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# Quantum Simulation of Generic Many-Body Open System Dynamics Using Classical Noise

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We introduce a scheme for the quantum simulation of many-body decoherence based on the unitary evolution of a stochastic Hamiltonian. Modulating the strength of the interactions with stochastic processes, we show that the noise-averaged density matrix simulates an effectively open dynamics governed by  $k$ -body Lindblad operators. Markovian dynamics can be accessed with white-noise fluctuations; non-Markovian dynamics requires colored noise. The time scale governing the fidelity decay under many-body decoherence is shown to scale as  $N^{-2k}$  with the system size  $N$ . Our proposal can be readily implemented in a variety of quantum platforms including optical lattices, superconducting circuits, and trapped ions.

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Understanding the nonequilibrium dynamics of a quantum system embedded in an environment is a long-standing problem at the core of the foundations of physics. Environmentally induced decoherence paves the way to the emergence of classical reality from a quantum substrate. The decoherence program and its extensions such as quantum Darwinism are focused on it [1]. The open quantum dynamics of a system is as well of relevance to quantum technologies.

While it is often desirable to beat decoherence and dissipation by suppressing system-environment interactions [2,3], new paradigms have emerged that fully embrace this coupling. To date, a variety of approaches have been put forward to simulate the reduced dynamics of an open quantum system [4–6], including the engineering of quantum jump operators via digital quantum simulation [7,8], or encoding the role of the environment in an auxiliary qubit [4,9]. Important instances also include dissipative state preparation and quantum computation [10–15]. Recent efforts focus on the possibility of engineering the environment to which the system is coupled [14,16,17], which provides new avenues for quantum simulation of exotic phases of quantum matter [4–6]. Engineering of artificial baths is also motivated by the need to compute thermal averages in a variety of fields ranging from statistical mechanics [18,19] to machine learning [20]. Further applications include the characterization and quantification of quantum non-Markovian behavior [21] and its experimental detection [22]. As an alternative, one can resort to a unitary quantum circuit [23], e.g., in combination with measurement of multitime correlation functions [24], for which efficient quantum algorithms have been developed [25].

In this Letter, we introduce a versatile scheme for the quantum simulation of the open dynamics of a many-body system embedded in an environment to which it couples via many-body interactions. The open-system dynamics is simulated in another, more controllable experimental platform, by adding appropriate classical noise processes. Our

scheme exploits current technologies for digital and analog quantum simulation of unitary dynamics, and can be readily implemented in various experimental platforms such as trapped ions, superconducting circuits, and cold atoms.

Our approach is based on the quantum simulation of an isolated many-body system described by a stochastic Hamiltonian, where classical noise is used as a tool to simulate many-body open-system dynamics. In particular, we focus on the addition of noise (understood as a stochastic modulation in time) to the coupling constants of  $k$ -body operators in the Hamiltonian, and show that the ensemble-average over noise realizations is described by a density matrix that evolves according to a master equation with many-body Lindblad operators. Markovian dynamics can be accessed modulating the coupling constants with a white noise; non-Markovian dynamics requires colored noise. The scheme is illustrated in Fig. 1. We characterize

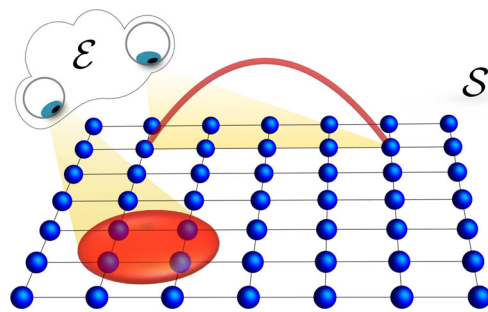


FIG. 1. Quantum simulation of many-body decoherence. The implementation in a quantum simulator of the unitary dynamics generated by a Hamiltonian with stochastic many-body terms is used to study the open dynamics induced by an environment  $\mathcal{E}$  that monitors many-body operators of the system  $\mathcal{S}$ . The red traits illustrate the  $k$ -body interactions which are general in our simulation scheme—specifically long-range and 4-body interaction in this illustration.

the resulting many-body decoherence dynamics by identifying the time scale governing the fidelity decay.

