Optimal Power Flow with Price-Elastic Demand

by

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Abstract

A MATLAB program for Optimal Power Flow (OPF) was implemented to include price elastic demand curves. The simulation results for a 39-buses electric power system indicate both higher economical and technical efficiencies in the price-elastic demand scenarios than those in a price-inelastic scenario. Iterative pricing scheme was then described, with simulation results for a 3-bus electric power system compared to those of coordinated OPF. The simulations for iterative pricing indicated the applicability of coordinated OPF in evaluating the optimality of a distributed decision making process on electric power systems.

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Chapter 1

Introduction

The deregulation of power industry has introduced competition in power supply. This, in turn, has led to the need for open non-discriminatory access to transmission grid by utility and non-utility owned power suppliers. The Federal Energy Regulatory Commission (FERC) introduced Open Access [1], [3] requirement on public utilities that own and/or control facilities used for the transmission of electric energy in interstate commerce to provide open access, non-discriminatory wholesale transmission services. The electric power demand side can no longer be assumed to be price inelastic either; there may be, in the future, customers willing to compromise power quality for lower price, and vice versa. Also, the actual power consumed by the customers may change in response to power prices. This thesis presents computer simulations for Optimal Power Flow under the condition that the demand-side is price sensitive.

1.1 Scope and Underlying Assumptions of this Thesis

This thesis is particularly concerned with the analysis of price elastic demand under open access. The standard optimal power flow (OPF) method, presently used for coordinated optimal scheduling of all power supplies to meet fixed demand, does not take into consideration of price elasticity on the demand side. While it is not clear how significant will be the effect of competitive supply on customers, in this thesis, we provide an analysis assuming that this is the case. It is possible to use statistical data from demand side management (DSM) experiments in the past to create some realistic cost curves on the demand side. In this thesis, only hypothetical cost curves are used.

It is furthermore assumed that the demand reacts instantaneously to price changes in power supply, and further studies are needed in the future to relax it. Static optimization tools, such as OPF, are only capable of providing analysis under this assumption. This thesis only deals with static optimization for economic operations of electric power systems. It should be noted that the power system is a very complicated dynamical system. This leads to many new open questions relating to power systems in a competitive industry. In this thesis, system dynamics are assumed to stable and only the real-power scheduling for stationary operation is of interest.

Another assumption made in this thesis is that significant contingencies on the system are dealt with in a preventive operation mode. Scheduling is done to remain within the specifications so that no single unexpected outage violates system integrity and reliable operation.

1.2 Organization of This Thesis

In Chapter 2, an overview of optimal power flow is first provided. Then, a MATLAB [7] program for Optimal Power Flow under the condition that the demand is price sensitive is implemented and the simulation results of for electric power systems as complex as a 39 buses example, are presented. Chapter 3 demonstrates how the OPF MATLAB program can be used in evaluating the results from recently proposed iterative pricing scheme. The concept of cost of ancillary services is introduced, followed by a description of the iterative pricing mechanism proposed in [4], [5], [12], and [13]. Then, the simulation of iterative pricing is presented, and result compared to the solutions obtained from centralized Optimal Power Flow (OPF) method introduced in Chapter 2. The numerical input data, MATLAB program and detailed simulation results of both Chapter 2 and Chapter 3 are presented in the Appendices.

The technical contribution of this thesis is the provision of a computationally efficient MATLAB OPF program, while the primary policy contribution is the evaluation of the feasibility of an iterative pricing scheme discussed in Chapter 3. The iterative pricing scheme is an elegant pricing mechanism that encourages competition while at the same time manages the cost of ancillary services to keep the electric power systems together.

Chapter 2

Optimal Power Flow with Price-elastic Demand

The present electric power system has been designed for qualitatively different types of input other than the market-driven demand and generation. However, unless otherwise decided, the technical performance objectives are likely to remain unchanged, which can be employed in the context of both present and changing operations. In a centralized (also known as coordinated) industrial structure, cost/benefit functions are fully known. The OPF as a coordinated static optimization can be used to achieve both *ideal technical efficiency* and *economic efficiency (that is social welfare maximization)* in real-time operations.

In this chapter, the Optimal Power Flow (OPF) technique is briefly summarized and simulations for a 39-buses electric power system under five demand scenarios is presented. The first section presents a review of the OPF method. OPF for price-inelastic demand, and price-elastic demand of a 39-buses example is then introduced. In the second section, simulation results under different scenarios of demand elasticity are presented and compared. Note that we do not study optimality issues related to investment; optimality is studied with respect to real-time operations only, taking real power generated and demanded as control variables for optimality.

2.1 Review of the Optimal Power Flow Method

2.1.1 Optimization for Economic Operations of Conventional Power Systems

In a regulated power industry, the demand is assumed to be fixed and price insensitive at a certain period of time as shown in Figure 2.1; here, $C_{Gi}(P_{Gi})$ is the total generation cost curve corresponding to the supply curve and $C_{Di}(P_{Di})$ is the total consumer's utility curve corresponding to the demand curve. Given such assumption, the amount of electric power generation is usually known with high certainty based on various demand forecast methods. Random, relatively small deviations from anticipated demand is compensated by generators participating in the automatic generation control (AGC) [14].

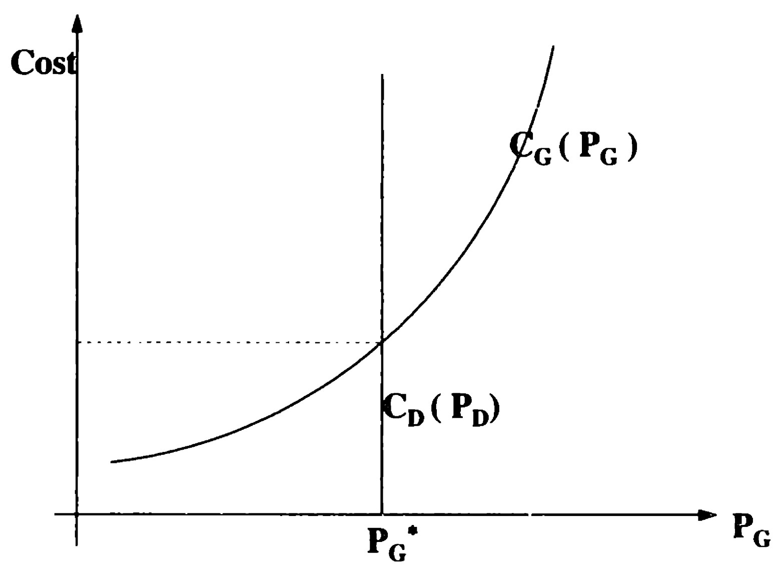

Figure 2.1 Supply And Demand Curve Assumed In A Regulated Power Industry

Under the assumption of a fixed power demand capacity at a certain period of time, static optimization techniques are used for economic operation of the whole system.

With the real power generated being the control variable for optimization, the most commonly used is the Optimal Power Flow (OPF) summarized as follows:

Minimize

$$Total\ Generation\ Cost = \sum_{i \in N_G} C_{Gi}(P_{Gi}) \qquad (2.1)$$

where

N_G = number of generation nodes;

P_{Gi} = the real power injected at each bus with net generation; and

$C_{Gi}(P_{Gi})$ = cost curve function of each generator.

subject to the following constraints:

<1> generation capacity limit at each bus: $P_{Gi}^{min} \le P_{Gi} \le P_{Gi}^{max}$ (2.2)

<2> real power flow equation for the whole system: $\mathbf{f}(\mathbf{P}_G, \mathbf{x}) = \mathbf{0}$, (2.3)

in which $\mathbf{P}_G$ denotes the vector composed of P_{Gi} and $\mathbf{x}$ is a vector composed of system variables such as the conductance of each individual transmission line.

<3> bus voltage limit: $V_i^{min} \le V_i \le V_i^{max}$ (2.4)

<4> real power transmission congestion constraint: $P_{ij} \le P_{ij}^{max}$ (2.5)

where $i, j \in N_G$. Note that transmission losses are taken into account in the real power equations. Further detailed description of OPF can be found in [2].

2.1.2 Optimization for Economic Operations in an Open-Accessed Transmission Grid Industry

Under open access of power transmission grid, the demand can no longer be price-inelastic. In fact, the demand curve will correspond to a "negative" utility curve, with negative slope as shown in Figure 2.2. Note in Figure 2.2, the demand curve is $C_D(P_G)$as shown in solid line if transmission losses are ignored. The demand curve $C_D(P_G - P_L)$, where P_L is the total power loss of the system, and is itself a function of P_G, as shown in dashed line, will be a more realistic one, in which the transmission power losses are considered.

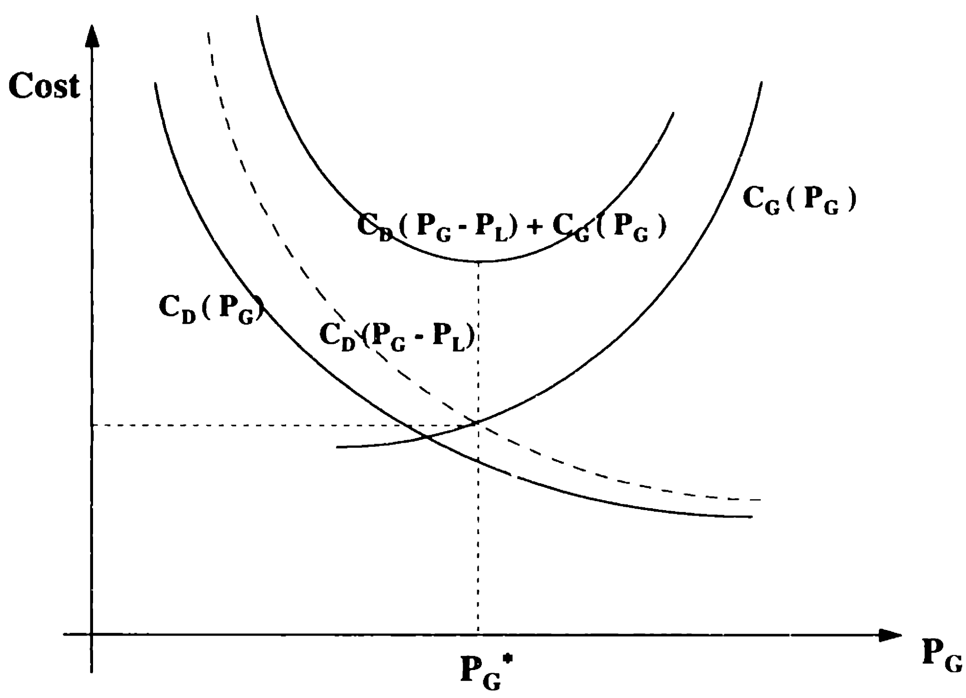

Figure 2.2 Supply-Demand Curve in a Generation Compctitive Industry

From the supply-demand curve shown in Figure 2.2, it can be constructed that in order to achieve maximum social welfare described in [15], optimization formulation introduced in [2] is summarized as follows:

Minimize

$$-Social\ Welfare = \sum_{i \in N_G} C_{Gi}(P_{Gi}) + \sum_{i \in N_D} C_{Di}(P_{Di}) \quad (2.6)$$

where N_G = number of generation nodes

and N_D = number of demand nodes

subject to

<1> generation capacity limit at each bus: $P_{Gi}^{min} \le P_{Gi} \le P_{Gi}^{max}$ (2.7)

<2> demand limit at each bus: $P_{Di}^{min} \le P_{Di} \le P_{Di}^{max}$ (2.8)

<3> real power flow equation for the whole system: $\mathbf{f}(\mathbf{P_G}, \mathbf{P_D}, \mathbf{x}) = \mathbf{0}$ (2.9)

in which $\mathbf{P_G}$ denotes the vector composed of P_{Gi}, $\mathbf{P_D}$ denotes the vector composed of P_{Di},and $\mathbf{x}$ is a vector composed of system variables such as the conductance of each individual transmission line.

