Workers' Heterogeneity and Job Search in the Flow Approach to Labor Markets: a Theoretical Analysis

by

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Abstract

This dissertation investigates the theoretical implications of heterogeneity of workers’ productivities for labor markets equilibrium, in the presence of allocative frictions and wage bargaining. The emphasis on both ex ante and ex post sources of individual heterogeneity, introduced simultaneously, allows to exploit many findings of the extensive microeconometric literature and to provide new explanations for several macroeconomic phenomena.

Chapter 1 considers a one-sector economy with differently skilled workers and matching heterogeneity. Good workers can create extra rents for firms, in excess of those offered by less skilled workers, in order to remain employed in bad times. The adjustment of job search and tenure policies partially insulates relative wages from aggregate shocks and amplifies the volatility of the employment of unskilled workers (“Fattening” in booms).

Chapter 2 considers a two-sector economy, where workers have heterogeneous sector-specific skills. Relatively specialized workers search selectively in one sector, while less specialized individuals search randomly to maximize the chance of finding a job. The proportion of random searchers is countercyclical, so that gross inter-industry mobility is less procyclical than net mobility. An increasing specialization of the labor force reduces inter-industry mobility, as recently observed in the US economy.

Chapter 3 combines the analysis of sector-specific skills and matching heterogeneity. High-wage, specialized workers contact fewer selected vacancies, but are accepted more often by firms; therefore, they can exit faster than industry movers from unemployment, and yet they may be trading off good jobs for longer unemployment spells.

Thesis Supervisor: Ricardo J. Caballero
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Introduction

The flow approach to labor markets recognizes explicitly the coexistence at any point in time of large flows of workers and jobs among different “states”, or stocks. Workers move in and out of employment, unemployment, and the labor force, as well as across industries. Jobs are created and destroyed at the same time, even within narrowly defined industries.¹

This approach relies upon a few key concepts. Allocative frictions and some source of real rigidity are commonly introduced, though they are not necessary, in order to explain the observed flows and stocks. Another typical notion is that of a “job”, based on a fixed-proportion technology. But a logical premise of this approach is the existence of firm and worker heterogeneity, at different levels. Indeed, in a representative agent world, we could not observe contemporaneous flows of jobs and workers in all directions.

In general, heterogeneity poses a formidable challenge to theoretical economists, due to its limited analytical tractability and to its logical complexity. At the same time, the problem of heterogeneity and aggregation should be the bone and flesh of Macroeconomics. The applied research has moved much faster on these tracks. The diffusion of panel data, as well as of the statistical methods and the computing facilities necessary to analyze them, has triggered an extraordinarily vast production of applied work. The possibility to follow over time single individuals or entire cohorts has reversed the growing mistrust in (macro)econometrics. However, most of these micro studies simply estimate reduced form; theoretical analysis is lagging behind, and the evidence appears “in search of a theory”.

Nowhere this is more true than in the analysis of labor markets. The labor microeconometric literature is already extensive and detailed, and has relied on many concepts

¹For a comprehensive empirical analysis of U.S. job flows, see Davis and Haltiwanger (1992); of U.S. worker flows, Blanchard and Diamond (1990); of job and worker flows in Europe, Burda and Wyplosz (1994).
typical of the flow approach, such as entry rates into and exit rates out of unemployment, matching functions, job creation and destruction. A complete theoretical treatment of heterogeneity within the flow approach is slowly emerging, but most of the valid attempts in this direction have been made on the side of firms' heterogeneity only.2

This dissertation embraces the flow approach to labor markets and develops a theoretical analysis that takes explicitly into account the major recognized dimensions of workers' heterogeneity. The goal of this work is two-fold. First, it is a fact-motivated exercise, an attempt to construct consistent mathematical models that account for the received micro empirical evidence in terms of rational behavior and expectations. Second, it is a macroeconomic analysis, in that it tries to explain aggregate phenomena, such as the cyclical pattern of productivity and the trends in labor mobility and unemployment, by means of micro heterogeneity, primarily through composition effects.

This dissertation contains no normative analyses nor policy recommendations. This might appear surprising in a moment when unemployment, especially among unskilled workers, is plaguing many developed and underdeveloped countries. But the theory of heterogeneity in labor markets is still in its infancy, and still inadequate even on the positive side.

The economic literature has isolated two main types of heterogeneity in workers' productivities. *Ex ante* heterogeneity refers to differences in workers' transferable abilities, skills that are not specific to the particular jobs currently taken, though they may be productive only in some type of sector or occupation. *Ex post* heterogeneity instead refers to the degree of specificity that develops in any employer-worker match, and that is lost completely outside of the job.

Each of these two classes of heterogeneity can be divided in two sub-classes. *Ex ante* heterogeneity can be meant either in terms of general human capital (ability independent of the type of job) or in terms of sector- or industry-specific skill, that can be transferred only to similar jobs. In turn, *ex post* heterogeneity can be either in Becker's (1962) form of firm-specific human capital, possibly accumulated over time through learning-

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by-doing; or in Jovanovic (1979) form of matching heterogeneity, the component of the success of the employer-worker match that cannot be foreseen ex ante and that must be "experienced" to be revealed to the two parties. Matching heterogeneity is literally a form of unobservable ex ante heterogeneity.

Both classes of heterogeneity have attracted considerable attention among economists for many years now, but rather independently: indeed, no one has ever explored their interaction. The first chapter of this dissertation tackles directly this task in the context of a one-sector economy, i.e. with one-dimensional ex ante and ex post differential productivities. I present a model of a production economy with search frictions and wage bargaining, and characterize its general equilibrium under the most plausible assumption on preferences for leisure. The main theoretical result is that workers who are more productive in an ex ante sense (more "skilled") accept matches that are worse on average in an ex post sense. The flip side of the coin is that acceptance rates are higher for more skilled job applicants.

The formal analysis provides a structural explanation for many empirical regularities observed in developed countries' labor markets. In particular, it is worth mentioning the rationale for the very different levels and volatilities of unemployment rates for different groups of workers. Skilled workers can exploit their productivity advantage to create extra rents for the firms, in excess of those produced by unskilled workers, in order to be accepted more often and to remain employed longer. In this context, a neutral aggregate labor demand shock exerts non neutral effects. After a negative shock, more productive workers inflate the extra rents given up to firms, in order to remain employed. Conversely, after a positive shock, they appropriate the reduction in the cost of unemployment and thus become relatively less attractive to firms. The latter then hire more often less skilled workers, determining the "Fattening" of the employed labor force that we observe systematically in market economies over aggregate expansions.

The one-dimensional heterogeneity of the first chapter, produced both by ex ante and ex post elements, orders workers by absolute advantages. The second chapter of the dissertation emphasizes ex ante heterogeneity only, but recognizes its multi-dimensionality,
giving rise to *comparative* advantages of workers in different sectors, industries or occupations. In particular, Chapter 2 explores the interaction of comparative advantages and allocative frictions in determining job search behavior, and thus inter-industry mobility.

In a two-sector economy, workers are endowed with a pair of sector-specific skills, each equally productive in any job, but only within the same sector. Due to search frictions, which make mobility across jobs and sectors time-consuming and costly, workers with relatively weak comparative advantages optimally pursue random search strategies, because they value more the need to find a job quickly. Specialized workers, instead, adopt more selective search strategies, in order to exploit their strong comparative advantage in one sector.

A large body of evidence on workers’ flows and relative wages is very naturally rationalized within this framework. For example, the model predicts that, during an expansion, a larger fraction of the labor force chooses to search selectively, because search frictions bite less. Accordingly, the proportion of random searchers, who are not really seeking comparative advantages and thus produce “spurious” inter-industry movements, falls. This might explain why net inter-industry labor mobility, measured by the changes in industries’ employment shares, is observed to be more procyclical than the gross one, which includes also “crossing” movements of workers that do not alter those shares. The model delivers also a natural prediction on the decline in inter-industry mobility, experienced by the US economy from the late 1960s to the mid-1980s; one major source may have been the increasing specialization of the new entrants into the labor force, who search more for a particular job and less for a job “whatever it is”.

The **third chapter** exploits and combines the results obtained in the first two chapters to investigate the inequality of exit rates from unemployment and unemployment durations. The issue is key to understand the origin of the secular increase in unemployment. The combination of ex ante comparative advantages and ex post (matching) heterogeneity provides a very rich and flexible framework to address the inequality of unemployment spells.

The results of Chapter 3 originate from a simple but illuminating decomposition of
the exit rate from unemployment. Given the two-step nature of the matching process, the exit rate is the product of the rate of contact of open vacancies, and of the conditional (on finding a vacancy) chance of being accepted by the firm at an acceptable wage offer for the worker. Specialized workers - industry stayers - apply to fewer open vacancies, as they search more selectively pursuing their strong comparative advantages. At the same time, they are accepted more often by the firms they contact, because they are quite productive in their sector and have a relatively weak outside option in other sectors. The net effect of these two forces is ambiguous, and may explain why high-wage workers are observed to be less mobile across industries and to exit faster from unemployment. In addition, some industry stayers can be trading off good jobs for longer unemployment spells, spells that they could reduce by searching randomly.

This flexibility has implications also for cyclical responses. In a recession, the perspective long-term unemployed - non specialized workers - absorb part of the decline in exit rates by choosing more random search strategies, while the short-term unemployed - specialized industry stayers - do not. This differential response moderates the increased inequality of exit rates that is produced by acceptance rates between the two groups. This phenomenon, combined with the lower level of all exit rates, may cause the spurious duration dependence to fall for the first months of durations, as documented on US and French data.

The same argument can help to explain why, at any point in time, exit rates from unemployment are much less unequal than entry rates across workers' groups. Following this logic, part of this "limited" inequality is the result, on the one hand, of higher choosiness of skilled workers among potential employers; and, on the other hand, of lower choosiness of firms, in terms of success of the match, when hiring a more skilled or specialized worker. This conclusion suggests that the observed limited inequality of exit rates from unemployment is the result of two forces that concur to widen the inequality of welfare among workers.
Chapter 1

Fattening Economies

1.1 Introduction

Labor is not a homogeneous factor of production. A host of reasons differentiate productivities across workers, independently of the jobs they are assigned to. At the same time, the substitutability between different types of workers appears surprisingly high (e.g. see Hamermesh 1993, Chapt. 3), though certainly not perfect (Murphy and Welch 1992). Heterogeneity and substitutability together raise a major issue: what determines the employment of different types of workers in similar jobs, and thus more importantly their unemployment?

Traditionally, researchers have taken up this issue in terms of prices rather than quantities, asking what determines the relative wages of workers, given their different endowments of general and firm-specific human capital. But, on this matter, the extensive body of empirical literature on relative wages is partially inconclusive, especially at business cycle frequencies. On the contrary, a robust picture emerges from the available evidence on non-wage measures of labor market performances. Virtually in any country and in any time period for which dependable data exist, more productive (skilled, educated, experienced) individuals experience lower and less cyclical unemployment rates. Also, their participation rates and weekly hours worked are both higher and less volatile. In terms of flows, exit rates from unemployment are higher, and entry rates into unem-
ployment are remarkably lower, for more skilled workers; over business cycles, both rates are much more volatile for less productive individuals. This empirical regularity is found both in aggregate figures and in panel data studies. Quantities appear more informative than prices.¹

If we believe in involuntary unemployment, we learn an important lesson: employers must consider the more productive workers also more profitable, because they employ them more. In terms of prices, some source of wage compression across types must be at work, in the following sense: in equilibrium, more productive workers give up to a larger part of their productivity in wages.

Understanding the source and the nature of this pattern is important on many dimensions, such as for the design of educational programs and micro policies aimed to labor markets, or for anticipating the consequences of cyclical conditions on the distribution of wages. Here, I am primarily concerned with the implications of wage compression for aggregate productivity over business cycles. The higher cyclical volatility of participation rates, unemployment rates, and weekly hours of less productive workers gives rise to a countercyclical composition effect in the quality of employed labor. Thus, the forces making average labor productivity (as it is well known) procyclical are even underestimated. To what extent, it obviously depends on individual productivity differentials, given the changes in composition.²

¹The orders of magnitudes for workers of different ages and education levels are similar in all industrialized countries (cf. OECD 1995, Johnson and Layard 1986, Nickel and Bell 1995). At any point in time, “primary” workers show from 5% to 45% higher labor participation rates, enjoy from 3% to 20% lower unemployment rates and from two to ten times lower entry rates into unemployment, as well as from 1.2 to two times higher exit rates from unemployment and a higher number of weekly hours worked. The absolute changes in the unemployment rates are much higher (mainly from 2% to 15% higher, more so in Europe) for low skill workers. In the U.S. the elasticity of labor participation to total unemployment (the cycle) for 18-24 men is about twice that of the 25-44 class, and the elasticity of hours worked is 1.6 times larger (Pencavel (1986)). For a similar picture using U.S. panel data (relying on the C.P.S.), see Blanchard and Diamond (1990). The available evidence within sector or industry is extremely scarce, yet seemingly consistent with this pattern.

²To have a feeling of the size of such composition effects, I considered as an example, in the U.S. manufacturing sector, the ratio of the total hours worked by workers aged 18 to 24 to the total hours worked by 25-44 years old individuals. Compounding participation rates, unemployment rates and weekly hours worked for each group, over post-war expansions the average through-to-peak change in this ratio has been in the order of 30%. For other groups of “peripheral” workers the numbers are even larger.
In this chapter I capture these "stylized" cross-sectional and dynamic patterns of employment by combining, in a theoretical framework, workers' ex ante heterogeneity with a mechanism for wage compression across types. I introduce a source of real rigidity in the form of decentralized wage bargaining, which takes place because of allocative (search) frictions in the labor market. I consider a very general technology specification, where the productivity of any worker depends both on general skills and on a match-idiosyncratic outcome, such as firm-specific human capital. Traditionally, theories of labor turnover and unemployment have considered heterogeneity either in an ex ante sense, as in the selection bias literature, or in an ex post sense, as in the matching hypothesis of Jovanovic (1979). This work is aimed to show that the interaction of this two types of heterogeneity produces a number of results that are both intuitive and empirically appealing.

In fact, in this context, a simple assumption on preferences suffices to generate all of the desired implications, cross-section and over time, and to provide for them an intuitive explanation: I assume that the alternative value of working time rises across workers at a lower rate than productivity (Fig. 1.1).

The first implication of this assumption is a very realistic picture of the search equilibrium allocation at any point in time. High types of workers accept a larger compression in wages, because they have more to lose from rejecting a wage offer and staying unemployed. Thus, they search more, have higher exit rates from unemployment and lower unemployment rates. Intuitively, general human capital and matching productivity are substitute inputs; since higher types produce higher rents, a firm is willing to employ them at a lower minimum acceptable match outcome, and thus the probability of im-

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3Let η(·) denote the marginal productivity and a(·) the alternative value of working time net of possible search costs, for each type-productivity of worker x. The assumption is: η(x) − a(x) increasing in types (Fig. 1.1, at the end of the chapter). A number of considerations make this assumption a compelling one; I will mention just a few. First, in most industrialized countries unemployment benefits are paid as a fraction of past wages. Second, middle age workers, normally included among the most productive, have more stringent consumption commitments, mainly due to the presence of young children in their families. Third, higher types are more career concerned, in the sense of Holmström (1982), because they have formed a higher market reputation, so that they have more to lose in absolute terms from being often unemployed.
plementing and keeping alive their matches is larger (Fig. 2.1). With free entry in the market, labor demand (vacancy openings) is set by firms according to the size and the quality of the labor supply, as summarized by the endogenous cross-section distribution of unemployed workers (jobless and searching; Fig. 3.1).

The second implication relates to the cross-sectional dynamics in the labor market. I consider a permanent and positive multiplicative shock to the profitability of all jobs, which mimics a neutral (across skills) increase in labor demand - a "boom". I show that the adjustment of job-tenure and hiring margins and wages is not neutral across types. In particular, market perspectives improve for all workers' types, reducing proportionally the loss from rejecting wage offers; but higher types have larger potential losses to begin with, hence their outside option in the bargaining process improves by more in absolute terms, and wages get "decompressed" across types. Along the whole transition, wage compression moves inversely to the inequality of unemployment rates and to the level of labor demand.

The effect of the boom on the ex ante component of aggregate productivity is straightforward: as wages rise and their structure is decompressed across types, less productive workers search more and are hired relatively more often, so that the composition of the employed labor force worsens along both margins. In a Walrasian economy, we observe a similar composition effect only on the participation margin: low types enter the labor force and get hired in an expansion. Allocative frictions and bargaining, in addition, tilt the pattern of relative unemployment rates, and re-distribute employment among the types already in the market (Fig. 4.1). This additional composition effect, that I call "Fattening", suggests a mean-reverting property of output paths in market economies.4

The effect of the boom on the total (general and firm-specific) average labor produc-

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4There is a natural connection with the Cleansing literature; Caballero and Hammour (1994) consider a related phenomenon in a model with heterogeneous (vintage) capital. Another important reference is to the literature on the duration dependence of business cycles (cf. Diebold and Rudebusch 1994), though my analysis suggests a dependence of the size of a recession on the length of the previous expansion. Not all of the real rigidity theories do well on the dimension of cross-sectional dynamics in labor markets. For example, the hypothesis of Implicit Contracts predicts increasing absolute risk aversion, usually considered uncommon: higher types, who are richer on average, experience less volatile employment, hence they should be asking for more insurance from the risk-neutral firms.
tivity is more ambiguous, but still appealing on empirical grounds. While the average quality of employed labor worsens, at the same time only better and better matches are implemented and kept alive, so that an expansion produces also some endogenous job destruction, interpreted as quits by workers who search for better matches. We know that quits are procyclical. The model suggests that part of the large contemporaneous job creation and destruction documented by Davis and Haltiwanger (1992) for the U.S. manufacturing sector, usually accounted for in terms of reallocative shocks, can be caused by aggregate shocks.

In what follows I formalize and establish rigorously the intuition presented in this Introduction, referring each theoretical result to the overall empirical picture. I introduce and analyze a dynamic model of a production economy, where ex ante heterogeneous workers and homogeneous capital are combined in fixed proportions to produce a single good. In Section 1.2 I describe the economy and characterize its unique equilibrium with a centralized Walrasian labor market. I also specify the details of a frictional decentralized trade mechanism. In Section 1.3 I derive the optimal search and job-tenure individual policies and the pattern of wages in any equilibrium of the frictional economy. In Section 1.4 I present a full characterization of stationary frictional equilibria. Finally, in Section 1.5 I perform the crucial comparative statics exercise on both the Walrasian and the frictional economies, and I study the dynamic response of the composition of the employed labor force to a neutral positive shock to labor demand. Section 1.6 concludes, presenting two additional implications of the model and discussing the robustness of the results to changes in the (few) simplifying assumptions.

I defer to the Appendix most of the proofs and some parts of the formal analysis that are not necessary to follow the line of the argument. I thereby present a partial

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5Relevant to this work are the results in Moscarini (1995). I prove that, in this class of models, for an open and dense set of parameters there exist multiple stationary frictional equilibria without any increasing returns to searching, what generalizes Diamond's (1982) intuition on coordination failures. I also prove that, if a Pareto dominated stationary equilibrium exists, it converges to a sequential equilibrium of the frictionless economy with zero output, whilst only the Pareto superior equilibrium converges to the Walrasian equilibrium, as expected. This shows that Gale's (1987) results on convergence to Walrasian equilibria in exchange economies do not necessarily extend to production economies, due to the coordination failure.
characterization of cross-sectional dynamics in the non stationary frictional equilibrium, necessary to describe the transition between steady states in Section 1.5.

1.2 A Model of a Production Economy with Ex Ante Heterogeneous Workers and Ex Post Heterogeneous Jobs

1.2.1 Technology, Factor Endowments, and Preferences

Time is continuous and runs from zero to infinity.

There are two goods, a non reproducible and non depreciating capital good and a consumption-numéraire good.

Labor is ex ante heterogeneous. A unit measure of workers is distributed across productivity (skill, general human capital) types \( x \in [0, 1] \) according to a given cross-section measure \( \Psi (\cdot) \), which has a smooth density \( \psi (\cdot) \) with respect to Lebesgue measure.

Capital is ex ante homogeneous. There exists a continuum of firms, of mass as large needed to always ensure entry in the market for the consumption good. Each firm owns one piece of capital.

The consumption good is produced with a Leontief technology, that requires a 1:1 match ("job") between a unit of capital and a worker. The flow productivity of any implemented job is assumed to depend both on the type of the worker \( x \), invariant over time and across matches, and on a purely idiosyncratic match-specific outcome \( z \), drawn once-and-for-all when the match occurs. \( z \) is distributed i.i.d. over time and across matches, according to a c.d.f. \( G (\cdot) \) with smooth density \( g (\cdot) \) and support \([-Z, Z]\),
$Z \in (0, \infty)$, $z \perp x$.\footnote{Any correlation between $z$ and $x$ can be embodied in the type $x$ by re-defining it properly. The boundedness of the support of $G(\cdot)$ is necessary to define an Arrow-Debreu economy.} A simple linear form is chosen:\footnote{All of the qualitative results of this paper hold with the following, more general production technology: the total flow productivity of a match is a strictly increasing, differentiable, weakly supermodular and bounded function of both $x$ and $z$. Hence any (non zero) substitutability between heterogeneous workers is allowed. The gain from working with a linear specification is two-fold: algebra is easier, and convexities arising in value and wage functions can be directly imputed to equilibrium interactions, rather than to the technology.} \[ \eta \cdot (x + z) \]

$\eta$ is an aggregate profitability parameter. It can be interpreted either as an exogenous process for the price of the produced good (tastes for working time) or as technological progress. All three variables $\eta$, $x$, $z$ are observed both by the firm and the worker upon meeting, whatever the allocation mechanism in the labor market.

The match-specific component $z$ makes the ex ante homogeneous jobs ex post heterogeneous. It is meant to capture any ex ante unobserved characteristics of workers and firms that determine the success of the match, such as firm-specific human capital of the worker.

Filled jobs are assumed to break exogenously at flow (Poisson) rate $\delta > 0$. Out of steady state, they will also break endogenously.

Both workers and firms are infinitely lived and risk-neutral, and choose actions in order to maximize expected returns in output units discounted at rate $r > 0$. When a worker $x$ is not working, he enjoys a flow benefit equal to $b > 0$, independent of $\eta$ and $x$.\footnote{As amply discussed in the Introduction, the results of this paper are robust when considering a type-dependent unemployed flow benefit $b(x)$ and job search costs $c(x)$, provided that the function $a(x) = b(x) - c(x)$ (the alternative value of working time) is less steep than the productivity schedule. I work with the simplest case $b(x) = b$, $c(x) = c$ to simplify the algebra.} While he holds a job $(x, z)$, he receives a flow wage denoted by $w_t(x, z)$.

### 1.2.2 Allocative Mechanism

The model presented so far describes only a labor market. Since only one good is produced, and there are neither savings nor foreign trade to decouple income and expendi-
ture, the same model can be seen as one of general equilibrium of a production economy. Technology and preferences are described above. The distribution of workers across types describes the endowments of labor. The endowments of capital are with the ex ante homogeneous firms. The equilibrium interest rate is always equal to the discount rate \( r \), by the risk-neutrality of all agents. To define an equilibrium, I just need to specify an allocative mechanism.

**The Walrasian Economy and its Equilibrium**

The first mechanism I propose is the Walrasian auctioneer. Trade is centralized: all agents meet in the same place and at the same time, face no transaction cost, and bid under perfect information. The auctioneer allocates workers to firms according to the successfulness of their matches \((z)\), and clears the market. The equilibrium of this Walrasian (Arrow-Debreu\(^9\)) economy is important as a benchmark for the frictional economy.

**Proposition 1 (Walrasian Equilibrium)** There exists a unique intertemporal competitive equilibrium of the Walrasian economy. In this equilibrium:

1. only matches with \( z = Z \) are implemented;

2. the equilibrium wage for worker \( x \) exhausts output: \( w^* (x) = \eta (x + Z) \);

3. the employed labor force is the measure \( \Psi ([x^*, 1]) \), with the searching cutoff type defined by \( x^* := \max (0, b/\eta - Z) \)

The proof in the Appendix gives the simple intuition. Firms are the long side of the market; if any match at \( z < Z \) were implemented, with wage at most \( \eta (x + z) \), an inactive firm could offer a higher wage and have a positive chance of making rents, while attracting a worker. Wages \( \{w^* (x) \mid x \geq x^*\} \) are the only set of prices consistent with price-taking maximization, free entry, and market-clearing.

\(^9\)Strictly speaking, the total measure of firms must be finite to define an Arrow-Debreu economy. This can be made consistent with free entry, requiring always excess supply of capital, by choosing a large enough, though finite, measure of firms. Throughout the paper, Walrasian equilibrium values and functions will be identified with a star.
In what follows, I specify an alternative class of economies by associating a different allocative mechanism, giving rise to decentralized trade.

Search Frictions and Decentralized Trade Economies

Let us suppose that trade is decentralized because of informational deficiencies and search costs, that make the meeting process time- and resource-consuming. Then the economy described above becomes a general equilibrium search model, with transferable utilities and ex ante heterogeneous agents on one side of the market. The details of decentralized trade are as follows.

A firm holding an idle unit of capital may open a vacancy at flow cost \( \kappa > 0 \). An open vacancy is contacted by searching workers, independently of their types, at flow Poisson rate \( q_t \), which is assumed to be a homogeneous of degree zero function of the measures of unemployed-searchers \( u_t \) and of open vacancies \( v_t \) ("linear" matching technology). Thus, let the vacancy rate \( \theta_t := v_t / u_t \) and write \( q_t = q(\theta_t) \), a differentiable and strictly decreasing function, with \( q(\infty) \to 0 \).

Workers can be either working, in which case they receive a wage set by bilateral bargaining over the surplus of the match; or searching for a job, in which case they enjoy unemployment benefits \( b > 0 \) and pay a search cost \( c \geq 0 \); or not searching at all, in which case they just receive \( b \). There is no on-the-job search. Unemployed workers are defined, like in official statistics, as jobless workers who are currently searching for a job. The (endogenous) measure of jobless workers not searching for a job is referred to as "out of the labor force".

Let the rate of meeting open vacancies for an unemployed worker be denoted by \( p(\theta_t) \). By the standard abuse of the LLN the flow of matches is \( v_t q(\theta_t) = p(\theta_t) u_t \), what implies \( p(\theta_t) = \theta_t q(\theta_t) \). Assume \( p'(\cdot) > 0 \) for all \( \theta \geq 0 \), and \( p(\infty) \to \infty \).
1.3 Equilibrium in a Frictional Economy: Optimal Individual Policies

In the next two sections, I study the Rational Expectations Equilibrium (REE) of the frictional economy. In this section, I characterize the optimal individual policy functions for (in order): match acceptance and tenure by a worker-firm pair, job search by workers, and vacancy creation by firms. The description of this equilibrium is completed by cross-sectional dynamics (see the Appendix).

Instead of presenting directly the wage schedule and the optimal policies, I go through the individual optimization problems and the arbitrage (Bellman) equations in some detail. Such equations provide the intuition for the optimal rules and depict a very clear picture of the opportunities offered by the market to any agent. The proofs are in the Appendix.

1.3.1 Arbitrage Equations and Wage Determination

Firms

Let $J_t(x, z)$ denote the present discounted value of profits, net of future search costs, obtained by the firm while holding a job filled by a worker $x$ at outcome $z$. This value satisfies the following arbitrage (accounting) equation:

$$r J_t(x, z) = J_t(x, z) + \eta (x + z) - w_t(x, z) - \delta [J_t(x, z) - V_t]$$

where $V_t$ denotes the shadow value of a vacancy. This relation equates, as usual, the flow value of the “asset” to the capital gain, plus productivity net of wage, minus the change in capital value determined by the exogenous dissolution of the job.