*Scheme for the quantum simulation of many-body decoherence.*—The reduced dynamics of a system embedded in an environment is generally described by a master equation of the form

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[\hat{H}_T(t), \rho(t)] + \mathcal{D}[\rho(t)], \quad (1)$$

where  $\rho(t)$  is the reduced density matrix of a “target” system, with Hamiltonian  $\hat{H}_T(t)$ , interacting with an environment. The first term on the right-hand side accounts for the unitary part of the evolution; the second term accounts for the nonunitary dynamics resulting from the interaction with the environment, which is described by the dissipator  $\mathcal{D}[\cdot]$ . We aim at the quantum simulation of this master equation when the  $\hat{H}_T(t)$  Hamiltonian describes a many-body quantum system. We shall see that our simulation scheme, which relies on the unitary evolution of a related stochastic simulator Hamiltonian  $\hat{H}_S(t)$ , generates a family of dissipators leading to many-body decoherence.

Specifically, our scheme utilizes the unitary dynamics of a stochastic wave function  $|\psi_{\text{st}}(t)\rangle$  and requires the experimental implementation of the stochastic Hamiltonian

$$\hat{H}_S(t) = \hat{H}_T(t) + \sum_{\alpha} \lambda_{\alpha}(t) \hat{L}_{\alpha}, \quad (2)$$

in the quantum platform. The Hamiltonian of the quantum simulator,  $\hat{H}_S(t)$ , is composed of the target Hamiltonian,  $\hat{H}_T(t)$ , describing the system one aims at simulating, and a stochastic part that includes a set of operators  $\hat{L}_{\alpha}$  with noisy coupling constants  $\lambda_{\alpha}(t)$ . This stochastic part will be used to engineer the dissipator in (1) leading to many-body decoherence.

For the sake of experimental implementation, we consider the simulator and target Hamiltonians to be Hermitian. Hermiticity carries over the stochastic term, yielding  $\sum_{\alpha} \lambda_{\alpha}(t) \hat{L}_{\alpha} = \sum_{\alpha} \lambda_{\alpha}^*(t) \hat{L}_{\alpha}^{\dagger}$ . As a result,  $\hat{L}_{\alpha}$  need not be Hermitian if the coupling constants  $\lambda_{\alpha}(t)$  take complex values. We choose the latter to be of the form  $\lambda_{\alpha}(t) \equiv \hbar\sqrt{\gamma_{\alpha}}\eta_{\alpha}(t)$ , with  $\gamma_{\alpha}$  a positive real constant, and  $\eta_{\alpha}(t)$  a complex stochastic field chosen as independent random Gaussian processes. The latter can be decomposed as  $\eta_{\alpha}(t) = \eta'_{\alpha}(t) + i\eta''_{\alpha}(t)$ , where its real  $\eta'_{\alpha}(t)$  and imaginary  $\eta''_{\alpha}(t)$  parts are two independent real Gaussian processes satisfying

$$\begin{aligned} \langle \eta'_{\alpha}(t) \rangle &= \langle \eta''_{\alpha}(t) \rangle = \langle \eta'_{\alpha}(t)\eta''_{\beta}(t) \rangle = 0, \\ K'_{\alpha\beta}(t, t') &= \langle \eta'_{\alpha}(t)\eta'_{\beta}(t') \rangle, \\ K''_{\alpha\beta}(t, t') &= \langle \eta''_{\alpha}(t)\eta''_{\beta}(t') \rangle, \end{aligned} \quad (3)$$

where the bracket denotes averaging over noise realizations. The simulator Hamiltonian (2) can then be written in an equivalent form (see [26] for details),

$$\hat{H}_S(t) = \hat{H}_T(t) + \sum_{\alpha} \hbar\sqrt{\gamma_{\alpha}}(\eta'_{\alpha}(t)\hat{A}_{\alpha} + \eta''_{\alpha}(t)\hat{B}_{\alpha}), \quad (4)$$

where the operators  $\hat{A}_{\alpha} \equiv (\hat{L}_{\alpha} + \hat{L}_{\alpha}^{\dagger})/2$  and  $\hat{B}_{\alpha} \equiv i(\hat{L}_{\alpha} - \hat{L}_{\alpha}^{\dagger})/2$  are now Hermitian by construction, i.e.  $\hat{A}_{\alpha}^{\dagger} = \hat{A}_{\alpha}$  and  $\hat{B}_{\alpha}^{\dagger} = \hat{B}_{\alpha}$ .

