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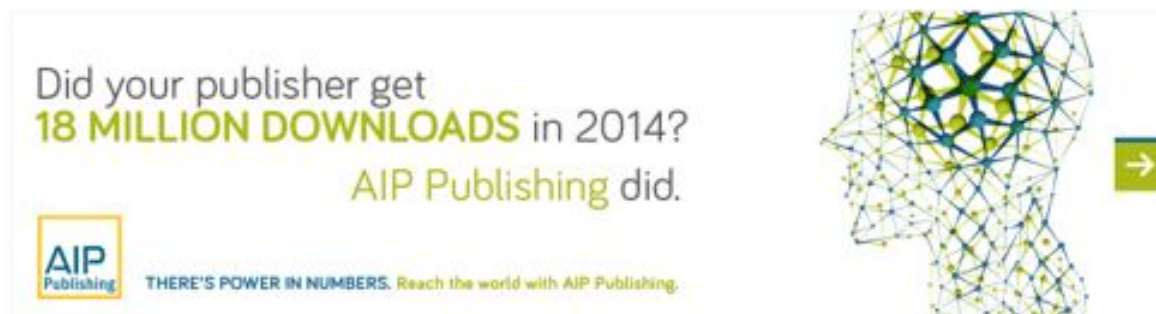
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## Partial coalescence of soap bubbles

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We present the results of an experimental investigation of the merger of a soap bubble with a planar soap film. When gently deposited onto a horizontal film, a bubble may interact with the underlying film in such a way as to decrease in size, leaving behind a smaller daughter bubble with approximately half the radius of its progenitor. The process repeats up to three times, with each partial coalescence event occurring over a time scale comparable to the inertial-capillary time. Our results are compared to the recent numerical simulations of Martin and Blanchette [“Simulations of surfactant effects on the dynamics of coalescing drops and bubbles,” *Phys. Fluids* **27**, 012103 (2015)] and to the coalescence cascade of droplets on a fluid bath. © 2015 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4923212>]

When a drop of fluid of radius  $R$ , density  $\rho$ , viscosity  $\mu$ , and surface tension  $\sigma$  is deposited on a bath of the same liquid, it can undergo complete or partial coalescence according to the relative magnitude of viscous and gravitational forces to capillary forces as are prescribed by the Ohnesorge number  $Oh_d = \frac{\mu}{\sqrt{\sigma R \rho}}$  and the Bond number  $Bo_d = \frac{\rho g R^2}{\sigma}$ , respectively. In partial coalescence, as arises when  $(Oh, Bo) \ll 1$ , the initial drop volume is not entirely transferred to the bath and a smaller daughter droplet remains. This phenomenon was first investigated by Charles and Mason<sup>1,2</sup> and later explored by Thoroddsen and Takehara,<sup>3</sup> who observed that the partial coalescence events can occur several times in succession, giving rise to a “coalescence cascade.” During each step, the droplet diameter is approximately halved, and the coalescence time scale is prescribed by the inertial-capillary time  $\tau_\sigma = \sqrt{\frac{\rho R^3}{\sigma}}$ . The time of partial coalescence for droplets, from initial droplet-bath contact to pinch-off, is typically in the range of  $(1.4 - 2.0)\tau_\sigma$ .<sup>4</sup>

Provided gravitational and viscous forces are negligible, the partial coalescence process is defined by a single time scale ( $\tau_\sigma$ ) and is so said to be self-similar.<sup>4</sup> In this regime, Blanchette and Bigioni<sup>5,6</sup> demonstrated that the partial coalescence phenomenon was a consequence of capillary waves vertically stretching the droplet, delaying vertical collapse, and allowing pinch-off to occur at the base. They predicted and observed a critical Ohnesorge number above which partial coalescence does not occur. Aryafar and Kavehpour developed a relationship for the daughter drop size ratio as a function of the Ohnesorge number, thus characterizing the dependence on drop viscosity.<sup>7</sup> The drop size range in which partial coalescence occurs was investigated experimentally by Chen *et al.*<sup>8</sup> and numerically by Yue *et al.*<sup>9</sup> Gilet *et al.* showed that partial coalescence can be observed in the absence of capillary waves and total coalescence even in their presence.<sup>4</sup> The presence of surfactants in the drop or bath can either inhibit or favor the phenomenon.<sup>10,11</sup> While the partial coalescence of drops has been widely investigated,<sup>12,13</sup> the analogous phenomenon for bubbles has only been considered by Martin and Blanchette,<sup>11</sup> whose numerical simulations indicate the possibility of partial coalescence. In this letter, we present the results of the first experimental investigation of the merger of a soap bubble with a soap film.

