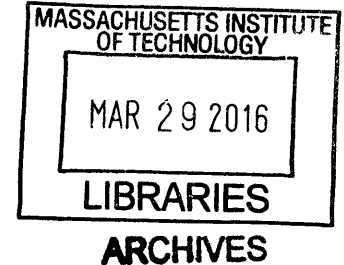


# The Roman Pantheon: Scale-Model Collapse Analyses

by

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Submitted to the Department of Architecture  
in partial fulfillment of the requirements for the degree of

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“The model has an answer to (nearly) everything.”

- Heinz Isler

# Scale-Model Collapse Analyses of the Roman Pantheon

by J. William Plunkett

Submitted to the Department of Architecture on January 15, 2016  
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## Abstract

The Roman Pantheon is among the largest unreinforced masonry dome ever built and is an unparalleled example of the construction capabilities of the ancient Romans. As one of the most well-known buildings in the world, its preservation remains important because of its cultural and societal significance, and the methods used to assess the safety of historic masonry structures continue to be developed, particularly for three-dimensional vaulted forms. Through a study of the Roman Pantheon, this thesis compares analytical and experimental results on a 1:100 scale model of the variable thickness, hemispherical dome. The model is created using additive manufacturing for accuracy.

This thesis, using a physical scale model, quantifies the safety of the Roman Pantheon against the two most probable causes of collapse (1) deformation of the building geometry and (2) seismic activity. The structural behavior of the model is compared to analytical predictions of (1) spreading supports, simulating leaning walls that result from the dome thrust or settling of the foundations, and (2) tilting, a first-order approximation of horizontal ground acceleration. The experimental tests lead to the formation of a mechanism and collapse due to instability. High-speed imagery captures the observed collapse mechanisms and failure limits. Experimental results are compared to analytical predictions for hemispherical masonry domes.

The results of the physical experiment demonstrate the potential for digitally fabricated scale models in approximating the behavior of three-dimensional structures with complex geometries. The low cost and rapid approach provides a useful method for validating analytical predictions of the limit states and collapse mechanisms of unreinforced masonry structures.

Thesis Supervisor: John A. Ochsendorf  
Professor of Architecture and Civil and Environmental Engineering

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# 1.0 Introduction

The Pantheon in Rome, Italy, is one of the great technological and architectural achievements in human history and one of the most recognizable structures in the world. Its influence on the architecture of both civic and religious buildings is likely unparalleled, and the construction of its clear span is unprecedented and has remained unsurpassed for more than a millennium. Continuing to preserve the Roman Pantheon for additional centuries is important not only for the safety of visitors but also for its significant cultural and architectural heritage.

## 1.1 Motivation

As one of the largest unreinforced concrete and masonry dome ever constructed, the Roman Pantheon has been the subject of numerous and contradictory structural analyses throughout its history (Mark 1986, Croci 2006, Lancaster 2009). Its span was not matched for more than a millennium until the 15<sup>th</sup> century dome of the Cathedral of Florence, Italy, and was not exceeded until the late 19<sup>th</sup> century with the iron trusses of the Devonshire Royal Hospital in Buxton, England. This research was inspired by a twofold need: first, to quantify the potential causes of future collapse of the Roman Pantheon due to foundation settlement or earthquakes using limit analysis, and second, to validate the results through physical experiments on a scale model.

## 1.2 Research Objectives

By exploring the collapse mechanisms of a physical scale model, this thesis uses limit analysis on a scale model to demonstrate (1) the safety of the Roman Pantheon against collapse due to deformations caused by support displacements, (2) the safety of the Roman Pantheon against collapse due to seismic activity, and (3) the validity of predictions from analytical models based on equilibrium methods. The combination of physical experiments and simple tools for analytical models applied to the Roman Pantheon supplies a new assessment methodology for the structural behavior of unreinforced masonry.

### **1.3 Outline of Chapters**

This thesis studies the safety of the Roman Pantheon through collapse analyses of a digitally fabricated scale model.

Chapter 2 provides an overview of published literature in three relevant areas. First, an overview is given of the assessment methods used in masonry structures with an emphasis on limit analysis for masonry domes. Second, recent developments in the use of digitally fabricated scale models as an experimental tool to validate analytical models are discussed. Third, the general history and structural analyses of the Roman Pantheon specifically are examined.

Chapter 3 presents the methodology of the physical experiments. The assumptions made in the creation of a dimensionally-accurate discretized virtual and physical model are discussed. The general testing setup and procedure are also described.

Chapter 4 finds the collapse mechanisms of the full-scale model and the dome alone due to spreading supports, simulating settling of the foundation or outward rotation of the walls of the drum, respectively. The increase in the span before collapse is found and compared to analytical predictions.

Chapter 5 derives the horizontal ground acceleration that causes collapse of the full-scale model, the dome alone, and the drum alone. Using a tilt test, seismic activity is approximated using a static technique to estimate the minimal horizontal acceleration required to form a collapse mechanism. The resulting critical lateral acceleration is compared to analytical predictions of collapse.

Chapter 6 presents the results of the experiments and discusses their limitations.

Chapter 7 generalizes the main conclusions of this thesis and outlines opportunities for further research.



## **2.0 Literature Review**

This chapter presents an overview of the literature regarding masonry dome analysis, scale model experiments, and the Roman Pantheon. First, it explores assessment methods for masonry structures; second, it presents developments in the use of digitally fabricated scale models as an experimental tool to validate analytical models; third, it examines the general history and structural analyses of the Roman Pantheon.

### **2.1 Masonry Analysis**

#### **Strength versus Stability**

Unreinforced masonry is the primary material for many historic structures, and it behaves differently from modern buildings in which the materials are more highly stressed. Masonry buildings rely primarily on their geometry for stability, owing to the fact that masonry is strong in compression and weak in tension. Contemporary building codes require the structure to absorb the energy of lateral loads, particularly seismic loads, through material failure. The use of equilibrium or stability analysis is often more appropriate than stress or strength analysis for unreinforced masonry structures (Heyman 1995, Boothby 2001, Lourenco 2002).

