Heavy Particle Modes and Signature of the I-Regime

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Abstract

The recently discovered properties of the I-confinement Regime are explained as resulting from the excitation of a heavy particle mode. The theoretically predicted mode phase velocity in the direction of the electron diamagnetic velocity and the induced confinement of impurities at the edge of the plasma column have been confirmed by the experiments. The direction of the mode phase velocity is consistent with that (opposite) of the spontaneous rotation in the plasma core. The mode is of the "ion-mixing" type, in that it does not produce any electron transport across the fields and it involves significant poloidal magnetic field fluctuations.

Keywords: I-Regime, impurity, heavy particle mode, fluctuations

1. Introduction

Recent investigations of the so called I-Regime by the Alcator C-Mod machine have brought to light features of this regime that we propose to explain by the excitation of a new kind of heavy particle mode [1, 2] at the edge of the plasma column. This mode involves both density and magnetic field fluctuations. The transport of heavy particles (impurities) that is produced by it is outward, while the main plasma ions are transported inward. This can lead to a high degree of plasma purity, a condition necessary to reach ignition in fusion burning plasma. These features are consistent with those of the observed mode having a frequency about 200 kHz and with the finding [3, 4, 5, 6, 7, 8] that, in the I-Regime, impurities are confined at the edge. The proposed mode has a phase velocity in the direction of the electron diamagnetic velocity, a feature was found first theoretically and that has been observed later experimentally [9].

The mode is "dissipative" in that its growth rate depends inversely on the effective finite longitudinal thermal conductivity of the main ion population for which the relative temperature gradient exceeds the relative density gradient. Under this condition, the mode driving factor is the main ion temperature gradient in association with a finite impurity temperature. The observed spontaneous rotation of the plasma column is in the direction of the ion diamagnetic velocity. This is consistent with the existence of, according to the theory presented in Ref. [10], a recoil process at the edge of the plasma column resulting from ejection of angular momentum, in the opposite direction, associated with the excited mode.

The electromagnetic fluctuations that are observed experimentally as being connected with the density fluctuations produced by the mode are justified as being associated with a plasma current density introduced to justify the reduction of the ion fluxes when the mode is saturated relative to the phase where the linearized description of the mode is valid. The effects of the longitudinal (to the magnetic field) thermal conductivity of the heavy population are required to be relatively small for the validity of the presented theory.

2. Plane Geometry Formulation

At first we consider a plane plasma configuration where the equilibrium magnetic field $\mathbf{B} \simeq B\,\mathbf{e_z}$ and all quantities depend on x. The components are the electrons with density $n_e(x)$, the main ion population with density $n_i(x)$ and $Z = 1$ and a heavy ion (impurity) population with density $n_I(x)$ and mass number $A_I \gg 1$. Clearly $n_e = n_i + Zn_I$. We consider the electric field perturbations with $\hat{\mathbf{E}} \simeq -\nabla\hat{\phi}$ and the simplest model of the modes that shall be identified is $\hat{\phi} = \tilde{\phi}\,(x)\sin(k_{\shortparallel} z)\exp(-i\omega t + ik_y y)$ as they are standing along the field and propagating across it. Since the expected dispersion relation depends on $k_{\shortparallel}^2$, we may analyze at

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first longitudinally propagating modes $\hat{\phi} = \tilde{\phi}(x)\exp(-i\omega t + ik_{\shortparallel}z + ik_y y)$ where, for the sake of simplicity, $k_y^2 \gg \left|\left(d^2\tilde{\phi}\big/dx^2\right)\big/\tilde{\phi}\right|$. The relevant longitudinal phase velocities are in the range

$$v_{thI}^2 \lesssim |\omega/k_{\shortparallel}|^2 < v_{thi}^2 < v_{the}^2 \tag{2.1}$$

where $v_{thj}^2 \equiv 2T_j/m_j$, $j = i$, e and I. Under these conditions the key equations that describe the modes of interest are $Z\hat{n}_I \simeq \hat{n}_e - \hat{n}_i$, $0 \simeq -ik_{\shortparallel}\hat{n}_e T_e + ik_{\shortparallel}en_e\hat{\phi}$, $0 \simeq -ik_{\shortparallel}\hat{n}_i T_i - ik_{\shortparallel}en_i\hat{\phi}$,

$$-i\omega n_I m_I \hat{u}_{I\shortparallel} \simeq -ik_{\shortparallel}\left(\hat{n}_I T_I + n_I\hat{T}_I\right) - ik_{\shortparallel}eZn_I\hat{\phi} \tag{2.2}$$

where $-i\omega\hat{n}_I + \hat{v}_{Ex}\,dn_I/dx + ik_{\shortparallel}n_I\hat{u}_{I\shortparallel} \simeq 0$ and $\hat{v}_{Ex} = -ik_y c\,\hat{\phi}\big/B$ and the thermal energy balance for the impurity population in the limit of negligible thermal conductivity

