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PAPER

Interactive diversity promotes the evolution of cooperation in structured populations

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Abstract

Evolutionary games on networks traditionally assume that each individual adopts an identical strategy to interact with all its neighbors in each generation. Considering the prevalent diversity of individual interactions in the real society, here we propose the concept of interactive diversity, which allows individuals to adopt different strategies against different neighbors in each generation. We investigate the evolution of cooperation based on the edge dynamics rather than the traditional nodal dynamics in networked systems. The results show that, without invoking any other mechanisms, interactive diversity drives the frequency of cooperation to a high level for a wide range of parameters in both well-mixed and structured populations. Even in highly connected populations, cooperation still thrives. When interactive diversity and large topological heterogeneity are combined together, however, in the relaxed social dilemma, cooperation level is lower than that with just one of them, implying that the combination of many promotive factors may make a worse outcome. By an analytical approximation, we get the condition under which interactive diversity provides more advantages for cooperation than traditional evolutionary dynamics does. Numerical simulations validating the approximation are also presented. Our work provides a new line to explore the latent relation between the ubiquitous cooperation and individuals' distinct responses in different interactions. The presented results suggest that interactive diversity should receive more attention in pursuing mechanisms fostering cooperation.

1. Introduction

Understanding the evolution of cooperation is a fundamental problem in socioeconomics and evolutionary biology [1, 2]. Resorting to the powerful mathematical framework of evolutionary game theory, researchers have invested much effort in exploring this problem [3–8]. Since the seminal work on spatial games [9], traditional explorations of the evolution of cooperation on ideal structureless (well-mixed) populations [3–6] are moved to structured cases where individuals are constrained to play with their first or second order of neighbors [10–15]. With the discovery of topologies embedded in human interactions in the real society [16–19], many further investigations have been carried out in various complex networks where nodes indicate individuals and links represent neighborhoods [20–23]. It is widely accepted that spatial structures promote the evolution of cooperation in terms of theoretical predictions [23–27], computation simulations [9, 28, 29], and behavioral experiments [30].

Besides exploring structural complexity [31], researchers set out to consider diversities recently [32–38]. For example, social diversity defines a scenario where the investment of each individual per game relies on the number of its social ties and individuals with more neighbors contribute less to each game. In particular, when social ties follow a scale-free distribution, individuals' investments per game are diverse, which leads to an

impressive boost of cooperation [34, 35]. Heterogeneous teaching activity models a situation where the rate of strategy imitation depends on both payoff difference and individuals' teaching rates, allowing that individuals with high teaching rates spread their strategies much faster than others do. The remarkable distinction of teaching rates enhances cooperation significantly [36]. Reacting diversity endows individuals with different rates to remove unwanted ties, by which a few individuals sever dissatisfied social ties (that link to defectors) swiftly while others tolerate neighbors' defection. The coexistence of different reacting rates promotes cooperation despite individuals' myopia in their own interests [37]. For a comprehensive review of the diversity, see [38].

Although various diversities receive much attention, so far, most researchers have just focused on the difference of individuals' social features [38], like the number of social ties [35], teaching rates [36] and reacting rates [37]. In these studies, each individual treats all its neighbors as equivalent and adopts an identical strategy against them in each generation [38], which is in stark contrast with many realistic situations [39–42]. For example, in epidemic dynamics, individuals may take distinct interactive strategies against neighbors to protect themselves from diseases, such as keeping the contact with the healthy while severing links to the infected [39]. In human society, heredity and inheritance endow individuals with distinct properties, like consanguinity, phenotypes and briefs. These individuals may live in harmony with those possessing similar properties while frequently in conflict with the others. A typical example is about parochial altruism that individuals benefit fellow group members at a cost to themselves and adopt a hostile attitude towards those from other groups [40, 41]. Besides, in biological systems, human disease missense mutations may cause specific perturbations in particular molecular interactions while leave other interactions undisturbed, which shows the distinct reactions of protein mutants in different interactions [42]. Also, one's emotions, like the sympathy or goodwill to the inferiors and the envy or spite to the superiors, may lead to its distinct acts towards neighbors in different social rank, which has been introduced to explore the evolution of cooperation [43, 44]. All the above examples indicate that individuals' actions in an interaction usually depend on both who they are and whom they interact with. Probably, the same individual behaves distinctly in different interactions in one generation.

Taken together, here we interpret the discriminative acts of an individual in different interactions as interactive diversity. From the perspective of networked systems, we investigate the dynamical effects of interactive diversity on the evolution of cooperation based on edge dynamics [45] rather than traditional nodal dynamics explored in other kinds of diversities [35, 37]. Put differently, instead of probing how cooperation disseminates over nodes, here we are more interested in how cooperation spreads from one edge to another. The well-mixed and structured populations represented by various complex networks are studied systematically. The obtained results show that interactive diversity fosters cooperation without invoking any other mechanisms.

2. Model

In a typical symmetric two-person game, two individuals simultaneously decide whether to cooperate (C) or to defect (D). Depending on their choices, they receive some payoffs, which can be expressed by a general 2×2 payoff matrix [29]. Without loss of generality, we use two parameters T, S to represent the payoff matrix as

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & S \\ T & 0 \end{pmatrix}. \end{array}$$

With $-1 < S < 1, 0 < T < 2$, three typical social dilemmas, the Prisoner's Dilemma [46], the Snowdrift game [47], and the Stag-hunt game [48], can be captured [49]. In this paper, we study interactive diversity and cooperation in these most used social dilemmas.

Figure 1 shows how interactive diversity is introduced in the structured population. Traditionally, without interactive diversity, each individual either cooperates or defects to all its neighbors in each generation [29]. While in the presence of interactive diversity, everyone is allowed to adopt cooperation and defection simultaneously against its different neighbors. As shown in figure 1(b), individual l_c cooperates with h_1 and meanwhile defects to h_2.

