Aero. eng'g
Thesis
1929

# THE DESIGN OF A MACHINE
# FOR
# THE TESTING OF STRUCTURAL TUBES
# IN COMBINED BENDING AND COMPRESSION

BY

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AND

W. D. JOHNSON

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENT
FOR DEGREE OF
BACHELOR OF SCIENCE IN AERONAUTICAL ENGINEERING
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
1929

SIGNATURES OF AUTHORS

CERTIFICATION BY THE AERONAUTICAL ENGINEERING STAFF

PROFESSOR IN CHARGE OF THESIS

HEAD OF COURSE

May 23, 1929

Professor A. L. Merrill,
Secretary of the Faculty,
Massachusetts Institute of Technology,
Cambridge, Mass.

Dear Sir:

Submitted herewith, in partial fulfillment of the requirement for the degree of Bachelor of Science in Aeronautical Engineering, is a thesis: The Design of a Machine for the Testing of Structural Tubes in Combined Bending and Compression.

Very truly yours,

# ACKNOWLEDGEMENT

The authors of this thesis desire to express their appreciation for the assistance and advice accorded them by Professor Joseph S. Newell of the Structures Department, Professor Harrison W. Hayward and Professor Irving H. Cowdrey of the Mechanical Engineering Department, and Mr. Zimmerman and Mr. Maeser of the Mechanical Engineering Department.

These gentlemen were ready and willing at all times to render any cooperation and courtesy within their power. Therefore, whatever merit is embodied in the final results is due in great part to them.

# GENERAL INFORMATION

# AND

# DESCRIPTION

# Design Of A Machine
# For The Testing of Structural Tubes In Combined Bending and Compression

## Object

The object of the thesis herein presented was the design of an apparatus for use in the testing of structural tubes in combined bending and compression. The design of this apparatus developed through the problem presented in the thesis first taken by the authors, namely that of the actual testing of tubes in combined bending and compression. When the original thesis was undertaken it became necessary to construct some sort of jig which would be used to put bending in the tubes at the same time compressions was being applied. Several plans for such a jig were presented for approval to Professors Hayward and Cowdrey of the Testing Materials Laboratory, but were rejected by them as unsuitable for use with the machines in that laboratory. It was then suggested to the authors that the design of some specific apparatus, self contained and outside the compression machine, should be attempted, such an apparatus to serve as a regular piece of laboratory equipment for use in future combination tests on structural tubes. Therefore the original idea of a temporary jig was entirely discarded, and the design of a regular laboratory testing apparatus was undertaken. This idea was presented to Professor Joseph Newell of the Structures Department who is in charge of this thesis and was approved by him.

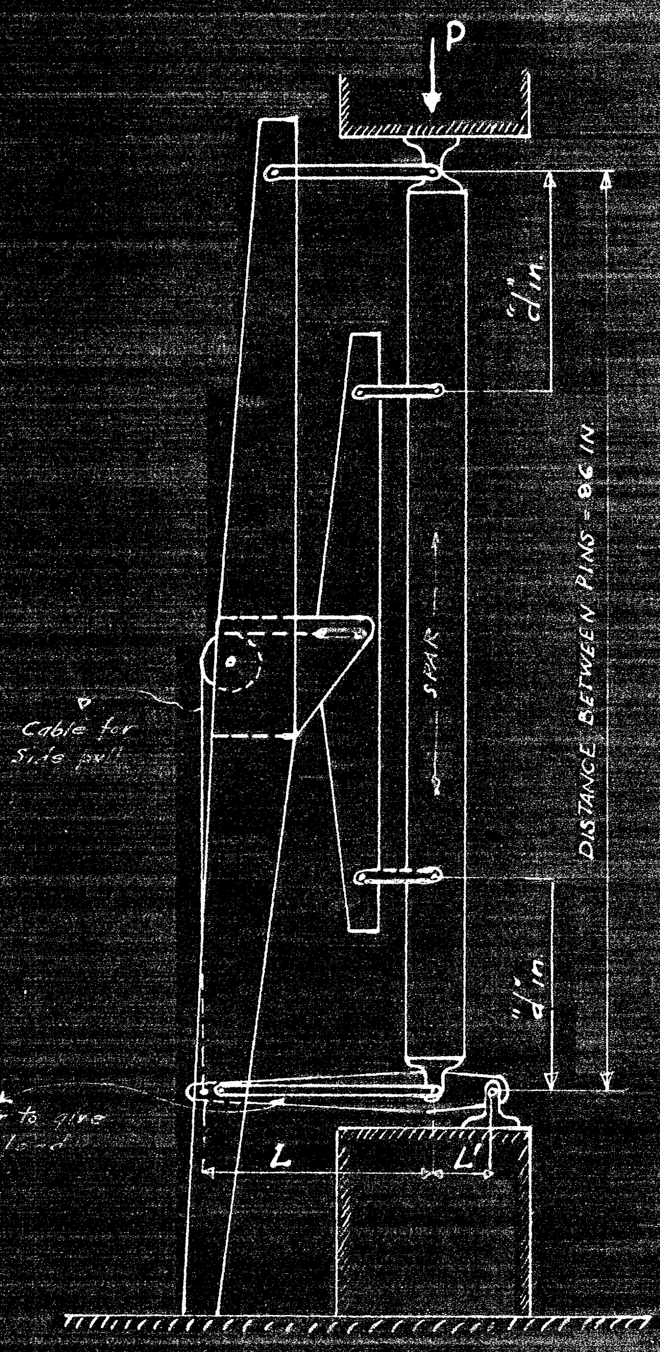
The requirements laid down by Professor Newell, and by Professors Hayward and Cowdrey were that the apparatus should be designed from the standpoint of material strength and flexibility with some consideration being made of weight factor together with convenience and ease of operation. The present thesis was undertaken therefore from a purely engineering standpoint.

## Procedure

The procedure followed in the design of the apparatus consisted in the drawing up of various preliminary schemes which were then considered from the standpoints of strenth, flexibility, ease of handling, adaptability to the standard testing machines, and interaction of the component parts of the apparatus itself. The main problem as seen by the authors was that of incorporating the extreme of simplicity in the final apparatus, that is to keep the number of parts down to a minimum and also to provide for the disassembling of the apparatus for carrying it from place to place for tests. Effort was made to provide a unit which would make it possible for one person to carry out a complete combined bending and compression test unassisted.

One of the most important considerations was to provide for various conditions of transverse loading and such provision has been made.

Mention should here be made of the apparatus used at McCook Field, Dayton, Ohio, for the testing of experimental spars in combination loading. This method of testing was one of those proposed for use in the original



SCHEMATIC ARRANGEMENT OF TESTING JIG

THE RATIO OF L TO L' DEPENDS ON THE RATIO OF SIDE TO END LOAD. DISTANCES d" FROM PIN TO LOAD POINTS VARY ACCORDINGLY AS THE BENDING MOMENT TO BE OBTAINED.

FIG~A

thesis of the authors and was not accepted as suitable for use on the laboratory machines of the Testing Material Laboratory. Figure A shows the arrangement used at McCook Field. The diagram is self-explanatory. By the use of this method the ratio of side to end load can be varied at the will of the tester. Also the distance "d" from pin to load points may be varied to obtain any desired bending moment. The advantage of this apparatus is that both loadings are applied simultaneously and the ratio is perfectly consistent so that at the moment of failure the exact side load is known through the compression reading, and the deflection corresponding can be easily noted.

Thesis Apparatus

As has been said above, several schemes were considered using a combination of metal, wood and cable. However, from consideration of bulk, strength, and deformation properties, it was finally judged that all metal construction would be the best. Therefore, steel members have been used throughout. For the least stressed members Mild steel was used, and for those more highly stressed cold rolled steel was employed . In the case of the tension rod, because it was desired to keep the diameter small coupled with the fact that a spline had been cut in the shaft, a special alloy steel was adjudged necessary and a commercial steel by the trade name of Elastuf having a yield point of ? per square inch was used. Each part of the apparatus was machined from standard stock with the exception of the shackles (part 12 on assembly sheet) which are steel forgings, and the tension nut (part 5 on assembly sheet) which is of brass.

In order to obtain accurate adjustment and ease of application of the side load a thrust bearing was used under the tension nut (see photograph).

One of the most important considerations was the use of some instrument for the accurate measurement of side load. The instrument chosen for this purpose was the tension dynamometer designed by Professor Frost of the Photo Elasticity Laboratory at the Massachusetts Institute of Technology. There were several of these instruments available, but of unknown capacity, therefore it was necessary to make calibration runs upon one of these dynamometers to obtain data which was used in plotting a curve of load against reading of the dial. A discussion of this calibration is given below.

For the end supports of the tubes under bending, two S. K. F. self aligning bearings were used in order to simulate point contact and also to allow for vertical sliding movement of the tube in bending and compression, and to hold down the restraining moment to a minimum.

The distance between the compression rods which governs the distance between the end supports is adjustable, the apparatus being designed to take a maximum

