SOME CALCULATIONS ON THE RANGE
AND STRAGGLING OF PROTONS

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Introductory Remarks

In the following pages frequent use is made of a number of terms which have been generally accepted in the literature on nuclear physics. Consequently, the author abstained, in most cases, from giving definitions in order to avoid stretching this thesis to an unjustifiable length and in order to emphasize the original material presented. Concise definitions of the fundamental terms are available in the works of R. D. Evans (1), G. Mano (2), and M. G. Holloway and M. S. Livingston (3). The numbers in this text refer to references contained in the bibliography at the end of this paper.
ABSTRACT

Part I of this work concerns the range energy relation of protons. The literature pertaining to this subject is summarized and discussed. It was attempted to obtain additional range energy data by deriving two empirical relations, which make it possible to obtain mean ranges from experimental proton Bragg curves. The procedure was as follows:

By means of the cinema integrator, two different curves of the ionization of the single proton were combined with a family of Normal Law curves to yield two sets of proton Bragg curves. (For this purpose, one of the single proton ionization curves had to be reduced to standard conditions, and it was necessary to prepare a summary of the literature on the atomic stopping power of helium.) The linear portions of these Bragg curves were extrapolated and for each set a linear relation was determined, which holds between \( y \) and \( \alpha \), and between \( \alpha \) and \( x \), where \( y \) measures the slope of the Bragg curves, \( \alpha \) is the range straggling parameter and \( x \) is the difference between the extrapolated ionization range and the mean range. The factors determining the probable
error of each of these relations are discussed.

The attempt to gain supplementary information on the range energy relation proved to be not feasible. Only one experimental proton Bragg curve (for which the initial energy of the proton beam is known) could be found in the literature. However, evidence is presented here which indicates that this Bragg curve is not sufficiently accurate to make a calculation of the corresponding mean range possible. Consequently, the empirical relations mentioned above could not be applied as yet to check the presently accepted range energy relation for protons ("Revised Cornell Relation 1937").

In Part II, the range straggling parameter as a function of residual proton range is computed for the following absorbers: hydrogen, helium, carbon, air, neon, and aluminum. Bethe's straggling equation was used in the calculations and the values of the constant $k$ were taken from the unpublished work of A. M. Clogston, who obtained the values of $k$ for elements of low atomic number.

Part III contains the description and photograph of the model which was built, as a part of this thesis, to help visualize the dependence of the range straggling on the range, the relative magnitudes of these two distances at different energies and the unsymmetrical
nature of the energy straggling. The calculations for the model are based on the "Revised Cornell Relation 1937" (i.e., the ranges are in air), on Bethe's straggling equation (with Clogston's value of k), and on Gaussian distributions of the ranges.
PART I

Some Calculations on the Range of Protons

A. INTRODUCTION

A great number of experiments in nuclear physics are devised to measure the mass, spin, or energy of a nuclear particle. In this paper, we shall first consider the methods which are available for determining the energy of the particles emitted in nuclear transformations. Obviously, our knowledge of nuclear reaction energies and often also of nuclear masses, depends on the success of these energy measurements. In the following, we shall be primarily interested in protons. But these general statements also apply to other heavy particles such as α rays, deuterons, H³ and He³ nuclei.

The direct method for the measurement of the energy (or of the velocity) of a beam of protons consists of a magnetic deflection of the protons in an evacuated chamber. By limiting the rays through a system of slits and using a photographic plate as a detector, the radius of curvature $\rho$ of the circularly deflected beam can be calculated from the geometry of the apparatus. Knowing the magnetic field strength $H$ and assuming the charge $E$ and the rest mass of the proton as given, the
velocity $V$ can be obtained from the elementary formula:

$$V = \frac{E}{M} H \rho \quad (I - 1)$$

where $M$ is the relativistic mass of the proton. From formula $(I - 1)$ we can easily understand the great difficulty involved in this method. Because of the large mass of the proton (as compared to an easily deflectable particle like the electron), the formula requires high field intensities of the order of a few thousand gauss. It is evident that for high energy protons we need either an unusually strong magnet or an apparatus of cumbersome dimensions. In spite of this disadvantage, however, the magnetic deflection technique must always be used for direct energy measurements.

For a number of years an indirect method has been developed which allows routine measurements of proton energies with the help of comparatively simple apparatus. This procedure is essentially an absorption method. It is based on the fact that protons of a particular energy have a certain well defined "mean range," which depends on the nature of the absorber. The relationship between the energy of a group of protons and their mean range in dry air at $15^\circ C$ and 760 mm Hg is called the range energy relation for protons. Once this relation is firmly established, measurements
of proton energies can be carried out by determining the mean range in standard air. This can be done by means of the ionization chamber, particle counter, or cloud chamber. Range measurements made in absorbers other than air can be reduced to ranges in air (i.e., expressed in units of "air-om") if the atomic stopping power of the absorbing substance is known.

It must be emphasized at this point that the range energy relation is by no means as accurately known for protons as it is for α rays. This is due to the fact that protons, unlike certain α rays, are not emitted by naturally radioactive substances. Protons must be obtained by artificial disintegration (usually by bombardment of paraffin with α rays) and hence the development of sources, which emit proton beams of sufficient intensity and homogeneity, constitutes a major problem. This probably explains the lack of an adequate number of direct range and energy measurements for protons as well as the absence of fairly accurate determinations of proton Bragg curves and of the specific ionization of the single proton.
B. THE LITERATURE ON THE RANGE ENERGY RELATION FOR PROTONS

1. The Theory

The theory of the range energy relation for protons has been worked out by Bethe (4,5) in his wave-mechanical treatment of the absorption of swift charged particles in their passage through matter. Bethe's basic differential equation for the rate of change of the kinetic energy of a nuclear ray with respect to the distance traversed has two serious limitations which restrict its practical value. In the following, we shall discuss these two limitations from the point of view of the usefulness of this equation for the range energy relation. No attempt will be made to reproduce the formulae and the method of integration since several references (1,2) are available on this subject.

First of all, Bethe's formula is not valid for protons of "very low" velocity because it was obtained by assuming that the velocity of the protons is much greater than the velocity of any of the electrons in the absorbing substance. Thus for every absorber there is a certain critical proton velocity below which the formula is not valid. (Part III of this thesis provides an example for this lower limit.) Consequently, Bethe's differen-
tial equation cannot be integrated over the range interval from $r = 0$ to $r = R$ and hence it does not enable us to calculate the initial energy of protons of mean range $R$. Similarly, we cannot integrate from $V = 0$ to some velocity $V$, so that absolute mean ranges cannot be calculated. We can, however, integrate Bethe's equation between two velocities which are greater than the critical velocity and thus obtain range differences.

The second drawback of this theory is its inability to predict (for absorbers other than hydrogen) the value of the "average excitation potential" $J$, which is a constant of the above-mentioned differential equation. Hence one has to evaluate $J$ for each absorber from experimental data. This is done essentially by assuming Bethe's equation to be correct. Two sets of data (corresponding to two experimentally determined points on the range energy relation) are substituted into the integrated form of the equation, which is then solved for $J$.

These two features of Bethe's equation show that it is far from being a truly theoretical formula. Nevertheless, "it does facilitate interpolation between measured ranges, extrapolation of range data to very high energy rays and the interpretation of data on the ab-
sorbing power of other materials relative to air." (1) It might be added here that the theoretical work of Bloch (6), which was carried out by a method different from that of Bethe, lead to the same result except for two additional correction terms. On account of the assumptions underlying Bloch's solution, however, it was soon found (5) that the correction terms become negligible in the domain of validity of the formula. The results of Bethe and Bloch are thus identical.