In each generation, each individual plays games with all its neighbors with corresponding strategies and accumulates payoffs from all games that it participates in. When it comes to strategy updating, each individual only knows its neighbors' actions towards itself owing to the interactions with these neighbors, while it is not informed how its neighbors interact with the third party. Therefore each individual would update its strategies by imitating how its neighbors behave to it. Here we take l_c updating the strategy against h_2 for example (figure 1(b)). First, a neighbor is randomly chosen from l_c's neighborhood, called h_1. The chosen one could be h_2, which means that l_c possibly treats h_2 in the same way as h_2 does to l_c. Next, if l_c's accumulated payoff P_{l_c} is less than h_1's accumulated payoff P_{h_1}, l_c imitates h_1 (l_c replaces its strategy against h_2 with h_1's strategy against l_c) with

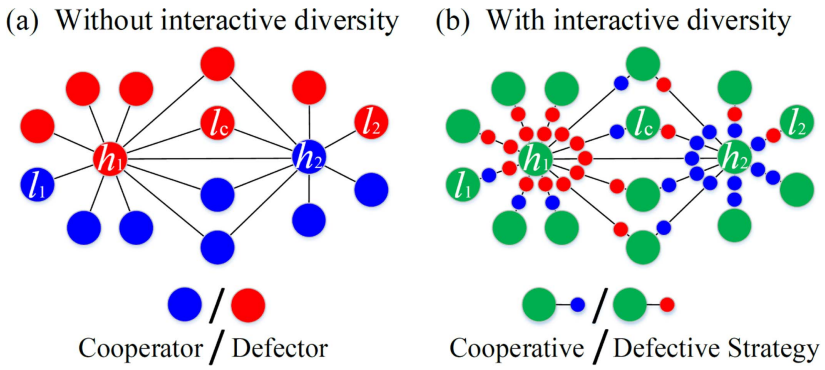

Figure 1. Illustrations of interactive diversity on a double-star graph. (a) Traditionally, without interactive diversity, each individual is assumed to adopt an identical strategy against all of its neighbors in each generation. For example, h_2 cooperates with all its neighbors while l_c defects. The corresponding individual is called a cooperator (blue) or a defector (red). (b) With interactive diversity, individuals (green) could adopt cooperative (blue) and defective (red) strategies simultaneously against different neighbors, where the strategies are marked with smaller cycles. For example, in the specific interaction happening between l_c and h_1, l_c chooses to cooperate while h_1 defects. Meanwhile, l_c adopts a defective strategy to another neighbor h_2, bringing an accumulative payoff $S + T$. In the presence of interactive diversity, individuals cooperating (defecting) in all interactions are still named as cooperators (defectors), like h_2 (h_1).

the probability $(P_{h_1} - P_{l_c})/(MK)$, where $M = \max\{T, 1\} - \min\{S, 0\}$ and K is the larger one between l_c's and h_1's degree. MK ensures the proper normalization of the probability [29]. Otherwise, l_c remains its strategy against h_2. l_c's strategies towards other neighbors are updated in the same way. In fact, all strategies of an individual are independent in some ways. Thus, it is reasonable for each individual to subsequently update its strategies independently by randomly choosing a neighbor to imitate, in accordance with the canonical updating rule [28, 29, 35]. In this paper, all individuals update their strategies synchronously in the same way as l_c (the example mentioned above). And asynchronous update, which usually best describes biological and human processes [31], is also studied for a comparison.

For a comprehensive research, we explore interactive diversity in both well-mixed and structured populations represented by complex networks with different degree of heterogeneity, including regular, random [50], small world [16], and scale-free networks [17]. To further understand the evolutionary dynamics from the microscopic perspective, a double-star graph with common leaves (figure 1), serving as the simplest abstraction of heterogeneous network [34, 35, 51], is constructed to show the detailed processes of evolution. The double-star graph has two centers, a left center h_1 with $N - 1$ leaves and a right center h_2 with $M - 1$ leaves. Among the leaves of h_1, $N - L - 1$ leaves l_1 are solely linked to h_1, and L leaves l_c are linked to both h_1 and h_2. Similarly, $M - L - 1$ leaves l_2 are just linked to h_2. In this paper, considering interactive diversity, we quantify the frequency of cooperation f_C with the average frequency of cooperative strategies [52, 53].

3. Results

By Monte Carlo simulations, we analyze the dynamics of interactive diversity in well-mixed and four structured populations represented by complex networks. Here the small world network is generated in terms of the Watts–Strogatz algorithm [16] with rewiring probability $p = 0.1$, and the scale-free network is generated via the mechanisms of growth and preferential attachment [17]. The population size is 200 for the well-mixed one and 1000 for structured ones with average degree 4. Initially, each individual randomly chooses cooperation or defection in each interaction, and the population has equal amount of cooperative and defective strategies. The mean frequency of cooperation is averaged over 5×10^3 generations after a transient time of 10^6 generations, and the equilibrium average is averaged over 10 simulations with 20 different realizations for each kind of network. The obtained results do not change in larger populations or with asynchronous update rules. Later, by theoretical analysis and Monte Carlo simulations on a double-star graph, we clearly elucidate the underlying microscopic mechanisms responsible for the outcomes in complex networks.

3.1. Interactive diversity in complex networks

Let us first focus on how interactive diversity affects the evolution of cooperation in the well-mixed population. In the SH domain ($-1 < S < 0, 0 < T < 1$), previous work has demonstrated that in the infinite well-mixed population without the interactive diversity, the critical value of the initial density of cooperators is $S/(T + S - 1)$ above which cooperators dominate while below which defectors take over the whole population