$$\frac{3}{2}n_I\left(-i\omega\hat{T}_I + \hat{v}_{Ex}\frac{dT_I}{dx}\right) + ik_{\shortparallel}n_I T_I\hat{u}_{I\shortparallel} \simeq 0 \tag{2.3}$$

where we assume that $T_I \simeq T_i \simeq T_e$. Moreover, if we consider the impurity density profile to be relatively peaked, that is $|(dT_I/dx)/T_I| \ll |(dn_I/dx)/n_I|$, Eq. (2.3) reduces simply to $\hat{T}_I \simeq 2k_{\shortparallel}T_I\hat{u}_{I\shortparallel}/(3\omega)$ and $\hat{T}_I\big/T_I \simeq \left(\hat{n}_I/n_I + Ze\hat{\phi}\big/T_I\right)$
$\cdot\, 2\omega_{IA}^2/\left(5\omega^2 - 2\omega_{IA}^2\right)$ where $\omega_{IA}^2 \equiv (5/3)\left(k_{\shortparallel}^2 T_I/m_I\right)$.

Considering that

$$Z\hat{n}_I \simeq e\hat{\phi}\,\bar{\bar{n}}\big/\bar{\bar{T}} \tag{2.4}$$

where $\bar{\bar{n}}\big/\bar{\bar{T}} \equiv n_i/T_i + n_e/T_e$, we are led to the dispersion relation

$$\left(\omega^2 - \omega_{IA}^2\right)(\omega + \omega_{*I}) = \omega_{SI}^2\left(\omega - \omega_{**}^I\right) \tag{2.5}$$

where

$$\omega_{*I} \equiv k_y\frac{c\bar{\bar{T}}}{eB}\frac{Z}{\bar{\bar{n}}}\frac{dn_I}{dx}, \quad \omega_{SI}^2 \equiv \frac{3}{5}\omega_{IA}^2\Delta, \quad \Delta \equiv \frac{Z^2 n_I}{\bar{\bar{n}}}\frac{\bar{\bar{T}}}{T_I} \text{ and } \omega_{**}^I \equiv \frac{\omega_{*I}}{\Delta}.$$

3. Discussion of the Model Dispersion Relation

Clearly, for the derivation of dispersion relation (2.5), the thermal conductivities of the main ion and the electron populations have been considered to be large while that of the impurity population has been considered to be negligibly small. Then we note that the dispersion relation for the known impurity [1] driven modes $\omega(\omega + \omega_{*I}) - \omega_{SI}^2 = 0$ does not involve the frequencies ω_{IA}^2 or ω_{**}^I as it is derived in the hypersonic limit where $\omega^2 > 2T_I\,k_{\shortparallel}^2\big/m_I$ and $\omega \sim \omega_{*I} \sim \omega_{SI}$. In this limit we have an impurity drift mode $\omega \simeq -\omega_{*I}$ for $\omega_{SI}^2 < \omega_{*I}^2/4$, that has a phase velocity in the direction of the main ion diamagnetic velocity v_{di} and an impurity sound mode $\omega \simeq \omega_{SI} - \omega_{*I}/2$ for $\omega_{SI}^2 > \omega_{*I}^2/4$, that has a phase velocity in the opposite direction requiring that $\Delta > 2$.

More realistically, if we consider the case where $\Delta < 1$ and the mode with phase velocity directed to the electron diamagnetic velocity ($\omega > 0$ for $k_y > 0$), $\omega \simeq \omega_{IA} + 3\Delta\left(\omega_{IA} - \omega_{**}^I\right)/10$.

As will be shown this mode can be driven unstable by the local temperature gradient of the main ion population for $\omega_{**}^I < \omega_{IA}$. In particular, we can argue that (see Section 5) the transport of the impurity population toward the edge of the plasma column raises the value of $dn_I/dx\,/n_I$ until the marginal stability condition $\omega = \omega_{IA} = \omega_{**}^I$ is reached.

4. Dissipative Effects

The analysis of the data concerning the I-Regime indicates that the main ion population has to be treated as collisionless. On the other hand, a good insight on the effects of the relevant mode-particle resonances can be gained by the effect of a finite longitudinal thermal conductivity in the limit where $D^{th}_{\shortparallel i} k^2_{\shortparallel} > |\omega|$. Then, the thermal energy balance equation for the main ion population reduces to

$$\hat{v}_{Ex} \frac{dT_i}{dx} \left(1 - \frac{2}{3\eta_i}\right) \simeq -k^2_{\shortparallel} D^{th}_{\shortparallel i} \hat{T}_i, \tag{4.1}$$

where $\eta_i \equiv (\,d \ln T_i/dx\,)/(\,d \ln n_i/dx\,)$, and

$$\frac{\hat{n}_i}{n_i} \simeq -\frac{e\hat{\phi}}{T_i} - \frac{\hat{T}_i}{T_i} \simeq -\frac{e\hat{\phi}}{T_i}\left(1 + \bar{\varepsilon}_i\right) \tag{4.2}$$

