AN INVESTIGATION OF THE RELEVANT PARAMETERS
OF AN
EXPONENTIAL PILE

by
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ABSTRACT

Thermal neutron flux measurements made on a recently constructed graphite-moderated natural uranium-fueled subcritical pile have led to values of $52.6 \text{ cm.}$ for the diffusion length of thermal neutrons in graphite, and $1.19 \times 10^{-2} \text{ cm.}^{-2}$ for the material buckling constant of the lattice.

In addition to the two parameters mentioned in the first paragraph, three more were considered: the cadmium ratio, the Fermi age of neutrons in the lattice, and the infinite multiplication factor. Physical significance was ascribed to each of these, and methods for making experimental determinations of them were given.

Furthermore, techniques for making the necessary thermal neutron flux measurements and applying harmonic corrections to these measurements were presented, along with a detailed description of the pile from a structural standpoint.

Finally, the data obtained and the calculations used in computing the diffusion length and the material buckling will be found in Appendices A and B.
Since the production of the first divergent chain reaction by Enrico Fermi at the University of Chicago in late 1942, much interest has been connected with atomic energy and its applications for both wartime and peacetime uses. Within the past fifteen years, many experimental reactors and piles have been constructed, and not a few commercial installations have been contemplated or planned, and several have already been completed. Here at M.I.T., a heavy water-moderated research reactor is in the final stages of construction.

In addition to the MITR, the Nuclear Engineering Department of M.I.T. has also fostered the development and construction of a natural uranium graphite-moderated exponential pile as a research and instruction implement.

In October of 1956 the author undertook the project of constructing the pile and making an initial determination of some of its parameters as the basis for his S.B. Thesis in Physics. Due to contractor delays in preparing the pile site, however, the construction phase of the project initially slated for the middle of November or early December 1956 had to be postponed until March 28th of this year, on which date the first graphite bar was laid in place.
By the end of the last week in April, the last bar of some thirty - odd tons of graphite had been coaxed into position, and, in the time remaining, what measurements could be obtained were made.

The author would like to acknowledge his very deep gratitude to Professor T. J. Thompson of the Department of Nuclear Engineering of M.I.T. for making available the opportunity of working on this project, and for much advice and encouragement received during the course of the work.

Additional thanks are due the members of Professor Thompson's Nuclear Engineering Laboratory Course who gave willingly of their time to aid in the work of stacking the graphite. Without their much valued assistance, the task would have been an impossible one.

Cambridge, Mass. Richard W. Knapp
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1. Introduction

An atomic pile is essentially a structure consisting of a lattice of uranium metal and graphite. If large enough, it will be critical - i.e., capable of sustaining a chain reaction. It is possible, though, to construct a pile using the same basic lattice structure but with only a fraction of the critical size, in which case a chain reaction cannot be sustained. However, by the addition to the pile of a small source of neutrons (such as a Pu-Be combination), it will be possible to realize a steady state neutron flux distribution, and by investigating this distribution, one can obtain many of the parameters relevant to the design and operation of the critical pile. (The flux in the smaller pile is found to decrease in an approximately exponential manner as one moves away from the source; hence the term "exponential pile").

2. Physical Description of the Pile and Foundation

A photograph and a schematic drawing of the completed pile in its foundation may be found on the two pages immediately following Section 11.

The pile has been constructed in the form of a cube with sides and height of seven feet seven inches, and rests on a base or pedestal seven feet seven inches square by two feet thick. The pedestal is almost completely below floor level,
resting on the bottom of an especially constructed reinforced concrete pit.

Considerable difficulty was experienced in levelling the pit floor. This was finally accomplished by the use of commercial grinding equipment, and covering the floor surface with a double layer of rubber-faced felt following the grinding operation.

The pit proper extends for several feet beyond the north face of the pile so that access may be had to the source channels in the pedestal. This open portion of the pit is normally bridged over with removable iron plates so that one may walk directly up to the pile face for loading, etc. (see photograph).

Referring to the schematic, it will be seen that the pile and pedestal are constructed from crossed layers of graphite bars. Every other bar in the layers running north and south is pierced for its entire length with a 1.25" diameter fuel channel. Following measurements of the diffusion length (see Section 7), these fuel channels were loaded with eight inch long by one inch diameter aluminum-jacketed natural uranium fuel slugs, with ten slugs being inserted in each channel.

The central bars of each layer are removable, being machined several thousandths undersized for that purpose, and have small circular depressions milled into them at six inch inter-
vals for holding indium foils, which may thus be inserted into the pile for making flux measurements (see Section 3).

In the pedestal, all the bars of layer no. 3 are pierced for the insertion of plutonium-beryllium neutron sources in various configurations. These sources, emitting roughly $1.5 \times 10^6$ neutrons per second, are canned in tantalum cylinders about an inch in diameter by an inch and a quarter in length, and have thin cords attached to hooks in one end so that they may be retrieved, if desired, after once being pushed into the proper location with a metal tape measure.

3. Flux Measurements

Neutron fluxes at various positions within the pile may be measured by means of a rather indirect but standard method - that of irradiating small indium foils introduced into the pile by the method described in Section 2, and measuring the induced beta activity.