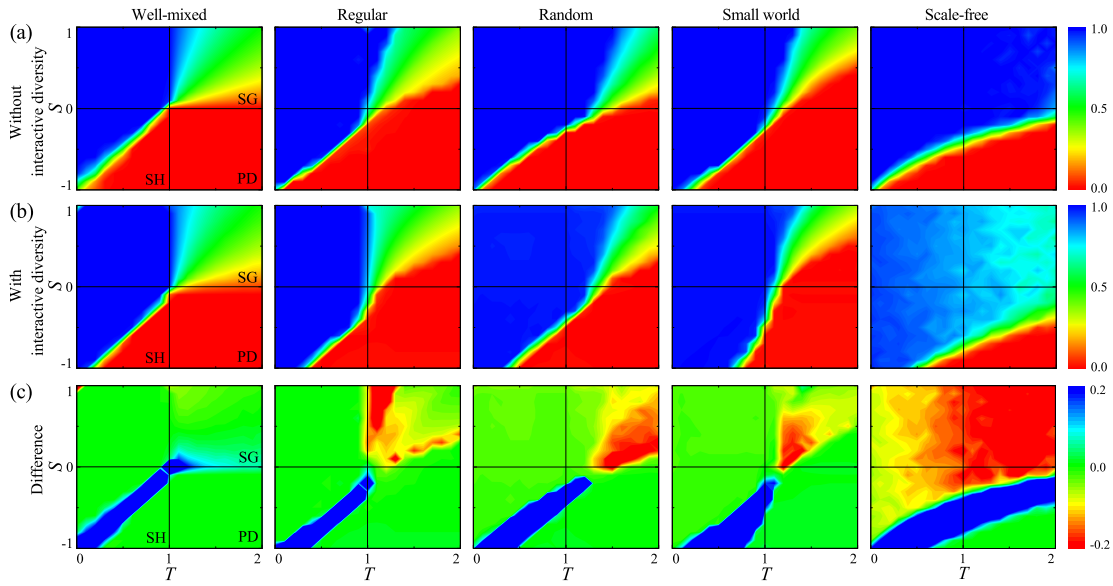

Figure 2. Interactive diversity favors cooperation in structured populations. For the evolution of cooperation without (a) and with (b) interactive diversity, the frequency of cooperation f_C in the well-mixed population, regular, random, small world, and scale-free networks are plotted as contours of T and S. Detailed difference of f_C shown in (c) is generated by subtracting those in (a) from those in (b) correspondingly. In each panel, the upper-right quadrant corresponds to the Snowdrift game (SG, $0 < S < 1 < T < 2$, $T + S < 2$), the lower-left one to the Stag-Hunt game (SH, $-1 < S < 0 < T < 1$), and the lower-right one to the Prisoner's Dilemma (PD, $-1 < S < 0$, $1 < T < 2$).

[54]. Here, we set the initial cooperator density to be 0.5 (apply to all the simulations in figure 2). When $T - S > (<)\ 1$, $S/(T + S - 1) > (<)\ 0.5$, meaning that under our initial setting cooperators finally die out (dominate). However, when interactive diversity is introduced, our results show that it can lead to the emergence of cooperation even when $T - S > 1$ (see the first column of figure 2), thus indicating that interactive diversity enlarges the survival ranges of cooperation. Similar results are also found in PD where the population inevitably evolves to full defection in the traditional model. When it comes to the SG domain, interactive diversity elevates the final level of cooperation. An intuitive explanation for these results is that, interacting diversely makes it possible for individuals to react to opponents' cooperative acts with cooperation and defective acts with defection. It helps to maintain individuals' incomes and reduce the exploitation by defectors, which weakens the definite advantage of defectors in the well-mixed population. We find a similar case in previous studies [55]: individuals with low strategy values and high image scores benefit from their mutual donation while refuse to contribute to defectors with the highest strategy values and low image scores, which leads to the extinction of defectors.

Then we employ the population structures represented by regular, random, small world, and scale-free networks. Due to the similar results obtained in the first three networks (middle three contours in figure 2(c)), here we analyze them together. We find that, in both the Prisoner's Dilemma and the Stag-hunt game, cooperation is elevated significantly beyond that supported network reciprocity alone. More to the point, the moderate topological heterogeneity embedded in random and small world networks seems to enhance the facilitative effects of interactive diversity on the evolution of cooperation. Here we do not stress this point too much. For these outcomes in the structured populations, the stable cooperative clusters are responsible. Inside the clusters, individuals preserve high payoffs by their mutual cooperation. Meanwhile, they defect to defectors outside to get out of exploitation. Consequently, individuals in the boundary of the clusters resist the invasion of defectors even in the harsh situation where the temptation to defect is sizable. Moreover, we observe the slight decrease of the cooperation level in SG, and a brief explanation is given below. The payoff structure of SG determines that it is better for individuals to adopt the opposite strategies of their opponents, which inhabits the formation of compact clusters of cooperators [53]. Meanwhile, since individuals are able to react differently to different neighbors in our model, the preferred opposite strategy pattern along a link is far more pervasive than that in the traditional case. These two together lead to the shrinkage in the level of cooperation.

Further we incorporate large topological heterogeneity in our study. By virtue of the greatest heterogeneity possessed by scale-free networks, interactive diversity guarantees the largest survival domain of cooperation in both PD and SH. However, in the relaxed social dilemma, such as the upper-left domain of PD, cooperation and defection coexist. Such coexistence is in contrast with the domination of cooperators in the traditional setting [29] as well as that in structured populations without large topological heterogeneity. In other words, when the

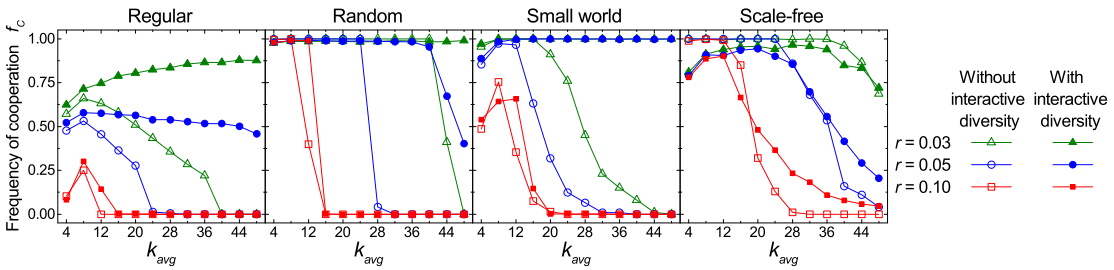

Figure 3. Interactive diversity maintains cooperation in the highly connected populations. We plot the frequency of cooperation f_C as a function of the average degree k_{avg} for different cost-to-benefit ratios r (0.03, 0.05, 0.10). Four populations structured in regular, random, small world, and scale-free networks are considered, respectively.