Consider a soap bubble of radius  $R$  approaching a horizontal soap film. The pressure above and below the film is  $P_a$ , while that inside the bubble is elevated by the capillary pressure  $4\sigma/R$ .

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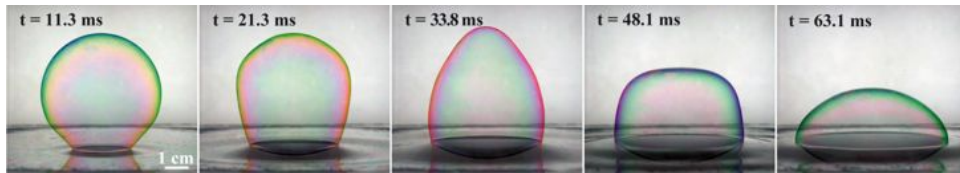


FIG. 1. Time sequence of a soap bubble becoming trapped in a planar soap film. (Multimedia view) [URL: <http://dx.doi.org/10.1063/1.4923212.1>]

Once the intervening air layer separating the film and bubble has drained to a critical thickness, coalescence will be initiated. If the film does not rupture beneath the bubble, the high pressure air inside the bubble cannot escape, and the bubble will simply become trapped in the soap film, forming a lens (Figure 1), the form of which minimizes the system's surface energy. However, if the film does break at the point of contact, the high pressure air inside the bubble can be evacuated. In this case, one might expect that the surface would simply return to a planar configuration in order to minimize the overall surface area. However, the total collapse of the bubble is resisted by capillary waves that sweep up the bubble, prompting the pinch-off of a smaller daughter bubble (Figure 2). The process can be repeated successively, giving rise to a coalescence cascade (Figure 3). Up to three partial coalescence events were observed in our experiments, with the bubble reducing in radius by approximately one half in each event.

A schematic of the experimental setup is shown in Figure 4. For the present experiments, we use a solution of tap water and Dawn<sup>®</sup> Professional Detergent at 9.1% by weight for both the bubble solution and soap film. The concentration of the solution is well above the critical micelle concentration, which was estimated to be less than 0.1%.<sup>14</sup> We found that soap concentrations below 5% were less favorable to partial coalescence. Once well mixed, the solution is poured both into a small reservoir and the base of a transparent tank. A lid (with a few access holes) is added to the top of the tank, reducing evaporation in the test chamber and the influence of air currents, thereby extending the life of the soap film noticeably. A circular acrylic frame (with diameter 18 cm) is suspended on guide wires and lowered into the solution. It is then extracted from the fluid, resulting in the formation of a circular planar soap film. The film is tilted to allow for gravitational drainage until a black film forms on its upper side, after which it is aligned with the horizontal. By viewing the fringes of reflected white light,<sup>15</sup> we estimate a mean thickness of  $h < 1 \mu\text{m}$ .

A cylindrical drink stirrer of inner diameter 2.5 mm is dipped into the small reservoir and draws in a small amount of the soap solution by capillary action. A single bubble is blown outside of the tank to remove excess fluid from the tip of the stirrer before the test bubble is blown. A centimeter scale bubble of radius  $R$  is then blown through a small access hole in the center of the lid which gently lands on the soap film. A white panel LED (light-emitting diode) lamp is placed behind the

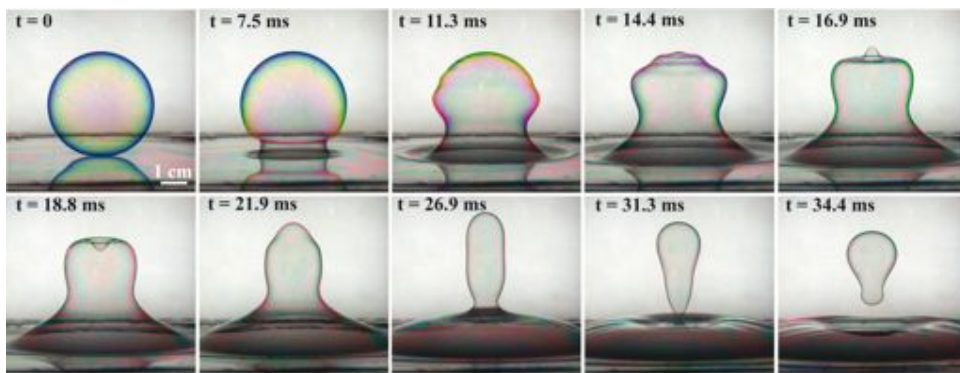


FIG. 2. Time sequence of the partial coalescence of a soap bubble. (Multimedia view) [URL: <http://dx.doi.org/10.1063/1.4923212.2>]