The stochastic density matrix corresponding to one realization of the Gaussian processes,  $\rho_{\text{st}}(t) = |\psi_{\text{st}}(t)\rangle\langle\psi_{\text{st}}(t)|$ , is given in terms of the pure state  $|\psi_{\text{st}}(t)\rangle$ , which is obtained from the exact solution of the Schrödinger equation generated by the stochastic Hamiltonian implemented in the simulator,  $\hat{H}_S(t)$  in Eq. (4). Its time evolution is described by the stochastic quantum Liouville equation

$$\begin{aligned} \frac{d\rho_{\text{st}}(t)}{dt} &= -\frac{i}{\hbar}[\hat{H}_T(t), \rho_{\text{st}}(t)] \\ &\quad - i \sum_{\alpha} \sqrt{\gamma_{\alpha}} [\eta'_{\alpha}(t)\hat{A}_{\alpha} + \eta''_{\alpha}(t)\hat{B}_{\alpha}, \rho_{\text{st}}(t)]. \end{aligned} \quad (5)$$

Averaging over different realizations of each of the stochastic processes  $\{\eta_{\alpha}(t)\}$  leads to the noise-averaged density matrix,  $\langle\rho_{\text{st}}(t)\rangle = \langle|\psi_{\text{st}}(t)\rangle\langle\psi_{\text{st}}(t)|\rangle$ , the dynamics of which is governed by the master equation

$$\frac{d}{dt}\langle\rho_{\text{st}}(t)\rangle = -\frac{i}{\hbar}[\hat{H}_T(t), \langle\rho_{\text{st}}(t)\rangle] + \mathcal{D}[\rho_{\text{st}}(t)], \quad (6)$$

where

$$\mathcal{D}[\rho_{\text{st}}(t)] = -i \sum_{\alpha} \sqrt{\gamma_{\alpha}} ([\hat{A}_{\alpha}, \langle\eta'_{\alpha}(t)\rho_{\text{st}}(t)\rangle] + [\hat{B}_{\alpha}, \langle\eta''_{\alpha}(t)\rho_{\text{st}}(t)\rangle]). \quad (7)$$

Comparison of (6) with the master equation describing the reduced dynamics of open systems (1) enables us to identify  $\mathcal{D}[\cdot]$  as a dissipator responsible for an effective nonunitary evolution of the noise-averaged density matrix. The explicit form of the dissipator can be evaluated using Novikov’s theorem, which gives the mean value of a product of a Gaussian noise with its functional [31,32]. We refer the reader to [26] for the derivation that yields

$$\begin{aligned} \mathcal{D}[\rho_{\text{st}}(t)] &= -\sum_{\alpha\beta} \sqrt{\gamma_{\alpha}\gamma_{\beta}} \int_0^t dt' \\ &\quad \times (K'_{\alpha\beta}(t, t') [\hat{A}_{\alpha}, \langle[\hat{U}_{\text{st}}(t, t')\hat{A}_{\beta}\hat{U}_{\text{st}}^{\dagger}(t, t'), \rho_{\text{st}}(t)]\rangle] \\ &\quad + K''_{\alpha\beta}(t, t') [\hat{B}_{\alpha}, \langle[\hat{U}_{\text{st}}(t, t')\hat{B}_{\beta}\hat{U}_{\text{st}}^{\dagger}(t, t'), \rho_{\text{st}}(t)]\rangle]), \end{aligned} \quad (8)$$

where the time-evolution operator  $\hat{U}_{\text{st}}(t, t') \equiv \mathcal{T} \exp[-(i/\hbar) \int_{t'}^t \hat{H}_S(s) ds]$  is defined in terms of the full stochastic Hamiltonian and  $\mathcal{T}$  denotes the time-ordering operator.