The saturation activity $A_\infty$ of the foil is calculated from

$$A_t = A_\infty e^{-\lambda t} - e^{-\lambda (t - T)} = \phi N \sigma_a (1 - e^{-\lambda T}) e^{-\lambda t}$$

where:
- $A_t$ = activity at time $t$ (cps) (measured)
- $\lambda$ = radioactive decay constant = $0.6931 / \text{half life}$
- $T$ = time of exposure (sec.)
- $t$ = time of delay before counting (sec.)
- $\sigma_a$ = microscopic abs. cross section = $191 \times 10^{-24} \text{cm}^2$
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\[ N = \text{atoms/cm}^3 \text{ of foil} = \frac{(\text{atoms/mole})(\text{moles/gm.})(\text{gm./cm}^3)}{114.8} \]

But the neutron flux \( \phi \) is given by \( \phi = \frac{A}{V N_{\text{f}} a} \), where \( V \) is the foil volume in cm\(^3\). In practice, the foils would be irradiated overnight to minimize the corrections.

However, it was decided to use boron trifluoride counters inserted into the pile via holes in spare fuel channel bars pushed into vacant foil stringer openings. This was considerably less time consuming than the foil irradiation method, and permitted more measurements to be made in the time available.

4. Material Buckling and the Thermal Neutron Flux Distribution

As has been previously stated, the addition to the pile of an extraneous source of neutrons will enable the pile to attain a steady state neutron flux distribution, since the difference between the neutron loss by leakage and absorption and the neutron production by fission will be made up by the source neutrons. The system is not critical, however, for the chain reaction is not self sustaining, and the flux density would decrease steadily after removal of the source.

In what follows, reference is made to Figure 1, page 5 for location of the source and coordinate axes. The \( x \) axis emerges from the north face of the pile, etc.

With the source present in the center of the pedestal and the fuel channels loaded with natural uranium fuel
Fig. 1: Location of Coordinate Axes
slugs, the thermal neutron flux distribution within the pile, at a distance from the boundaries, satisfies to a close approximation

\[ \nabla^2 \phi + B_m^2 \phi = 0 \]

where \( \phi \) is the thermal neutron flux, and \( B_m^2 \) is a quantity defined as the material buckling of the particular lattice of fuel and graphite moderator. We can obtain solutions to this equation by the method of assuming a product solution and separating the variables, subject to the boundary conditions that 1) the flux be everywhere finite and non-negative, 2) the flux vanish at the extrapolated boundaries of the pile*, and 3) the flux distribution must be symmetric in both \( x \) and \( y \) about the \( z \) axis, since the neutron source lies on the \( z \) axis.

*Note: The term "extrapolated boundaries" is used, since the neutron flux does not vanish at the physical boundaries due to the fact that the neutrons have a finite mean free path. However, for mathematical purposes, it is convenient to assume the existence of a boundary slightly outside of the physical boundaries of the system at which the neutron flux does go to zero. The so-called extrapolated boundary may be determined experimentally by irradiating indium foils along, say, a line parallel to the \( x \) axis of the pile and half way between the pedestal and the top of the structure, and plotting their activities as a function of \( x \) and thereby determining where the flux extrapolates to zero. Taking the distance so obtained beyond the edge of the pile, doubling it, and adding that quantity to the

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x and y measurements of the pile, one obtains, respectively, a and b, the lateral extrapolated dimensions of the assembly.

We write

\[ \varphi (x,y,z) = X(x)Y(y)Z(z) \]

and introduce this expression into (4.1). Upon separating the variables, the result is

\[ \frac{1}{X} \frac{d^2X}{dx^2} + \frac{1}{Y} \frac{d^2Y}{dy^2} + \frac{1}{Z} \frac{d^2Z}{dz^2} + B_m^2 = 0. \]

With the terms in x, y, and z set respectively equal to \(-\varphi^2\), \(-\beta^2\), and \(\gamma^2\), we have

\[ \alpha^2 + \beta^2 - \gamma^2 = B_m^2. \]

Depending, for example, on the sign of \(\alpha^2\), the solutions for \(X(x)\) could be of the form \(A \cos \alpha x + C \sin \alpha x\), or \(A \sinh \alpha x + C \cosh \alpha x\), where A and C are arbitrary constants. However, the flux symmetry condition (3) rules out the \(\sin\) and \(\sinh\) functions, while condition (2) requiring that the flux vanish at the extrapolated boundaries rules out \(\cosh \alpha x\) as an allowable solution. Hence

\[ X(x) = A \cos \alpha x \]

and it is seen that \(\alpha^2\) is a positive quantity. However, to meet the requirements of condition (2) more fully, it is necessary that \(\alpha^2 = (m\pi/a)^2, m = 1, 3, 5, \ldots\) in order that \(\varphi (x = a/2) = 0\).