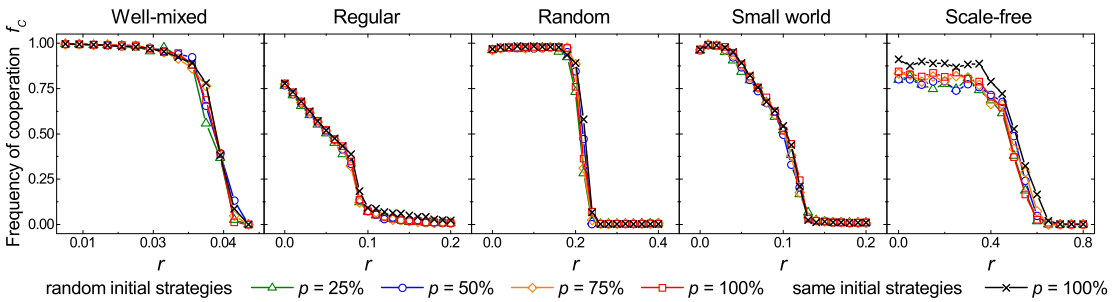

Figure 4. Evolution of cooperation with different strategy updating proportions under two initial states, say, with random initial strategies (open symbols) and with same initial strategies (×). Here the frequency of cooperation f_C is plotted as a function of r for four strategy updating proportions p (25%, 50%, 75%, 100%).

temptation to defect is small, the combination of interactive diversity and large topological heterogeneity reversely decreases cooperation level, implying that combining many promotive factors may make a worse outcome [25]. The evolution on the double-star graph later helps to make this mechanism clear, as figure 5 shows.

Next, we explore interactive diversity in the highly connected populations. Taking the Prisoner's Dilemma as a typical example, we consider this problem with rescaled payoff matrix

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & -r \\ 1+r & 0 \end{pmatrix}, \end{array}$$

where r denotes the ratio of the cost over the net benefit of cooperation [56].

Figure 3 shows that interactive diversity maintains cooperation in the highly connected populations irrespective of population structures. Traditionally, the evolution of cooperation is extremely susceptible to the average degree. Increasing the number of social ties makes cooperators possibly be exploited by more defectors, which weakens the resistance of cooperator clusters to defectors' invasion. In particular, the approximation to the well-mixed limit consistently causes the sharp decline of cooperation [57]. However, in our model, increasing average degrees towards the fully connected networks does not offer defectors such an advantage to exploit more cooperators. As we have mentioned, individuals inside the clusters reduce the exploitation by defectors outside, which actually consolidates the clusters [9]. Therefore, considerable cooperative acts remain in the population.

The results above are obtained in the setting that individuals randomly choose strategies against different neighbors initially (with random initial strategies) and everyone updates all its strategies synchronously. Considering that individuals know little about their neighbors in the beginning, instead of treating neighbors differently, they may adopt an identical strategy against neighbors initially (with same initial strategies). Furthermore, we probe how strategy updating proportions p affect the evolution of cooperation. For $p = 25\%$, each individual is set to update one fourth of its strategies in each generation. The presented results in figure 4 demonstrate that the evolution is robust for strategy updating proportions and individuals' initial states.