*Markovian limit.*—The form of the dissipator greatly simplifies when the stochastic variables  $\{\eta_{\alpha}(t)\}$  are described by independent white noises such that  $K'_{\alpha\beta}(t, t') = K''_{\alpha\beta}(t, t') = \delta_{\alpha\beta}\delta(t - t')$ . In particular, the

dissipator now only depends on the average density operator  $\langle \rho_{st}(t) \rangle$ , that we hereafter denote by  $\rho(t)$  to simplify the notation. Equation (8) reduces in this case to

$$\begin{aligned} \mathcal{D}[\rho(t)] &= -\sum_{\alpha} \gamma_{\alpha} ([\hat{A}_{\alpha}, [\hat{A}_{\alpha}, \rho(t)]] + [\hat{B}_{\alpha}, [\hat{B}_{\alpha}, \rho(t)]]) \\ &= \sum_{\alpha} \gamma_{\alpha} \left( \hat{L}_{\alpha} \rho(t) \hat{L}_{\alpha}^{\dagger} - \frac{1}{2} \{ \hat{L}_{\alpha}^{\dagger} \hat{L}_{\alpha}, \rho(t) \} \right. \\ &\quad \left. + \hat{L}_{\alpha}^{\dagger} \rho(t) \hat{L}_{\alpha} - \frac{1}{2} \{ \hat{L}_{\alpha} \hat{L}_{\alpha}^{\dagger}, \rho(t) \} \right) \\ &= \sum_{\mu} \gamma_{\mu} \left( \hat{L}_{\mu} \rho(t) \hat{L}_{\mu}^{\dagger} - \frac{1}{2} \{ \hat{L}_{\mu}^{\dagger} \hat{L}_{\mu}, \rho(t) \} \right), \end{aligned} \quad (9)$$

where the  $\mu$  index in the last line includes the sum over the set  $\{\hat{L}_{\alpha}\} \cup \{\hat{L}_{\alpha}^{\dagger}\}$ . This form corresponds to the diagonal Lindblad form [33,34] of a Markovian dynamics, i.e., to the form the dissipator of the reduced dynamics in (1) would take whenever the time scale of the system is much longer than that of the environment. In this case, the equivalence between the master equations (6) and (1) and the form of the dissipator (9) shows that our scheme allows for the quantum simulation of an open system, upon identifying the noise-averaged density matrix  $\langle \rho_{st}(t) \rangle$  with the reduced density matrix  $\rho(t)$ . Notice that requiring each term in the sum to be associated with its conjugate follows from the Hermiticity of the stochastic part of the simulator Hamiltonian—second term on the right-hand side of Eq. (2). Lifting this condition would require the implementation of a non-Hermitian Hamiltonian in the simulator, which is outside the scope of our proposal since we are interested in a scheme readily implementable in current experimental platforms.

Notice that, if the stochastic processes are taken to be real from the beginning ( $\eta_{\alpha}''(t) = 0$ ), the  $\hat{L}_{\alpha}$  operators in (2) then fulfill Hermiticity. The resulting dissipator

$$\mathcal{D}[\rho(t)] = -\sum_{\alpha} \gamma_{\alpha} [\hat{L}_{\alpha}, [\hat{L}_{\alpha}, \rho(t)]], \quad (10)$$

becomes unital, i.e.,  $\mathcal{D}(\mathbb{1}) = 0$ , where  $\mathbb{1}$  is the identity operator on the Hilbert space of the target system. The noise-averaged dynamics thus leads to a monotonic decay of purity [35].