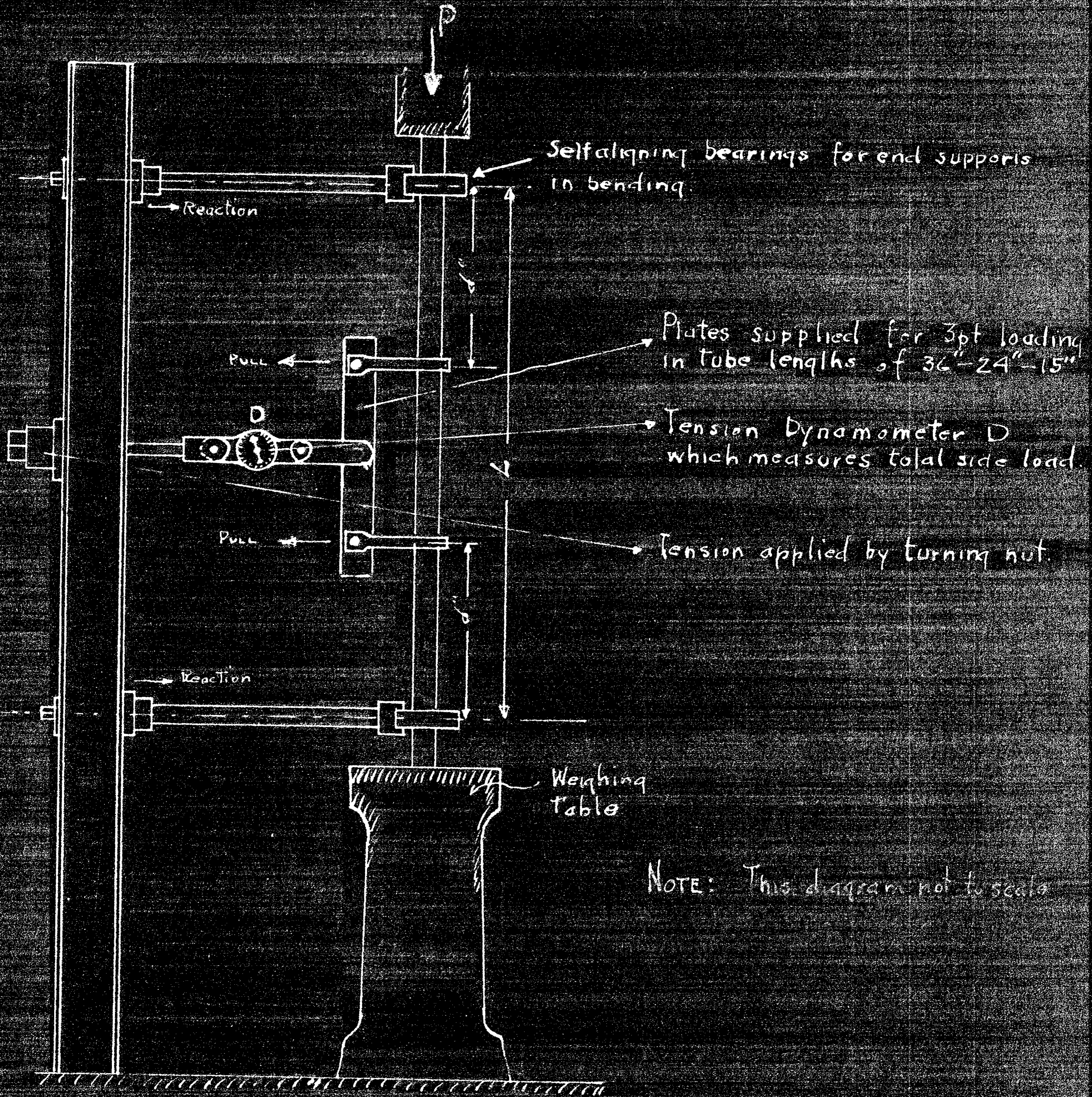
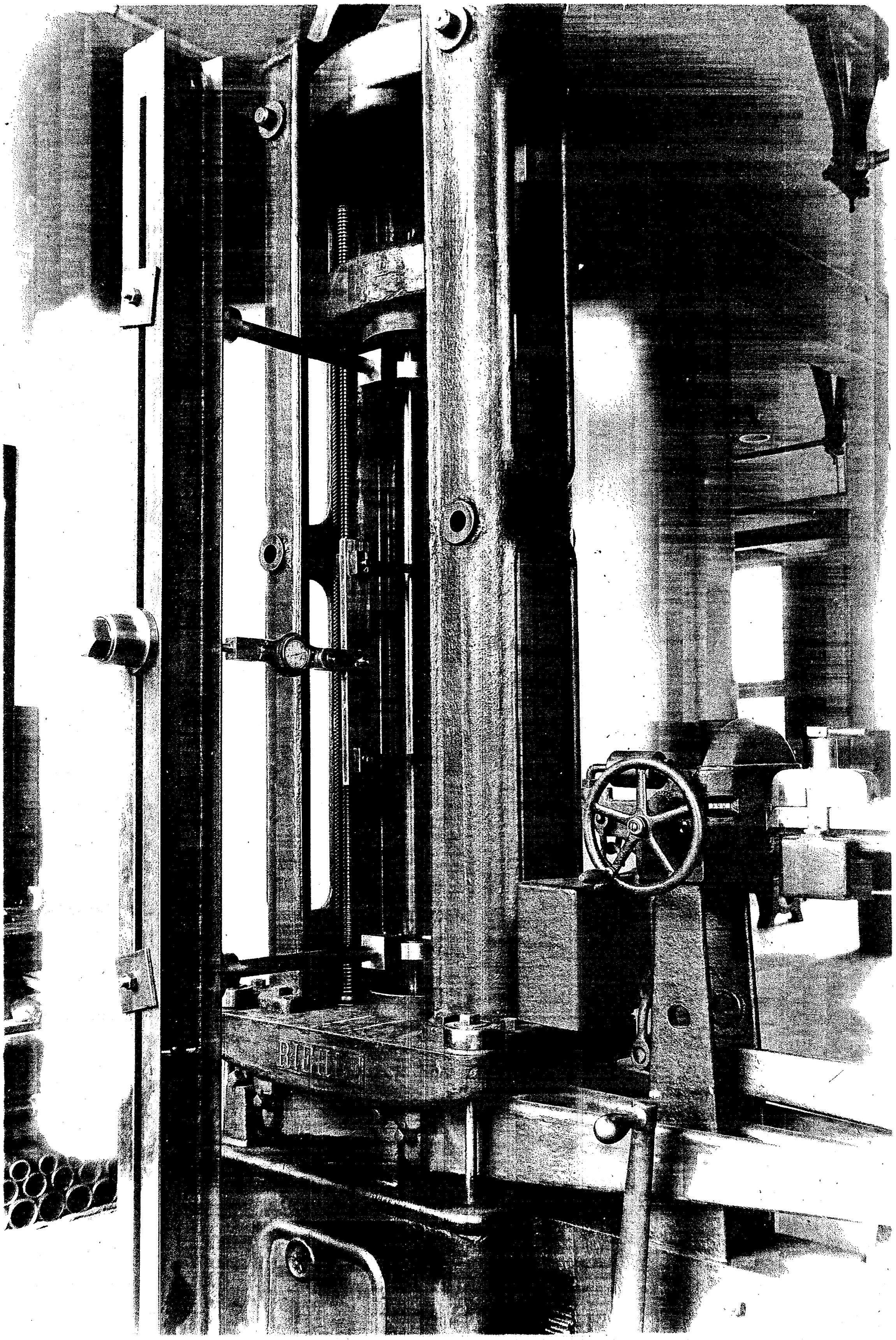
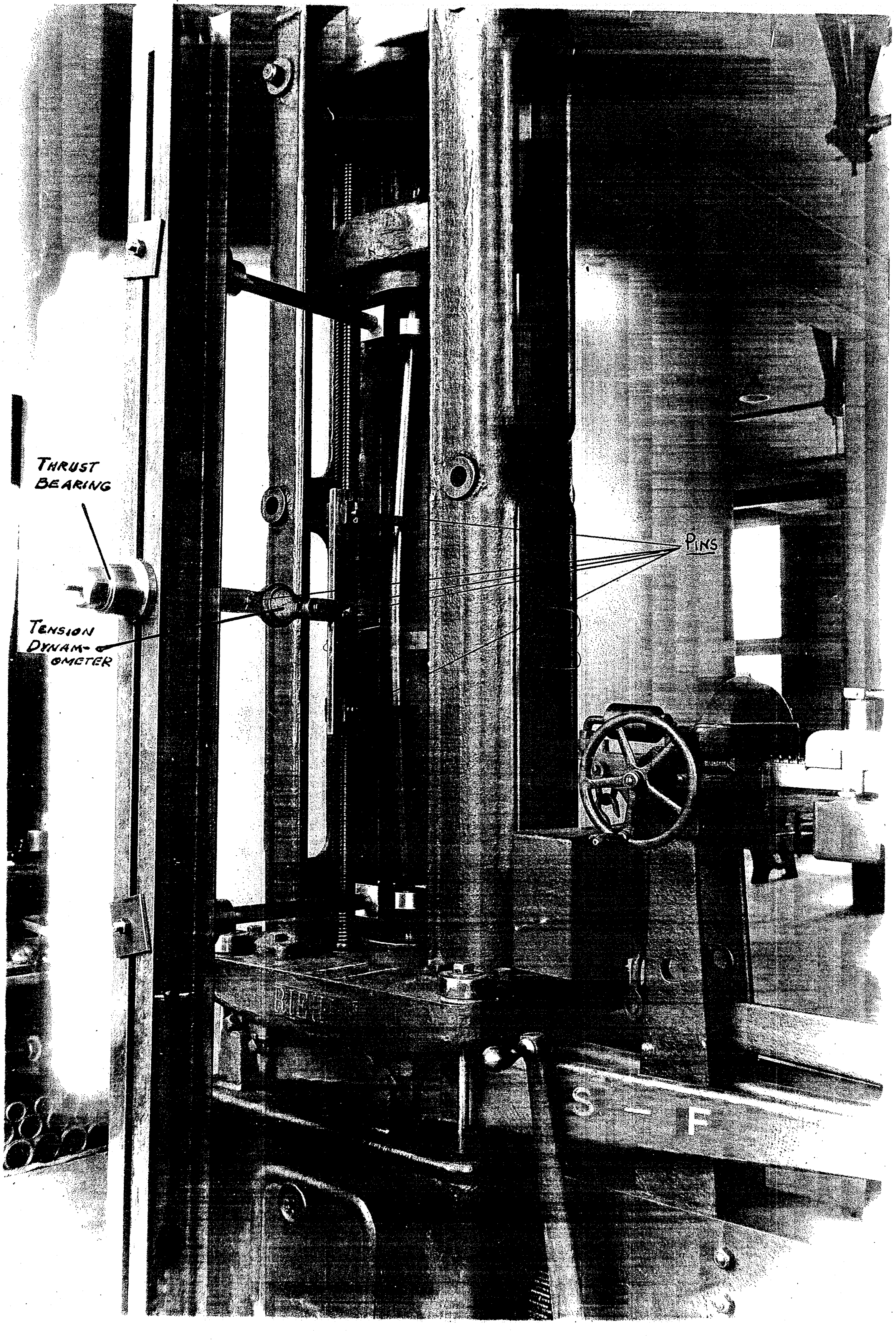


FIG. 1

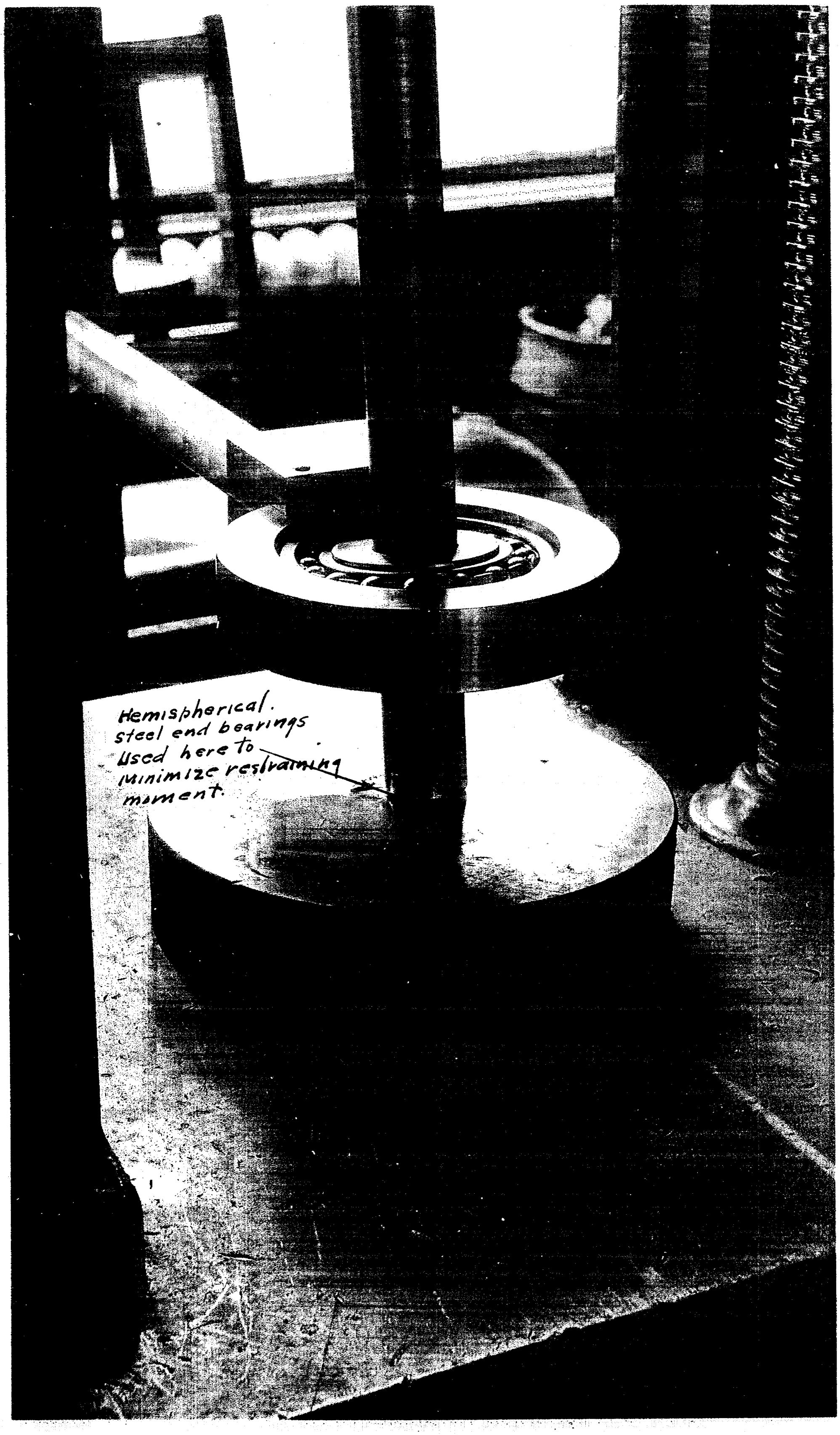
Method of Operation of Apparatus



Method of Setting Up Apparatus
Fig P-A



TUBE UNDER COMBINED LOADING
FIG P-B



ACTION of SELF ALIGNING SUPPORT BEARINGS

FIG P-C

length of 36" and a minimum length of 8" between supports. However, it is possible to obtain a maximum length of 60" if desirable.