#### **Limit Analysis of Masonry Domes**

Stability methods are commonly referred to as limit analysis or limit state analysis. Coulomb (1776) is attributed with first formalizing the concept of minimum and maximum states of stability. Heyman, who presented a modern formulation for limit analysis of masonry structures, describes three primary assumptions for using the theory of limit analysis in unreinforced masonry structures (Heyman 1966; 1995). First, masonry does not resist tensile forces. While this assumption is typically applied to the structure as a whole to be conservative, the masonry itself can transfer some small amount of tension, although joint mortar is often weaker. The frequent appearance of cracks within masonry structures indicates a tensile failure of either the material or the joint but does not

necessarily compromise the entire system. In reality, masonry has some nominal tensile strength, though it is a conservative assumption to neglect this limited tensile capacity. Second, it can be assumed that masonry has infinite compressive strength. This assumption, certainly not conservative, is applicable in practice because masonry structures rarely experience high material stresses in compression. The working stresses are typically on the order of only 5% of their crushing strength. Studies have tested this hypothesis to determine the capacity of arches, including the compressive strength of the masonry, and have demonstrated capacity, even with weak or old masonry, far beyond the height or applied loads in most historic masonry structures. (Foce 2003, Aita et. al. 2012). Moreover, when local material failure does occur, the integrity of the structure is rarely compromised as the forces redistribute across a larger interface. The third assumption is that masonry blocks have enough friction and do not slide. This assumption should always be considered carefully since sliding may occur in particular geometric configurations, often if joints are not perpendicular to the dominant force transfer and have relatively low thrusts across their interface. These three basic assumptions allow for the study of structures as rigid blocks that depend on their geometry for their stability (Heyman 1995). More thorough examinations of the differences between plastic and elastic analysis are presented by Huerta and Aroca (1989) and Block (2006).

Historically, graphic statics, or the representation of forces within a structure as vectors, has been used to apply the principles of limit analysis by various authors. The first application of a graphical method of forces as vectors was Stevin, a Dutch mathematician in the 17<sup>th</sup> century (Allen and Zalewski 2009.) For developments that apply to domes, Poleni famously assessed the stability of the dome of St. Peter's Basilica in Rome ignoring the impact of any hoop forces (Poleni 1748). Bossut (1776) proposed an analysis of domes without hoop stresses. Durand-Claye (1880) introduced indeterminacy, creating a method for drawing all possible thrust lines and introduced an early form of a factor of safety, but like Bossut he did not consider hoop forces. Wolfe (1921) introduces hoop forces in his derivation of a line of thrust. Domes were traditionally studied as two-

dimensional arches, though Heyman extended the method to spherical domes using membrane analysis (Heyman 1967).

There are two primary causes of a loss of stability in a masonry structure. One cause of collapse in historic buildings is deformation from the original building geometry typically caused by settling of foundations or supports (Danyzy 1732, Viollet-le-Duc 1854). A second cause of collapse is horizontal ground acceleration. Numerous recent studies have been dedicated to the collapse of arches (Oppenheim 1992, Smars 2000, Boothby 2001, Ochsendorf 2002), buttresses (De Luca 2004), barrel vaults (DeJong 2009), and domes (Lau 2006, Cooke and Ochsendorf 2012, Zessin 2012), and vaults (Shapiro 2012, van Mele et 2012.)

## **2.2 Physical Experiments**

The use of small-scale physical experiments provides an effective method to create and validate results. The importance of physical testing in the study of unreinforced masonry structures is particularly important, as computational approaches using strength-based assumptions can lead to inaccurate or infeasible results (Block 2006). Additionally, finite element analysis models do not easily accommodate large support displacements that are common in unreinforced masonry structures. Because most historical masonry structures have low stresses, their safety is governed by stability and not by the compressive strength of the materials (Heyman 1995). Several recent authors have demonstrated the ability of scale models to investigate the stability of masonry vaults and domes (Quinonez et al. 2010, van Mele et al. 2011, Shapiro 2012).

## **2.3 The Roman Pantheon**

The construction of the unprecedented dome of the Pantheon is attributed to Emperor Hadrian (117-138 AD), though the names of the master builders responsible for its construction are unknown (Lancaster 2005). Hadrian's most likely inspiration for building such a large dome came from the domes he knew from the popular ancient Roman resort town of Baiae (Yourcenar 2005). There is also a connection with Marcus

Agrippa, whose name is famously inscribed on the current Pantheon constructed by Hadrian, and Baiae because Agrippa was likely involved with the construction of Portus Julius adjacent to Baiae. Between 30BC and 12BC, Agrippa built or expanded numerous public works in the Campus Martius of Rome, including the first of the great baths constructed in Rome which likely contain the largest dome span in the world when completed and the first building known as the Pantheon. While the construction history of the Pantheon is unknown, it is believed that Agrippa constructed the first version that was destroyed by fires and rebuilt in 80 AD by Domitian. The third and current Pantheon was constructed around 110 AD by Hadrian (Moore 1995, MacDonald 2002).

Ancient Roman architecture placed great significance on the form of the dome, particularly as a representation of imperial power. Both octagonal and round domes with long spans built of different materials including brick (Baths of Trajan), concrete (Pantheon), and hollow terracotta (Baths of Caracalla) demonstrate the mastery and experimentation of the ancient Romans. Numerous authors have illustrated the influence of the Pantheon in subsequent architecture throughout the world (Pliny 79, Terenzio 1934, MacDonald 2002, Belardi 2006).

Many recent scholars have studied the structural behavior of the Pantheon, often with the intent of understanding the stability of the structure and how the Ancient Romans could have constructed the span (Mark 1986, Croci 2006, Lancaster 2009, Vogel 2009, Zessin 2012). Mark and Hutchinson (1986) outline the results of several finite element models representing an uncracked dome, cracked dome, stepped rings, and coffering. Their exploration considered the possible tension forces that led to the development of large, evenly spaced meridional cracks running from the bottom of the dome. These cracks are assumed to have formed soon after construction as repairs utilize the same bricks as the original structure (Terenzio 1934, Lancaster 2005). Zessin (2012) proposes an alternate means for the formation of the meridional cracks using limit analysis that also matches the cracks observed in the Pantheon. Lancaster (2009) applied thrust line analysis to study various Roman vaulted structures, and she investigated a range of questions surrounding the construction of the Pantheon, including the role of the step

rings and the use various densities of concrete, *caementa*, in the structure. Her study is the most complete analysis of the Pantheon structure using static analysis, though it does not address the question of collapse limits for the current building.

In summary, previous authors have conducted numerous studies of the history and structure of the Roman Pantheon. However, none have attempted to quantify the limits of collapse due to tilting, an equivalent static analysis of horizontal acceleration due to earthquakes. Zessin (2012) calculated the first theoretical predictions of the stability of the Pantheon due to spreading supports, but did not validate the results with a numerical model or a physical experiment. To address these gaps in the existing literature, this thesis investigates the stability of the dome of the Pantheon through a scale model.

## 3.0 Methodology

This chapter describes the design and fabrication of the physical model and test experiments and documents assumptions used in creating the discretized scale model.