<4> bus voltage limit: $V_i^{min} \le V_i \le V_i^{max}$ (2.10)

<5> real power transmission congestion constraint : $P_{ij} \le P_{ij}^{max}$ (2.11)

where $i, j \in N_G, N_D$.

Compared to the optimization formulation in Section 2.1.1 , the demand price-elastic situation has the additional control variable P_{Di}, and hence introduces more computational complexity. The optimality conditions for solving (2.6) to (2.11) are discussed in the next section.

2.1.3 Optimality Conditions

When fully coordinated OPF is used that attempts to simultaneously fulfill the constraints (2.6) to (2.11), the process begins with an initial guess for the power levels (denoted as control vector $\mathbf{u}$). The Lagrangian L is formulated to be

$$L = C(\mathbf{x}, \mathbf{u}) + \lambda^T f(\mathbf{x}, \mathbf{u})$$

where $\mathbf{x}$ is a vector of system states (such as load bus voltages, bus voltage angles except that of slack bus..etc.) and f stands for the load flow equations.

The initial value for $\mathbf{u}$ and $\mathbf{x}$ is obtained by solving the load flow equations. The vector λ_0 is calculated as

$$\lambda_0 = \left[\left(\frac{\partial f}{\partial \mathbf{x}}\right)\right]^{-1} \frac{\partial C}{\partial \mathbf{x}} \qquad (2.12)$$

and then $\frac{\partial L}{\partial \mathbf{u}}$ is determined by

$$\frac{\partial L}{\partial \mathbf{u}} = \frac{\partial C}{\partial \mathbf{u}} + \left(\frac{\partial L}{\partial \mathbf{u}}\right)^T \lambda_0 \qquad (2.13)$$

With a step in the direction of $\frac{\partial L}{\partial \mathbf{u}}$, a new control value $\mathbf{u}_1$ may be obtained such that the cost C is reduced. The process then repeats until convergence, i.e. $\frac{\partial L}{\partial \mathbf{u}} = 0$. In this thesis, these numerical steps are implemented by means of MATLAB code [7]. In particular, the **CONSTR** function in the **MATLAB *Optimization Toolbox*** is used.

2.2 Numerical Simulations Using OPF

2.2.1 Introduction of the 39-buses Numerical Data

The 39-buses example is an electric power system consists of 10 generation units, 29 loads, and 48 transmission lines. The bus data is shown in Appendix A, and the line data in Appendix B. The cost functions for the ten generation units are shown in Table 2.1. They are obtained by using typical costs for various technologies.

Table 2.1 Cost Functions for the Ten Generation Units of 39-buses Example

Name	**Type of Generation**	**Maximum Capacity (MW)**	**Cost Function Coefficient *a***	**Cost Function Coefficient *b***	**Cost Function Coefficient *c***
Conn	***Nuclear***	600	454.63	8.73	0
Northfield	***Hydro***	846	1.26	0.02	0
Millstone	***Nuclear***	662	376.02	6.55	0
Brayton 3	***Coal***	643	0	21.78	-0.01
Brayton 1&2	***Coal***	482	0	23.04	-0.01
Pilgrim	***Nuclear***	678	286.40	4.87	0
Canal	***Oil***	1072	0	30.43	-0.01
Vermont	***Nuclear***	563	353.61	7.24	0
Maine	***Nuclear***	864	412.47	5.5	0
NY&West	***(N/A)***	110	0	0.02	0

* *The Cost Function is* $C_{Gi} = a + bP_{Gi} + cP_{Gi}^{\ 2}$

2.2.2 Rationale Behind Setting the Congestion Constraint

In order to evaluate the numerical convergence using MATLAB, reasonably stringent power congestion is imposed. Discussion of how the constraint is chosen is described below:

The profile of real power transmission obtained from a first-run-through of load-flow program is shown in Figure 2.3. In Figure 2.3, the horizontal axis represents bus numbers; the y-axis corresponds to the bus from which the power is injecting into a transmission line, and x-axis corresponds to the bus into which the power receiving is receiving a transmission line. A value along the z-axis represents the amount of real power delivered from y to x; a positive means that y corresponds to the bus with generation unit, and x corresponds to the bus with the load, and vice versa for negative values of along the z-axis.

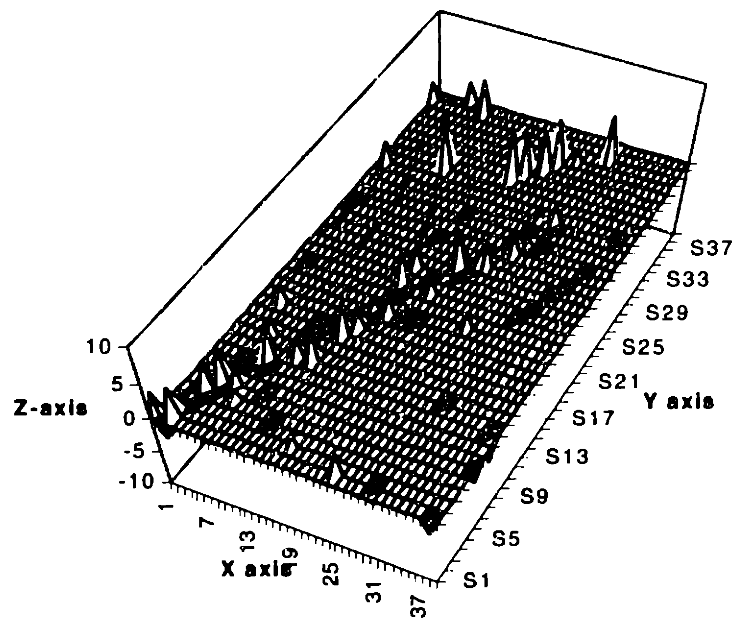

Figure 2.3 The Profile of Nominal Power Transmission

With careful observation of the two graphs in Figure 2.3, the region $+5 < -x + y < -5$ and $-4x + y > 29$ have most of the "spikes" resided. Therefore it can be concluded that the buses corresponding to $+5 < -x + y < -5$ and $-4x + y > 29$ will be connected by transmission lines that have most effect on the OPF sensitivity of power transmission constraint. Therefore, those transmission lines are selected as those lines on which transmission congestion constraint will be imposed so that evaluation of OPF convergence when subject to the most stringent transmission constraint can be tested.

Care should also be taken in determining the threshold for congestion constraint; it should be stringent enough to evaluate the robustness of the program used, but should also be such that feasible solution can be obtained. The transmission constraint of 80% of power flow under the first run-through of load-flow program is applied in the simulation.

2.2.3 Rationale for Demand Curve Assumptions

The demand curve data is not given from the 39-buses example, and hence should be assumed. From [8], a demand utility curve is $u(P_l) = 34.166P_L - P_L^2$, where P_L is the power consumed by the load. Hence it is reasonable to make the assumption that at each load bus, the nominal demand "cost" curve is $C_i = P_{D_i}^2 - 34.166P_{D_i}$ where $i \in N_D$, N_D is the number of loads , and P_{D_i} is the power demanded at each load bus. It should be noted that the main purpose of this thesis is to implement an OPF code sufficiently functional to generate optimal solution for systems as complex as a 39-buses example. Hence, hypothetical demand curve such as $C_i = P_{D_i}^2 - 34.166P_{D_i}$ is used.

In addition to the nominal demand curve, different scenarios for demand curve should also be evaluated with the simulation, and results should be compared including with that of fixed-demand conditions. In summary, the simulations are performed for the following 5 scenarios of demand situation:

Scenario 1: OPF under price inelastic demand situations

Scenario 2: demand curve with nominal elasticity: $C_i = P_{D_i}^{\ 2} - 34.166 P_{D_i}$, where $i \in N_D$, N_D is the number of loads , and P_{D_i} is the power demanded at each load bus.

Scenario 3: demand curve with slope 50% of nominal elasticity: $C_i = 0.5 P_{D_i}^{\ 2} - 17.083 P_{D_i}$, where $i \in N_D$, N_D is the number of loads , and P_{D_i} is the power demanded at each load bus.

Scenario 4: demand curve with slope 200%: $C_i = 2 P_{D_i}^{\ 2} - 68.322 P_{D_i}$, where $i \in N_D$, N_D is the number of loads , and P_{D_i} is the power demanded at each load bus.

Scenario 5: demand curve with nominal elasticity with minimum demand: $C_i = P_{D_i}^{\ 2} - 34.166 P_{D_i}$, and $P_{D_i} \geq 50$ MWh, where $i \in N_D$, N_D is the number of loads , and P_{D_i} is the power demanded at each load bus.

2.2.4 Simulation Results

Various interesting facts can be observed from the simulations of OPF under the five scenarios described in Section 2.2.3. This section summarizes many of the important results.

The optimal power injected in each individual bus under different scenario is shown in Table 2.2 and Table 2.3 . Note that a negative value denotes the power demanded, a positive value denotes the power supplied by that particular bus. From the tables, it can be observed that the more in-elastic demand, the more likely the generation hit generation will reach its limit. This observation is, however, hard to generalize for all possible demand situation.

Table 2.2 Optimal Power Injected at the Generation Side

Generator Names	Scenario 1 (Fixed Demand)	Scenario 2 (Nominally Elastic)	Scenario 3 (Highly Elastic)	Scenario 4 (Lowly Elastic)	Scenario 5 (Minimum Demand)	Electric Power Limit
Conn	***6***	***6***	2.6466	***6***	***6***	***6***
Northfield	***8.46***	***8.46***	8.2421	***8.46***	***8.46***	***8.46***
Millstone	***6.62***	***6.62***	4.5376	***6.62***	***6.62***	***6.62***
Brayton 3	***6.43***	3.6405	0	***6.43***	3.6607	***6.43***
Brayton1&2	2.8273	3.531	0	***4.82***	3.5365	***4.82***
Pilgrim	***6.78***	***6.78***	5.4006	***6.78***	***6.78***	***6.78***
Canal	0	0.2049	0	***10.72***	0.6325	***10.72***
Vermont	5.3905	***5.63***	3.9296	***5.63***	***5.63***	***5.63***
Maine	8.0723	***8.63***	6.6836	***8.63***	***8.63***	***8.63***
NY&West	***1.1***	***1.1***	***1.1***	***1.1***	***1.1***	***1.1***
Total	**51.6801**	**50.5964**	**32.5401**	**65.19**	**51.0497**	***65.19***