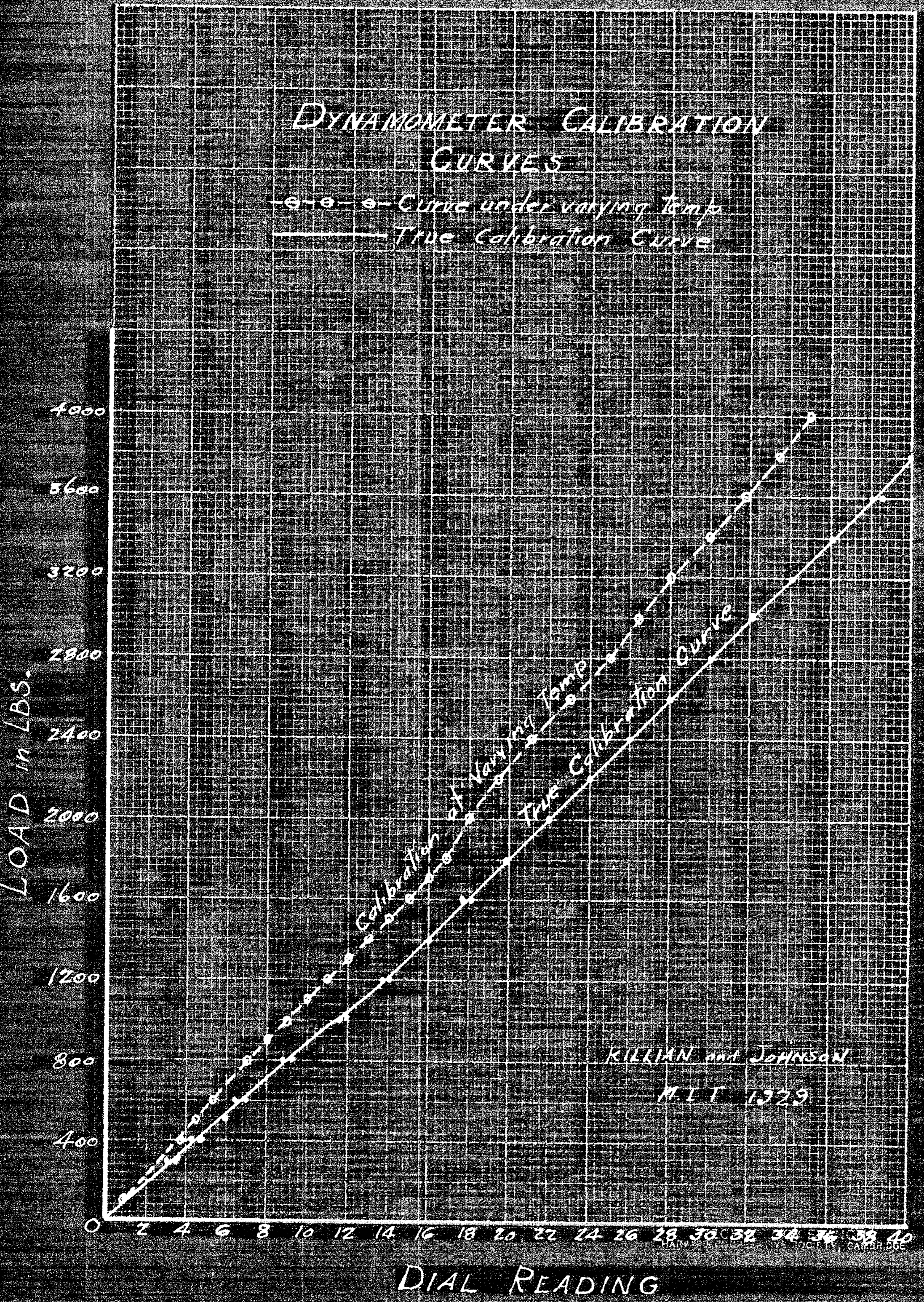
The apparatus is self contained and statically balanced so that the only load imposed on the weighing table is a straight vertical load due to compression and the load is applied directly at the center of the table.

Approximately one to one and one half inches of tubing is allowed to extend beyond the end supports to provide clearance for the compression rods in the application of axial loads. This extension is so short that no detrimental effect is imposed on the value of the tests.

For the bearings of the tube ends against the head and table of the testing machine two hemispherical hardened steel bearings are used. This arrangement will cut down the restraining moment at the tube ends to a minimum so that this end moment can safely be neglected in the computation of results.

## Calibration of Dynamometer

Method of Calibration - The dynamometer was set up in the 20,000# Riehle testing machine and loads were applied in 100# increments up to an ultimate value of 4,000#. The calibration was carried only to this point for two reasons, first because the design load for the subject of this thesis was 4,000#, and second because of


DYNAMOMETER CALIBRATION CURVES
-o-o- o- Curve under varying Temp.
———— True Calibration Curve
Calibration at Varying Temp
True Calibration Curve
KILLIAN and JOHNSON
M I T 1929
LOAD in LBS.
4000
3600
3200
2800
2400
2000
1600
1200
800
400
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
DIAL READING

the danger of producing a permanent set in the dynamometer since its capacity was unknown.

In making the first runs any effect of temperature change was neglected, and the test was performed directly before an open window. Readings taken on this run could not be checked on subsequent trials. This aroused the suspicion that temperature change was vitiating the calibration. On this theory the same procedure was carried out on the following day, precaution being taken to see that the work was performed at a constant temperature. Under these conditions successive runs checked very closely.

A curve of load against dial reading was plotted and except for a slight curvature between 200 and 1,000 lbs. was virtually a straight line. Where combined bending and compression tests were run off on the tubes the amount of side load recorded on the dial was shown actually by the corresponding point on the plotted curve.

By inspection of the curve of dial reading against load on the dynamometer it is apparent that the deflection is not proportional to the load over the entire range. The graph is a straight line between loads of 0-900, a slight curvature between 900-1300, straight line between 1300-3200, and slight curvature between 3200-4000. This of course makes it necessary that for accurate results any setting of the dynamometer for any particular load must be done by reference to the calibration curve.

A second curve of the readings at varying temperature conditions has been drawn on the calibration graph to show the effect of temperature fluctuation in the instrument. It can be seen that the readings of deflection on specific loads varies widely from those obtained at constant temperature of 78° or at 80°. The reason for this could only be found in the temperature fluctuation theory. Therefore as it is apparent that temperature fluctuations tend to throw off the action of the dynamometer, it is recommended that tests be run at as near constant temperature conditions as possible.

## Sample Tests on Steel Tubes

In order to obtain some idea of the results obtainable through the use of the apparatus a test was made upon two 36" 10-20 point carbon steel, 1" - .0625 tubes in combination bending and compression. Cold rolled tubing was used because of the fact that alloy steel or Duralumin tubes could not be obtained.

### Test to Obtain Value of EI for this Tubing

To obtain a value of EI, one of the tubes was placed in the small 10,000# Riehle machine and loaded in simple transverse loading, the load being applied at the center point in a length of 36" between supports. The deflection at the center point was measured for use in computing the value of EI. Four readings were taken under loads of

10, 20, 30 and $50^{\#}$.

## Tests in Combined Bending and Compression.

The object of this test was to obtain data for the computations of maximum combined bending moment on the tube (Primary and Secondary)) and to also compute this bending moment by the use of the precise formula given on page 26 of Air Corps Information circular No. 622, this formula being changed somewhat to conform to the specific case under consideration. These two computed moments should check to prove the accuracy of the method of testing.

The tube was placed in the apparatus and fixed in the compression machine in the manner shown in Figure I-A to be tested in combined axial and side loading. Following the plan pursued in the spar tests of the same character at McCook Field, the side and end loads were kept proportional and in a ratio $\frac{\text{Axial Load}}{\text{Side Load}} = \frac{2}{1}$ or two to one ratio of axial to side load. The loads were put on in increments of $50^{\#}$ from a combination of $100^{\#}$ end load, $50^{\#}$ side load to $750^{\#}$ axial, $375^{\#}$ transverse at which point or a little beyond which the tube failed. Readings of deflection against loads were taken at each $50^{\#}$ increment until the tube failed. The ratio of side to end load being maintained constant.

Precise Formula for Maximum Moment at Center of Span:

$$M = \frac{M_1}{\cos\frac{L}{2j}} = \frac{Wj \sin\frac{a}{j}}{\cos\frac{L}{2j}}$$

Now as $M_1$ in our case was negligable due to proper end bearings, the formula reduces to:

$$M = \frac{Wj \sin\frac{a}{j}}{\cos\frac{L}{2j}}$$

Ordinary Bending Formula for Maximum Moment at Center of Span:

$$M = R_1 \left(a - \frac{b}{2}\right) + dp - \frac{b}{2} W$$

See Appendix B for Symbols.

If these two computed moments check, the accuracy of this method of testing will be justified.

The fiber stress under each load combination has been computed to be compared with the ultimate tensile strength value for cold rolled steel or $80{,}000^{\#}$ per square inch.

Tables of Precise Moments and Moments computed by ordinary beam theory, together with differences between them have been drawn up so that the accuracy of the experimental data as obtained on the designed apparatus may be checked. This table is included in the Results.

# RESULTS

# AND

# DISCUSSION

## TABLE OF RESULTS OBTAINED FROM TESTS ON TUBES

EI For Tubing = 596,000 Average.

Note: Precise Moment taken as standard in all cases and differences between Precise Moment and Moment computed from ordinary beam formula recorded as positive or negative as case may be.

### TUBE #1

| End Load | Side Load | d | Moment as computed from ordinary beam formula | Precise Moment | Differenc |
|---|---|---|---|---|---|
| 100 | 50 | .09 | 309 | 295 | 14"# |
| 150 | 75 | .14 | 441 | 465 | - 24"# |
| 200 | 100 | .19 | 638 | 628 | 10"# |
| 250 | 125 | .26 | 825 | 764 | 61"# |
| 300 | 150 | .30 | 990 | 966 | - 34"# |
| 350 | 175 | .34 | 1062 | 1138 | - 76"# |
| 400 | 200 | .38 | 1352 | 1325 | 27"# |
| 450 | 225 | .44 | 1534 | 1495 | 35"# |
| 500 | 250 | .49 | 1745 | 1680 | 65"# |
| 550 | 275 | .60 | 1980 | 1890 | 90"# |
| 600 | 300 | .65 | 2190 | 2080 | 110"# |
| 650 | 325 | .79 | 2462 | 2287 | 195"# |
| 700 | 350 | .90 | 2815 | 2510 | 300"# |
| 750 | 375 | .99 | 2992 | 2485 | 507"# |

Fiber Stress 77,560

## TUBE #2

| End Load | Side Load | d | Moment as computed from ordinary beam formula | Precise Moment | Difference |
|---|---|---|---|---|---|
| 200 | 100 | .11 | 622 | 629 | - 7"# |
| 300 | 100 | .20 | 960 | 967 | - 7"# |
| 350 | 175 | .24 | 1134 | 1137 | - 3"# |
| 400 | 200 | .34 | 1336 | 1320 | 16"# |
| 450 | 225 | .38 | 1520 | 1500 | 20"# |
| 500 | 250 | .44 | 1720 | 1694 | 26"# |
| 550 | 275 | .59 | 1976 | 1890 | 86"# |
| 600 | 300 | .66 | 2196 | 2080 | 116"# |
| 650 | 325 | .77 | 2300 | 2294 | 6"# |

Fiber stress = 60,010 $\text{lbs/in}^2$

## DISCUSSION

It is apparent from inspection of the above tables of moment differences as computed by the precise method and ordinary method, that the results are unsatisfactory except those obtained at low loads. However, it will be noted that the run performed on the second tube is better than that performed on the first tube. This immediately gave us a clue as to one source of difficulty, namely:

(1) Unfamiliarity with handling of apparatus.

(2) Very inefficient method of measuring deflection.

(3) Poor working conditions when tests were being run.

The condition of unfamiliarity with the manipulation of the apparatus is of course understood, as the test on tube #1 was the first time the apparatus had been tried as a testing machine. It was apparent to the authors when the second tube was tested that the handling of the apparatus was considerably easier.

During the first test, the method for measuring deflection consisted in using a rough wood reference block tied to the frame of the compression machine, and a steel scale graduated in 1/100 inches. The light was very poor during the tests, and consequently it was extremely difficult to record such a small scale with any degree of accuracy. Now, it is the opinion of the authors that the main reason for the poor results obtained lies in this inaccuracy of deflection readings.

This opinion is strengthened by the fact that when in the test on the second tube a slider was fitted to the steel scale to readings, the results show the effect clearly.

Another source of error may possibly lie in the dynamometer, but the calibration on the instrument was done with very great care, and when loads were set on the dynamometer, reference was constantly made to the data on the calibration. Also, the dynamometer would certainly not introduce the magnitude of error found in our results.