where $\bar{\varepsilon}_i \equiv \left[\omega^T_{*i}/\left(k^2_{\shortparallel} D^{th}_{\shortparallel i}\right)\right]\left[1 - 2/(3\eta_i)\right]$ and $\omega^T_{*i} \equiv (\,ck_y/eB\,)\,(\,dT_i/dx\,)$. Thus the quasi-neutrality condition becomes

$$Z\hat{n}_I = \frac{e\hat{\phi}}{\bar{\bar{T}}} \bar{\bar{n}} \left(1 + i\varepsilon_i\right) \tag{4.3}$$

where $\varepsilon_i \equiv \left(n_i \bar{\bar{T}} \middle/ \bar{\bar{n}} T_i\right) \bar{\varepsilon}_i$. Consequently, the dispersion relation becomes

$$\left(\omega^2 - \omega^2_{IA}\right)\left[\omega\left(1 + i\varepsilon_i\right) + \omega_{*I}\right] = (\,3/5\,)\left(\omega - \omega^I_{**}\right)\omega^2_{IA}\Delta. \tag{4.4}$$

We consider $\Delta < 1$ and take $\omega = \omega_{IA} + \delta\omega$ where $\mathrm{Im}\delta\omega \simeq -(\,3/5\,)\,\varepsilon_i \Delta \left(\omega_{IA} - \omega^I_{**}\right) \cdot \left[\left(1 + \omega_{*I}/\omega_{IA}\right)^2 + \varepsilon^2_i\right]^{-1}$. Therefore, for $dT_i/dx < 0$, $\eta_i > \frac{2}{3}$ and $k_y > 0$ the dispersion equation gives a positive growth rate when $\omega_{IA} > \omega^I_{**}$.

The longitudinal thermal conductivity of the impurity population, when included in the theory introduces a damping for the considered mode. In particular, if we consider $D^{th}_{\shortparallel I} k^2_{\shortparallel} / \omega_{IA} < 1$, $D^{th}_{\shortparallel I}$ being the longitudinal thermal diffusion coefficient for the impurity population, the damping is $\left(\mathrm{Im}\delta\omega\right)_D \simeq -2k^2_{\shortparallel}\, D^{th}_{\shortparallel I}/15$.

Finally, considering collisionless regimes we obtain, for the one-dimensional plane model,

$$\frac{\hat{n}_i}{n_i} \simeq -\frac{e\hat{\phi}}{T_i}\left[1 + i\sqrt{\pi}\frac{\omega^T_{*i}}{|k_{\shortparallel}|\, v_{thi}}\left(\frac{1}{2} - \frac{1}{\eta_i}\right)\right]. \tag{4.5}$$

Clearly the one-dimensional nature of the mode-particle resonance, that needs be rectified as in Section 6, leads to increase the critical value of η_i by a factor 3 relative to that found with the finite thermal conductivity model.

5. Quasi-linear Transport

The directions of the particle flows produced by the considered mode can be estimated by the quasi-linear approximation. Taking conventional averages over y and z denoted by $\langle\ \rangle$, we observe that, since $\hat{n}_e$ and $\hat{\phi}$ are in phase, $\Gamma_e \equiv \langle \hat{n}_e \hat{v}_{Ex} \rangle = 0$ that is, there is no net electron flow. Therefore, $Z\langle \hat{n}_I \hat{v}_{Ex}\rangle = -\langle \hat{n}_i \hat{v}_{Ex}\rangle$ and if the transport of the main ion population is inward that of the impurity population is outward. In particular, since $\langle \hat{n}_i \hat{v}_{Ex}\rangle \simeq -n_i \left\langle \hat{T}_i \hat{v}_{Ex} \right\rangle \middle/ T_i$ we obtain

$$\langle \hat{n}_i \hat{v}_{Ex}\rangle \simeq \left(\frac{1}{T_i}\frac{dT_i}{dx}\right)\frac{n_i}{k^2_{\shortparallel} D^{th}_{\shortparallel i}}\left\langle |\hat{v}_{Ex}|^2 \right\rangle \left(1 - \frac{2}{3\eta_i}\right)$$

after using Eq. (4.1). This shows that the main ion transport is inward for $\eta_i > 2/3$, while the impurity transport is outward, a feature consistent with the fact that the impurities are observed to be expelled toward the edge of the plasma column in the presence of the considered mode.

6. Toroidal Modes

We refer to the simplest model of a toroidal configuration having magnetic surfaces with circular cross sections and large aspect ratios ($\epsilon_0 \equiv r/R_0 < 1$) represented by

$$\mathbf{B} \simeq \left[B_\zeta\,(r)\,\mathbf{e}_\zeta + B_\theta\,(r)\,\mathbf{e}_\theta\right]/(1 + \epsilon_0 \cos\theta)\ . \tag{6.1}$$

The electrostatic modes that are radially localized around a given rational magnetic surface $r = r_0$ and can be excited in this configuration, where magnetic shear is present, are described by the potential

$$\hat{\phi} \simeq \tilde{\phi}(r_0, \theta) \exp\left\{-i\omega t + in^0 [\zeta - q(r)\theta] + in^0 \left[q(r) - q_0^0\right] F(\theta)\right\}. \tag{6.2}$$

Here $q(r) = rB_\zeta/(RB_\theta)$ is the unwinding parameter, n^0 is the toroidal mode number and $\tilde{\phi}(r_0, \theta)$ is a periodic function of θ with $\left|\frac{d\tilde{\phi}}{d\theta}\middle/\tilde{\phi}\right| \ll n^0 q_0^0$, $q_0^0 \equiv q(r_0) = m^0/n^0$, $m^0 \gg 1$ is an integer.

