6.453 Quantum Optical Communication — Lecture 1

- Handouts
  - Syllabus, schedule/policy, probability chapter, lecture notes, slides, problem set 1
  - Sign-up on class list

- Introductory Remarks
  - Subject organization
  - Subject outline

- Technical Overview
  - Optical eavesdropping tap — quadrature-noise squeezing
  - Action at a distance — polarization entanglement
  - Long-distance quantum state transmission — qubit teleportation
**Optical Homodyne Detection — Semiclassical**

- Signal is weak, LO is strong
- Energy conservation
  \[ a_\pm = \frac{a_s \pm a_{LO}}{\sqrt{2}} \]
- Detectors are noisy square laws
  \[ i_\pm(t) \text{ Poisson distributed} \]
  \[ \text{mean} = |a_\pm|^2 \]
- Output mean and variance
  \[ \langle K \Delta i(t) \rangle = 2K \text{Re}(a_s a_{LO}^*) \]
  \[ \text{var}(K \Delta i(t)) = K^2 |a_{LO}|^2 \]

**Optical Waveguide Tap — Semiclassical**

- Coupler is a beam splitter
  \[ a_{s_{\text{out}}} = \sqrt{T} a_{s_{\text{in}}} + \sqrt{1 - T} a_{t_{\text{in}}} \]
  \[ a_{t_{\text{out}}} = \sqrt{1 - T} a_{s_{\text{in}}} - \sqrt{T} a_{t_{\text{in}}} \]
- Tap input is zero
- Homodyne SNR at signal input
  \[ \text{SNR}_{\text{in}} = 4 |a_{s_{\text{in}}}|^2 \]
- Homodyne SNR at signal output
  \[ \text{SNR}_{\text{out}} = 4T |a_{s_{\text{in}}}|^2 \]
- Homodyne SNR at tap output
  \[ \text{SNR}_{\text{tap}} = 4(1 - T) |a_{s_{\text{in}}}|^2 \]
Quantum Homodyne Detection and Waveguide Tap

Balanced Homodyne Receiver

Fused Fiber Coupler

Homodyne SNR at signal output

$$\text{SNR}_{\text{out}} \approx 4|a_{\text{sin}}|^2$$

Homodyne SNR at tap output

$$\text{SNR}_{\text{tap}} \approx 4|a_{\text{sin}}|^2$$

Billiard-Ball Photons and the Poincaré Sphere

- Polarization of $+z$-going photon:

$$\mathbf{i} \equiv \begin{bmatrix} \alpha_x \\ \alpha_y \end{bmatrix}, \quad \mathbf{i}^{\dagger}\mathbf{i} = 1$$

- Poincaré sphere representation

$$\mathbf{r} \equiv \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} 2\text{Re}(\alpha_x^*\alpha_y) \\ 2\text{Im}(\alpha_x^*\alpha_y) \\ |\alpha_x|^2 - |\alpha_y|^2 \end{bmatrix}$$

- $\pm \mathbf{r}_m$ polarization measurement

$$\text{Pr}($$

$$\text{polarized } \pm \mathbf{r}_m$$

$$\text{)} = \frac{1 \pm \mathbf{r}_m^{\dagger}\mathbf{r}}{2}$$
Classical Correlation vs. Quantum Entanglement

- Classical-Correlated, Randomly-Polarized Photons
  - Source produces $\pm r$ photon pair with $r$ completely random
    \[ \Pr(\text{photon } 1 = \pm r_m) = \Pr(\text{photon } 2 = \mp r_m) = 1/2 \]
  - Conditional probability given photon $1$ is $r_m$ instead of $-r_m$
    \[ \Pr(\text{photon } 2 = -r_m \mid \text{photon } 1 = r_m) = 2/3 \]

- Maximally-Entangled Photons
  - Source produces $\pm r$ photon pair with $r$ completely random
    \[ \Pr(\text{photon } 1 = \pm r_m) = \Pr(\text{photon } 2 = \mp r_m) = 1/2 \]
  - Conditional probability given photon $1$ is $r_m$ instead of $-r_m$
    \[ \Pr(\text{photon } 2 = -r_m \mid \text{photon } 1 = r_m) = 1 \]

Properties of Single-Photon Polarization States

- Polarization cannot be perfectly measured $\rightarrow$

- $\leftarrow$ Polarization cannot be perfectly cloned

- Photons can be lost in propagation:
  \[ \Pr(\text{photon loss in 50 km of low-loss fiber}) = 0.9 \]
Photon Polarization States Can Be Teleported

The Road Ahead: Problem Set 1, Lectures 2 and 3

- Problem Set 1
  - Reviews of essential probability theory and linear algebra

- Lectures 2 and 3:
  - Fundamentals of Dirac-Notation Quantum Mechanics
    - Quantum systems
    - States as ket vectors
    - State evolution via Schrödinger’s equation
    - Quantum measurements — observables
    - Schrödinger picture versus Heisenberg picture