2. Experiments and Calculations

The literature dealing with experimental investigations on the range energy relation for protons will now be discussed. The papers containing calculations by means of the Bethe theory will be summarized, following the experimental articles on which they are based.

In 1930 Gerthsen (7) performed the first set of correlated range and energy measurements for protons. He used an ionization chamber in which the pressure of the absorbing gas (air, and also hydrogen) was varied. Knowing the radius of the chamber, the range at standard pressure of protons of a certain energy (determined by magnetic deflection) was deduced from the pressure at which the protons "just reach" the wall of the ioniza-
tion chamber. Obviously, these ranges depend on the apparatus and have no relation to mean or extrapolated ranges. Temperature corrections were not applied in these experiments. It was found that between 27 and 57 ekv. the "range" is proportional to the 1.5 th. power of the velocity.

By means of similar apparatus, H. Batzner (8) verified Gerthsen's relation for the absorbers aluminum, copper, silver, tin, and gold in the region from 4 to 60 ekv.

Blackett and Lees (9) determined the ranges of slow protons (up to about 471 ekv. energy) in standard air. Their method consisted of producing protons by collisions of \( \alpha \) particles with hydrogen nuclei and measuring the ranges of the incident \( \alpha \) rays, the deflected \( \alpha \) rays and the emitted protons in a cloud chamber. The energy of the protons was not directly measured but calculated from the energy of the \( \alpha \) rays by means of classical collision theory, which gives the ratio of the \( \alpha \) ray energies after and before collision as a function of the angle of deflection. Knowing the energy of the incident \( \alpha \) rays from their ranges, the energy of the protons immediately follows. Blackett and Lees measured 69 hydrogen forks which were taken from about a million tracks and so obtained 69 points through which they drew their range energy relation for protons. Since this procedure did not (and prob-
ably could not) involve many range measurements for the same proton energy, the straggling effect was not taken into account. This should have been done by determining the mean range from a Gaussian distribution of the ranges, in the manner described by King and Rayton (10) for α particles. Consequently, Blackett and Lees' ranges do not represent mean ranges even though the straggling is comparatively small at such low energies. (See Part II for the values of the range straggling coefficient at different energies.) The greatest drawback of this procedure arises from the fact that the range values are based on the assumed correctness of the α ray range energy relation which was used in the calculations. Since Brigg's relation had to be used (no other α ray range energy relation being available at the time), Blackett and Lees' values of the energy at a given range are now increased by 9% (5) in order to correct for the discrepancy between Brigg's relation and the more reliable modern determinations. The errors in Blackett and Lees' experiments are due to inaccuracies in measurements of the angles and ranges and to the necessity for using approximate values of the stopping power of hydrogen in reducing the data taken in an air-hydrogen mixture to standard air.

According to a review paper by Becker (11), several other experiments have been performed in which the pro-
ton energy was derived from the $\alpha$ ray range energy relation in a manner similar to Blackett and Lees. The results are in approximate agreement with the proton range energy relations which were calculated on the basis of Blackett and Lees' data. These calculations will now be considered.

Blackett (12) and Duncanson (13) used Bethe's equation (with corrections for variations of charge but not for relativity) to calculate, on the basis of Blackett and Lees' data, a range energy relation for protons extending into the domain of higher energies. They regarded the average excitation potential $J$, as well as the atomic number $Z$, as an adjustable parameter.

Manzo (14) calculated by means of Bloch's formula (including also the relativity correction) a table of the range energy relation up to energies of about 19.3 Mev. He used for the integration one uncorrected point of Blackett and Lees' original curve and proceeded essentially as explained in Section 1.

Livingston and Bethe's (15) range energy relation was obtained in the following manner: The theoretical range energy curve for $\alpha$ rays was recalculated by making better approximations in Bethe's equation (including the relativity correction) and obtaining a new value for the
constant J. Certain empirical corrections were then applied to bring the relation in agreement with observations on \( \alpha \) particle ranges. From the final relation, the proton range energy relation was calculated by using the fact that, according to ionization theory, \( \alpha \) rays and protons of equal velocity have approximately equal ranges: 

\[
R_p(v) = 1.0072 \ R_\alpha(v) - 0.20 \text{ cm.}
\]

The value of the constant (0.20 cm.) is purely empirical. It is taken directly from the experimental results of Blackett and Lees and hence it is probably not very accurate. In the low energy region (up to 0.3 Mev.), Blackett and Lees' results (corrected for the 9%) were used. Unfortunately, some mistakes were made in the calculation of the theoretical part of this range energy relation. However, the curve so obtained passes "very near" (not within the claimed accuracy) to two experimental points which were obtained by Cockcroft and Walton (16) in preliminary measurements. The procedure followed in these calculations obviously depends on the correctness of the \( \alpha \) ray range energy relation. Since the latter was changed by the experimental work of Holloway and Livingston (3) Bethe's proton range energy relation is certainly not correct.

Realizing the need for more accurate experimental
determinations, Parkinson and his collaborators (17) measured the ranges of protons in aluminum and in air as a function of proton energy up to about 2 Mev. In the case of aluminum, this was done by measuring the proton current incident on the absorbing foil and the current due to those protons, which penetrated this foil, by using a null method electrical circuit. The ratio of these two currents was then plotted as a function of proton energy (measured by a voltmeter method) for each of the various thicknesses of the aluminum foil. Finally, a range curve was constructed by extrapolating these curves down to a ratio of zero and plotting the value of energy so obtained against the foil thickness. This method evidently amounts to determining "number-energy curves." (However, the published data only show the fraction of the incident protons transmitted by various thicknesses of aluminum.) The ranges mentioned above are thus "extrapolated number-energy ranges." By plotting the values of the energy at half-maximum (i.e., at 50% transmission) against the corresponding thicknesses, mean ranges in aluminum could have been obtained.

The data for the determination of the ranges in air were taken as follows: After traversing an aluminum foil of known air equivalent, a predetermined thickness of air and another aluminum foil, the protons were made to
enter an ionization chamber of 0.4 mm. depth. Using several values for the length of the air path, measurements were then made of the ratio of the current in the ionization chamber to the incident proton current as a function of proton energy. The values so obtained were plotted and every curve (corresponding to a certain air path, i.e., range) was extrapolated to a ratio of zero. Finally, the extrapolated energy values were plotted against the ranges, which were reduced to 0°C and 760 mm Hg. At energies above 0.7 Mev. there is a remarkably good agreement with Mano's calculated curve (14), in which the customary mean ranges are given. The visually estimated range curve of Tuve (18) and his collaborators is certainly not very accurate (according to the authors, errors of 1 mm. are possible in the range values) and hence it is not significant that it disagrees considerably with Parkinson's range curve.