Let $J_t(x, z)$ the Bellman value of the same job, i.e.

$$J_t(x, z) = \max \langle V_t, J_t(x, z) \rangle$$
The optimal hiring-tenure policy of the firm is clearly: hire a worker \( x \) and keep him employed if and only if \( J_t(x, z) \) is no less than the value of search (value of a vacancy). This occurs whenever the outcome of the match happens to be in the set:

\[
A_t := \{(x, z) \in [0, 1] \times [-Z, Z] \mid J_t(x, z) \geq V_t\}.
\]

In this circumstance \( J_t(x, z) = J_t(x, z) \) and thus \( J_t(x, z) \) satisfies the same arbitrage equation (2.3), now a Bellman equation in continuous time. Otherwise, it is worth destroying the job and search for a new, more profitable match.

The shadow value of a vacancy \( V_t \) satisfies

\[
\begin{aligned}
rv_t &= \hat{V}_t - \kappa + q(\theta_t) \left[ \tilde{J}_t - V_t \Pr((x, z) \in A_t) \right] \\
\end{aligned}
\]  

(1.2)

where \( \tilde{J}_t \) is the expected surplus from a new feasible and acceptable match \( (x, z) \in A_t \) times the probability of the match occurring and being acceptable. The expectation is taken both in terms of types to be met (only jobless workers with types in the set \( LP_t \subseteq [0, 1] \) are searching for a job) and in terms of match-specific outcomes, which are acceptable with a type \( x \) only if they belong to the \( x \)-cross-section set \( H_t(x) := \{z \mid (x, z) \in A_t\} \):

\[
\tilde{J}_t = E_{x,z} [J_t(x, z) \mid x \in LP_t, z \in H_t(x)] \Pr(z \in H_t(x))
\]

Free entry in vacancy creation ensures at all times:

\[
V_t = 0
\]

(1.3)

Workers

Let \( U_t(x) \) be the value to an \( x \)-type worker of being unemployed and searching, and \( W_t(x, z) \) be the human wealth for a worker \( x \) employed at match-specific outcome \( z \in \)
$H_t(x)$. For all $(x, z) \in A_t$, $W_t(x, z)$ satisfies

$$rW_t(x, z) = \bar{W}_t(x, z) + w_t(x, z) + \delta [U_t(x) - W_t(x, z)] \quad (1.4)$$

An unemployed worker $x$ who knows in equilibrium to be acceptable, and thus searches for a job (has type in the set $LP_t$), is hired as soon as he is interviewed by a firm if the outcome of the meeting is $z \in H_t(x)$. Thus, for all types searching $x \in LP_t$, $U_t(x)$ satisfies

$$rU_t(x) = \bar{U}_t(x) + b + \max (0; -c + p(\theta_t) [W_t(x, z) - U_t(x)]^e) \quad (1.5)$$

where $[W_t(x, z) - U_t(x)]^e := E [W_t(x, z) - U_t(x) \mid z \in H_t(x)] \Pr(z \in H_t(x))$ is the expected surplus for a worker from a new match acceptable to both the firm and the worker, times the probability of the match being acceptable. Unlike the hiring firm, the worker knows (and hence can condition on) his own ex ante productivity $x$ and takes the expectation on the r.h.s. of (1.5) only with respect to the match-specific outcome $z$.\(^{10}\)

**Wage Determination under Nash Bargaining**

The bilateral monopoly indeterminacy is resolved by assuming that the two parties bargain continuously over the surplus of the match. I adopt the Nash bargaining solution. If the worker gets a share $\beta$ of the total surplus from a filled job $(x, z)$, this solution yields:

$$W_t(x, z) - U_t(x) = \frac{\beta}{1 - \beta} J_t(x, z)$$

As usual, the firm and the worker always agree on accepting the match or not. Taking expectations with respect to acceptable match outcomes on both sides, this sharing rule links in the same way the expected surpluses. Using this property, differentiation with respect to time and substitution from (1.4) and (1.5) give an expression for the wage schedule as a function of flow values and of the expected surplus from next match,\(^{10}\)

---

\(^{10}\)This is typical of the standard Mortensen-Diamond-Pissarides equilibrium search model with homogeneous workers and firms, where match productivity is purely idiosyncratic.
\[ \beta J_t^c(x) \]:

\[ w_t(x, z) = (1 - \beta)(b - c) + \beta \eta(x + z) + \beta p(\theta_t) J_t^c(x) \] (1.6)

Using free entry (1.3) and the wage equation (1.6) the arbitrage equation can be rewritten as:

\[ \dot{J}_t(x, z) = (r + \delta) J_t(x, z) - (1 - \beta)(\eta(x + z) - b + c) + \beta p(\theta_t) J_t^c(x) \] (1.7)

### 1.3.2 Optimal Job Search and Tenure Policies

**Hiring-Tenure Policy**

The optimal hiring and job-tenure rule for a firm is of the reservation type: hire a worker of type \( x \) or keep him employed at time \( t \) if and only if the success of the match \( z \) is not below a type-dependent and time-dependent cutoff \( h_t(x) \), strictly decreasing in types (the proofs are in the Appendix). By Nash bargaining, this is also the optimal rule for the worker to accept the job and to keep it.

To prove the reservation property, I impose a feasibility condition, necessary for the Bellman equation in integral form to deliver a finite surplus \( J_t(x, z) \) at any point in time, given that total resources are indeed finite. Since the effective discount rate for firms is equal to \( r + \delta \) due to exogenous match destruction, I restrict attention to the solution of the arbitrage Equation (1.7) satisfying

\[ \lim_{t \to \infty} e^{-(r+\delta)t} J_t(x, z) = 0 \] (1.8)

for all \((x, z) \in A_t\). As shown in the Appendix, the border condition (1.8) selects the

\[ \text{11To clarify the notation, notice that the \textit{ex ante} asymmetric information between the worker, who can condition on his own type \( x \), and the firms, who ignore the type of the worker they are going to meet, makes the expected surplus of the worker (\( J_t^e(x) \)) different from that of any firm holding a vacancy (\( \dot{J}_t \)): \( \dot{J}_t = E[J_t^e(x) \mid x \in LP_t, x \text{ unemployed}] \cdot \Pr(x \text{ unemployed} \mid x \in LP_t) \). On the contrary, once the job is filled the firm and the worker are symmetrically informed. Since they both entertain, in a REE, correct expectations about the worker's outside option, the wage depends on the type-specific expected surplus \( J_t^e(x) \) and not on the firm's expected surplus \( \dot{J}_t \).} \]
following forward-looking solution \( J_t(x, z) \) for the differential equation (1.7):

\[
J_t(x, z) = \frac{1}{r + \delta} \left( \eta (x + z) - b + c \right) - \beta \int_t^\infty p(\theta_r) e^{-(r+\delta)(r-t)} J_r^c(x) \, dr. \tag{1.9}
\]

This is not an explicit solution. Though, it imposes a restriction on the time path of \( J_t(x, z) \) that, joined to the unrestricted arbitrage (differential) Equation (1.7), produces all the results on asset values and policy functions.

First, it is strictly increasing in \( z \), what proves immediately the following:

**Proposition 2 (Hiring-Tenure Reservation Policy)** There exists a function \( h : \mathbb{R}_+ \times [0, 1] \to [-Z, Z] \) such that the match between a firm and a type-\( x \) worker is acceptable to both parties if and only if the realization of the random component \( z \) of the match productivity is no less than the cutoff value \( h(t, x) \equiv h_t(x) \).

Using the identity \( J_t(x, h_t(x)) = 0 \), Proposition 2 yields also the simpler and intuitive expression for the firm's surplus

\[
J_t(x, z) = \frac{1 - \beta}{r + \delta} \eta [z - h_t(x)] \tag{1.10}
\]

This expression indicates that the additive separability of \( x \) and \( z \) in the flow productivity translates into its expected present discounted value. Hence the difference in profitability between any two jobs filled by identical workers is proportional to the difference in match fortunes (or firm-specific human capital).

**Job Search Policy and the Labor Force**

While jobless, workers also follow an optimal search rule of the reservation type: all and only workers with type \( x \) not below some cutoff \( \underline{x}_t \) search for an open vacancy, when not employed. The proof is non trivial, due to the forward-looking nature of the optimization (cf. (1.9)).
Proposition 3 *(Workers’ Search Reservation Rule)* There exists a function \( z : \mathbb{R}_+ \rightarrow [0, 1] \) such that a jobless worker of type \( x \) searches for a job at time \( t \) if and only if \( x \geq z(t) = z_t \).

In the light of Propositions 2 and 3, and of the Nash bargaining solution, the condition determining the searching cutoff type \( z_t \) can be rewritten as

\[
\frac{1 - \beta}{\beta} \int_{h_t(z_t)}^{z} [W_t(z, z) - U_t(z_t)] dI(z) = \int_{h_t(z_t)}^{z} J_t(z, z) dG(z) = J_t^e(z_t) \geq \frac{(1 - \beta) c}{\beta p(\theta_t)} \tag{1.11}
\]

with equality if \( z_t \in [0, 1] \), strict inequality if \( z_t < 0 \), a reversed inequality if no one is searching \( (z_t > 1) \). Hence, the set of searching types (if any) is the interval \( LP_t = [z_t, 1] \subseteq [0, 1] \).

### 1.3.3 Wage Compression Across Types

Once more, the forward-looking solution (1.9) gives the following characterization results (see the Appendix). The predictions for wages and exit rates are reassuring both on theoretical and empirical grounds. The title of the proposition refers in particular to the last claim.

Proposition 4 *(Wage Compression across Types in non Steady State Equilibrium)* In any REE of the frictional economy, on the whole set of acceptable matches \((x, z) \in A_t\):

- the value function \( J_t(\cdot, z) \) is strictly increasing and strictly concave in types;
- the hiring schedule \( h_t(\cdot) \) is strictly decreasing and strictly convex in types;
- the expected value function \( J_t^e(\cdot) \) is strictly increasing in types;
- the wage schedule \( w_t(\cdot, z) \) is strictly increasing in types;

finally, the difference between the present discounted values of productivity and wages increases in productivity across workers.

High types, who have more to lose from rejecting a match, compress their wages and accept worse matches \((h_t(\cdot) < 0)\). This intuitive result has two strong testable
predictions:

- **General** and average firm-specific human capital are negatively correlated across workers employed on the same job, because high types accept matches that are worse on average.

- **Exit rates from unemployment increase in types**: a higher type, enjoying a larger acceptance set \([h_t(\cdot), Z]\), by random matching has a higher exit rate from unemployment \(p(\theta_t)[1 - G(h_t(\cdot))].^{12}\)

### 1.3.4 Labor demand: Entry and the Creation of Vacancies

Let \(f_t(\cdot) = \frac{u_u(\cdot)}{u}\) denote the normalized (by the mass of unemployed) density function of unemployed workers at time \(t\), with respect to types. The corresponding c.d.f. is denoted by \(F_t(\cdot)\), with support on the set \(LP_t\) ("Labor Participation").

Restricting attention to equilibria with positive output \((\zeta_t \leq 1)\), the free entry condition determines the equilibrium vacancy rate and thus labor demand by equalizing the total expected costs and the expected surplus from searching for unemployed workers:

\[
\frac{\kappa}{q(\theta_t)} = \int_{\zeta_t}^1 J^e_t(x) \, dF_t(x) = \int_{\zeta_t}^1 \int_{h_t(x)}^Z J_t(x, \zeta) \, dG(\zeta) \, dF_t(x)
\]  \hspace{1cm} (1.12)

By definition, the cross-section density of unemployed workers \(f_t(\cdot)\) sums up exactly to one over the set of searchers \([\zeta_t, 1]\), and is therefore a probability density for firms.

Also, notice that, for \(\zeta_t \leq 1\),

\[
\frac{\kappa}{q(\theta_t)} = \int_{\zeta_t}^1 J^e_t(x) \, dF_t(x) \geq J^e_t(\zeta_t) \geq \frac{(1 - \beta)c}{\beta p(\theta_t)}
\]

with the first inequality strict if \(\zeta_t < 1\), because \(J^e_t(\cdot)\) is strictly increasing (Proposition

---

\(^{12}\text{Furthermore, if } z \text{ were not firm-specific human capital, but a shock hitting the match continuously and independently over time, the reservation policy } h_t(\cdot) \text{ would hold basically unaltered and higher types would exhibit lower entry rates into unemployment (what is certainly true for different skill levels; cf. Nickell and Bell 1995).}\)
4) and $F_t(\cdot)$ is a c.d.f. with full support $[x_t, 1]$. Substituting for $p(\theta_t) = q(\theta_t) \theta_t$ and rearranging yields the following conclusions:

**Lemma 1 (Lower Bound of the Vacancy Rate)** In any REE of the frictional economy with some job search ($x_t \leq 1$), the vacancy rate $\theta_t$ is bounded below by

$$\theta_t \geq \theta(\beta, \kappa, c) := \frac{1 - \beta}{\beta} \cdot \frac{c}{\kappa} > 0$$

and the frictional economy tends to shut down as $\theta_t$ tends to its lower bound: $\lim_{\theta_t \to \theta} x_t = 1$

Intuitively, the lower bound $\theta$ is equal to the relative bargaining share of firms with respect to workers ($\frac{1-\beta}{\beta}$) divided by the relative search cost ($\frac{c}{\kappa}$). It does not depend on the matching function $q(\cdot)$, nor on the time cost ($r + \delta$), nor on $\eta$. The independence of $\eta$ is particularly striking, because a large $\eta$ is equivalent to negligible search costs ($c, \kappa, r, \delta$). The intuition, however, is clear. A rate $\theta_t < \theta$ keeps expected wages too low to induce any job search whatever $\eta$ (hence even at almost zero search costs); on the other hand, if no worker is searching, it does not pay to post vacancies. This coordination failure is illustrated formally in the Appendix.

**Equilibrium in a Frictional Economy: Cross-Sectional Dynamics**

The definition and the characterization of the frictional equilibrium are completed by the evolution of the cross-section distributions of employed and unemployed workers. I defer this description to the Appendix.

## 1.4 Steady State Equilibria in the Frictional Economy

In this section I fully characterize the *steady state* equilibria of the frictional economy. This will allow me in the next section to perform the desired analysis of comparative statics and transitional dynamics that illustrate the Fattening hypothesis. In addition
to the results obtained in the previous section, when imposing stationarity I can derive explicit expressions for policy functions and cross-section distributions, and I can prove that, in types space, the steady state wage schedule is flatter than the Walrasian schedule and is convex.

1.4.1 Optimal Policies and Equilibrium Cross-Sections

Hiring-Tenure Policy and Wage Schedule

Dropping the time subscript to denote steady state values, after some manipulations of (1.10), the wage schedule in steady state is expressed by a weighted average, of flow productivity and of another weighted average, of the flow value of being searching, \((b - c)\), and of the share of the expected flow productivity of a new match for the same worker:

\[
w(x, z) = \beta \eta(x + z) + (1 - \beta) \frac{(r + \delta)(b - c) + \beta p(\theta) \int_{h(x)}^{z} \eta(x + s) \, dG(s)}{r + \delta + \beta p(\theta)} \frac{\beta p(\theta) \int_{h(x)}^{z} \eta(x + s) \, dG(s)}{[1 - G(h(x))]} \tag{1.13}
\]

Using the identity \(J(x, h(x)) = 0\) and Equation (1.13) in (1.10), upon rearranging I get an equation that defines implicitly and uniquely the stationary optimal hiring-tenure schedule:

\[
h(x) = \frac{(r + \delta)(b - c - \eta x) + \eta \beta p(\theta) \int_{h(x)}^{z} \eta(x + s) \, dG(s)}{\eta \{r + \delta + \beta p(\theta)\} [1 - G(h(x))]} \tag{1.14}
\]

From the Implicit Function theorem, it is also a unique function of the vacancy rate \(\theta\), pointwise in \(x\):

\[
\frac{dh(x)}{d\theta} = \frac{\beta p'(\theta) \int_{h(x)}^{z} (s - h(x)) \, dG(s)}{r + \delta + \beta p(\theta) [1 - G(h(x))]} > 0 \tag{1.15}
\]

The sign of this effect is intuitive: coeteris paribus, an increase in the vacancy rate improves workers’ outside options and thus puts pressure on wages, making firms choosier on the dimension of firm-specific human capital \((z)\). One can wonder whether the response of choosiness \(h(\cdot)\) could more than offset the increase in the vacancy rate, so that exit rates from unemployment \(E(x) := p(\theta) [1 - G(h(x))]\) fall in the vacancy rate \(\theta\). Intuition rules out this possibility: the increase in wages that raises the \(h(\cdot)\) cutoff schedule

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occurs only if exit rates from unemployment rise in the first place. The following result confirms the intuition.

Lemma 2 Coeteris paribus, exit rates from unemployment are strictly increasing in the vacancy rate \( \theta \) for all types in the participating labor force \( x \in [x, 1] \).

Now, I can look again at wage compression. In addition to the results of Proposition 4, valid for any REE, for stationary equilibria I can obtain the following explicit expressions, very useful to capture the intuition:

Proposition 5 (Wage Compression across Types in Steady State Equilibrium) In any steady state equilibrium, in the whole set of acceptable matches \([x, 1] \times [h(x), Z]\), search frictions \( r + \delta > 0 \) and \( p(\theta) < \infty \) imply all of the following:

1. the hiring-tenure schedule \( h(\cdot) \) is strictly decreasing and strictly convex in types:

\[
h'(x) = -\frac{r + \delta}{r + \delta + \beta p(\theta)[1 - G(h(x))]} \in (-1, 0) \tag{1.16}
\]

\[
h''(x) = \frac{[h'(x)]^2 \beta p(\theta) g(h(x))}{r + \delta + \beta p(\theta)[1 - G(h(x))]} > 0
\]

2. the expected surplus from a new match \( J^e(\cdot) \) is strictly increasing and strictly convex in types:

\[
J^{eu}(x) = \frac{1 - \beta}{\beta p(\theta)} h''(x) > 0
\]

3. the surplus from a filled job \( J(\cdot, z) \) is strictly increasing and strictly concave in types:

\[
\frac{\partial^2 J(x, z)}{\partial x^2} = -\frac{(1 - \beta) \eta}{r + \delta} h''(x) < 0
\]

4. the wage schedule \( w(\cdot, z) \) is strictly increasing and strictly convex in types, with slope strictly lower than the Walrasian slope \( \eta \):

\[
\frac{\partial w(x, z)}{\partial x} = \beta \eta \frac{r + \delta + p(\theta)[1 - G(h(x))]}{r + \delta + \beta p(\theta)[1 - G(h(x))]} \in (\beta \eta, \eta) \tag{1.17}
\]
\[
\frac{\partial^2 w(x, z)}{\partial x^2} = (1 - \beta) \eta h''(x) > 0
\]

Once more, the predictions of the model for the pattern of wages deserve a comment.

- **Wage Compression Across Types.** A slope of the wage schedule below \(\eta\) indicates directly that an increase in \(x\) guarantees a larger output at rate \(\eta\), but only a fraction of it goes to the worker. As a consequence, higher types are more profitable to the firms and experience higher exit rates from unemployment and lower unemployment rates.

- **Convexity of the Wage Schedule.** Given the linear technology and sharing rule, a higher type increases linearly the share of output he gets. In addition, he enjoys a higher exit rate from unemployment, because of wage compression, what gives an extra kick to his outside option and creates the convexity in wages. The formulas confirm that wages are convex if and only if they are compressed across types. The convexity of wages in human capital is indeed a well documented fact. It might certainly be produced by a convex technology; but, unlike technology, this exit rates effect, arising from wage compression, goes always in the direction of the convexity that we observe.

- **Wage Dispersion Conditional on a Worker’s Type.** This is also a widely measured and studied phenomenon. The assumption in this framework is that dispersion derives from match dimensions of heterogeneity, such as firm-specific human capital. The testable prediction of the model is that the dispersion is larger in absolute terms for higher types, because the set of implemented \(z'\)’s is larger for them (the cutoff \(h(\cdot)\) is lower).

**Job Search**

Using Equation (1.10), the searching cutoff type \(z\) is implicitly and *uniquely* defined by:

\[
\int_{h(z)}^{z} (s - h(x)) dG(s) = \frac{c(r + \delta)}{\eta \beta p(\theta)}
\]  

(1.18)
Uniqueness again follows from the Implicit Function Theorem: using (1.15)

\[
\frac{d\bar{z}}{d\theta} = -\frac{p'(\theta) c (r + \delta)}{\beta n p^2 (\theta) \left[ 1 - G (h (\bar{z})) \right]} : 0
\]  

(1.19)

Intuitively enough, as the vacancy rate \( \theta \) increases, coeteris paribus the outside option improves for any worker, what raises wages and attracts more types into job search.

Vacancy Creation

Turning to the equilibrium vacancy rate \( \theta \), the zero profit condition for vacancy creation in steady state equalizes again total expected search costs to the expected surplus:

\[
\frac{\kappa}{q (\theta)} = \bar{J} = \int_{1}^{\bar{z}} \int_{h(x)}^{z} J (x, z) dG (z) dF (x)
\]

Cross-Section Distribution of Unemployed Workers

In any steady state equilibrium we have the important simplification that the labor force is exactly in the set of types \([\underline{z}, 1]\).\(^{13}\) Furthermore, I can describe explicitly:

**Lemma 3 (Stationary Distribution of Unemployed Workers)** The cross-section distribution of unemployed workers in steady state, for \( x \in [\underline{z}, 1] \), is equal to

\[
f (x) = \left[ \int_{\underline{z}}^{1} \frac{\psi (\xi) d\xi}{\delta + p (\theta) [1 - G (h (\xi))]} \right]^{-1} \cdot \left[ \frac{\psi (x)}{\delta + p (\theta) [1 - G (h (x))]} \right]
\]  

(1.20)

Recall that \( \psi (\cdot) \) is the cross-section density of existing workers, and \( p (\theta) [1 - G (h (x))] \) is the exit rate for type \( x \). It is clear that \( f (\cdot) \) integrates up to 1, so that firms view

\(^{13}\)It is easy to see why this is true. If we imagine the system in steady state since time 0, then types below \( \underline{z} \) never search and never searched for a job. If instead we study the convergence to a s.s. with a \( \bar{z} \) increasing toward \( \underline{z} \), there can be some workers with type \( \underline{z} \leq x < \bar{z} \), who searched and found at time \( t \) a job at outcome \( z \geq h (x) > h_1 (x) \) such that their match remains profitable in the eventual steady state. These workers remain employed as long as they can, but, when the match breaks exogenously, they just leave the labor force. Clearly, the total measure of such workers decays exponentially to zero, due to zero creation and positive destruction. Therefore, in both cases, all and only the workers in \([\underline{z}, 1]\) are either searching for a job or working, the others are inactive.
it as a probability distribution from which applications to their vacancies arrive. Total unemployment is equal to

$$u = \delta \int_{\bar{z}}^{1} \frac{\psi(\xi)}{\delta + p(\theta) [1 - G(h(\xi))]} d\xi$$

(1.21)

The natural or frictional rate of unemployment, defined as the total measure of unemployment divided by the labor force (both endogenous), is simply \( \tilde{u} := u/\Psi([\bar{z}, 1]) \). The unemployment density is \( u(x) = u f(x) \). One can immediately verify that the point unemployment rate \( u(x)/\psi(x) \) is decreasing in workers’ types, as we observe in the “stylized” empirical pattern sketched in the Introduction. In general, we obtain typical frictional unemployment expressions, but where the average exit rate from unemployment matters.

### 1.4.2 Existence and Multiplicity of Steady State Equilibria

The issues of existence and uniqueness are important both per se and for signing comparative statics effects of parameters. In Moscarini (1995) I prove that, in this class of models, multiple stationary equilibria may exist with no increasing returns to matching. Furthermore, if multiple equilibria exist, then the Pareto worst equilibrium does not converge to the Walrasian equilibrium as search frictions vanish.

As a consequence of these (non)convergence properties, in this paper I always consider the stationary equilibrium with highest vacancy rate \( \tilde{\theta} \). In Moscarini (1995) I also show that the effect of changes in \( \eta \) on the equilibrium vacancy rate \( \tilde{\theta} \) is ambiguous. I show that in a neighborhood of the Pareto undominated equilibrium the expected net profitability from posting an additional vacancy must be decreasing in the vacancy rate; hence, under the reasonable assumption on parameters that an increase in productivity also raises the profitability of posting an additional vacancy, the total comparative statics effect of an increase in \( \eta \) is an increase in the vacancy rate.

Before examining in detail comparative statics and dynamics, I find it useful to compare piece by piece the stationary REE of frictional economies described in this section.
to the unique equilibrium of the corresponding Walrasian economy, described in Section 1.2.2.

1.4.3 Comparing Frictional and Walrasian Equilibrium Allocations

This comparison highlights the general equilibrium effects of search frictions. The first effect listed below is obvious and well-known, as direct consequence of adjustment costs at the individual level; the other effects are far less obvious, as they arise from general equilibrium interactions, and exist only in economies with ex ante heterogeneous agents.

1. **Frictional unemployment.** In the frictional economy there is a positive mass of involuntary unemployed.

2. **Pattern of unemployment rates.** The unemployment rate by type is decreasing in types.

3. **Lower labor participation.** Labor participation is strictly smaller in the frictional economy: direct inspection reveals that \( x^* = b/\eta - Z < x = b/\eta - h(x) \).

4. **Wage compression across types occurs only in the frictional economy.**

5. **Creation of low-productivity jobs.** In the frictional economy, a mass of matches with \( z \in [h(x), Z) \) are implemented, that would be hidden away in a Walrasian economy, where only matches with \( z = Z \) are implemented in equilibrium.

Notice that 1, 3, 4, and 5 hold in any frictional REE, even non stationary. The reasons for the existence of involuntary unemployment and for its cross-sectional pattern in the model are straightforward. They are certainly welcome implications on empirical grounds.

The reason for lower labor participation in the frictional economy is two-fold. First, obviously, search is costly: hence, given expected wages, some low types will not enter the market. Second, expected wages are lower, because of the creation of low-productivity
jobs. Since implementing matches is hard and costly, some matches activated at $h_t(\cdot) < z < Z$ cannot be bidden away: hence, on average, acceptable matches are less productive, and accordingly expected wages are lower than in the Walrasian economy.

Finally, the creation of low-productivity jobs originates from a pure search-theoretic (optimal stopping) aspect of firms policies. Simply, the net continuation value of search for firms in a frictional economy is lower than the continuation value of bidding in a centralized market, hence firms must be less choosy on firm-specific human capital $z$. The positive rents that search frictions generate for firms, and that would be bidden away by costless retrading in the Walrasian economy, are never sufficient to make up for this effect. On the contrary, firms face expected rents from search that are lower than the rents from the job in hand for $z \geq h_t(x)$; in this case, it is not worth scrapping the match to improve it.

1.5 Fattening Frictional Economies

1.5.1 Comparative Statics Effects of a Positive Permanent Shock

Finally, I am in the position to address formally the main topic of the Chapter, namely the composition of the employed labor force over aggregate fluctuations.

I interpret a small exogenous and permanent increase in aggregate profitability from jobs ($d\eta > 0$) as a neutral shock to the demand for labor (a "boom"). I study the change in the cross-sectional pattern of hiring and tenure margins, showing that they determine a "Fattening" of the employed labor force, additional to the change in participation that we would observe in a Walrasian economy. To this purpose, I contrast the comparative statics responses of the two equilibrium allocations, the Pareto superior frictional allocation ($\bar{\theta}$) and the Walrasian one.