### 3.1 Model

The dimensional measurement of the Pantheon has been highly varied throughout its history, dating back to Serlio and Palladio in the 16<sup>th</sup> century. For example, measurements of the horizontal diameter of the dome taken in the second half of the 21<sup>st</sup> century vary from 43.3 meters to 44.08 meters (de Fine Lict 1966, Geertman 1980, Pelletti and Martines 1989, Bartoli 1994, Aliberti and Altozano 2011, Fernandez-Cabo 2013). Most studies state 43.8 meters as the horizontal diameter of the dome because it equates to a measure of 148 Roman feet. The most accurate digital survey of the Roman Pantheon has been the collaboration on the Bern Digital Pantheon Project (Grabhoof 2009). In addition to providing accurate measurements of the current building, the survey demonstrates the clear deviations that refute the common misconception that the dome and base form a perfect hemisphere. The discretized virtual model used to 3D print the physical model was derived from the dimensions of the geometric model created by Lancaster (2005). A section elevation of the Lancaster model is shown in Figure 3.1

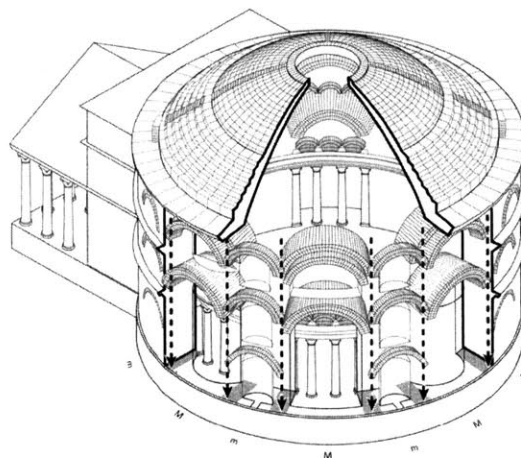


Figure 3.1 : Model illustrating the construction of the Pantheon by Lancaster (2005).

In order to create a 3D printed scale model of the Roman Pantheon, a virtual model is created to explore the stereotomy. A virtual model of the Roman Pantheon was created using Rhinoceros 3D, a computer aided drawing and modeling software (rhino3d.com). Particular attention was paid to preserving the voids within the walls of the drum.

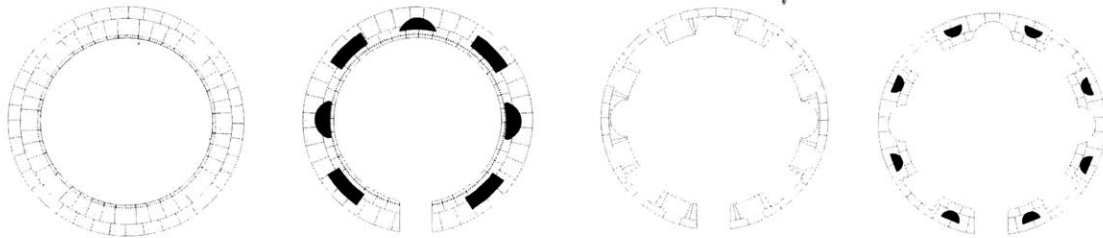


Figure 3.2: Four plan sections taken from different heights of the Pantheon drum

The model was divided into eleven horizontal courses, four courses for the drum and seven courses for the dome. Each of the four courses of the drum and the lowest layer of the dome were divided into an inner and outer layer, which also aided the printing of the voids within the walls. The model does not include the porch of the structure as it does not provide structural support of the dome.

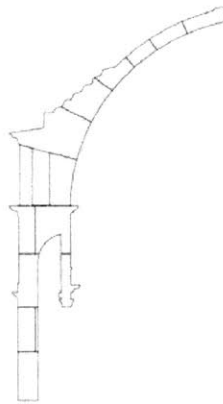


Figure 3.3: A typical cross section showing the discretization of the eleven courses, four horizontal courses for the drum and seven radial courses for the dome

The virtual model was discretized into 492 blocks, 200 for the dome and 292 for the drum. All joints were placed radially normal to the interior surface of the dome. The horizontal discretization of the dome followed the ribs of the coffers, a choice made for visual clarity and to aid in constructing the model. The vertical discretization alternated between the ribs of the coffers and the midpoint of the coffers.

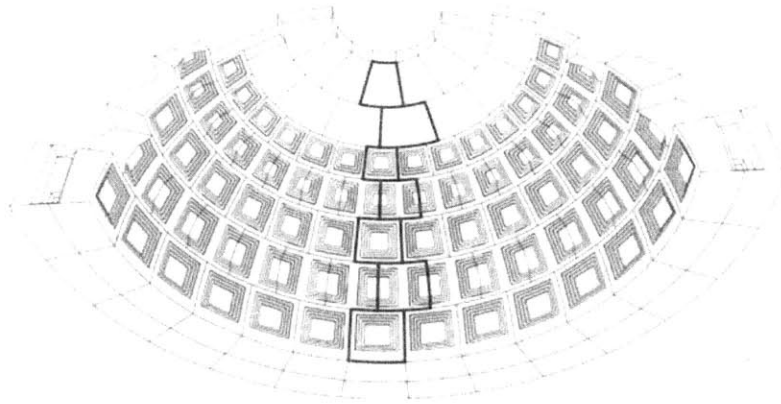


Figure 3.4: Axonometric view showing the offset discretization of the seven radial courses in relation to the coffers of the Pantheon dome

The discretized model was 3D printed using a ZCorp 350 printer. The model is 1/100<sup>th</sup> of full scale: one centimeter in the model represents one meter in the Roman Pantheon. All blocks were printed with a uniform density as a conservative assumption, since the density of the concrete in the Roman Pantheon likely has lighter aggregate than the base. All printed blocks were infiltrated with ZBond 90 to cure and harden the blocks. The density and friction of the blocks were measured empirically to be 0.5 g/cm<sup>3</sup> with a friction angle of 43 degrees. Blocks were labeled according to the course and position based on the radial symmetries within a particular layer.



Figure 3.5: Image of the labeled blocks for courses 10 and 11 during construction



### 3.2 Physical Experiments

The model was constructed atop a piece of 10 mm thick oriented strand board. Two test bases were constructed: a tilting assembly and a spreading assembly. Both used the same setup with the exception that the base of the spreading assembly is divided into quadrants while the base for the tilting assembly is a single board. A cylinder matching the inner radius of the drum was cut from the base board in order to allow for the removal of the supporting formwork and to view collapse experiments from below. The cylinder was affixed to the original board using brackets. Sand paper was affixed on top of the board in order to increase the coefficient of friction at the base to prevent sliding between the blocks of the base layer and the support.