*Generalization to non-Markovian dynamics.*—While the use of white noise leads to a Lindblad dissipator simulating Markovian dynamics, many interesting processes follow a non-Markovian evolution. Such a general evolution can be obtained using colored noise. Solving the master equation (6) with the dissipator (8), although written locally in time because the dynamics generated by (2) remains unitary, requires the stochastic unraveling over different trajectories, or the use of perturbative schemes [31,36]. The latter approach allows us to describe the time evolution of the density matrix by a perturbative integrodifferential equation: To second order in the strength of the noise,

after approximating  $\hat{U}_{st}(t, t')$  by the deterministic time-evolution operator  $\hat{U}_T(t, t') \equiv \mathcal{T} \exp[-(i/\hbar) \int_{t'}^t \hat{H}_T(s) ds]$ , Eqs. (6)–(8) simplify to

$$\begin{aligned} \frac{d}{dt} \rho(t) &= -\frac{i}{\hbar} [\hat{H}_T(t), \rho(t)] - \sum_{\alpha\beta} \sqrt{\gamma_{\alpha}\gamma_{\beta}} \int_0^t dt' \left( K'_{\alpha\beta}(t, t') \right. \\ &\quad \left. \times [\hat{A}_{\alpha}, [\hat{A}_{\beta}^{\dagger}(t, t'), \rho(t)]] + K''_{\alpha\beta}(t, t') [\hat{B}_{\alpha}, [\hat{B}_{\beta}^{\dagger}(t, t'), \rho(t)]] \right), \end{aligned} \quad (11)$$

where  $\hat{A}_{\beta}(t, t') \equiv \hat{U}_T(t, t') \hat{A}_{\beta} \hat{U}_T^{\dagger}(t, t')$ . A specific non-Markovian evolution can thereby be simulated from a specific type of colored noises, which can be designed using a filter function convoluted with a white noise signal, as in signal analysis, or via a Cholesky decomposition as described in [37].

*Many-body decoherence.*—We next focus on a quantum simulator of  $N$  particles with many-body operators  $\hat{L}_{\alpha}$  invariant under the permutation of particles, i.e., fulfilling

$$[\hat{P}, \hat{L}_{\alpha}] = 0, \quad (12)$$

where  $\hat{P}$  is the permutation operator. Specifically, we consider the general case of symmetric  $k$ -body Lindblad operators of the form

$$\hat{L}_{\alpha} = \sum_{i_1 < \dots < i_k} \mathbb{L}_{i_1, \dots, i_k}^{(\alpha, k)}, \quad (13)$$

where the sum runs over all possible tuples of  $k$  particles. Our quantum simulation scheme then yields a broad class of dissipators which we associate with many-body decoherence, and which directly inherit the symmetrization over particle indices. To appreciate this, it suffices to consider the Hermitian case with a single coupling constant, taken as a real Gaussian process. Equation (10) readily gives the dissipator

$$\mathcal{D}[\rho(t)] = -\sum_{\alpha} \sum_{i_1 < \dots < i_k} \sum_{i'_1 < \dots < i'_k} \gamma_{\alpha} \left[ \mathbb{L}_{i_1, \dots, i_k}^{(\alpha, k)}, \left[ \mathbb{L}_{i'_1, \dots, i'_k}^{(\alpha, k)}, \rho(t) \right] \right]. \quad (14)$$

The structure of this dissipator radically differs from that customarily encountered in the study of decohering many-particle systems. Indeed, the customary dissipators introduced in the study of decohering many-body systems result from coupling  $k$  subsets of particles to independent environments, which gives rise to a single sum over the particle indices  $\{i_1, \dots, i_k\}$ , and is distinctly different from our result. As we shall discuss below, similar features are found in lattice systems where the symmetrization is over the lattice index. But let us first characterize the many-body dynamics.

A natural question concerns the time scale in which many-body decoherence alters the evolution of the system. We propose the use of quantum speed limits for arbitrary physical processes [38,39] to address this question. The

notion of speed relies on the distance traveled during the evolution, which can be quantified by the Bures length,  $\mathcal{L}[\rho(0), \rho(t)]$ , defined in terms of the fidelity between the initial and the time-evolving states. Assuming the initial state to be deterministically prepared in a pure state  $|\psi(0)\rangle$  at  $t=0$ , the fidelity simply reads  $F(t) = \langle \psi(0) | \rho(t) | \psi(0) \rangle = \cos^2 \mathcal{L}[\rho(0), \rho(t)]$ . It is well known that the short-time dynamics of the fidelity decay follows a quadratic dependence for unitary dynamics,  $F(t) = 1 - |\dot{F}(0)|^2 t^2 / 2 + \mathcal{O}(t^3)$ , and a linear decay for Markovian dynamics. Here, we recover the linear dynamics for the noise-averaged dynamics under stochastic Hamiltonians such as (2), but with a decoherence time that now reveals a strong signature of many-body decoherence. For the sake of illustration, we focus on the real white-noise case, Eq. (10). It is found that  $F(t) = 1 - t/\tau_D + \mathcal{O}(t^2)$ , where