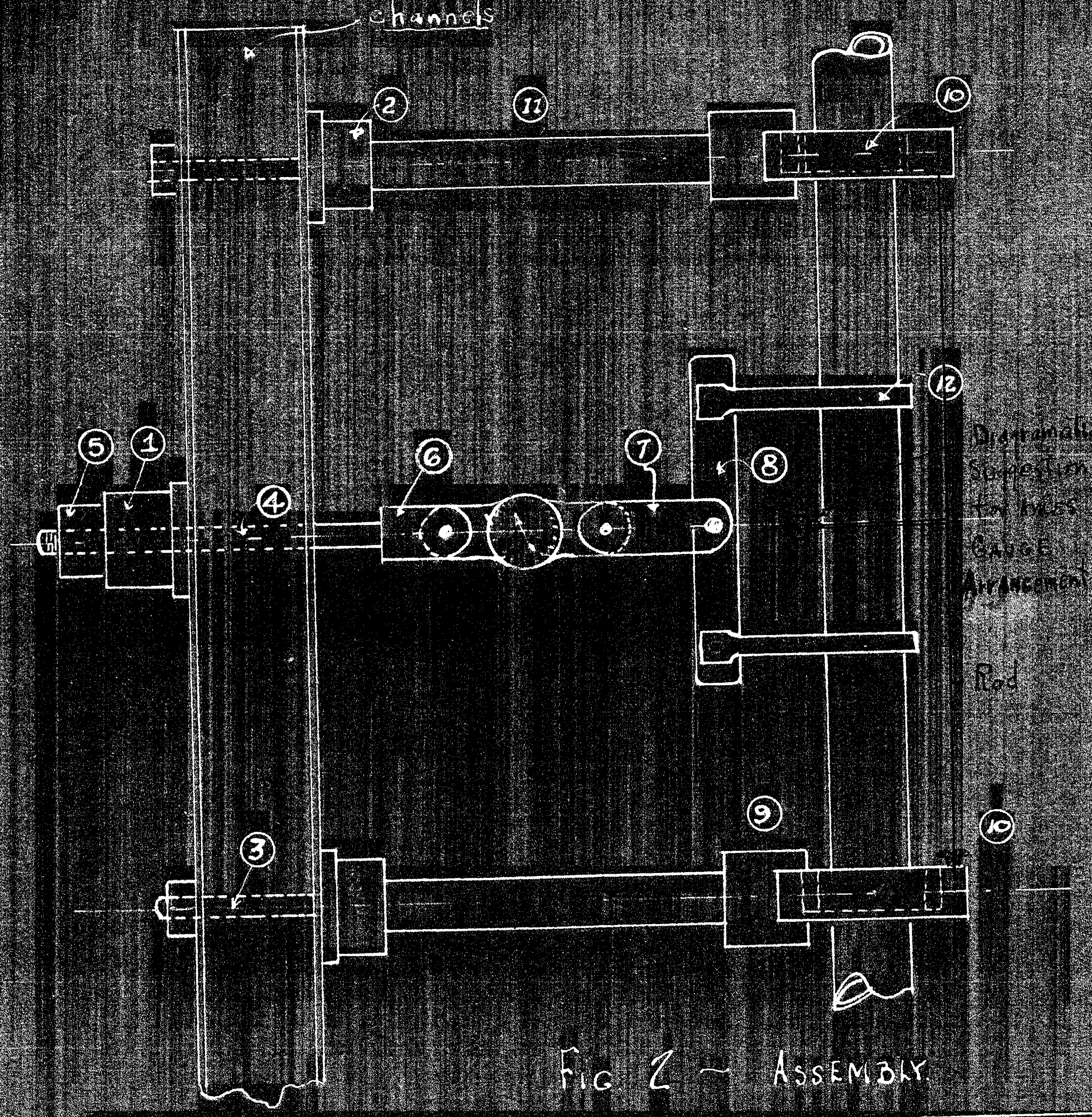
The writers of this thesis recognize the fact that the end restraining moments are not zero as has been assumed in the computations. Even with the use of a hemispherical end bearing there is bound to be a small bending moment introduced, and this might contribute to the source of error.

## Suggested Improvements:

Notwithstanding these other sources of possible error, which in the opinion of the authors are negligable, the error introduced by the erratic deflection is by far the most important. Therefore, some appliance must be added to the designed apparatus which will enable accurate deflection readings to be taken. This could be done by boring two holes on the outside top face of the Self aligning bearing holders and running a rod between them to which could be fastened an arrangement using an Ames Gage for measuring deflections. (This suggested improvement is shown on the accompanying blue print.)

It is also true that two tests do not constitute a fair trial of the apparatus, and the writers regret that time did not suffice for more tests which would probably show better results.

As the apparatus stands now, neither of the authors of this thesis are satisfied that it is quite as efficient as it might be with a few minor changes. In the



1. Main Bearing Socket
2. Compression Rod Socket
3. Clamp Rod
4. Tension Rod
5. Tension Nut
6. Socket Lug
7. Link
8. Spacing Plate
9. Bearing Socket Lug
10. Self Aligning bearing Socket
11. Compression Rod
12. Shackles
13. Thrust Bearing
14. Two Self Aligning Bearings.

Fig. 2 ~ Assembly.

first place, the apparatus should have a means of support to do away with the necessity of steadying the whole thing when loads are released, and to facilitate the setting of the instrument while the tube is being held in the big testing machine.

Some means should also be provided for the partial support of the tension arm unit. This unit is not easy to handle as it is made up of several sections loosely connected, and the whole thing is rather heavy. The weight of the entire apparatus could be cut down considerably if the lower part of the channels were cut off and a wooden stand or tripod were filled to the jig.

If this testing jig should finally prove to be unsatisfactory, it could easily be made into an apparatus working on the same principle as that at McCook Field mentioned above. However, the authors are emphatically of the opinion that the main source of trouble in the present apparatus is the introduction of erroneous deflection readings. The results were very good in both cases at the lower loads, and there is no reason why this dependability could not be extended farther up the load scale.

Of course there is one inherent disadvantage to this apparatus, and that is that it is very difficult to apply the loads continuously and keep the proportionality between side and end load constant. The apparatus used at McCook Field had this property, it being automatically supplied by the ratio of the lever system. Therefore, in the McCook Field apparatus it is possible to steadily increase the axial load until the member fails and when that occurs the side load is absolutely known. The authors feel that a very close approximation to this result can be obtained on our apparatus if the increments of loading increase up around the expected failure point of the member are kept sufficiently small. The error introduced would then in our opinion be negligable.

The apparatus submitted has a great many possibilities for use in other sorts of testing. If some sort of special universal jaw were devised for the torsion machine, the apparatus could be well used in combined torsion and bending tests. The possibility of using this jig for fatigue tests on rotating shafts has also been suggested to the authors. If this was to be done, ball-bearings would have to be fitted into the shackle eyes.

As a last word, the writers wish to express their faith in the possibilities of this apparatus, and to point out that an absolutely unbiased criticism has been made. If fault has been found, it has not been done through lack of confidence that the machine will fulfill the purpose for which it was designed, but rather that its usefulness will be increased by the suggested improvements.

# APPENDIX

# DISCUSSION OF DESIGN COMPUTATIONS

In appendix A is given a series of computations on the design of the various parts of the apparatus. These computations amount to check figures on the final parts used. It was adjudged by the authors as unecessary to include the total mass of preliminary calculations gone through in the design; for in the method of design used by the authors, namely that of trial and error, the majority of these preliminary computations would mean nothing. As has been said before, effort was continually made to cut the number of parts to a minimum and to simplify those remaining. As this was the case, the preliminary computations were used as means of determining just what members could be cut or left out altogether.

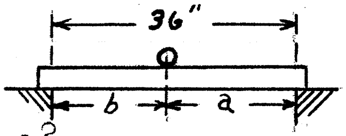
In the computations as given in the Appendix, a great many of the parts will show up as greatly over strength. However, these members were purposely used over strength in order to insure the maximum of rigidity in the apparatus. In the case of the Compression Rods, the 1" C. R. Steel Rods used are very greatly over strength, but as we have a component of the supporting force at the bearings tending to put bending into these compression arms, such over strength was adjudged necessary.

Computations are given for the design of 5/8 and

1/2 inch pins used throughout the apparatus. These pins are not included in the details as they were made up to convenient lengths before the assembly of the apparatus.

# DESIGN COMPUTATIONS

## CHANNELS



### Safe Load Per Channel:

Ryerson's Catalogue used.

Safe load = that given in table x $\frac{l^2}{8ab}$

Value given in table = 6770 lbs.

$$\text{Safe load} = 6770 \times \frac{1296}{(8)(18)(18)} = 3383 \text{ lbs}$$

Safe load = 3383 lbs per channel

$$\text{F.S.} = \frac{3383}{2000} = 1.69$$

### Maximum Bending Moment:

$$M_o = \frac{wl}{4}$$

Then for a load of 2,000 lbs on each channel:

$$M_o = \frac{(2{,}000)(36)}{4} = 18{,}000 \text{ inch pounds}$$

$M_o$ = 18,000 inch pounds per channel

### Maximum Fiber Stress in Bending:

$$f = \frac{My}{I}$$

$Y = 4"$
$I = 3.8 \text{ in}^4$

$$f = \frac{(18{,}000)(4)}{3.8} = 18{,}950 \text{ lbs/in}^2$$

$f_o$ = 18,950 lbs/in$^2$ per channel

### Factor of Safety:

Assuming V.T.S. = 60,000 lbs/in$^2$

$$\text{F.S.} = \frac{60{,}000}{18{,}950} = 3.16$$

F.S. = 3.16 per channel

### Maximum Deflection:

$$D_o = \frac{WP^3}{48EI}$$

$$D_o = \frac{(2{,}000)(36)^3}{(48)(29{,}000{,}000)(3.8)} = .1763 \text{ inches}$$

$$D_o = 0.1763 \text{ in. per channel}$$

## COMPRESSION ROD

### Intensity of Stress:

$$p = \frac{P}{A}$$

$P = 2{,}000$

$A = .7854$ sq. in.

$$p = \frac{2{,}000}{.7854} = 2550 \text{ lbs/in}^2$$

$$p = 2550 \text{ lbs/in}^2 \text{ per strut}$$

### Factor of Safety:

$$\text{F.S.} = \frac{80{,}000}{2550} = 31.39$$

$$\text{F.S.} = 31.39 \text{ per strut}$$

## TENSION NUT

### Threads Necessary

$$\text{NO. threads} = \frac{(1.910)(W)}{f(D-P)P}$$

where: $W$ = lead
$f$ = working fiber stress
$D$ = outside diameter
$P$ = pitch

Want 20 th/in.

$$\text{Then } P = \frac{1}{20} = .05$$

$$W = 4{,}000 \text{ lbs}$$

$$f = \frac{30{,}000}{3} = 10{,}000 \text{ lbs/in}^2$$

$$D = 1/2''$$

$$\text{Then } N = \frac{1.910 \times 4{,}000}{(10{,}000)(.45)(.05)} = 8.46 \text{ th} \quad \text{necessary}$$

## DYNAMOMETER PIN

Diameter of Pin:

$$R_s = \frac{2\,\pi\;d^2}{4}\,f_s$$

$$4{,}000 = \frac{d^2}{2}\;\frac{(45{,}000)}{5}$$

$$\frac{40}{45} = d^2$$

$$d^2 = .283 \qquad d = .531$$

$$R_c = d\;t\;f_c$$

$$2{,}000 = d\;(.5)\;\frac{(90{,}000)}{5}$$

$$d = \frac{10{,}000}{45{,}000} = .222''$$

## SPACING PLATE

Maximum Bending Moment:

$$M_o = \frac{(4{,}000)(12)}{4} = 12{,}000 \text{ inch lbs.}$$

$$M_o = 12{,}000 \text{ inch pounds}$$

### Maximum Fiber Stress:

$$f = \frac{M\,Y}{I}$$

$$M = 12{,}000 \text{ in. lbs.}$$

$$Y = 1 \text{ in.}$$

$$I = \frac{bh^3}{12} = \frac{(.5)(2)^3}{12} = \frac{.5 \times 8^2}{3} = \frac{1}{3} = .333$$

$$f = \frac{12{,}000 \times 1}{.333} = 36060 \text{ lbs/in}^2$$

$$F.S. = \frac{80{,}000}{36{,}050} = 2.215$$

12" plate only computed as others are of same width and thickness and are shorter, therefore less stressed at the same design load.