Foam formwork matching the curvature of the interior surface of the dome (i.e. the surface if no coffers were presented) was milled using a computer numerical control router. The formwork was subdivided into four quadrants and trimmed to provide a suitable gap for ease of removal when lowering the formwork after construction. Temporary spacers were used to ensure the formwork was in the correct location. Threaded rods, which allowed the supporting formwork to be lowered after assembling the dome, were used to support the temporary foam formwork. After the model was assembled, the formwork was lowered using the threaded rods. The removal of the formwork was made easier because of access through the oculus of the dome, which allowed for the removal of the temporary spacers maintaining the location of the interior formwork. The formwork, threaded rods, and base cylinder were then lowered allowing for a view from below the dome. The formwork, assembly process, spacers, completed model, and lowering of the formwork are shown in Figure 3.6 and 3.7. The interior view of the completed dome including the coffers and oculus is shown in Figure 3.8

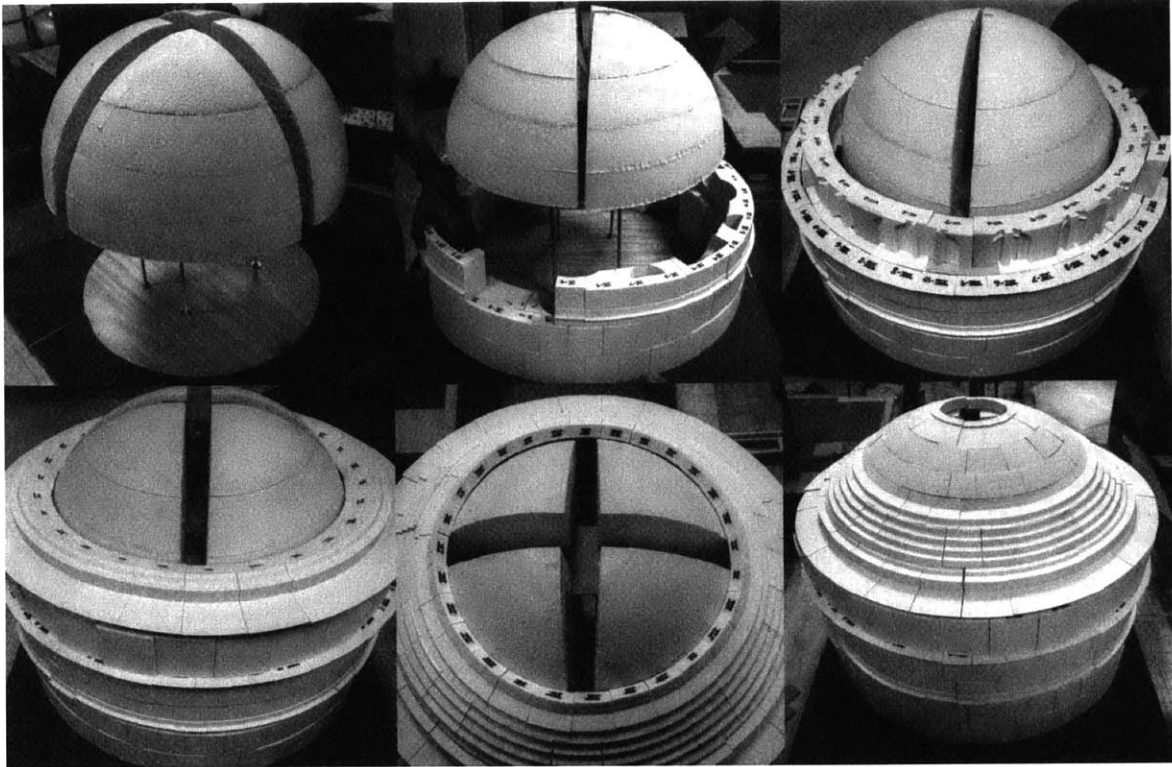


Figure 3.6: Construction process of the full model showing the use of formwork

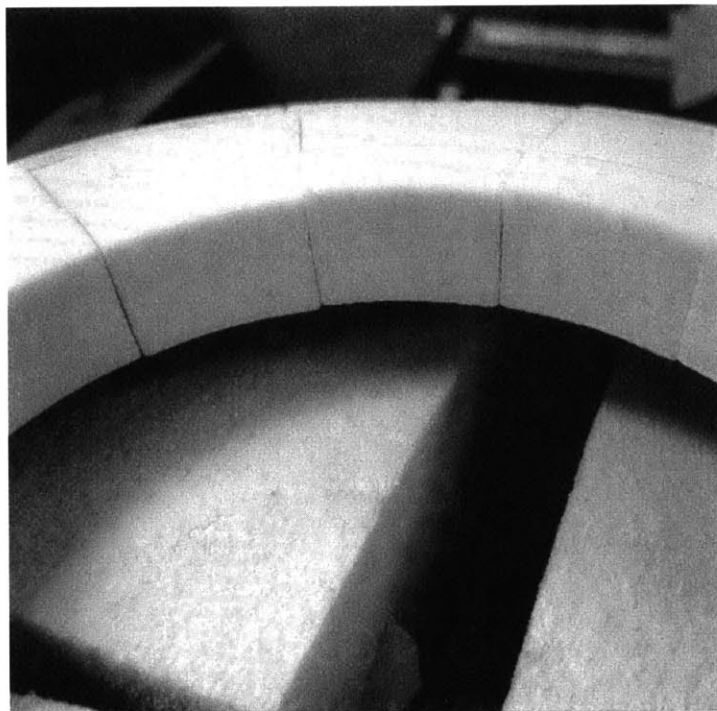


Figure 3.7: View of dome from oculus showing the initial lowering of the formwork