Table 2.3 Optimal Power Injected at the Demand Side

	Scenario 1 (Fixed Demand)	Scenario 2 (Nominally Elastic)	Scenario 3 (Highly Elastic)	Scenario 4 (Lowly Elastic)	Scenario 5 (Minimum Demand)
Bus1	0	0	0	0	-2.307
Bus2	0	-3.6482	-3.0557	-5.2154	-2.127
Bus3	-3.22	-2.2337	-1.6822	-2.842	-1.7313
Bus4	-5	-0.9589	-0.396	-0.8083	-1.5532
Bus5	0	-0.0983	0	0	-1.5242
Bus6	0	-0.6961	-0.1406	-0.2817	-1.5275
Bus7	-2.338	-2.6189	-2.161	-2.8397	-1.5171
Bus8	-5.22	-3.6517	-3.2299	-4.2608	-1.5552
Bus9	0	-1.2609	-0.2594	-0.5168	-2.0224
Bus10	0	-1.0685	-0.5826	-1.0615	-1.663
Bus11	0	-0.9196	-0.4207	-0.7575	-1.5773
Bus12	-0.085	-0.9412	-0.4622	-0.7908	-1.5109
Bus13	0	-1.0525	-0.55	-0.9871	-1.5892
Bus14	0	-1.0834	-0.5412	-1.0197	-1.5497
Bus15	-3.2	-1.5205	-0.8861	-1.7584	-1.6493
Bus16	-3.294	-1.7841	-1.0636	-2.2462	-1.7801
Bus17	0	-1.7594	-1.167	-2.2626	-1.6198
Bus18	-1.58	-1.8914	-1.3295	-2.3623	-1.6261
Bus19	0	-2.2906	-1.2542	-3.3789	-2.2814
Bus20	-6.8	-2.3532	-1.21	-3.6371	-2.337
Bus21	-2.74	-1.8689	-1.0864	-2.378	-1.8634
Bus22	0	-2.0359	-1.1581	-2.7286	-2.0271
Bus23	-2.475	-2.1116	-1.2567	-2.8498	-2.1007
Bus24	-3.086	-1.7948	-1.069	-2.2449	-1.791
Bus25	-2.24	-1.6598	-1.37	-5.4541	-1.2038
Bus26	-1.39	-1.5388	-1.1659	-3.0142	-1.4035
Bus27	-2.81	-1.5892	-1.1293	-2.482	-1.4438
Bus28	-2.06	-2.3813	-1.5434	-2.764	-1.6187
Bus29	-2.835	-2.8389	-1.7885	-3.143	-1.817
Total	**-50.373**	**-49.6503**	**-31.9592**	**-64.0854**	**-50.3177**

The total power generation and demand can be visualized in Figure 2.4, which provided the information that the higher the demand elasticity, the lower the total power traded.

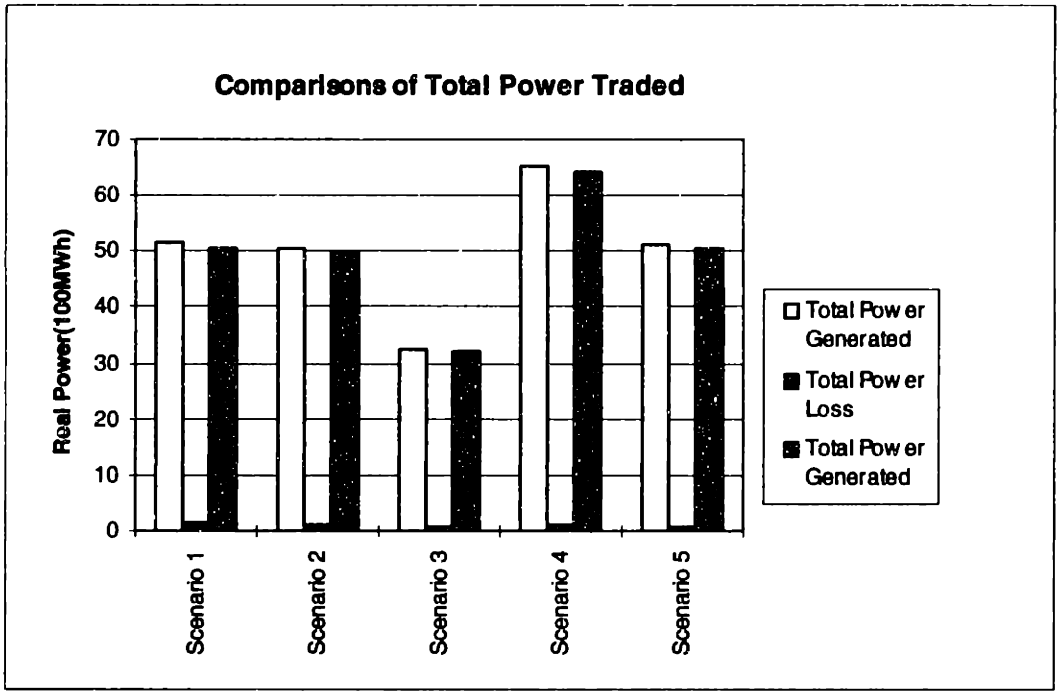

Figure 2.4 Comparisons of power generation, losses and demand under different scenarios

An interesting observation can be made from the transmission losses information as shown in Figure 2.5. Given more price-sensitive assumption, the total power transmission losses are all lower than that of fixed demand situation. A look at Figure 2.6, where the total demand is divided by total power generated, further confirmed the fact that with centralized OPF, a price sensitive demand situation actually results in better technical efficiency (in term of real power used) than that in price-insensitive situation. Again, it should be noted that this observation is hard to generalize for any operating situations.

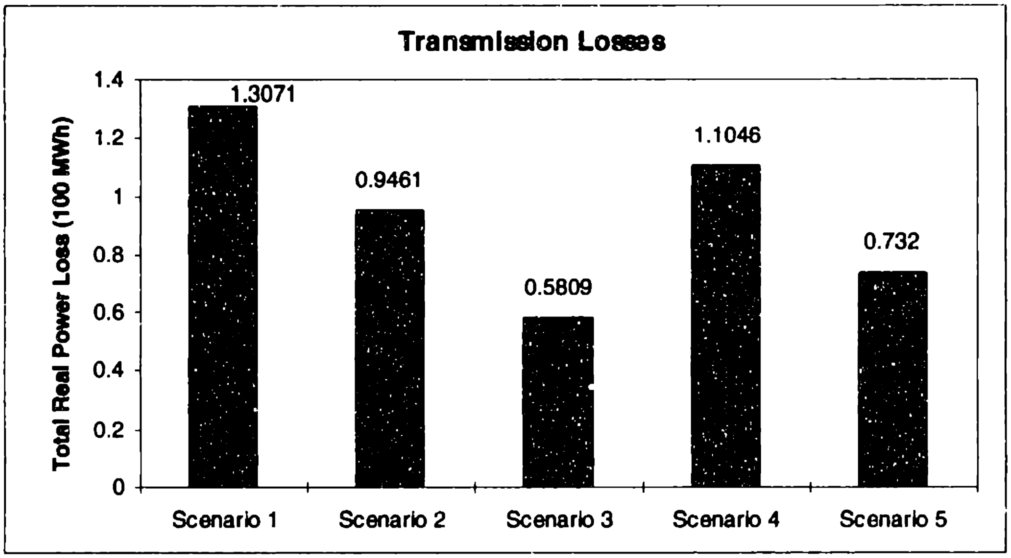

Figure 2.5 Total Real Power Losses Under Different Scenarios

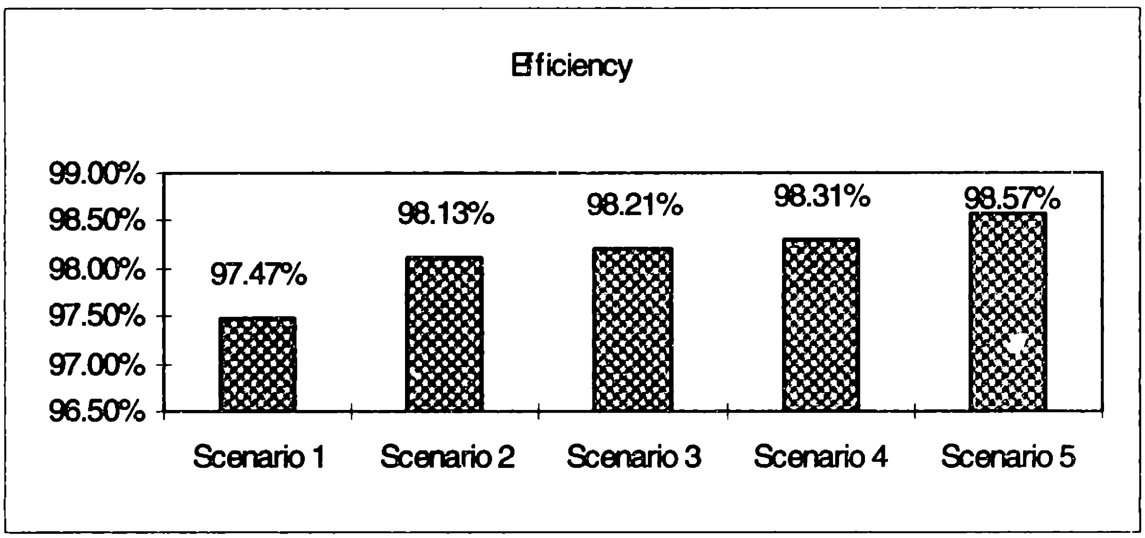

Figure 2.6 Efficiency of power used at the demand side

Economically, price-sensitive demand also resulted in lower "negative social welfare". In other words, total social welfare is higher in price sensitive situation. This claim is supported by the result shown in Table 2.4 and Table 2.5. By comparing the result shown on the two tables, it can be shown that under all demand elastic scenarios, the demand-fixed assumption will have lower social welfare.

Table 2.4 Relative Negative Social Welfare for Price-elastic Demand

	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Relative Negative Social Welfare	1009.10	1695.30	-1016.70	983.76

Table 2.5 Comparison on Economic Efficiency

Systems Cost for Inelastic Demand: *14365.00*

	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Relative Negative Social Welfare for Inelastic Demand:	12832.156	13598.5782	11299.313	12832.16

Finally, the evaluation of the relationship of price-elasticity of demand versus simulation time is compared in Table 2.5. The result is obtained by running the OPF program on a Pentium(90Mhz) PC with 8MB RAM. As indicated in the figure, no relationship can be drawn between the time for optimization and price-elasticity.