## TENSION ROD

$$P = 4{,}000\#$$

$$A = .1962$$

$$p = \frac{4{,}000}{.1962} = 20{,}400$$

$$F.S. = \frac{200{,}000}{20{,}400} = 9.8$$

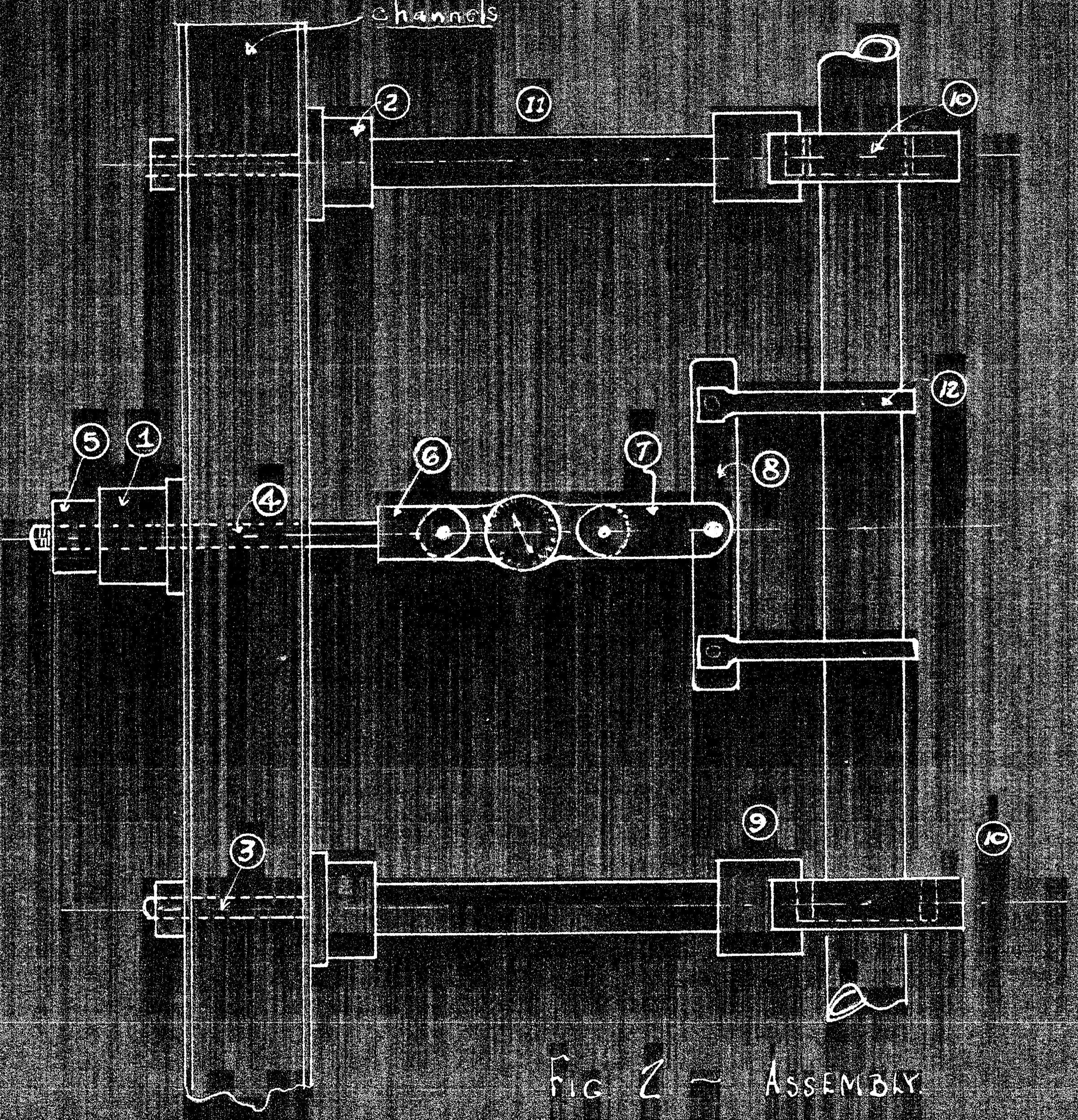
## SHACKLE PINS

$$R_s = \frac{2\,d^2\pi}{4}\,f_s$$

$$\frac{2{,}000}{\text{[illegible]}} = \pi\frac{d^2}{2} = \left(\frac{45{,}000}{5}\right)$$

$$\frac{20{,}000}{\pi(45{,}000)} = d^2 = .1415 \qquad d = .376$$

use 1/2" pins



1. Main Bearing Socket
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10. Self Aligning bearing Socket
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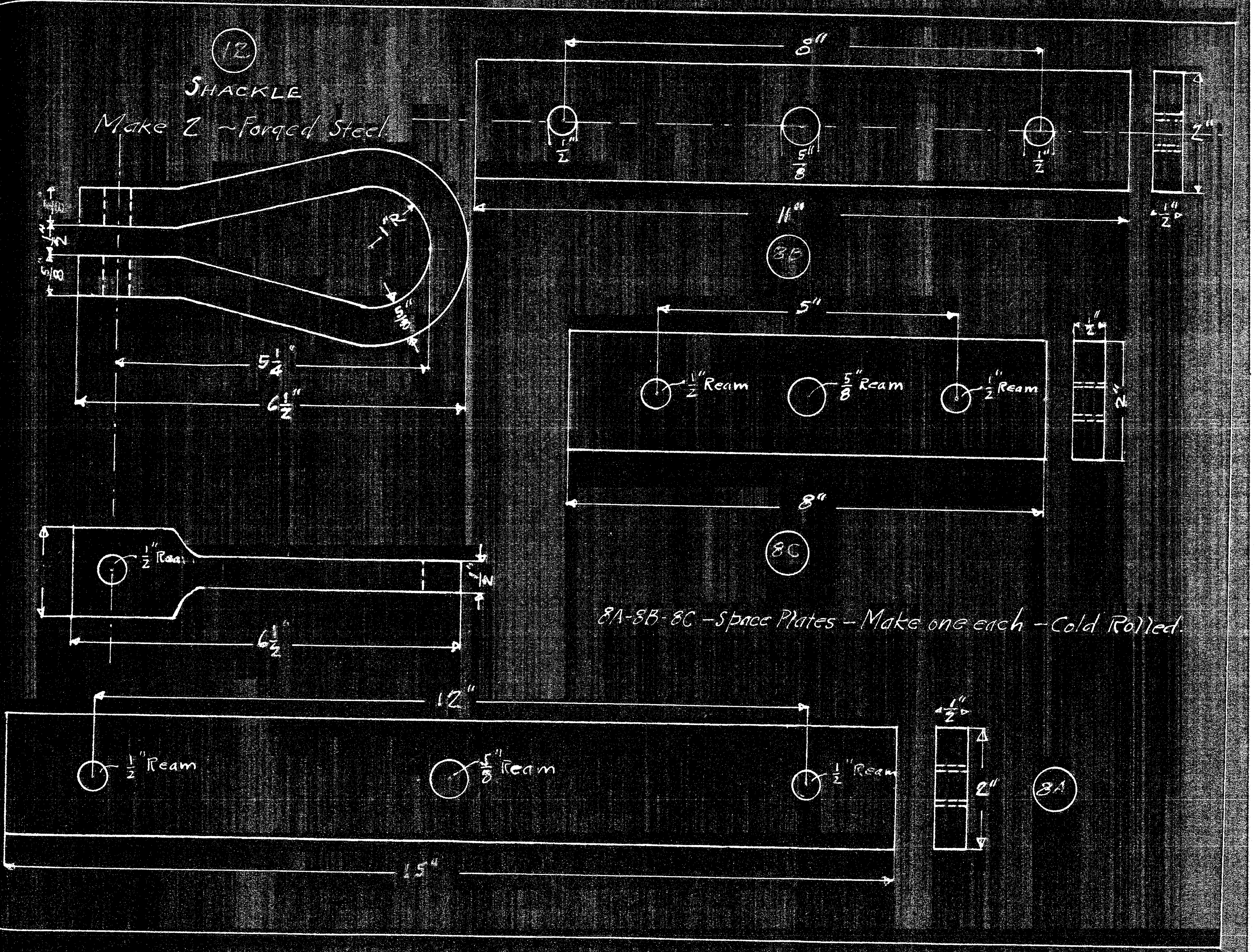
12. Shackles
13. Thrust Bearing
14. Two Self Aligning Bearings.

Fig 2 — Assembly.

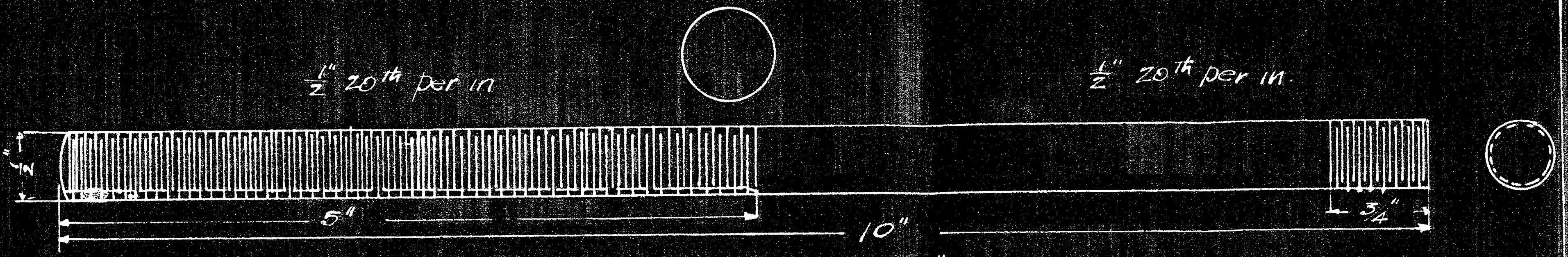
⑫

**SHACKLE**

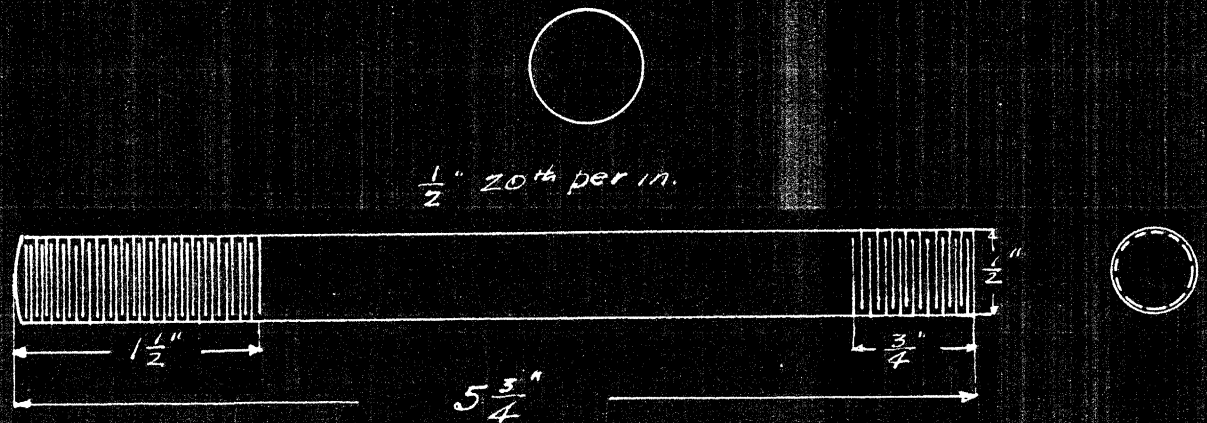
Make 2 – Forged Steel



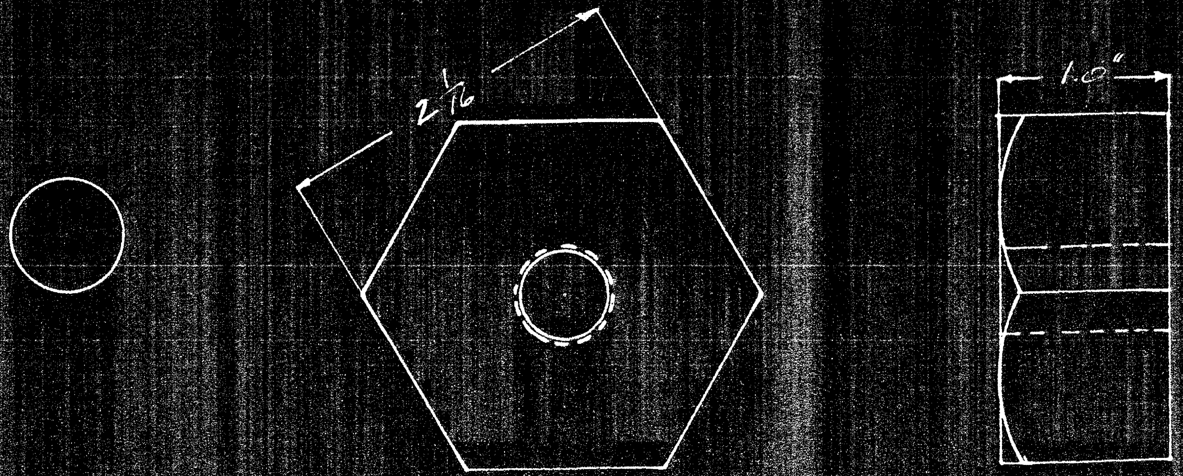
8A-8B-8C – Space Plates – Make one each – Cold Rolled.