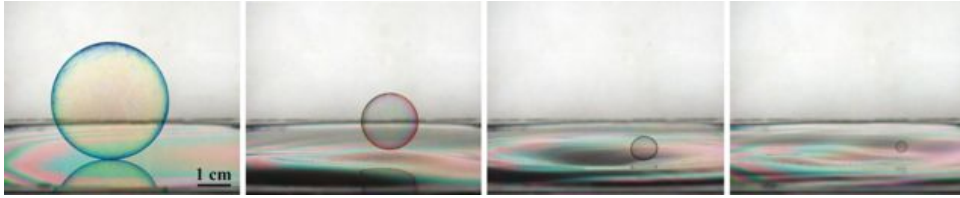


FIG. 3. Sequence of three successive steps of a coalescence cascade, from left to right.

box and light is diffused by a translucent sheet. A fast camera placed in front of the box acquires videos at 1600 frames/s.

If the film is not drained, we observed partial coalescence in only approximately 10% of attempts, however, after draining, this rate increased to nearly 80%. In cases where partial coalescence is not observed, the bubble typically attaches to the film, as shown in Figure 1. Our experiments suggest that both a thin film and a thin bubble ( $\leq 1 \mu\text{m}$ ) are conducive to partial coalescence, presumably because the soap film must rupture at the contact point in order for the air in the bubble to be expelled across it. The simulations of Martin and Blanchette<sup>11</sup> are initialized after film rupture, so the air is allowed to escape in all cases, regardless of film or bubble thickness.

Our soap-water solution has density  $\rho = 10^3 \text{ kg/m}^3$  and surface tension  $\sigma = 24.3 \pm 2.2 \text{ mN/m}$ , the latter being inferred from the period of oscillation of a free-falling droplet of known size.<sup>16</sup> With a film thickness  $h < 1 \mu\text{m}$ , we estimate the Bond number  $Bo = \frac{\rho R h g}{2\sigma} < 0.01$ , indicating that gravity plays a negligible role, as is confirmed by the near sphericity of the initial bubble. Using  $\mu_a = 1.8 \cdot 10^{-5} \text{ Pa s}$  and  $\rho_a = 1.2 \text{ kg/m}^3$  as the viscosity and density of air, respectively, we estimate the Ohnesorge number to be  $Oh = \frac{\mu_a}{\sqrt{2\sigma R \rho_a}} < 0.001$ : the viscous effects of the air are also negligible. In our experiments, the behavior is thus dominated by surface tension and inertia: the bubble is evacuated by the capillary overpressure and the flow is resisted by the inertia of the air.

We expect the partial coalescence process to be dominated by two time scales, specifically, the time necessary for the bubble to deflate and the time required for the capillary waves to span the bubble. First, we estimate the typical time scale of evacuation of the air in the bubble. Note that for a soap bubble, we can neglect the film mass compared to the air inside the bubble provided that  $\rho R^2 h \ll \rho_a R^3$  or  $R \gg \rho h / \rho_a \sim 1 \text{ mm}$ , as was always the case for the centimetric bubbles considered. Once coalescence is initiated, air is expelled by the capillary overpressure in the bubble  $4\sigma/R$  which is resisted by the inertia of the air being evacuated. Bernoulli's equation relates the overpressure to the velocity  $u$  of air exiting the bubble,