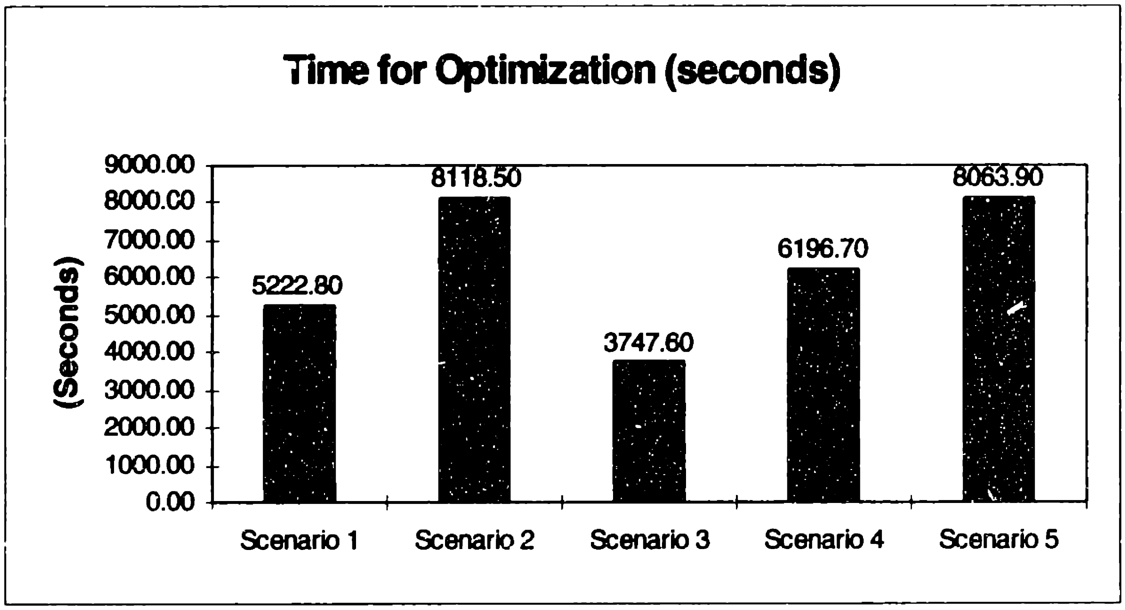

Figure 2.7 Simulation Time for Different Scenarios

In conclusion, the simulation results shown above demonstrate, first of all, the applicability of the MATLAB optimization toolbox solving OPF for systems up to 39 buses. Second, they suggest that price-elastic demand with a centralized OPF optimization will lead to both higher economical and technical efficiencies. The economical efficiencies are not surprising. More research effort is needed to investigate if technical benefits seen in these specific scenarios can be generalized.

Chapter 3

Iterative Pricing Scheme

The OPF with price-elastic demand operation described in Chapter 2 is necessary for evaluating any kind of proposals relevant to distributed decision making on the power systems. This chapter demonstrates how the program can be used in evaluating the results from recently proposed iterative pricing scheme. The concept of cost of ancillary services is introduced, followed by a description of the iterative pricing mechanism proposed in [4], [5], [12], and [13]. Then, the simulation of iterative pricing is presented, and result compared to the solutions obtained from centralized Optimal Power Flow (OPF) method introduced in Chapter 2.

3.1 The Concept of Cost Of Ancillary Services

In determining the operation cost of keeping the system together, consider the ideally achievable technical efficiency. Assuming that there is no real power loss, and that there is no voltage and real power constraint, then the optimization process can be mathematically formulated as follows:

Minimize

$$Total\ Generation\ Cost = \sum_{i \in N_G} C_{Gi}(P_{Gi}) \qquad (3.1)$$

where N_G = number of generation nodes

subject to the following constraints:

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} \qquad (3.2)$$

$$\sum_{i \in N_G} P_{Gi} = \sum_{i \in N_D} P_{Di} \qquad (3.3)$$

By using Lagrangian function with Khun-Tucker optimization method [9] , the necessary condition for an optimal solution of the above formulation is known to be [10] :

$$\frac{\partial C_{G1}}{\partial P_{G_1}} = \frac{\partial C_{G2}}{\partial P_{G2}} = \ldots = \frac{\partial C_{Gn}}{\partial P_{Gn}} = \lambda^* \qquad (3.4)$$

where λ^* is known as the short run marginal cost (SRMC). From this condition, the optimal solution will be $\underline{P}_G^{ideal^*} = (P_{G1}^{ideal^*}, P_{G2}^{ideal^*}, P_{G3}^{ideal^*}, \ldots, P_{GN}^{ideal^*})$. Hence, the ideally achievable optimal cost of specific generating unit will be denoted as

$$C_{Gi}^{ideal^*} = C_{Gi}(\underline{P}_G^{ideal^*}) \qquad (3.5)$$

in which the subscript i denotes the i-th generation facility.

3.1.1 Cost for Transmission Loss Compensation

By modifying the second constraint of equations (3.1), (3.2) and (3.3) to

$$\sum_{i \in N_G} P_{Gi} - P_{loss} = \sum_{i \in N_D} P_{Di} \tag{3.6}$$

where P_{loss} denotes the system-wide transmission power loss. One control analyze the cost of transmission loss while facilitating desired supply/demand profile. The transmission loss is referred to as anscillary service by FERC. The optimality in this case is defined as

$$\frac{\frac{\partial C_{G1}}{\partial P_{G1}}}{1-\frac{\partial P_{loss}}{\partial P_{G1}}} = \frac{\frac{\partial C_{G2}}{\partial P_{G2}}}{1-\frac{\partial P_{loss}}{\partial P_{G2}}} = \ldots = \frac{\frac{\partial C_{GN}}{\partial P_{GN}}}{1-\frac{\partial P_{loss}}{\partial P_{GN}}} = \lambda_i^{loss\,*} \tag{3.7}$$

where $\lambda_i^{loss\,*}$ is the short run marginal cost (SRMC) for generator i when P_{loss} is taken into consideration. The above optimality condition will yield the optimal solution as $\underline{P}_G^{loss*}$ = ($P_{G1}^{loss*}, P_{G2}^{loss*}$, ... , P_{GN}^{loss*}). Therefore, the values for the optimal cost $C_{Gi}^{loss*} = C_{Gi}(\underline{P}_G^{loss*})$ at each generation facility when system-wide transmission loss is taken into account. The cost incurred for loss compensation will be

$$\Delta C_{Gi}^{loss*} = C_{Gi}(\underline{P}_G^{loss*}) - C_{Gi}(\underline{P}_G^{ideal*}) \tag{3.8}$$

Note that the values of ΔC_{Gi}^{loss*} are different at different buses.

3.1.2 Generation Cost of Eliminating Power Transmission Constraint

Now if the constraint of equations (3.1), (3.2) and (3.3) were added by the real power transmission constraint (i.e. the real power congestion constraint):

$$P_{ij} \le P_{ij}^{\max} \qquad (3.9)$$

while all other constraint remain the same, and again, by constructing Lagrangian function and Khun-Tucker optimization method, the optimal solution will become $\underline{P}_G^{cong*} = (P_{G_1}^{cong*}, P_{G_2}^{cong*}, \ldots, P_{G_N}^{cong*})$, yielding the optimal cost $C_{Gi}^{cong*} = C_{Gi}(\underline{P}_G^{cong*})$ at each generation facility when power congestion constraint is take into consideration. The generation-based cost at each generation unit for eliminating transmission line congestion will be

$$\Delta C_{Gi}^{cong*} = C_{Gi}(\underline{P}_G^{cong*}) - C_{Gi}(\underline{P}_G^{ideal*}) \qquad (3.10)$$

Note that the values of ΔC_{Gi}^{cong*} are also non-uniform at different buses.

From the discussions in section 3.1.1, Error! Reference source not found., and, and equations (3.8), (3.9) and (3.10) it can be concluded that the cost at each individual generating unit for keeping the system together will be

$$C_{Gi}^{together*} = \Delta C_{Gi}^{loss*} + \Delta C_{Gi}^{cong*} \qquad (3.11)$$

which is also known as the cost for main ancillary services. As noted in [4], in the centralized controlled electric power industry, there is no strong distinction between "ancillary ser···es" and the rest of the generation. But under the deregulation and transmission grid open access requirement, the electric power industry has to shift from a vertically integrated industry structure to a competitive one, and the unbundling between transmission and generation of electric power makes the definition of ancillary service necessary. The ancillary services are vital for keeping the systems together, and hence a pricing scheme, which proposese a method of how to provide cost information of these services to the supply/demand market players, is described next.

3.2 Iterative Pricing for Transmission Losses and Line Congestion

In order to allow for the presence of competitive supply/demand activity, while at the same time have coordinated management and pricing of ancillary generation, an iterative pricing mechanism was proposed in [4]. Briefly, the pricing mechanism is based on two-level system hierarchy composed by primary and secondary levels as shown in Figure .13.1:

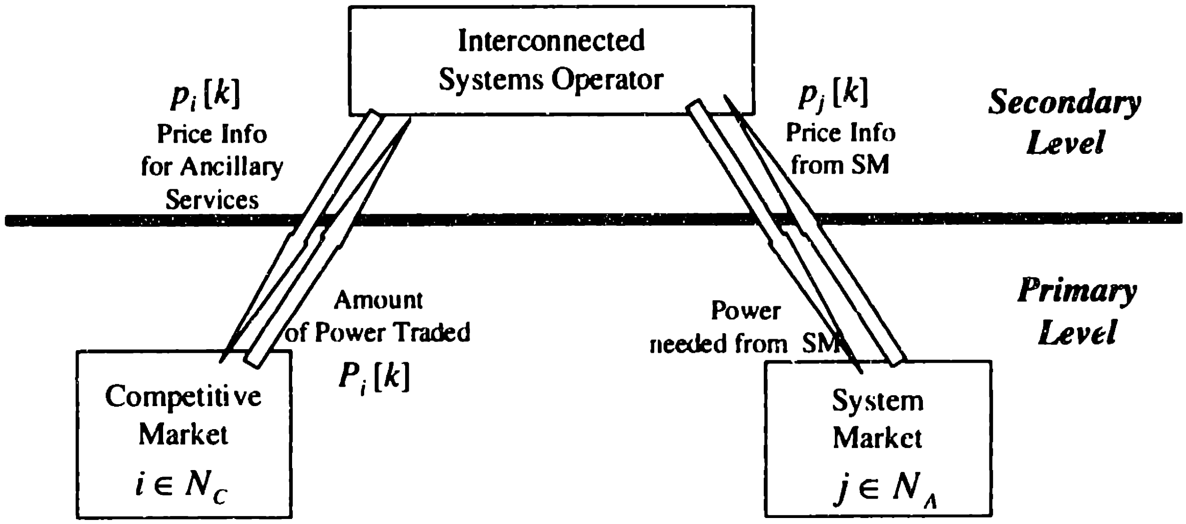

Figure .13.1 The two-level hierarchy for iterative pricing

In the two-level hierarchy, the competitive market player at each k-th iterative step provide to Interconnected Systems Operator (ISO) the information on the amount of power $P_i\,[k]$ (where $i \in N_C$ and N_C is the number of participants in the competitive market) traded. Next, all market participants are given the price signal $p_i\,[k]$(where $i \in N_C$) for ancillary generation under the market activities according to equation (3.11)

, which is then fully paid back to the ancillary generators in the systems market (SM) through $p_j[k]$ (where $j \in N_A$ and N_A is the number of ancillary generators). The individual competitive market participants then, with the knowledge of their own cost and benefit for the given power quantity, use the price signal from above to adjust their power quantities to $P_i\,[k+1]$ (where $i \in N_C$). Depending on the sign of gradient of their local profit function with respect to the power quantity, the either increase or decrease the power quantity they are trading. The aforementioned process continues until no further changes in power quantity treaded are seen.

Non-discriminatory cost allocation for ancillary generation is essential in the iterative pricing mechanism. According to [12], with a known set of simultaneous price-driven power transactions at the primary level of the competitive market, a total out-of-merit cost of $C_1^{together}$, $C_2^{together}$, ..., $C_{N_A}^{together}$ is incurred. Typical linearized approach, such as the MW-mile method [13], allocates this cost to individual members of the price-driven market according to $\frac{\partial\ C_i^{together}}{\partial\ P_j} \Delta P_i$ so that

$$\sum_{j \in N_C} \left(\frac{\partial\ C_i^{together}}{\partial\ P_j} \Delta P_j \right) = (1+r)\, C_i^{together} = p_i^{together} \qquad \text{for all } i \in N_A \qquad (3.12)$$

The actual cost of serving location j from the ancillary location i is then determined as $\frac{\partial\ C_i^{together}}{\partial\ P_j} \Delta P_j$ (for $j \in N_D$). Coefficients $\frac{\partial\ C_i^{together}}{\partial\ P_j}$ are computed using distribution factors-like formula at the optimum solution which determines ancillary generation necessary to balance the profit driven power mismatch.