In order to obtain a proton range energy relation based on the most reliable experimental data, Bethe (19) made two obvious corrections on Parkinson's range curve. First of all, the ranges in air were increased by 0.2 mm. (i.e., half the depth of the ionization chamber) to obtain mean ranges. These were then reduced to the standard temperature of 15°C by multiplying by the factor \( \frac{288}{273} \).
Secondly, "mean voltages" had to be determined. This was done by assuming that the above-mentioned ionization curves can be used to obtain mean energies (with an error of less than 5%) by taking the values of the energies at half maximum ionization. Since these ionization curves are completely analogous to Bragg curves, this procedure seems to be justified. (α-ray Bragg curves are known to have the property just mentioned.) Considering that these "Bragg curves" have approximately constant slope, the extrapolated energy values had to be increased by about 20 eKv. to obtain mean energies. (In Bethe's paper it is not explained how the second correction was made. But the method is clearly understood by comparing Parkinson's range table with the final range energy relation.) It is easily seen that these corrections were not applied quite rigorously. They represent, however, the best possible evaluation of Parkinson's measurements. Based on these data, Bethe calculated the range energy relation for the high energy region up to 15 Mev. According to Bethe, the so obtained "Revised Cornell Relation 1937" has a probable error of 5%. (Bethe used this range energy relation in his recalculation of the binding energy of the deuteron and the mass of the neutron and several nuclei (19).) For simplicity, this relation will now be referred to as the range energy rela-
tion. At the present time (summer, 1939) it is generally accepted. Since the range energy relation has not been published (20), we shall reproduce it in the region from zero to 1.8 Mev. (Fig. I - 1)

C. THE EMPIRICAL RELATIONS FOR THE CALCULATION OF MEAN RANGES FROM EXPERIMENTAL PROTON BRAgg CURVES

1. The Method

In order to obtain independent data for the range energy relation, it will now be attempted to derive empirical linear equations which make it possible to calculate mean ranges from experimental proton Bragg curves. Knowing the initial energy of the proton beam which was used in the determination of a Bragg curve, this method enables us to obtain from each Bragg curve one point for the range energy relation.

The procedure to be used has often been discussed in the $\alpha$-ray literature. Most recently, Holloway and Livingston (3) made use of this method to obtain such "empirical relations" on the basis of their redeterminations of the specific ionization of the single $\alpha$-ray. (In the following, the previously mentioned linear equations will be referred to as the "empirical relations.")

It is a well known feature of $\alpha$-ray Bragg
curves that their straight line part can be extrapolated linearly (to zero ionization) with a high degree of accuracy. Several investigators have also noticed the empirical fact that the relation between \( x \) (\( x \) = the difference between the so obtained extrapolated ionization range and the mean range) and the experimental range straggling parameter \( \alpha \) can be approximated with "sufficient" accuracy by a linear equation. Similarly, it was found that \( \alpha \) is a linear function of the slope of the Bragg curve. Following Holloway and Livingston, this slope is expressed by the distance \( y \), measured on the range axis \( r \), between the points \( P_1 \) and \( P_2 \): \( P_1 \) is the intercept of the straight line \( s \), which was used for the extrapolation, with the axis \( r \) and \( P_2 \) represents the projection on \( r \) of the point \( P'_2 \), where \( P'_2 \) is the intersection of \( s \) with a straight line drawn parallel to \( r \) through the ordinate of maximum ionization.

Knowing these empirical relations, the value of \( \alpha \) and of the mean range can thus be obtained from an experimental Bragg curve. The accuracies attained for \( \alpha \)-rays are great enough to use these mean ranges for the range energy relation.

The method of obtaining the empirical relations is the same for protons and \( \alpha \)-rays. In view
of the following investigations, we shall give the explanation in terms of the word "proton."

If the specific ionization curve of the single proton is experimentally known, we can construct a series of Bragg curves each of which corresponds to a certain chosen value of the range straggling parameter $\alpha$. This is done on the basis of equation (I - 2) which can be regarded as being the definition of a Bragg curve, i.e., of the specific ionization curve for a beam of homogeneous protons.

$$I(z) = \sum_{x-z}^{x=\infty} f(x) I(x-z) \quad (I - 2)$$

Where: $I(z) =$ specific ionization at $z$ of a beam of homogeneous protons (i.e., ordinate of the Bragg curve at $z$).

$I(x-z) =$ ionization produced by a single proton in a short interval at an average distance $x-z$ from the end of its track (i.e., ordinate of the specific ionization curve of the single proton at $x-z$ residual range).

$f(x) =$ ordinate of the Gaussian distribution function characterized by the chosen
value of $\alpha$. $x$ and $z$ are measured from the same origin.

The meaning of equation $(I - 2)$ is obvious. At the distance $z$, the average specific ionization $I(z)$ is composed of the sum of the ionizations at $z$ of all those protons, which have a range $x$ greater than $z$. The factor $f(x)$ determines how many protons (or rather what fraction of the total number of protons) have a range $x$ so that their contribution $I(x-z)$ to the ionization at $z$ must be multiplied with $f(x)$. $I(z)$ thus represents one point of the Bragg curve which is characterized by a chosen value of $\alpha$. By making this summation for several values of $z$ (chosen in suitably small intervals) we obtain the whole Bragg curve.

It is evident that these calculations are very laborious, especially if (as in our case) several Bragg curves have to be computed. However, this work can be done in comparatively short time by means of the cinema integrator. This machine is based on a photoelectric device. The multiplications are performed by means of a logarithmic amplifier method.

A family of Gaussian distribution curves was first plotted on a large sheet of graph paper. The values of the standard deviation $\sigma \quad (\sigma = \frac{\alpha}{\sqrt{2}})$ were
deduced from the following chosen values of \( \alpha \): 0.062, 0.080, 0.100, 0.115, 0.124, 0.135, 0.150, and 0.200.

On a similar sheet and using the same range scale, two different experimental curves were then drawn, which represent two determinations of the specific ionization of the single proton. These curves were combined by means of the cinema integragraph and yielded two sets of Bragg curves. After extrapolating all the Bragg curves, two sets of empirical relations were obtained (see section C3), which were then applied to an experimental Bragg curve (see section D).

Because of the relatively high accuracy of the cinema integragraph, the usefulness of the empirical relations depends mainly on the accuracy of the specific ionization curves of the single proton. Consequently, we shall now briefly summarize the experimental methods on which these two ionization curves are based. In addition, certain corrections will be discussed which had to be applied to one of these curves.

2. The Specific Ionization of the Single Proton

Apparently only two determinations of the specific ionization of the single proton have been made up to the present time.

The curve of Schmidt and Stetter (21) was obtained in 1930 by means of an ionization method. A
vacuum tube electrometer was used for the measurement of the ionization and great care was taken to eliminate the effect of straggling. However, the data on the electrometer deflections were evaluated in such a way (p. 128 of the reference) that the result does not represent the specific ionization of the proton of mean range. The last part of the range (about the last 2\(\frac{1}{2}\) mm.) was obtained by extrapolation since the deflections became too small to be measured. Data on the relative number of deflections (assumed to be inversely proportional to the specific ionization in domains of approximately equal deflections) and linear extrapolation were used. This procedure leads to an uncertainty of about one mm. in the residual range values. The data were taken at standard conditions and they are accurate, according to the investigators, to within 10% of their values. It will be noticed that the published curve (Fig. 13 of the paper) does not pass through the point zero on the residual range scale. This is obviously wrong and is probably due to an error in printing. In reproducing this curve for use with the cinema integragraph, the whole curve was shifted 0.4 mm. to the right. This was preferred to making a new linear extrapolation through zero, because Stetter and Jentschke's (22) reproduction of Fig. 13 does show the curve to pass through
zero and because the position of the maximum (3.5 mm.) checks with the value which is obtained from Fig. 13 after adopting the procedure mentioned above. A similar argument holds for a comparison with Fig. 12 of the original paper. Bower (23), however, quotes 3.9 mm. as being the residual range for the position of the maximum in the work of Schmidt and Stetter, but this is just the value one obtains by measuring from the end of the scale in Fig. 13, so that our correction is entirely justified and we can safely assume that Bower quoted the wrong value. Fig. (I-2) of this thesis shows the specific ionization of the single proton. Curve "S" of this graph shows the results of Schmidt and Stetter. The scale of the ionization is in relative units and the ordinates of curves "S" and "P" are matched at the maximum.