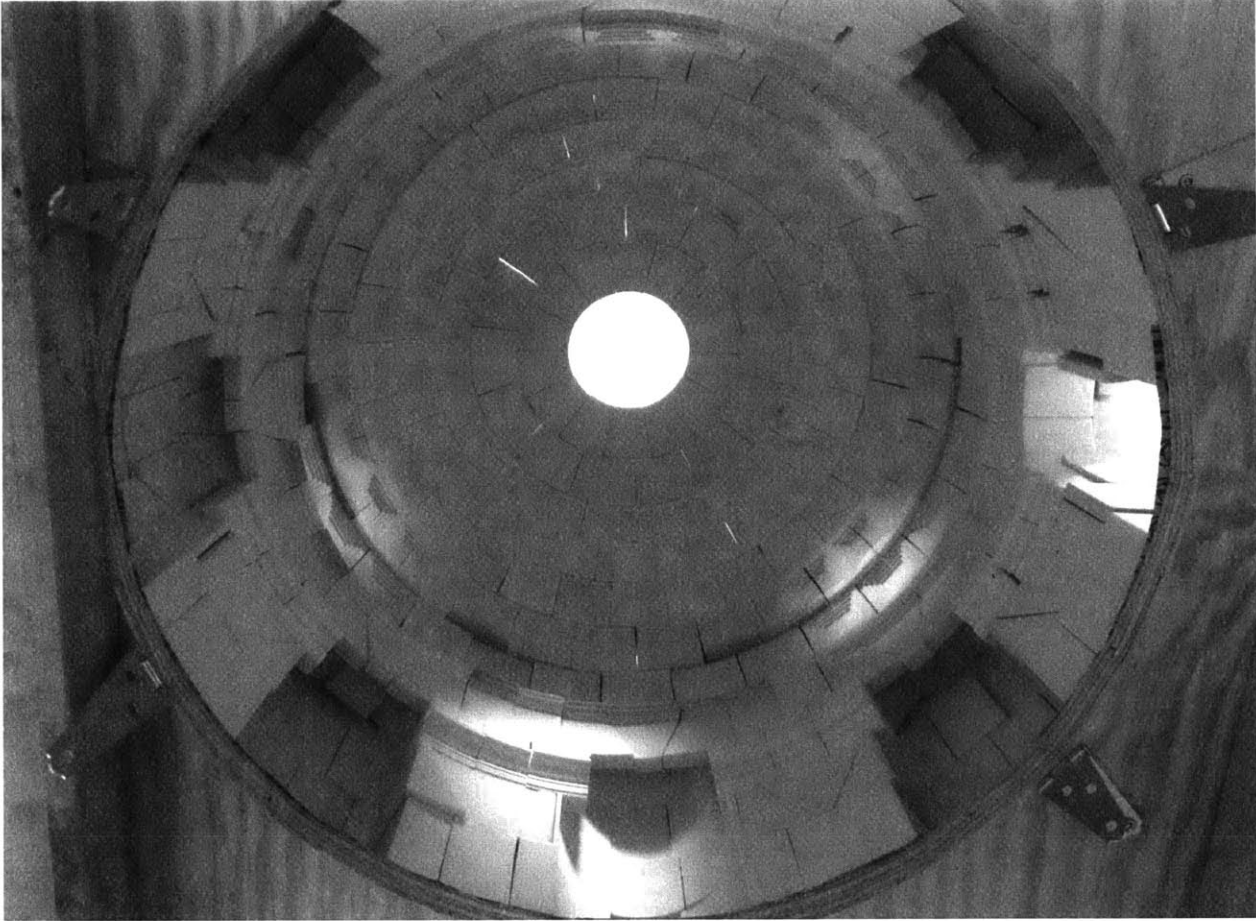


Figure 3.8: Interior view of the completed model assembly

## 4.0 Spreading Supports

### 4.1 Analytical Prediction

Zessin (2012) found the Rankine factor of the Roman Pantheon to be 3.2 in its current state, building on the work of Lancaster (2005) who found using graphical methods the Rankine factor to be 4.2. Additionally, Zessin proposed an analytical prediction that the span increase that would cause the dome to collapse is 4.43 meters, which corresponds to a 6 degree lean of the supporting walls. It follows that the analytical prediction for the span increase that would cause collapse of the scale model is 4.4 centimeters.

### 4.2 Scale Model Collapse Results

Two trials of a spreading test were conducted on the scale model.

Trial	Span Increase
1	3.4 cm
2	3.0 cm
Average	3.2 cm

Table 4.1

A span increase of 3.2 cm is observed to cause collapse of the dome. The critical span increase is observed to be 7.3% of the clear span. The experimental result is conservative for three reasons. First, the analytical prediction assumes two densities of materials in the dome, while the scale model blocks have uniform density. Second, the discretization of the dome in the analytical prediction has twice as many courses, which allows for the hinge location to form closer to the actual location. Third, the physical model is susceptible to imperfections in construction and printing. This last assumption is often addressed using an effective thickness modification to account for the rounding of edges the blocks after multiple tests. The effect was observed and quantified during tilting tests of arch models by DeJong (2009).

## 5.0 Horizontal Acceleration through Tilting

### Analytical Predictions

Equivalent static analysis is frequently used to approximate an earthquake load. This allows for a tilt analysis to be a first-order approximation. Horizontal ground acceleration is the equivalent of gravity multiplied by the tangent of the tilt angle. The dome of the Roman Pantheon has a variable thickness, at the top of the oculus, the thinnest portion of the dome is one meter thick, while at the base of the dome where the dome meets the drum, the dome is four meters thick. Using the conservative assumption that the dome could be approximated as a uniform thickness dome with a thickness-to-radius ratio of .25, the dome is predicted to collapse at 30 degrees, or at a horizontal acceleration of .58g (Zessin 2012). The critical lateral acceleration can be calculated using the tangent of the tilt angle at collapse.

### Experimental Results

Three trials of a tilting test were conducted on the dome of the scale model. Images of the collapse mechanisms are shown in Figure 5.1 and Figure 5.2.

Trial	Collapse Angle
1	27.1
2	26.8
3	28.0
Average	27.3

Table 5.1

The average tilt angle of 27.3 yields a collapse at a critical lateral acceleration of .51g. The results of the third trial indicate that the quality of the assembly impacts collapse as it would be accepted that the collapse angle would be lower than previous trials due to slight deterioration of the blocks resulting from multiple collapses.