First, a useful fact is that a larger profitability $\eta$ per se lowers the frictional cutoff
schedule, because it raises all types’ productivities and makes worse matches affordable:

$$\frac{\partial h(x)}{\partial \eta} = -\frac{(r + \delta) (b - c)}{\eta^2 [r + \delta + \beta p(\theta) (1 - G(h(x)))]} < 0$$ (1.22)

The assumption $b > c$ is implicit. This fact and Lemma 2 prove:

**Lemma 4** Steady state exit rates from unemployment increase after a “boom”. For all $x \geq \bar{x}$:

$$\frac{dE(x)}{d\eta} = \frac{dE(x)}{d\bar{\theta}} \cdot \frac{d\bar{\theta}}{d\eta} - p(\bar{\theta}) g(h(x)) \frac{\partial h(x)}{\partial \eta} > 0$$

Then, I can analyze separately the three sources of change in average productivity and verify how the predictions of the model fit in the empirical evidence on labor markets over expansions.

1. **Tilting of the Hiring-Tenure Schedule.** The relative choosiness of firms across types falls after the boom (Fig. 4.1):

$$\frac{dh'(x)}{d\eta} = -\frac{\beta h'(x)}{r + \delta + \beta E(x)} \cdot \frac{dE(x)}{d\eta} > 0$$

The intuition is the following. In equilibrium a higher vacancy rate reduces the expected loss from rejecting a match for all workers. Since, by assumption, high types have more to lose from being unemployed, they are favored relatively more by the increase in vacancy rate, and their wages rise faster. In other words, rising exit rates imply that wages rise faster than productivity after a “boom” $d\eta > 0$, hence wage differentials increase and the convexity of the wage schedule is reduced. As a consequence, firms find it relatively more profitable to hire low types when meeting them.

2. **Changes in Total Productivities:** the effect of a “boom” on the level of choosiness is ambiguous.

$$\frac{dh(x)}{d\eta} = \frac{dh(x)}{d\theta}igg|_{\theta=\bar{\theta}} \frac{d\bar{\theta}}{d\eta} + \frac{\partial h(x)}{\partial \eta} <= 0?$$

38
For large $\eta$, the frictional stationary equilibrium converges to the Walrasian allocation where $h(x) = Z$, so that the total effect is positive. This means that Sclerosis is dampened. The lower quality of the employed labor force is balanced by a better ex post job allocation.

3. **Increase in Labor Participation.** Some Fattening occurs through this channel as well: the frictional participation margin $x$ is always more volatile across steady states than the Walrasian participation margin $x^*$. Using the expression for $h'(x)$ yields

$$\frac{dx}{d\eta} = -\frac{b}{\eta^2} + \frac{\partial h(x)}{\partial \eta} \frac{d\theta}{d\eta} = -\frac{b}{\eta^2} + \text{(negative term)} < -\frac{b}{\eta^2} = \frac{dx^*}{d\eta}$$

All the extra action in $x$ comes from search frictions, showing up through the flexible margin $h(x)$: the steeper the tenure schedule in a right neighborhood of $x$ (the closer $h'(x)$ to $-1$), the higher the marginal gain in terms of exit rate from entering job search, hence the larger the measure of new searchers.

The first effect, the change in the cross-sectional picture of acceptance rates, is what distinguishes most sharply the frictional economy from its Walrasian counterpart. It replicates the behavior of cross-sectional exit and unemployment rates observed over expansions. The second effect is harder to relate to observables, yet it helps reconciling Fattening with the observed procyclicality of average productivity. The third effect is just an amplification of the only effect of the shock on the Walrasian economy; yet, it goes in the direction of Fattening and it is perfectly consistent with the facts.

### 1.5.2 Transitional Dynamics: from the “Jobless Expansion” to Fattening

The transitional dynamics between steady states, before and after the positive aggregate shock, are interesting in themselves, as they add economic intuition to the workings of
this artificial economy and provide a possible rationale for some empirical regularities observed in market economies during expansions.

The Appendix presents a partial characterization of transitional dynamics. There, I first derive the general law of motion for the equilibrium vacancy rate. Differentiating the free entry condition (1.12) with respect to time, I show that \( \theta_t \) is the sum of five effects:

1. average capital gain on prospective matches;

2. average exogenous job destruction across types (at rate \( \delta \));

3. average endogenous job creation (hires of different types);

4. change in job search participation (in the cutoff \( z_t \));

5. average endogenous job destruction (quits); this occurs for each type \( x \) only if the match-tenure cutoff \( h_t (x) \) is rising, so that some existing matches become no longer acceptable.

The third effect, the average effect of hiring, is always negative: intuitively, as high types exit faster from unemployment, the quality of the unemployed pool has a negative “drift”. The signs of the other four effects depend on the changes in job search participation and in the asset value of filled jobs.

Unfortunately, the full characterization of the transition of the frictional economy is complicated by cross-sectional dynamics: the REE is a system of both ordinary and partial differential equations. The descriptive results that I have established so far concern a sub-class of dynamics, identified by two properties: convergence to the new (Pareto superior) steady state, and monotonicity. For this sub-class of models I can characterize

---

\(^{14}\)The same results can be read in terms of “conditional” convergence: given a monotone path for the vacancy rate \( \theta_t \), I prove that this path is increasing after a positive shock and I show how all other variables comove with \( \theta_t \) while converging to their new steady state values. In general, I could not rule out non monotone convergence and limit cycles. Given the large number of degrees of freedom in the parametrization of the model, it seems that such possibilities cannot be ruled out at all in general (cf. Guckenheimer and Holmes 1983, ch.1.9).
fully the transition of the system between steady states, following any positive aggregate shock. Figure 5.1 depicts the paths of the main variables. In the rest of this section, I provide the intuition behind the pictures, building on the results proven in the Appendix.

The first result (Lemmata 6 and 7) is that the asset value of all filled jobs \( J_t(x, z) \), \((x, z) \in A_t\), and the search cutoff type \( z_t \) move in opposite direction to the vacancy rate \( \theta_t \) while converging to the new steady state. Intuitively, given parameters, as the vacancy rate increases, market perspectives improve for all workers, what raises wages: as a consequence, filled jobs are less profitable (the cost of labor increases, making firms choosier on the \( z \) dimension) and more people are attracted into job search.

Thus, to prove that the paths in Fig. 5.1 describe the correct overall transitional dynamics I just have to establish that the path of the vacancy rate is increasing. Since the comparative statics effect of a \( d\eta > 0 \) on the vacancy rate \( \theta \) is positive, this is equivalent to assert that \( \theta_t \ does \ not \ reach \ at \ once \) its new s.s. value. This result sharply differs from the predictions of search models with ex ante homogeneous agents (cf. Pissarides 1985). I will now show that this important difference indeed follows from the presence of agents' ex ante heterogeneity, which indeed plays a very similar role to the convex costs of creation in models with heterogeneous capital (Caballero Hammour 1994).

The intuition is the following. Due to endogenous job search and tenure decisions, the measure and the cross-section of unemployed workers are not predetermined variables, in spite of search frictions. Hence they can change at once, and then evolve slowly, so as to absorb some of the action of the aggregate shock and to prevent the vacancy rate from jumping all the way. Just after a boom \( d\eta > 0 \), a bunch of new low types enter job search, and they are \textit{all} unemployed (the shaded area in Fig. 6.1). Nothing else happens in the shortest run if the job-tenure margin \( h(\cdot) \) falls. If instead this margin initially rises, \textit{some} good types quit their jobs immediately, as market perspectives improve and induce them to search for better matches. Thus, only in this second case the net effect on the quality of the unemployed pool seems ambiguous, but I prove that it must be negative. Hence, a moment after the boom, the cross-section distribution of unemployed workers is skewed towards low types (Fig. 6.1 again).
Over time, low types get hired faster than high types because they are in "excess supply". In relative terms, due to slower hiring and to exogenous job destruction, the relative availability of high types in the unemployment pool increases. This effect keeps raising the profitability of vacancy posting, in spite of the continuous inflow of lower and lower types, of the capital loss on filled jobs and of the increasing congestion of firms. Due to search frictions, it takes some time to see the excess supply of low types being absorbed by the market; in the meanwhile, firms hold off their vacancies, waiting for better times to hire. This keeps wages temporarily compressed and high types still much more attractive: the delay strategy is self-fulfilling.\textsuperscript{15}

This pattern motivates the expression "Jobless Expansion" to indicate the first phase. Firms know that delaying new hires means losing the first part of the expansion, when profits from filled jobs are maximum. But it does not pay enter a market congested by low types, and waiting to create jobs keeps the market congested. This phenomenon introduces a form of aggregate labor hoarding, that complements naturally the private labor hoarding stories of short-run increasing returns. At the onset of a recovery, the average productivity of jobs increases, both exogenously, by $d\eta > 0$, and endogenously, due to quits of workers from the worst matches; instead, employment hardly responds. Also, firms are still choosy across types, so that the new hires are still very unevenly distributed and do not worsen sensibly the quality of the unemployment pool.

Then, the Fattening phase of the expansion takes over. As vacancy posting and job creation expand over time, wages are decompressed across workers' types and firms become less and less choosy. The employment of good workers (which was already relatively high) expands slowly at first, then faster, but only because the stock of their relative unemployment is increasing; in relative terms the employment of low types grows faster and indeed takes off, fattening the economy.

\textsuperscript{15}One prediction is that, in terms of employee's skills, the onset of a recovery is the worst time to hire, because lots of bad workers are searching for jobs; the best time is the through of a recession, when only relatively good workers are searching actively. The peak of an expansion is not such a bad time to hire, because high types are relatively less employed, due to their high wages. In terms of profits, of course, the timing is different, because of procyclical wages.
1.6 Further Implications and Extensions

In this final section, I briefly describe two interesting further implications of the model and discuss two main extensions of the previous analysis.

- **Changes in the workers' cross-section distribution** \( (\psi) \). The model can be used to derive predictions on the response of labor demand and of the market equilibrium to changes in the relative supply of skills. Recall that the expected surplus function \( J^e (\cdot) \) is strictly increasing and strictly convex. Any FSD upward shift in the cross-section distribution of workers \( \psi (\cdot) \) raises in equilibrium the profitability of posting vacancies and then the exit rates from unemployment for all workers. Similarly, a Mean Preserving Spread in \( \psi (\cdot) \) raises the profitability of posting vacancies, also because some of the spread falls below the search cutoff \( x \). Thus, a more unequal distribution of skills is able to raise labor demand and wages for everybody.\(^{16}\)

- **Allocative efficiency and wage inequality.** The analysis on the role of search frictions (in the Appendix) indicates that allocative efficiency has distributional implications. In particular, a faster matching mechanism raises labor demand in the Pareto undominated equilibrium and, through this channel, “de-compresses” wages, driving their distribution to the more unequal Walrasian one. Since it is easier to receive applications to a vacancy when matching is faster, given that the implementation cutoff for match outcomes \( h (\cdot) \) is decreasing in types, a firm can implement a larger proportion of good applications, what raises ex ante the profitability of posting vacancies. Thus all workers have less to fear from rejecting a match, but high types gain more, given the assumption on the preferences for being unemployed. A natural conjecture arises, that the recent surge in wage inequality in the U.S. (cf. Katz and Murphy 1992) might be in part due to an improved

\(^{16}\)Clearly, the cross-section distribution of unemployed workers \( f (\cdot) \), the search type cutoff \( x \) and the expected surplus function from a match \( J^e (\cdot) \) adjust to such changes in \( \psi (\cdot) \), possibly offsetting some of the increase in the profitability of vacancy creation. However, these other adjustments are second order.
efficiency in the process of allocating unemployed workers to open vacancies, from which higher skills have benefited most. The inequality of income, instead, depends also on the average duration of unemployment; on this side, the effect of an increase in matching speed works in the opposite direction, because exit rates across types tend to be leveled.

Let me finally turn to the main extensions of the model.

- **Asymmetric information on the workers’ types.** This possibility is interesting but introduces a number of ambiguities. First, adverse selection in a dynamic setting is hardly an understood problem in the crisp air of the partial equilibrium Agency framework. The complexity of a macroeconomic environment multiplies the difficulties. Hence I will limit myself to discuss it in stationary equilibria. Next, it is not clear who should have the bargaining power. If it is all on the workers, then asymmetric information is irrelevant. If it is split, then we have to specify an extensive form of the bargaining game, and I see no clear criterion to be followed. If it is all to the uninformed party, the firm, then this model configures a problem of optimal Mechanism Design with reservation utilities that are both *type-dependent* and *endogenous*. This is a possibility that arises only in general equilibrium adverse selection, hence so far it has been (almost) completely ignored by the Agency literature. This model might provide a general equilibrium framework to tackle this problem.

- **Type-specific matching technology.** The search literature is still relatively young, and the matching process is definitely still a black box. Some preliminary work along these lines is in Acemoglu (1995). I consider far too strong to imagine that a firm can attract exactly the type of worker it desires, there is always some risk when hiring (Lazear (1995) makes the same point). But it is also true that firms do set specific type (age, education, work experience) requirements when posting vacancies, which is the main objection to the analysis of this paper. There are three complementary answers to this objection.
First, as said in the Introduction, there is strong evidence for a large substitutability between different types of workers for the same kind of jobs. Hence, even with segregated labor markets, a neutral increase in labor demand decompressing wages will lead firms to substitute high to low types.

Second, the present analysis can be read in terms of unobservables, such as innate skills, whose heterogeneity is well known and highly relevant for work performances. In this case, the empirical evidence mentioned in the introduction is of no direct help, but the analysis applies entirely.

Third and last, the “types” of this model can be interpreted as unobserved but observable differences in productivities, conditional on firms’ formal requirements. Let me take up an example. Suppose that a firm posts a vacancy for 22 to 34 years old individuals holding a College degree. Then, applicants will hold degrees from many different Colleges; as long as the general skills of a worker are correlated with the quality of the educational institution attended, the “high types” will be graduate of the top Universities. This model predicts that they will do better than their colleagues from less valuable institutions, and that firms will adjust their choosiness on the quality of their employees’ education according to market conditions. There are many other similar examples. Under this interpretation, random matching is the only sensible assumption.
1.7 Appendix

Characterization of Equilibrium Dynamics

First, I deal with equilibrium dynamics of the frictional economies. First, I present the aggregate dynamics of the cross-section distribution of workers and close the definition of the Rational Expectations Equilibrium. Second, I characterize the different components determining the evolution of the vacancy rate along any REE smooth path. Third, I characterize the dynamics of the overall system conditional on a given path of \( \theta_t \), under the assumption that the steady state is an attractor and that any REE converges to it monotonically.

Equilibrium in the Frictional Economy: Aggregate Dynamics

The cross-section density of workers \( \psi(\cdot) \) is fixed. Due to the dynamics of the optimal search margins \( x_t \) and \( h_t(\cdot) \), the cross-section distributions of employed and unemployed workers evolve endogenously. Hence, I need to track the dynamics of the bi-variate cross-section distribution of employed workers \( e_t(\cdot, \cdot) \), of the marginal density of employed workers \( e_t(\cdot) \), of the density \( u_t(\cdot) \) and of the measure \( u_t \) of unemployed workers, and finally of the normalized density of unemployed workers \( f_t(\cdot) \). Assuming no aggregate uncertainty, the description of aggregate dynamics is simply an exercise in accounting.

Density of Employed Workers: Job Creation, Job Destruction, and Quits

Among all workers previously employed in matches that are now unacceptable \((x, z) \notin A_t\) no one is employed and \( e_t(x, z) = 0 \).

For all workers employed in strictly profitable matches \((z > h_t(x))\) the employment density has a smooth time path according to the following vector field:

\[
\dot{e}_t(x, z) = T(e_t(x, z), x_t) := \begin{cases} 
-\delta e_t(x, z) & x < x_t \\
-\delta e_t(x, z) + p(\theta_t) g(z) u_t(x) & x \geq x_t 
\end{cases}
\]

The former are the workers who searched for a job when it was profitable to do so,
and found one which is still valid, but who would not search again were they laid off exogenously \((x < \bar{x}_t)\). The latter are the employed workers who, if laid off, would search again.

Finally, marginal matches \((x, h_t(x))\) get entirely and endogenously destroyed if and only if the cutoff \(h_t(x)\) is rising, in which case the path of the bivariate employment density is discontinuous on the \(z = h_t(x)\) manifold.

\[
\begin{align*}
\dot{e}_t(x, h_t(x)) &= T(e_t(x, h_t(x)), \bar{x}_t) \quad \text{if } \dot{h}_t(x) \leq 0 \\
\Delta e_t(x, h_t(x)) &= -e_t(x, h_t(x)) \quad \text{if } \dot{h}_t(x) > 0
\end{align*}
\]

Aggregation across \(z\)'s eliminates the discontinuity and yields the dynamics of the marginal density of employed workers:

\[
\dot{e}_t(x) = -\dot{h}_t(x) e_t(x, h_t(x)) \mathbb{1}_{h_t(x) > 0} + \begin{cases} 
-\delta e_t(x) & \text{for } x < \bar{x}_t \\
-\delta e_t(x) + p(\theta_t) u_t(x) [1 - G(h_t(x))] & \text{for } x \geq \bar{x}_t
\end{cases}
\]

(1.23)

The term \(-\dot{h}_t e_t(x, h_t(x))\), measuring the endogenous destruction of jobs, or "quits", must be added only when the hiring cutoff is rising, as the indicator function \(\mathbb{1}_{h_t(x) > 0}\) shows. In fact, a rising \(h_t(x)\) entails by (1.10) a falling surplus \(J_t(x, \cdot)\), inducing a quit when this value falls too much.

**Density of Unemployed Workers**

**Lemma 5** The normalized cross-section density of unemployed workers evolves as follows:

\[
\begin{align*}
\dot{f}_t(x) &= \frac{\delta}{u_t} \{\psi(x) - f_t(x) \Psi([\bar{x}_t, 1])\} + \\
&+ f_t(x) p(\theta_t) \left[ G(h_t(x)) - \int_{\bar{x}_t}^{1} G(h_t(\xi)) f_t(\xi) d\xi \right] + \\
&+ f_t(x) \left[ \tilde{\Lambda}_t(x, \tilde{x}_t) \right] + \\
&+ \frac{1}{u_t} \left\{ \dot{h}_t(x) e_t(x, h_t(x)) \mathbb{1}_{h_t(x) > 0} - f_t(x) \int_{\bar{x}_t}^{1} \dot{h}_t(\xi) e_t(\xi, h_t(\xi)) \mathbb{1}_{h_t(\xi) > 0} d\xi \right\}
\end{align*}
\]

Type of flow

- **exog. EU**
- **UE**
- **EN or NU**

\[(1.24)\]
for $x \in [\bar{x}_t, 1]$, with

$$
\Lambda (\bar{x}_t, \hat{x}_t) := \begin{cases} 
\psi (\bar{x}_t) - e_t (\bar{x}_t) = u_t f_t (\bar{x}_t) & \text{if } \hat{x}_t > 0 \\
0 & \text{if } \hat{x}_t = 0 \\
\psi (\bar{x}_t^-) - e_t (\bar{x}_t^-) = \lim_{x \to \bar{x}_t^-} [\psi (x) - e_t (x)] & \text{if } \hat{x}_t < 0
\end{cases}
$$

**Proof.** By definition, the normalized density $f_t (\cdot)$ with support $[\bar{x}_t, 1]$ satisfies:

$$
\hat{f}_t (x) = \left[ \frac{\dot{u}_t (x)}{u_t (x)} - \frac{\dot{u}_t}{u_t} \right] f_t (x)
$$

By Large Numbers the unemployment density satisfies, for $x \in [\bar{x}_t, 1]$,

$$
\dot{u}_t (x) = \delta e_t (x) - u_t (x) p (\theta_t) [1 - G (h_t (x))] + \dot{h}_t (x) e_t (x, h_t (x)) \mathbb{1}_{h_t (x) > 0}
$$

with the obvious constraint $u_t (x) = \psi (x) - e_t (x)$ for $x \in [\bar{x}_t, 1]$. If the schedule $h_t (x)$ is rising, we have to add the last term quantifying the endogenous destruction of jobs that employ workers $x \geq \bar{x}_t$.

The mass of unemployed workers $u_t = \int_{\bar{x}_t}^{1} u_t (x) \, dx$ can be expressed also as

$$
u_t = \int_{\bar{x}_t}^{1} [\psi (x) - e_t (x)] \, dx \quad (1.25)$$

The effect of changes in $\bar{x}_t$ on $u_t$ is generally asymmetric. If $\bar{x}_t$ rises, at the margin a density $u_t (\bar{x}_t^+) = u_t f_t (\bar{x}_t^+) = \psi (\bar{x}_t^+) - e_t (\bar{x}_t^+)$ of unemployed workers quit searching and thus leave the labor force; if $\bar{x}_t$ falls, a measure $\psi (\bar{x}_t^-) - e_t (\bar{x}_t^-)$ of new workers enter the labor force. In general, the density $\psi (x) - e_t (x)$ is discontinuous at $\bar{x}_t$. More precisely, since by assumption the indifferent marginal worker searches, this density is right-continuous: $\psi (\bar{x}_t^+) - e_t (\bar{x}_t^+) = \psi (\bar{x}_t) - e_t (\bar{x}_t) = u_t f_t (\bar{x}_t) \neq \psi (\bar{x}_t^-) - e_t (\bar{x}_t^-)$. For ease of notation I deal with this fundamental discontinuity by defining $\Lambda (\bar{x}_t, \hat{x}_t)$ as in the claim. Equation (1.25) helps writing the differential equation that $u_t$ obeys by Large
Numbers:
\[ u_t = - \int_{\bar{x}_t}^{1} \dot{\epsilon}_t (x) \, dx - \hat{\theta}_t \Lambda (\bar{x}_t, \hat{x}_t) \]
the first term being the change in unemployment due to creation and destruction (both endogenous and exogenous) of jobs involving workers in the searching pool \([\bar{x}_t, 1]\); the second term indicating the change in the pool of searchers. Substituting from above and integrating

\[ u_t = \delta (\Psi [\bar{x}_t, 1] - u_t) - p (\theta_t) u_t \int_{\bar{x}_t}^{1} [1 - G (h_t (x))] \, f_t (x) \, dx + \]
\[ + \int_{\bar{x}_t}^{1} h_t (x) \epsilon_t (x, h_t (x)) \mathbb{I}_{h_t (x) > \hat{x}} \, dx - \hat{\theta}_t \Lambda (\bar{x}_t, \hat{x}_t) \]

Substituting and rearranging, the normalized density satisfies the differential equation in the claim. ■

Blanchard and Diamond (1990a) propose a convenient shorthand to indicate these four types of job flows. Let E=“Employment”, U=“Unemployment”, and N=“Not in the labor force”.

1. The first term, the “exogenous EU” flow, represents the contribution of exogenous destruction to the availability of workers; its sign depends whether the density of unemployed \(f_t (x)\) is above or below the density of the same type, \(\psi (x)\), normalized by the total mass \(\Psi ([\bar{x}_t, 1])\).

2. The second term, the UE flow, represents the contribution of the endogenous creation of jobs to the availability of workers. Since \(G (h_t (x))\) is differentiable and strictly decreasing in \(x\), and \(f_t (\cdot)\) is a density function with full support on \([\bar{x}_t, 1]\), clearly there exists a type \(\hat{x}_t \in (\bar{x}_t, 1)\) such that \(G (h_t (\hat{x}_t)) = \int_{\bar{x}_t}^{1} G (h_t (\xi)) \, f_t (\xi) \, d\xi\) and \(G (h_t (x)) > \int_{\bar{x}_t}^{1} G (h_t (\xi)) \, f_t (\xi) \, d\xi\) if and only if \(x < \hat{x}_t\). Hence this UE flow always tends to increase the relative abundance of low types in the unemployment pool.

3. The third term represents the effect of changes in labor participation on total un-
employment. It is a EN or NU flow whenever \( z_t \) is (resp.) increasing or decreasing, and has always the sign of \( \dot{z}_t \).\(^{17}\)

4. The fourth term, the "endogenous" EU, represents the effect of the endogenous destruction of jobs or quits, which occurs only if the scrapping margin \( h(\cdot) \) is rising.

Rational Expectations Equilibrium

Having described the arbitrage equations, the optimal policies, and the aggregate dynamics, I can define:

**Definition 1** A Rational Expectations Equilibrium (REE) is a ten-tuple of functions

\[
\begin{align*}
\theta : \mathbb{R}_+ & \to \mathbb{R}_+ \\
w : [0,1] \times [-Z, Z] \times \mathbb{R}_+ & \to \mathbb{R}_+ \\
W : [0,1] \times [-Z, Z] \times \mathbb{R}_+ & \to \mathbb{R}_+ \\
h : [0,1] \times \mathbb{R}_+ & \to [-Z, Z] \\
e : [0,1] \times [-Z, Z] \times \mathbb{R}_+ & \to \mathbb{R}_+ \\
x : \mathbb{R}_+ & \to [0,1] \\
J : [0,1] \times [-Z, Z] \times \mathbb{R}_+ & \to \mathbb{R}_+ \\
U : [0,1] \times \mathbb{R}_+ & \to \mathbb{R}_+ \\
u : \mathbb{R}_+ & \to [0,1] \\
f : [0,1] \times \mathbb{R}_+ & \to \mathbb{R}_+
\end{align*}
\]

satisfying the arbitrage Equations (1.3), (1.4), (1.5), and (1.7), the optimal policy Equations (1.10), (1.11), and (1.12), the Nash bargaining solution (1.6), and the aggregate dynamics equations (1.23) and (1.24), subject to the feasibility constraint (1.8) and to the normalization constraint \( \int_{z_t}^{1} f_t(x) dx = 1 \).

Evolution of the Equilibrium Vacancy Rate

Since the free entry condition (1.12) must hold almost everywhere (with respect to the Lebesgue measure of time), I can differentiate it on both sides to get the evolution of the

\(^{17}\)Notice that, since workers have to search to be hired, and they are all classified as unemployed when searching, strictly speaking in this model there is no NE flow. This flow, which appears to be empirically important in the U.S., can be identified with the workers who start searching and get hired in the time interval between two consecutive collections of statistics.
vacancy rate $\theta_t$ over any REE smooth path. After differentiation, using the expressions for $J^e_\xi (x)$ and $f_t (x)$ derived previously and collecting terms yields the following law of motion of the equilibrium vacancy rate:

$$\dot{\theta}_t = \alpha_t [\Theta_{1t} + \Theta_{2t} + p (\theta_t) \Theta_{3t} + \hat{x}_t \Theta_{4t} + \Theta_{5t}]$$

(1.26)

where

$$\alpha_t := -\frac{\sigma^2 (\theta_t) \kappa (\theta_t)}{\kappa' (\theta_t)} > 0$$

$$\Theta_{1t} := -\frac{1-\sigma}{\tau + \delta} \int_{x_t}^1 \dot{h}_t (x) \left[ 1 - G (h_t (x)) \right] f_t (x) \, dx$$

$$\Theta_{2t} := \frac{\delta}{u_t} \left\{ \int_{x_t}^1 G_t (x) \psi (x) \, dx - \Psi ([x_t, 1]) \right\} \int_{x_t}^1 J^e_\xi (x) f_t (x) \, dx$$

$$\Theta_{3t} := \int_{x_t}^1 f_t (x) G (h_t (x)) f_t (x) \, dx - \left[ \int_{x_t}^1 G (h_t (\xi)) f_t (\xi) \, d\xi \right] \cdot \int_{x_t}^1 J^e_\xi (x) f_t (x) \, dx$$

$$\Theta_{4t} := \frac{1}{u_t} \Lambda (x_t, \hat{x}_t) \left[ \int_{x_t}^1 J^e_\xi (x) f_t (x) \, dx - J^e_\xi (x_t) \right]$$

$$\Theta_{5t} := \frac{1}{u_t} \left\{ \int_{x_t}^1 f_t (x) \dot{h}_t (x) e_t (x, h_t (x)) \Omega_{h_t (x) > 0} \, dx - \frac{\kappa}{\kappa' (\theta_t)} \int_{x_t}^1 \dot{h}_t (\xi) e_t (\xi, h_t (\xi)) \Omega_{h_t (\xi) > 0} \, d\xi \right\}$$

(1.27)

This seemingly messy expression for $\dot{\theta}_t$ isolates the five components driving the vacancy rate and thus, eventually, job creation. I have commented on each component in the text. I can sign two of these five terms (namely the third and fourth) unambiguously, i.e. independently of the particular path the system is undertaking. The sign of the other three terms depends on the particular path the system is on. $\Theta_{3t}$ quantifies the direct effect of endogenous job creation on vacancy profitability through $f_t (\cdot)$. I can prove by First Order Stochastic Dominance arguments (proof not shown) that it is always strictly negative, because high types always go faster out of unemployment. $\Theta_{4t}$ represents the total effect of the change in labor participation, through both the thick market effect and the induced change in the measure of unemployed. From Proposition 1 we already know that $\Theta_{4t} = \int_{x_t}^1 J^e_\xi (x) f_t (x) \, dx - J^e_\xi (x_t) > 0$ (unless $x_t = 1$, a trivial case), due to the quality search externality.