The general solution for \(X(x)\) subject to boundary conditions (1), (2), and (3) is thus

\[ X_m(x) = A_m \cos \left( m\frac{\pi x}{a} \right). \]
By the identity of the requirements on the x and y solutions, it is also true \( p^2 \) is a positive number equal to 
\[ (n \pi/b)^2, \text{ with } n = 1, 3, 5, \ldots \] and
\[
y_n = C_n \cos (n \pi y/b)
\]

Turning to the solutions for \( Z(z) \), we must first determine the sign of \( y^2 \). To do this, it is merely necessary to note that, in the exponential experiment, the \( a \) and \( b \) extrapolated dimensions of the pile are a good deal less than in the critical structure, and therefore \( a^2 \) and \( p^2 \) are considerably larger than the corresponding components of the critical geometric buckling, given by
\[
B_m^2 = (\pi/a')^2 + (\pi/b')^2 + (\pi/c')^2,
\]
with \( a' \), \( b' \), and \( c' \) the dimensions of the critical pile. In fact, the \( a \) and \( b \) magnitudes will cause \( a^2 + p^2 \) to exceed \( B_m^2 \). But \( B_m^2 \) is just equal to \( B_g^2 \) for criticality. Hence, remembering (4.14), it is seen that \( y^2 \) must be a positive number.
Therefore, the solutions of
\[
\frac{1}{Z} \frac{d^2 Z(z)}{dz^2} = y^2 \text{ must be of the form of}
\]
\[
Z(z) = C_1 \cosh yz + C_2 \sinh yz
\]
But boundary condition (2) requires that \( Z(z) \) vanish when \( z = c \), and thus we must have that 
\[ 0 = C_1 \cosh yc + C_2 \sinh yc \]
and therefore 
\[ C_2 = -C_1 \cosh yc / \sinh yc \]. Placing this value of \( C_2 \) into (4.9), we have
\[
Z(z) = C_1 \cosh yz - C_1 (\cosh yc / \sinh yc) \sinh yz
\]
\[ = (C_1 / \sinh yz)(\sinh yc \cosh yz - \cosh yc \sinh yz) \]
\[ \phi = A_{mn} \cos (m \pi x/a) \cos (n \pi y/b) \sinh \gamma_{mn} (c-z) \]

where the constant \( \sinh \gamma_c \) has been included in the value of \( C_3 \). But we see from (4.4) that there will be a definite value of \( \gamma \) for each value of \( m \) and \( n \), which we shall term \( \gamma_{mn} \), defined by

\[ (m \pi/a)^2 + (n \pi/b)^2 - \gamma_{mn}^2 = E_m^2 \]

Thus, the general solution of (4.9) is finally

\[ Z(z) = C_{mn} \sinh \gamma_{mn} (c-z). \]

We can now write the solution of (4.1) as the product of \( Z(z) \), \( X(x) \), and \( Y(y) \). Upon combining all the constants into \( A_{mn} \), we have

\[ \phi = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \cos (m \pi x/a) \cos (n \pi y/b) \sinh \gamma_{mn} (c-z). \]

As can be seen from (4.13), each term in this series falls off approximately as \( e^{-\gamma_{mn} z} \). But it will be noted from (4.11) that \( \gamma_{mn} \) increases as the sum of the squares of \( m \) and \( n \). Hence the higher terms fall off more rapidly than the first, and the contribution of the higher harmonics to the flux may be neglected beyond about a hundred cm. from the source. Retaining only the mode for \( m = n = 1 \) leaves

\[ \phi = A_{11} \cos (\pi x/a) \cos (\pi y/b) \sinh \gamma_{11} (c-z). \]

Along the \( z \) axis, the two cosines assume the value of unity, and we can write

\[ \phi (z) = C \sinh \gamma_{11} (c-z) = C e^{-\gamma_{11}^2} \left[ 1 - e^{-2 \gamma_{11}^2 (c-z)} \right] \]

where we include the constant term \( e^{\gamma_c} \) in \( C \). At distances not too close to the top of the pile, i.e., \( c > z \), the bracketed...
term in (4.15) is approximately unity, and we have

\[ \phi = C e^{-\gamma_{11}^2} \]  

Thus, along the z axis, or for that matter any line parallel to the z axis and not too close to the vertical boundaries of the pile, the flux is seen to decrease in an approximately exponential fashion.

Now, if we make an experimental determination of the neutron fluxes along the z axis of the pile by the techniques of Section 3, we can plot a quantity proportional to \( \log \phi(z) \) vs. \( z \), and obtain a straight line of slope \( -\gamma \), as can be seen by taking logarithms of both sides of (4.16). However, there will be departures from the linearity of the plot when a) one is very near the base of the pile, due to contributions of the higher harmonics of the flux distribution, and b) one is very near the top of the pile, due to the neglecting of the bracketed quantity in (4.15), known as the end correction factor. If necessary, the extrapolated block height \( c \) can be obtained by a method similar to that used for determining \( a \) and \( b \), and the fluxes divided through by the end correction factor using a provisional value of \( \gamma \) as determined by applying (4.16). By making successive approximations in this manner, we finally obtain a sufficiently accurate value of \( \gamma \). Since this is actually \( \gamma_{11} \), and we have from (4.11) that

\[ B_m^2 = (\pi/a)^2 + (\pi/b)^2 - (\gamma_{11})^2 \]

remembering that \( a \) and \( b \) are the extrapolated dimensions of
the pile. We could now equate $B_m^2$ to $B_e^2$, whose value was given by (4.8), and thus obtain $a'$, $b'$, and $c'$, the dimensions of a critical pile having the same lattice structure as the exponential pile.

We obtained experimentally a $B_m^2$ of $1.19 \times 10^{-4}$ cm$^{-2}$ (see Appendix B), which would indicate that a critical structure with this lattice would have to be at least 16.4 feet on a side.

5. **Cadmium Ratio**

We have based our treatment so far on the assumption that the flux inside the pile was that of thermal neutrons - i.e., that the source neutrons had been completely thermalized in their passage through the upper half of the pedestal and into the base of the pile proper. Hence, it would be desirable to perform an experiment to verify our assumption. Such an experiment can be readily performed if we make use of cadmium shielded indium foils and the variation of the cadmium and indium cross sections with energy, as displayed in the graph found on the following page.