In 1938, Bower (23) redetermined the "specific ionization of the single proton." He used a cloud chamber and measured the density along photographs of a few proton tracks by means of a microphotometer which had an effective slit width of 0.10 - 0.12 mm. After averaging the measurements, a density versus residual range curve is obtained (Curve 5 on Fig. 3 of the original paper). This curve refers to a mixture of 93% He and
THE SPECIFIC IONIZATION OF THE SINGLE PROTON

Figu. 2.

LEGEND:

S = SEMIACTIVITY TABLE
B = BOLIER
S AND B ARE MATCHED AT THE MAXIMUM ORDINATE.
ARROW ON B MARKS BEGINNING OF EXTRAPOLATION.

EXTRAPOLATION

SPECIFIC IONIZATION IN RELATIVE UNITS

IN AIR AT 76 CM Hg AND 25°C

FORM 1 T
7% air at 38 cm Hg and 15°C. In a similar experiment on x-rays (Fig. 1), the density as well as the relative intensity of the scattered light was measured. From these data we can obtain the constants of the photographic plates and, assuming that the same type of plate was used for protons and x-rays, we can deduce, from Fig. 3, the intensity of scattered light versus residual range curve for the individual proton. If we reduce the data to standard air and assume (as Bower did) that the ionization is proportional to the light scattering power of the track, we obtain a curve which can be regarded as the specific ionization curve of the average proton. The curve (Fig. 1 - 2, curve B) has no tail due to straggling since only the last 1.9 air-mm. of the range were considered. In order to be of use for the cinema integrator, Bower's curve was extrapolated to 1.6 air-cm. residual range. This was done rather arbitrarily by drawing a smooth curve which joins Bower's curve on one side and approaches Schmidt and Stetter's curve at 1.6 air-cm. This extrapolation makes Bower's curve of little value. However, for the sake of interest and for a comparison with the results of Schmidt and Stetter, it was also used for the derivation of an empirical relation. This procedure can be justified by the argument that Bower's curve includes the point of maximum ionization and that the ex-
trapolation should not effect appreciably the linear portion of the Bragg curves. As a matter of fact, this proves to be the case (see Section C3). However, the two sets of empirical relations will be shown to differ considerably. This is due to the difference in shape between the two ionization curves.

Bower's curve is not very reliable (much less reliable than even Schmidt and Stettler's curve), because Bower's measurements were made mainly for determining the position of maximum ionizing efficiency (believed to be accurate within 10%). According to Bower, the slope of the curve, particularly on the short range side of the maximum, cannot be regarded as being more than an approximation to the correct slope.

We shall now take up the corrections which had to be added to Bower's curve in order to obtain the form shown in Fig. (I - 2). It will be seen that one of these corrections reduces materially the accuracy of Bower's curve.

After reducing the ranges to 76 cm. Hg pressure (this involves a factor of 0.5) they have to be converted to air ranges. For this purpose, we make use of a formula connecting R (= mean range in air) and L (= mean range in some substance, in this case in a mixture m).
Bethe's differential equation for the loss of proton energy in an absorber of atomic number \( Z \) can be written (the relativity correction can be neglected for protons usually met in nuclear disintegrations):

\[
- \frac{dT}{dr} = \frac{4 \pi Z^2 e^4}{m V^2} N B \quad (I - 3)
\]

where

\[
B = Z \ln \frac{2 m V^2}{J} \quad (I - 4)
\]

and the symbols mean:

- \( T \) = kinetic energy of the proton
- \( r \) = distance or range traversed by the proton in the absorber
- \( z = 1 \) for protons
- \( e \) = charge of the electron
- \( m \) = mass of the electron
- \( V \) = velocity of the proton
- \( N \) = number of atoms per unit volume of absorber
- \( B \) = atomic stopping number
- \( J \) = average excitation potential of the absorber

From (I - 3) we obtain

\[
\frac{dr_{air}}{dr_{m}} = \frac{N_m}{N_{air}} \frac{B_m}{B_{air}} \quad (I - 5)
\]
By definition

\[
S_m = \frac{(dT_{m})}{(dT_{air})_m} \quad \frac{N_{air}}{N_m} = \frac{B_m}{B_{air}} \quad (I - 6)
\]

where \(S_m\) = atomic stopping power of the medium \(m\).

By \((I - 5)\) and \((I - 6)\) we now have

\[
R = \int_0^R dr_{air} = \frac{N_m}{N_{air}} \int_0^L S_m dr_m \quad (I - 7)
\]

and

\[
L = \int_0^L dr_m = \frac{N_{air}}{N_m} \int_0^R \frac{dr_{air}}{S_m} \quad (I - 8)
\]

For \(S_m\) we use an obvious formula which in our case is

\[
S_m = f_{He} S_{He} + f_{air} S_{air} \quad (I - 9)
\]

where \(f\) stands for the atomic fractions.

For the calculations we assume that the mixture \(m\) contains 93% He and 7% air by volume. Consequently, in 100 molecules of \(m\) we have 93 He molecules and 7 air molecules. Considering air to be a diatomic gas, we obtain for the atomic fractions:
\[ f_{He} = \frac{23}{107} = 0.218 \]
\[ f_{air} = \frac{14}{107} = 0.131 \]

Now we calculate the values of \( N \). From the gas equation we get \( n = 4.33 \times 10^{-5} \) moles for 1 cc. at 76 cm Hg and 15°C. Using the fact that there are on the average 1.07 atoms per molecule of \( m \) (this follows from the above considerations), we can write:

\[ N_{air} = 4.33 \times 10^{-5} \times 6.023 \times 10^{23} \times 2 = \frac{5.22 \times 10^{19} \text{ atoms/cc.}}{\text{atoms/cc.}} \]

\[ N_m = 4.33 \times 10^{-5} \times 6.023 \times 10^{23} \times 1.07 \text{ atoms/cc.} \]

\[ \therefore \frac{N_m}{N_{air}} = \frac{1.07}{2} = 0.535 \]

For the range interval in question we can take \( S_{He} \) constant. From the literature (see appendix) and on the basis of one converted point (the maximum) of Bower's work we choose \( S_{He} = 0.366 \). By (I - 9) we now obtain \( S_m = 0.449 \) and by (I - 7) \[ \Delta R = \frac{N_m}{N_{air}} S_m \Delta L \]

\[ \therefore \Delta R = 0.24 \Delta L \quad (I - 10) \]

Applying the factor 0.24 to the previous range values...
gives ranges in standard air. The probable error is about 3% due to the uncertainty in $S_{He}$.

Next we compute the intensity of scattered light $I$ from the values of the density $D$. Following Bower, we have, from Fig. 3:

$$I = 4.75 \, e^\frac{D}{P}$$

where 12 was arbitrarily taken as the maximum value of $I$. By making several measurements on Fig. 1, we obtain

$P_{\text{average}} = 75.5$

$$\therefore I = 4.75 \, e^\frac{D}{75.5}$$  \hspace{1cm} (I - 11)

which gives us the values of the ionization. The probable error is about 16% due to the fact that $P$ is not constant.