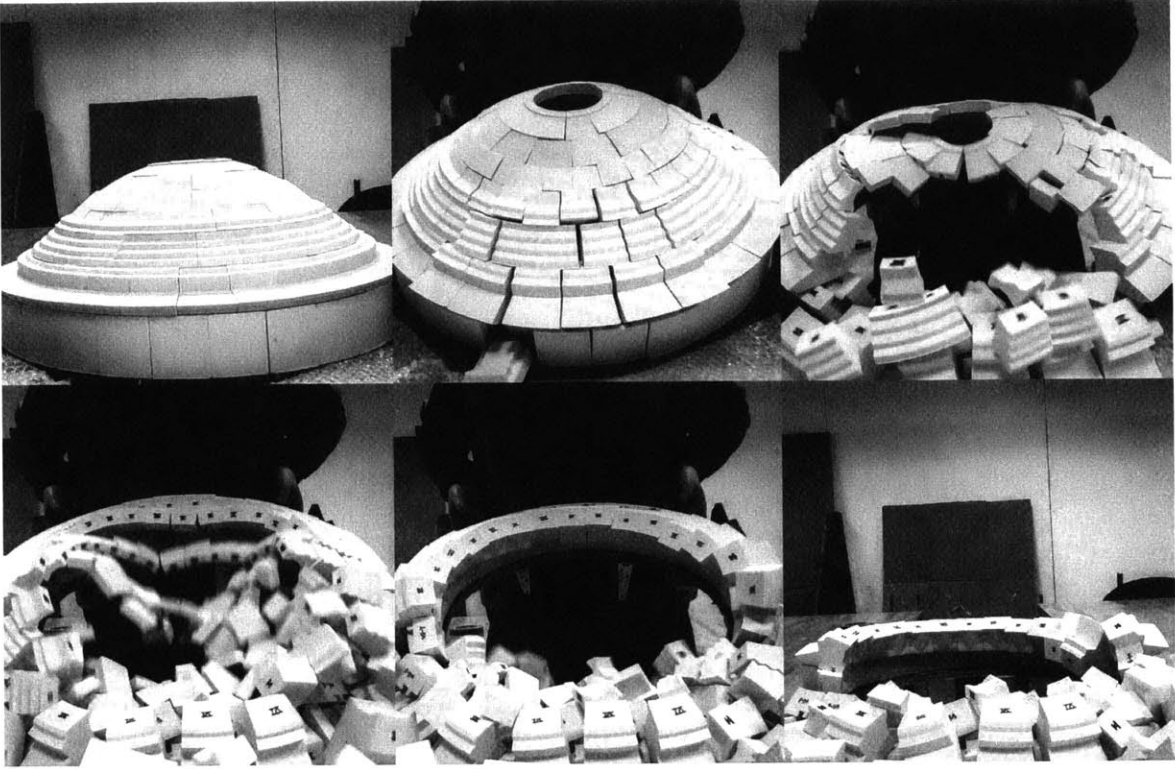


Figure 5.1: Dome collapse due to tilting front view

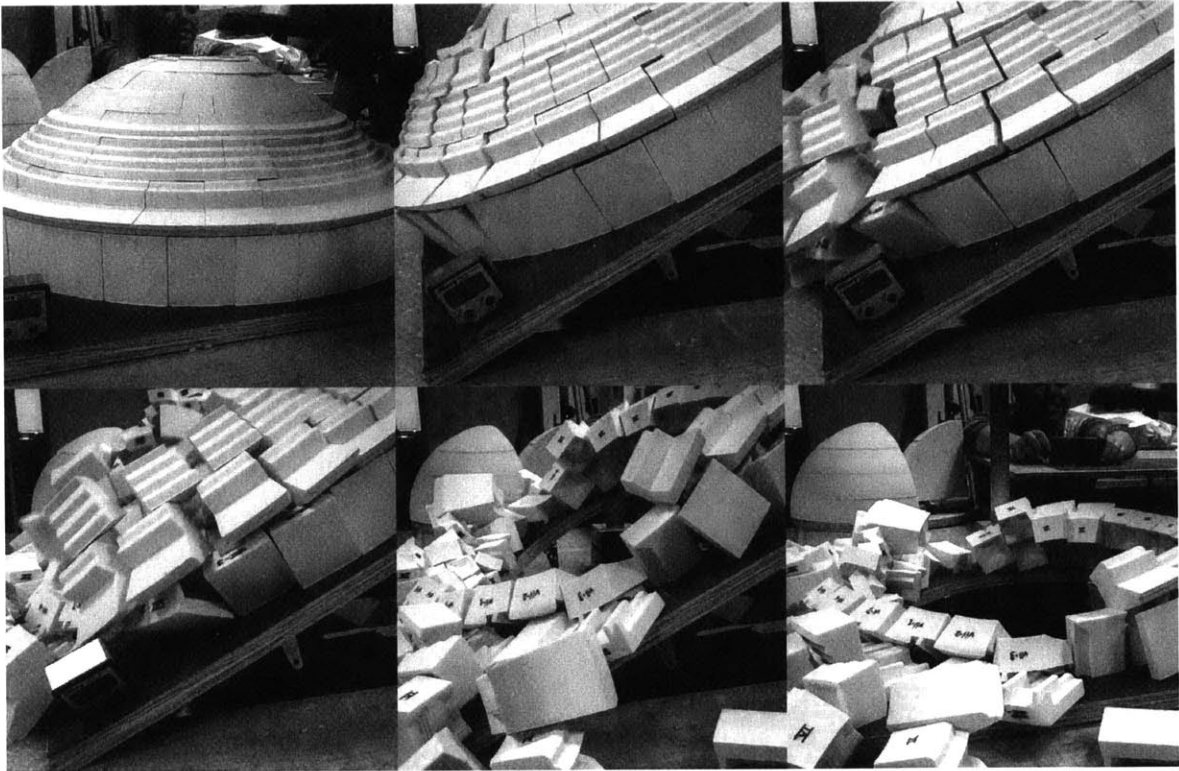


Figure 5.2: Dome collapse due to tilting side view

## 6.0 Discussion

### 6.1 Spreading Supports

The conservative assumptions used in testing the scale model indicate the experiment results support the analytical prediction using limit analysis for the dome. The critical span increase before collapse is observed to be 7.3% of the clear span compared to 10.2% in the analytical prediction. As the meridional cracking within the dome of the Roman Pantheon has already increased the span by approximately one meter, the physical experiments indicate that the current, deformed structure has a geometric factor of safety against collapse of at least 3, versus 4.5 indicated based on the analytical factor of safety. Assuming the scale model experiment is conservative, the results indicate alignment between the analytical Rankine (4.2), experimental geometric (3.1), and analytical geometric (4.5) factors of safety. The results of the scale model experiments indicate that the dome span could increase by an additional two meters before collapse, confirming the high degree of stability of the structure.

### 6.2 Horizontal Acceleration through Tilting

Comparing the result to the analytical predictions of Zessin (2012), the critical lateral acceleration of a uniform thickness dome with a thickness to radius ratio of .25, approximating the thickness of the Roman Pantheon, is .58g. The average tilt angle of collapse 27.3 yields a critical lateral acceleration of .51g. This result validates the analytical predictions using limit analysis to calculate the collapse mechanisms of domes subject to horizontal ground acceleration. It also provides another result to support using a uniform thickness dome as an approximation for a variable thickness dome to simplify calculations. Zessin (2012) finds analytically the critical tilt angle for a .2 thickness to radius ratio dome is 26 degrees, a critical lateral acceleration of .49g, which was confirmed using a scale model with a collapse tilt angle of 24.8 degrees (.46g) (Zessin 2012).

## **7.0 Conclusion**

### **7.1 Summary of Findings**

- The use of scale models provides a method to validate analytical models.
- Analytical predictions of collapse due to spreading supports using limit analysis are validated by the scale model experiments on the Roman Pantheon.
- The experimental factor of safety against spreading is conservative compared to the analytical prediction.
- The stability of the Roman Pantheon against collapse to spreading supports or leaning walls is confirmed.
- Approximating the critical lateral acceleration of a variable thickness dome as a uniform thickness dome is shown to be a valid simplifying assumption due to the high lateral stability of the form.
- The scale model experiments validate the analytical predictions of the high lateral stability of unreinforced masonry domes.

### **7.2 Future Research**

This thesis outlines the collapse mechanisms of a specific unreinforced masonry dome, using a 1:100 scale model of the Roman Pantheon. Future research can be performed using scale physical experiments to validate analytical models 1) specific to the Roman Pantheon and 2) more broadly on non-uniform thickness masonry domes.