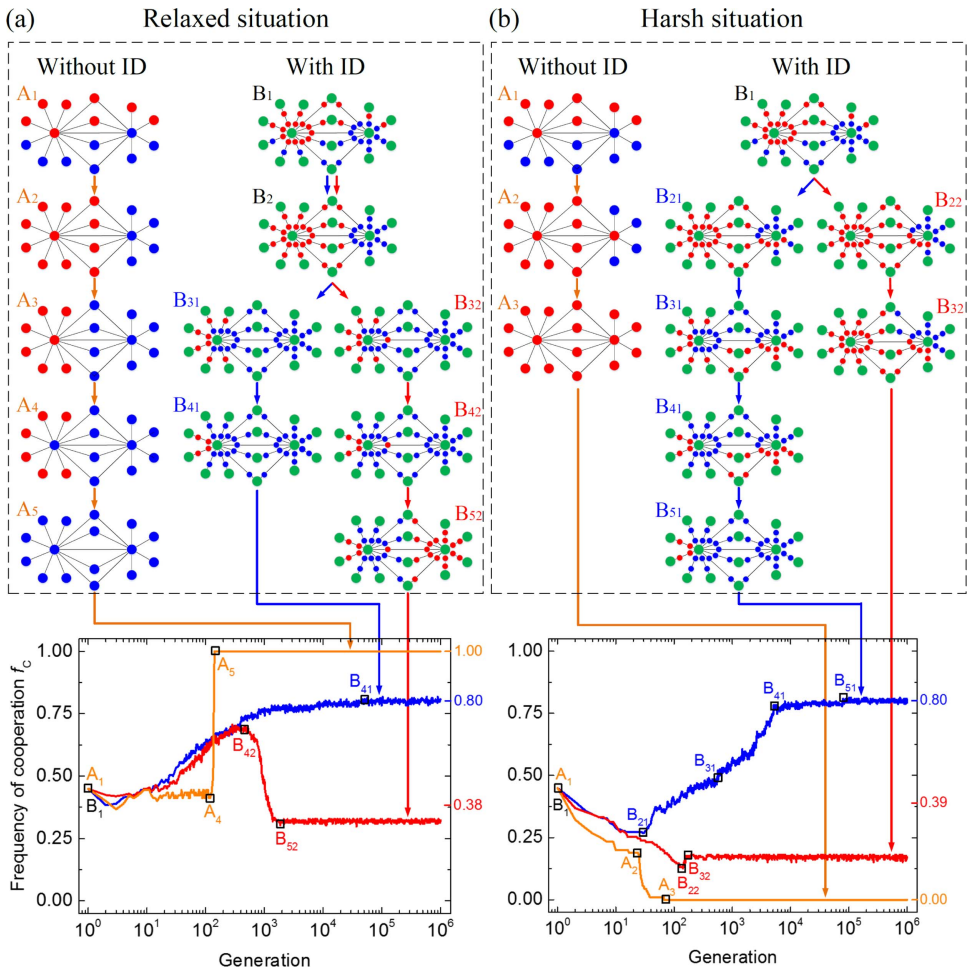

Figure 5. Typical scenarios for the evolution of cooperation on the double-star graph. Two sets of graphs are plotted in two situations, (a) relaxed ($r = 0.19$) and (b) harsh ($r = 0.90$). In each situation, we consider the evolution without and with interactive diversity (ID). Each set of graphs shows the main evolutionary process (dashed box), and analytical stable frequencies of cooperation are shown in the right side of solid boxes. The results are validated by the simulations (lines in the solid box), and the corresponding states are marked (open squares in each line). Initially, on the double-star graph, the left center is occupied by a defector and the right center by a cooperator, separately. Over leaves, defectors and cooperators are distributed equally. The population starts to evolve and finally reaches the equilibrium states. The graph size is $N = 60$, $M = 40$, $L = 10$.

3.2. Interactive diversity on the double-star graph

By scrutinizing the detailed evolutionary process on the double-star graph, we make two questions understood: how cooperative clusters form and expand; why cooperation shrinks in the relaxed situations when interactive diversity and topological heterogeneity are combined together. The answer for the first question well explains the emergence of cooperation in the harsh situations. Here the payoff structure is the same as that in figure 3. Two situations, the relaxed one for a small r and the harsh one for a large r, are quantitatively analyzed respectively in the traditional setting (without interactive diversity, see appendix A) and in our model (with interactive diversity, see appendix B). In figure 5, in each situation, three most common evolutionary processes, including one in the traditional setting and two in our model, are elaborated. The theoretical cooperation level corresponding to each process is shown in table 1.

We start with the evolution in the relaxed situation. In the traditional setting, defectors on hubs easily become the victims of their own success due to the dissemination of defection in their neighborhoods, and then lose the occupation of hubs [35, 58, 59]. Here on the double-star graph, we get a critical cost-to-benefit ratio

$$r^* = \frac{L}{N - L} \tag{1}$$

below which cooperators easily wipe out all defectors and dominate in the population ($A_1 \rightarrow A_5$ in figure 5(a), see appendix A).

In the presence of interactive diversity, each individual is allowed to adopt cooperation and defection simultaneously. Especially in the games played by individuals sitting on hubs, cooperation and defection are observed simultaneously. Indeed, the dissemination of defection causes inevitably losses to them. Nevertheless,

Table 1. Theoretical frequencies of cooperation corresponding to evolutionary processes in figure 5. f_C^R (f_C^H) is the cooperation level in the traditional setting (without ID) in the relaxed (harsh) situation. Similarly, f_{C1}^R and f_{C2}^R (f_{C1}^H and f_{C2}^H) are the cooperation levels in our model (with ID) in the relaxed (harsh) situation.

Situations	f_C (Without ID)	f_C (With ID)	
Relaxed	$f_C^R = 1$	$f_{C1}^R = \frac{2M-1}{M+N-1}$	$f_{C2}^R = \frac{M-2}{M+N-1}$
Harsh	$f_C^H = 0$	$f_{C1}^H = \frac{2M-1}{M+N-1}$	$f_{C2}^H = \frac{M-1}{M+N-1}$

by virtue of the natural advantage to participate in more games and the mutual cooperation with some neighboring cooperators, individuals on hubs still keep advantages over those on nearby sites. In other words, defection survives in the population under the shelter of individuals on hubs ($B_{31} \rightarrow B_{41}$ in figure 5(a)). $B_{32} \rightarrow B_{52}$ in figure 5(a) illustrates how an individual on the hub leads to dissemination of defection over its neighbors. Both two equilibrium values f_{C1}^R and f_{C2}^R in the table 1 are lower than f_C^R, identifying the inhibitive effects caused by the combination of interactive diversity and large topological heterogeneity.

In the harsh situation, the sizeable temptation to defect and the great loss from exploitation weaken the viability of cooperators. In the traditional setting, defectors on hubs directly invade their nearby sites, resulting in the extinction of cooperators ($A_1 \rightarrow A_3$ in figure 5(b)).

With interactive diversity, cooperation however emerges. The formation and expand of cooperative clusters are responsible for this. The former one can be explained through the process $B_{22} \rightarrow B_{32}$ in figure 5(b). The latter one is shown in $B_{21} \rightarrow B_{51}$. On the one hand, relying on the mutual cooperation inside the clusters and the resistance to exploitation by defectors outside, individuals in the boundary of the clusters hold their advantages over individuals outside. These advantages make the dissemination of cooperation feasible. On the other hand, for individuals outside the clusters, in the traditional setting, imitating cooperative behaviors means to be exploited by all neighboring defectors. However, interactive diversity enables them to imitate cooperative acts just in some interactions, which reduces the risk to adopt cooperation and facilitates the further spread of cooperation.

Moreover, the analytical dynamics above (the graph sets in the dashed box and the stable values in the right side of solid box) is validated by our simulation results (the lines in the solid box of figure 5) in both relaxed and harsh situations. Here we have to note that the steady trapped states shown in figure 5 actually result from the update rule by which individuals only imitate those performing better. When an irrational choice for one to learn the less successful individuals is allowed, as induced by the application of the Fermi rule, population escapes from such trapped states. Despite the different equilibriums, both the average frequency of cooperation and the microscopic mechanisms responsible for these results still remain consistent.