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Comovements along Monotone Paths and the Expansion

I characterize the dynamics of the search policies $h_t(\cdot)$ and $z_t$ conditional on a given path of $\theta_t$, under the assumption that the steady state is an attractor for the overall system and that any REE converges to it monotonically. The main results are that the hiring-tenure schedule $h_t(\cdot)$ comoves with $\theta_t$ for all searching workers, while the labor participation cutoff $z_t$ moves in the opposite direction of $\theta_t$.

Assumption (Monotone Convergence). The Rational Expectations Equilibrium converges monotonically to a steady state.

Formally, if $\lambda_t$ denotes any of the components of the vector field and $\lambda$ denotes one steady state value, the assumption is equivalent to $\dot{\lambda}_t (\lambda_t - \lambda) < 0$. This assumption lets me characterize the comovements of all variables conditional on a (monotone) path of the “reference” state variable $\theta_t$.

Lemma 6 (Dynamics of the Value of Filled Jobs) Under the Assumption of Monotone Convergence, along any REE path, the vacancy rate and the asset value of each filled job move in opposite directions:

$$\dot{J}_t(x, z) \cdot \dot{\theta}_t < 0$$

Proof. First, consider again the arbitrage Equation (1.7). Imposing stationarity in it to obtain an expression for $J(x, z)$ and substituting it back, one can rewrite the arbitrage equation as follows:

$$\dot{J}_t(x, z) = (r + \delta) [J_t(x, z) - J(x, z)] + \beta p(\theta_t) \int_{h_t(x)}^{Z} J_t(x, s) dG(s) - \beta p(\theta) \int_{h(x)}^{Z} J(x, s) dG(s)$$

(1.28)

By assumption,

$$J_t(x, z) - J(x, z) < 0 \iff \dot{J}_t(x, z) > 0$$

(1.29)
Also \( J_t(x, z) < J(x, z) \Rightarrow J_t(x, h_t(x)) = 0 < J(x, h_t(x)) \Rightarrow h_t(x) > h(x) \), and therefore

\[
\int_{h_t(x)}^{z} J_t(x, s) \, dG(s) < \int_{h(x)}^{z} J_t(x, s) \, dG(s) < \int_{h(x)}^{z} J(x, s) \, dG(s) \quad (1.30)
\]

From (1.29) and (1.28) it follows also that

\[
J_t(x, z) - J(x, z) < 0 \quad \text{and} \quad J_t(x, z) > 0 \quad \Rightarrow \quad p(\theta_t) \int_{h_t(x)}^{z} J_t(x, s) \, dG(s) > p(\theta) \int_{h(x)}^{z} J(x, s) \, dG(s)
\]

This last inequality, together with (1.30), finally implies \( p(\theta_t) > p(\theta) \), and therefore \( \theta_t > \theta \). By a similar argument,

\[
J_t(x, z) - J(x, z) > 0 \Rightarrow \frac{p(\theta_t)}{p(\theta)} < \frac{\int_{h_t(x)}^{z} J(x, s) \, dG(s)}{\int_{h(x)}^{z} J_t(x, s) \, dG(s)} < 1 \Rightarrow \theta_t < \theta
\]

Since the same argument is valid for any type \( x \), and \( \theta_t - \theta \) has the same sign whatever \( x \), this also proves the opposite direction of implication and thus the claim. ■

This Lemma and (1.10) immediately imply the following Corollary.

**Corollary 1 (Dynamics of the Hiring-Tenure Policy)** Under the Assumption of Monotone Convergence, along any REE path and for any \( x \)

\[
\dot{h}_t(x) \cdot \dot{\theta}_t > 0
\]

**Lemma 7 (Dynamics of Labor Participation)** Under the Assumption of Monotone Convergence, along any REE path the vacancy rate and the job search type-cutoff move in opposite directions:

\[
\dot{x}_t \cdot \dot{\theta}_t < 0
\]
Proof. The condition determining the marginal searchers \( z_t \) and \( z \) can be written also as
\[
\beta p (\theta) \int_{h(z_t)}^Z J_t (z_t, s) \, dG(s) = c (1 - \beta) = \beta p (\theta) \int_{h(z)}^Z J (z, s) \, dG(s)
\]
(1.31)

Applying Proposition 6 to \( z_t \) yields \( \theta_t - \theta < 0 \iff J_t (z_t, z) - J (z_t, z) > 0 \), what implies
\[
p (\theta_t) \int_{h(z_t)}^Z J_t (z_t, s) \, dG(s) < p (\theta) \int_{h(z)}^Z J (z_t, s) \, dG(s)
\]
(1.32)

(1.31) and (1.32) together imply
\[
J^e (z) = \int_{h(z)}^Z J (z, s) \, dG(s) < \int_{h(z_t)}^Z J (z_t, s) \, dG(s) = J^e (z_t)
\]
and therefore \( z_t > z \). By a similar argument \( \theta_t - \theta > 0 \) implies \( z_t < z \). \( \blacksquare \)

Building on these results, I can characterize the transition from one steady state equilibrium to the new one after a positive aggregate shock \( d\eta > 0 \), under the assumption of monotonicity. Fig. 5.1 depicts the monotone paths. Given the assumption of monotone convergence and the results of Lemmata 6 and 7, all I need to prove is that the path for \( \theta_t \) is increasing just after the shock, that is at time \( 0^+ \) (and thus, by monotonicity, along the whole adjustment path). This implies that \( \theta_t \) does not overshoot nor it attains at once its new s.s. value \( \theta_\infty = \theta \).

Lemma 8 The vacancy rate is increasing after a \( d\eta > 0 \) occurring at time \( 0^+ \): \( \dot{\theta}_{0^+} > 0 \)

Proof. By contradiction, suppose not. Then, by monotonicity, necessarily \( \theta_{0^+} \geq \theta_\infty = \theta \). Also, by the Corollary, \( \dot{h}_t (z) \leq 0 \) for all \( z \in [z_{0^+}, 1] \) and hence \( \Theta_{5,0^+} = 0 \). By Lemmata 6 and 7 we have that \( J^e_{0^+} (\cdot) > 0 \) and \( z_{0^+} > 0 \), so that \( \Theta_{1,0^+} > 0 \) and \( z_{0^+} \cdot \Theta_{4,0^+} > 0 \). Then, by 1.26, \( \dot{\theta}_{0^+} \leq 0 \) requires the sum of the remaining two terms to be negative (and negative enough):
\[
\Theta_{2,0^+} + p (\theta_{0^+}) \Theta_{3,0^+} = \left. \int_{z_{0^+}}^1 J^e_{0^+} (x) \, \dot{f}_{0^+} (x) \right|_{z_{0^+} \text{ const.}} \, dx < 0
\]
where the equality follows because the effect of \( \dot{z}_{0^+} \) on \( \dot{f}_{0^+} (x) \) is already embodied in \( \Theta_{4,0^+} \). The remaining effects driving \( \dot{f}_{0^+} (x) \) are the endogenous hiring and the exogenous
job creation. Since \( h_{0}(x) > h_{0+}(x) > h(x) \) for all \( x \) in the labor force, and also \( x_{0+} < x \), both due to the assumption \( \theta_{0+} > \theta \) and to monotonicity, there are no endogenous job destructions and the only initial change in the cross-section of unemployed workers comes from the additional participation (Fig. 6.1). Then \( u_{0+} = u + \Psi([x_{0+}, x]) \), and

\[
f_{0+}(x) = \begin{cases} 
  \frac{u(x)}{u_{0+}} & \text{for } x \in [x, 1] \\
  \frac{\psi(x)}{u_{0+}} & \text{for } x \in [x_{0+}, x] 
\end{cases}
\]

Then, use the expressions for \( \Theta_{2,0+} \) and \( \Theta_{3,0+} \) to compute

\[
\Theta_{2,0+} + p(\theta_{0+}) \Theta_{3,0+} = \frac{\delta}{u_{0+}} \left\{ \int_{x_{0+}}^{1} J_{0+}(x) \psi(x) \, dx - \Psi([x_{0+}, 1]) \right\} \int_{0+}^{1} J_{0+}(x) f_{0+}(x) \, dx + p(\theta_{0+}) \cdot \\
\left\{ \int_{x_{0+}}^{1} J_{0+}(x) f_{0+}(x) \, dx \right\} \left[ \int_{x_{0+}}^{1} [1 - G(h_{0+}(\xi))] f_{0+}(\xi) \, d\xi \right] - \int_{x_{0+}}^{1} J_{0+}(x) [1 - G(h_{0+}(x))] f_{0+}(x) \, dx
\]

Subtracting the fourth term from the first and using the expressions for \( f_{0+}(x) \) and for \( u(x) \) yields

\[
\int_{x_{0+}}^{1} J_{0+}(x) \left[ \frac{\delta \psi(x)}{u_{0+}} - p(\theta_{0+}) [1 - G(h_{0+}(x))] f_{0+}(x) \right] \, dx = \\
= \int_{x_{0+}}^{1} J_{0+}(x) \frac{\delta \psi(x)}{u_{0+}} \left[ 1 - \frac{p(\theta_{0+}) [1 - G(h_{0+}(x))]}{\delta + p(\theta)[1 - G(h(x))]} \right] \, dx = \int_{x_{0+}}^{1} J_{0+}(x) \tilde{f}_{0+}(x) \, dx
\]

where \( \tilde{f}_{0+}(\cdot) \) is implicitly defined. Similarly, subtracting the second term from the third yields

\[
\left[ \int_{x_{0+}}^{1} J_{0+}(x) f_{0+}(x) \, dx \right] \left[ \int_{x_{0+}}^{1} p(\theta_{0+}) [1 - G(h_{0+}(\xi))] f_{0+}(\xi) \, d\xi - \frac{\delta \Psi([x_{0+}, 1])}{u_{0+}} \right] \propto \\
\propto \int_{x}^{1} \left\{ p(\theta_{0+}) [1 - G(h_{0+} + (\xi))] f_{0+}(\xi) - \frac{\delta \psi(\xi)}{u_{0+}} \right\} \, d\xi = - \int_{x_{0+}}^{1} \tilde{f}_{0+}(x) \, dx
\]

Notice that the interval of integration can be taken to be equivalently \([x, 1]\) or \([x_{0+}, 1]\), because on \([x_{0+}, x]\) the density of unemployed workers \( u_{0+}(\cdot) \) coincides with \( \psi(\cdot) \). Thus,
summing up

$$\Theta_{2,0^+} + p(\theta_0+) \Theta_{3,0^+} = \int_{\xi_0^+}^1 J_0^\circ_+ (x) \tilde{f}_0^+ (x) \, dx - \left[ \int_{\xi_0^+}^1 J_0^\circ_+ (x) f_0^+ (x) \, dx \right] \left[ \int_{\xi_0^+}^1 \tilde{f}_0^+ (x) \, dx \right]$$

and to prove the claim by contradiction I just need to prove that this is positive. This is equivalent to

$$\int_{\xi_0^+}^1 J_0^\circ_+ (x) \frac{\tilde{f}_0^+ (x)}{\int_{\xi_0^+}^1 \tilde{f}_0^+ (\xi) \, d\xi} \, dx := \int_{\xi_0^+}^1 J_0^\circ_+ (x) \tilde{f}_0^+ (x) \, dx > \int_{\xi_0^+}^1 J_0^\circ_+ (x) f_0^+ (x) \, dx \quad (1.33)$$

Notice that $\tilde{f}_0^+ (x)$, as implicitly defined, is a proper density on $[\xi_0^+, 1]$, with c.d.f. $\tilde{F}_0^+ (\cdot)$. Since $J^\circ (\cdot)$ is always a strictly increasing function in any REE (Proposition 4), then a sufficient condition for (1.33) to hold is that $\tilde{F}_0^+ \succ_{sFSD} F_0^+$ (Strict FSD), i.e. that for all $y < 1$

$$\int_{\xi_0^+}^y \left[ \tilde{f}_0^+ (x) - f_0^+ (x) \right] \, dx < 0 \quad (1.34)$$

with equality at $y = 1$. Let

$$C (x) := p(\theta) \left[ 1 - G (h (x)) \right] - p(\theta_0+) \left[ 1 - G (h_{0^+} (x)) \right] = E (x) - E_0^+ (x)$$

Using the expression for $\tilde{f}_0^+ (x)$ and rearranging, (1.34) is equivalent to

$$E [C (x) \mid x \leq y] = \int_{\xi_0^+}^y C (x) \frac{\tilde{f}_0^+ (x)}{\int_{\xi_0^+}^y \tilde{f}_0^+ (\xi) \, d\xi} \, dx < \int_{\xi_0^+}^1 C (\xi) f_0^+ (\xi) \, d\xi = E [C (x)]$$

for all $y < 1$, where the expectations are taken with respect to the measure $dF_0^+ (\cdot)$. Thus, a sufficient condition is that the function $C (\cdot)$ is strictly increasing. By assumption, $\theta_0^+ > \theta$, so that wages get decompressed and the slope of the exit rate density $E (\cdot)$ must decrease in order to converge monotonically to the even lower new s.s. slope (see Lemma 4 and its corollaries). This gives $C' (x) = E' (x) - E_0' (x) > 0$ and the desired contradiction. □
Remaining Proofs

Proof of Proposition 1 (Walrasian Equilibrium) To see that \((w^*(\cdot), x^*)\) is indeed a competitive equilibrium, notice that, given \(w^*(\cdot)\), each atomistic firm maximizes profits by offering this same wage schedule and makes zero profits in equilibrium; all workers with \(w^*(x) \geq b\), or \(x \geq w^*^{-1}(b) = x^*\) maximize by accepting it.

As of uniqueness, consider any candidate equilibrium wage \(w(x, z)\). Clearly \(w(x, z) < w^*(x)\) because no wage schedule weakly above \(w^*(x)\) is feasible. The flow profits from offering a wage \(w(x, z) < w^*(x)\) are \(\Pi = \eta (x + z) - w(x, z)\). Since the firm cannot make negative profits, as it is wealth constrained, and this is common knowledge, the expected total profits (scaled by \(\delta\) to take into account match dissolution) are

\[
E[\Pi] = E[\Pi | \Pi \geq 0] \Pr(\Pi \geq 0) + 0 \cdot \Pr(\Pi < 0) = \\
= E[\Pi | \eta z \geq w(x, z) - \eta x] \Pr(\eta z \geq w(x, z) - \eta x) = \\
= (\eta x - w(x, z))(1 - G\left(\frac{w(x, z)}{\eta} - x\right)) + \eta \int_{\frac{w(x, z)}{\eta} - x}^{\eta z} s dG(s)
\]

Let \(\frac{w(x, z)}{\eta} - x = k(x)\). To be an equilibrium, these profits must be zero by free entry

\[
E[\Pi] = 0 \Rightarrow w(x, z) = \eta x + \eta \int_{k(x)}^{Z} \frac{s dG(s)}{1 - G(k(x))} \Rightarrow k(x) = \int_{k(x)}^{Z} \frac{s dG(s)}{1 - G(k(x))}
\]

The only way the last equation can be satisfied is clearly \(k(x) = Z\), implying \(w(x, z) - \eta x = \eta Z\) or \(w(x, z) = w^*(x)\), a contradiction. ■

Proof of Proposition 2 (Hiring-Tenure Reservation Policy) Consider the arbitrage equation in the form (1.7):

\[
\dot{J}_t(x, z) = (r + \delta) J_t(x, z) - (1 - \beta) (\eta (x + z) - b + c) + \beta p(\theta_t) J^*_t(x) \quad (1.35)
\]

As usual, I just need to prove that \(J_t(x, z)\), solving (1.35) subject to (1.8), is strictly increasing in \(z\) for any \(x\), given that the alternative is the value of search, both for the
firm \((V_t = 0)\) and for the worker \((U_t(x) > 0)\) independent of \(z\). Integration of the partial differential equation (1.35) with respect to time between 0 and \(t\) by variation of the arbitrary constant yields:

\[
J_t(x, z) = \xi(x, z) e^{(r+\delta)t} + (1 - \beta) \frac{1 - e^{(r+\delta)t}}{r + \delta} (\eta(x + z) - b + c) + \beta \int_0^t p(\theta_\tau) e^{-(r+\delta)(t-\tau)} J_\tau(x) \, d\tau
\]

where \(\xi(x, z)\) is a constant of integration, function of \((x, z)\) but not of time. Using the feasibility condition \(\lim_{t \to \infty} e^{-(r+\delta)t} J_t(x, z) = 0\) discussed in the text and computing limits gives

\[
\xi(x, z) - \frac{1 - \beta}{r + \delta} (\eta(x + z) - b + c) + \beta \int_0^\infty p(\theta_\tau) e^{-(r+\delta)\tau} J_\tau(x) \, d\tau = 0
\]

Hence I can solve for the constant of integration \(\xi(x, z)\) and rewrite the solution to the firms' arbitrage equation in the following implicit forward-looking form:

\[
J_t(x, z) = \frac{1 - \beta}{r + \delta} (\eta(x + z) - b + c) - \beta \int_0^\infty p(\theta_\tau) e^{-(r+\delta)(t-\tau)} J_\tau(x) \, d\tau \tag{1.36}
\]

Since \(J_t(x, z)\) is strictly increasing in \(z\), an optimal reservation rule exists.

If the reader is worrying about corner solution of the type \(h_t(x) = \pm Z\), notice that \(h_t(x) < Z\) for any type to be willing to search and work, while \(h_t(x) = -Z\) can be avoided by setting parameters so as to stay away from an annoying kink in the hiring schedule.

This proposition has several handy implications. It lets me rewrite the firm's expected ex post surplus as \(J_t^e(x) = \int_{h_t(x)}^Z J_t(x, \zeta) \, dG(\zeta)\), and the forward-looking solution as

\[
J_t(x, z) = \frac{1 - \beta}{r + \delta} (\eta(x + z) - b + c) - \beta \int_0^\infty p(\theta_\tau) e^{-(r+\delta)(t-\tau)} \int_{h_\tau(x)}^Z J_\tau(x, \zeta) \, dG(\zeta) \, d\tau \tag{1.37}
\]

Next, \(J_t(x, z)\) is clearly \(C^\infty\) in \(z\). Finally, the capital gain \(\dot{J}_t(x, z)\) is independent of the current \(z\)

\[
\frac{\partial \dot{J}_t(x, z)}{\partial z} = \frac{\partial^2 J_t(x, z)}{\partial t \partial z} = \frac{\partial}{\partial t} \frac{1 - \beta}{r + \delta} \eta = 0 \tag{1.38}
\]

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what turns out to be useful to prove Proposition 3. First, I need:

**Lemma C.1. (Smoothness of the Value Function)** $J_t(\cdot, z)$ is $C^\infty$ in its first argument for all $t$ and on the whole set of acceptable matches $(x, z) \in A_t$.

**Proof.** By definition, $J_t(x, z) = \max \langle J_t(x, z), V_t \rangle = \max \langle J_t(x, z), 0 \rangle$. For $z < h_t(x)$, $J_t(x, z) = 0$ and the claim is true. For $z = h_t(x)$, $J_t(x, h_t(x)) \equiv 0$ identically across $x$'s, and hence again the claim follows. For $z > h_t(x)$, $J_t(x, z)$ is equal to the forward-looking solution (1.36), which makes clear that the claim is true if and only if the discounted expected surplus $p(\theta, \tau) e^{-r \delta(\tau-t)} J^*_\tau(\cdot)$ is $C^\infty$ for all $\tau > t$. By $J_t(x, h_t(x)) \equiv 0$ and the Implicit Function Theorem, the claim is equivalent to $h_t(\cdot)$ being $C^\infty$. Hence, differentiating $J^*_\tau(\cdot)$

$$J'^*_\tau(x) = -J_t(x, h_t(x)) h'_t(x) + \int_{h_t(x)}^z \frac{\partial}{\partial x} J_t(x, z) dG(z) = \int_{h_t(x)}^z \frac{\partial}{\partial x} J_t(x, z) dG(z)$$

and so forth for higher order derivatives of $J^*_\tau(\cdot)$. Therefore, $J^*_\tau(\cdot)$ is $C^\infty$ if and only if $J_t(\cdot, z)$ has the same property $\forall z \in [h_t(\cdot), Z]$, because the existence of the higher order derivatives of $h_t(\cdot)$ is equivalent to the existence of those of $J_t(\cdot, z)$.

Summing up, for $z > h_t(x)$, I have shown that $J_t(\cdot, z)$ is $C^\infty$ in $x$ if and only if $e^{-r \delta(\tau-t)} p(\theta, \tau) J_t(\cdot, z)$ is such for all $\tau > t$. Now I can prove the claim by contradiction, exploiting the recursive structure of the Bellman equation (1.36). Suppose that there exists a $(x', \tau)$, $\tau > t$, such that $J_t(\cdot, z)$ is not $C^\infty$ in $x$ at $x = x'$. From the above reasoning, this is equivalent to the existence of at least one $s$, $s > \tau > t$, such that $e^{-r \delta(s-t)} p(\theta, \tau) J_t(\cdot, z)$ (i.e. $J_t(\cdot, z)$) is not $C^\infty$ in $x$ at $x = x'$. Iterating the reasoning, this is also equivalent to the existence of a strictly increasing and unbounded sequence of times $\{r_n\}_{n=0}^\infty$, with $r_0 > s$, such that $e^{-r \delta(r_n-s)} p(\theta, r_n) J_{r_n}(\cdot, z)$ is not $C^\infty$ in $x$ at $x = x'$ for all $n$. Since the sequence is unbounded, this is equivalent to $J_\infty := \lim_{\tau \to \infty} p(\theta, \tau) e^{-r \delta(\tau-t)} J_\tau(x, z)$ being not $C^\infty$ in $x$ at $x = x'$. But we know that $J_\infty = 0$, because $p(\theta, \tau)$ is bounded above by feasibility constraints and the rest of the limit is driven by the non-bubble condition; clearly $J_\infty = 0$ is $C^\infty$ in $x$ at $x = x'$, a contradiction. ■
Simply put, the boundary discounted value function $J_\infty = 0$ is $C^\infty$, hence by backward induction $J_t (\cdot, z)$ has the same property.

**Proof of Proposition 3 (Workers’ Search Reservation Rule)** As usual, it is sufficient to prove that the function

$$\int_{h_t(x)}^{Z} [W_t (x, z) - U_t (x)] dG (z) = \frac{\beta}{1 - \beta} J_t^* (x)$$

is strictly increasing in $x$. The function $h_t (\cdot)$ is differentiable because $J_t (x, z)$ is (in $x$). Differentiate to get

$$J_t'' (x) = -h_t' (x) J_t (x, h_t (x)) g (h_t (x)) + \int_{h_t(x)}^{Z} \frac{\partial J_t (x, z)}{\partial x} dG (z) = \int_{h_t(x)}^{Z} \frac{\partial J_t (x, z)}{\partial x} dG (z)$$

where the second equality follows from $J_t (x, h_t (x)) = 0$. Therefore the claim is equivalent to the rightmost term in (1.39) being strictly positive. From here, the proof goes in two steps.

First, from (1.10) it follows that $\frac{\partial J_t (x, z)}{\partial x}$ is independent of $z$, so that

$$\int_{h_t(x)}^{Z} \frac{\partial J_t (x, \zeta)}{\partial x} dG (\zeta) = [1 - G (h_t (x))] \frac{\partial J_t (x, z)}{\partial x}$$

Therefore from (1.39) and (1.40) the claim is equivalent to

$$DJ_t (x) := \frac{\partial J_t (x, \cdot)}{\partial x} > 0 \quad \forall x \in [0, 1]$$

Consider again the arbitrage equation and use (1.40) to get a differential equation for $DJ_t (x)$:

$$\dot{J}_t (x) = \{r + \delta + \beta \theta_t [1 - G (h_t (x))]\} DJ_t (x) - (1 - \beta) \eta$$

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The second step in the proof consists in using (1.42) to prove (1.41), and thus the claim, by contradiction. Contrary to the claim, suppose that \( DJ_t(x) \leq 0 \) for some type \( x \) and time \( t \). From (1.42) it follows immediately that \( \dot{D}J_s(x) < 0 \) for all \( s \geq t \), because
\[
p(\theta_s) [1 - G(h_s(x))] \geq 0, \rho + \delta > 0, \text{ and } (1 - \beta) \eta > 0.
\]
Hence
\[
\exists (t, x) \ni DJ_t(x) \leq 0 \Rightarrow DJ_s(x) < 0 \quad \forall s > t
\]
(1.43)

Differentiate both sides of the forward-looking solution (1.36) with respect to \( x \) to get:
\[
DJ_t(x) = \frac{1 - \beta}{r + \delta} \eta - \beta \int_t^\infty p(\theta_s) [1 - G(h_s(x))] e^{-(r+\delta)(s-t)} DJ_s(x) \, ds
\]
Using (1.43), I have the following implication
\[
\exists (t, x) \ni DJ_t(x) \leq 0 \Rightarrow \int_t^\infty p(\theta_s) [1 - G(h_s(x))] e^{-(r+\delta)(s-t)} DJ_s(x) \, ds < 0
\]
and then the contradiction
\[
0 \geq DJ_t(x) = \frac{1 - \beta}{r + \delta} \eta - \beta \int_t^\infty p(\theta_s) [1 - G(h_s(x))] e^{-(r+\delta)(s-t)} DJ_s(x) \, ds > 0
\]
\[
\blacksquare
\]

**Proof of Proposition 4 (Compression of the Equilibrium Wage Schedule across Types)** For the first two claims, following the same steps as in Proposition 3 I can prove that
\[
DDJ_t(x) := \frac{\partial^2 J_t(x, z)}{\partial x^2} = \frac{dDJ_t(x)}{dx} < 0
\]
Differentiate the arbitrage equation twice with respect to \( x \) on both sides and use (1.42) to obtain
\[
\dot{DD}J_t(x) = \{r + \delta + \beta p(\theta_t) [1 - G(h_t(x))]\} DDJ_t(x) - \beta p(\theta_t) g(h_t(x)) h_t'(x) DJ_t(x)
\]
Consider the forward-looking solution (1.36) and differentiate it twice on both sides to get

$$DDJ_t(x, z) = -\beta \int_{t}^{\infty} p(\theta \tau) e^{-(r+\delta)(\tau-t)} J_t^{\nu}(x) \, d\tau$$

We know from Proposition 3 that $DJ_t(x) > 0$ and $h_t'(x) < 0, \forall (x,t)$, so that the last term on the r.h.s. is always strictly positive. Also, I used $J_t^{\nu}(x) = [1 - G(h_t(x))] DDJ_t(x) - g(h_t(x)) h_t'(x) DJ_t(x)$.