**APPENDIX. THE ATOMIC STOPPING POWER OF He**

This section is not intended to be a complete summary of the literature on the atomic stopping power of helium. We shall only justify the selection of the value $S_{He} = 0.366$ which was used in the calculations.

Fig. (I - 3) shows the experimental determinations of $S_{He}$. The upper abscissae represent $\alpha$ -ray residual ranges, the lower scale shows residual ranges
THE ATOMIC STOPPING POWER OF HELIUM

\[ a \text{ RAY \hspace{1cm} RESIDUAL \hspace{1cm} RANGE} \]

\[
\begin{align*}
0.340 & 0.330 & 0.320 & 0.310 & 0.300 & 0.290 & 0.280 & 0.270 & 0.260 & 0.250 & 0.240 & 0.230 & 0.220 & 0.210 & 0.200 & 0.190 & 0.180 & 0.170 & 0.160 & 0.150 & 0.140 & 0.130 & 0.120 & 0.110 & 0.100 \\
\end{align*}\]

\[
\begin{align*}
\text{PROTON \hspace{1cm} RESIDUAL \hspace{1cm} RANGE} \hspace{1cm} \text{THE RANGES ARE IN AIR-CH} \hspace{1cm} \text{LEGEND:} \hspace{1cm} \text{Fig. (1-3)}
\end{align*}\]

- \( \Delta \) BATES
- \( \times \) GURNEY
- \( \Diamond \) MANQ
- \( \otimes \) NAIDU
- \( \Box \) VALUE USED BY BOWER
- \( \text{T} \) VALUE USED IN THIS THESIS
of protons. These scales differ by 0.2 cm. because
\[ \frac{R_p(v)}{R_a(v)} = 1.0072 \quad R_a(v) = 0.20 \text{ cm.}, \] as was men-
tioned in Section B2. (In the literature the values
of \( S_{He} \) are quoted for certain \( \alpha \) -ray ranges, whereas
we are interested in values of \( S_{He} \) for proton ranges.)
The results of the following investigators were
plotted: Bates (24), Gurney (25), Mano (2), and
Naidu (26). It is apparent that we cannot draw a
curve which will pass through all these points. Con-
sequently, the straight line shown in the graph is not
very accurate, but it can be used for our purposes since
an error of 0.010 in the value of \( S_{He} \) changes the cal-
culated residual proton ranges (for Bower's curve) only
by 2%. In the cases where the experimental determina-
tions of \( S_{He} \) refer to a range interval, we have indi-
cated the whole interval and marked the center. Some
authors gave values of "stopping power" instead of
atomic stopping power. These values had to be multi-
plied by two since air (the reference substance) can be
considered as diatomic, i.e., \( \frac{N_{air}}{N_{He}} = 2 \). (This pro-
cedure follows from equation (I - 6).)

Bower calculated that the maximum of his curve
(situated at 6.3 mm. residual range at the experimental
conditions) would lie at 0.78 mm. in standard air. From
this we can calculate (by "going backwards") that Bower used the value $S_{He} = 0.381$. Now we can estimate that Bower's curve, which included 16 mm. residual range in the original gas mixture at 38 cm. Hg, would cover about 2 air-mm. at standard conditions. Since $S_{He}$ is nearly constant in this region, we use the value of $S_{He}$ at 1 air-mm, which is the center of the interval. Fig. (I - 3) then gives $S_{He} = 0.366$. The value of $S_{He}$ used by Bower seems to lie within the probable error of the atomic stopping power curve, the uncertainty of which causes the computed air-mm. residual ranges to have a probable error of 3%. Hence it is not significant that we obtain 0.76 air-mm. instead of Bower's 0.78 air-mm. for the position of the maximum of Bower's curve.

3. The Empirical Relations

In Section CI it was mentioned that the Bragg curves obtained by means of the cinema integrator were extrapolated and that $x$ and $y$ were plotted against $\lambda$. Now we shall derive the empirical relations.

First we consider the relations obtained on the basis of Schmidt and Stetter's ionization curve. Fig. (I - 4) shows a plot of $\alpha$ versus $y$ and Fig. (I - 5)
$y$ vs. $x$ for PROTONS

Based on Schmidt and Stetter's curve of the specific ionization of the single proton.

Fig. (1-4)
X vs. \( \lambda \) for Protons

Based on Schmidt and Stetter's curve of the specific ionization of the single proton.

Fig. (x-5)
represents $x$ as a function of $\alpha$. From Fig. (I - 4) we derive the relation

$$\alpha = 1.14 y - 0.212 \text{ air-cm}. \quad (I - 12)$$

and from Fig. (I - 5) we obtain

$$x = 0.521 \alpha - 0.051 \text{ air-cm}. \quad (I - 13)$$

Fig. (I - 5) shows that the points do not lie on a straight line at all. In fact, the dotted line shows a parabola (with the vertex at $P$), which is seen to represent the actual relation between $x$ and $\alpha$ much more accurately than a straight line. But we must consider here that the cinema integragram is less accurate at high values of $\alpha$ than at low values. Consequently, we can safely neglect the point at $\alpha = 0.2 \text{ air-cm}$ and draw a straight line through the remaining data. This is justified by the fact that Mr. S. C. Brown checked Holloway and Livingston's $x$ vs. $\alpha$ relation for $\alpha$-rays by using the same method and neglecting the point at $\alpha = 0.2 \text{ air-cm}$.

The probable errors of the above relations (due to the straight line approximations) cannot be stated generally since they obviously depend on the magnitude of the range straggling parameter and hence on the proton
range. (For \( \alpha \)-rays, Holloway and Livingston considered the errors in the region of the natural-\( \alpha \)-particles.) In any application of this relation, however, the errors can be estimated from the graphs.

The errors inherent in the extrapolations of the Bragg curves are negligible (much smaller than 1\%).

Tabulation of probable errors for relations (I - 12) and (I - 13):

- Probable error of Schmidt and Stetter's ionization curve: 10\%
- Probable error of the Bragg curve extrapolations: negligible
- Probable error of the straight line approximations: depends on \( \alpha \), i.e., on the proton range

The relations obtained on the basis of Bower's ionization curve are derived from Fig. (I - 6) and Fig. (I - 7).

\[
\begin{align*}
\alpha &= 0.700 y - 0.014 \text{ air-cm.} \quad (I - 14) \\
x &= 1.03 \alpha - 0.028 \text{ air-cm.} \quad (I - 15)
\end{align*}
\]

These equations are seen to differ considerably from equations (I - 12) and (I - 13). However, the relations based on Bower's curve are much less reliable than those based on Schmidt and Stetter's curve. This
\( \alpha \) vs \( \beta \) for Protons

Based on Bower's curve of the specific ionization of the single proton.

Fig (2-4)

\( \alpha \) in Air-CM

\( \beta \) in Air-CM
\(X\) vs. \(x\) for PROTONS

Based on Bower's curve of the specific ionization of the single proton.

Fig. (5-7)
will be apparent from the following tabulation of probable errors.