In figure 6, we show the cooperation level throughout the entire range of r on double-star graphes with two different graph sizes. When individuals interact identically, as r increases, f_C is predicted to decrease sharply near r^* since the cooperator suffers from its occupation to a hub and this suffering inhibits the further spread of cooperation to its neighbors. With interactive diversity, individuals adjust their strategies partly and cooperation spreads easily even for large r, resulting in the smooth curve of f_C. Analytically, r^* well predicts when interactive diversity provides more advantages for cooperation than the traditional dynamics does (see appendix B). As figure 6 shows, no matter who occupies the larger center initially, interactive diversity always benefits cooperation for r above r^*. Moreover, the promotion is enhanced further in both relaxed and harsh situations when a cooperator is located on the larger center in the beginning. Other graph sizes lead to similar results. Although we study this problem on the double-star graph, we have checked that the microscopic mechanisms explained above are also applicable in both well-mixed and general structured populations.

To this end, we have explained the non-monotonic results shown in figure 2. Here we want to stress that the impact of interactive diversity on the evolution of cooperation is less straightforward than that of other diversities like topological diversity [29]. In case of topological diversity, individuals' difference helps to establish a sort of strategy homogeneity around hubs, where successful hubs spread strategies to their neighborhoods. This strategy homogeneity is harmful to defector hubs while it is beneficial to cooperator hubs, leading to a higher cooperation level around the highly-connected nodes and then the boom of cooperation in the whole population [35, 60]. However, interactive diversity enables successful individuals to enforce cooperation and defection simultaneously to their neighbors, breaking the strategy homogeneity. Hence the largest cooperation level may not accompany individuals with many social ties, and the diverse system behavior is observed.

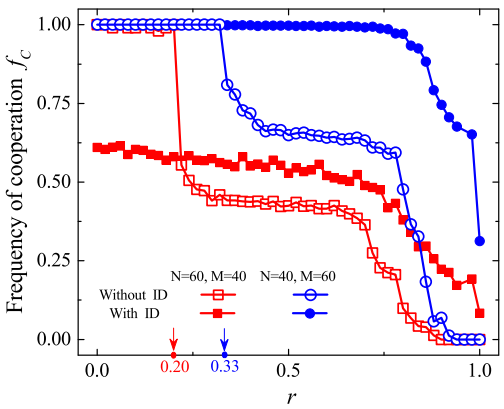

Figure 6. The effects of interactive diversity (ID) on the evolution of cooperation are predicted analytically. We plot the frequency of cooperation f_C as a function of r in the populations with (solid symbols) and without (open symbols) interactive diversity. The red lines are plotted as a defector occupies the larger center initially ($N = 60$, $M = 40$, $L = 10$) with $r^* = 0.20$. The blue lines are plotted as a cooperator occupies the larger center initially ($N = 40$, $M = 60$, $L = 10$) with $r^* = 0.33$. The value of r^* at which f_C decreases sharply could be predicted analytically (arrows) by $r^* = L/(N - L)$. The initial strategy distribution is the same as that in figure 5. Each point is averaged over 500 simulations.

4. Discussion and conclusions

We have investigated interactive diversity in both well-mixed and structured populations represented by various networks, focusing specifically on the effect of individuals' distinct responses against different neighbors on the evolution of cooperation. Without invoking other mechanisms like recognizing each other from past interactions [61] or being informed how neighbors interact with the third party [55], individuals independently update each of their strategies by imitating what successful neighbors apply to them.

We find that, in the well-mixed population, with interactive diversity, cooperation emerges in a wide parameter range under three most adopted game metaphors. In the structured populations, interactive diversity drives the cooperation to a high level in PD and SH. The facilitative effects are also identified on networks associated with high average degrees, which provides an instrumental clue on explaining how cooperation thrives on highly connected social populations [16, 62, 63]. All of these have demonstrated that interactive diversity benefits cooperation. When interactive diversity is introduced in the scale-free network, cooperation is elevated in the harsh situation while reduced in the relaxed case. Note that both interactive diversity and large topological heterogeneity facilitate cooperation solely. The reduction implies that combining many promotive factors together may make the outcome worse than that with each single one, verifying the conclusion in previous studies [25].

We elaborate on the evolutionary process on a double-star graph which serves as the simplest abstraction of heterogeneous networks, to explore the underlying mechanisms leading to outcomes above. On the one hand, interactive diversity helps to form stable cooperative clusters in the boundary of which individuals cooperate with cooperators inside and meanwhile defect to defectors outside. Compared with traditional cooperator clusters [9], here individuals in the boundary can reduce the exploitation and resist the invasion of defectors in harsh social dilemmas. On the other hand, interactive diversity enables individuals to imitate cooperative behaviors in a fraction of interactions, which lowers the loss caused by the strategy turning from defection to cooperation. Therefore, in the harsh social dilemma, dissemination of cooperation is still feasible. These two together well explain the promotive effects of interactive diversity on the emergence of cooperation.

Actually, these mechanisms can be easily understood from the perspective of edge dynamics. Different from the traditional dynamics where edges connecting to a specific node are closely related, here interactive diversity considers the independence of edges linking to a focal node to some extent. In the network, cooperation disseminates from one edge to another rather than from one node to another, which effectively reduces the fluctuation of relevant properties of the associated node. Put differently, interactive diversity facilitates the permeation of edges of mutual cooperation, indicating that how two individuals treat each other may have a more profound impact on the evolution of cooperation than whether they are cooperators or defectors.

As for the combination of topological heterogeneity and interactive diversity, the former one endows individuals on hubs with chance to participate in more games, and the latter one enable them to adopt cooperation and defection simultaneously. Although these individuals behave defectively in some interactions and incur losses, they still keep advantages over their neighbors. Consequently, defection survives under the shelter of individuals on hubs.