$$\frac{4\sigma}{r} = \frac{1}{2} \rho_a u^2. \quad (1)$$

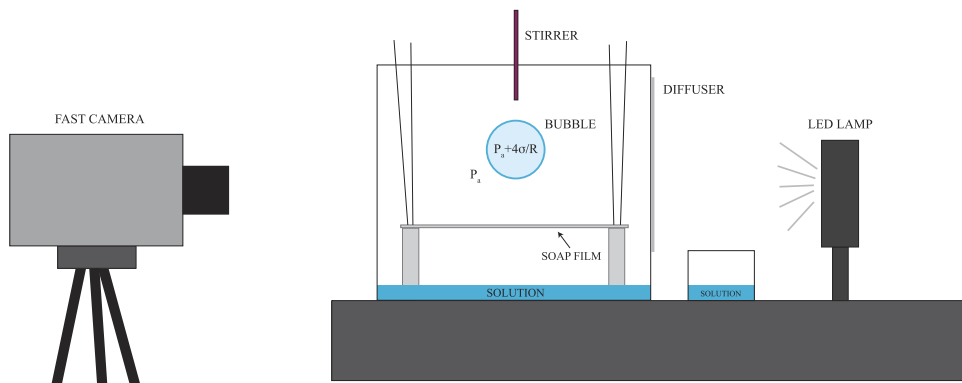


FIG. 4. Schematic of experimental setup. A circular soap film is generated by dipping a circular frame into a soap solution. A bubble is then released and settles onto it.

The rate of change of the bubble radius is related to the typical exit velocity  $u$  and the film hole size  $r_h$  by continuity:  $4\pi r^2 \frac{dr}{dt} = \pi r_h^2 u$ . Assuming that the hole size  $r_h$  is equal to the radius of the bubble  $r$  (see Figure 2 beyond 7.5 ms), we find that  $u = 4 \frac{dr}{dt}$ . We can then substitute this back into Eq. (1) and integrate to find

$$\frac{r}{R} = \left[ 1 - \frac{3t}{4t_c} \right]^{2/3}, \quad (2)$$

where  $t_c = \sqrt{\frac{\rho_a R^3}{2\sigma}}$ . Thus, if the bubble is reduced in radius by a constant ratio during each event, we expect the time scale of the deflation process to scale with  $t_c$ . For example, if the bubble is reduced by half its initial radius ( $r/R = 1/2$ ), we predict that the evacuation should occur over a time  $t_e = 0.9t_c$ . To calculate the time required for the capillary waves generated at coalescence to span the bubble, we use an estimate for the wave speed  $v$  on a soap film,<sup>17</sup>  $v = \sqrt{\frac{2\sigma}{\rho h + \rho_a \lambda}}$ , where  $\lambda$  is the wavelength of the capillary waves. Since  $\rho_a R \gg \rho h$ , the capillary wave speed is dominated by the inertia of the air. Taking  $\lambda \sim R$ , we find  $v \sim \sqrt{\frac{2\sigma}{\rho_a R}}$  which yields the relevant time scale for wave propagation  $\frac{R}{v} \sim t_c$ . Note that  $t_c$  is also the relevant time scale for normal-mode oscillations of a soap bubble when the film inertia is negligible.<sup>18</sup> The evacuation and wave-induced distortion of the centimetric bubble thus occur over the same inertial-capillary time scale. We refer to this process as self-similar since the dynamics is dominated by a single time scale ( $t_c = \sqrt{\frac{\rho_a R^3}{2\sigma}}$ ). We expect this self-similarity to break down for small bubbles ( $R < 1$  mm), when the inertia of the film becomes comparable to that of the air.

We filmed 166 partial coalescence events, many of which were part of a series of multiple events, and measured the daughter and initial bubble radii,  $R_D$  and  $R$ , respectively. Figure 5(a) shows that  $R_D/R$  is largely independent of  $R$  for the parameter regime examined.  $R_D/R \approx 0.47$  on average, remarkably similar to the case of droplets coalescing into a bath at low  $Bo$  and  $Oh$ .<sup>3,4,7</sup> The ratio is always in the range  $0.35 < R_D/R < 0.6$ , in good agreement with the simulations of Martin and Blanchette.<sup>11</sup> The distribution of  $R_D/R$  in Figure 5(b) indicates a peak centered at  $0.48 < R_D/R < 0.50$ .

We also measured the time of partial coalescence  $t_p$  defined as the time between film-film contact and pinch-off of the daughter bubble.  $t_p$  is plotted as a function of the initial bubble radius  $R$  in Figure 6(a), validating that  $t_p \sim R^{3/2}$ . The distribution in Figure 6(b) indicates that  $t_p \approx 2t_c$ .