If we irradiate a cadmium covered indium foil at a given position on the z axis of the pile with the source in position, the cadmium shield (which does not itself become radioactive) will absorb all neutrons with energies in the region under the dotted cadmium cross section line, and essentially the only neutrons indicated by measurements of the saturation activity of the indium foil will be those at the indium resonance energy, i.e., 1.44 ev. If we now place a fresh foil (unshielded)
Cross sections of cadmium and indium as functions of neutron energy
in the same position, and irradiate it, we shall obtain the flux of all neutrons at and below the indium resonance energy. The difference of these two quantities gives then the flux only of thermal neutrons at the given position. It is a measure of the neutron density, since indium has a cross section which varies as 1/ν in the thermal region, where ν is the neutron velocity. The same pair of operations performed at another point along the axis of the pile would yield another value of the thermal neutron flux, larger or smaller than the previous value depending on whether the measurements were made nearer to or farther from the source, respectively.

However, instead of taking the differences of the shielded and unshielded activities at various points along the pile axis, if one takes the ratio of the unshielded to the shielded activities, one obtains the so-called cadmium ratio, which has the following desired significance: if the cadmium ratio decreases as our points are taken farther and farther away from the source, then we know that a portion of the neutrons present are still slowing down from production to thermal energies, since the fraction of higher energy neutrons is decreasing. If, however, the ratio is a constant and large as we move away from the source, it is apparent that essentially all the neutrons have slowed down to thermal energies, and are then diffusing through the lattice.

It is worthwhile noting that the fluxes determined with the cadmium shielding surrounding the foils are exactly the
ones required for determining the Fermi age of neutrons (see Section 10), which gives a measure of the distance travelled by neutrons in slowing down from their energies at birth to the indium resonance of 1.44 ev. Thus, one method for determining age is to plot unshielded indium foil activity minus shielded foil activity versus \( r^2 \) from the source and calculate the age from the slope of the plot and equation (10.1).

6. Diffusion Length in Graphite

We have considered the flux distribution of neutrons within the loaded pile, and thereby obtained the material buckling \( E_m^2 \). Next, a method was given for verifying that the flux at a distance from the source and boundaries was indeed that of thermal neutrons. It will now be of interest to discuss the behavior of these thermal neutrons as is demonstrated by a quantity proportional to the root mean square distance travelled by them in going from formation to capture. This quantity is known as the diffusion length, and is denoted by \( L \).

What we must essentially consider is the problem of obtaining a solution to the diffusion equation

\[
D \nabla^2 \phi - \Sigma_a \phi + S = 0
\]

where \( D \) is the diffusion coefficient, \( \Sigma_a \) is the macroscopic cross section, \( \phi \) is the flux of thermal neutrons, and \( S \) is the source term, which is zero everywhere except at the position of the source in the medium. If we set \( S = 0 \), then (6.1) becomes

2) Ibid., p. 106
(6.2) \[ D \nabla^2 \phi - \Sigma_a \phi = 0 = \nabla^2 \phi - \chi^2 \phi, \]
where \( \chi^2 = \frac{\Sigma_a}{D} \), which is valid everywhere except at the source. We note that
D has the dimension of a length, and \( \Sigma_a \) that of a reciprocal length, so that \( \chi^2 \) has the dimension of a reciprocal length squared, and it is the reciprocal of \( \chi \) which we associate with the diffusion length \( L \).

We can ascribe a physical significance to \( L \) by solving (6.2) in spherical coordinates subject to the boundary conditions that the flux is finite everywhere except at the source, and that the total number of neutrons passing through the surface of a sphere of radius \( r \) must equal the source strength as \( r \) tends to zero. The solution is
(6.3) \[ \phi = e^{-\chi t / 4\pi Dr}. \]
Introducing this value of \( \phi \) into the expression for the mean square distance travelled by a neutron from the source to the point of absorption,
(6.4) \[ \bar{r}^2 = \int_0^\infty r^2 (4\pi r^2 \Sigma_a) \phi dr / \int_0^\infty 4\pi r^2 \Sigma_a \phi dr, \]
we obtain \( \bar{r}^2 = 6/\chi^2 = 6L^2 \), or \( L^2 = \frac{r^2}{6} \), which tells us that the square of the diffusion length is the mean square distance (divided by six) travelled by a thermal neutron in going from its point of formation to the point at which it is captured by the medium.

3) Ibid., p. 107
3') Ibid., p. 115
7. Experimental Determination of the Diffusion Length

It will be possible to use the structure of the exponential pile to determine the diffusion length in graphite, provided the fuel channels are filled in with graphite in order to make the structure homogeneous.