Moreover, $F(\theta)$ is an odd function of θ that is vanishing for $-\pi < \theta < \pi$, with $F(\theta = \pm\pi) = \pm 1$ and ensures that $\hat{\phi}$ is a periodic function of θ while $q(r) \neq q_0^0$. In particular, we take $\tilde{\phi} = 0$ and $d\tilde{\phi}\big/d\theta = 0$ for $|\theta| = \pi$. In the previous literature [11, 12, 13] this has been referred to as the disconnected mode approximation. For the radially localized modes we consider $q(r) \simeq q_0^0 [1 + \hat{s}(r/r_0 - 1)]$ where $\hat{s} \equiv d\ln q/d\ln r$ is the magnetic shear parameter. Then, for $-\pi < \theta < \pi$, $\mathbf{B}\cdot\nabla\hat{\phi} \simeq (B_\theta/r_0)\left(d\tilde{\phi}\big/d\theta\right)\left(\hat{\phi}\big/\tilde{\phi}\right)$.

The considered equilibrium distribution for the main ion population is $f_i \simeq f_{Mi}(1 + \Delta_i)$ where $f_{Mi} = [2\pi\, T_i(r)/m_i]^{-3/2} n_i(r) \exp[-\,\mathcal{E}/T_i(r)]$, $\Delta_i \simeq -\left(v_\zeta/\Omega_\theta^i\right)\cdot[(1/n_i)\, dn_i/dr - (1/T_i)\, dT_i/dr\,(3/2 - \mathcal{E}/T_i)]$, $\mathcal{E} = (m_i/2)\left(v_\parallel^2 + v_\perp^2\right)$ and $\Omega_\theta^i = eB_\theta/(m_i c)$. Then, following a standard procedure,

$$\hat{f}_i = -\frac{e}{T_i} f_{Mi}\left[\hat{\phi} + i\left(\omega - \omega_{*i}^T\right)\int_\infty^t dt'\hat{\phi}(t')\right] \tag{6.3}$$

where the integration is taken along unperturbed particle orbits,

$$\omega_{*i}^T(\mathcal{E}) \equiv \omega_*^i\left[1 - \eta_i\left(\frac{3}{2} - \frac{\mathcal{E}}{T_i}\right)\right], \quad \omega_*^i \equiv -\frac{n^0}{R_0}\frac{cT_i}{eB_\theta}\frac{1}{n_i}\frac{dn_i}{dr} \tag{6.4}$$

and $\eta_i \equiv (d\ln T_i/dr)/(d\ln n_i/dr)$. Modes for which $\tilde{\phi}(r_0, \theta)$ is even or odd in θ have different characteristics. In particular, the average potential

$$\tilde{\phi}^{(0)}(\mathcal{E}, \mu) = \oint \frac{dl}{|v_\parallel|}\tilde{\phi}(l) \Bigg/ \oint \frac{dl}{|v_\parallel|}, \tag{6.5}$$

where $dl = qR_0 d\theta$, vanishes when $\tilde{\phi}(r_0, \theta)$ is an **odd** function of θ that is the considered case. Therefore, the relevant modes will not see the locally (in θ) unfavorable curvature seen by the trapped (main) ions. In particular we take

$$\tilde{\phi}(r_0, \theta) = \tilde{\phi}_0(r_0)\sin(l_0\theta)\left\{1 - \exp\left[-\left(\pi - |\theta|\right)^2\big/\delta_0^2\right]\right\} \tag{6.6}$$

where $\delta_0 \ll 1$ and l_0 is an integer, the modes we consider have frequency $\omega^2 < l_0^2\bar{\omega}_{ti}^2$ where $\bar{\omega}_{ti} = v_{thi}/(qR_0)$ is the average transit frequency for circulating particles.

Then, referring to the circulating particle population and adopting the form (6.2) for the considered modes we employ the decomposition

$$\tilde{\phi}(r_0, \theta) = \sum_{p^0 \geq 1}\tilde{\phi}^{(p^0)}(\mathcal{E}, \Lambda, r_0)\exp\left[ip^0\sigma\omega_t t(\theta)\right], \tag{6.7}$$

where $\Lambda \equiv \mu B_0/\mathcal{E}$, $\omega_t(\mathcal{E}, \Lambda) \equiv 2\pi/\tau_t$, $\tau_t = \int_{-\pi}^{\pi} d\theta\Big/\left|\dot{\theta}\right|$ is the transit period for the main ion population, $t(\theta) = \int_0^\theta d\theta\Big/\left|\dot{\theta}\right|$, $\dot{\theta} = v_\parallel/(qR_0)$, $v_\parallel = \sigma[2m_i]^{1/2}(1 - \Lambda B/B_0)^{1/2}$ and $\sigma = \mathrm{sgn}(v_\parallel)$. Clearly,

$$\tilde{\phi}^{(p^0)}(\mathcal{E}, \mu) = \frac{1}{\tau_t}\int_{-\pi}^{\pi} d\theta\left|\dot{\theta}\right|^{-1}\tilde{\phi}(\theta)\exp\left[-ip^0\sigma\omega_t t(\theta)\right]. \tag{6.8}$$