Probable error due to the extrapolation of Bower's ionization curve (see Fig. I-2): unknown

Probable error of Bower's curve: (See Section C2) unknown, but well over 10%

Probable error of the range values due to the uncertainty in the value of $S_{He}$: 3%

Probable error of the ionization values due to the variation of the constant of the photographic plate: 16%

Probable error of the Bragg curve extrapolations: 0.3 - 2%

Probable error of the straight line approximations: (Smaller than in the case of Schmidt-Stetter) depends on $\alpha$, i.e., on the proton range

We can now conclude that equations (I - 12) and (I - 13) represent, within their uncertainty, the correct empirical relations.

D. **APPLICATION OF THE EMPIRICAL RELATIONS**

We shall now apply the empirical relations to an experimental proton Bragg curve. Only one such curve could be found in the literature. (It is extremely unlikely but not quite impossible that other determinations have been published.)
The Bragg curve of Blau and Rona (27) was determined by means of an ionization chamber and electrometer. A magnetic field was used for the energy measurements and the field strength was adjusted for 2.395 Mev. protons. Curve A in Fig. 7 of the original paper shows the experimental Bragg curve. (Curve B was calculated on the basis of number-distance data. We shall not use it because it is based on Brigg's curve of the specific ionization of the single \( \lambda \) -ray and because it assumes that the specific ionization of the single proton is a constant fraction of the specific ionization of the single \( \lambda \) -ray.) Using the empirical relations, we now obtain from Blau and Rona's curve (after reducing the data to standard conditions):

\[
\begin{align*}
\lambda &= 3.29 \text{ air-cm.} \quad \text{(S.S.)} \\
\lambda &= 2.14 \text{ air-cm.} \quad \text{(B.)} \\
R &= 8.08 \text{ air-cm.} \quad \text{(S.S.)} \\
R &= 7.57 \text{ air-cm.} \quad \text{(B.)}
\end{align*}
\]

On the other hand, the range-energy relation gives \( R = 9.625 \text{ air-cm.} \) for the energy in question. This disagreement cannot be used to correct the range energy relation because of the following reasons:

Blau and Rona's curve is probably very inaccou-
imrate, because both relations give entirely "unreasonable" values for \( \alpha \). (The values of \( \alpha \) are at least 10 times too large. See Part II.) Furthermore, Blau and Rona's proton beam was not sufficiently homogeneous (70\% of all protons had a residual range \( R \) of 9.75 ± 1.5 cm.) and the authors state that their measurements have several serious sources of error: Non-detection of a small group of short range protons, straggling in the mica foils, and increasing air equivalence of the mica foils for protons of greater velocity.

E. SUMMARY AND RESULTS

The literature on the range energy relation for protons has been summarized and discussed. Equations (I - 12) and (I - 13) are suggested for the determination of mean ranges from experimental proton Bragg curves. These "empirical" relations were applied to one Bragg curve but it was not feasible to obtain additional data for the range energy relation.
PART II

Calculation of the Range Straggling Parameter $\delta$ for Protons

A. INTRODUCTION

Up to the present time the phenomenon of the straggling of protons has not been investigated experimentally. This is probably due to the same reasons which have been advanced in Part I, Section A, for the lack of sufficient experimental data on the ranges of protons.

The theory of proton straggling, however, has been worked out by Bethe (28) for the general case of a nuclear ray. The resulting wave-mechanical formula contains a constant $k$ which is characteristic of the absorber. Since this constant cannot be calculated by means of the theory, it is necessary to determine it by empirical methods. Such an empirical evaluation of $k$ was recently carried out by Clogston (29). On the basis of these results we shall now calculate the range straggling of protons for several absorbers. Before reporting the actual calculations, however, we shall review the formulae to be used and discuss Clogston's unpublished results.
B. THE FORMULAE

1. The Straggling Equation and the Constant \( k \)

According to Clogston, Bethe's straggling equation can be written

\[
P = \frac{d\sigma^2}{dx} = 4\pi z^2 e^4 N \left[ \sum \frac{J}{mV^2} \frac{4}{3} \sum k T_k \ln \left( \frac{2mV^2}{J} \right) \right]
\]

where \( \sigma^2 \) = the standard deviation of the energy

\( P \) = symbol for \( \frac{d\sigma^2}{dx} \)

\( T_k \) = the average kinetic energy of the \( k \)-th electron in its orbit

and all the other quantities are as defined in equations (I - 3) and (I - 4).

\( P \) can also be written in the equivalent form

\[
P = 4\pi z^2 e^4 N Z \left[ I + k \frac{J}{mV^2} \ln \left( \frac{2mV^2}{J} \right) \right] \tag{II - 1}
\]

where \( k = \frac{4}{3} \frac{T_G}{JZ} \) with \( T_G = \sum k \frac{T_k}{k} \) \tag{II - 2}

To obtain \( P \) in terms of the kinetic energy \( T \) we substitute \( V^2 = \frac{2T}{m_p} \) (where \( m_p \) = mass of the proton) into equation (II - 1). This is permissible for velocities below \( 5 \times 10^9 \) cm/sec (corresponding to \( T < 13.0 \text{ Mev} \)) because the relativity correction can
be neglected (5). We now have

\[ P = 4\pi z^2 e^N Z \left[ \frac{\kappa J m_e}{2 m} \frac{\ln \left( \frac{4m}{2m} \frac{T}{T} \right)}{T} \right] \]  

(II-3)

or in abbreviated form

\[ P = P_0 (1 + \Theta) \]  

(II-3a)

The evaluation of the constant \( k \) is based on equation (II - 2), in which the total kinetic energy \( T_G \) of the electrons in the absorbing atom is equal (but of opposite sign) to the total ground state energy of the absorbing atom. The values of \( T_G \) were obtained by Clogston from the spectroscopic literature and the data on \( J \) were taken from table (4 - 2) of reference (1).

Fig. (II - 1) shows a plot of \( J \) vs. \( Z \) (used for interpolations) and Fig. (II - 2) represents the values of \( k \) for different atoms as obtained from table (II - 1).
MEAN EXCITATION POTENTIAL $J$

vs.

ATOMIC NUMBER $Z$

Fig (x-1)
<table>
<thead>
<tr>
<th>Absorbing atom</th>
<th>$T_G$ in ev.</th>
<th>$J$ in ev.</th>
<th>$k$ dimensionless</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>13.54</td>
<td>18</td>
<td>1.0</td>
</tr>
<tr>
<td>He</td>
<td>78.5</td>
<td>39.5</td>
<td>1.3</td>
</tr>
<tr>
<td>Li</td>
<td>203</td>
<td>44</td>
<td>2.1</td>
</tr>
<tr>
<td>Be</td>
<td>397</td>
<td>56 (1)</td>
<td>2.4</td>
</tr>
<tr>
<td>B</td>
<td>667</td>
<td>69 (1)</td>
<td>2.6</td>
</tr>
<tr>
<td>C</td>
<td>1020</td>
<td>82</td>
<td>2.8</td>
</tr>
<tr>
<td>N</td>
<td>1470</td>
<td>91</td>
<td>3.1</td>
</tr>
<tr>
<td>O</td>
<td>2020</td>
<td>111</td>
<td>3.0</td>
</tr>
<tr>
<td>F</td>
<td>2600 (1)</td>
<td>121 (1)</td>
<td>3.2</td>
</tr>
<tr>
<td>Ne</td>
<td>3420</td>
<td>147</td>
<td>3.1</td>
</tr>
<tr>
<td>Na</td>
<td>4340</td>
<td>147 (1)</td>
<td>3.6</td>
</tr>
<tr>
<td>Mg</td>
<td>5330</td>
<td>160 (1)</td>
<td>3.7</td>
</tr>
<tr>
<td>Al</td>
<td>6500 (e)</td>
<td>171</td>
<td>3.9</td>
</tr>
</tbody>
</table>

In this table "i" means "interpolated value" and "e" stands for "extrapolated value."