Formulae aforementioned are easy to produce since most of the control centers have a version of distribution factor algorithm. They are non-discriminatory since they reflect the sensitivities of the ancillary generator supporting competitive injection at each bus.

The following section demonstrates the iterative pricing mechanism on a three buses example, and presents the role of a coordinated OPF program in evaluating the

3.3 Simulations on a Three Buses Example

3.3.1 Description Of The Simulation Input Data

The iterative pricing simulation was performed given a three bus example as shown in

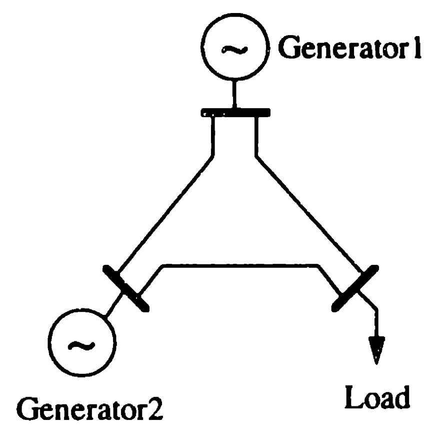

Figure 3.2 Topology of a Three-buses Power System

Figure 3.2, the line data and node data shown in Appendix F, and the generator cost curves as

$$C_1 = \cos t(P_{G_1}) = P_{G_1}^{\ 2} + P_{G_1} + 0.5 \qquad \text{for Generator 1} \tag{3.13}$$

$$C_2 = \cos t(P_{G_2}) = 2P_{G_2}^{\ 2} + 0.5P_{G_2} + 1 \qquad \text{for Generator 2} \tag{3.14}$$

and the utility curve at the load as

$$u(P_L) = 34.1666P_L - P_L^{\ 2} \tag{3.15}$$

.

Generator will act as the system market participant (SMP) and generate the power needed to balance the system in response to power injected at Generator 1 and elastic demand at the load (Bus 3). Both Bus 3 and Generator 2 were competitive market participants (CMP).

From equation (3.12) the nodal price for the three bus example, with only transmission loss consideration, were calculated according to the following formulae:

$$p_{L-(k+1)} = p_{L-k} + \left(\frac{\partial C_2}{\partial P_L} - p_{L-k} \right) \times \frac{f_L P_{Loss}}{P_L} \quad \text{where} \quad f_L = \frac{P_L}{2P_L} \qquad (3.16)$$

$$p_{G_1-(k+1)} = p_{G_1-k} + \left(\frac{\partial C_2}{\partial P_{G_1}} - p_{G_1-k} \right) \times \frac{f_{G_1} P_{Loss}}{P_{G_1}} \quad \text{where} \quad f_{G_1} = \frac{P_{G_1}}{2P_L} \qquad (3.17)$$

Similarly, for transmission line congestion instead of losses being considered, the nodal prices are calculated using the formulae:

$$p_{L-(k+1)} = p_{L-k} + \left(\frac{\partial C_2}{\partial P_L} - p_{L-k} \right) \times \frac{f_L (P_{ij} - \overline{P}_{ij})}{P_L} \qquad (3.18)$$

$$p_{G_1-(k+1)} = p_{G_1-k} + \left(\frac{\partial C_2}{\partial P_{G_1}} - p_{G_1-k} \right) \times \frac{f_{G_1} (P_{ij} - \overline{P}_{ij})}{P_{G_1}} \qquad (3.19)$$

where $\overline{P}_{ij}$ is the transmission congestion limit from bus i to bus j .

with the cost curve C_2 adjusted as

$$C_2 = \text{cost}\left(P_{G_2}\right) + \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \left(P_{ij} - \overline{P}_{ij}\right)^2 \qquad (3.20)$$

After receiving the new price information, the market players adjust their power levels accordingly, the slack bus computes a new generation level and new price signals, and the process iterates until convergence.

3.3.2 Simulation Results

Iterative Pricing simulations[*] were performed for two different conditions: (1) transmission line losses are considered while congestion not considered; (2) transmission congestion considered while transmission losses were ignored. With the program shown in Appendix F , the result for the (1) situation is shown in Figure 3.3 and Figure 3.4:

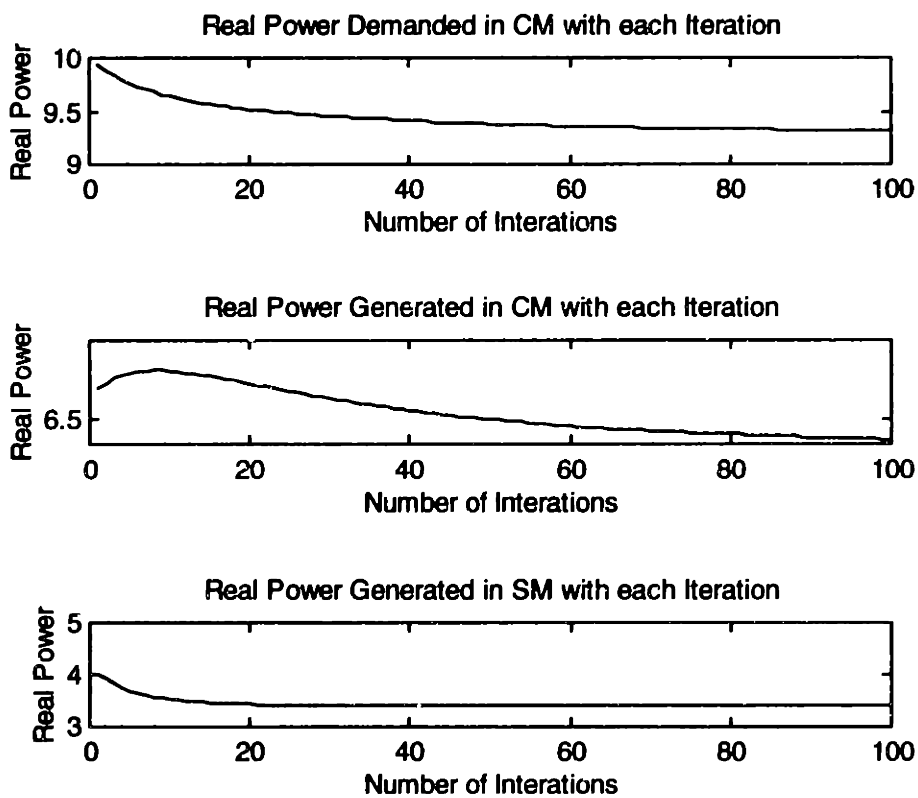

Figure 3.3 Iteration profile of power demanded/generated with transmission losses

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[*] *The code for iterative pricing for a 3-buses system was originally written by Eric Allen, who is at present a PhD candidate of the Department of Electrical Engineering and Computer Science.*

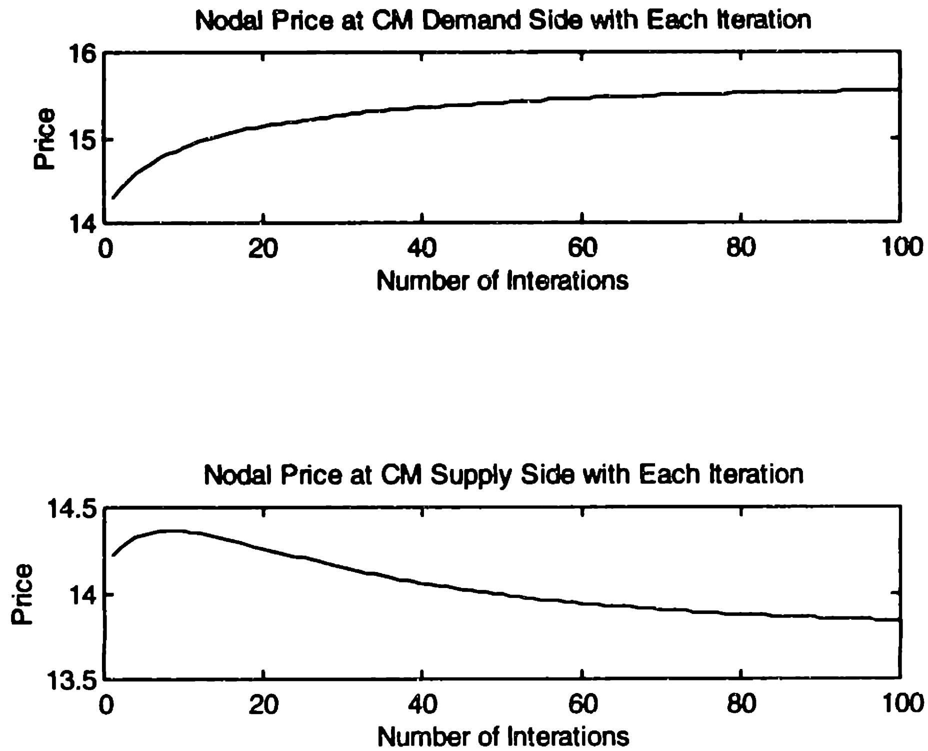

Figure 3.4 Iteration profile for nodal prices of the CMPs with transmission losses

With the program shown in Appendix G , the result for the (2) situation is shown in Figure 3.5 and Figure 3.6 :