Then,

$$\begin{aligned}\hat{n}_i \quad \simeq \quad & -\frac{e}{T_i}\left\{n_i\hat{\phi} + i\pi\omega_*^i\left[\int d^3\mathbf{v} f_{Mi}\left(1 - \frac{3}{2}\eta_i + \frac{\mathcal{E}}{T_i}\eta_i\right)\right.\right. \\ & \left.\left.\cdot\sum_{p^0 \geq 1}\tilde{\phi}^{(p^0)}\exp\left[ip^0\sigma\omega_t t(\theta) - i\omega t + in^0(\zeta - q\theta)\right]\delta\left(\omega - p^0\sigma\omega_t\right)\right]\right\}.\end{aligned} \tag{6.9}$$

For the analysis that follows (Section 8) it is convenient to evaluate the quadratic form

$$\int_{-\pi}^{\pi} d\theta \hat{\phi}^* \hat{n}_i \simeq -\frac{en_i}{T_i}\left\{ I_0 + i\sqrt{\pi}^3 \omega_*^i \sum_\sigma \int_0^{1-\epsilon_0} d\Lambda L\left(\Lambda, \epsilon_0\right) \right. \tag{6.10}$$

$$\left. \int_0^\infty \frac{d\mathcal{E}}{T_i}\sqrt{\frac{\mathcal{E}}{T_i}} \exp\left(-\frac{\mathcal{E}}{T_i}\right)\left[1-\left(\frac{3}{2}-\frac{\mathcal{E}}{T_i}\right)\eta_i\right] \sum_{p^0 \geq 1} \left|\tilde{\phi}^{(p^0)}\right|^2 \delta\left(\omega - p^0\sigma\omega_t\right)\right\}$$

where $L\left(\Lambda, \epsilon_0\right) \equiv \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta \left[1-\Lambda(1+\epsilon_0\cos\theta)^{-1}\right]^{-1/2}$ and $I_0 \equiv \int_{-\pi}^{\pi} d\theta \left|\tilde{\phi}\left(r_0, \theta\right)\right|^2$ $= \pi\left|\tilde{\phi}_0\left(r_0\right)\right|^2$ for the eigenfunction given by Eq. (6.6). Finally, we observe that given the sign chosen in Eq. (6.2), the wave number used for the plane one-dimensional model k_y corresponds to $-m^0/r$. Therefore, the phase velocity of a mode is $\omega R_0/n^0$ in the toroidal direction and $-\omega r_0/m^0$ in the poloidal direction.

7. Simplest Toroidal Dispersion Relation

We consider the same set of equations as those introduced in Section 2. Retracing the steps that lead to the dispersion relation (2.5), we obtain

$$(\omega + \omega_{*I})\left(\omega^2 + \bar{\omega}_{IA}^2 \frac{d^2}{d\theta^2}\right)\tilde{\phi} + \bar{\omega}_{SI}^2\left(\omega - \omega_{**}^I\right)\frac{d^2}{d\theta^2}\tilde{\phi} = 0, \tag{7.1}$$

where $\bar{\omega}_{IA}^2 \equiv (5/3)\, T_I/\left(m_I q^2 {R_0}^2\right)$ and $\bar{\omega}_{SI}^2 \equiv \left(Z^2 n_I/\bar{\bar{n}}\right)\bar{\bar{T}}\Big/\left(m_I q^2 {R_0}^2\right)$. Therefore, we may consider the appropriate solution to be of the form represented in Eq. (6.6), where $\omega^2 < l_0^2\bar{\omega}_{ti}^2$. We refer to the experiments carried out by the Alcator C-Mod machine, taking $R_0 \simeq 68$ cm, $T_i \simeq 600$ eV and $q \simeq 2.5$ we have $\bar{\omega}_{ti} \simeq 1.4 \times 10^5$ Hz $\simeq 8.9 \times 10^5$ rad $\cdot$ sec^{-1}. Therefore we may assume $l_0 \geq 9$. The relevant dispersion relation is

$$\left(\omega^2 - \bar{\bar{\omega}}_{IA}^2\right)(\omega + \omega_{*I}) = \left(\omega - \omega_{**}^I\right)\bar{\bar{\omega}}_{SI}^2, \tag{7.2}$$

where $\bar{\bar{\omega}}_{IA}^2 \equiv \bar{\omega}_{IA}^2 l_0^2$, and $\bar{\bar{\omega}}_{SI}^2 \equiv \bar{\omega}_{SI}^2 l_0^2$, and we have returned to consider a dispersion equation that is very close to that obtained for the simplest plane geometry model.