$$\frac{1}{\tau_D} = \sum_{\alpha} \gamma_{\alpha} \Delta L_{\alpha}^2 \leq \frac{1}{4} \sum_{\alpha} \gamma_{\alpha} \|\hat{L}_{\alpha}\|^2, \quad (15)$$

and  $\Delta L_{\alpha}^2 = \langle \hat{L}_{\alpha}^2 \rangle - \langle \hat{L}_{\alpha} \rangle^2$ . The inequality follows from using the seminorm of the Hermitian operator  $\hat{L}_{\alpha}$ —the difference between its largest and lowest eigenvalue—as an upper bound for the variance [40].

The seminorm of the symmetrized  $k$ -body Lindblad operator (13) can be upper-bounded as  $\|\hat{L}_{\alpha}\| \leq \sum_{i_1 < \dots < i_k} \|\mathbb{L}_{i_1, \dots, i_k}^{(\alpha, k)}\| = \binom{N}{k} \|\mathbb{L}^{(\alpha, k)}\|$ , where  $\binom{N}{k}$  is the binomial coefficient. It follows that

$$\frac{1}{\tau_D} \leq \binom{N}{k}^2 \sum_{\alpha} \frac{\gamma_{\alpha}}{4} \|\mathbb{L}^{(\alpha, k)}\|^2 \sim \frac{N^{2k}}{k!^2} \sum_{\alpha} \frac{\gamma_{\alpha}}{4} \|\mathbb{L}^{(\alpha, k)}\|^2; \quad (16)$$

i.e. the decoherence time  $\tau_D$  scales as  $N^{-2k}$  where  $N \gg k$  is the number of particles in the quantum simulator and  $k$  denotes the range of the interaction terms. As a result, the rate of decoherence characterizing the noise-averaged dynamics generated by  $k$ -body stochastic Hamiltonians with  $k > 1$  greatly surpasses that under local environments ( $k = 1$ ). For the sake of illustration, we next discuss the implementation of our scheme with ultracold atoms trapped in an optical lattice and with spin chains.

*Local Lindblad operators and long-range dissipator.*—We first consider a Lindblad operator symmetrized over a single lattice index. This scenario naturally arises in the quantum simulation of the Bose-Hubbard model [41], which we use as our target Hamiltonian, taking

$$\hat{H}_T \rightarrow \hat{H}_{\text{BH}} = -J \sum_{\langle i, j \rangle} \hat{b}_i^{\dagger} \hat{b}_j + \sum_i \frac{U_i}{2} \hat{n}_i (\hat{n}_i - 1), \quad (17)$$

where  $\hat{b}_i$  and  $\hat{b}_i^{\dagger}$  are annihilation and creation operators at site  $i$ ,  $\hat{n}_i = \hat{b}_i^{\dagger} \hat{b}_i$  being the site occupation number operator. The constant  $J$  denotes the hopping amplitude and  $U_i$  the on-site interaction. Such a model can be implemented in an analog quantum platform formed by an optical lattice loaded with ultracold atoms. In the most common setting,

the interaction strength is site independent,  $U_i = U$ , and can be tuned via a Feshbach resonance [42]. It then acts as a coupling constant of an operator symmetrized over the particle index. Our scheme shows that its stochastic modulation via a single real white noise,  $U \rightarrow U + 2\hbar\sqrt{\gamma}\eta(t)$ , makes the dynamics of the noise-averaged density matrix effectively open. The evolution is then dictated by the master equation (6) with the dissipator

$$\mathcal{D}[\rho(t)] = -\gamma \sum_{i, j} [\hat{n}_i (\hat{n}_i - 1), [\hat{n}_j (\hat{n}_j - 1), \rho(t)]]. \quad (18)$$