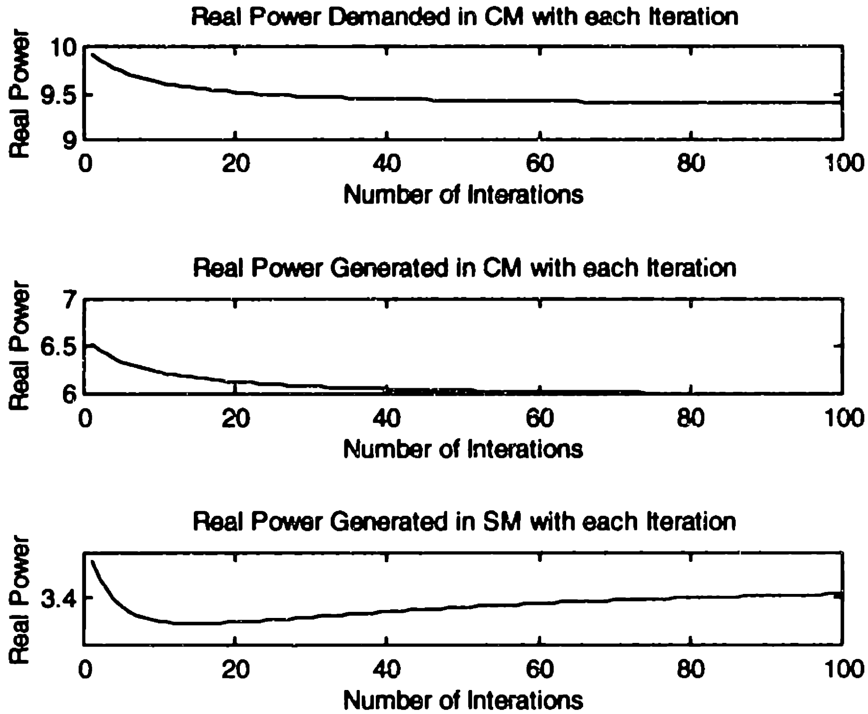

Figure 3.5 Iteration Profile of Power Demanded/Generated with Line Congestion

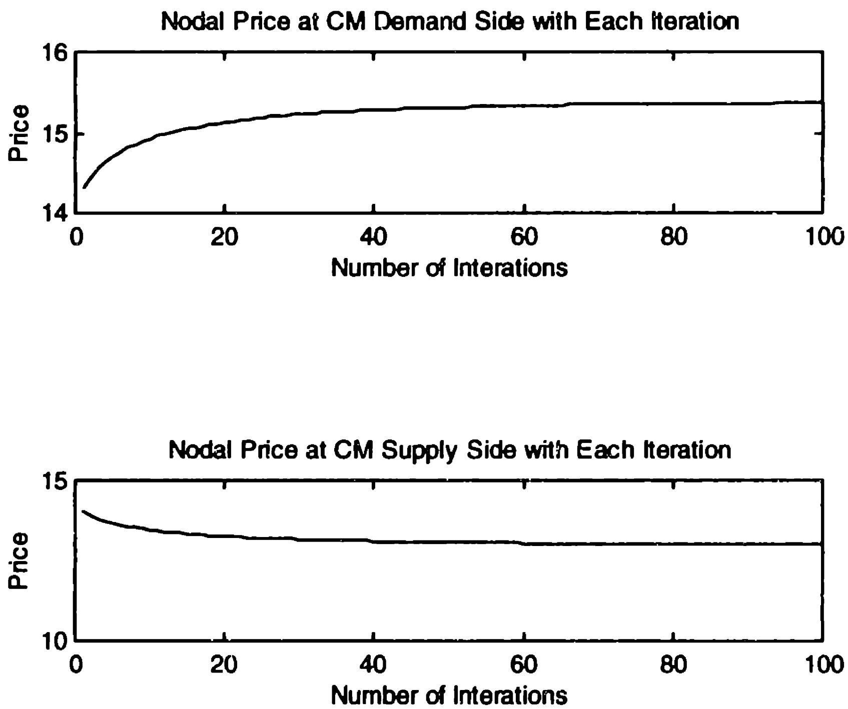

Figure 3.6 Iteration Profile for Nodal Prices of the CMPs with Line Congestion

The result of iterative pricing for transmission losses is $P_L = 9.3139$, $P_{G_1} = 6.4199$ and $P_{G_2} = 3.4027$. With minor modification of the OPF program the optimal result of a centralized optimization is also $P_L = 9.3139$, $P_{G_1} = 6.4199$ and $P_{G_2} = 3.4027$. Similarly, the iterative pricing simulation program yield a final converged result of $P_L = 9.4009$, $P_{G_1} =$ 5.9990 and $P_{G_2} = 3.4019$ which is the same as that from centralized OPF simulation. From the fact that the iteration process reach convergence fairly quickly, and that the result is the same as that obtained from centralized OPF, it can be concluded that the iterative pricing proposed in [4] and [5] will be a practical solution to help meet economic efficiency (in a competitive market) while at the same keep the system together.

Congruent to the above simulation on three buses example, one could also construct iterative pricing for 39 buses example, and use the optimal results in Chapter 2 for comparison purposes. However, the numerical algorithm for iterative pricing for a 39-buses power system requires further developemnt to become computationally efficient. This task was not attempted in this thesis. Nevertheless, the demonstration of the 3-buses example provided a numerical presentation of how the OPF program be used for examining the result of a distributed decision making process in a complex electric power system.

Chapter 4

Conclusion

It has been shown in Chapter 2 that the OPF is numerically stable, and in Chapter 3 that the OPF code implemented can be used for evaluating the result of the iterative pricing scheme. The OPF simulation results in Chapter 2 also demonstrate the outcome that with price elastic demand under centralized OPF optimization, both higher economical and technical efficiencies can be achieved as compared to that in fixed demand situation. It should be noted, however, that such an observation is hard to generalize. In Chapter 3, the iterative pricing yields the same optimal result as that from centralized OPF, and hence demonstrated the applicability of iterative pricing for a 3-bus system. Thus, the technical contribution of this is the provision of a computationally efficient MATLAB OPF program, while the primary policy contribution will be on the evaluation of the feasibility of an iterative pricing scheme discussed in Chapter 3.

Recommendation for Future Research

Iterative pricing simulation for the 39-buses example used in Chapter 2, will be helpful in further demonstrating the applicability of iterative pricing. Moreover, while hypothetical demand curve is used in Chapter 2 for simulation, further study on the power market consumer behavior will also be helpful in coming up with a more realistic demand curve. In chapter 2, the simulation result showed that, with compassion under different scenarios, both economical efficiency and technical efficiency are higher than price-inelastic demand situation. While higher economic efficiency is not surprising, the observation of higher technical efficiency with price-elastic demand is hard to generalize to any operating situations, and further effort on study how price-elastic demand affects technical efficiency.

Appendix A Bus Data for 39-buses Example

Bus Number	Bus Type *	Voltage (P.U.)	Voltage Angle	Load Real Power	Load Reactive Power	Generated Real Power	Generated Reactive Power	Desired voltage	Votalge Upper Limit	Voltage Lower Limit
0	2	1.0481	-9.43	0	0	0	0	0	0	0
1	2	1.0505	-6.88	0	0	0	0	0	0	0
2	2	1.0341	-9.73	322	2.4	0	0	0	0	0
3	2	1.0116	-10.53	500	184	0	0	0	0	0
4	2	1.0165	-9.38	0	0	0	0	0	0	0
5	2	1.0173	-8.68	0	0	0	0	0	0	0
6	2	1.0067	-10.84	233.8	84	0	0	0	0	0
7	2	1.0057	-11.34	522	176	0	0	0	0	0
8	2	1.0322	-11.15	0	0	0	0	0	0	0
9	2	1.0235	-6.31	0	0	0	0	0	0	0
10	2	1.0201	-7.12	0	0	0	0	0	0	0
11	2	1.0072	-7.13	8.5	88	0	0	0	0	0
12	2	1.0207	-7.02	0	0	0	0	0	0	0
13	2	1.0181	-8.66	0	0	0	0	0	0	0
14	2	1.0194	-9.06	320	153	0	0	0	0	0
15	2	1.0346	-7.66	329.4	32.3	0	0	0	0	0
16	2	1.0365	-8.65	0	0	0	0	0	0	0
17	2	1.0343	-9.49	158	30	0	0	0	0	0
18	2	1.0509	-3.04	0	0	0	0	0	0	0
20	2	0.9914	-4.45	680	103	0	0	0	0	0
21	2	1.0337	-5.26	274	115	0	0	0	0	0
22	2	1.0509	-0.82	0	0	0	0	0	0	0
23	2	1.0459	-1.02	247.5	84.6	0	0	0	0	0
24	2	1.0399	-7.54	308.6	-92.2	0	0	0	0	0

* 0 denotes P,Q load
1 denotes PV generation with P fixed
2 denotes PV generation with V fixed
3 denotes slack bus

Bus Data for 39-buses Example (*continued*)

Bus Number	Bus Type *	Voltage (P.U.)	Voltage Angle	Load Real Power	Load Reactive Power	Generated Real Power	Generated Reactive Power	Desired voltage	Votalge Upper Limit	Voltage Lower Limit
25	2	1.0587	-5.51	224	47.2	0	0	0	0	0
26	2	1.0536	-6.77	139	17	0	0	0	0	0
27	2	1.0399	-8.78	281	75.5	0	0	0	0	0
28	2	1.0509	-3.27	206	27.6	0	0	0	0	0
29	2	1.0505	-0.51	283.5	126.9	0	100	1.0505	9999	-9999
30	2	1.0475	-4.47	0	0	250	136.2	1.0475	9999	-9999
31	2	0.9831	1.63	0	0	650	175.9	0.9831	9999	-9999
32	2	0.9972	2.18	0	0	632	103.34	0.9972	9999	-9999
33	2	1.0123	0.74	0	0	508	164.39	1.0123	9999	-9999
34	2	1.0493	4.14	0	0	650	204.83	1.0493	9999	-9999
35	2	1.0635	6.83	0	0	560	96.88	1.0635	9999	-9999
36	2	1.0278	1.27	0	0	540	-4.43	1.0278	9999	-9999
37	2	1.0265	6.55	0	0	830	19.38	1.0265	9999	-9999
38	2	1.03	-10.96	0	0	1000	68.45	1.03	9999	-9999
0	3	0.982	0	9.2	4.6	572.86	170.34	0.982	9999	-9999

* 0 denotes P,Q load
1 denotes PV generation with P fixed
2 denotes PV generation with V fixed
3 denotes slack bus

Appendix B Line Data for 39-buses Example

From Bus#	To Bus#	Number of Circuits	Branch Type	R	X	B
1	2	1	0	0.0035	0.0411	0.6987
1	38	2	0	0.002	0.05	0.375
1	38	1	0	0.002	0.05	0.375
2	3	1	0	0.0013	0.0151	0.2572
2	25	1	0	0.007	0.0086	0.146
3	4	1	0	0.0013	0.0213	0.2214
3	18	1	0	0.0011	0.0133	0.2138
4	5	1	0	0.0008	0.0128	0.1342
4	14	1	0	0.0008	0.0129	0.1382
5	6	1	0	0.0002	0.0026	0.0434
5	8	1	0	0.0008	0.0112	0.1476
6	7	1	0	0.0006	0.0092	0.113
6	11	1	0	0.0007	0.0082	0.1389
7	8	1	0	0.0004	0.0046	0.078
8	9	1	0	0.0023	0.0363	0.3804
9	38	1	0	0.001	0.025	1.2
10	11	1	0	0.0004	0.0043	0.0729
10	13	1	0	0.0004	0.0043	0.0729
13	14	1	0	0.0009	0.0101	0.1723
14	15	1	0	0.0018	0.0217	0.366
15	16	1	0	0.0009	0.0094	0.171
16	17	1	0	0.0007	0.0089	0.1342
16	19	1	0	0.0016	0.0195	0.304

Line Data for 39-buses Example (*continued*)

From Bus#	To Bus#	Number of Circuits	Branch Type	R	X	B
16	21	1	0	0.0008	0.0135	0.2548
16	24	1	0	0.0003	0.0059	0.068
17	18	1	0	0.0007	0.0082	0.1319
17	27	1	0	0.0013	0.0173	0.3216
21	22	1	0	0.0008	0.014	0.2565
22	23	1	0	0.0006	0.0096	0.1846
23	24	1	0	0.0022	0.035	0.361
25	26	1	0	0.0032	0.0323	0.513
26	27	1	0	0.0014	0.0147	0.2396
26	28	1	0	0.0043	0.0474	0.7802
26	29	1	0	0.0057	0.0625	1.029
28	29	1	0	0.0014	0.0151	0.249
2	30	1	1	0	0.0181	0
6	0	2	1	0	0.05	0
6	0	1	1	0	0.05	0
10	31	1	1	0	0.02	0
12	11	1	1	0.0016	0.0435	0
12	13	1	1	0.0016	0.0435	0
19	20	1	1	0.0007	0.0138	0
19	32	1	1	0.0007	0.0142	0
20	33	1	1	0.0009	0.018	0
22	34	1	1	0	0.0143	0
23	35	1	1	0.0005	0.0272	0
25	36	1	1	0.0006	0.0232	0
29	37	1	1	0.0008	0.0156	0