8. Mode-particle Resonance Effects

The considered mode can acquire a growth rate by a resonant interaction with the main ion population corresponding to

$$\omega = p^0\sigma\omega_t\left(\mathcal{E}, \mu\right) = p^0\sigma\ \frac{2\pi}{qR_0}\Bigg/\int_{-\pi}^{\pi}\frac{d\theta}{|v_{\shortparallel}|}\ . \tag{8.1}$$

Then we obtain, instead of ε_i,

$$\bar{\bar{\varepsilon}}_i = -\alpha_i^c \frac{\omega_*^i}{\bar{\omega}_{ti}} \left(\eta_i - \eta_i^c\right), \tag{8.2}$$

where $\alpha_i^c \equiv \sqrt{\pi}\, \Pi_1/(I_0 \eta_i^c)$, $\eta_i^c \equiv (3/2 - \Pi_2/\Pi_1)^{-1}$,

$$\Pi_1 \equiv 2\pi \sum_\sigma \sum_{|p^0|>1} \frac{1}{|p^0|} \int_0^{1-\epsilon_0} d\Lambda L^2(\Lambda, \epsilon_0)\, \bar{\bar{v}}_{res}^2 \exp\left(-\bar{\bar{v}}_{res}^2\right) \left|\tilde{\phi}^{(p^0)}\left(\bar{\bar{v}}_{res}^2, \Lambda, r_0\right)\right|^2,$$

$$\Pi_2 \equiv 2\pi \sum_\sigma \sum_{|p^0|>1} \frac{1}{|p^0|} \int_0^{1-\epsilon_0} d\Lambda L^2(\Lambda, \epsilon_0)\, \bar{\bar{v}}_{res}^4 \exp\left(-\bar{\bar{v}}_{res}^2\right) \left|\tilde{\phi}^{(p^0)}\left(\bar{\bar{v}}_{res}^2, \Lambda, r_0\right)\right|^2$$

and $\bar{\bar{v}}_{res} \equiv \omega L(\Lambda, \epsilon_0) \big/ \left(p^0 \sigma \bar{\omega}_{ti}\right)$.

Referring to large aspect ratios ($\epsilon_0 \equiv r/R_0 < 1$) and $l_0 \epsilon_0 < b < 1$ where $b \equiv \omega^2 \big/ (l_0 \bar{\omega}_{ti})^2$, we obtain $\alpha_i^c \simeq (\sqrt{\pi}/2l_0)\left[1 + 9\epsilon_0^2/\left(4b^2\right) - 3b\right]$ and $\eta_i^c \simeq 2\left[1 - 3\,\epsilon_0^2/b^2 + 2b\right]$. Clearly, when $3\epsilon_0^2 > 2b^3$ we have $\eta_i^c < 2$.

9. Poloidal Magnetic Field Fluctuations

In order to justify the observed electromagnetic fluctuations, we note that $\hat{E}_\shortparallel = -ik_\shortparallel \hat{\phi} + i\omega\, \hat{A}_\shortparallel \big/ c$ and up to this point we have neglected $i\omega\, \hat{A}_\shortparallel \big/ c$. The estimated density fluctuations are $\tilde{n}_e/n_e \simeq 10^{-2} \times \alpha_n$ [14], where $\alpha_n \sim 1$, while the estimated poloidal magnetic field fluctuations are $\tilde{B}_\theta \simeq (3-8) \times 10^{-4}\,\mathrm{T}$ [15]. Then we may compare

$$\tilde{n}_e/n_e \quad \text{to} \quad |\omega/k_\shortparallel|\,[e/(cT_e)] \int \tilde{B}_\theta dr. \tag{9.1}$$

If we consider $|\omega/k_\shortparallel| \sim v_{thi}$ and $T_e \simeq T_i \simeq 600\,\mathrm{eV}$, we may take $|\omega/k_\shortparallel| \simeq 2 \times 10^7\,\mathrm{cm/sec}$. Moreover, for

$$\tilde{A}_\shortparallel \simeq \int \tilde{B}_\theta dr \simeq \bar{\bar{\tilde{B}}}_\theta \Delta r \tag{9.2}$$

where $\Delta r \sim 1\,\mathrm{cm}$ [14] the comparison (9.1) becomes

$$\frac{\tilde{n}_e}{n_e} \qquad \text{to} \qquad 1.7 \times 10^{-3} \left(\frac{|\omega/k_\shortparallel|}{2 \times 10^7\,\mathrm{cm/\,sec}}\right)\left(\frac{\bar{\bar{\tilde{B}}}_\theta}{5\,\mathrm{G}}\right) \cdot \left(\frac{\Delta r}{1\,\mathrm{cm}}\right)\left(\frac{600\,\mathrm{eV}}{T_e}\right).$$

Thus the electrostatic approximation used in the previous sections is acceptable but the estimated values of $\tilde{B}_\theta$ have to be justified. Clearly,

$$\hat{A}_\shortparallel \simeq \frac{4\pi \hat{J}_\shortparallel}{ck^2} \simeq -i4\pi \frac{\nabla \cdot \hat{\mathbf{J}}_\perp}{ck^2 k_\shortparallel}. \tag{9.3}$$

In particular, $\nabla \cdot \hat{\boldsymbol{J}}_\perp = \nabla \cdot \left[(n_i + Zn_I - n_e)\, \hat{\boldsymbol{v}}_E\right] = 0$ at the stage where the linear description of the mode is valid.