Because of the uncertainty in the values of $T_G$ and $J$, it is believed that $k$ is accurate to two figures only. (Beyond Mg no data on $T_G$ are available.)
sidering that the third figure might be as high as 4, the error in \( k \) is probably between 1 and 4%. In all the calculations we shall use the computed values of \( k \) because the smooth curve of Fig. (II - 2) is too uncertain.

2. **The Range Straggling in Air**

The standard deviation \( \sigma_R \) of the range straggling can be obtained from the calculated values of the energy straggling by means of the equation

\[
\sigma_r^2 = \int_0^R P \left( \frac{dT}{d\tau} \right)^2 d\tau \quad (II - 4)
\]

where \( P \) is given by equation (II - 3) and \( \frac{dT}{d\tau} \) is the slope of the range energy relation. In the form quoted above, equation (II - 4) can only be used for the case of air to obtain \( \sigma_R \) in units of air-cm. The derivation of this equation is given in reference (1), page 405, and the value of the integral must be computed by a graphical integration.

The range straggling parameter \( \alpha \) for protons of a certain mean range \( R \) is given by \( \alpha = \sigma_R \sqrt{2} \).
3. The Range Straggling in Absorbers Other Than Air

Assuming that equation (II - 4) governs the straggling in absorbers other than air, we can write

\[ \sigma_{R_i}^2 = \int_0^R P_i \left( \frac{dT}{d\tau_i} \right)^{-2} d\tau_i \]  

(II - 5)

where the subscript "I" refers to the absorber in question. In this form, equation (II - 5) would lead to lengthy computations. Consequently, we shall now derive a formula which allows us to compute the straggling in any absorber on the basis of the straggling in air. For this purpose, we have to express all distances in air-cm. We then obtain (the subscript "a" stands for air):

By (II - 3a) and (II - 3)

\[ P_I = P_a \frac{P_{0I}}{P_{0a}} \frac{(1 + Q_I)}{(1 + Q_a)} = P \frac{N_I}{N_a} \frac{Z_I}{Z_a} \frac{(1 + Q_I)}{(1 + Q_a)} \]

By (I - 3) and (I - 6)

\[ \left( \frac{dT}{d\tau_I} \right)^{-2} = \left( \frac{dT}{d\tau_a} \right)^{-2} \left( \frac{N_a}{N_I} \right)^2 \left( \frac{1}{S_I} \right)^2 \]

By (I - 5) and (I - 6)

\[ dr_I = dr_a \frac{N_a}{N_I} \frac{1}{S_I} \]
By (I - 5) and (I - 6)

\[ \sigma_{R_1}^2 = \sigma_{R_a}^2 \left( \frac{N_a}{N_1} \right)^2 \left( \frac{1}{S_1} \right)^2 \]

Now we substitute these last four expressions into (II - 5).
After rearranging the terms and making cancellations, we have:

\[ \sigma_{R_a}^2 \left( \frac{1}{S_1} \right)^2 = \int_0^R \mathcal{P}_a \left( \frac{dT}{d\tau_a} \right)^{-2} \left[ \frac{Z_i}{Z_a} \left( \frac{1 + \Theta_i}{1 + \Theta_a} \right) \left( \frac{1}{S_1} \right)^3 \right] d\tau_a \]

If we perform the integration over sufficiently small intervals the value of \( S_1 \) can be regarded as a constant and we obtain:

\[ \sigma_{R_a}^2 = \int_0^R \mathcal{P}_a \left( \frac{dT}{d\tau_a} \right)^{-2} f \ d\tau_a \quad (II - 6) \]

where

\[ f = \left[ \frac{Z_i}{Z_a} \left( \frac{1 + \Theta_i}{1 + \Theta_a} \right) \frac{1}{S_1} \right] \quad (II - 7) \]

A comparison of equations (II - 4) and (II - 6) shows that they are identical except for the factor \( f \). This obviously simplifies our calculations because we now simply multiply the integrand of (II - 4) by \( f \) (at about
3 energies) and perform a new graphical integration. The upper limit of this integration is that range \( R \) (in air-cm.) which corresponds to \( R_1 \) (in cm.). Fortunately, we do not have to know the value of \( R_1 \) since equation (II - 6) gives \( \bar{\sigma}_{Ra} \) in units of air-cm. We might emphasize again that \( \bar{\sigma}_{Ra} \), as obtained from equation (II - 6), is the value of the S.D. of the range straggling (in air-cm.) at a range \( R \) air-cm. in an absorber 1.

4. Some Conversion Formulae

In the calculations of the following section repeated use will be made of two conversion formulae:

\[
\begin{align*}
V_p &= 4.39 \times 10^{-2} \sqrt{t} \quad 10^9 \text{ cm/sec} \\
T_p &= 5.13 \times 10^{2} \times v^2 \quad \text{kev}
\end{align*}
\]

(II - 8)

where \( t \) = energy of a proton in kev.

and \( v \) = velocity of a proton in units of \( 10^9 \) cm/sec.

As was mentioned in section 34, the relativity correction is negligible up to velocities of \( 5 \times 10^9 \) cm/sec. Consequently, these formulae are based on \( T = \frac{m_p v^2}{2} \), where the value \( m_p = 1.661 \times 10^{-24} \) g was used.

C. CALCULATION OF \( \alpha \) FOR AIR

Because of the basic importance of the straggling in air we shall now outline the numerical calculations.
We use equation (II - 3) to calculate $P$ at several energies (i.e., at several mean ranges). It is important to note that this equation refers to e.s.u. units. Consequently, $T$ and $J$ must be expressed in ergs. The value of $k$ for air was obtained from Fig. (II - 2). Because of the uncertainty of the curve, the two neighboring points ($N$ and $O$) were joined by a straight line and $k_{air}$ was determined on this line at $Z = 7.22$. This procedure is permissible because neither $k_N$ nor $k_O$ is based on interpolated values of $J$ or $T_g$. We thus obtain $k_{air} = 3.08$. $N_{air}$ for standard air had already been calculated in Part I, Section C2, in the paragraph on Bower's ionization curve. $J_{air}$ was taken as $98$ ev $= 1.57 \times 10^{-10}$ ergs. The other numerical constants were the following:

\[
m = m_0 = 0.9097 \times 10^{-27} \text{ g. (relativity neglected)}
\]
\[
e = 4.803 \times 10^{-10} \text{ e.s.u.}
\]
\[
z = 1 \text{ for protons}
\]
\[
m_p = 1.661 \times 10^{-24} \text{ g.}
\]

(As shown in Part III, Bethe's formula for protons in air is valid for a minimum energy of 178.4 ev.) We now obtain the numerical equation

\[
P_{air} = 2.51 \times 10^{-16} (1 + 2.76 \times 10^2 \frac{\ln T - 3.8}{T})
\]
where $T$ is in e.k.v., but proper conversion factors are included so that $P_{\text{air}}$ is still in c.g.s. units.

The above equation shows the interesting fact that at 1 Mev. the second term is as much as 80% of the first (i.e., classical) term. At higher energies the second term decreases. (For other absorbers $P$ behaves similarly.)