Now, contrary to the claim suppose that there exists a $(x,t)$ such that $DDJ_t(x) \geq 0$. It follows

$$DDJ_t(x) \geq 0 \Rightarrow DDJ_t(x) > 0 \Rightarrow DDJ_t(x) > DDJ_t(x) \geq 0 \ \forall s > t \Rightarrow$$

$$\Rightarrow J_t^{\nu}(x) > 0 \ \forall s > t \Rightarrow DDJ_t(x,z) < 0$$

a contradiction. From (1.10) it follows easily $h_t''(x) = -\frac{r+\delta}{(1-\beta)\eta} \frac{\partial^2 J_t(x,z)}{\partial x^2} > 0$.

The third claim follows immediately from (1.6) and Proposition 3. The fourth claim follows from the previous claim and from differentiating the wage schedule (1.6) with respect to $x$. The last claim is just a corollary of Proposition 3:

$$\frac{\partial}{\partial x} \left[ \frac{\eta(x+z)}{r+\delta} - \int_{t}^{\infty} p(\theta \tau) e^{-(r+\delta)(\tau-t)} w(x,z) \, d\tau \right] > 0 \iff DJ_t(x) > 0.$$

\textbf{Proof of Lemma 2} The claim is in only doubt because an increase in the vacancy rate $\theta$, while raising the probability rate of meeting open vacancies, puts also pressure on wages and raises the implementation cutoff $h(x)$ and hence the probability of rejecting a match.

Suppose that the claim is false for some $x \in [z,1]$. Then

$$\frac{dE(x)}{d\theta} \leq 0 \iff p'(\theta) [1 - G(h(x))] - p(\theta) g(h(x)) \frac{dh(x)}{d\theta} \leq 0$$
Substituting from (1.15) and dividing by \( p'(\theta) > 0 \), the last inequality is equivalent to

\[
\frac{1 - G(h(x))}{g(h(x))} \leq \frac{\beta p(\theta) \int_{h(x)}^{Z} (s - h(x)) dG(s)}{r + \delta + \beta p(\theta) [1 - G(h(x))]} 
\]

(1.44)

The exit rate from unemployment of type \( x \) can decrease in \( \theta \), contrary to the claim, only if the increase in \( h(x) \) is large enough. Hence, for each \( x \), \( E(x) \) and \( h(x) \) should move in opposite directions: \( dh(x)/dE(x) \leq 0 \). To compute this derivative, let us rewrite (1.14) as follows:

\[
h(x) = \frac{(r + \delta)(b - c - \eta x) + \eta \beta E(x) \int_{h(x)}^{Z} \frac{dG(z)}{1 - G(h(x))}}{\eta \{r + \delta + \beta E(x)\}}
\]

and use implicit differentiation to get

\[
dh(x)\over{dE(x)} = \frac{\int_{h(x)}^{Z} \frac{\beta(z-h(x))}{r+\delta+\beta E(x)} \frac{dG(z)}{1-G(h(x))}}{1 - \beta \frac{E(x)}{r+\delta+\beta E(x)} \frac{g(h(x))}{1-G(h(x))} \int_{h(x)}^{Z} (s - h(x)) \frac{dG(s)}{1-G(h(x))}} > 0
\]

Hence:

\[
\frac{dE(x)}{d\theta} \leq 0 \Rightarrow \frac{dh(x)}{dE(x)} \leq 0 \Rightarrow \frac{1 - G(h(x))}{g(h(x))} \geq \frac{\beta p(\theta) \int_{h(x)}^{Z} (s - h(x)) dG(s)}{r + \delta + \beta p(\theta) [1 - G(h(x))]} 
\]

Since, generically in parameters, the last inequality can hold only strictly, this contradicts (1.44).

\[\blacksquare\]

**Proof of Lemma 3 (Stationary Distribution of Unemployed Workers)**: Workers in the labor force \((x \geq \underline{x})\) but unemployed are a mass equal to

\[
u = \frac{\delta \Psi([\underline{x}, 1])}{\delta + p(\theta) \int_{\underline{x}}^{1} [1 - G(h(x))] f(x) dx}
\]

(1.45)

In steady state, by the LLN applied to each type of worker, the (normalized by \( u \)) cross-section density of unemployed workers \( f(\cdot) \) satisfies:

\[
f(x) = \frac{\psi(x)}{\Psi([\underline{x}, 1]) + \frac{\psi}{\delta} p(\theta) \int_{\underline{x}}^{1} [G(h(\xi)) - G(h(x))] f(\xi) d\xi}
\]

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\[
\frac{\psi(x)}{\Psi([\xi, 1])} \left[ 1 + \frac{p(\theta) \int_{\xi}^{1} [G(h(\xi)) - G(h(x))] f(\xi) d\xi}{\delta + p(\theta) \int_{\xi}^{1} [1 - G(h(\xi))] f(\xi) d\xi} \right]^{-1} = \\
= \frac{\psi(x)}{\Psi([\xi, 1])} \frac{\delta + p(\theta) \int_{\xi}^{1} [1 - G(h(\xi))] f(\xi) d\xi}{\delta + p(\theta) [1 - G(h(x))]} \\
\tag{1.46}
\]

where the second equality follows from substituting for \( u \) from (1.45), the third from using \( F(1) - F(\xi) = 1 - 0 = 1 \). Thus, the fraction of type \( x \) unemployed is higher than the fraction of existing type \( x \) workers if and only if their exit rate from unemployment is lower than the average one in the labor force.

In equation (1.46), however, \( f(\cdot) \) appears on both sides. I can get rid of it by using the property that \( f(\cdot) \) integrates up to one between \( \xi \) and 1. Integrate both sides of (1.46) between \( \xi \) and 1 to get

\[
\int_{\xi}^{1} f(x) dx = 1 = \frac{\delta + p(\theta) \int_{\xi}^{1} [1 - G(h(s))] f(s) ds}{\Psi([\xi, 1])} \int_{\xi}^{1} \frac{\psi(x) dx}{\delta + p(\theta) [1 - G(h(x))]} \\
= \frac{\delta + p(\theta) \int_{\xi}^{1} [1 - G(h(s))] f(s) ds}{\Psi([\xi, 1])} = \left( \int_{\xi}^{1} \frac{\psi(x) dx}{\delta + p(\theta) [1 - G(h(x))]} \right)^{-1} \\
\tag{1.47}
\]

and substituting it back into (1.46) gives the claim. \( \blacksquare \)
Wage compression across workers' types: \( \eta'(x) > w'(x) \)

In a frictional economy with decentralized wage bargaining, the wage is a weighted average of productivity and of the alternative value of working time:

\[
\text{w}(x) = m \, \eta(x) + (1-m) \, a(x)
\]

The weight on productivity \( (m) \) depends on effective allocative frictions, scaled by the state of labor demand. The lower labor demand, the more frictions bite, the lower the weight \( m \), the less steep the wage schedule and the lower the distance between the productivity and the wage schedules, measuring wage compression across types.
$z = \text{match-specific productivity outcome, distributed on } [-Z, Z] \text{ i.i.d. with c.d.f. } G(.)$

$h(x) = \text{type-dependent match-implementation cutoff: accept the match with type } x \text{ iff } z \geq h(x)$

$AB = Z - h(x') = \text{set of acceptable match outcomes for a match with type } x' \text{ at time } t$

$1 - G(h(x)) = \text{probability of implementing a match with type } x' \text{ at time } t$
out of the labor force $[0,x)$

in the labor force $[x,1]$

Fig. 3.1

$\psi(x) =$ cross-section distribution of workers' productivities (types)

$u(x) =$ equilibrium cross-section distribution of unemployed workers
The hiring-tenure schedule $h(.)$ rises for all types, more so for high types ($h'(.)$ increases)
The match-acceptance sets and probabilities shrink more for high types.
The exit rates from unemployment $p\theta (1-G(h(.)))$ increase because the vacancy rate $\theta$ rises, but they increase more for low types because of the change in the acceptance margin $h(.)$. 

Fig. 4.1

Effect of a permanent increase in $\eta$ (a "boom")
Fig. 5.1

Monotone transitional dynamics
Fig. 6.1

Short-run effect of a "boom" on the cross-section of unemployed workers

$\psi(x) = \text{cross-section distribution of workers' productivities (types)}$

$u(x) = \text{equilibrium cross-section distribution of unemployed workers}$
Chapter 2

Comparative Advantages and Systematic Job Search

2.1 Introduction

Over the 1968-1988 period, in the U.S. economy, gross inter-industry labor mobility was sharply declining and slightly procyclical; net mobility appeared only mildly decreasing and highly procyclical. At the same time, unemployment spells, and with them the unemployment rate, rose steadily. This chapter proposes an explanation of these trends and cycles that combines the Roy’s (1951) self-selection theory of labor allocation with search frictions in labor markets.

In a two-sector economy where workers are endowed with heterogeneous sector-specific skills, individuals with “weak comparative advantages” search randomly to maximize the chance of finding a job, while “specialized” workers always search in the same sector. Gross mobility is produced by the former, random job searchers, and I impute its decline to the increasing specialization of the labor force, what is also consistent with the average decline in exit rates from unemployment. Instead, net mobility is produced by sector-specific shocks, that have exhibited a very systematic pattern over aggregate fluctuations (the so-called Cyclical Upgrading of labor), but no significant trend. Net mobility is more procyclical than gross mobility because search frictions bite more in recessions, so
that the proportion of non-selective searchers is countercyclical.

The coexistence of multi-dimensional ex ante heterogeneity, in the Roy's tradition, and of mobility costs, in the form of search frictions à la Mortensen-Diamond-Pissarides, is a novelty of this work. Its merit is to preserve the appealing predictions and the powerful graphical apparatus of the Walrasian story of Roy and his followers (Heckman and Sedlacek 1985, Heckman and Honoré 1990), and to improve upon it where it manifestly fails. From this point of view, this is a complementary approach to the theories of labor turnover based on matching (ex post) heterogeneity, in the line of Jovanovic (1979). The model provides a multi-market foundation to the trade-off, first emphasized by Salop (1973), that workers face between soliciting high wage offers and increasing the probability of an offer.

The model is also consistent with many stylized facts concerning the relative employment shares and the real wages of stayers and movers, both at any point in time and over business cycles. One important ingredient is the explicit treatment of labor participation as a separate sector. The empirical importance of this sector is stressed quite strongly by Heckman and Sedlacek (1985) and by Keane, Moffitt and Runkle (1988).

The plan of the Chapter is as follows. In the next section I sketch the empirical picture under investigation, presenting the main stylized facts that any theory of inter-industry labor mobility is required to match. In Sections 2.3 and 2.4 I formalize the trade-off that workers face, when searching for a job, between comparative advantages and cost of waiting. To this purpose, I present and analyze a general equilibrium model of a two-sector production economy, and show how we can partition the labor force into non-participating, stayers in one sector, and non-specialized searchers moving across sectors. Concluding Section 2.4, I also mention the available empirical evidence on job search behavior, that supports this tri-partition. Next, I build on the formal analysis to propose my explanation for the trends (Section 2.5) and cycles (Section 2.6) in labor mobility. Finally, in Section 2.7, I address the observed strong comovement in the real wages of industry stayers; in the context of the same model, I emphasize a synchronizing force arising from mobility costs and heterogeneity, additional to the forces already discussed.
by other authors. The Appendix contains most of the proofs and the complete derivation of the equilibrium of the frictional economy.

2.2 Inter-Industry Labor Mobility: Some Evidence

2.2.1 Quantities

It is well known that the bulk of workers' sectoral movements occur simultaneously in both directions, so that on average net flows are always a small (10-25%) fraction of gross flows of workers across sectors or two-digit industries.\(^1\) The major source of this phenomenon is the widespread ongoing reallocation at the firm level, caused by idiosyncratic shocks that hit firms continuously across the border.\(^2\)

It is also well known that net workers' flows across industries are very systematic over aggregate fluctuations (e.g. Bils and McLaughlin 1992). High wage two-digit industries - such as durables, construction, transportation, utilities - have very volatile employment as a share of total employment; the opposite holds for low wage industries - e.g. retail trade, textiles, services. Workers flow from low wage to high wage industries and mostly from out of the labor force to low wage industries during an expansion (Cyclical Upgrading of labor). Instead, employment and output levels in all industries co-move: unemployment and non participating population fuel the economy with the extra labor input. The intensive margin mimics closely, both in levels and in shares, the extensive margin across industries.

Across workers, employment and exit rates from unemployment of movers are relatively low and highly volatile, when compared to those of industry stayers. Individual mobility across industries definitely falls with age, work experience, education and wage, while the effect of tenure is disputed (see Thomas 1996). Incidence of unemployment - the percentage of the class of workers who experienced some unemployment in a given

\(^1\) Murphy and Topel (1987), Jovanovic and Moffitt (1990).
\(^2\) For the evidence, see the references quoted in footnote 1 in the Introduction to this dissertation.
period - is much (more than three times, according to Murphy and Topel) higher for movers than for stayers.

These are well established regularities. But one episode stands as particularly striking. Murphy and Topel (1987) and Jovanovic and Moffitt (1990) document convincingly that gross and, to a lesser extent, net inter-industry mobility have been falling dramatically in the U.S. from the 1960s to the 1980s. The total reduction in gross mobility is in the order of 50-60%, and the result does not depend on the fineness of industry classification.

This phenomenon has drawn much attention because we observe a simultaneous upsurge in unemployment spells and rate, suggesting a causal link from lower mobility to higher unemployment. As a consequence, scholars have paid attention only to trends, in order to formulate and verify their hypotheses, neglecting the information on the cyclical behavior of these magnitudes.

At business cycle frequencies, the available evidence depicts a richer pattern. Gross mobility was mildly but consistently procyclical; as of net mobility, we have very few datapoints, but the picture definitely suggests an even stronger procyclicality. To see why, the following simple decomposition is useful:

\[ G_t = N_t + S_t \]

\( G_t \) is gross mobility, \( N_t \) is net mobility, so that \( S_t \) is the ("Spurious") mobility of workers across industries that washes out in aggregation. The evidence of Murphy and Topel says that \( G_t \) has been falling more than \( N_t \), in absolute terms, so that \( S_t \) must have been falling; and that \( G_t \) and \( N_t \) are procyclical. The ratio \( N_t/G_t \) has been slightly but insignificantly decreasing, and has moved inversely to unemployment both in the 1971 and 1975-76 slumps and in the following recoveries.

Other indirect evidence strongly supports the conclusion that \( E_t \) was countercyclical. Bowlus (1995) documents from micro data that the match quality, as measured by its following duration, is procyclical: in recessions, workers accept jobs that dissolve quicker, creating more mismatch. The mismatch is higher in high-wage volatile industries, that
contract by more in recessions. In expansions, formed jobs appear more long-lasting, which suggests that workers search more selectively. Fallick (1993) concludes, also from micro data, that workers search more in industries with above-average recent employment growth, but show on average considerable attachment to the last job’s industry. Since the dispersion of industry employment growth rates is countercyclical (Lilien 1982), the relative weight of macroeconomic incentives (exit rates) over individual incentives (comparative advantages) increases in a recession.

One major goal of this paper is to exploit this additional information on cyclical behaviors to discriminate among competing hypotheses and to propose a new explanation of both trends and cycles in labor mobility.

2.2.2 Real Wages

On the price side, a fairly wide array of panels have been analyzed by labor and macro economists in order to resolve two old debates: one on the aggregation biases in the computation of real wage cyclicality, and the other on the relationship between the distribution of skills and the distribution of earnings.³

The analysis of real wage cyclicality based on aggregate data is affected by two composition biases. One is procyclical: workers flow to high wage industry in expansions. The other is countercyclical: workers entering the labor market in expansions have lower than average wages. If we stick to the traditional shock-propagation view, in a Walrasian spot labor market real wages are countercyclical following an aggregate demand shock, that shifts labor supply (Keynes’ argument); they are procyclical after an aggregate supply shock, that shifts labor demand (as in RBC models). But a number of contractual and

³The most popular panels are the CPS, DWS, NLSY, MID and PSID in the U.S., and NLFS in the U.K.. The literature on real wage cyclicality is surveyed in Abraham and Haltiwanger (1995), with reference both to aggregate and panel data studies. The literature on the earnings function in the context of multi-dimensional skills (assignment models) is surveyed by Sattinger (1993), and includes the selection bias of Heckman as one important case. The questions asked in the process of compiling those surveys often involved also non-wage aspects of labor market performance, such as current and past employment status, change of industry and so forth, so that this body of evidence has become a precious source of information also to understand the determinants of workers’ flows.
bargaining aspects of the wage-determination mechanism point to systematically pro-
cyclical real wages. On the basis of the panel data studies, there is now a rather general 
consensus that individual real wages were moderately but consistently procyclical over 
the last two decades both in the U.S. and in many E.E.C. economies.

We can also look separately at industry stayers and movers. The real wages of industry 
stayers comove substantially across industries. A surprising implication is that, unlike 
employment shares, relative industry wages are consistently acyclical (e.g. Murphy and 
Topel 1987, Wood and Solon 1990, Bils and McLaughlin 1992), even between high-
wage/volatile and low-wage/stable industries.

The real wages of industry movers are always lower than both the average wage in the 
industry they leave and the one in the industry they join: inter-industry flows are mostly 
low-to-low movements in terms of wage ranks. Such stayers-movers wage differentials 
survive, though considerably reduced, when controlling for observable heterogeneity (age, 
education; see Bils and McLaughlin 1992). Also industry movers experience a sizable 
wage gain upon joining a high wage industry when its employment share is growing, 
and a loss when it is declining. The opposite happens to movers to low-wage industries. 
These jumps cannot be explained only by compensating differentials or industry effects 
(Krueger and Summers 1988).

The available empirical picture here summarized, though drawn from heterogeneous 
sources, appears very robust. In order to rationalize these findings, in the next two 
sections I lay out and analyze a formal setup, that I propose as my structural form of 
the reduced forms estimated by these authors. The main focus will be on the trends and 
cycles in labor mobility, but the complete empirical picture of quantities and wages will 
always be in the background.

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2.3 A Two-Sector Economy with Ex Ante Heterogeneous Workers

2.3.1 Technology, Factor Endowments, and Preferences

Consider the following artificial economy.

Time is continuous and runs from zero to infinity.

There are four goods, two non reproducible and non depreciating capital goods, and two consumption goods. Any of the four goods can be chosen as the numeraire. Labor is ex ante heterogeneous. A unit measure of workers is distributed across bi-dimensional productivity (skill) types $x \in [0, 1]^2$ according to a given cross-section measure $\Psi(\cdot)$. I assume that the measure has a smooth bi-variate density $\psi(\cdot, \cdot)$ with respect to Lebesgue measure. Capital is heterogeneous. Each firm owns one piece of capital of either type. There exist a continuum of firms of each type, ensuring free entry in both sectors.

Each consumption good is produced with a Leontief technology, that requires a 1:1 match ("job") between a unit of capital and a worker. The flow productivity of any implemented job in sector $i$ is $\eta_i \cdot x_i$, the type of the worker in that sector $x_i$ multiplied by a sectoral parameter $\eta_i$. The latter can be interpreted either as an exogenous process for the price of the produced good or as technological progress. The $\eta_i$'s are public knowledge, while the type $x$ of the worker is known to the worker and is observed by the firm only upon meeting.

Filled jobs in both sectors are assumed to break exogenously at flow (Poisson) rate $\delta > 0$; my preferred interpretation is in terms of job- or firm-idiosyncratic productivity shocks, that force the closure of the production unit. Out of steady state, jobs may dissolve also endogenously.

Workers and firms are infinitely lived, are risk-neutral, have linear preferences over

---

4This functional form is without loss in generality. Following Heckman and Sedlaceck (1985), the most general form defines the type $y$ as a bundle of diverse skills, and maps all of them into the output of each good $i$ by means of a task function $f_i(\cdot)$. Then just let $x_i = f_i(y)$. Any non-linearity can be taken away by rescaling the $f_i(\cdot)$'s.
the two consumption goods and choose actions in order to maximize expected utility discounted at rate \( r > 0 \). When a worker \( x \) is not working, he enjoys a flow benefit equal to \( b \geq 0 \), independent of \( \eta_1 \) and \( x \). While he holds a job in sector \( i \) he receives a flow wage, denoted by \( w_t^*_i (x) = w_{it} (x) \).

This model describes only a labor market. Since there are neither savings nor foreign trade to decouple income and expenditure, the same model can be seen as one of general equilibrium of a production economy. Technology and preferences are described above. The distribution of workers across types is a synthetic description of the endowments of labor. The endowments of capital are with the ex ante homogeneous firms. The equilibrium interest rate is always equal to the discount rate \( r \), by the risk-neutrality of all agents. The equilibrium relative price \( \eta_1 / \eta_2 \) is indeterminate, because of linear preferences, hence its dynamics are assumed to be exogenous. To define an equilibrium, I just need to specify an allocative mechanism.

### 2.3.2 The Walrasian (Roy) Economy and its Equilibrium

The first mechanism I propose is the Walrasian auctioneer. Trade is centralized: all agents meet in the same place and at the same time, face no transaction cost, and bid under perfect information. The auctioneer allocates workers to firms according to the types \( x \), and clears the market. This economy is an intertemporal extension of the model pioneered by Roy (1951) in his self-selection theory of income distribution.

**Proposition 6 (Roy Equilibrium)** There exists a unique intertemporal competitive equilibrium of the Walrasian economy. In this equilibrium:

1. in each sector, the equilibrium wage for worker \( x \) exhausts output: \( w_t^*_i (x) = \eta_i x_i \);
2. a worker chooses to participate and work iff \( \max (\eta_1 x_1, \eta_2 x_2) \geq b \);
3. a participating worker chooses sector 1 iff \( \eta_1 x_1 \geq \eta_2 x_2 \).

The proof is straightforward: firms are the long side of the market, and workers sort according to comparative advantages. Fig. 1.2 describes the equilibrium. The
curves are the iso-density loci of one possible bivariate distribution $\psi$. This framework has been heavily exploited to explain income distribution and inter-industry flows.\textsuperscript{5} It is a very useful benchmark to understand the equilibrium properties of an alternative class of economies, characterized by a different allocative mechanism, that gives rise to decentralized trade.

### 2.3.3 Search Frictions and Decentralized Trade Economies

Let us suppose that trade is decentralized because of informational deficiencies and search costs, that make the meeting process time- and resource-consuming. Then the economy described in Subsection 2.3.1 becomes a general equilibrium search model, with transferable utilities, two sectors and ex ante heterogeneous agents on one side of the market. The details of decentralized trade are as follows.

A firm holding an idle unit of capital may open a vacancy in sector $i$ at flow cost $\kappa > 0$ in numeraire units. An open vacancy in sector $i$ is contacted by workers searching for a job $i$, independently of their types, at flow Poisson rate $q_{it}$.\textsuperscript{6} This rate is assumed a homogeneous of degree zero function of the measures of unemployed-searchers in sector $i$, $u_{it}$, and of open vacancies, $v_{it}$ ("linear" matching technology). Thus, I let the sectoral vacancy rate $\theta_{it} := v_{it}/u_{it}$ and always consider a function $q_{it} = q(\theta_{it})$, differentiable and strictly decreasing, with $q(\infty) \to 0$.

Workers can be either working in a sector, in which case they receive a wage, set by Nash bilateral bargaining giving to the worker a share $\beta$ of the surplus of the match;

\textsuperscript{5}As well known (cf. also Heckman Sedlacek 1985), the main implications of the Roy model pertain to the equilibrium distribution of earnings. Due to self-selection, the latter is always skewed to the right with respect to the latent distribution of skills in each sector, provided that this is log-normal or, more generally, that skills are not too positively correlated. In this case there is positive selection in both sectors, as in Fig. 1: the conditional mean of skills allocated to sector $i$ is higher than the population mean, due to self-selection. This should explain the well-known "fat" (Pareto) right tail in the distribution of income, and also why workers close to indifference, and then first movers, are always relatively low-skilled and low-wage in both sectors.

\textsuperscript{6}This random matching technology does not imply a random matching process. In fact, in a Rational Expectations Equilibrium, firms opening vacancies in sector $i$ anticipate the types of workers searching for those jobs and post corresponding vacancy requirements. Given the nature of the bargaining solution (Nash), a hiring firm has a credible threat not to hire a worker with type differing from the required one, even if he applies.
or searching for a job, in either sector or just randomly, in which case they enjoy un-
employment benefits \( b \geq 0 \) and pay a search cost \( c \geq 0 \); or not searching at all, in
which case they just receive \( b \). There is no on-the-job search. Unemployed workers are
defined, like in official statistics, as jobless workers who are currently searching for a job.
The (endogenous) mass of jobless workers not searching for a job is referred to as the
“non-participating” population.

Workers find open vacancies in sector \( i \) at flow rate \( p_{it} \) if they search selectively in
sector \( i \); they find a vacancy of either type at rate \( \pi_t \) when searching randomly.\(^7\)

2.4 Equilibrium of the Frictional Economy

After describing the economy, I can now characterize the optimal individual policies for
a given path of exit rates from unemployment \( \{p_{1t}, p_{2t}, \pi_t\}_{t \in [0, \infty)} \). In the Appendix I
present the remaining conditions that define a Rational Expectation Equilibrium; free
entry and cross-sectional dynamics pin down the equilibrium path of exit rates, given the
optimal individual policies.

The equilibrium picture that emerges in this section is an extremely simple and useful
tool to interpret the evidence summarized in Section 2.

2.4.1 Wage Schedule and the Value of a Filled Job

Let \( J_{it}(x) \) denote the Bellman value to the firm of a job filled by a worker \( x \) in sector
\( i \) at time \( t \). By free entry, the outside option of the firm (the value of an open vacancy
in sector \( i \), \( V_{it} \)) is zero, hence \( J_{it}(x) \) is also the firm’s surplus from the match. By Nash
bargaining, \( J_{it}(x) \) is \((1 - \beta)/\beta \) times the surplus of the worker, which is equal to the
human wealth when filling the job \( W_{it}(x) \) minus the reservation wage \( U_{t}(x) \).

\(^7\)I discuss later the reasonable assumptions that can be made about these arrival rates, and in the
Appendix the restrictions that the Law of Large Numbers imposes on these arrival rates, given the
matching function \( q(\cdot) \).
The wage schedule set by Nash bargaining is given by:

\[ w_{it}(x) = (1 - \beta)(b - c) + \beta \eta_i x_i + \beta \Phi_t(x) \]

Let \( \alpha_t \) denote the proportion of type-1 vacancies open in the economy at time \( t \). The term

\[ \Phi_t(x) := \max \left\{ \frac{1 - \beta}{\beta} c, p_{1t} J_{1t}(x), p_{2t} J_{2t}(x), \pi_t \{ \alpha_t J_{1t}(x) + (1 - \alpha_t) J_{2t}(x) \} \right\} \quad (2.1) \]

represents the expected returns from the next search decision of worker \( x \) in the event he were laid off. The worker can either:

1. stop searching and leave the labor force, what saves him the search cost.
2. Search only in sector 1.
3. Search only in sector 2.
4. Search randomly.