Appendix C Code for OPF under the Condition that Demand is Fixed

```
function [optim_pg, f_series, p1_s, p2_s, p3_s, p4_s, elasp_time] =
jeffopf(dummy)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% optim_pg : optimal power generated
% t_cost   : total cost
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

ttt= clock;

%get all the parameters from data files
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%
[busno,gens,Pnet,Qnet,Imax,a,b,g,v,pick] = loadIEEE(1);

[lins,junk] = size(Imax);
[ca,cb,cc,tk,vlim,pglim,qglim] = loadopf(gens,lins,busno);

%defining up
%%%%%%%%%%%%
lds = busno - gens;
up = Pnet(lds+1:busno);

%define the control variable x
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
x = up;


%Using constr function
%%%%%%%%%%%%%%%%%%%%%%
x0 = x;
OPTIONS(1) = 1;
%OPTIONS(2) = 1;
%OPTIONS(3) = 1;
%OPTIONS(4) = 0.1;
OPTIONS(13) = 1;
OPTIONS(16) = 0.5;
xlb = pglim(:,1);
```

```
xub = pglim(:,2);
[optim_pg, dmm1, dm2, dm3, ...
f_series, p1_s, p2_s, p3_s, p4_s]...
= j_constr('opf_func', x0, OPTIONS, xlb, xub);

t_cost= sum( ca.*(optim_pg.^2) + cb .* optim_pg + cc );
elasp_time = etime(clock,ttt);
```

(Note: j_constr is an revised version of the function constr of the MATLAB's Optimization Toolbox, which is identical with constr in terms of functionality)

```
function [F,G] = opf_func(x)

%get coefficient ca, cab and cc
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[busno,gens,Pnet,Qnet,Imax,a,b,g,v,pick] = loadIEEE(1);

[lins,junk] = size(Imax);
[ca,cb,cc,tk,vlim,pglim,qglim] = loadopf(gens,lins,busno);
load nom_flw.dat;

%Define the goal function F
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%F = ( ...
%  sum( ca(1:29).*(x(1:29).^2) + cb(1:29).*x(1:29) + cc(1:29) ) + ...
%  sum( ca(30:39).*(x(30:39).^2)+cb(30:39).*(-1*x(30:39))+cc(30:39))...
%    )


% for fixed demand
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
F = sum( ca.* (x.^2) + cb.*x + cc ) ;


%set some part of Pnet to be x
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
lds = busno - gens;
Pnet(lds+1:busno) = x;


%get the power flow at each line
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[pflinear,d,v,pfm,qfm] = ldflwaux(busno,gens,Pnet,Qnet,Imax,a,b,g,v,pick);
```

```
%Calculating Total P Loss
%%%%%%%%%%%%%%%%%%%%%%%%%%%
t_p_loss = 0;
for i = 1: size(pfm)
  for j = 1: size(pfm)

        t_p_loss = t_p_loss + abs( pfm(i,j) + pfm(j,i) );

  end
end


%defining constraints
%%%%%%%%%%%%%%%%%%%%%%%%
G(1) = sum(x) - t_p_loss + sum(Pnet(1:lds));
[row_no, col_no] = size(pfm);
for ii = 1: row_no
        for jj = 1: col_no
          if (((ii-jj) < 5) &  ((ii-jj) > -5)) | ((ii-4*jj) > 29)
         G(ii+jj) = pfm(ii,jj) - 0.8*nom_flw(ii, jj);
          end
        end
end



function [pflinear,d,v,pfm,qfm] =
ldflwaux(busno,gens,Pnet,Qnet,Imax,a,b,g,v,pick)

% This load flow program loads data in IEEE common format and solves
the problem using
% newton-raphson complete solution

% Modified program (Eric H. Allen) for use with Optimal Power Flow

% THIS PROGRAM IS WRITTEN BY ASSEF ZOBIAN AT MIT,
COPYRIGHT 1991 - 1995
% YOU CAN USE IT ON THE CONDITIONS YOU ACKNOWLEDGE ITS
USE AND THE AUTHOR WORK
% AND NOT TO TAKE THIS NOTICE OUT
% [busno,gens,Pnet,Qnet,a,b,g,v,pick] = loadIEEE(a);
```

```
%Pnet= [Pload;Pgen];
inj=[Pnet(1:busno-1); Qnet(1:busno-gens)];
d=zeros(busno,1);
pfsolution = solv(a,b,g,d,v,inj,pick);
d(1:busno-1,1)=pfsolution(1:busno-1,1);
%d=d*180/pi
v(1:busno-gens,1)=pfsolution(busno:2*busno-gens-1,1);
% vl = v(1:busno-gens);
% calculate the jacobian at the solution
pflinear = pfjac(a,b,g,d,v);

loads= busno-gens;
dpda= pflinear(1:busno,1:busno);
Jll = dpda(1:loads, 1:loads);
Jlg = dpda(1:loads, loads+1:busno);
Jgl = dpda(loads+1:busno, 1:loads);
Jgg = dpda(loads+1:busno, loads+1:busno);
 % calculate the power flow in the lines
 [pfm, qfm] = linepf2(Imax,a,b,g,d,v,busno);


function [ca,cb,cc,tk,vlim,pglim,qglim]= loadopf(gens,lins,busno)

load datao.dat
mat = datao;

cf = mat(1,1:2).';

ca = cf(1)*mat(2:gens+1,1);
cb = cf(1)*mat(2:gens+1,2);
cc = cf(1)*mat(2:gens+1,3);

tk = mat(gens+2:gens+lins+1,1:2);
tk(:,2) = cf(2)*tk(:,2);

vlim = mat(gens+lins+2:gens+lins+busno+1,1:2);

pglim = mat(gens+lins+busno+2:2*gens+lins+busno+1,1:2);
qglim = mat(2*gens+lins+busno+2:3*gens+lins+busno+1,1:2);

function [busno,gens,Pnet,Qnet,Imax,a,b,g,v,pick]= loadIEEE(a)
% this program load the bus data in IEEE format with some modification
% generators should be last, NO text for bus names

load datan.dat          % change Sbase also.
```

```
nodes = datan;

[busno, colno] = size(nodes);

i=1;
for k= 1:busno,
  if (nodes(k,4) == 0)
    i= i+1;
  end
end
loads = i-1;

gens = busno-loads;

%Sbase
Sbase = 100;

Pl= nodes(1:busno,7)/Sbase;
Ql= nodes(1:busno,8)/Sbase;
Pg= nodes(1:busno,9)/Sbase;
% you can add shunts (in mvar) at  a given bus if you want
Qg= nodes(1:busno,10)/Sbase;

Pnet = Pg-Pl;
Qnet = Qg-Ql;

v= nodes(1:busno,5);
% take slack bus then ready for inj


% filename = input(' enter the file name', 's');

load datal.dat
lines = datal;

[lineno, colno] = size(lines);
R= lines(1:lineno,7);
X= lines(1:lineno,8);

i= sqrt(-1);
Z = R+ i*X;
Y= ones(lineno,1) ./ Z;
g= real(Y);
b= imag(Y);
% you need to add shunt admitances
```

```
a = zeros(busno,lineno);
i=0;
j=0;

for count=1:lineno,
 for count2= 1:busno,
      if lines(count,1) == nodes(count2,1),
       i= count2;
      end

      if lines(count,2) == nodes(count2,1),
       j= count2;
      end
 end

a(i,count)=  1;
a(j,count)= -1;
lmax(count,1) = i; lmax(count,2) = j;
end

temp =eye(2*busno);
pick =[temp(1:busno-1,:); temp(busno+1:2*busno-gens,:)];



function [pflow,qflow] = linepf(lmax,a,b,g,del,v,busno)

pflow = zeros(busno, busno);
qflow = zeros(busno, busno);

[lins,junk] = size(lmax);

for l = 1:lins,
  vi = v(lmax(l,1)); vj = v(lmax(l,2));
  di = del(lmax(l,1)); dj = del(lmax(l,2));
  G = g(l); B = b(l);
  pflow(lmax(l,1),lmax(l,2)) = ...
    vi^2*G - vi*vj*(G*cos(di - dj) + B*sin(di - dj));
  pflow(lmax(l,2),lmax(l,1)) = ...
    vj^2*G - vi*vj*(G*cos(di - dj) - B*sin(di - dj));
  qflow(lmax(l,1),lmax(l,2)) = ...
    -vi^2*B - vi*vj*(G*sin(di - dj) - B*cos(di - dj));
  qflow(lmax(l,2),lmax(l,1)) = ...
    -vj^2*B - vi*vj*(-G*sin(di - dj) - B*cos(di - dj));
```

```
end



function miss = mism(a,b,g,del,v,inj,pick)
miss = (pick*pf(a,b,g,del,v))-inj;



function flow = pf(a,b,g,del,v)

i=sqrt(-1);
V= v .* exp(i*del);
G= (a*diag(g)*a');
B= (a*diag(b)*a');
Y=  G + i* B;
Sflow = (diag(V))* conj(Y) * conj((V));
pn=real(Sflow);
qn=imag(Sflow);
flow=[pn;qn];
```

```
function jac = pfjac(a,b,g,del,v)
aa=abs(a);
vl=vline(a,v);
vldir=dvldv(a,v);
sig=a'*del;
lsin=sin(sig);
lcos=cos(sig);
dpda=(aa*diag(prod([g';vl';lsin']))-a*diag(prod([b';vl';lcos'])))*a';
dpdv=-
(aa*diag(prod([g';lcos']))+a*diag(prod([b';lsin'])))*vldir+diag(prod([2*v';(aa*
g)']));
dqdv=(aa*diag(prod([b';lcos']))-a*diag(prod([g';lsin'])))*vldir-
diag(prod([2*v';(aa*b)']));
dqda=(-a*diag(prod([g';vl';lcos']))-aa*diag(prod([b';vl';lsin'])))*a';

%dpda
%dpdv
%dqdv
%dqda'
jac=[dpda dpdv;dqda dqdv];



function xsol = solv(a,b,g,d0,v0,inj,pick)
del=d0;
v=v0;
nn=size(del);
xsol=pick*[d0;v0];
miss = mism(a,b,g,d0,v0,inj,pick);
while norm(miss) > 0.001
dx = - inv(pick*pfjac(a,b,g,del,v)*pick')*miss;

%%%%%%%%%%%%%%%%%%
%disp('**** dx')
%%%%%%%%%%%%%%%%%%

temp=pick'*dx;
del=del + temp(1:nn,1);
kk=nn+1;
ns=2*nn;
v = v + temp(kk:ns,1);
miss = mism(a,b,g,del,v,inj,pick);
end
xsol=pick*[del;v];
```