Diffusion theory considerations lead to the conclusion that an investigation of the neutron flux distribution in the homogeneous structure will lead to the knowledge of \( \chi \), and hence to \( L \), by means of the relationship (see 6.2)

\[
\nabla^2 \phi - \chi^2 \phi = 0.
\]

Note the formal analogy between the flux distribution expressions leading to the values of \( \chi^2 \) and \( B_m^2 \), as evidenced by a comparison of (7.1) and (4.1). The use of this analogy and the identity of the boundary conditions in the two cases allow us to write directly

\[
\phi = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \cos \left( \frac{m \pi x}{a} \right) \cos \left( \frac{n \pi y}{b} \right) e^{-\gamma_{mn} z}.
\]

where

\[
(\gamma_{mn})^2 = \chi^2 + \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2.
\]

Similarly, contributions of higher harmonics beyond about two diffusion lengths from the source may be neglected, and thus \( \phi(z) \) and \( \chi^2 \) are given respectively by

\[
\phi(z) = \text{const} \ e^{-\gamma_{11} z}, \text{ and }
\]

\[
\chi^2 = \gamma_{11}^2 - \left( \frac{\pi}{a} \right)^2 - \left( \frac{\pi}{b} \right)^2.
\]
It should be observed in (7.5) that the terms \((\pi/a)^2\) and \((\pi/b)^2\) are really corrections to the value of \(\chi^2\) allowing for the leakage of neutrons through the vertical sides of the structure. This can be seen by allowing \(a\) and \(b\) to approach infinity, at which values there would be no leakage, and \((\pi/a)^2\) and \((\pi/b)^2\) would vanish. In this case, \(\gamma_{11}^2\) would equal \(\chi^2\) which in turn equals \(1/L^2\).

\(L\) is obtained experimentally as was \(B^2_m\) - i.e. by inserting \(BF_3\) counters into the pile at various levels along the \(z\) axis and plotting the logarithms of the measured count rates versus \(z\), and noting that the slope of the line so obtained will be \(-\gamma_{11}\), with the value of \(L\) following from (7.5) and (6.1).

We obtained an \(L\) of 52.6 cm. (See Appendix A.)

8. Harmonic Corrections

As was stated previously, beyond about two diffusion lengths from the source, the first harmonic was the only one making any appreciable contribution to the flux. However, in the determination of \(L\), it will be desireable to make measurements as close as one diffusion length from the source, in which case, the \(\phi_{13}\), \(\phi_{31}\), and \(\phi_{33}\) harmonics would make a considerable contribution to the neutron flux. Therefore, we may represent \(\phi(z)\) by calculating

\[
\phi(z) = \text{const} \, e^{-\gamma_{11}z} \left[ 1 + \gamma_{11} e^{11z} \left( e^{-\gamma_{13}z} + e^{-\gamma_{31}z} + e^{-\gamma_{33}z} \right) \right]
\]
from (7.2), and term the bracketed quantity the **harmonic correction factor**.

In practice, an approximate value of $X^2$ will be obtained from (7.5), and $\gamma_{31}, \gamma_{13},$ and $\gamma_{33}$ calculated from (7.3). The measured fluxes $\phi$ are then divided by the harmonic correction factor, and the logarithms of these corrected fluxes are again plotted as functions of $z$, giving a new value of $\gamma_{11}$, and thus of $L$. The process of making successive approximations may be continued until values of the desired accuracy are obtained.

9. **Fermi Age of Neutrons**

In our treatment of the diffusion length, we have assumed that the neutrons involved all possessed energies in the thermal region, i.e., about 0.025 ev. The source and fission neutrons, however, are fast, but slow down into the thermal range by means of collisions with the nuclei of the moderating material. It is important to know how far a neutron travels between the time of its formation and its reaching thermal energy, because this distance determines neutron leakage during the slowing down process, and it will be remembered that only neutrons which have slowed down to thermal energies are capable of causing the fission process in natural uranium.

We will now present a condensation of an approach to the problem due originally to Fermi.\(^4\) If we consider a neutron

produced by the fission process and having the energy $E_o$, it will move for a certain interval of time through the medium until it collides with a nucleus and loses energy to that nucleus. It will then continue on with a lower energy, and, consequently, a lower velocity, until it collides with another nucleus, and etc. The time between collisions will increase from collision to collision because of the decreasing velocities, however. Although this process is a stepwise one, and differs slightly for each neutron, we can replace the steps by a continuous curve which averages the behaviour of all the neutrons. This approximation is valid only for moderators with relatively high mass numbers, however, and would not apply to a moderator containing such low mass number elements as H or D, since the neutron could then conceivably lose all its energy in one collision.

The equation describing the behaviour of neutrons in the model was shown by Fermi to be

\[ \nabla^2 q = \frac{3q}{\tau}, \]

known as the Fermi age equation. $\tau$ is the Fermi age, and $q$ is the slowing down density - a function of the average logarithmic energy change per collision, the velocity of a neutron at a particular energy, the scattering mean free path, the lethargy of the neutron (defined as $\ln E_0/E$, where $E_0$ is the fission

5) Ibid., p. 176
energy, and \( E \) is the energy at which the lethargy is calculated, and the number of neutrons per \( \text{cm}^3 \) per unit lethargy.

The distance travelled from formation at fission energy to the point of thermalization is given by

\[
1 = \sqrt{\lambda}
\]

where \( 1 \) is the slowing down length.

The solution to (9.1) for the case of a point source of fast neutrons in a region of large extent is

\[
q(r, \tau) = e^{-r^2/4\lambda} \left(\frac{4\pi\lambda}{r}\right)^{3/2}
\]

where \( q \) is again the slowing down density, evaluated at a distance \( r \) from the source.