### **Future Work on the Roman Pantheon**

#### **Variable Material Density**

In order to demonstrate the worst-case performance of the dome, the fabricated model has a uniform material density. Most researchers agree that concrete of the Roman Pantheon becomes lighter closer to the oculus as the density of the pozzolanic concrete is reduced (de Fine Licht 1968, Lancaster 2005, Belardi 2006); however, the height when the



material change occurs, reduction in density, and number of different materials are not well established. This lower density concrete makes the dome more stable by reducing the thrust on the supporting walls. Digital fabrication of the scale model allows for the density to be controlled, providing a quick method to model and test hypotheses of how much the lighter concrete increases structural stability. This could be modeled by introducing voids in the central blocks of the dome.

### **Stepped Rings**

The purpose of the “stepped rings” of the dome remains an open question. Exploring physical models where the stepped rings are printed separately from the uniform thickness hemispherical dome, as performed in the photo-elasticity experiments of Mark and Hutchinson (1986), could provide insight into whether the rings increase the overall structural stability or if they serve perhaps only as an aid to the construction – reducing the steep angle to place concrete at the base of a hemispherical dome – or if they are purely an aesthetic choice.

### **Discretization**

The discretization of the scale model of the Roman Pantheon is derived from logical divisions of the building geometry. The use of large blocks may cause an overestimate of stability of the structure and produce greater error – simulating larger cracks than in the actual structure – where blocks are not dimensionally perfect. Determination of an appropriate discretization size through comparison of results through blocks of different sizes would provide additional validation. This study could also influence future scale models where the block size is not known or for which it would be too labor-intensive to replicate the actual structure. Similarly, aligning rather than offsetting the joints between blocks could approximate the large cracks present in the Roman Pantheon, causing the dome to act more like a series of arches.

## **Future Work on Scale Models of Masonry Domes**

### **Additional Case Studies**

An immediate extension of this research is its application to other iconic structures and dome geometries. The potential to replicate and to validate historic collapses of structures – something that is not necessary or possible for the Roman Pantheon, which has not experienced a documented collapse – is particularly promising. For example, replication on model scale of the partial collapses of the Hagia Sophia in Istanbul, Turkey, could provide validation of possible collapse mechanisms. Moreover, studying the collapse of scale models through physical experiments of other dome typologies, such as Islamic double-shell domes, will yield additional information on the accuracy of results to domes more broadly.

### **Dynamic Analysis**

As the results of this study confirm, seismic loads are more likely than deformation to cause a surprise collapse of the Roman Pantheon. Further study of the dynamic behavior, including rocking, sequential linear analysis, and time history analysis would be an area of additional research to explore scale effects between the virtual and physical model.

### **Three-Dimensional and Two-Dimensional Analytical Models**

The structural behavior of unreinforced masonry vaults throughout history has been studied using two-dimensional slices, simplified from the three-dimensional forms due to the mathematical complexity and indeterminacy of masonry structures. Domes, particularly those with primary vertical cracking patterns like the ones seen in the Roman Pantheon, allow for further research into whether two-dimensional analytical models remain a valid tool or if more recently developed three-dimensional models yield more accurate results.

## References

- Aita, D., R. Barsotti, and S. Bennati. "Equilibrium of Pointed Circular and Elliptical Masonry Arches Bearing Vertical Walls." *Journal of Structural Engineering* 138, no. 7 (2012): 880-8.
- Belardi, G. *Il Pantheon: storia, tecnica e restauro*. Viterbo, Italy: BetaGamma, 2006.
- Beltrami, L. *Il Pantheon*. Milan: Allegrretti, 1898.
- Block, P., T. Ciblac, and J. Ochsendorf. "Real-time Limit Analysis of Vaulted Masonry Buildings." *Computers and Structures*. 84, no. 29-30 (2006): 1841-1852.
- Block, P., and L. Lachauer. "Three-dimensional equilibrium analysis of gothic masonry vaults." In *8th International Conference on Structural Analysis of Historical Constructions* (2012).
- Block, P., and L. Lachauer. "Three-dimensional funicular analysis of masonry vaults." *Mechanics Research Communications* 56 (2014): 53-60.
- Block, P., L. Lachauer, and M. Rippmann. "Validating Thrust Network Analysis using 3D-printed, Structural Models." *Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium* (2010).
- Boothby, T.E. "Analysis of masonry arches and vaults." *Progress in Structural Engineering and Materials* 3, no. 3 (2001): 246-56.
- Bossut, C. "Recherches sur l'équilibre des voûtes." In *Histoire de l'Académie Royale des sciences*. Paris: Imprimerie royale, 1774.
- Bossut, C. "Nouvelles recherches sur l'équilibre des voûtes en dôme." In *Histoire de l'Académie Royale des sciences*. Paris: Imprimerie royale, 1776.
- Caliò, I., F. Cannizzaro, M. Marletta. "A discrete element for modelling masonry vaults." *Advanced Materials Research* 133-134 (2010): 447-52.
- Cerasoli, F. "I restauri del Pantheon dal secolo XV al XVIII." *Buletino della commissione archeologica comunale di Roma* (1909): 280-89.
- Cooke, T. and J. Ochsendorf, "The Temple of Diana at Baiae: History and Structure of an Imperial Roman Dome." *Nuts and Bolts of Construction History: Culture, Technology and Society*. Paris: Picard, 2012.

- Coulomb, C. "Essai sur une application des règles de maximis et minimis à quelques problèmes de Statique, relatifs à l'architecture." *Memoires de Mathematique et de Physique presents a l'Academie Royale des Sciences*. Savans, Paris, 1776.
- Croci, G. *Il Pantheon: Storia, Tecnica e Restauro*. Betagamma, 2006.
- De Fine Licht, K. *The Rotunda in Rome: A study of Hadrian's Pantheon*. Copenhagen: Gyldendal, 1968.
- DeJong, M. "Seismic Assessment Strategies for Masonry Structures." PhD thesis. Massachusetts Institute of Technology. Cambridge, MA, 2009.
- De Jong, M., and J. Ochsendorf. "Analysis of vaulted masonry structures subjected to horizontal ground motion." *Structural Analysis of Historical Constructions*. Edited by P. Lourenco, P. Roca, C. Modena, S. Agrawal. New Delhi (2006): 973-80.
- Durand-Claye, A. "Note sur la vérification de la stabilité des voûtes en maçonnerie et sur l'emploi des courbes de pression." *Annales des ponts et chaussées* (1897): 63-96.
- Durand-Claye, A. "Vérification de la stabilité des voûtes et des arcs: Application aux voûtes sphériques." *Annales des ponts et chaussées* (1880).
- Eddy, H.T. *New constructions in graphical statics*. New York: D. Van Nostrand, 1877.
- Huerta, S. (2004). *Arcos bóvedas y cúpulas. Geometría y equilibrio en el cálculo tradicional de estructuras de fábrica*. Madrid: Instituto Juan de Herrera, 2004.
- Fernandez, S. Huerta. "Designing by Geometry. Rankine's Theorems of Transformation of Structures." In *Geometría y proporción en las estructuras. Ensayos en honor de Ricardo Aroca*. Lampreave, 2010.
- Fernandez, S. Huerta. "Oval Domes: History, Geometry and Mechanics." *Nexus Network Journal* 9, no. 2 (2007): 211-48.
- Fernandez, S. Huerta. "The Analysis of Masonry Architecture: A Historical Approach." *Architectural Science Review* 51, no. 4 (2008): 297-328.
- Fernandez-Cabo, Miguel Carlos. "Analysis of Different Hypotheses about the Geometric Pattern of the Pantheon's Coffered Dome." *Nexus Network Journal* 15, no. 3 (2013): 527-47.