A previous work [52] has probed the effects of simultaneous adoption of cooperation and defection in a square lattice and ring lattice in the Prisoner's Dilemma. However, previous conclusions are incomplete due to the lack of further investigations in other structured populations. Here we systematically study these problems in structured populations represented by various complex networks with three typical two-player games. We find the close relation between interactive diversity and edges, such as the connectivity stressing edges' number and heterogeneity stressing degree distribution. And we understand the evolutionary dynamics on networks from the point of edges. We have to point out that, when individuals are modeled to behave differently, they need to remember their strategies to each neighbor, which might be costly. The consideration of a cognition cost assigned to memorize the information may enrich the work and bring some interesting results [64–66]. Moreover, the combination with other factors, like individuals' reputation [67] and the evaluation to the payoffs [68, 69] may provide an efficient response to different opponents' strategies. Especially, with intention recognition, individuals recognize opponents's intentions on the basis of the previous interactions or in some other ways, and then make decisions according to such recognized intentions [66, 70, 71], which well achieves interactive diversity and is worth a further investigation. Interactive diversity, consisting of diverse interaction types [72], constitutes realistic representations of the real world and should receive more attention.

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Appendix A. Evolutionary dynamics on the double-star graph without interactive diversity

In this part, we provide a theoretical analysis about the evolutionary dynamics on the double-star graph in the traditional setting. The double-star graph is depicted in the main text, where the number of common leaves l_c (L) is much less than the connectivity of two centers (N, M). We begin with the assumption of $N > M$. Initially, a defector occupies the left center h_1 and a cooperator occupies the right center h_2. Over leaves l_1, l_c, and l_2, defectors and cooperators are distributed equally. The evolutionary processes in relaxed and harsh situations are studied and the details are provided below. For the convenience of description, we use h_1 (h_2, l_1, l_2, l_c) to represent the individual on h_1 (h_2, l_1, l_2, l_c).

In the beginning (A_1 of figure 5(a)), the payoffs (P) of h_1 and h_2 are given by

$$P_{h_1} = \frac{N+1}{2}r + \frac{N+1}{2}, \tag{A.1}$$

$$P_{h_2} = -\frac{M+1}{2}r + \frac{M-1}{2}. \tag{A.2}$$

The payoff of a cooperator (defector) l_1 is $P_{l_1} = -r$ ($P_{l_1} = 0$). $P_{h_1} > P_{l_1}$ and $P_{h_1} > P_{h_2}$ make it possible for **D**s (defective strategies) of h_1 to spread to both l_1 and h_2. However, considering

$$(P_{h_1} - P_{l_1}) - (P_{h_1} - P_{h_2}) > 0 \Leftrightarrow r < \frac{M-1}{M+1} \equiv r_1 \tag{A.3}$$

and the update rule with which each individual imitates successful neighbors with the probability proportional to their payoff difference, **D**s are more likely to spread to l_1 than to h_2. A similar case happens to l_c. This means that h_1's neighbors on leaves will turn into defectors before h_2, leading to the deterioration of h_1's neighborhood. Meanwhile, h_2 spreads **C**s to its neighbors.

At this stage (A_2 of figure 5(a)), both l_c and h_1 have payoffs $1 + r$. By

$$P_{h_2} - P_{l_c} = P_{h_2} - P_{h_1} = -(L+2)r + (M-L-2) > 0 \tag{A.4}$$

$$\Leftrightarrow r < \frac{M-L-2}{L+2} \equiv r_2, \tag{A.5}$$

both h_1 and l_c reversely imitate h_2. Compared with h_1, l_c has less neighbors to imitate. In other words, l_c will imitate h_2 more frequently and turn into a cooperator before h_1 (A_3 of figure 5(a)). After that, h_1 is also inevitably invaded. Once it happens, the invader on h_1 obtains $P_{h_1} = L + 1 - r(N - L - 1)$. Given that defectors l_1 get $P_{l_1} = 1 + r$, the invader can stabilize on h_1 (A_4 of figure 5(a)) whenever

$$P_{h_1} - P_{l_1} = -(N - L)r + L \geqslant 0 \Leftrightarrow r \leqslant \frac{L}{N - L} \equiv r^*, \tag{A.6}$$

where r^* is a critical cost-to-benefit ratio. Since both N and M are much larger than L, we get $r_1 > r^*$ and $r_2 > r^*$, which demonstrates the existence of r^*. For $r < r^*$, $P_{h_1} > P_{l_1}$ whereby **C**s of h_1 spread to l_1 further (A_5 of figure 5(a)). If r is slightly above r^*, due to $P_{h_1} < P_{l_1}$, **C**s can never spread to l_1, resulting in $f_C = \frac{2M+L-1}{2(M+N-L)}$ (A_3 of figure 5(a)). Here we stress that the common leaves l_c play an important role in the evolutionary dynamics. The traditional double-star graph exhibiting no loops makes the simple analytic treatments possible [35]. However, when it comes to PD rather than PGGs (Public Goods Games), without the mutual cooperation with common neighboring cooperators, the cooperator can never stabilize on the left center, which actually can not explain the boost of cooperation in heterogeneous networks [28, 29].

As r increases, the initial difference between h_1's and h_2's payoff

$$P_{h_1} - P_{h_2} = \frac{N + M + 2}{2}r + \frac{N - M + 2}{2} \tag{A.7}$$

rises. Meanwhile, the value

$$(P_{h_1} - P_{l_1}) - (P_{h_1} - P_{h_2}) \leqslant -\frac{M - 1}{2}r + \frac{M - 1}{2} \tag{A.8}$$

decreases, which indicates that **D**s of h_1 may not spread to l_1 before to h_2. In particular, in the harsh situation, h_2 is invaded initially and defectors dominate in the population ($A_1 \rightarrow A_3$ of figure 5(b)).

Appendix B. Evolutionary dynamics on the double-star graph with interactive diversity

All settings are similar to those in appendix A except that individuals are allowed to behave differently against different neighbors. Because various types of individuals is involved in the graph and their states are diverse, the evolutionary dynamics is quite intricate. Here we simplify the evolution scenarios and give two most typical processes separately in both relaxed and harsh situations. In each situation, the average cooperation level lies between the theoretical values of these two processes.