10. Experimental Determination of Age

We may again use the structure of the exponential pile with its channels filled in with graphite for the determination of the Fermi age, provided we insert a cadmium curtain between the pedestal and the pile by means of the shutter slots provided, and fill in the source channels in the proximity of the source in the pedestal with uranium slugs. The source neutrons will collide with the nuclei of the atoms in the uranium slugs producing neutrons at fission energy, and the cad-

6) Ibid., p. 180
mium curtain will serve to prevent any thermal neutrons produced in the pedestal from entering the pile. We may now irradiate cadmium-covered indium foils along the axis of the pile, and their saturation activities will then be proportional to the slowing down density $q$ for the indium resonance energy, which is 1.44 ev. The slowing down density distribution around a point source is given by (9.3), and when the logarithms of both sides of (9.3) are taken, we obtain that, for a given energy (in which case $\tau$ is constant)

$\ln q(r) = \text{const} - \frac{r^2}{4\tau}$.

Thus, if we plot the logarithms of the saturation activities of the indium foils, we obtain a straight line of slope $-1/4\tau$, from which the age from fission to indium resonance energy will be obtained. By extrapolation, it is then possible to calculate $\tau$ from fission to thermal energies.

11. Infinite Multiplication Factor

We have now arrived at methods for determining three of the parameters governing the operation of a critical pile by essentially measuring the flux distributions in various set-ups of the exponential pile. They are 1) the material buckling $B_m^2$, from which the critical size of a pile with the same basic lattice structure as the exponential pile could be determined by equating the critical geometric buckling $B_g^2$ to $B_m^2$; 2) the diffusion length in graphite, which gives a measure of the root mean square distance travelled by neutrons.
from the point of their formation to the point of capture; and 3) the Fermi age in graphite (for all practical purposes equivalent to the Fermi age in the fuel-moderator lattice), which measures the distance travelled by neutrons in going from fission to thermal energies through relation (9.2).

These three parameters may be related to one another by means of the introduction of a fourth - the infinite multiplication factor $k_\infty$, and this relation takes the form of the critical equation

$$ k_c (1 + \frac{E_m^2}{B_m^2 L^2 + \zeta^2}) = 1, $$

where $E_m^2$ is again the material buckling, $L^2$ is the square of the diffusion length in the moderator corrected for the effect of the fuel present in the lattice, and $\zeta$ is again the Fermi age of neutrons in the lattice.

---

8) Glasstone and Edlund, op. cit., p. 280
**MIT Graphite Exponential Pile**

- **4" Square Hole Down Thru Center**
- **14" Dia. Thru Hole**
- **7'-6"**
- **14" Dia. Holes**
- **10'-3½"**
- **7'-7"**
- **3'-0"**
- **2'-0"**
- **Shutter Slots**
- **Removable Foil Blocks**
- **Cadmium Shutter Slots**
- **Cover Plate**
- **Floor Level**

Typical Fuel Stringer
Appendix A: DATA ON MEASUREMENT OF THE DIFFUSION LENGTH

With a single plutonium-beryllium neutron source located half way from front to back of the pile in the central source channel, measurements proportional to the neutron flux $\phi$ were taken with a standard tubular BF$_3$ counter inserted at various levels of the pile into vacant fuel channels adjacent to the foil stringers in such a way as to bring the active portion of the counter as near as possible to the z-axis of the pile. The counter output was fed into an Atomic Instruments Corp. Model 255 preamplifier, and from there into an AIC Model 215 non-overloading amplifier, and finally into an AIC glow-tube scaler.

At each level, five one-minute counts were taken and the results averaged. From these measurements, the average of the background counts taken at various levels was subtracted to give corrected count rates proportional to the flux $\phi$. These results are presented in table A1, below.

Table A1 - Diffusion Length Measurements

<table>
<thead>
<tr>
<th>Level</th>
<th>$\phi$ (cpm)</th>
<th>Bckgrnd. (cpm)</th>
<th>$\phi$ - Bckgrnd.</th>
<th>cm. from source</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>114047</td>
<td>34</td>
<td>114013</td>
<td>57.2</td>
</tr>
<tr>
<td>4</td>
<td>25688</td>
<td>41</td>
<td>25647</td>
<td>75.5</td>
</tr>
<tr>
<td>6</td>
<td>15067</td>
<td>34</td>
<td>15027</td>
<td>94.0</td>
</tr>
<tr>
<td>8</td>
<td>3822</td>
<td>41</td>
<td>3782</td>
<td>112.5</td>
</tr>
<tr>
<td>10</td>
<td>5066</td>
<td>34</td>
<td>4724</td>
<td>121.0</td>
</tr>
<tr>
<td>12</td>
<td>3008</td>
<td>34</td>
<td>2964</td>
<td>149.2</td>
</tr>
<tr>
<td>14</td>
<td>1762</td>
<td>41</td>
<td>1721</td>
<td>167.8</td>
</tr>
<tr>
<td>16</td>
<td>1013</td>
<td>41</td>
<td>973</td>
<td>186.1</td>
</tr>
<tr>
<td>18</td>
<td>625</td>
<td>34</td>
<td>591</td>
<td>204.5</td>
</tr>
<tr>
<td>20</td>
<td>373</td>
<td>34</td>
<td>333</td>
<td>223.0</td>
</tr>
<tr>
<td>22</td>
<td>229</td>
<td>34</td>
<td>189</td>
<td>241.3</td>
</tr>
<tr>
<td>24</td>
<td>126</td>
<td>34</td>
<td>86</td>
<td>260.0</td>
</tr>
</tbody>
</table>
In Graph A1 on the following page the natural logarithms of const \( \phi (z) \) were plotted as a function of \( z \) - the resulting slope of the curve being a constant for measurements taken at a distance from the source and boundaries, as was anticipated. The slope of the linear portion of the curve was measured from the graph to be \(-0.0295 \text{ cm}^{-1}\), which is \( \chi_{11} \). The x and y dimensions of the pile, \( a \) and \( b \), are almost exactly 231 cm., and hence we have (see Section 7)

\[
(A.1) \quad \chi^2 = (\chi_{11})^2 - (\pi/a)^2 - (\pi/b)^2,
\]

which leads to a value of \( 5.00 \times 10^{-4} \text{ cm}^2 \).