The firm's value function satisfies the arbitrage equation

\[ \dot{J}_{it}(x) = (r + \delta) J_{it}(x) - (1 - \beta)(\eta_i x_i - b + c) + \beta \Phi_t(x) \]

subject to the optimal policy \( J_{it}(x) \geq 0 = V_{it} \) which tells when to scrap the match.

The following proposition gives a complete characterization of the value function. The proof is in the Appendix.

**Proposition 7** The value function \( J_{it}(\cdot, \cdot) \) is strictly increasing in \( x_i \) and weakly decreasing in \( x_{-i} \).

This powerful result allows me to characterize all of the optimal policies.
2.4.2 Characterization of Equilibrium Search Behavior

Suppose that the worker, if laid off, would search again (the maximum in (2.1) is not the first term). Then he chooses his search method according to two reservation rules.

**Proposition 8 (Selective vs. Random Job Search)** Let \( R_{1t} := \frac{\pi(1-\alpha_t)}{p(1-\pi_t)\alpha_t} \) and \( R_{2t} := \frac{p_t - \pi_t(1-\alpha_t)}{\pi_t\alpha_t} \). Then, a worker \( x \), when unemployed, searches:

1. in sector 1 if and only if \( \frac{J_{1t}(x)}{J_{3t}(x)} > R_{1t} \), which is equivalent to

\[
x_1 > \xi_{1t}(x_2)
\]

with \( \xi_{1t}(0) > 0 \), \( \xi_{1t}(\cdot) \) increasing;

2. in sector 2 if and only if \( \frac{J_{1t}(x)}{J_{3t}(x)} < R_{2t} \), which is equivalent to

\[
x_1 < \xi_{2t}(x_2)
\]

with \( \xi_{2t}(0) < 0 \), \( \xi_{2t}(\cdot) \) increasing;

3. at random if and only if \( R_{2t} \leq \frac{J_{1t}(x)}{J_{3t}(x)} \leq R_{1t} \) i.e.

\[
\xi_{2t}(x_2) \leq x_1 \leq \xi_{1t}(x_2)
\]

**Proof.** Follows immediately from Proposition 7.

Fig. 2.2 depicts two possible lines delimiting the regions of selective (in either sector) and random searchers. The region of random search could well be empty, and later I explore the conditions under which this is not true. The bottomline is: in any rational expectations equilibrium, if there exist workers who search randomly, they must have weak comparative advantages. The term “weak” is clearly relative to the size of search frictions.
Finally, the decision whether to search at all. From (2.1) it is clear that a worker decides to search and stay in the labor force according to another reservation rule. This rule is also depicted in Fig. 2.2.

**Proposition 9 (Labor Participation)** A worker $x$ searches for a job when unemployed if and only if at least one of the following conditions holds:

1. $x_{i} \geq x_{it}$ for some $i$

2. $x_{1} \geq \xi_{3t}(x_{2})$ with $\xi_{3t}(\cdot)$ strictly decreasing.

**Proof.** A worker searches for a job if and only if $c < \frac{\theta}{1-\beta} \Phi_{t}(x)$. Then the claim follows immediately from Proposition 7. ■

**Equilibria with Random Search**

The next question is: when do people search randomly in equilibrium? A necessary and sufficient condition is that the ratio of surpluses from jobs in the two sectors is in the interval $[R_{2t}, R_{1t}]$. Hence we can ask when is this interval non-empty. Notice that, if $\lambda$ denotes the share of unemployed workers searching in sector 1 (either selectively or randomly), then the economy-wide vacancy rate can be written as $\theta = \theta_{1}\lambda + \theta_{2}(1 - \lambda)$.

The most reasonable restrictions I can impose on the matching functions are:

**Assumption 1. Effectiveness of Non-Selective Search:** the probability of finding an open vacancy per unit time is higher with non-selective search than with selective search.

$$\pi_{t} > \max(p_{1t}; p_{2t}) \forall \lambda \in (0, 1)$$

with equality only if $\lambda = 0, 1$.

**Assumption 2. Precision of Selective Search:** the probability per unit time of finding a vacancy of a specific type is higher with selective search than with non-selective search.

$$\begin{align*}
\alpha \pi_{t} &< p_{1t} \\
(1 - \alpha) \pi_{t} &< p_{2t}
\end{align*}$$

$\forall \lambda, \alpha \in (0, 1)$
These two assumptions are sufficient (but not necessary) to generate some random search in equilibrium.

**Proposition 10 (Non Specialized Job Searchers)** Given Assumptions 1 and 2, there exists always a non empty subset $X_R \subset [0,1]^2$ of types who prefer random search to selective search.

**Proof.** For notational simplicity drop the time index. It is easy to show that Assumptions 1 and 2 imply $R_1 > R_2$. In fact

$$ R_1 > R_2 \iff \pi^2 \alpha (1 - \alpha) > (p_1 - \pi \alpha)[p_2 - \pi (1 - \alpha)] \iff p_1 (\pi - p_2) > \pi \alpha (p_1 - p_2)$$

in the last inequality, $p_1 > \pi \alpha$, and $\pi > p_1$ so that $\pi - p_2 > p_1 - p_2$. ■

Thus, we can distinguish three classes of workers’ flows:

1. **WITHIN-SECTOR FLOWS OF SPECIALIZED WORKERS.** These workers are separated from their jobs by exogenous firm-idiosyncratic shocks (at rate $\delta$), then they search in the same sector and get eventually re-hired there.

2. **ACROSS-SECTORS (GROSS) FLOWS OF NON SPECIALIZED WORKERS.** These workers are separated from their jobs by exogenous firm-idiosyncratic shocks (at rate $\delta$), then they search randomly and the first vacancy they find is in the other sector.

3. **ACROSS-SECTORS NET FLOWS OF WORKERS.** These flows are due to changes in comparative advantages (sectoral shocks, changes in the $\eta_i$’s).

I should add also the within-sector flows of random searchers, who happen to be hired in the same sector or industry by chance. In a multi-industry context, it is clear that the probability of such an event and thus the size of these flows is negligible.
Just as an example, in steady state it is easy to derive explicit expressions for the reservation rules under the simplifying assumption \( b = c = 0 \). They take a linear form:

\[
x_1 > \xi_1 x_2 := \left[ \frac{\eta_2}{\eta_1} \frac{\pi (1 - \alpha) (r + \delta + \beta p_1)}{(r + \delta)(p_1 - \alpha \pi) + \beta \pi p_1 (1 - \alpha)} \right] x_2
\]

\[
x_1 < \xi_2 x_2 := \left[ \frac{\eta_2}{\eta_1} \frac{(p_2 - (1 - \alpha) \pi) (r + \delta) + \beta \pi p_2 \alpha}{\pi \alpha (r + \delta + \beta p_2)} \right] x_2 \tag{2.2}
\]

Under the usual two assumptions, \( \xi_1 > \xi_2 \). Furthermore:

\[
\lim_{r + \delta \to 0} \xi_1 = \lim_{r + \delta \to 0} \xi_2 = \frac{\eta_2}{\eta_1}
\]

which is the slope of the Roy frictionless case. Finally, \( \xi_1 > \frac{\eta_2}{\eta_1} > \xi_2 \), i.e. the slope of the Roy equilibrium borderline is included between the two frictional slopes, provided that Assumption 1 holds, for any \( r + \delta > 0 \).

The measures of unemployed are: \( u_{it}^S \) specialized searchers in sector \( i \), and \( u_{it}^R \) random searchers. The effective measure of unemployed searching for vacancies of type 1 is \( u_{1t} = u_{it}^S + \alpha_t u_{it}^R \), because random searchers create congestion in sector 1 in proportion to the relative dimension of sector 1's vacancies. To compute the equilibrium, we must develop the previous argument using these definitions. The equilibrium is completed by the cross-section distribution of unemployed workers for each type of vacancy and the two zero profit conditions for vacancy creation (see the Appendix).

### 2.4.3 Equilibrium Wage Dispersion

The equilibrium just characterized formalizes and substantially enriches an early intuition of Salop (1973), who first saw the trade-off, when searching systematically for a job, between the probability of receiving an acceptable wage offer and the level of this offer. The firm-specific wage distributions, posited exogenously by Salop in the fashion of early search theory, are generated endogenously as a market equilibrium distribution of wages from sector-specific skills.
In fact, workers devoted to random search still have a comparative advantage in one sector. This implies that a fraction of them is rationally accepting wage offers that are not the maximum available for workers of their type. Simply, the extra wage is not high enough to justify selective search. This result goes back to the original spirit of the search literature: there exists an equilibrium distribution of prices for the same factor, and searchers do not necessarily accept the best offer. The innovation is that an equilibrium distribution of wages is supported by a distribution of different occupations, and that to generate it we do not need to resort necessarily to ex post heterogeneity, which was indeed introduced exactly to this purpose.

2.4.4 Some Evidence on Job Search Behavior

Is this picture empirically robust? A conclusive answer is not at hand, yet. A recent study concludes: “A thorough investigation of the existence and importance of systematic search or job queuing awaits good micro data on the type of firms contacted during the course of the unemployment spell.” (Thomas 1996, p. 150). But we can refer to a fairly large amount of micro empirical evidence on job search methods and attitudes, across workers, that can provide an indirect test.

The evidence can be summarized as follows. Workers choose mainly among nine different methods of job search; each worker uses on average around three of these methods simultaneously. Directed search methods produce fewer job contacts than job search delegated to Government or private agencies (Chirinko 1982). Search intensity declines with age and tenure. According to Fallick (1993), search across industries responds to macro incentives: workers search more in industries with faster employment growth. But industry effects are important in predicting also the probability of re-entry in the previous industry.

The bulk of evidence concerns on-the-job versus unemployed search (Pissarides and Wadsworth 1994 for UK, Blau and Robins 1990 for the US). Employed search produces fewer contacts but more job offers; it is lower but more effective for high-wage, mature, long-tenured workers. Also, it happens mostly through personal contacts, while unem-
ployed people rely more upon job centers, also because of the provisions of unemployment insurance programs. The results of the model are supported by this evidence if on-the-job search is correlated with within-industry search. This is indeed suggested by the fact that employed search is done mostly through personal contacts, and by the simple observation that the most frequent contacts in everyday working life are made with potential employers in the same two-digit industry.

Osberg (1993) analyzes Canadian longitudinal data for 1981, 1983, and 1986, in order to extract information on the search methods adopted by jobless workers. He reports that the number of search methods adopted is countercyclical, expressing harsher competition among workers in times of high unemployment (1983) as compared to times of low unemployment (1981 and, especially, 1986). More importantly, direct job search methods, such as direct applications and personal contacts, are procyclical, while use of public and private agencies, suggesting more random strategies, is strongly countercyclical.

The only direct tests of the present hypothesis are performed by Thomas (1993, 1996) on respectively UK and Canadian data. The author finds that high wage workers are more selective and less mobile, but exit faster from unemployment, while long-tenured workers are more mobile but exit slower from unemployment. From these findings, the author concludes that the hypothesis here formalized and previously formulated by other researchers has no empirical content.

The interpretation of these two studies has a few serious problems. First, data concern displaced workers, who sign in some unemployment record; my previous argument suggests that most of selective searchers might search on-the-job. In fact, the proportion of selective searchers in Thomas' samples is quite low, never exceeding 20%. Second, tenure has little to do with ex ante heterogeneity and much to do with ex post factors, such as matching success and firm-specific human capital; hence tenure is not a good proxy for ex ante comparative advantages.

Third, and most importantly, Thomas and other authors fail to recognize the two-step nature of the matching process. My hypothesis explains the pattern of contact rates with open vacancies across workers, not necessarily of exit rates from unemployment. The
difference between the two rates is in the conditional (on finding a vacancy) probability of receiving an acceptable job offer. This probability is affected by firm- or match-specific heterogeneity. In Chapter 3 I combine the results from the first two Chapters, to show that the latter probability is generally increasing in "types", and thus may well offset the effect of selective search on exit rates without altering any of the results on mobility presented here. Simply, high wage workers are less mobile across industries because they search more selectively to exploit their specializations; but also they exit faster from unemployment because, being good types within their sector, their applications are much more likely to be accepted.

2.5 A Hypothesis for The Decline in Labor Mobility

I can finally use the model to shed some light over the measured trends and cycles in labor mobility. The evidence is reviewed in Section 2.2.

Gross and, to a much smaller extent (if any), net inter-industry mobility have declined. One proposed explanation is based on the matching theory of labor turnover (Jovanovic 1979); workers try different jobs in different sectors, and movers are those who learn over time that their match was poor and worth being scrapped, what explains also why they have lower than average wages. Based on this hypothesis, Jovanovic and Moffitt (1990) find a significant positive relationship between the trends of mobility and the dispersion of wages; the more concentrated the dispersion of wages, the lower the incentive to scrap the current match to find a better one.

The matching hypothesis is extremely appealing when it comes to account for the large difference between gross and net worker flows at any point in time. Also, there is no denying that employer-worker matches are characterized by an important component of specificity, here ignored. But, in this context, the matching hypothesis appears incomplete on many dimensions. The argument is based on the ex ante homogeneity of workers, with all heterogeneity occurring ex post at the job level. Jovanovic and Moffitt admit that sectoral shocks still play a role, suggesting the existence of comparative advantages.
Indeed, Bull and Jovanovic (1988) explore the interaction between matching and sectoral shocks. But the latter are modelled as common shocks to the distribution of matching outcomes, and then they do not encompass individual comparative advantages.

If we let matching interact with one-dimensional ex ante heterogeneity, within a single sector I show in Chapter 1 that in general the exit rates from unemployment are higher for more productive workers. But allowing for more than one sector and corresponding skill dimensions would still induce self-selection of workers according to comparative advantages, with or without match-specific productivity. To see why, just observe that, by definition, the expected value of the matching component of productivity must be the same in all sectors: any difference in mean ex post productivity should be embodied in ex ante heterogeneity. Thus, there is no reason for ex ante heterogeneous workers to flow simultaneously in both directions across sectors when their current job turns out to be a poor match.

In addition, the correlation of mobility and wage dispersion that Jovanovic and Moffitt find does not seem to fit in the order of magnitude of the decline in mobility. The concentration in the distribution of wages in their sample was a general phenomenon in the US economy only in the 1970s, and was strongly reversed in the 1980s (Katz and Murphy 1992), while mobility was still declining. Finally, they do not comment on cycles.

Murphy and Topel (1987) notice that about one half of the 1968-1985 decline happened around the mid-1970s. Therefore, they argue that the inflow of young people in the labor market, due both to the baby-boom and to the end of the Vietnam war, substituted for the reallocation flows of incumbent workers and drove down the returns to mobility. This hypothesis, for which admittedly they do not provide strong evidence, does not explain why the decline in mobility, that hit all classes of workers, continued in the 1980s. Also, they document, but do not comment on, the cyclical patterns.

To put the existing theories in perspective, consider again the equilibrium partition of workers into non-participants, selective and random searchers. The Jovanovic and Moffitt (1990) story can be referred mainly to gross flows of non specialized workers, who search randomly. The theory of Lucas and Prescott (1974) and Lilien (1982) addresses
only the net flows, ignoring both the within-sector flows of specialized workers and the
across-sector flows of random searchers. No wonder why, on empirical grounds, sectoral
shocks can account for only a small fraction of unemployment (Murphy and Topel 1987,
Lael Brainard and Cutler 1993).

In the present framework, it is very easy to identify the factors that can be responsible
for the observed trends in mobility. Gross flows are proportional to the equilibrium
measure of random searchers, not seeking comparative advantages. Two factors can
shrink permanently this measure.

1. Polarization of the distribution of skills. This phenomenon shifts weight
away from the middle and towards the outer regions of the support of skills. This
“specialization” of the new labor force is hard to measure. But higher education
nowadays is synonymous of specialization, and the massive increase in the propor-
tion of highly educated people in the US labor force, especially women, is a well
documented fact.\(^8\)

2. Improvement in skill-matching technology. The other potential factor is a
reduction in relative search frictions, in the form of a change in market institutions
that improves the process of matching job requirements to the available skills and
thus discourages random search. In formal terms, this is equivalent to an increase
in the ratio of exit rates \(p_i/\pi\) for given measures of unemployed and vacancies.

As of the first factor, Murphy and Topel themselves attribute an important role to the
evolution of industry-specific skills. They point mainly to \(ex \ post\) heterogeneity, either
à la Jovanovic (1979) or as a product of learning-by doing. In their view, the original
shock was the inflow of young labor force in the mid-1970s, that drove down the returns
to education. Then, they suggest that the decline in mobility might have been self-
reinforcing: the less people move across industries, the more specific human capital they

\(^8\)For example, Freeman (1995) reports that the in 1970 only 12% of the 25-64 years old workforce
had more than 12 years of schooling; in 1992 this figure was 52%. In the same period, the proportion of
College graduates in the same pool more than doubled.
acquire, the lower incentives they have to switch industry. It is however peculiar that a younger labor force might become less mobile, considering that young people are by far the most mobile workers. That is, Murphy and Topel emphasize the effect on individual behavior, but neglect the composition effect, which works in the opposite direction and makes the puzzle even stronger. Also, the decline in mobility started manifestly in the late 1960s.

My explanation, not necessarily inconsistent with the two just mentioned, relies on increasing diversification of ex ante heterogeneity. Due to the difficulty in measuring it, at this stage there exists no direct evidence pro or contra this claim. Verifying the claim amounts to verify whether higher education leads to increased comparative advantages in one specific industry or activity. If this is the case, then the inflow of young highly educated workers in the 1970s explains the sharp drop in mobility. The causal link, though, is different from Murphy and Topel’s: these workers are less mobile than the average incumbent ones because they are more specialized. In fact, the mobility of older workers does not fall meaningfully (Murphy and Topel, Table 9).

This factor seems the natural candidate because it fits also with the other secular trends observed. First, a polarization of skills does reduce net flows for a given change in relative skill price, even though the effect is strongest on gross mobility. In fact, we observed a milder decline in net than in gross inter-industry mobility. Second, it predicts that average exit rates from unemployment, even for observationally equivalent classes of workers, should fall over time, what happened with no shade of a doubt (again Murphy and Topel 1987). This last point is taken up more carefully in Chapter 3.

A more selective matching technology, while reducing gross mobility, does not affect meaningfully the net mobility that follows sectoral shocks.

2.6 The Procyclical Selectivity of Job Search

I turn now to the detected cyclical behavior of gross and net mobility. Within my model, I can depict a typical business cycle building on the very systematic pattern of employment
shares across industries called Cyclical Upgrading of labor. I will present an informal argument, then I will make it tighter with an example.

Let industry 1 be a high-wage industry with volatile employment and procyclical employment share, such as durables; and industry 2 be a low-wage industry with stable employment and countercyclical employment share, such as retail trade. Empirically, a typical expansion corresponds to an increase in both skill prices, but more than proportional in sector 1 (durables). In terms of job creation, both vacancy rates and exit rates from unemployment increase, in such a way that the exit rate in durables $p_{1t}$, initially below the one in services $p_{2t}$, rises above it. It is easy to show that, given the random search rate of success $\pi_t$, the threshold $R_{1t}$ decreases in $p_{1t}$ and $R_{2t}$ increases in $p_{2t}$. Obviously, $\pi_t$ and also the proportion of vacancies in sector 1 are increasing, but let us assume that their effects are second order. We observe two events:

1. **Reduction in random search.** The higher exit rates of unemployment and higher wages in both industries make search frictions less binding; workers pursue more directed search strategies, and the area of random searchers shrinks, i.e. the two lines come closer together. In addition, as exit rates rise, given entry rates, unemployment rates fall in both sectors, so that the measure of random searchers shrinks for both reasons.

2. **Amplification of net flows.** The two lines delimiting the region of random search tilt with the Roy frictionless line and come closer to it, producing an extra net flow. On the contrary, in a recession, some of the inverse flow from sector 2 to sector 1 goes into the pool of random searchers, hence the backward net individual mobility is lower.

Simply put, in a highly frictional or depressed economy workers go where jobs are; in a frictionless or booming economy they go only where the *good* (for them) jobs are, i.e. where the wages are high. Fig. 3.2 depicts the two cases.

This typical pattern produces procyclical gross and especially net flows, while their
difference, namely the offsetting flows arising merely from random search \( S_t \), are countercyclical. This is exactly the empirical regularity mentioned in the Introduction.

Making the argument tight requires tracking comparative statics and dynamics effects of a change in both good prices. A typical business cycle corresponds to a cycle in the price of services \( (\eta_2) \) and to a more amplified cycle in the price of durables \( (\eta_1) \). The complete analysis is extremely complex.

Rather than resorting to simulations, I present a simple partial equilibrium exercise. Suppose that the price of durables increases, given the price of services: \( d\eta_1 > 0 = d\eta_2 \). Assume that parameters are such that the comparative statics response, across steady states, is the following: the proportion of vacancies of type 1, namely \( \alpha \), increases; the exit rate \( p_1 \) increases, but to a lower extent, what is reasonable as we expect new workers to create congestion; and the random exit rate \( \pi \) increases by even less, what is reasonable provided that \( p_2 \) changes little or does not change at all (what I will assume). To make things more precise:

\[
d\alpha > dp_1 > d\pi > 0 = dp_2
\]

Consider the simple case \( b = c = 0 \), so that all workers search when jobless. The lines delimiting the three regions in steady state have slopes \( \xi_1 \) and \( \xi_2 \), found in the previous section (Equations 2.2). Both lines tilt down because of the decrease in the relative price of services \( \eta_2/\eta_1 \). In addition:

\[
d\xi_1 = \frac{\partial \xi_1}{\partial p_1} dp_1 + \frac{\partial \xi_1}{\partial \alpha} d\alpha + \frac{\partial \xi_1}{\partial \pi} d\pi \alpha
\]

after computing the derivatives and substituting this is proportional to

\[
\xi \left( p_1 (d\pi - d\alpha) + \pi (d\alpha - dp_1) - \beta \pi^2 \frac{1 - \alpha}{r + \delta} dp_1 \right)
\]

The only positive term is the second one. In order for the whole response to be negative we just need \( d\alpha - dp_1 \) to be small enough, and it is clear that in general this response is
negative. On the other hand:

\[ d\xi_2 = \frac{\partial \xi_2}{\partial \alpha} d\alpha + \frac{\partial \xi_2}{\partial \pi} d\pi \propto (\pi - p_2) \frac{d\alpha}{\alpha} - p_2 \frac{d\pi}{\pi} \]

so that a last negative term is missing, and the total effect may well be positive.

The bottom line is: both lines tilt down, because of the change in the relative price, but the upper line tilts more and reduces the area of random search, due to the effect of exit rates. This is the case depicted in Fig. 3.2, where I take into account also the inflows of workers from out of the labor force.

### 2.7 The Correlation of the Wages of Industry Stayers

After dealing with quantities, in this section I attempt to match my hypothesis with the evidence on real wages. I eventually focus on one fact, namely the strong synchronization of industry stayers’ wages and the resulting acyclicality of industry wages; the goal is to emphasize a force explaining this phenomenon and completely ignored so far by the literature.

As in any bargaining model, individual wages are procyclical. With positive selection of skills in both sectors, wages of movers (both random and specialized searchers) are always low-to-low wage moves. The jump in pay of a mover when joining a high-wage expanding industry is explained both by the higher skill price and by mobility costs, that do require such a jump to produce a move. The loss in wage when joining a low-wage industry from another industry, an event that occurs mostly in recessions, must reveal that the reservation wage is even lower than the new wage, so that the worker prefers a wage cut to entering unemployment.

As of the wages of stayers, we can find in the literature three channels through which comovements in the real wages of stayers in each sector can be produced:

1. **Capital intensity.** With a variable capital/labor ratio technology, the outflow of
workers increases this ratio in the non-expanding sector and raises the marginal productivity of stayers.

2. **Price of the good.** Given the demands for the two goods, after demand of good $i$ increases and workers arrive from sector $-i$, the supply of good $i$ increases and the supply of good $-i$ falls, driving back down partially the relative price of good $i$ and up the wages of stayers in sector $-i$.

3. **Scale of operations effect.** Sattinher (1993) dubs with this expression the change in the opportunity cost of concurrent scarce factors that are employed with labor. For example, if the total number of jobs in the economy is fixed, as the price of good $i$ rises we need an increase in the price of labor in sector $-i$ to induce destruction in that sector and creation in sector $i$ with consequent flow of jobs and workers across sectors. Similarly, in the Schumpeterian “Creative Destruction” theory of Caballero and Hammour (1994, 1995) jobs are not in fixed supply, but capital is employed in fixed proportions with homogeneous labor in two sectors; creation in one sector and destruction in the other are synchronized (more or less efficiently) by the shadow wage, or opportunity cost of labor, so that wages rise in the shrinking sector and fall in the expanding one.

The latter effect, though, in a Roy economy with sector-specific skills and fixed-proportions technology is to be ascribed entirely to adjustment (e.g. search) costs. In fact, with Walrasian allocation the change in the (shadow) wage in the shrinking sector would occur only at the margin, and would not affect the wages of inframarginal stayers. Search costs, instead, introduce an outside option that enters wage determination even if it is not exercised, because it would be exercised were the current match dissolved. Thus, in this context, the third effect must be qualified as follows:

3a. **Differential Outside Option Effect.** Suppose sector $i$ expands, drawing employment away from the other. Stayers in the shrinking sector $-i$ might decide not to move simply because of search frictions: it does not pay to quit the current job for
a higher wage in the other sector, because it would take too long to find one. But some of them, if they were laid off and had to pay the mobility cost anyway, would move to search in the other sector, pursuing comparative advantages. Hence even for these stayers the outside option in the other sector enters the determination of their wages, though it is not directly exercised. As wages rise in the expanding sector $i$, some workers in sector $\neg i$ decide to stay but only at higher wages, given the better alternative.

To see how the argument works, suppose for simplicity that $\pi = 0$ so that there is only selective search (the argument does not depend on this).

**Proposition 11** Suppose that $p_i$ and $\eta_i$ are constant, and that $p_{-i}$ is increasing. Then there exists always a non-empty set of stayers in sector $i$ whose wages comove with those of stayers in sector $\neg i$.

**Proof.** The claim is equivalent to the existence of a subset of workers satisfying:

1. No-scraping condition:

   $$J_{it}(x) > 0$$

2. Industry-switch condition:

   $$p_{-i}J_{-it}(x) > p_i J_{it}(x)$$

Given the properties of the surplus function (Proposition 7), there exists always a subset of types such that the two inequalities hold at the same time.

$$\frac{p_{-i}J_{-it}(x)}{p_i} > J_{it}(x) > 0$$

\[\square\]
This subset is always closer to the 45 degree line than the rest of the stayers who do not enjoy the outside option effect on wages. Hence some workers, while staying in sector \( i \), receive the same wage change that they would obtain by moving to sector \(-i\):

\[
\dot{w}_{it}(x) = \beta \frac{d}{dt} [p_{-it}J^{-it}(x)] = \dot{w}_{-it}(x)
\]

If wages in sector \(-i\) are rising, so are the wages of these stayers in the shrinking sector. In spite of the equal increases in wages, the worker will sooner or later move, when their matches break exogenously.

Caballero and Hammour (1995) present a model of costly reallocation of heterogeneous jobs and homogeneous workers across sectors, focusing on job flows. All workers, due to their homogeneity, benefit from the outside option effect, and in fact it is the co-movement of the wages of stayers that coordinate creation and destruction. The present analysis emphasizes that, in the presence of workers' comparative advantages, very specialized workers are insulated from this form of "wage resonance" across sectors.