We argue that in the saturated state $\hat{n}_e/n_e$ remains $\simeq e\hat{\phi} \big/ T_e$ and the electron radial velocity is $\hat{v}_{Ex}$. Thus $\Gamma_{ex} \equiv \langle \hat{n}_e \hat{v}_{Ex} \rangle = 0$. Consistently with the experimental observation that injected impurities are promptly expelled, the impurity radial flux is considered to become greatly reduced relative to that estimated by the quasi-linear theory that is valid at the start of the mode evolution. Consequently, $|\Gamma_{ix}| \equiv \left|\langle \hat{n}_i \hat{v}_x^i \rangle\right| \ll |\langle \hat{n}_i \hat{v}_{Ex} \rangle|$. We assume that $\hat{n}_i$ remains of the form $\hat{n}_i \simeq -\, e\hat{\phi} n_i \left(1 + i\varepsilon_i^s\right) \big/ T_i$ in the saturated state, starting from the initial stage where $\hat{n}_i \simeq -e\hat{\phi} n_i \left(1 + i\bar{\bar{\varepsilon}}_i\right)$ and $\bar{\bar{\varepsilon}}_i$ is given by Eq. (8.2). Then we argue that ε_i^s remains close to $\bar{\bar{\varepsilon}}_i$ and $\hat{v}_x^i \simeq \hat{v}_{Ex} + \Delta \hat{v}_x^i$, where $\Delta \hat{v}_x^i \simeq -i\varepsilon_i^s \hat{v}_{Ex}$. Correspondingly, $\hat{v}_x^I \simeq \hat{v}_{Ex} + \Delta \hat{v}_x^I$, and since $Z\hat{n}_I \simeq \hat{n}_e - \hat{n}_i$ we have $\Delta \hat{v}_x^I \simeq -i\varepsilon_i^s \hat{v}_{Ex}\, n_i \bar{\bar{T}} \big/ (\bar{\bar{n}} T_i)$. Thus a current density

$$\hat{J}_x \simeq e\left(n_i \Delta \hat{v}_x^i + Zn_I \Delta \hat{v}_x^I\right) = -i\varepsilon_i^s e n_i \hat{v}_{Ex}\left[1 + Zn_I \bar{\bar{T}} \big/ (\bar{\bar{n}} T_i)\right] \simeq -i\varepsilon_i^s e n_i \hat{v}_{Ex}$$

has to be considered. We shall take $\nabla \cdot \hat{\boldsymbol{J}}_\perp \simeq \alpha_J \, \partial \hat{J}_x \Big/ \partial x \simeq \alpha_J \, \hat{J}_x \Big/ \Delta r$ and in view of Eqs (9.2) and (9.3) we obtain, for $d_i^2 \equiv c^2/\omega_{pi}^2$,

$$\bar{\bar{B}}_\theta \simeq i\alpha_J \frac{4\pi\varepsilon_i^s e n_i \tilde{\phi}}{k\Delta r k_{\shortparallel} \Delta r B} \simeq i \frac{1}{k\Delta r k_{\shortparallel}\Delta r} \frac{cT_i}{eB} \frac{B}{d_i^2 \Omega_{ci}} \times \varepsilon_i^s \times \alpha_n \times \alpha_J \times 10^{-2}. \tag{9.4}$$

For $k_{\shortparallel} \simeq l_0/(qR_0) \simeq 2/35 \text{ cm}^{-1} \, (l_0/10) \, (2.5/q) \, (70 \text{ cm}/R_0)$, $\Delta r \sim 1 \, \text{cm}$, $k \sim 2 \, \text{cm}^{-1}$, $T_i \simeq 600 \, \text{eV}$, $n_i \simeq 10^{14} \, \text{cm}^{-3}$ and $B \simeq 5 \times 10^4$ G, referring to the relevant experiments, Eq. (9.4) reduces to

$$\left| \frac{\bar{\bar{B}}_\theta}{B} \right| \simeq \frac{1}{2} \times \varepsilon_i^s \times \alpha_n \times \alpha_J \times 10^{-4} \left[\frac{2}{k\Delta r} \right] \left[\frac{2/35}{k_{\shortparallel}\Delta r} \right] \left[\frac{cT_i}{eB} \Big/ \left(1.2 \times 10^6 \, \text{cm}^2/\text{sec} \right) \right]$$
$$\cdot \left[\frac{3 \text{ cm}}{d_i} \right]^2 \left[\frac{2.4 \times 10^8 \text{ rad/sec}}{\Omega_{ci}} \right]$$

and we can see that values of $\bar{\bar{B}}_\theta$ are in the range that has been estimated from experimental observations.