By means of the range energy relation we then calculate the values of the integrand of equation (II - 4). The resulting curve is shown in Fig. (II - 3), where $f = 1$ for air. (It is very important to determine accurately the slope of the range energy relation because it enters as a second power into equation (II - 4). The deviation of 2 points from the "air" curve of Fig. (II - 3) is due to an "unsmoothness" in the range energy relation.) Graphical integration of this curve leads to values of $\sigma^2_R$ from which the curve of $\alpha$ vs. $R$ can be derived. The final results are contained in the "air" curve of Fig. (II - 4).

As mentioned previously, no experimental data are available to check these results. But we can make an approximate check by means of the following rough calculation:

At the range $R = 3.84$ air-cm., the $\alpha$-ray range straggling is about 0.05 air-cm., as computed by means of
the Bethe equation and checked by experiments (10).
For protons of this range our results give the value
0.129 air-cm. We know, however, that \( \frac{z^4}{z_p^2} = 4 \) and
that \( \frac{\sigma}{\sigma_p} \) = 3.66 at this range. Since the ve-
locities are approximately equal in both cases, the in-
tegrand of equation (II - 4) will be 0.3 times as large
for \( \alpha \) -rays as for protons. Assuming now that the
"air" curve of Fig. (II - 3) is approximately a straight
line (through the origin), we find that at \( R = 3.84 \) air-cm.
the values of \( \sigma^2 \) for \( \alpha \) -rays and protons are in the
ratio \( \frac{0.3}{1} \). Consequently, the range straggling parameter
for \( \alpha \) -rays comes out to be about 0.07 air-cm. instead of
the actual 0.05 air-cm. These superficial considera-
tions show that at one range our result agrees within a
factor of 1.5 with the experimentally verified \( \alpha \) -ray
calculation. It is thus probable that our calculations
of \( \alpha \) are accurate within the limitations of the strag-
gling theory. The latter is believed to be correct
within 10% of its prediction.

D. CALCULATION OF \( \alpha \) FOR HYDROGEN, HELIUM, CARBON,
NEON, AND ALUMINUM

Between \( Z = 1 \) and \( Z = 13 \) the elements most
useful for proton absorption experiments are the follow-
ing: hydrogen, helium, carbon, neon, and aluminum. The values of the range straggling parameter \( \alpha \) for these absorbers were therefore calculated by means of the method discussed. Data on the atomic stopping power were obtained from Table (4-2), reference (1). (For reasons of consistency, the values of \( S_{He} \) were also taken from this table and not from Fig. (I - 3).) Three points were used in the calculations, corresponding to proton ranges of 0.9, 2.9, and 7.5 air-cm. Fig. (II - 3) shows the curves obtained by equation (II - 6) and Fig. (II - 4) represents the results of the integrations. The straggling in mica was not calculated because different consignments of mica, even from the same quarry, possess stopping powers varying by 3%.

E. SUMMARY AND RESULTS

The range straggling parameter \( \alpha \) for protons was calculated for the absorbers hydrogen, helium, carbon, air, neon, and aluminum. The calculations are based on Bethe's equation and Clogston's values of the constant \( k \). Fig. (II - 4) shows the resulting curves of \( \alpha \) (in air-mm.) as a function of the mean residual range \( R \) (in air-cm.). Experimental determinations of \( \alpha \) are not available, but a rough calculation shows
that the values of $\alpha$ for protons in air agree with the corresponding results of experimentally verified calculations of the straggling of $\alpha$ rays.
PART III

Description and Explanation of the Model: "The Straggling of Protons in Air"

The results of Part II show that for rays of a given mean range the straggling of protons (as determined by the value of $\alpha$) is more than twice as large as the straggling of $\alpha$ particles. It is certainly interesting to obtain a clear "picture" of the magnitude of the range straggling for protons of different energies. In order to achieve this aim, it was thought that a three-dimensional representation of the straggling would best help in the visualization of the relative magnitudes of the range straggling and the range.

It was thus decided to build, as a part of this thesis, a model which represents the straggling of protons in air. This model consists of the following parts:

The range energy relation for protons (energy in Mev. vs. residual mean range in air-cm.) was plotted on graph paper of 65 x 45 cm. size and this sheet was then pasted on a slightly larger wooden board. Ten centimeters on this paper were called "1 air-cm." so that the apparent range scale represents a magnifica-
tion of exactly ten times. The plotted curve extends over about 6 air-cm. (i.e., 1.8 Mev.) of the range energy relation. This is a natural upper limit because only in this region is the "Revised Cornell Relation 1937" based on experimental data alone (Parkinson et al.).

The range straggling of protons of a certain energy was represented by means of Gaussian distribution curves which were sawed out of plywood (3/16 inch thick) and pasted vertically on the plot of the range energy relation parallel to the range axis. To understand the meaning of these distribution curves, let us consider a beam of protons which have an initial energy $T'$ and a mean range $R'$. Our model shows at $T'$ a Gaussian curve the maximum of which is situated perpendicularly "above" the range energy relation at the range $R'$. An ordinate $z$ of this Gauss curve represents the probability that an individual member of this proton beam possesses a range which is indicated by the position of $z$ on the range scale. ($z$ is perpendicular to the plane of the model.) In other words, the range straggling is on the same scale as the range. To obtain the energy straggling distribution we have to imagine a cut parallel to the energy axis through this group of Gauss curves.
It is seen that this distribution is unsymmetric, as it should be.

For the calculation of the Gaussian curves we plotted, in Fig. (III - 1), the values of $\sigma_R$ versus the energy $T$. This curve is easily obtained from the calculations of Part II, Section C, and it enables us to find the S.D. of the distribution curves for energy intervals of 20 ekv. (The thickness of the plywood was so chosen that it covered nearly 20 ekv. on the energy scale.) The Gaussian curves were then calculated by means of tables. They were drawn wide enough to cover 99% of their actual area. (The area of all these curves is constant, corresponding to a probability of 1.) Because of practical limitations it was necessary to cut off the broad curves at an actual height of about 0.5 mm. This was done in such a way that the two curves which can be thought to connect the end points of these distribution curves, will still be smooth. The calculations refer to the high energy side of the "thickness" of the Gauss curves. Fig. (III - 2) shows in true size the highest, the lowest, and an intermediate distribution curve.

For the passage of protons through air, Bethe's straggling equation (II - 3) is valid for $T > 4 T_c$,

where $T_c = \frac{m_p}{4m} J_{air}$. Hence we obtain $T = 178.4$ ekv.
THE STRAGGLING OF PROTONS IN AIR

THE STANDARD DEVIATION OF THE RANGE DISTRIBUTION

VS.

PROTON ENERGY

Fig. (3r - 1)

ENERGY IN MEV.
"THE STRAGGLING OF PROTONS IN AIR"

(See Fig. III-1)

Gaussian curves showing the distribution of the ranges of protons of a particular energy around the (corresponding) mean range \( R \).

These curves are of the same size as those used for the model.

The ordinates represent the probability that a proton has a certain range.

On the model, very flat curves are cut off at about 8.5 scale units. Ordinates above these points are indicated on the two lower curves.

Fig. (III-2)
Fig. (III-3)
for the lower limit. In the model, the straggling was calculated between 190 and 1900 ekv. Consequently, 82 Gaussian curves were constructed.

Fig. (III - 3) shows a photograph of the model.
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