Appendix D Code for OPF under the Condition that Demand is Elastic

The program is similar to that in Appendix C, with slight modification to the function `opf_func` as shown below:

```
function [F,G] = opf_func(x)

%get coefficient ca, cab and cc
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[busno,gens,Pnet,Qnet,Imax,a,b,g,v,pick] = loadIEEE(1);
[lins,junk] = size(Imax);
[ca,cb,cc,tk,vlim,pglim,qglim] = loadopf(gens,lins,busno);
load nom_flw.dat;
%Define the goal function F
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
F = ( ...
   sum( ca(11:39).* (x(11:39).^2) + cb(11:39).*x(11:39) + cc(11:39) ) + ...
   sum( ca(1:10).* (x(1:10).^2) + cb(1:10).*(-1*x(1:10)) + cc(1:10) ) ...
   );

% for fixed demand
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%F = sum( ca.* (x.^2) + cb.*x + cc ) ;
%set some part of Pnet to be x
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
lds = busno - gens;
Pnet(lds+1:busno) = x;
%get the power flow at each line
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[pflinear,d,v,pfm,qfm] = ldflwaux(busno,gens,Pnet,Qnet,Imax,a,b,g,v,pick);
%Calculating Total P Loss
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
t_p_loss = 0;
for i = 1: size(pfm)
  for j = 1: size(pfm)
       t_p_loss = t_p_loss + abs( pfm(i,j) + pfm(j,i) );
  end
```

```
        end
        %defining constraints
        %%%%%%%%%%%%%%%%%%%%%%%
        G(1) = sum(x) - t_p_loss + sum(Pnet(1:lds));
        [row_no, col_no] = size(pfm);
        for ii = 1: row_no
                for jj = 1: col_no
                  if   (((ii-jj) < 5) &  ((ii-jj) > -5)) | ((ii-4*jj) > 27)
                  G(ii+jj) = pfm(ii,jj) - 0.8*nom_flw(ii, jj);
                  end
                end
end
```

Appendix E Input Data for 3 Buses Example

Line data for 3-bueses example

For congestion constraint only,

From Bus#	To Bus#	Number of Circuits	Branch Type	R	X	B
2	1	1	0	0	0.099	0
3	1	1	0	0	0.099	0
3	2	1	0	0	0.099	0

For power loss compensation situation,

From Bus#	To Bus#	Number of Circuits	Branch Type	R	X	B
2	1	1	0	0.01	0.099	0
3	1	1	0	0.01	0.099	0
3	2	1	0	0.01	0.099	0

Node data for 3-bueses example

Bus Number	Bus Type *	Voltage (P.U.)	Voltage Angle	Load Real Power	Load Reactive Power	Generated Real Power	Generated Reactive Power	Base KV
3	2	1	0	10	0	0	0	0
1	2	1	0	0	0	5	0	0
2	3	1	0	0	0	5.42344	0	0

Appendix F MATLAB Program for Iterative Pricing Method under Transmission Losses

```
function [pd, ps, pmat] = iprice(a)
% IPRICE -- Iterative Pricing Method for Transmission Losses, etc.
%
% Note:  Be sure load "generators" are first in circuit/opf files
%
% Eric H. Allen -- December 11, 1995
% Modified by Jeffrey Kuan for ploting functions
% This program is to be used for MIT research purpose ONLY;
% all other uses are forbidden.

epsilon = 1e-2; % tolerance
lds = 1; % no. of loads; NOT given in .dat files
tgens = 2; % no. of total generators; NOT given in .dat files
cgens = 1; % no. of competitive generators; NOT given in .dat files

[busno,gens,Pnet,Qnet,lmax,a,b,g,v,pick] = loadIEEE2(a);

uv = v(busno-gens+1:busno);
[lins,junk] = size(lmax);
[ca,cb,cc,tk,vlim,pglim,qglim] = loadopf(gens,lins,busno);

cpa = 1./(2*ca);
cpb = cb./(2*ca);

tda = sum(cpa(1:lds));
tdb = sum(cpb(1:lds));
tsa = sum(cpa(lds+1:lds+tgens));
```

```
tsb = sum(cpb(lds+1:lds+tgens));

pr = (tsb + tdb)/(tsa + tda)
pd = cpb(1:lds) - cpa(1:lds)*pr
ps = cpa(lds+1:lds+tgens)*pr - cpb(lds+1:lds+tgens)

priced = pr*ones(lds,1); prices = pr*ones(cgens,1); diff = epsilon + 1;
% cs1 = 1/tsa; cs2 = tsb/tsa;
% cd1 = -1/tda; cd2 = tdb/tda;
J = 0; pmat = [pd.' ps.' priced.' prices.'];

while ((diff > epsilon) & (J < 100)),
 Pnet(1:lds) = -pd; Pnet(lds+1:lds+tgens) = ps;
 % diary off
 [pflinear,d,v,pfm,qfm] = ldflwaux(busno,gens,Pnet,Qnet,lmax,a,b,g,v,pick);
 % diary
 pl = sum(pfm(:))

 up = [-pd; ps(1:tgens-1)];
 [sens, dd] = dcdu(up, uv);
 sens = -(2*ca(busno)*sum(pfm(busno,:)) + cb(busno))*dd;

 for l=1:lds,
%   pmin = pd(l) - pl/2*pd(l)/sum(pd);
  fd = pd(l)/(2*sum(pd));
  pricedn(l) = priced(l) + (sens(l) - priced(l))*fd*pl/pd(l);
 end
 for l = 1:cgens;
  fs = ps(l)/(2*sum(pd));
  pricesn(l) = prices(l) + (sens(l+lds) - prices(l))*fs*pl/ps(l);
 end
 pdn = cpb(1:lds) - cpa(1:lds).*pricedn(l);
```

```
  psn = cpa(lds+1:lds+cgens).*pricesn(I) - cpb(lds+1:lds+cgens);
  diff = abs(sum(pricesn - prices + psn - ps) + ...
          sum(pricedn - priced + pdn - pd));
  pd = pdn
  ps = [psn; sum(pfm(busno,:))]
  prices = pricesn;
  priced = pricedn;

  J = J+1; pmat = [pmat; pd.' ps.' priced.' prices.'];

  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
  pdemd(J) = pd;   %power demanded
  pcm(J) = ps(1); %power supplied by CM
  psm(J) = ps(2); %power supplied by SM
  d_price(J) = priced;
  s_price(J) = prices;
  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
end

num_iteration = 1:J;

subplot(5,1,1);
plot(num_iteration, pdemd,'w-');
xlabel('Number of Interations');
ylabel('Real Power');
title('Real Power Demanded in CM with each Iteration');


subplot(5,1,3);
plot(num_iteration, pcm,'w-');
xlabel('Number of Interations');
ylabel('Real Power');
```

```
title('Real Power Generated in CM with each Iteration');


subplot(5,1,5);
plot(num_iteration, psm,'w-');
xlabel('Number of Interations');
ylabel('Real Power');
title('Real Power Generated in SM with each Iteration');


figure(2);
subplot(3,1,1);
plot(num_iteration, d_price,'w-');
xlabel('Number of Interations');
ylabel('Price');
title('Nodal Price at CM Demand Side with Each Iteration');

subplot(3,1,3);
plot(num_iteration, s_price,'w-');
xlabel('Number of Interations');
ylabel('Price');
title('Nodal Price at CM Supply Side with Each Iteration');
```

Appendix G MATLAB Program for Iterative Pricing Method under Line Congestion

```
function [pd, ps, pmat] = iprice2(a)
% IPRICE2 -- Iterative Pricing Method for Congestion, etc.
%
% Note:  Be sure load "generators" are first in circuit/opf files
%
% Eric H. Allen -- December 11, 1995
% Modified by Jeffrey Kuan for ploting functions
% This program is to be used for MIT research purpose ONLY;
% all other uses are forbidden.

epsilon = 1e-2; % tolerance
lds = 1; % no. of loads; NOT given in .dat files
tgens = 2; % no. of total generators; NOT given in .dat files
cgens = 1; % no. of competitive generators; NOT given in .dat files

[busno,gens,Pnet,Qnet,lmax,a,b,g,v,pick] = loadIEEE2(a);

uv = v(busno-gens+1:busno);
[lins,junk] = size(lmax);
[ca,cb,cc,tk,vlim,pglim,qglim] = loadopf(gens,lins,busno);

cpa = 1./(2*ca);
cpb = cb./(2*ca);

tda = sum(cpa(1:lds));
tdb = sum(cpb(1:lds));
```

```
tsa = sum(cpa(lds+1:lds+tgens));
tsb = sum(cpb(lds+1:lds+tgens));

pr = (tsb + tdb)/(tsa + tda)
pd = cpb(1:lds) - cpa(1:lds)*pr
ps = cpa(lds+1:lds+tgens)*pr - cpb(lds+1:lds+tgens)

prices = pr; priced = pr; diff = epsilon + 1;
% cs1 = 1/tsa; cs2 = tsb/tsa;
% cd1 = -1/tda; cd2 = tdb/tda;
J = 0; pmat = [pd.' ps.' priced.' prices.'];

while ((diff > epsilon) & (J < 100)),
  Pnet(1:lds) = -pd; Pnet(lds+1:lds+tgens) = ps;
  % diary off
  [pflinear,d,v,pfm,qfm] = ldflwaux(busno,gens,Pnet,Qnet,lmax,a,b,g,v,pick);
  % diary
  pl = pfm(2,1) - tk(2,1)
%  pl = 0.4799

  up = [-pd; ps(1:tgens-1)];
  [sens, dd] = dcdu(up, uv);
  sens = -sens - (2*ca(busno)*sum(pfm(busno,:)) + cb(busno))*dd;

  for l=1:lds,
%    pmin = pd(l) - pl/2*pd(l)/sum(pd);
    fd = pd(l)/(2*sum(pd));
    pricedn(l) = priced(l) + (sens(l) - priced(l))*fd*pl/pd(l);
  end
  for l = 1:cgens;
    fs = ps(l)/(2*sum(pd));
    pricesn(l) = prices(l) + (sens(l+lds) - prices(l))*fs*pl/ps(l);
```

```
  end
  pdn = cpb(1:lds) - cpa(1:lds).*pricedn(I);
  psn = cpa(lds+1:lds+cgens).*pricesn(I) - cpb(lds+1:lds+cgens);
  diff = abs(sum(pricesn - prices + psn - ps) + ...
          sum(pricedn - priced + pdn - pd));
  pd = pdn
  ps = [psn; sum(pfm(busno,:))]
  prices = pricesn;
  priced = pricedn;
  J = J+1; pmat = [pmat; pd.' ps.' priced.' prices.'];

  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
  pdemd(J) = pd;   %power demanded
  pcm(J) = ps(1); %power supplied by CM
  psm(J) = ps(2); %power supplied by SM
  d_price(J) = priced;
  s_price(J) = prices;
  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


end

num_iteration = 1:J;

subplot(5,1,1);
plot(num_iteration, pdemd,'w-');
xlabel('Number of Interations');
ylabel('Real Power');
title('Real Power Demanded in CM with each Iteration');


subplot(5,1,3);
```

```
plot(num_iteration, pcm,'w-');
xlabel('Number of Interations');
ylabel('Real Power');
title('Real Power Generated in CM with each Iteration');


subplot(5,1,5);
plot(num_iteration, psm,'w-');
xlabel('Number of Interations');
ylabel('Real Power');
title('Real Power Generated in SM with each Iteration');


figure(2);
subplot(3,1,1);
plot(num_iteration, d_price,'w-');
xlabel('Number of Interations');
ylabel('Price');
title('Nodal Price at CM Demand Side with Each Iteration');

subplot(3,1,3);
plot(num_iteration, s_price,'w-');
xlabel('Number of Interations');
ylabel('Price');
title('Nodal Price at CM Supply Side with Each Iteration');
```

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