In Figure 7(a), we compare the maximum width  $w$  of the bubble as a function of time for three different bubbles undergoing a single partial coalescence event. When appropriately non-dimensionalized (Figure 7(b)), the curves collapse onto one another, again indicating the self-similarity of the process for the parameters considered. We see that the dynamics of  $w$  is characterized by two regimes, the duration of which both scale with the capillary time  $t_c$ . During the

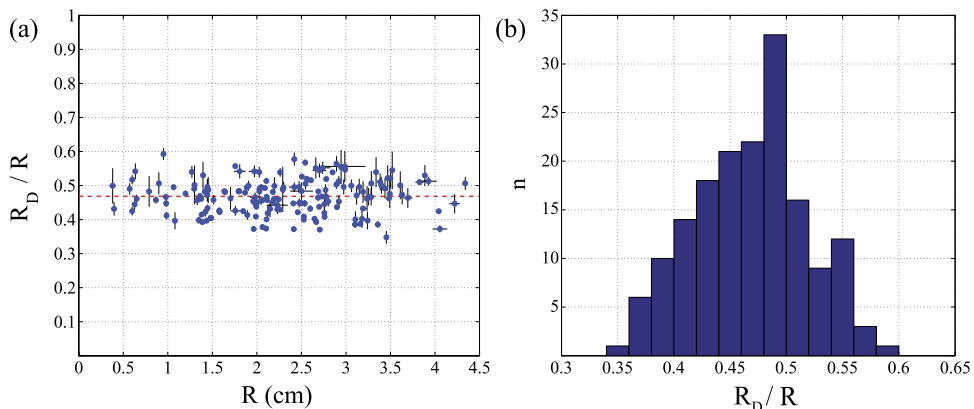


FIG. 5. (a) Ratio of the radius of the daughter bubble  $R_D$  to the initial bubble  $R$  as a function of  $R$ . (b) Distribution of ratios  $R_D/R$ . Total number of partial coalescence events  $N = 166$ .



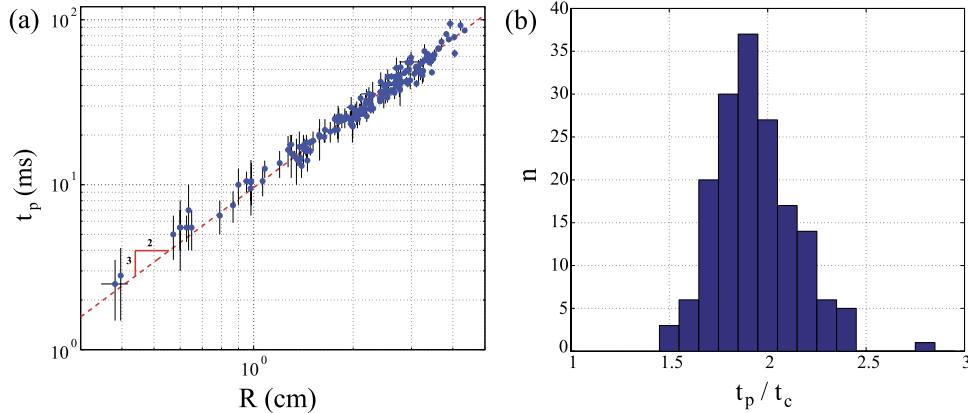


FIG. 6. (a) Time of partial coalescence  $t_p$  as a function of the initial bubble radius  $R$ . (b) Distribution of ratios  $t_p/t_c$ , where  $t_c = \sqrt{\frac{\rho_d R^3}{2\sigma}}$  is the inertial-capillary time scale. Total number of partial coalescence events  $N = 166$ .

first, the opening phase, which lasts for approximately  $0.8t_c$ , the bubble maintains nearly constant width. This opening time is related to the time required for the capillary waves to span the height of the bubble, that is,  $\sim t_c$ . This is followed by the evacuation phase, in which the bubble width decreases with time in a manner consistent with Eq. (2). If we include an appropriate time delay to account for the opening phase, the simple model does very well in describing the evolution of the bubble width during the evacuation phase. A delay of approximately  $t_d = 0.8t_c$  best fits the data, which is the sole fitting parameter used. Combining this delay time with our earlier prediction for the evacuation time to reduce the bubble radius by one-half ( $t_e = 0.9t_c$ ), we arrive at an estimate of  $t_p = t_d + t_e = 1.7t_c$  which is consistent with the measured values (Figure 6(b)).

We also compare the evolution of the width to that observed in the analogous droplet system,<sup>5</sup> shown as the magenta thin line in Figure 7(b). Note that the time scale is normalized by  $\tau_\sigma$  rather than the  $t_c$  used for the bubbles. The evacuation stage is analogous to that observed for bubbles, but the delay time over which the drop radius remains nearly constant is significantly decreased.