10. Electron Temperature Fluctuations

If we include the electron temperature fluctuations in the plasma quasi-neutrality condition, this becomes

$$Z\hat{n}_I = \frac{e\hat{\phi}}{\bar{\bar{T}}} \bar{\bar{n}} - n_e \frac{\hat{T}_{e\shortparallel}}{T_e} + n_i \frac{\hat{T}_{i\shortparallel}}{T_i}. \tag{10.1}$$

It is clear that when considering the ratio of the relevant collisional longitudinal thermal conductivities or of the relevant linearized mode-particle resonances $\hat{T}_{e\shortparallel}/T_e$ can be neglected. On the other hand it is possible that, since other microscopic modes may be present, the longitudinal electron thermal conductivity may be depressed and that electron temperature fluctuations become significant. In fact, recently A. White concluded that electron temperature fluctuations of 1 % ∼ 2 % were observed in the Alcator C-Mod experiments in the I-Regime [16]. In this context we note that electron temperature fluctuations are particularly important for collisional modes that can be found with phase velocity $(k_{\shortparallel}\lambda_i) \, v_{thi} < \omega/k_{\shortparallel} < v_{thi}$ where $k_{\shortparallel}\lambda_i < 1$ and λ_i is the main ion mean free path. In this case $\hat{T}_i$ can be evaluated in the limit of negligible longitudinal thermal conductivity.

Here we consider regimes for which $k_{\shortparallel}\lambda_i \simeq k_{\shortparallel}\lambda_e \simeq 4.2 \times 10^2 \left[k_{\shortparallel} \big/ \left(2/35 \text{ cm}^{-1} \right) \right]$ $\cdot \left[\lambda_i \simeq 7.0 \times 10^3 \text{ cm } (T_i/600 \text{ eV})^2 \Big/ \left(n_i/10^{14} \text{cm}^{-3} \right) \right]$ (λ_e being the electron mean free path) referring to the series of relevant Alcator experiments. Therefore we may exclude this kind of mode from the present consideration and write the effective longitudinal thermal energy balance equation for the electron as

$$k_{\shortparallel}^2 D^e_{\shortparallel\text{eff}} \hat{T}_{e\shortparallel} + \hat{v}_{Ex} \frac{dT_e}{dx} + \eta_e^c T_e \nabla_{\shortparallel} \hat{u}_{e\shortparallel} = 0. \tag{10.2}$$

Then we have

$$\frac{\hat{T}_i}{T_i} n_i - \frac{\hat{T}_{e\shortparallel}}{T_e} n_e \simeq -\frac{\hat{v}_{Ex}}{k_{\shortparallel}^2} \left\{ \frac{n_i}{D^i_{\shortparallel\text{eff}}} \frac{1}{T_i} \frac{dT_i}{dx} \left(1 - \frac{\eta_i^c}{\eta_i} \right) - \frac{n_e}{D^e_{\shortparallel\text{eff}}} \frac{1}{T_e} \frac{dT_e}{dx} \left(1 - \frac{\eta_e^c}{\eta_e} \right) \right\} \tag{10.3}$$

where $\eta_e \equiv (d \ln T_e/dx)/(d \ln n_e/dx)$ and η_i^c and η_e^c have to be evaluated on the basis of the processes that determine the values of $D^i_{\shortparallel\text{eff}}$ and $D^e_{\shortparallel\text{eff}}$. Therefore the condition for the instability of the mode we consider is evident from Eq. (10.3).

11. Spontaneous Rotation

The spontaneous rotation observed in the I-Regime is in the direction of the main ion diamagnetic velocity (co-current) and, following the accretion theory [10], a process to scatter angular momentum to the wall generating a recoil in the opposite direction, that of the ion diamagnetic velocity, and an inflow process to transport the generated angular momentum from the edge of the plasma column toward the center, should be present.

The heavy particle mode that is excited at the edge of the plasma column can provide the scattering of angular momentum in the same direction as that of electron diamagnetic velocity.

12. Conclusions

We have identified a new mode that can be excited in a multi-component plasma with a massive particle population. This has: i) a phase velocity in the direction of the electron diamagnetic velocity, ii) produces an outward transport of the heavy particle (impurity) population and iii) involves the light ion population temperature gradient and the effective longitudinal thermal conductivity as the driving factors. These characteristics and the fact that the mode is shown to produce measurable electromagnetic fluctuations make it a suitable candidate to explain the features of the 200 kHz mode found experimentally in the I-confinement Regime where the plasma thermal energy is well confined while the impurities are segregated at the edge of the plasma column. A limitation of the presented theory is its formulation for a toroidal confinement configuration with circular magnetic surfaces. A closer comparison with the experiments would require a more accurate description of the mode poloidal profiles.

Given the promising confinement characteristics of the I-Regime in view of future experiments on fusion burning plasmas, it is worth considering the possibility to influence the onset and the evolution of the heavy particle mode that we have investigated. In fact an experimental effort in this direction has been undertaken already by T. Golfinopoulos.

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