- Foce, F. and D. Aita. "The masonry arch between 'limit' and 'elastic' analysis. A critical re-examination of Durand-Claye's method." *Proceedings, First International Congress on Construction History* Vol. 2. 895-908. Edited by S. Huerta. Madrid: Instituto Juan DeHerrera, 2003.
- Heyman, J. *Equilibrium of Shell Structures. Oxford Engineering Science Series*. Oxford: Clarendon Press, 1977.
- Heyman, J. "On Shell Solutions for Masonry Domes." *International Journal of Solids and Structures* 3 (1967): 227-41.
- Heyman, J. *Structural Analysis: a Historical Approach*. Cambridge: Cambridge University Press, 1998.
- Heyman, J. *The Stone Skeleton: Structural Engineering of Masonry Architecture*. Cambridge: Cambridge University Press, 1995.
- Heyman, J. "The Stone Skeleton." *International Journal of Solids and Structures* 2 (1966): 249-79.
- Lancaster, L. *Concrete Vaulted Construction in Imperial Rome: Innovations in Context*. Cambridge, UK: Cambridge University Press, 2005.
- Lancaster, L. "Materials and Construction of the Pantheon in Relation to the Developments in Vaulting in Antiquity." In *The Pantheon in Rome. Contributions to the Conference Bern*. 117-26. Bern: Bern Studies, 2009.
- Lau, W. "Equilibrium Analysis of Masonry Domes." Master's thesis. Massachusetts Institute of Technology. Cambridge, MA, 2006.
- Lourenco, P. "Computations on historic masonry structures." *Progress in Structural Engineering and Materials* 4, no. 3 (2002): 301-19.
- Mark, R. and P. Hutchinson. "On the Structure of the Roman Pantheon." *The Art Bulletin* 68, no. 1 (1986): 24-34.
- MacDonald, W. *The Pantheon*. Cambridge, MA: Harvard University Press, 2002.
- Moore, D. *The Roman Pantheon: The Triumph of Concrete*. University of Guan Station, 1995
- Montani, C. "Il Pantheon e I suoi recenti restauri, Capitolium." *Capitolium* 8 (1932): 417-26.

- Ochsendorf, J. "Collapse of Masonry Structures." PhD thesis. University of Cambridge, Cambridge, UK, 2002.
- Ochsendorf, J. "The masonry arch on spreading supports." *Structural Engineer* 84, no. 2 (2006): 29-35.
- O'Dwyer, D. "Funicular analysis of masonry vaults." *Computers and Structures* 73 no. 1-5 (1999): 187-97.
- Oppenheim, J., D.J. Gunaratnam, and R.H. Allen. "Limit state analysis of masonry domes." *Journal of Structural Engineering* 115, no. 4 (1989): 868-82.
- Palladio, A. *Four Books of Architecture*. Cambridge, MA: MIT Press, 2002.
- Pliny the Elder. *Natural History*. Translated by H. Rackham. Cambridge, MA: Harvard University Press, 1938-63.
- Poleni, G. *Memorie istoriche della Gran Cupola del Tempio Vaticano*. Padova: Nella Stamperia del Seminario, 1748.
- Pollio, Vitruvius. *Ten Books on Architecture*. Cambridge, MA: Harvard University Press, 1914.
- Quinonez, A., J. Zessin, J. Ochsendorf. "Small-Scale Models for the Analysis of Masonry Structures." *Proceedings of the 7th International Conference on Structural Analysis of Historical Construction*. Edited by P.B. Lourenço, and P. Roca. Shanghai 2010.
- Quinonez, A., J. Zessin, A. Nutz, and J. Ochsendorf. "Small-Scale Models for Testing Masonry Structures." *Advanced Materials Research* 133-134 (2010): 497-502.
- Rankine, W.J.M. *A Manual of Applied Mechanics*, 17<sup>th</sup> Edition. London: Griffin, 1904.
- Shapiro, E. "Collapse Mechanisms of Small-Scale Unreinforced Masonry Vaults." Masters thesis. Massachusetts Institute of Technology, Cambridge, MA, 2012.
- Smars, P. "Etudes sur la stabilité des arcs et voûtes - confrontation des méthodes de l'analyse limite aux voûtes gothiques en Brabant." Thèse de doctorat. Université Catholique de Louvain. Leuven, 2000.
- Taylor, R. *Roman Builders: Roman Builders A Study in Architectural Process*. Cambridge: Cambridge University Press, 2003.

- Terenzio, A. "La restauration du pantheon de rome." *La conservation des monuments d'art et d'histoire* (1934): 280-85.
- Van Mele, T., J. McInerney, M. DeJong, and P. Block. "Physical and Computational Discrete Modeling of Masonry Vault Collapse." *Proceedings of the 8<sup>th</sup> International Conference on Structural Analysis of Historical Constructions, Wroclaw*. 2012.
- Vogel, Thomas. "A Structural Evaluation of the Pantheon: Envisaged Tasks." Digital Repository of the Bern Digital Pantheon Project. Edited by Gerd Graßhoff, Michael Heinzelmann, and Markus Wäfler. Bern: Bern Studies, 2008.
- Wolfe, W. S. *Graphical analysis: A Textbook on Graphic Statics*, 1<sup>st</sup> Edition. New York: McGraw-Hill, 1921.
- Yourcenar, Marguerite. *Memoirs of Hadrian*. New York: Farrar, Straus and Giroux, 2005.
- Zessin, J. "Collapse analysis of unreinforced masonry domes and curving walls." PhD thesis. Massachusetts Institute of Technology, Cambridge, MA, 2012.
- Zessin, J., W. Lau, and J. Ochsendorf. "Equilibrium of Cracked Masonry Domes." *Proceedings of the ICE - Engineering and Computational Mechanics* 163, no. 3 (2010): 135-45.