### 2.8 Conclusions

The study of inter-industry labor mobility calls for better and more abundant data. This is even more true for European countries, where the immobility of labor, not only among regions, is seen as one major hurdle to the solution of the unemployment problem. Indeed, industrial districts - the concentration of certain industries in specific geographical regions - align the problems of sectoral and spatial mobility of workers. The increasing diffusion of "in-home" jobs, supported by the new telecommunication technologies, might well reverse the trend in inter-industry labor mobility. On the other hand, the tendency of the labor force to the polarization of comparative advantages is likely to continue. The formal framework presented in this chapter should provide a flexible tool to interpret the evidence and predict future trends.

To conclude this chapter I wish to mention what is missing from it, that I consider
important. McCall (1990) finds that the tenure in the previous job predicts much better the one in the subsequent job when the two jobs are in the same occupation. The author suggests the existence of yet another dimension of heterogeneity, that he calls "Occupational Matching". In fact, workers in the US change not only industry but also occupation with some frequency. Is the specialization of workers occurring in terms of occupation or of industry? The analysis of this chapter was run under the implicit assumption that workers would not hold their occupation across industries. If we define an occupation as a bundle of tasks to be performed, then the industry level might not be the only interesting one.

Once again, more detailed evidence on systematic job search would be of great help. However, the choosiness of workers among potential employers might be reflected not only in search behavior, but also in the decision whether to accept or reject job offers. Every worker searching for a job receives unexpected or ex ante undesired offers with some probability; it is the essence of the search problem. Also, there is a strong impression that on-the-job search is the preferred method by qualified or "specialized" searchers; if this impression were confirmed, the existing empirical studies based on samples of displaced workers would be misleading, and we should look also among employed searchers to identify the trade-off between unemployment spells and comparative advantages.
2.9 Appendix

Derivation of the Equilibrium of the Frictional Economy: Firms’ Arbitrage Equations

For notational convenience, from now on let $X_{it} \subseteq [0,1]^2$ denote the set of types of workers who search for a vacancy in sector $i$, if unemployed, and $L_t = X_{1t} \cup X_{2t} \subseteq [0,1]^2$ the set of types of workers who search for some vacancy if unemployed.

Let $J_{it}(x)$ denote the present discounted value of profits, net of future search costs, obtained by the firm while holding a job $i$ filled by a worker $x$. This value satisfies the following arbitrage (accounting) equation:

$$ rJ_{it}(x) = \dot{J}_{it}(x) + \eta_i x_i - w_{it}(x) - \delta [J_{it}(x) - V_{it}] 
$$

(2.3)

where $V_{it}$ denotes the shadow value of a vacancy in sector $i$. This equation equates, as usual, the flow value of the “asset” to the capital gain, plus productivity net of wage, minus the change in capital value determined by the exogenous dissolution of the job.

Let $J_{it}(x)$ the Bellman value of the same job, i.e.

$$ J_{it}(x) = \max \langle V_{it}, J_{it}(x) \rangle $$

The optimal hiring-tenure policy of the firm is clearly: hire a worker $x$ and keep him employed in sector $i$ if and only if $J_{it}(x)$ is no less than the value of search (value of a vacancy). This occurs whenever the type is contained in the set

$$ P_{it} := \{ x \in [0,1]^2 | J_{it}(x) \geq V_{it} \} $$

In this circumstance $J_{it}(x) = J_{it}(x)$ and thus $J_{it}(x)$ satisfies the same arbitrage equation (2.3), now a Bellman equation in continuous time. Otherwise, it is worth destroying the
job and search for a new, more profitable match.

The shadow value of a vacancy $V_{it}$ satisfies

$$rV_{it} = \bar{V}_{it} - \kappa + q (\theta_{it}) \left( \bar{J}_{it} - V_{it} \right)$$  \hfill (2.4)

where $\bar{J}_{it}$ is the expected surplus from a new acceptable match. The expectation is taken over unemployed workers with types searching in sector $i$ ($x \in X_{it}$).

Free entry in both sectors ensures at all times

$$V_{it} = 0$$

It follows, for all active types $x \in L_t$:

$$(\tau + \delta) J_{it} (x) = \dot{J}_{it} (x) + \eta_i x_i - w_{it} (x)$$  \hfill (2.5)

Workers’ Arbitrage Equations

Let $U_t (x)$ be the value to an $x$-type worker of being unemployed and searching, and $W_{it} (x)$ be the human wealth for a worker $x$ employed in sector $i$. For all $x \in P_{it}$, $W_{it} (x)$ satisfies

$$r W_{it} (x) = W_{it} (x) + w_{it} (x) + \delta [U_t (x) - W_{it} (x)]$$  \hfill (2.6)

and the worker keeps the job as long as $W_{it} (x) \geq U_t (x)$.

An unemployed worker $x$ who knows to be acceptable, and thus searches for a job $i$ (has type in the set $X_{it}$), is hired as soon as he is interviewed by a firm. Let $\alpha_i$ denote the proportion of open vacancies in sector 1. For all types searching $x \in X_{it}$, $U_t (x)$ satisfies

$$r U_t (x) = \dot{U}_t (x) + b - c + \max_{t=1,2} \left\{ p_{it} \left[ W_{lt} (x) - U_t (x) \right] ; \pi_t [\alpha_i W_{it} (x) + (1 - \alpha_i) W_{2t} (x) - U_t (x)] \right\}$$  \hfill (2.7)

Unlike the hiring firm, the worker knows in advance what will be his own ex ante productivity $x_i$, hence he can condition on it.
Wage Determination under Nash Bargaining

The bilateral monopoly indeterminancy is resolved by assuming that the two parties bargain continuously over the surplus of the match. I adopt the Nash bargaining solution. If the worker gets a share $\beta$ of the total surplus from a filled job $(i, x)$, this solution yields:

$$W_{it}(x) - U_{i}(x) = \frac{\beta}{1 - \beta} J_{it}(x);$$

as usual, the firm and the worker always agree on whether to accept the match or not.

Differentiation with respect to time and substitution from (2.6) and (2.5) gives an expression for the wage schedule as a function of the expected surplus from next match:

$$w_{it}(x) = (1 - \beta)(b - c) + \beta\eta_i x_i + \beta \Phi_i(x)$$  \hspace{1cm} (2.8)

where $\Phi_i(x)$ is defined in the text.

Characterization of the Value Function

For workers who would not leave the labor force if separated from the job, the arbitrage equation can be rewritten as follows:

$$J_{it}(x) = (r + \delta) J_{it}(x) - (1 - \beta) \phi_i(x) + \beta \Phi_i(x)$$ \hspace{1cm} (2.9)

where $\phi_i(x) = \eta_i x_i - b + c$.

I impose:

$$\lim_{t \to -\infty} e^{-(r+\delta)t} J_{it}(x) = 0$$ \hspace{1cm} (2.10)

Integration of the partial differential Equation (2.9) with respect to time between 0 and $t$ by variation of the arbitrary constant yields

$$J_{it}(x) = \xi_i(x) e^{(r+\delta)t} + (1 - \beta) \frac{1 - e^{(r+\delta)t}}{r + \delta} \phi_i(x) + \beta \int_{0}^{t} \Phi_s(x) e^{-(r+\delta)(s-t)} ds$$

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where \( \xi_i(x) \) is a constant of integration, function of \( x \) but not of time. I can determine it using the feasibility condition (2.10). Multiplying both sides by \( e^{-(r+\delta)t} \) and taking limits for \( t \to \infty \) gives:

\[
\xi_i(x) - \frac{1 - \beta}{r + \delta} \phi_i(x) + \beta \int_0^\infty \Phi_s(x) e^{-(r+\delta)s} ds = 0
\]

Hence I can solve for the constant of integration \( \xi_i(x) \) and rewrite the solution to the firms’ arbitrage equation in the following implicit forward-looking form:

\[
J_{it}(x) = \frac{1 - \beta}{r + \delta} (\eta_i x_i - b + c) - \beta \int_t^\infty \Phi_s(x) e^{-(r+\delta)(s-t)} ds
\]  

(2.11)

**Proof of Proposition 7.** The value function \( J_{it}(\cdot) \) is strictly increasing in \( x_i \) and weakly decreasing in \( x_{-i} \).

**Proof.** First, substituting from (2.11):

\[
J_{it}(x) - J_{2t}(x) = \frac{1 - \beta}{r + \delta} (\eta_1 x_1 - \eta_2 x_2)
\]  

(2.12)

which is strictly increasing in \( x_1 \) and strictly decreasing in \( x_2 \). By contradiction, suppose the claim is false: there exist \( (i, x_i > y_i, t) \) such that

\[
J_{it}(x_i, x_{-i}) - J_{it}(y_i, x_{-i}) \leq 0
\]

By (2.12) this implies

\[
J_{-it}(x_i, x_{-i}) - J_{-it}(y_i, x_{-i}) = J_{it}(x_i, x_{-i}) - J_{it}(y_i, x_{-i}) + \frac{1 - \beta}{r + \delta} \eta_i (y_i - x_i) < 0
\]

Clearly the last two inequalities imply \( \Phi_t(x_i, x_{-i}) \leq \Phi_t(y_i, x_{-i}) \). Using (2.9):

\[
J_{-it}(x_i, x_{-i}) - J_{-it}(x_i, x_{-i})
\]

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\[ (r + \delta) [J_{-it}(x_i, x_{-i}) - J_{-it}(y_i, x_{-i})] + \beta [\Phi_t(x_i, x_{-i}) - \Phi_t(y_i, x_{-i})] < 0 \]

Hence, for all \( s > t \)

\[ J_{-is}(x_i, x_{-i}) - J_{-is}(y_i, x_{-i}) < 0 \Rightarrow \Phi_s(x_i, x_{-i}) \leq \Phi_s(y_i, x_{-i}) \]

From (2.9) and (2.11) this implies:

\[ 0 < \int_t^\infty [\Phi_s(x_i, x_{-i}) - \Phi_s(y_i, x_{-i})] e^{-(r+\delta)(s-t)} ds \leq 0 \]

a contradiction.

The second claim is proven in a similar manner. ■

Cross-Sectional Dynamics

The density of specialized searchers unemployed in sector 1 satisfies by Large Numbers

\[ \dot{u}_{1t}^S(x) = \delta \left[ \psi(x) - u_{1t}^S(x) \right] - p_{1t}(x) u_{1t}^S(x) \]

The total measure of specialized searchers in sector 1 is

\[ u_{1t}^S = \int_{\xi_{2t}}^1 \int_{\max(\xi_{1t}, \xi_{1t}(x_2))}^1 u_{1t}^S(x) \, dx_1 \, dx_2 \]

Similarly for sector 2.

The density of random searchers satisfies

\[ \dot{u}_t^R(x) = \delta \left[ \psi(x) - u_t^R(x) \right] - \pi_t u_t^R(x) \]

and their total measure is

\[ u_t^R = \int_{\xi_{2t}}^1 \int_{\xi_{2t}(x_2)}^1 u_t^R(x) \, dx_1 \, dx_2 \]

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Let $f_i(\cdot)$ the normalized density of unemployed workers searching for vacancies $i$, that is

$$f_{1t}(x) = \frac{u_{1t}^S(x) + \alpha_i u_{1t}^R(x)}{u_{1t}^S + \alpha_i u_{1t}^R}$$

since the two sets never overlap, one of the two terms in the numerator is always zero.

**Free Entry**

The zero profit condition ensured by free entry in each sector pins down the equilibrium vacancy rates in the two sectors.

$$\kappa = q(\theta_{1t}) \int_0^1 \left[ \int_{\max(z_{1t}, z_{2t}(x_2))}^1 J_{1t}(x) f_{1t}(x) \, dx_1 \right] \, dx_2$$

$$\kappa = q(\theta_{2t}) \int_{z_{2t}}^1 \left[ \int_0^{\xi_{2t}(x_2)} J_{2t}(x) f_{2t}(x) \, dx_2 \right] \, dx_1$$

Given the measures of unemployed workers, predetermined by search frictions, the vacancy rates also pin down the measures and proportion of vacancies open in the two sectors.

**Equilibrium Exit Rates from Unemployment**

By Large Numbers, the following equations pin down the exit rates. First, the number of vacancies filled must be equal to the number of workers hired in each sector.

$$q(\theta_{1t}) v_{1t} = p_{1t} u_{1t}^S + \pi_i \alpha_i u_{1t}^R$$

$$q(\theta_{2t}) v_{2t} = p_{2t} u_{2t}^S + \pi_i (1 - \alpha_i) u_{2t}^R$$

Second, the same must hold economy-wide

$$q_t v_t = p_{1t} u_{1t}^S + p_{2t} u_{2t}^S + \pi_i u_{2t}^R$$

Given the vacancy rates and measures of unemployed workers from above, these three
equations determine the three exit rates $p_{1t}$, $p_{2t}$, and $\pi_t$.

It is immediate to show that, in general, $q_t \neq q(\theta_t)$. This is not surprising, as the matching technology is required to apply only within each labor market. Hence we have one additional unknown $q_t$, or the last equation cannot be used to pin down the three exit rates. We need another independent relationship between $\pi_t$ and the two $p_{it}'s$, or the measures of vacancies and unemployed.

The most reasonable way is to specify a matching function for random search $\pi(\cdot)$, function of sector vacancy rates and measures ($\theta_{it}, v_{it}$). In fact, random search across sectors is considered an intrinsically different activity from job search in a labor market, and thus must generate job offers according to a different technology. The correct formulation of such a function satisfying Assumptions 1 and 2 (effectiveness and precision of random search) is an open research issue.

This completes the description of the general equilibrium of the frictional economy.
Fig. 1.2

Equilibrium of the Roy economy with endogenous labor participation
Equilibrium of the frictional economy (non steady state)
Fig. 3.2

Cyclical behavior of the frictional economy

Dashed lines = recession
Solid lines = expansion

Sector 1: low wage, low volatility of employment (e.g. retail trade)
Sector 2: high wage, high volatility of employment (e.g. durables)

In an expansion, workers flow from sector 1 to sector 2 and from out of the labor force to both sectors. The area of non-specialized (random) job searchers shrinks.
Chapter 3

Workers’ Heterogeneity and Unemployment Duration

3.1 Unemployment Duration and Unemployment Rate

The flow approach to labor markets addresses the problem of unemployment in terms of entry rates into and exit rates out of this state. Indeed, in steady state, the unemployment rate for each type of worker is given by:

\[ u(x) = \frac{\delta(x)}{\delta(x) + E(x)} \]

where \( \delta(x) \) is the entry rate into unemployment and \( E(x) \) is the exit rate to employment. Hence, in order to understand the sources and the nature of unemployment, we may analyze separately its two determinants, or equivalently their inverse, the expected tenure on the job and the unemployment duration or spell, respectively.\(^1\)

The available empirical evidence on entry and exit rates - levels, volatilities, and cross-

\(^1\)Out of steady state, we need to add to the denominator the change in the unemployment rate \( \dot{u}_t(x) \). Entry and exit rates are also important when referring to the out-of-the-labor-force state: we know that flows between this state and either unemployment or employment are large. In the following discussion, I concentrate only on the flows between employment and unemployment, aware of the disputes that have arisen on the existence of separate joblessness states. The models presented in this thesis, though, make this distinction, and then could accommodate for more facts than I discuss here.
sections - varies across countries and time periods. There is, however, a rather general consensus on several facts.²

First, most of the time series variation in entry rates occurs at seasonal, rather than cyclical frequencies. The opposite holds for exit rates, which are highly procyclical.

Second, for all classifications of workers in terms of observable characteristics - education, age (from middle age to young to close to retirement), work experience, manual/non manual - the profile of entry rates across groups is monotone and very steep, much in favor of more qualified workers; also, the ratios of entry rates across groups are almost constant over time. Vice versa, the profile of exit rates is monotone but flatter, still in favor of primary workers; and the ratios of exit rates across types fluctuate substantially over business cycles. In fact, most of the total absolute increase in unemployment in recessions is due to longer spells of less qualified workers. Hence the quality composition of the inflow is acyclical, while that of the outflow is clearly countercyclical (implying Fattening in expansions).

Third, at lower frequencies, entry rates in developed countries have exhibited no meaningful trend; the long-term increase in unemployment rates observed since the late 1960s until today both in the US and, especially, in Europe, is due (in an accounting sense) to a dramatic increase in unemployment spells, which affected non-movers' categories, though not uniformly.³

Fourth, unemployment exhibits a remarkable negative duration dependence: the longer a worker has been unemployed, the lower his or her chance of getting out of this state in the current period.

A host of conflicting hypotheses have been advanced to interpret this evidence. This dissertation attempts a richer treatment of heterogeneity, on many dimensions, and thus

²We have very detailed evidence on unemployment duration only for the US and UK economies, more scattered evidence for other European countries. Excellent references are: Murphy and Topel (1987) for trends, and Blanchard and Diamond (1990) for cyclical responses, in the US; Johnson and Layard (1986) for the UK; Layard, Nickell and Jackman (1991, Chapter 6) for both the US and the UK.

³For the US, see Murphy and Topel (1987). A recent report by CENSIS (1996) in Italy indicates that the average time to the first job for young workers has tripled, from 9 to 27 months, over the 1967-1995 period.
generates (at least in part) new hypotheses. In Chapter 1 I presented an explanation for the longer and more volatile unemployment spells of less qualified workers, based on the wage compression that arises from the interaction of ex ante and ex post heterogeneity. In this chapter I exploit the previous analysis to identify the sources of two other empirical regularities concerning exit rates and duration. Entry into unemployment is not addressed directly.

In particular, in Section 3.2 I address the pattern of exit rates between industry stayers and movers, completing the discussion of Chapter 2. I show that, when ex ante comparative advantages and ex post heterogeneity coexist, specialized workers who search selectively are accepted more often by firms, so that they can exit faster from unemployment in spite of their lower rate of contact with open vacancies. In addition, a subset (possibly all of) of the specialized searchers could exit even faster from unemployment than they actually do if they gave up to selective search, which means that they are trading off longer spells for good jobs. In Section 3.3 I address "spurious" duration dependence and its mild decline in recessions observed by van den Berg and van Ours (1996) in France and the US. This decline seems to contradict both the Ranking hypothesis of Blanchard and Diamond (1994) and the Fitting hypothesis of Chapter 1. Instead, building on the model and the logic of Chapter 2, I argue that it can be explained by an increase in random search by non specialized workers, who can absorb part of the increased level and inequality in their exit rates by sacrificing some of their (weak) comparative advantages, and by the increase in the spells of qualified workers, whose exit from their cohort of unemployed is less massive in the first months.

3.2 A Useful Decomposition of the Exit Rate from Unemployment

In Chapter 2 I pointed at the increasing specialization of the labor force as one major potential cause of the decline in labor mobility. This hypothesis was based on the as-
sumption that selective search generates more job offers of a desired type, while random search generates more offers of all types, so that specialized workers privilege the former method and the other workers choose the latter.

The only piece of evidence contradicting this hypothesis, and indeed its only direct test, is that high wage workers, though less mobile across industries - as predicted - appear to exit faster from unemployment (Thomas 1993, 1996). From this finding, the author concludes that good workers do not really trade unemployment spells for better jobs. I argue that this interpretation is flawed.

Rather than comparing the exit rates of stayers and movers, we should compare the observed exit rates of stayers to the exit rates that these same workers would experience if they took the first available job, even in a different industry. This second comparison might be impossible, because the latter rates are unobservable, but it is the correct one: movers and stayers are likely to be intrinsically different workers, and hence they cannot be compared. Indeed, it might well be that high-wage workers move less across industries and exit faster from unemployment for the same reason: because they are more specialized and more productive in the industry where they work most of the time.

In this section I show that the interaction of workers’ comparative advantages and ex post (matching) heterogeneity leads precisely to this conclusion. The intuition is found in a simple but illuminating decomposition of the exit rate from unemployment. The contact rate with open vacancies is lower for more specialized workers, because they search more selectively and thus move less across industries. But, once a vacancy is found, the conditional acceptance rate is higher for these workers, due to wage compression across types much in the fashion of Chapter 1. In fact, wage compression is even reinforced by specialization: specialized workers are productive in “their” sector and have a weak outside option in the other sector, hence they are the preferred workers by the firms and are accepted most often. The higher acceptance rate may well more than compensate the lower contact rate and produce shorter spells for industry stayers.

This combination of effects is remarkable, in that the force that leads a worker to search more selectively and to move less across industries, namely specialization, also
leads firms to accept these workers more often. The reason is that any bargaining solution that is Pareto optimal within the bargaining parties (no surplus is wasted), such as Nash, aligns partially the interests of workers and firms: specialized workers go where the expected surplus is highest, which means most often where it is highest also for the firms. This explains why stayers may exit faster from unemployment despite the fact that they contact fewer vacancies. In what follows I make this argument rigorous and provide some details.

A Model of Comparative Advantages and Matching Heterogeneity

The matching process for a worker engaged in job search happens in two steps. First, the worker searches for an open vacancy. Second, conditional on finding one, he decides whether to accept or not the match, i.e. the wage offer. Therefore, as said, we can decompose the exit rate of unemployment for type $x$, say $E_t(x)$, into the product of the contact rate $C_t(x)$, and the conditional acceptance rate $\Pr[z \in A_t(x)]$, where $z$ stands for matching dimensions of heterogeneity and $A_t(x)$ for the set of acceptable matches with worker $x$:

$$E_t(x) = C_t(x) \cdot \Pr[z \in A_t(x)]$$

By definition, given market labor demand, the contact rate $C_t(\cdot)$ is affected only by the ex ante characteristics of the worker $x$, through his choice of the search method and the personal requirements that firms might associate to the jobs. For example, in Chapter 2 I showed that the closer the skill ratio to the relative price of the skills, the more likely is the worker to search randomly rather than selectively, what increases his contact rate. Ex post factors cannot enter the determination of the contact rate, because this determination happens prior to experience the match.

By contrast, the conditional acceptance rate $\Pr[z \in A_t(x)]$ depends both on ex ante $(x)$ and ex post $(z)$ factors: the acceptance decision is made after the contact with the

---

4The qualification “most often” is necessary because non specialized workers sacrifice comparative advantages, thus sometimes they offer to firms sub-optimal (yet acceptable) personal abilities.
firm has occurred, which means after (at least an estimate of) the outcome of the match has been observed. For example, in Chapter 1 I showed that a higher type is more likely to be accepted, because he can substitute for a mediocre matching with his own ex ante ability. The following table summarizes these considerations.

\[
\begin{align*}
\text{Ex ante productivity } x & \rightarrow \text{ Contact rate } C_t(x) \\
\text{Ex post match success } z & \rightarrow \text{ Acceptance rate } \Pr[z \in A_t(x)]
\end{align*}
\Rightarrow \text{ Exit rate } E_t(x) = C_t(x) \cdot \Pr[z \in A_t(x)]
\]

How can we formalize these causal relations? Consider the same economy as in Chapter 2, with two-dimensional skills and two sectors; as in Chapter 1, add to the flow productivity of the job an ex post match productivity outcome \(z\), drawn, once-and-for-all when the match occurs, from a given distribution \(G(\cdot)\) with support \([-Z, Z]\). The notation is the usual one.

Skipping the familiar steps, once the Nash bargaining solution is imposed, with workers obtaining a share \(\beta\) of the surplus from a match, the value at time \(t\) of a job filled in sector \(i\) by a worker \((x_i, x_{-i})\) at match outcome \(z\) satisfies:

\[
\hat{J}_t(x, z) = (r + \delta) J_t(x, z) - (1 - \beta) [\eta_i(x_i + z) - b + c] + \beta \Phi_t(x)
\]

Similarly to the analysis in Chapter 2, the last term on the right hand side

\[
\Phi_t(x) := \max \left\{ \frac{1 - \beta}{\beta} c; \ p_{1t} J^c_{1t}(x); \ p_{2t} J^c_{2t}(x); \ \pi_t [\alpha_t J^c_{1t}(x) + (1 - \alpha_t) J^c_{2t}(x)] \right\} \tag{3.1}
\]

is the expected surplus from the next search decision, which can be any of the following: no search, selective search in one of the two sectors, random search. As in Chapter 1, the expected surplus \(J^c_t(x)\) is the expectation of the value function \(J_t(x, z)\) over the acceptable match outcomes, times the probability that the match outcome is acceptable.

In the Appendix, I prove the following results.
• The match acceptance and tenure rule is a cutoff rule on $z$: at time $t$, it is optimal to form and keep alive a job $i$ filled by a worker $x$ if and only if the success of the match is not below a reservation value $h_{it}(x)$.

\[ J_{it}(x, z) \geq 0 \iff z \geq h_{it}(x) \]

It follows that the probability of being hired (the conditional acceptance rate) in a job $i$ is $1 - G(h_{it}(x))$, the value function has the form $J_{it}(x, z) = \frac{1-\theta}{r+\delta} [z - h_{it}(x)]$, and the expected surplus has the form $J_{it}^e(x) = \int_{h_{it}(x)}^z J_{it}(x, z) dG(z)$.

• The value function $J_{it}(\cdot, \cdot, z)$ is strictly increasing in $x_i$ and weakly decreasing in $x_{-i}$ (Lemma 9 in the Appendix). This important result has the following two implications: the reservation schedule $h_{it}(\cdot, \cdot)$ is strictly decreasing in $x_i$ and weakly increasing in $x_{-i}$, while the expected value function $J_{it}^e(\cdot, \cdot)$ is strictly increasing in $x_i$ and weakly decreasing in $x_{-i}$.

The first implication says that the more specialized a worker is in sector $i$ (the larger his $x_i$ and the smaller your $x_{-i}$), the more likely he is to be hired by a firm $i$. The effect of a lower outside skill $x_{-i}$ on the acceptance decision in sector $i$ is intuitive: it weakens the outside option and keeps the product wage low, what makes the worker more profitable to the firm, and thus more acceptable.

The second implication, the monotonicity of the expected surplus function $J_{it}^e(\cdot, \cdot)$ in both skills, lets me characterize the optimal search strategy as a function of the type. As in Chapter 2:

1. workers search or stay out of the labor force according to a reservation rule on own types only. Higher types are more likely to participate to the labor force.

2. Workers with weak comparative advantages search randomly, while types off the relative skill price line search selectively. Again, the partition is pictured by two increasing curves (as in Fig. 2.2, Chapt. 2).
The Unemployment Durations of Stayers and Movers

With these results in hand, it is easy to show why a high-wage, specialized worker is less mobile and yet can exit faster from unemployment. Consider two workers: \( x = (x_1, x_2) \) and \( y = (y_1, y_2) \), with

\[
x_1 > y_1; \quad x_2 = y_2; \quad y_1 \sim \frac{n_2}{n_1}.
\]

They have the same skills in sector 2, but the first worker is better in sector 1. Also, the second worker would be almost indifferent between sectors in a Walrasian equilibrium, as the ratio of his skills is close to the relative price of skills.