TENSION ROD ~ MAKE 1 ~ "ELASTUF" STEEL



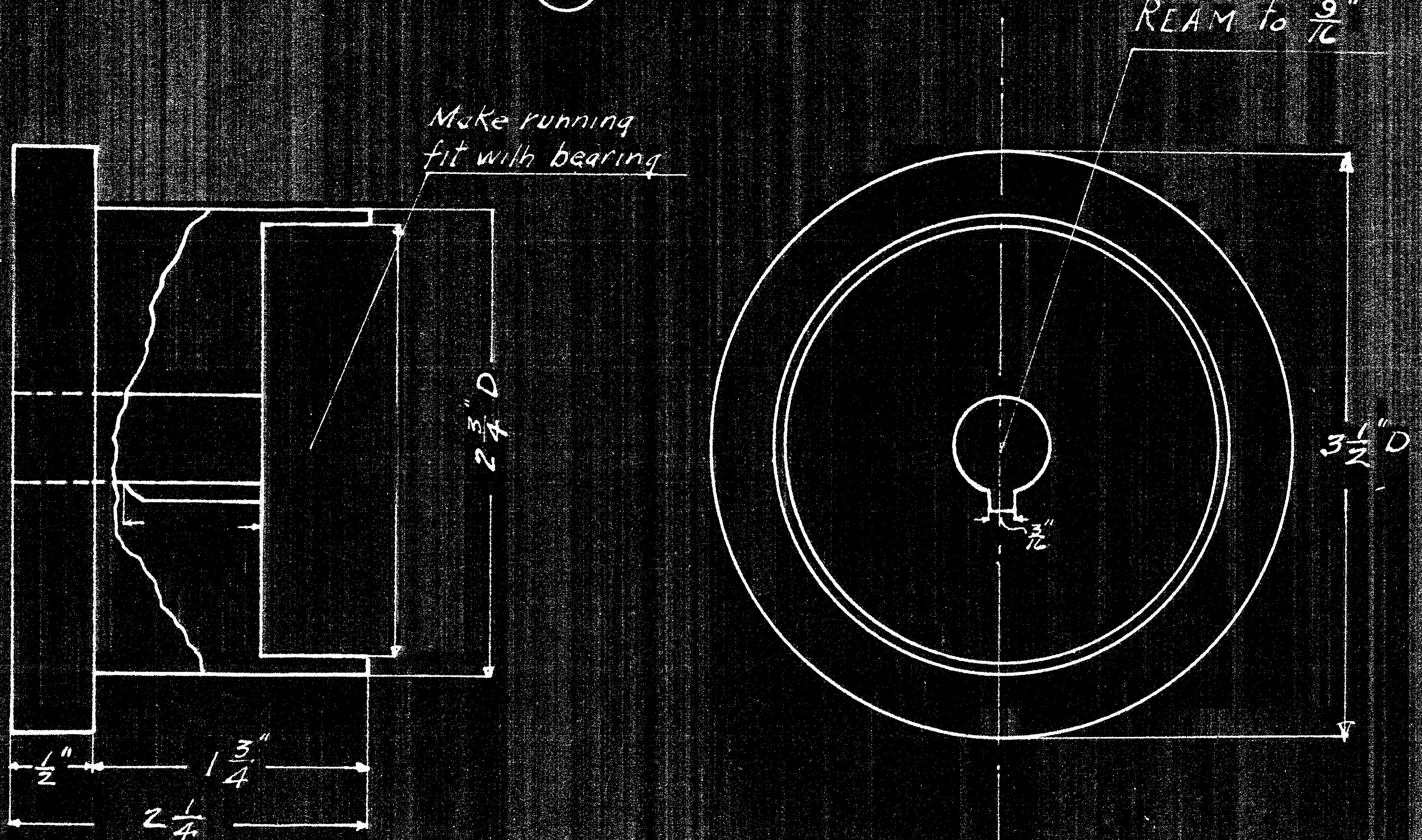
CLAMP ROD MAKE 2 MED. ST.



TENSION NUT
MAKE 1 BRONZE.

# MAIN BEARING SOCKET

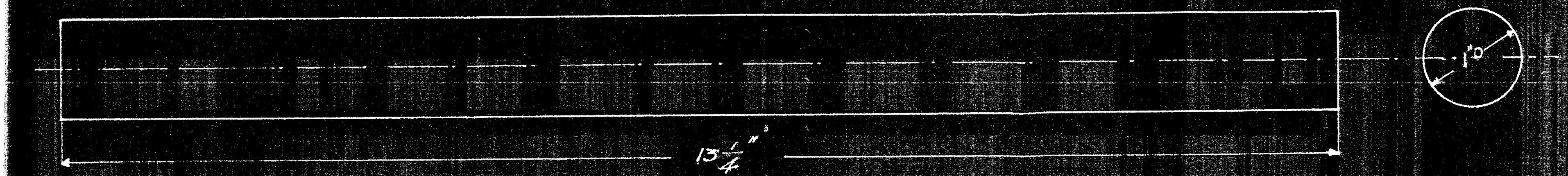
①



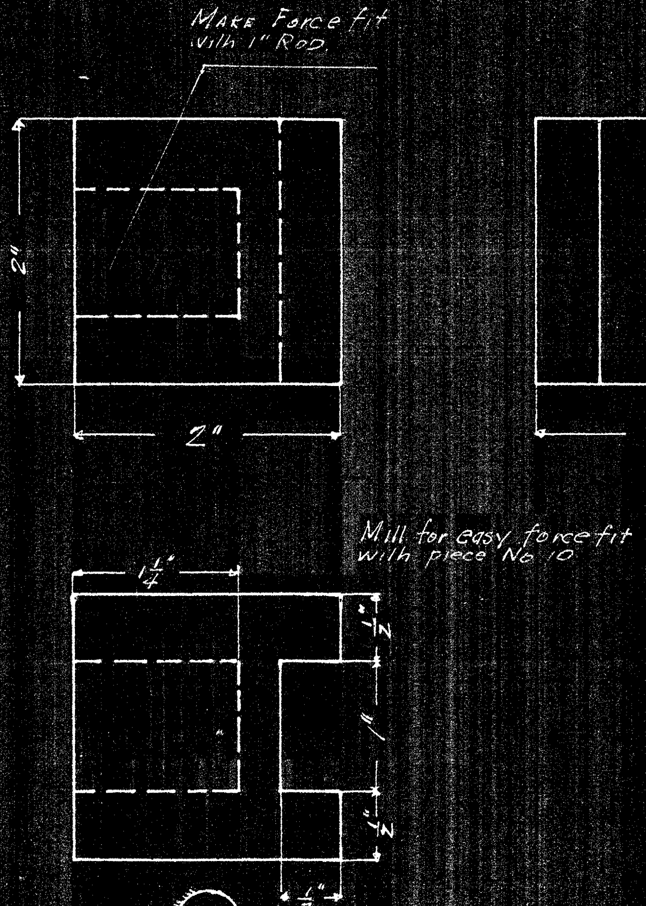
MAKE 1 ~ COLD ROLLED STEEL

# COMPRESSION ROD

⑪

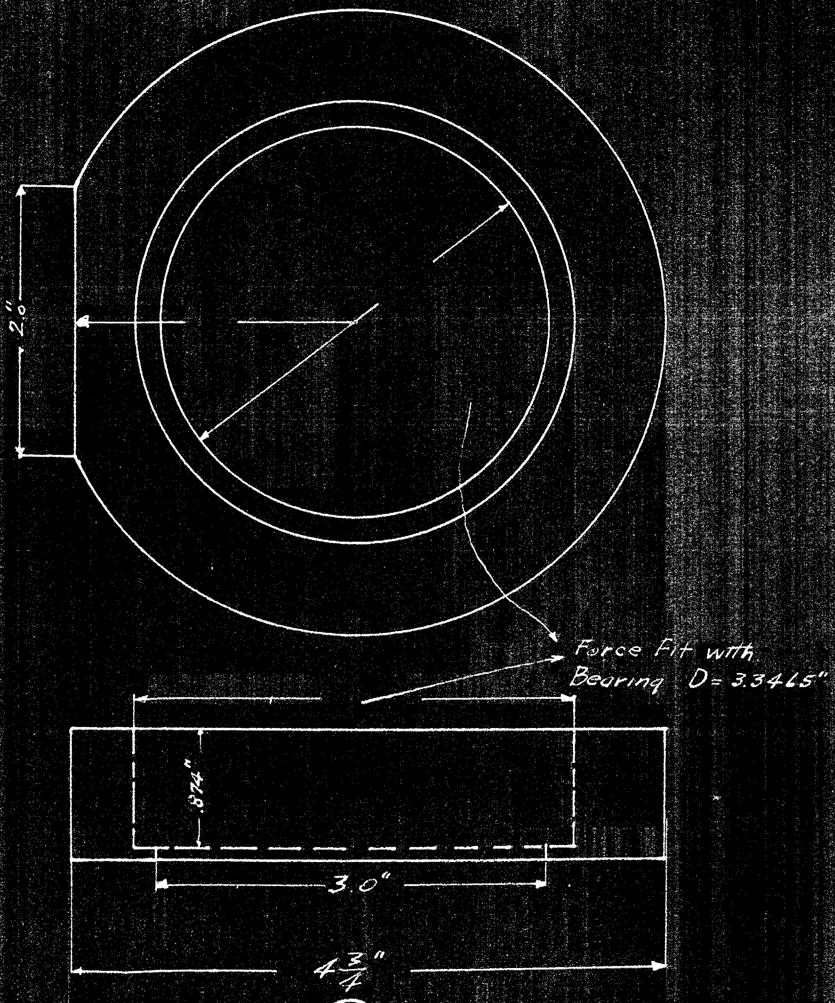


MAKE 2 ~ COLD ROLLED.



⑨

MAKE 2 ~ COLD ROLLED

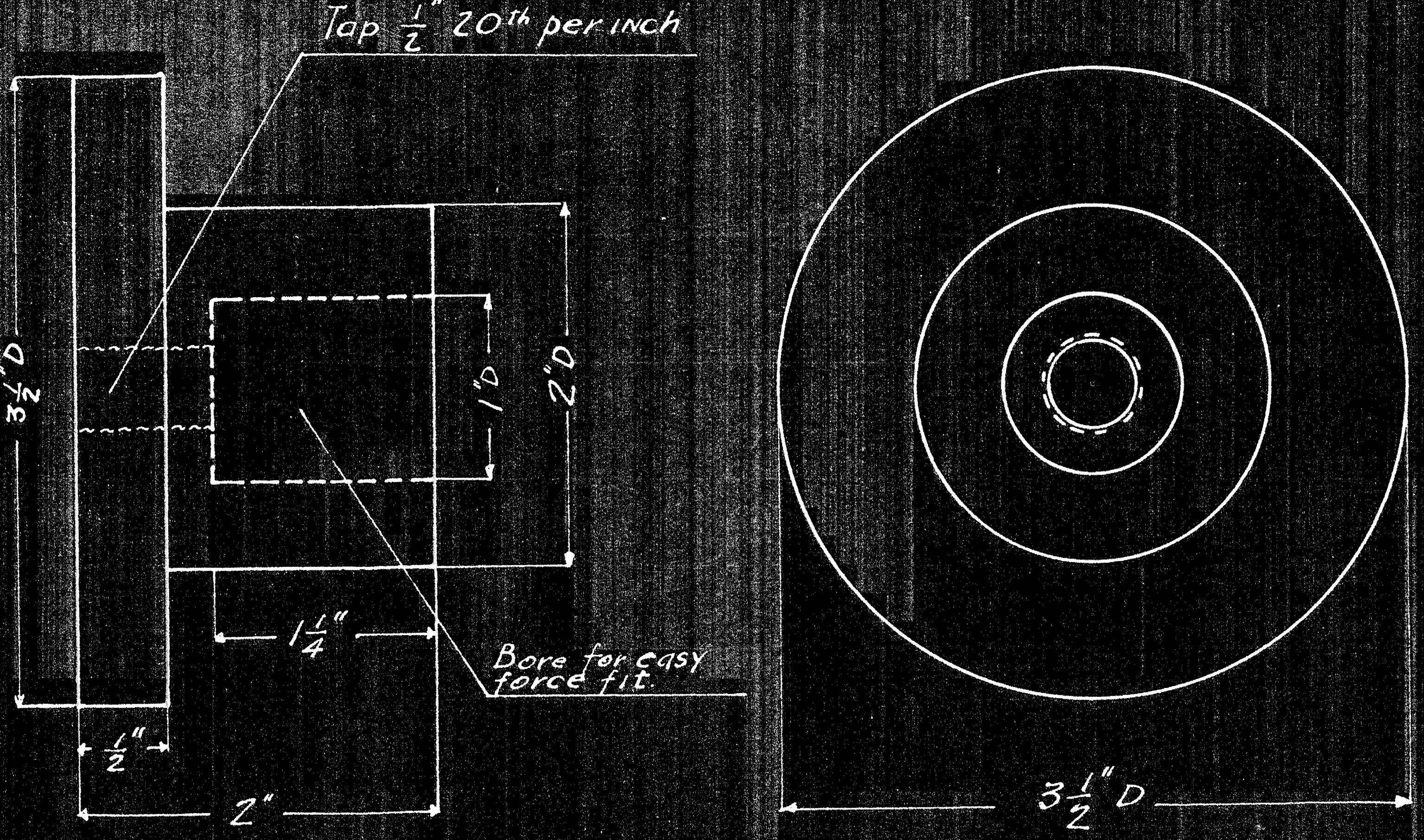


⑩

MAKE 2 - MILD STEEL

# COMPRESSION ROD SOCKET

②



MAKE 2 ~ COLD ROLLED STEEL

# APPENDIX B

## DATA ON DYNAMOMETER CALIBRATION

The following runs were made at constant temperature and in still air:

### RUN #1

8:30 A.M. - Temp. 78° F

| Load | Dial Reading (up) | Dial Reading (down) |
|---|---|---|
| 0 | 0 | 0 |
| 100 | 1.0 | 1.0 |
| 200 | 2.2 | 2.2 |
| 300 | 3.2 | 3.5 |
| 400 | 4.5 | 4.8 |
| 500 | 5.7 | 6.0 |
| 600 | 6.8 | 7.0 |
| 700 | 7.8 | 8.0 |
| 800 | 9.0 | 9.0 |
| 900 | 10.1 | 10.2 |
| 1000 | 11.5 | 12.0 |
| 1200 | 14.0 | 14.0 |
| 1400 | 16.0 | 16.1 |
| 1600 | 18.0 | 18.0 |
| 1800 | 20.0 | 20.0 |
| 200 | 22.0 | 22.0 |

(Calibration Data Cont'd)

## RUN #2

8:45 A.M. Temp 78° F

| Load | Dial Reading (up) | Dial Reading (down) |
|---|---|---|
| 0 | 0 | 0 |
| 100 | 1.0 | 1.0 |
| 200 | 2.0 | 2.1 |
| 400 | 4.2 | 4.5 |
| 600 | 6.5 | 7.0 |
| 800 | 9.0 | 9.0 |
| 1000 | 12.0 | 12.0 |

## RUN #3

9:50 A.M. Temp 80° F

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 100 | 1.0 | 1.0 |
| 200 | 2.1 | 2.1 |
| 300 | 3.1 | 3.5 |
| 400 | 4.2 | 4.8 |
| 600 | 6.4 | 7.0 |
| 800 | 8.8 | 9.4 |
| 1000 | 11.2 | 12.0 |
| 1200 | 13.8 | 14.1 |
| 1400 | 15.9 | 16 |
| 1600 | 17.9 | 18.1 |
| 1800 | 19.9 | 20.0 |
| 2000 | 21.9 | 22.0 |
| 2200 | 23.9 | 24.0 |
| 2400 | 25.9 | 26.0 |
| 2600 | 27.9 | 28.1 |
| 2800 | 30.0 | 30.0 |
| 3000 | 32.0 | 32.3 |
| 3200 | 34.0 | 34.2 |
| 3400 | 36.0 | 36.1 |
| 3600 | 38.0 | 38.5 |
| 3800 | 40.0 | 40.3 |
| 4000 | 42.0 | 42.0 |

(Calibration Data Cont'd)

RUN #4

3:30 P.M. Temp. 80° F

| Load | Dial Reading (up) | Dial Reading (down) |
|---|---|---|
| 0 | 0 | 0 |
| 100 | 1.0 | 1.1 |
| 200 | 2.1 | 2.2 |
| 300 | 3.3 | 3.7 |
| 400 | 4.5 | 4.9 |
| 500 | 5.5 | 6.0 |
| 600 | 6.8 | 7.0 |
| 800 | 9.0 | 9.3 |
| 1000 | 11.8 | 12.0 |
| 1200 | 14.0 | 14.0 |
| 1400 | 16.0 | 15.9 |
| 1600 | 18.0 | 17.8 |
| 1800 | 19.9 | 19.8 |
| 2000 | 21.9 | 21.9 |
| 2200 | 23.9 | 23.9 |
| 2400 | 25.9 | 26.0 |
| 2600 | 27.9 | 28.0 |
| 2800 | 30.0 | 30.0 |
| 3000 | 32.0 | 32.1 |

(Calibration Data Cont'd)

## SPECIAL RUN

Note - This run was performed at conditions of varying temperature, a fresh breeze from an open window blowing periodically across the dynamometer.

| Load | Dial Reading (up) | Dial Reading (down) |
|---|---|---|
| 0 | 0 | 0 |
| 100 | 1.0 | 0.8 |
| 200 | 2.0 | 2.0 |
| 300 | 2.75 | |
| 400 | 3.75 | 3.8 |
| 500 | 4.4 | |
| 600 | 5.3 | 5.4 |
| 700 | 6.1 | |
| 800 | 7.0 | 7.0 |
| 900 | 8.0 | |
| 1000 | 9.0 | 9.0 |
| 1100 | 10.0 | |
| 1200 | 11.0 | 11.0 |
| 1300 | 12.0 | |
| 1400 | 13.0 | 12.8 |
| 1500 | 13.95 | |
| 1600 | 15.0 | 14.3 |
| 1700 | 16.0 | |
| 1800 | 16.6 | 16.0 |
| 2000 | 18.0 | 18.0 |
| 2200 | 19.0 | 19.4 |
| 2400 | 21.4 | 21.2 |

(Calibration Data Cont'd)

SPECIAL RUN cont'd

| Load | Dial Reading (up) | Dial Reading (down) |
|---|---|---|
| 2600 | 23.0 | 23.0 |
| 2800 | 25.0 | 24.9 |
| 3000 | 26.3 | 26.7 |
| 3200 | 28.0 | 28.0 |
| 3400 | 30.0 | 30.0 |
| 3600 | 31.8 | 31.8 |
| 3800 | 33.0 | 33.5 |
| 4000 | 35.0 | 35.0 |

# APPENDIX C

# DATA ON COMBINED BENDING AND COMPRESSION TEST

## TUBE #1

Zero Reading on Deflection Scale = 5.22"

| End Load | Side Load | Deflection Read | 0 | True Deflection |
|---|---|---|---|---|
| 100 | 50 | 5.31 | 5.22 | 0.09 |
| 150 | 75 | 5.36 | 5.22 | 0.14 |
| 200 | 100 | 5.41 | 5.22 | 0.19 |
| 250 | 125 | 5.48 | 5.22 | 0.26 |
| 300 | 150 | 5.52 | 5.22 | 0.30 |
| 350 | 175 | 5.56 | 5.22 | 0.34 |
| 400 | 200 | 5.60 | 5.22 | 0.38 |
| 450 | 225 | 5.66 | 5.22 | 0.44 |
| 500 | 250 | 5.71 | 5.22 | 0.49 |
| 550 | 275 | 5.82 | 5.22 | 0.60 |
| 600 | 300 | 5.87 | 5.22 | 0.65 |
| 650 | 325 | 6.01 | 5.22 | 0.79 |
| 700 | 350 | 6.12 | 5.22 | 0.90 |
| 750 | 375 | 6.21 | 5.22 | 0.99 |

800 Failure in combined loading occurred at some indeterminate point between this last loading and the next condition of 800# end load and 400# side load.

## TUBE #2

Zero Reading onDeflection Scale = 5.40"

| End Load | Side Load | Deflection Read | 0 | True Deflection |
|---|---|---|---|---|
| 200 | 100 | 5.51 | 5.40 | 0.11 |
| 300 | 150 | 5.60 | 5.40 | 0.20 |
| 350 | 175 | 5.64 | 5.40 | 0.24 |
| 400 | 200 | 5.74 | 5.40 | 0.34 |

(Date on Combined Bending & Compression Test cont'd)

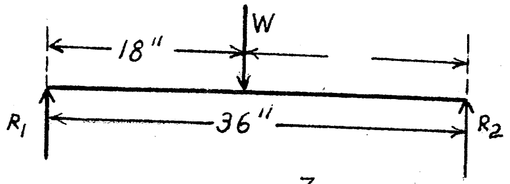
TUBE #2 cont'd

| End Load | Side Load | Deflection<br>Read | <br>0 | <br>True Deflection |
|---|---|---|---|---|
| 450 | 225 | 5.78 | 5.40 | 0.38 |
| 500 | 250 | 5.84 | 5.40 | 0.44 |
| 550 | 275 | 5.99 | 5.40 | 0.59 |
| 600 | 300 | 6.06 | 5.40 | 0.66 |
| 650 | 325 | 6.17 | 5.40 | 0.77 |

Failure in bending occurred at some indeterminate point between this last loading and the next condition of 700# end load and 350# side load.

# CALCULATIONS ON TEST DATA

Solution for Average EI



(1) $W = 10\#$ Basic Formula $d = \frac{Wl^3}{48EI}$

$d = .017"$

$$EI = \frac{Wl^3}{48d}$$

$$EI = \frac{(10)\ (36)^3}{(48)\ (.017)}$$

$$EI = 571{,}000$$

(2) $W = 20\#$

$d = .032"$

$$EI = \frac{(20)\ (36)^3}{(48)\ (.032)}$$

$$EI = 607{,}000$$

(3) $W = 30\#$

$d = .0475$

$$EI = \frac{(30)\ (36)^3}{(48)\ (.0475)}$$

$$EI = 613{,}000$$

(4) $W = 50\#$

$d = .0820$

$$EI = \frac{(50)\ (36)^3}{(48)\ (.082)}$$

$$EI = 593{,}000$$

$$\begin{array}{r} 571{,}000 \\ 607{,}000 \\ 613{,}000 \\ \underline{593{,}000} \\ 4\ \overline{)\underline{2{,}384{,}000}} \\ 596{,}000 \end{array}$$

Average Value of EI = 596,000

$A = .185$

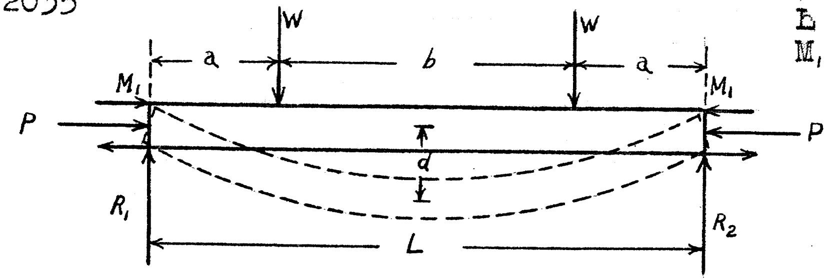
$I = .02035$

$a = 12''$

$b = 12''$

$L = 36''$

$M_1 = 0$



Basic Formula - $M_{L/2} = \dfrac{M_1}{\cos\frac{L}{2j}} - \dfrac{Wj\sin\frac{a}{j}}{\cos\frac{L}{2j}}$

Tube No. 1 Special Formula $M_{\frac{L}{2}} = -\dfrac{Wj\sin\frac{a}{j}}{\cos\frac{L}{2j}}$

| End Load | Side Load | Deflection |
|---|---|---|
| 100 lbs | 50 | .09" |

(1) $M = (18)(25) + (.09)(100) - (6)(25)$

$= 450 + 9 - 15.0$

$= 309$ inch pounds

(2) By Precise Method:

$$j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596{,}000}{100}} = \sqrt{5960} = 77$$

$$\frac{a}{j} = \frac{12}{77} = .156$$

$$\frac{L}{2j} = \frac{36}{154} = .234$$

$$\sin\frac{a}{j} = .14937$$

$$\cos\frac{L}{2j} = .97229$$

$$M = \frac{(25)(77)(.14937)}{.97229}$$

$= 295.5$ inch pounds

| End Load | Side Load | Deflection |
|---|---|---|
| 150 | 75 | .14 |

$$(1)\ M = (18)\ (37.5) + (.14)\ (150) - (6)\ (37.5)$$

$$= 675 + 21 - 255$$

$$= 441 \text{ inch pounds}$$

$$(2)\ j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596{,}000}{100}} = \sqrt{3970} = 63$$

$$\frac{a}{j} = \frac{12}{63} = .190$$

$$\frac{L}{2j} = \frac{36}{126} = .286$$

$$\sin \frac{a}{j} = .18886$$

$$\cos \frac{L}{2j} = .959$$

$$M = \frac{(37.5)\ (63)\ (.18886)}{.959}$$

$$M = 465 \text{ inch pounds}$$

---

| End Load | Side Load | Deflection |
|---|---|---|
| 200# | 50# | .19" |

$$(1)\ M = (18)\ (50) + (.19)\ (200) - (6)\ (50)$$

$$= 900 + 38 - 300$$

$$= 638 \text{ inch pounds}$$

$$(2)\ j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596{,}000}{200}} = \sqrt{2980} = 54.5$$

$$\frac{a}{j} = \frac{12}{54.5} = .22$$

$$\frac{L}{2j} = \frac{36}{109} = .33$$

$\sin \frac{a}{j} = .21823$

$\cos \frac{L}{2j} = .94604$

$$M = \frac{(50)\ (54.5)\ (.21823)}{.94604}$$

$M = 628$ inch pounds

---

| End Load | Side Load | Deflection |
|---|---|---|
| 250 # | 125# | .26 |

(1) $M = (18)\ (62.5) + (.26)\ (250) - (6)\ (62.5)$

$M = 1125 + 65 - 375$

$M = 825$ inch pounds

(2) $j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596{,}000}{250}} = \sqrt{2382} = 48.7$

