown, and Thomas Stützle. Paramils: An automatic algorithm configuration framework. *J. Artif. Int. Res.*, 36(1):267–306, September 2009.

[38] Frank Hutter, Holger H. Hoos, and Thomas Stützle. Automatic algorithm configuration based on local search. In *Proceedings of the Twenty-Second AAAI Conference on Artificial Intelligence, July 22-26, 2007, Vancouver, British Columbia, Canada*, pages 1152–1157. AAAI Press, 2007.

[39] Frank Hutter, Marius Thomas Lindauer, Adrian Balint, Sam Bayless, Holger H. Hoos, and Kevin Leyton-Brown. The configurable SAT solver challenge (CSSC). *CoRR*, abs/1505.01221, 2015.

[40] Jeevana Priya Inala, Rohit Singh, and Armando Solar-Lezama. Synthesis of domain specific encoders for bit-vector solvers. In *Theory and Applications of Satisfiability Testing, 19th International Conference, SAT 2016. Bordeaux, France, July 5-8, 2016*. Springer, 2016.

[41] Susmit Jha, Rhishikesh Limaye, and Sanjit A. Seshia. Beaver: Engineering an efficient smt solver for bit-vector arithmetic. In *Proceedings of the 21st International Conference on Computer Aided Verification*, CAV '09, pages 668–674, Berlin, Heidelberg, 2009. Springer-Verlag.

[42] Norbert Manthey, Marijn Heule, and Armin Biere. Automated reencoding of boolean formulas. In Armin Biere, Amir Nahir, and Tanja E. J. Vos, editors, *Hardware and Software: Verification and Testing - 8th International Haifa Verification Conference, HVC 2012, Haifa, Israel, November 6-8, 2012. Revised Selected Papers*, volume 7857 of *Lecture Notes in Computer Science*, pages 102–117. Springer, 2012.

[43] Ruben Martins, Vasco M. Manquinho, and Inês Lynce. Exploiting cardinality encodings in parallel maximum satisfiability. In *IEEE 23rd International Conference on Tools with Artificial Intelligence, ICTAI 2011, Boca Raton, FL, USA, November 7-9, 2011*, pages 313–320. IEEE Computer Society, 2011.

[44] Matthew W. Moskewicz, Conor F. Madigan, Ying Zhao, Lintao Zhang, and Sharad Malik. Chaff: Engineering an efficient sat solver. In *Proceedings of the 38th Annual Design Automation Conference*, DAC '01, pages 530–535, New York, NY, USA, 2001. ACM.

[45] A. Pnueli and R. Rosner. On the synthesis of a reactive module. In *Proceedings of the 16th ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*, POPL '89, pages 179–190, New York, NY, USA, 1989. ACM.

[46] Koushik Sen, Swaroop Kalasapur, Tasneem Brutch, and Simon Gibbs. Jalangi: A selective record-replay and dynamic analysis framework for javascript. In *Proceedings of the 2013 9th Joint Meeting on Foundations of Software Engineering*, ESEC/FSE 2013, pages 488–498, New York, NY, USA, 2013. ACM.

[47] Rohit Singh and Armando Solar-Lezama. Automatic generation of formula simplifiers based on conditional rewrite rules,arxiv:1602.07285, 2016.

[48] Armando Solar-Lezama. *Program Synthesis By Sketching*. PhD thesis, EECS Dept., UC Berkeley, 2008.

[49] Armando Solar-Lezama, Liviu Tancau, Rastislav Bodik, Vijay Saraswat, and Sanjit Seshia. Combinatorial sketching for finite programs. In *ASPLOS '06*, San Jose, CA, USA, 2006. ACM Press.

[50] Saurabh Srivastava, Sumit Gulwani, and Jeffrey S. Foster. From program verification to program synthesis. In *Proceedings of the 37th Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*, POPL '10, pages 313–326, New York, NY, USA, 2010. ACM.

[51] Aaron Stump, Geoff Sutcliffe, and Cesare Tinelli. Introducing starexec: a cross-community infrastructure for logic solving. In Vladimir Klebanov, Bernhard Beckert, Armin Biere, and Geoff Sutcliffe, editors, *COMPARE*, volume 873 of *CEUR Workshop Proceedings*, page 2. CEUR-WS.org, 2012.

[52] Haruto Tanno, Xiaojing Zhang, Takashi Hoshino, and Koushik Sen. Tesma and catg: Automated test generation tools for models of enterprise applications. In *Proceedings of the 37th International Conference on Software Engineering - Volume 2*, ICSE '15, pages 717–720, Piscataway, NJ, USA, 2015. IEEE Press.

[53] Grigori S Tseitin. On the complexity of derivation in propositional calculus. In *Automation of reasoning*, pages 466–483. Springer, 1983.

[54] Mauro Vallati, Frank Hutter, Lukás Chrpa, and Thomas Leo McCluskey. On the effective configuration of planning domain models. In Qiang Yang and Michael Wooldridge, editors, *Proceedings of the Twenty-Fourth International Joint Conference on Artificial Intelligence, IJCAI 2015, Buenos Aires, Argentina, July 25-31, 2015*, pages 1704–1711. AAAI Press, 2015.

[55] Miroslav N. Velev. Efficient translation of boolean formulas to cnf in formal verification of microprocessors. In *Proceedings of the 2004 Asia and South Pacific Design Automation Conference*, ASP-DAC '04, pages 310–315, Piscataway, NJ, USA, 2004. IEEE Press.

[56] Xi Wang, Nickolai Zeldovich, M. Frans Kaashoek, and Armando Solar-Lezama. A differential approach to undefined behavior detection. *Commun. ACM*, 59(3):99–106, 2016.