This value must be corrected, however, for the contributions of the \( \phi_{13} \), \( \phi_{21} \), and \( \phi_{33} \) harmonics (see Section 8). In Table A2, below, will be found the values of \( \phi \) from Table A1, the level numbers at which the corrections were applied, the calculated correction factor at that level, and the corrected flux \( \phi_c \). (Note: \( \phi_c \) is obtained through dividing the \( \phi \)'s from Table A1 by the correction factor.)

<table>
<thead>
<tr>
<th>Level</th>
<th>( \phi ) (cpm) (from Table A1)</th>
<th>Correction Factor</th>
<th>( \phi_c ) (cpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>144,007</td>
<td>1.51</td>
<td>29,160</td>
</tr>
<tr>
<td>6</td>
<td>15,027</td>
<td>1.24</td>
<td>12,130</td>
</tr>
<tr>
<td>10</td>
<td>5,026</td>
<td>1.11</td>
<td>4,535</td>
</tr>
<tr>
<td>14</td>
<td>1,722</td>
<td>1.05</td>
<td>1,640</td>
</tr>
<tr>
<td>18</td>
<td>585</td>
<td>1.03</td>
<td>568</td>
</tr>
<tr>
<td>22</td>
<td>189</td>
<td>1.01</td>
<td>187</td>
</tr>
</tbody>
</table>

In Graph A2 on p. 27 the natural logarithms of \( \phi_c \) were
Graph A1: Measurement of Diffusion Length

\[
-\chi'' = \frac{9.21 - 4.605}{264 - 105} = 4.4 \\
= 0.0293 \text{ cm}^{-1}
\]
Graph A2: Measurement of Diffusion Length

\[ y'' = \frac{-2.30(4-x)}{183 - 100} \]

\[ = 0.377 \text{ cm}^{-1} \]
plotted as functions of $z$, the resulting slope being equal to $-0.0277 \text{ cm}^{-1}$, which is $-\frac{1}{11}$. Applying (A.1) again leads to a $\chi$ of $1.97 \times 10^{-2} \text{ cm}^{-1}$, which, it will be remembered, is the reciprocal of $L$. Hence we obtain an $L$ of 50.8 cm.

Two further corrections must be applied to this value, however. The first is a correction required because of the fact that the measurements were made with the fuel channels empty, and hence the effective density of the pile is lower than that of the graphite in which the value of the diffusion length is desired. Since the diffusion length may be regarded as giving a measure of the distance in which the flux decreases by $1/e$, it can be readily seen that the diffusion length, to first order, should vary inversely as the density. The corrected value of $L$ is thus

$$L_{\text{corr.}} = \frac{L_{\text{meas.}} (\text{pile density})}{(\text{graphite density})} = \frac{L_{\text{meas.}} (\text{pile volume} - \text{channel vol.})}{(\text{pile volume})} = \frac{50.8}{(12.4 \times 10^6 - 263 \times 10^6)} = 49.8 \text{ cm.}$$

The second correction is again for density, but in this case it is to normalize the actual density to the anticipated density, which was 1.6 gm./cm.$^3$. A small sample cube of graphite was machined and weighed. Its density turned out to be 1.69 gm./cm.$^3$.

Our corrected and normalized value of the diffusion length is thus

$$L = (49.8)(1.69/1.6) = 52.6 \text{ cm.}$$
Appendix B: DATA ON MEASUREMENT OF THE MATERIAL BUCKLING

The pile was loaded with natural uranium fuel slugs in the following manner: as one views the north face of the pile, the extreme right and left vertical rows of fuel channels received seven slugs per channel, the next vertical rows on each side in towards the center received eight slugs per channel, and the remaining channels a full loading of ten slugs per channel. This procedure was necessary in order to distribute uniformly the available fuel in the optimum configuration.

The tubular BF$_3$ counter was again used in conjunction with the Atomic Instruments Corp. electronic equipment as outlined in Appendix A, and inserted into the pile through holes in spare fuel channel bars pushed into vacant foil stringer openings in the east face of the pile.

In addition to flux measurements made with the Pu-Be source in the same position as for the measurement of the diffusion length, additional counts were taken at each level with the source absent from the pedestal, in order to determine the magnitude of the flux resulting from neutron production due to spontaneous fission of the uranium nuclei. This flux can be regarded essentially as a background effect which must be subtracted from the count rate obtained with the source in position to obtain a corrected flux usable
for calculating $B_m^2$. These results are presented in Table B1, below.