While the corresponding Lindblad operator,  $\hat{L} = \sum_i \hat{n}_i (\hat{n}_i - 1)$ , is a local one-body operator, the double sum in (18) is not restricted to nearest neighbors and makes the dissipator  $\mathcal{D}[\cdot]$  effectively long range. The obtained master equation is exact to all orders in  $U$ . Notice that such dynamics is distinctively different from a standard dissipator, that would commonly display a single sum, and could be obtained here by setting  $i = j$  in (18), e.g., from the stochastic modulation of the interaction strength at each site. Clearly, our approach is not restricted to optical lattices and can be applied to ultracold atoms and polar molecules, including scenarios governed by three-body interactions [43]. Nor is it restricted to local Lindblad operators, as exemplified below.

*Long-range 2-body Lindblad operators.*—We next show how the stochastic modulation of the coupling constants in systems with (symmetrized) two-body interactions can be used to simulate the open quantum dynamics under long-range Lindblad operators. As an example, consider the long-range Ising chain in a transverse field  $h$ ,

$$\hat{H}_I = -\sum_{i < j} J_{ij} \sigma_i^z \sigma_j^z - h \sum_{i=1}^N \sigma_i^x. \quad (19)$$

Its experimental realization has recently been reported [44,45] with pairwise interactions exhibiting a power-law decay  $J_{ij} \propto |r_i - r_j|^{-a}$ , as a function of the distance  $r$  between two arbitrary sites  $(i, j)$  of the 1D chain. By adding a white-noise contribution to the interactions,  $J_{ij} \rightarrow J_{ij} + \hbar\sqrt{\gamma}\eta(t)$ , our results predict that the noise-averaged density matrix then obeys a master equation (6), where the target Hamiltonian is that of the Ising chain (19) and the dissipator takes a many-body nonlocal form given by

$$\mathcal{D}[\rho(t)] = -\gamma \sum_{i < j} \sum_{i' < j'} [\sigma_i^z \sigma_j^z, [\sigma_{i'}^z \sigma_{j'}^z, \rho(t)]]. \quad (20)$$

The associated dynamics is detailed in [26]. Up to parity effects, the decoherence time scales quadratically with the particle number,  $\tau_D \sim 1/N^2$ , for large  $N$  for an initial product state. By contrast, for maximally entangled states, the bound (16) is saturated and the enhancement scales as  $\tau_D \sim 1/N^4$ , a telltale sign of many-body decoherence. We emphasize that the 2-body long-range nature of the corresponding Lindblad operator,  $\hat{L} = \sum_{i < j} \sigma_i^z \sigma_j^z$ , is directly

inherited from the addition of noise to the coupling constant of the symmetrized two-body spin-spin interactions.

To summarize, we have developed a scheme for the quantum simulation of many-body decoherence, where classical noise is a tool used to facilitate the experimental realization of such a simulation. Our proposal relies on the unitary evolution generated by a many-body Hamiltonian that includes stochastic terms resulting from the addition of controlled noise to the interaction couplings. Averaging over the noise realizations yields an effectively open dynamics, which describes a wide variety of master equations characterized by many-body decoherence. In particular, the white-noise limit leads to Markovian dynamics, where the many-body Lindblad operators correspond to the operators introduced in the stochastic part of the simulator Hamiltonian. Non-Markovian effects can be accessed using colored noise. The characteristic time scale of evolution, as estimated from the fidelity decay, exhibits a strong signature of many-body decoherence as a function of the system size. Finally, we note that our scheme allows for the quantum simulation of a broad class of master equations that includes instances whose physical origin from first principles would be worth investigating via specific models of a system coupled to an environment. Because the addition of noise in the Hamiltonian is relatively easier than engineering specific dissipations, our proposal should find broad applications in environmental engineering for quantum technologies, including dissipation-assisted state preparation and quantum computation. Further, it can be readily implemented in a variety of platforms, including ultracold atoms in an optical lattice, trapped ions, and superconducting qubits.

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