$\frac{a}{j} = \frac{12}{48.7} = .246$

$\frac{L}{2j} = \frac{36}{97.4} = .37$

$\sin \frac{a}{j} = .2342$

$\cos \frac{L}{2j} = .932$

$$M = \frac{(62.5)\ (48.7)\ (.234)}{.932}$$

$M = 764$

---

| End Load | Side Load | Deflection |
|---|---|---|
| 300# | 150# | .30 |

(1) $M = (18)\ (75) + (.3)\ (300) - (6)\ (75)$

$= 1350 + 90 - 450$

$= 990$ inch pounds

(2) $$j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596{,}000}{300}} = \sqrt{1986} = 44.4$$

$$\frac{a}{j} = \frac{12}{44.4} = .270$$

$$\frac{L}{2j} = \frac{36}{88.8} = .406$$

$$\sin \frac{a}{j} = .2667$$

$$\cos \frac{L}{2j} = .9182$$

$$M = \frac{(75)\ (44.35)\ (.2668)}{.9182}$$

$= 966$ inch pounds

---

| End Load | Side Load | Deflection |
|---|---|---|
| 350# | 175# | .34 |

(1) $M = (18)\ (87.5) + (.34)\ (350) - (6)\ (87.5)$

$= 1575 + 11.9 - 525$

$M = 1062$

(2) $$j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596{,}000}{350}} = \sqrt{1702} = 41.3$$

$$\frac{a}{j} = \frac{12}{41.3} = .291$$

$$\frac{L}{2j} = \frac{18}{41.3} = .436$$

$$\sin \frac{a}{j} = .2859$$

$$\cos \frac{L}{2j} = .906$$

$$M = \frac{(87.5)\ (41.3)\ (.2859)}{.906}$$

M = 1138 inch pounds

---

| End Load | Side Load | Deflection |
|---|---|---|
| 400# | 200# | .38 |

(1) $M = (18)\ (100) + (.38)\ (400) - (6)\ (100)$

$= 1800 + 152 - 600$

$M = 1352$

(2)

$$j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596,000}{400}} = \sqrt{1490} = 38.6$$

$$\frac{a}{j} = \frac{12}{38.6} = .311$$

$$\frac{L}{2j} = \frac{18}{38.6} = .467$$

$$\sin \frac{a}{j} = .306$$

$$\cos \frac{L}{2j} = .892$$

$$M = \frac{(100)\ (38.6)\ (.306)}{.892}$$

$M = 1325$

---

| End Load | Side Load | Deflection |
|---|---|---|
| 450# | 225# | .44 |

(1) $M = (18)(112.5) + (.44)(450) - (6)(112.5)$

$= 2011 + 198 - 675$

$= 1534$

(2) $j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596,000}{450}} = \sqrt{1325} = 36.7$

$\frac{a}{j} = \frac{12}{36.7} = .327$

$\frac{L}{2j} = \frac{18}{36.7} = .490$

$\sin \frac{a}{j} = .321$

$\cos \frac{L}{2j} = .88233$

$M = \frac{(112.5)(36.7)(.321)}{.8823}$

$= 1495$ inch pounds

---

| End Load | Side Load | Deflection |
|---|---|---|
| 500# | 250# | .49 |

(1) $M = (18)(125) + (.49)(500) - (6)(125)$

$= 2250 + 245 - 750$

$= 1745$

$j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596,000}{500}} = \sqrt{1192} = 34.5$

$\frac{a}{j} = \frac{12}{34.5} = .348$

$\frac{L}{2j} = \frac{18}{34.5} = .521$

$\sin \frac{a}{j} = .341$

$\cos \frac{L}{2j} = .868$

$M = \frac{125 \times 34.2 \times .341}{.868}$

$M = 1680$

---

| End Load | Side Load | Deflection |
|---|---|---|
| 550# | 275# | .6 |

(1) $M = (18)(137.5) + (.\underline{6})(550) - (6)(137.5)$

$= 2475 + 330 - 825$

$= 1980$

(2) $j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596,000}{550}} = \sqrt{1085} = 33.$

$\frac{a}{j} = \frac{12}{33.} = .364$

$\frac{L}{2j} = \frac{18}{33.} = .545$

$\sin \frac{a}{j} = .356$

$\cos \frac{L}{2j} = .854$

$$M = \frac{(137.5)\ (33.)\ (.356)}{.854}$$

$$M = 1890$$

---

| End Load | Side Load | Deflection |
|---|---|---|
| 600# | 300# | .65 |

(1) $M = (18)\ (100) + (.65)\ (600) - (6)\ (150)$

$= 2700 + 390 - 900$

$M = 2190$

(2) $j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596,000}{600}} = \sqrt{994} = 31.5$

$$\frac{a}{j} = \frac{12}{31.5} = .381$$

$$\frac{L}{2j} = \frac{18}{31.5} = .572$$

$$\sin\frac{a}{j} = .3710$$

$$\cos\frac{L}{2j} = .842$$

$$M = \frac{(150)\ (31.5)\ (.3710)}{.842} = 2080$$

---

| End Load | Side Load | Deflection |
|---|---|---|
| 650# | 325# | .79 |

(1) $M = (18)(162.5) + (.79)(650) - (6)(162.5)$

$= 2922 + 513 - 975$

$= 2462$ inch pounds

(2) $j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596{,}000}{650}} = \sqrt{913} = 30.2$

$\frac{a}{j} = \frac{12}{30.2} = .396$

$\frac{L}{2j} = \frac{18}{30.2} = .594$

$\sin\frac{a}{j} = .386$

$\cos\frac{L}{2j} = .829$

$M = \frac{(162.5)(30.2)(.386)}{.829}$

$= 2287$

---

| End Load | Side Load | Deflection |
|---|---|---|
| 700# | 350# | .90 |

(1) $M = (18)(175) + (.9)(700) - (6)(162.5)$

$= \quad 3150 + 630 - 975$

$= \quad 2815$

(2) $j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596{,}000}{700}} = \sqrt{852.5} = 29.15$

$\frac{a}{j} = \frac{12}{29.15} = .412$

$\frac{L}{2j} = \frac{18}{29.15} = .618$

$\sin \frac{a}{j} = .400$

$\cos \frac{L}{2j} = .815$

$M = \frac{(175)\ (29.2)\ (.4)}{.815} = 2510$

---

| End Load | Side Load | Deflection |
|---|---|---|
| 750# | 375# | .99 |

(1) $M = (18)\ (187.5) + (.99)\ (750) - (6)\ (187.5)$

$= 3375 + 742 - 1125$

$= 2992$

$j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596{,}000}{750}} = \sqrt{795} = 38.2$

$\frac{a}{j} = \frac{12}{38.2} = .314$

$\frac{L}{2j} = \frac{18}{38.2} = .471$

$$\sin \frac{a}{j} = .3094$$

$$\cos \frac{L}{2j} = .891$$

$$M = \frac{(187.5)\ (38.2)\ (.3094)}{.891}$$

$$= 2485$$

$$\text{Fiber Stress} = \frac{750}{.185} + \frac{(2992)(15)}{.02035}$$

$$= 4060 + 73500$$

$$= 77{,}560$$

## TEST ON TUBE #2

| End Load | Side Load | Deflection |
|---|---|---|
| 200# | 100# | 0.11 |

(1) $M = 18 \times 50 + 0.11 - 6 \times 50$

$= 900 + 22 - 300 =$

$=$ 622 inch pounds

(2) $j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596,000}{200}} = \sqrt{2980} = 54.5$

$\frac{a}{j} = \frac{12}{54.5} = .220$

$\frac{L}{2j} = \frac{36}{109.0} = .33$

$\sin \frac{A}{j} = .21823$

$\cos \frac{L}{2j} = .94604$

$M = \frac{50 \times 54.5 \times .21823}{.94604} =$

$=$ 629 inch pounds

---

| End Load | Side Load | Deflection |
|---|---|---|
| 300# | 150# | .20 |

(1) $M = 18 \times 75 + .2 \times 300 - 6 \times 75$

$= 1350 + 60 - 450$

$=$ 960 inch pounds

(2) $j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596,000}{300}} = \sqrt{1986} = 44.4$

$$\frac{a}{j} = \frac{12}{44.4} = .270$$

$$\frac{L}{2j} = \frac{36}{88.8} = .406$$

$$\sin \frac{a}{j} = .2667$$

$$\cos \frac{L}{2j} = .9182$$

$$M = \frac{75 \times 44.4 \times .2667}{.9182}$$

$$= 967$$

---

| End Load | Side Load | Deflection |
|---|---|---|
| 350# | 175# | .24 |

(1) $M = 18 \times 87.5 + .24 \times 350 - 6 \times 87.5$

$$= 1575 + 84 - 525$$

$$= 1134$$

(2) $j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596,000}{350}} = \sqrt{1700} = 41.2$

$$\frac{A}{j} = \frac{12}{41.2} = .291$$

$$\frac{L}{2j} = \frac{36}{82.4} = .435$$

$$\sin \frac{a}{j} = .286$$

$$\cos \frac{L}{2j} = .907$$

$$M = \frac{87.5 \times 41.2 \times 286}{.907}$$

$$= 1137$$

| End Load | Side Load | Deflection |
|---|---|---|
| 400# | 200# | .34 |

(1) $M = 18 \times 100 + .34 \times 400 - 6 \times 100$

$= 1800 + 136 - 600$

$= 1336$ inch pounds

(2) $j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596{,}000}{400}} = \sqrt{1490} = 38.6$

$\frac{a}{j} = \frac{12}{38.6} = .311$

$\frac{L}{2j} = \frac{36}{77.2} = .466$

$\sin \frac{a}{j} = .305$

$\cos \frac{L}{2j} = .892$

$M = \frac{100 \times 38.6 \times (.305)}{.892}$

$= 1320$

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| End Load | Side Load | Deflection |
|---|---|---|
| 450# | 225# | .38 |

(1) $M = 18 \times 112.5 + .38 \times 450 - 6 \times 112.5$

$= 2024 + 171 - 675$

$= 1520$ inch pounds

(2) $j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596{,}000}{450}} = \sqrt{1325} = 36.7$

$\frac{a}{j} = \frac{12}{36.7} = .327$

$$\frac{L}{2j} = \frac{36}{73.4} = .490$$

$$\sin\frac{a}{j} = .321$$

$$\cos\frac{L}{2j} = .8823$$

$$M = \frac{112.5 \times 36.7 \times .321}{.8823} =$$

$$= 1500 \text{ inch pounds}$$

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| End Load | Side Load | Deflection |
|---|---|---|
| 500# | 250# | .44 |

(1) $M = 18 \times 125 + .44 \times 500 - 6 \times 125$

$= 2250 + 220 - 750 = 1720$

(2) $j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596,000}{500}} = \sqrt{1192} = 34.5$

$$\frac{a}{j} = \frac{12}{34.5} = .348$$

$$\frac{L}{2j} = \frac{36}{69.0} = .521$$

$$\sin\frac{a}{j} = .341$$

$$\cos\frac{L}{2j} = .868$$

$$M = \frac{125 \times 34.5 \times .341}{.868}$$

$$= 1694 \text{ inch pounds}$$

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| **End Load** | **Side Load** | **Deflection** |
|---|---|---|
| 550# | 275# | .59 |

(1) $M = 18 \times 137.5 + .59 \times 550 - 6 \times 137.5$

$= 2477 + 324 - 825$

$= 1976$ inch pounds

(2) $j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596,000}{550}} = \sqrt{1085} = 32.9$

$\frac{a}{j} = \frac{12}{32.9} = .364$

$\frac{L}{2j} = \frac{36}{65.8} = .547$

$\sin \frac{a}{j} = .35601$

$\cos \frac{L}{2j} = .85387$

$M = \frac{137.5 \times 32.9 \times .35601}{.85387}$

$= 1890$ inch pounds

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| **End Load** | **Side Load** | **Deflection** |
|---|---|---|
| 600# | 300# | .66 |

(1) $M = 18 \times 150 + .66 \times 600 - 6 \times 150$

$= 2700 + 396 - 900$

$= 2196$ inch pounds

(2) $j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596,000}{600}} = \sqrt{994} = 31.5$

$$\frac{a}{j} = \frac{12}{31.5} = .381$$

$$\frac{L}{2j} = \frac{36}{63.0} = .570$$

$$\sin \frac{a}{j} = .371$$

$$\cos \frac{L}{2j} = .842$$

$$M = \frac{150 \times 31.5 \times .371}{.842} = 2080$$

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| End Load | Side Load | Deflection |
|---|---|---|
| 650# | 325# | .77 |

$$(1)\ M = 18 \times \frac{325}{2} + .77 \times 650 - 6 \times \frac{375}{2}$$

$$= 2925 + 500 - 1125$$

$$= 2300 \text{ inch pounds}$$

$$(2)\ j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{596{,}000}{650}} = \sqrt{918} = 30.3$$

$$\frac{a}{j} = \frac{12}{30.3} = .396$$

$$\frac{L}{2j} = \frac{36}{60.6} = .594$$

$$\sin \frac{a}{j} = .38573$$

$$\cos \frac{L}{2j} = .82870$$

$$M = \frac{162.5 \times 30.3 \times .386}{.829} = 2294$$

<u>Fibre Stress</u> $= \dfrac{650}{.185} + \dfrac{2300 \times 5}{.02035}$

$= 3510 + 56500 = 60010$ lbs per sq in