If both workers are employed in sector 1, clearly the first worker is receiving a higher wage. Suppose that they are both laid off exogenously at time \( t \) and, in equilibrium, \( x \) decides to search selectively, while \( y \) searches randomly, due to his weak comparative advantages. The exit rates from unemployment for such a pair of workers, decomposed into contact rates and conditional acceptance rates, are:

1. Specialized worker \( x \), searching selectively for a job 1:

\[
E_t(x) = p_{1t} [1 - G(h_{1t}(x))]
\]

2. Non specialized worker \( y \), searching randomly:

\[
E_t(y) = \pi_t \left\{ \alpha_t [1 - G(h_{1t}(y))] + (1 - \alpha_t) [1 - G(h_{2t}(y))] \right\}
\]

Let us compare these exit rates component-wise. By assumption, random search generates a higher number of contacts per unit time, and that is why this method is adopted by non specialized workers like \( y \):

\[
C_t(x) = p_{1t} < \pi_t = C_t(y)
\]

As of the conditional acceptance rate, \( x_1 > y_1 \) and \( x_2 = y_2 \) imply a lower cutoff for \( x \) and thus a higher conditional acceptance rate in sector 1: \( 1 - G(h_{1t}(x)) > 1 - G(h_{1t}(y)) \).
The acceptance rate for the random searcher $y$ in sector 2 is similar to the rate he enjoys in sector 1: $1 - G(h_{2t}(y)) \approx 1 - G(h_{1t}(y))$. In fact, from the forward-looking expression derived in the Appendix for the match value we obtain:

$$h_{1t}(y) - h_{2t}(y) = \frac{1 - \beta}{r + \delta} (\eta_1 y_1 - \eta_2 y_2) \approx 0$$

so that a worker like $y$ is almost indifferent also for the firms in the two sectors. In fact, such a worker is not welcome in either sector for two reasons: first, he is not very productive in either sector ($y_1 < x_1$ and $x_2 = y_2$), second, he always has a good outside option in the other sector ($y_1 \approx \frac{y_2}{\eta_1}$). On the contrary, the specialized worker $x$ is very welcome in sector 1 for the same two reasons: he is productive in that sector, and he has not a comparable outside option in the other sector.

Therefore, taking averages with weights equal to the shares of open vacancies in the two sectors, $\alpha_t$ and $1 - \alpha_t$, the high-wage (more specialized) worker enjoys a higher acceptance rate:

$$\Pr[z \in A_t(x)] = 1 - G(h_{1t}(x)) > \alpha_t [1 - G(h_{1t}(y))] + (1 - \alpha_t) [1 - G(h_{2t}(y))] = \Pr[z \in A_t(y)]$$

(3.3)

More generally, since $\Pr[z \in A_t(x)]$ is strictly increasing in $y_1$, given $y_2$, each industry stayer $x$ in sector 1 has a conditional acceptance rate that strictly exceeds those of all industry movers $y$ with same skills in sector 2 (graphically, all points in the set of random searchers on a vertical line below $x_1$).

**Proposition 12 (Acceptance Rates of Stayers and Movers)** Fix any worker $x$ in the set of selective searchers; then, in equilibrium, all workers $y = (y_1, y_2)$ in the set of random searchers and with $y_2 = x_2$ have a lower conditional acceptance rate than $x$, i.e. the inequality in (3.3) holds strictly in equilibrium.

The bottomline is that the total effect of a change in type on the product of (3.2) and (3.3), the exit rate, is ambiguous. If the effect of the acceptance rate is stronger, we
observe that a high-wage, specialized worker, typically less mobile across industries (he searches always in the same market), exits faster from unemployment.

Trading Off Unemployment Spells for Good Jobs

As stated earlier, we can verify whether the specialized worker \( x \) is actually trading-off unemployment spells for a good job in sector 1. Worker \( x \) searches selectively if and only if

\[
p_{1t} J_{11}^x (x) > \pi_t [\alpha_t J_{11}^x (x) + (1 - \alpha_t) J_{21}^x (x)]
\]

(3.4)

I can rewrite this inequality in terms of contact rates, conditional (on contacting a vacancy) acceptance rates, and conditional (on accepting the match) surpluses. After doing so and rearranging, if:

\[
\frac{\int_{h_{11}(x)} Z J_{11} (x, z) \frac{dG(z)}{1 - G(h_{11}(x))}}{\int_{h_{21}(x)} Z J_{21} (x, z) \frac{dG(z)}{1 - G(h_{21}(x))}} > \frac{\pi_t (1 - \alpha_t) [1 - G(h_{21}(x))] }{(p_{1t} - \pi_t \alpha_t) [1 - G(h_{11}(x))]} > 1
\]

(3.5)

then the worker is searching selectively (first inequality, equivalent to 3.4) and is giving up to a strictly higher exit rate from unemployment (second inequality). In order for the two inequalities to hold simultaneously for some worker \( x \), we need the ratio between the conditional expectations of the surpluses to be much larger than one. In other words, if the worker has a sufficiently higher conditionally expected surplus from working in sector 1, then he can afford to trade off the expected capital gain with a longer unemployment spell.

The second inequality can be signed for each worker. By assumption, \( \pi_t > p_{1t} \), so that the effect of contact rates is the right one. But a specialized searcher has \( x_1 > \xi_{11} (x_2) > \frac{\pi}{\eta} x_2 \) and thus would be accepted less often in sector 2: \( h_{21} (x) > h_{11} (x) \). As a consequence, the ratio of conditional acceptance rates \( 1 - G(h_{11}(x)) \) is smaller than 1 and goes the opposite way. By monotonicity of the cutoff rules \( h_{11} (x) \), all specialized workers \( x \) below some threshold curve \( x_1 < \omega_{11} (x_2) \) satisfy the second inequality in (3.5).

As of the first inequality, it holds for all specialized searchers. Therefore we have the
following condition (recall from Chapt. 2 that $\xi_{1t} (\cdot )$ denotes the boundary line between random searchers and selective searchers in sector 1)

**Proposition 13** Let the function $\omega_t (\cdot )$ exactly equate exit rates with selective and random search, i.e. uniquely solve

$$p_{1t} [1 - G (h_{1t} (x_1, \omega_t (x_2)))] = \pi_t \alpha_t [1 - G (h_{1t} (x_1, \omega_t (x_2)))] + \pi_t (1 - \alpha_t) [1 - G (h_{2t} (x_1, \omega_t (x_2)))]$$

In equilibrium, at time $t$ there exists a non empty set of specialized searchers $x$ in sector 1 ($x_1 > \xi_{1t} (x_2)$) who trade off longer unemployment spells for selective search if and only if $\omega_t (x_2) > \xi_{1t} (x_2)$ for some $x_2$.

Depending on parameters, this set can be empty or, conversely, it can even include the whole set of selective searchers (when $\omega_t (\cdot ) > 1$), so that all of the latter are actually giving up to a potentially shorter spell in order to go for the jobs they prefer. Once more, it is a consideration of comparative rather than absolute advantages that determines the trade-off between values attached to different jobs and unemployment duration.

### 3.3 Genuine vs. Spurious Duration Dependence

Duration dependence of unemployment is a well documented fact. Two main hypotheses have been put forth to explain this phenomenon. **Genuine** duration dependence refers to changes in characteristics of the worker caused by the permanence in the unemployment state: for example, loss of skills, discouraged search, unemployment stigma. **Spurious** duration dependence is instead a mere composition effect: if some workers exit faster from unemployment for some unobserved (in the data) personal characteristics, then the remaining unemployed in a given cohort of entrants are those who exit slower and have longer spells.

This dichotomy is just an example of the classical distinction in statistics between state dependence and unobserved heterogeneity. The policy implications are different. Genuine dependence implies hysteresis in unemployment, spurious dependence does not.
A possible remedy is a subsidy aimed at long-term unemployed, to give them a job in the first case, to educate and train them in the second case.\footnote{Training could be a solution also in the case of skill loss, but it is important to distinguish this type of \textit{ex post} heterogeneity from the \textit{ex ante} heterogeneity of spurious dependence.}

The two hypotheses are not mutually incompatible, and their relative importance is mostly an empirical issue, as confirmed by the substantial wealth of applied work on this topic. Once more, the theoretical analysis has been lagging behind in identifying the mechanisms that can produce either genuine or spurious duration dependence.

The most recent, and careful, empirical investigations of this topic have been performed by Åbbring, van den Berg and van Ours (1995) on French data, and by van den Berg and van Ours (1996) on US data. They estimate the popular Mixed Proportional Hazard model, which posits that the exit rate from unemployment is a multiplicatively separable function of genuine dependence (the duration of unemployed), cyclical conditions (a function of calendar time), and unobserved heterogeneity. This restrictive assumption has the advantage of permitting non-parametric estimates of the relative magnitudes of the three effects on unemployment spells. The authors look separately at men and women, at black and white workers. The results indicate that, in both countries, unobserved heterogeneity matters much more than genuine duration dependence for all groups but for white males, for which the two effects are of similar importance. The procyclical pattern of exit rates is also confirmed.

Previous and less sophisticated studies had given slightly different answers, none denying the importance of heterogeneity.\footnote{E.g. Layard, Nickell and Jackman (1991), Chapter 6.} The next question then is: what types of unobserved heterogeneity affect the exit rate from unemployment, and through which channels and in what direction?

Based on their evidence, van den Berg and van Ours exclude the Ranking hypothesis of Blanchard and Diamond (1990, 1994) and Shimer (1995) from the number of the "suspects". Ranking occurs when heterogeneous workers apply for the same job, and firms hire the best applicant. In times of high unemployment, since there are more
good workers (in number) unemployed, the probability for each worker that a better competitor applies to the same vacancy is higher, hence duration dependence should be deepened. In fact, since it takes just one good workers to displace many less qualified applicants, good workers have a strong crowding-out potential. But the evidence says the opposite: duration dependence is slightly milder in recessions, at least for the first months of unemployment, within each of the four demographic groups.

The model presented in Chapter 1, which translates basically unaltered into a multi-sector framework (as proven in the previous section), also predicts spurious dependence. Like the Ranking hypothesis, the Fattening hypothesis seems to deliver the counterfactual prediction that, in bad times, duration dependence should be worsened, as exit rates from unemployment are more unequal. But the framework of Chapter 2, with choice of search methods according to comparative advantages, can help us understand the decline of duration dependence in recessions. To conclude this Chapter, I develop these two points in some detail, to verify how the hypotheses formalized in the previous chapters can cope with this evidence.

Spurious Duration Dependence

To see how spurious duration dependence is generated, let us track a cohort of entrants into unemployment in the one-sector economy of Chapter 1. I assume that the economy is in steady state, though this is inessential to the argument.\(^7\) At time 0, a cohort of workers with types in the participating population set \([\bar{z}, 1]\) are separated from their jobs. Whatever the composition of the inflow, I can parametrize it with the normalized density of workers in this cohort, say \(c(\cdot)\), summing up to one over its support \([\bar{z}, 1]\).\(^8\)

I can measure duration dependence by the rate of change (with duration) of the 

\(^7\)I drop the calendar time index \(t\), to concentrate on the duration of unemployment \(\tau\), but only for convenience of exposition. The next arguments hold both in and out of steady state, with minor modifications, because they follow from the fact that in equilibrium higher types exit faster from unemployment, which is always true in the analysis of Chapter 1.

\(^8\)In Chapter 1, separations occur both exogenously at rate \(\delta\), independent of the type of the worker, and endogenously due to shifts in the cutoff schedule \(h_t(\cdot)\). The argument, however, goes for any composition of the cohort \(c(\cdot)\) provided that it has full support on \([\bar{z}, 1]\), what is guaranteed by exogenous destruction.
average exit rate from unemployment of the remaining workers in the cohort. If \( E(x) \) denotes the steady state exit rate from unemployment of type \( x \), at time 0, when just entered, the average exit rate in the cohort is:

\[
E_0 = \int_1 E(x) \, c(x) \, dx
\]

Over time, workers get hired at type-dependent rates and leave the cohort, so that the latter shrinks in size. More precisely, by Large Numbers, the non-normalized composition of the cohort changes at duration \( \tau \) of unemployment according to the following differential equation:

\[
\dot{c}_\tau(x) = -c_\tau(x) \, E(x)
\]

Using the initial condition \( c_0(x) = c(x) \),

\[
c_\tau(x) = c(x) \cdot e^{-E(x)\tau}
\]

To obtain the normalized composition of the cohort, we must divide this density by its integral over the support. Therefore, the average exit rate in the cohort at duration \( \tau \) is

\[
E_\tau = \int_1 E(x) \frac{c(x) \, e^{-E(x)\tau}}{\int_1 c(\xi) \, e^{-E(\xi)\tau} \, d\xi} \, dx := \int_1 E(x) \, \ddot{c}_\tau(x) \, dx
\]

Since the exit rate from unemployment is strictly increasing in types, the cohort is “skimmed” from top to bottom, and we observe:

**Proposition 14 (Spurious Duration Dependence)** The average exit rate from unemployment in the cohort falls monotonically with the duration of unemployment:

\[
\dot{E}_\tau < 0.
\]

The proof, in the Appendix, exploits the strict monotonicity in types of the exit rates from unemployment to generate a strict First Order Stochastic Dominance ordering.
at all durations \( \tau \), which in turn is sufficient to generate a cohort-average exit rate that decreases strictly with duration. No genuine duration dependence is introduced, as individual exit rates are constant over time (and over duration) in this steady state exercise.

Also, from the expression for \( E_{\tau} \) it can be verified that a higher slope of the exit rate schedule \( E(\cdot) \) has two opposite effects on duration dependence. On the one hand, it enhances the inequality of exit speeds and thus the apparent duration dependence (direct effect, \( E(x) \)); on the other hand, given the composition of the inflow, it reduces the proportion of more qualified workers in the unemployment pool after the first month (indirect effect, \( e^{-E(\tau)} \)), so that their exit has a lower impact on the composition and the average exit rate of the remaining cohort.

**Pro cyclical Duration Dependence**

As already argued, if we believe that duration dependence is produced in a spurious way by ex ante unobserved heterogeneity, the Fattening model of Chapter 1 could suffer from the same counterfactual prediction as the Ranking hypothesis, namely that duration dependence should be countercyclical. Contrary to van den Berg and van Ours (1996) interpretation, the proportion-composition effect just mentioned says that the effect of a steeper exit rate schedule on spurious dependence is in fact not unambiguous.

In addition, the increased inequality of acceptance rates (see Chapt. 1) and thus exit rates in a recession can be partially offset by a change in the choice of the search method (see Chapt. 2). In times of high unemployment, when all exit rates fall across all workers’ types, each worker has the option to pursue less selective search strategies. The workers who exit unemployment less fast, namely marginal industry stayers and industry movers, are willing to give up to comparative advantages in order to absorb some of the fall in exit rates, hence they do exercise this option. Instead, specialized workers, typically industry stayers who exit first from unemployment, do not exercise this option and stick to selective search strategies, suffering the entire drop in exit rates. As a consequence, the inequality of exit rates among short-term and long-term unemployed, which is increased
in a recession by the tilting of acceptance rates (Chapt. 1), is moderated; the slow exit of qualified workers in the first months of unemployment might do the rest.

This is the only way to reconcile the finding of van den Berg and van Ours (1996) that, within demographic groups, spurious dependence is slightly but significantly procyclical, with the well documented fact (Blanchard and Diamond 1990) that the inequality of exit rates across age groups is procyclical. Notice that both studies are based on the CPS and cover overlapping periods (1968-86 and 1967-91, respectively).

More generally, if the selectivity of search is a continuous choice, following this same logic we would expect that the less specialized a worker, i.e. the longer his expected spell, the more he would try to adjust to the worse market conditions by searching less selectively. The very worst workers have no margin to adjust, because they were already searching completely randomly, and thus the decline in their exit rates cannot be compensated. In fact, those studies estimate that spurious duration dependence is milder in recessions only for the first months of duration.
3.4 Appendix

Proof of the results presented in Section 3.2

The value function satisfies:

\[ \hat{J}_{it}(x, z) = (r + \delta) J_{it}(x, z) - (1 - \beta) \eta_i (x_i + z) - b + c + \beta \Phi_t(x) \quad (3.6) \]

where \( \Phi_t(x) \) is defined in the text.

As in the previous chapters, imposing \( \lim_{t \to \infty} e^{-(r+\delta)t} J_{it}(x, z) = 0 \), I can write the solution to the firm arbitrage equation in the following implicit forward-looking form:

\[ J_{it}(x, z) = \frac{1 - \beta}{r + \delta} [\eta_i (x_i + z) - b + c] - \beta \int_t^{\infty} \Phi_s(x) e^{-(r+\delta)(s-t)} ds \quad (3.7) \]

This function is strictly increasing in \( z \), which immediately proves the existence of a cutoff rule to scrap the match: \( J_{it}(x, z) \geq 0 \iff z \geq h_{it}(x) \) and \( J_{it}^e(x) = \int_{h_{it}(x)}^\infty J_{it}(x, z) dG(z) \).

The crucial result is the monotonicity of the value function in both types.

**Lemma 9** The value function \( J_{it}(\cdot, \cdot, z) \) is strictly increasing in \( x_i \) and weakly decreasing in \( x_{-i} \).

**Proof.** First, by definition \( J_{it}(x_i, x_{-i}, h_{it}(x_i, x_{-i})) \equiv 0 \), so that substituting for \( h_{it}(x_i, x_{-i}) \) in (3.7) and rearranging yields

\[ h_{it}(x) = \frac{1 - \beta}{r + \delta} (\eta_i x_i - b + c) - \beta \int_t^{\infty} \Phi_s(x) e^{-(r+\delta)(s-t)} ds \]

and hence substituting back in (3.7)

\[ J_{it}(x, z) = \frac{1 - \beta}{r + \delta} [z - h_{it}(x)] \quad (3.8) \]

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Second, substituting from (3.7):

\[ J_{1t}(x, z) - J_{2t}(x, z) = \frac{1 - \beta}{r + \delta} (\eta_1 x_1 - \eta_2 x_2). \]  

(3.9)

Now, by contradiction, suppose the claim is false: there exist \((i, x_i > y_i, t)\) such that

\[ J_{it}(x_i, x_{-i}, z) - J_{it}(y_i, x_{-i}, z) \leq 0 \]  

(3.10)

**Step 1.** By (3.8), (3.10) implies \(h_{it}(x_i, x_{-i}) \geq h_{it}(y_i, x_{-i})\). Combining the last two inequalities in the following chain:

\[
J_{1t}^e(x_i, x_{-i}) - J_{1t}^e(y_i, x_{-i}) = \int_{h_{it}(x_i, x_{-i})}^{Z} J_{it}(x_i, x_{-i}, z) dG(z) - \int_{h_{it}(y_i, x_{-i})}^{Z} J_{it}(y_i, x_{-i}, z) dG(z) \leq \\
\leq \int_{h_{it}(y_i, x_{-i})}^{Z} J_{it}(x_i, x_{-i}, z) dG(z) - \int_{h_{it}(y_i, x_{-i})}^{Z} J_{it}(y_i, x_{-i}, z) dG(z) \leq \\
\leq \int_{h_{it}(y_i, x_{-i})}^{Z} J_{it}(y_i, x_{-i}, z) dG(z) - \int_{h_{it}(y_i, x_{-i})}^{Z} J_{it}(y_i, x_{-i}, z) dG(z) = \\
= 0
\]

Also, by (3.9), the following two terms are equal, and by (3.10) and the assumption \(x_i > y_i\) the second (and then both) strictly negative:

\[
J_{-it}(x_i, x_{-i}, z) - J_{-it}(y_i, x_{-i}, z) = J_{it}(x_i, x_{-i}, z) - J_{it}(y_i, x_{-i}, z) + \frac{1 - \beta}{r + \delta} \eta_i (y_i - x_i) < 0
\]

Similarly to the above, the negativity of the leftmost difference implies \(h_{it}(x_i, x_{-i}) > h_{it}(y_i, x_{-i})\) and thus with the same chain of inequalities \(J_{-it}^e(x_i, x_{-i}) - J_{-it}^e(y_i, x_{-i}) < 0\). To conclude Step 1 of the proof, notice that by substitution from (3.7) this negative difference is also equal to:

\[
0 > J_{-it}(x_i, x_{-i}, z) - J_{-it}(y_i, x_{-i}, z) = \beta \int_{t}^{\infty} [\Phi_s(y_i, x_{-i}) - \Phi_s(x_i, x_{-i})] e^{-(r+\delta)(s-t)} ds
\]  

(3.11)

**Step 2.** Summing up, in Step 1, arguing by contradiction (3.10), we have obtained
the inequality (3.11) and:

\[ J_t^s (x_i, x_{-i}) - J_t^s (y_i, x_{-i}) \leq 0 \]
\[ J_{-it}^s (x_i, x_{-i}) - J_{-it}^s (y_i, x_{-i}) < 0 \]

By definition of \( \Phi_t (x_i, x_{-i}) \), these two inequalities together imply \( \Phi_t (x_i, x_{-i}) \leq \Phi_t (y_i, x_{-i}) \).

Using (3.6) for \( x_i \) and \( y_i \) and subtracting:

\[ \dot{J}_{it} (x_i, x_{-i}, z) - \dot{J}_{it} (y_i, x_{-i}, z) = (r + \delta) [J_{it} (x_i, x_{-i}, z) - J_{it} (y_i, x_{-i}, z)] + \\
- (1 - \beta) \eta_t (x_i - y_i) + \beta [\Phi_t (x_i, x_{-i}) - \Phi_t (y_i, x_{-i})] < 0 \]

The negative sign follows because the first and third terms on the r.h.s. are non positive, and \( x_i - y_i > 0 \). Similarly:

\[ \dot{J}_{-it} (x_i, x_{-i}, z) - \dot{J}_{-it} (y_i, x_{-i}, z) = \\
= (r + \delta) [J_{-it} (x_i, x_{-i}, z) - J_{-it} (y_i, x_{-i}, z)] + \beta [\Phi_t (x_i, x_{-i}) - \Phi_t (y_i, x_{-i})] < 0 \]

Hence, iterating the reasoning, for all \( s > t \)

\[ J_{ts} (x_i, x_{-i}, z) - J_{ts} (y_i, x_{-i}, z) < 0 \]
\[ J_{-ts} (x_i, x_{-i}, z) - J_{-ts} (y_i, x_{-i}, z) < 0 \]

\[ \Rightarrow \Phi_s (x_i, x_{-i}) < \Phi_s (y_i, x_{-i}) \]

This implies:

\[ \beta \int_t^\infty [\Phi_s (y_i, x_{-i}) - \Phi_s (x_i, x_{-i})] e^{-(r+\delta)(s-t)} ds > 0 \]

which contradicts (3.11).

The second claim is proven in a similar manner. \( \blacksquare \)
Proof of Proposition 14: Spurious Duration Dependence

Proof. The claim is that the average exit rate of the unemployed cohort is strictly decreasing in duration. Differentiating this rate with respect to duration I obtain:

\[
\dot{E}_r = \frac{dE_r}{d\tau} = - \int_{\xi}^{1} [E(x)]^2 \frac{c(x) e^{-E(x)\tau}}{\int_{\xi}^{1} c(\xi) e^{-E(\xi)\tau} d\xi} dx - E_r \frac{\int_{\xi}^{1} E(\xi) c(\xi) e^{-E(\xi)\tau} d\xi}{\int_{\xi}^{1} c(\xi) e^{-E(\xi)\tau} d\xi} = \\
= - \int_{\xi}^{1} [E(x)]^2 \frac{c_r(x)}{\int_{\xi}^{1} c_r(\xi) d\xi} dx + E_r^2
\]

Rearranging terms and simplifying, this term is negative and the claim is true if and only if

\[
\int_{\xi}^{1} E(x) \frac{E(x) c(x) e^{-E(x)\tau}}{\int_{\xi}^{1} E(\xi) c(\xi) e^{-E(\xi)\tau} d\xi} dx > E_r \quad (3.12)
\]

Let

\[
\chi_\tau(x) := \frac{E(x) c(x) e^{-E(x)\tau}}{\int_{\xi}^{1} E(\xi) c(\xi) e^{-E(\xi)\tau} d\xi}
\]

which is a proper density on \([\xi, 1]\), because it is strictly positive and sums up to one. Then the claim (3.12) is equivalent to

\[
\int_{\xi}^{1} E(x) \chi_\tau(x) dx > \int_{\xi}^{1} E(x) \tilde{c}_\tau(x) dx
\]

Since the exit rate from unemployment is a strictly increasing function of types (both in and out of steady state, see Chapt. 1 or Chapt. 2), a sufficient condition is that at all durations \(\tau\):

\[
\chi_\tau(x) \overset{FSD}{\succ} \tilde{c}_\tau(x)
\]

or, applying the definition of strict First Order Stochastic Dominance, for all \(y < 1\),

\[
\int_{\xi}^{y} [\chi_\tau(x) - \tilde{c}_\tau(x)] dx < 0
\]

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with equality at $y = 1$. Substituting for the corresponding expressions the last inequality is equivalent to

$$\int_y^x \left[ \frac{E(x) c(x) e^{-E(x)r}}{\int_x^1 E(\xi) c(\xi) e^{-E(\xi)r} d\xi} - \frac{c(x) e^{-E(x)r}}{\int_x^1 c(\xi) e^{-E(\xi)r} d\xi} \right] dx < 0$$

and, rearranging, to

$$\frac{\int_y^x E(x) c(x) e^{-E(x)r} dx}{\int_y^x c(x) e^{-E(x)r} dx} < \frac{\int_x^1 E(\xi) c(\xi) e^{-E(\xi)r} d\xi}{\int_x^1 c(\xi) e^{-E(\xi)r} d\xi}$$

To avoid confusion with the symbol $E(\cdot)$, which stands for the exit rate function, let $\varepsilon_r [\cdot]$ denote the expectation operator with respect to the measure induced by the density $c(\cdot) e^{-E(\cdot)r}$. Then, for all $y < 1$, the last inequality can be rewritten as:

$$\varepsilon_r [E(x) | x \leq x \leq y < 1] < \varepsilon_r [E(x)]$$

This inequality, and then the claim, is always true for all $y < 1$ because $E(\cdot)$ is a strictly increasing function. ■

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Epilogue

In this dissertation I have investigated the implications of different types of heterogeneity of workers' productivities for equilibria in labor markets, in the presence of allocative frictions and consequent real rigidity of wages. I have introduced ex ante and ex post heterogeneity both separately and jointly, in order to study their independent roles and their interaction. I have considered ex ante heterogeneity both in terms of absolute (one-dimensional) and comparative (multi-dimensional) advantages. I have introduced ex post heterogeneity only in the form of matching. The major implications that I have emphasized concern average labor productivity (Chapter 1), inter-industry labor mobility (Chapter 2), and unemployment duration (Chapter 3). In the process, I have related these theoretical implications to the results of an extensive body of microeconometric literature.

The main focus of the analysis has been on the causes of exit from unemployment, primarily the choice of the search method and the conditional acceptance rate. In fact, the title of the dissertation refers explicitly to job search, not to job tenure. As a theory of unemployment within the flow approach, this analysis is missing an important aspect: an explicit investigation of the effects of heterogeneity on entry into unemployment. We know that entry and exit are different phenomena, determined by factors that overlap to a very limited extent. This is confirmed by the empirical regularities on entry, which differ remarkably from those on exit.
Traditionally, the economic analysis of job attachment and tenure is based on the only dimension of heterogeneity that I have mentioned in the Introduction but have not considered in this work: the firm-specific human capital that is accumulated on the job, through training or learning-by-doing. This very important diversification of productivities develops after exit from unemployment has occurred; but in a world of forward-looking agents, it affects exit as well. For example, we observe that some types of workers have systematically lower entry rates than others; both firms and workers certainly take this fact into account ex ante, when deciding whether to implement a match.

The existing analyses of entry, though, suffer from the same limitations that this work has tried to emphasize on the side of exit: the different dimensions of heterogeneity are either ignored or they are considered separately. We know very little on the mechanisms through which better or more specialized workers develop a larger attachment to their jobs. We know certainly no more about the implications of such mechanisms for cross-sections of unemployment, average labor productivity, inter-industry mobility and other macroeconomic phenomena of interest. Therefore, a most promising direction of future research appears the investigation of entry into unemployment of heterogeneous workers, along the lines of the analysis of exit from unemployment performed in this dissertation.
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