Coalescence cascades are less frequently observed in the bubble system than with drops deposited on a bath, likely due to the increase in film thickness following each step, which tends to

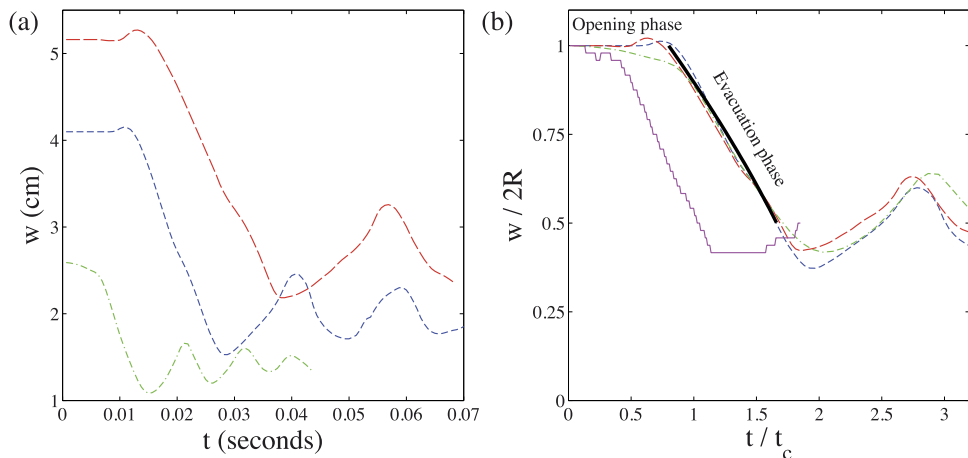


FIG. 7. Maximum width of bubble  $w$  as a function of time for three different partial coalescence events,  $R = 2.58$  cm (red, long dashes),  $R = 2.05$  cm (blue, short dashes), and  $R = 1.30$  cm (green, dashed-dotted). The black thick line represents the result from Eq. (2) taking  $w = 2r$  and adding a time delay of  $t_d = 0.8t_c$ :  $w(t) = 2R \left[ 1 - \frac{3(t-t_d)}{4t_c} \right]^{2/3}$ . The time  $t = 0$  corresponds to the point at which coalescence is initiated. The magenta thin line in (b) represents width data for a droplet of ethanol of radius  $R = 0.535$  mm coalescing with an ethanol bath, extracted from the video available in the supplementary materials of Blanchette and Bigioni.<sup>5</sup> The time scale for this curve is normalized by  $\tau_\sigma$  rather than  $t_c$ .

favor trapping over partial coalescence. Moreover, the weakly damped residual waves on the film generated by prior events interfere with the process and may displace the bubble from the film. Martin and Blanchette<sup>11</sup> predict that up to 10 steps should be possible in such a cascade, but they assume that the film is quiescent and do not consider the possibility of trapping. Another scenario not considered by Martin and Blanchette<sup>11</sup> but evident in our experiments is an intermediate situation between trapping and partial coalescences, in which a delayed partial coalescence occurs. Here, the bubble begins to transition towards a trapped state but the film then breaks. As the film rupture occurs significantly after initial contact, the capillary wave propagation is delayed and the radius of the daughter bubble ejected is significantly smaller than half that of its progenitor. In a few cases, we also observed the ejection of a millimetric bubble as a result of the convergence of the capillary waves at the top of the bubble. This tiny granddaughter bubble is then trapped inside the daughter bubble. A similar effect has been observed for droplets coalescing with a flat liquid surface<sup>6</sup> or impacting on a solid surface.<sup>19</sup>

We have reported the first experimental investigation of the coalescence cascade of a soap bubble. Several key features are shared with the analogous phenomenon for droplets. In the inertial-capillary regime, the size of the bubble or drop is reduced by approximately half during each event ( $R_D/R \approx 0.5$ ) and the process occurs over an inertial-capillary time scale  $t_c = \sqrt{\frac{\rho_a R^3}{2\sigma}}$ . We also observed new scenarios peculiar to the bubble-film case including trapping and delayed partial coalescence. A thicker film favors trapping behavior due to the increased resistance of the film to rupture. Furthermore, the limits of self-similarity in the bubble and droplet case differ. As the bubble size decreases, the self-similarity breaks down when the inertia of the film becomes significant. For droplets, the self-similarity breaks down when viscous effects become significant. The coalescence cascade may arise for relatively large bubbles since the effects of gravity are diminished. For bubbles, the time between coalescence events is typically on the order of seconds, meaning that a cascade can be witnessed with the naked eye. The resulting visibility along with the relatively simple experimental setup make this phenomenon suitable for tabletop demonstrations.

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