* * *

Table B1 - Material Buckling Measurements

<table>
<thead>
<tr>
<th>Level</th>
<th>$\phi$ meas. (cpm)</th>
<th>Bckgrnd (cpm)</th>
<th>$\phi$ - bckgrnd (cpm)</th>
<th>Cm. from source</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>115,556</td>
<td>969</td>
<td>114,587</td>
<td>66.0</td>
</tr>
<tr>
<td>5</td>
<td>71,631</td>
<td>1087</td>
<td>70,544</td>
<td>84.5</td>
</tr>
<tr>
<td>7</td>
<td>49,758</td>
<td>1205</td>
<td>48,553</td>
<td>102.9</td>
</tr>
<tr>
<td>9</td>
<td>36,061</td>
<td>1246</td>
<td>34,815</td>
<td>121.3</td>
</tr>
<tr>
<td>11</td>
<td>26,301</td>
<td>1287</td>
<td>25,014</td>
<td>139.7</td>
</tr>
<tr>
<td>13</td>
<td>19,749</td>
<td>1316</td>
<td>18,433</td>
<td>158.1</td>
</tr>
<tr>
<td>15</td>
<td>14,576</td>
<td>1308</td>
<td>13,268</td>
<td>176.5</td>
</tr>
<tr>
<td>17</td>
<td>10,806</td>
<td>1240</td>
<td>9,566</td>
<td>195.0</td>
</tr>
<tr>
<td>19</td>
<td>8,251</td>
<td>1171</td>
<td>7,080</td>
<td>213.4</td>
</tr>
<tr>
<td>21</td>
<td>5,765</td>
<td>933</td>
<td>5,832</td>
<td>231.0</td>
</tr>
</tbody>
</table>

In Graph B1 on page 31 the natural logarithms of the corrected fluxes were plotted as a function of $z$, and it was observed that the slope was a constant at a distance from the source and top of the pile. Near the source, the deviations from linearity of the plot are due to the presence of higher harmonics than the first in the flux distribution, and the deviations near the top of the pile are due to a neglect of the end correction factor (see Section 4).

The slope of the linear portion of the plot was measured to be $-1.1 \times 10^{-2}$ cm$^{-1}$, which is $-\gamma_{11}$. Since the $x$ and $y$ dimensions of the pile are 231 cm., we have from Section 4 that

(B.1) \[ B_m^2 = (\pi/a)^2 + (\pi/b)^2 - (\gamma_{11})^2 \]

which gives a $B_m^2$ of $80 \times 10^{-4}$ cm$^{-2}$. 

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Graph B1: Measurement of $B_{mm}^2$

$$y_1 = -\frac{(2.30)(5.4)}{(192.5 - 61)} = -\frac{12.3}{135.5}$$

$$m = 1.7 \times 10^{-3} \text{ cm}^{-1}$$

Distance from source in cm.

In C.M.
We must make a correction, however, to $B_m^2$ for the contributions of the $\phi_{13}$, $\phi_{31}$, and $\phi_{33}$ harmonics (see Section 8) as was done in the case of the diffusion length. In Table B2, below, will be found the values of the fluxes from Table B1, the level numbers at which the corrections were applied, the calculated correction factors at each level, and the corrected fluxes $\phi_c$, again obtained by dividing the fluxes in column 2 by the correction factor.

<table>
<thead>
<tr>
<th>Level</th>
<th>$\phi$(cpm)(from Table B1)</th>
<th>Correction Factor</th>
<th>$\phi_c$(cpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>71,631</td>
<td>1.185</td>
<td>60,500</td>
</tr>
<tr>
<td>7</td>
<td>49,758</td>
<td>1.061</td>
<td>46,900</td>
</tr>
<tr>
<td>9</td>
<td>36,061</td>
<td>1.040</td>
<td>34,650</td>
</tr>
<tr>
<td>11</td>
<td>26,301</td>
<td>1.024</td>
<td>25,470</td>
</tr>
<tr>
<td>13</td>
<td>19,479</td>
<td>1.015</td>
<td>19,500</td>
</tr>
<tr>
<td>15</td>
<td>14,576</td>
<td>1.0095</td>
<td>14,450</td>
</tr>
<tr>
<td>17</td>
<td>10,806</td>
<td>1.000</td>
<td>10,806</td>
</tr>
<tr>
<td>19</td>
<td>8,251</td>
<td>1.000</td>
<td>8,251</td>
</tr>
<tr>
<td>21</td>
<td>5,765</td>
<td>1.000</td>
<td>5,765</td>
</tr>
</tbody>
</table>

In graph B2 on page 33, the natural logarithms of $\phi_c$ were plotted as functions of $z$, the resulting slope being $-1.585 \times 10^{-2} \text{cm}^{-1}$, which is $\gamma_{11}$. Inserting this value in (B.1), we obtain a corrected $B_m^2$ of $1.19 \times 10^{-4} \text{cm}^{-2}$.

It is now possible to calculate the critical size by equating this value of the material buckling to the critical geometric buckling $B_g^2$, remembering that for a cubical critical pile of side $a'$, $B_g^2 = \left(\frac{\pi}{a'}\right)^2$. 
GRAPH B2: MEASUREMENT OF $C_m$

\[- \gamma' = - \frac{(2.30)(5.4)}{200 - 55} \approx -1.585 \times 10^{-2} \text{ cm}^{-1} \]
From this relationship, we learn that a critical pile having the same lattice structure as the M.I.T. Exponential Pile would be 499 cm. = 16.4 feet on a side.