Before our analysis, we give an intuitive understanding about how interactive diversity affects individuals' evolution. p ($0 < p < 1$) denotes the probability that a cooperator h_2 with M neighbors imitates a defector h_1. h_2 is assumed to have a larger payoff than all its neighbors except h_1, suggesting that h_2 can only change its strategy by imitateing h_1. In the traditional setting, h_2 turns to a defector with probability p and remains unchanged with probability $1 - p$. With interactive diversity, in one generation, the probability for h_2 turning into a defector is p^M (updating M strategies successfully) and the probability for h_2 to remain unchanged is $(1 - p)^M$ (failing to update any strategy). The relations $p > p^M$ and $(1 - p) > (1 - p)^M$ indicate that such cases are less likely to happen. Instead, h_2 is likely to update its strategies in Mp interactions in average.

Let us first discuss the evolution in the relaxed situation. The initial stages are similar to those in appendix A: **D**s of h_1 spread to l_1, l_c, and **C**s of h_2 spread to l_2 (B_2 in figure 5(a)). Next, both l_c and h_1 imitate h_2. Because of the connectivity of l_c, l_c becomes a cooperator before h_1.

Here we consider the first process: h_1 imitates h_2 successfully and adopts **C** against h_2 (B_{31} in figure 5(a)). Besides, h_1 is expected to adopt **C**s to additional L neighbors l_c (the exact number may not be L while it does not affect our analysis) and n_1 neighbors l_1. The expected value of n_1 is

$$\overline{n_1} = (N - L - 1)\frac{P_{h_2} - P_{h_1}}{N^2(1 + 2r)} < 1, \tag{B.1}$$

where $P_{h_2} = M$ and $P_{h_1} = L + 1 - rn_1$. The value of $\overline{n_1}$ suggests that n_1 is small. The payoff of l_1 associated with those n_1 interactions is $P_{l_1} = 1 + r$. We easily get the payoff difference

$$P_{h_1} - P_{l_1} \geqslant L - r(1 + n_1) > 0. \tag{B.2}$$

Sooner, h_1 places **C**s on n_1 corresponding leaves, which increases h_1's payoff in return and meanwhile enhances h_1's advantage over leaves l_1. Since the payoff of h_1 is still lower than that of h_2, h_1 continues imitating h_2 and adopting **C**s in some interactions. The cycle of imitation and spread of **C**s gradually increases h_1's payoff until it rises to or slightly exceeds h_2's payoff (B_{41} in figure 5(a)). At this stage, the payoffs of h_1, h_2 can be approximated as $P_{h_1} = P_{h_2} = M$. That is, h_1 adopts **C**s to L leaves l_c, $M - L - 1$ leaves l_1 and h_1. The stable frequency of cooperation is

$$f_{C1}^R = \frac{2M - 1}{M + N - 1}. \tag{B.3}$$

The second process is similar except that h_1 insists on defecting to h_2 (B_{32} in figure 5(a)). Once h_1 gets the highest payoff in its neighborhood (B_{42} in figure 5(a)), it reversely turns h_2 into a defector (B_{52} in figure 5(a)).

Actually, P_{h_1} may exceed $M - 1 - r$ (the maximum payoff of a cooperator h_2) slightly. That is, h_1 cooperates in $M - 2$ interactions and defects to h_2. The stable frequency of cooperation is

$$f_{C2}^{R} = \frac{M-2}{M+N-1}. \tag{B.4}$$

When r is a little above r^*, in the traditional setting, $f_C = \frac{2M+L-1}{2(M+N-L)}$ is near f_{C2}^{R} (both N and M are much larger than L). However, in our model, the average f_C lies between f_{C1}^{R} and f_{C2}^{R}. Equivalently, the average f_C is larger than f_C in the traditional setting. Therefore, r^* is also a critical value above which interactive diversity provides more advantages for cooperation.

In the harsh situation, although the cooperator h_2 is more vulnerable to nearby defector h_1, h_2 just adopts **D**s in a few interactions. Assume that h_2 adopts **D**s in n_2 interactions. Due to the rapid deterioration of h_1's neighborhood, h_1 will not be influential in h_2. Two typical processes are considered. In the first one, h_2 insists on cooperating with h_1 (B_{21} in figure 5(b)). At this stage, $P_{h_1} = P_{l_c} = 1 + r$ and h_2's payoff satisfies

$$P_{h_2} - P_{h_1} \geqslant M - (L+2)(1+r) - n_2 > 0 \Leftrightarrow r < \frac{M-L-2-n_2}{L+2}. \tag{B.5}$$

In such a case, **C**s spread to h_1 from h_2 and then to l_c through h_1. The subsequent process ($B_{31} \rightarrow B_{41}$ in figure 5(b)) is similar to the first process in the relaxed situation ($B_{31} \rightarrow B_{41}$ in figure 5(a)). The stable frequency of cooperation is

$$f_{C1}^{H} = \frac{2M-1}{M+N-1}. \tag{B.6}$$

The second process considers that h_2 adopts **D** to h_1 (B_{22} in figure 5(b)). In such a case, h_2 and its $M - n_2$ neighbors form a stable cooperative cluster while **C**s can never spread to h_1. Since h_2 adopts **D**s against h_1, n_2 is 1 at least. Taking $n_2 = 1$, the stable frequency of cooperation is

$$f_{C2}^{H} = \frac{M-1}{M+N-1}. \tag{B.7}$$

Actually, n_2 relies on the cost-to-benefit ratio and the time length of the evolution. Both these values increase n_2 and the actual cooperation level deviates much from the theoretical value as r increases.

Furthermore, if a cooperator occupies the larger center initially ($M > N$), $r^* = \frac{L}{N-L}$ still exists and the evolutionary dynamics remains nearly unchanged in the traditional case. With interactive diversity, in the relaxed situation, since h_1's payoff never exceeds h_2 ($M > N$), h_1 imitates h_2 until it precludes all **D**s. Finally, defection becomes extinct. In the harsh situation, cooperation is still more favorable compared with the case without interactive diversity. Taken together, interactive diversity always supports the evolution of cooperation.

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