Numerical Study of a near-Zero-Index Acoustic Metamaterial

Citation: Jing, Yun, Jun Xu, and Nicholas X. Fang. "Numerical study of a near-zero-index acoustic metamaterial." Physics Letters A, 376:45 (1 October 2012), pp. 2834–2837.

As Published: http://dx.doi.org/10.1016/j.physleta.2012.08.057

Publisher: Elsevier B.V.

Persistent URL: http://hdl.handle.net/1721.1/108497

Version: Author's final manuscript: final author's manuscript post peer review, without publisher's formatting or copy editing

Numerical study of a near-zero-index acoustic metamaterial

Yun Jing,[1,*] Jun Xu,[2] and Nicholas X. Fang[2]

[1]*Department of Mechanical and Aerospace Engineering, North Carolina State University, Raleigh, NC 27695, U.S.A.*

[2]*Department of Mechanical Engineering, MIT, Cambridge, MA 02139, U.S.A.*

(Dated: August 14, 2012)

Abstract

This paper studies a two-dimensional, membrane-based acoustic metamaterial with near-zero refractive index. It yields a frequency-dependent effective density that is near-zero at a certain narrow frequency band centered around the first resonance frequency, which results in a near-zero refractive index. Numerical simulations are shown here to demonstrate that the phase in this metamaterial undergoes small changes, and it functions as an angular filter, such that only a wave with a near-zero incident angle can transmit through. Its ability to tailor acoustic phase pattern is also discussed in this letter.

keywords: Acoustic metamaterials, near-zero-index

[*]Electronic address: `yjing2@ncsu.edu,phone:919-513-4673,fax:919-515-7968`

I. INTRODUCTION

Previous studies on acoustic metamaterial have been focusing on achieving negative acoustic properties, such as a negative bulk modulus [1, 2], negative density [3–5], and refractive index [6]. These extraordinary acoustic properties have potential applications of insulating noise of a broad-band frequency [7], achieving sub-wavelength imaging [8], and hiding an object from sonar detections [9]. Near-zero-index is another interesting phenomenon that can be realized by metamaterials and has recently been reported in numerous articles. Unfortunately, most of these studies concern the zero index metamaterials (ZIMs) for electromagnetic waves. Theoretically, a near-zero-index indicates a medium with infinitely large phase velocity and wavelength. In other words, a wave in a ZIM undergoes no phase change. This remarkable property has led to many interesting applications such as directive emission [10], self-collimation [11], tailoring radiation phase pattern [12], super-reflecting and cloaking [13]. In addition, a near-zero-index means only a wave with a near-zero incident angle (normal incidence) can be transmitted through the ZIMs, while other waves will be totally reflected. Therefore, a ZIM can also function as a highly selective angular filter.

One-dimensional (1D) acoustic ZIMs, whose effective density and compressibility are simultaneously equal to zero, have been proposed [14]. The designs are based on membranes and open channels built inside a waveguide. However, acoustic ZIMs can also be achieved with only effective density being zero. Theoretically, clamped membranes introduce frequency-dependent effective density while open channels (or branch openings) introduce frequency-dependent effective compressibility (or bulk modulus). Both the density and compressibility can be negative, zero, and positive, depending upon the frequency. A two-dimensional (2D) membrane-based acoustic metamaterial which accomplished negative density was recently reported and claimed to be able to amplify evanescent waves [8], thereby leading to sub-wavelength images. We show in this letter that, a similar metamaterial can also be a ZIM in a certain narrow frequency band, and can be used as an angular filter.

II. THEORY AND SIMULATION RESULTS

It has been both theoretically and experimentally shown that membrane-based metamaterials exhibit negative density approximately below a resonance frequency ω_0 determined by the elastic property of the membrane [4, 5, 14],

$$\omega_0 = \sqrt{2}\pi/a\sqrt{\mu_0/\rho_0}, \tag{1}$$

where a is the width of the membrane, μ_0 is the shear modulus, ρ_0 is the density of the membrane.

A negative density essentially results in a purely imaginary phase velocity (given that the compressibility is unaltered) and converts propagating waves into evanescent waves. Negative density can also be used to amplify evanescent waves. This has been experimentally verified in a recent paper using a 2D (x-z plane) membrane-based acoustic metamaterial, which is essentially a repetition of a 1D metamaterial proposed by the same authors in the z-direction [8]. Around the resonance frequency, the density in the membrane-based metamaterial is effectively near-zero, leading to an infinitely large phase velocity, and therefore, a near-zero index. This is numerically validated for the 1-D case [14]. It is hypothesized that, the 2D metamaterial can also behave as a ZIM around the resonance frequency.

Numerical simulations are presented here to validate this hypothesis. A full 3D simulation is extremely computationally consuming, therefore, 2D simulations are carried out without a loss of generality. Figure 1 illustrates the rectangular 2D metamaterial in our simulations, which has a size of $53cm \times 58cm$. It consists of a number of small unit cells where each one includes a square air cavity and four membrane surrounding it. The left side of this metamaterial is coupled to free space containing incident waves while the other three sides are non-reflecting, which can be physically achieved by highly absorptive materials, e.g., foams. The membranes are ideally clamped and assumed to be rubber-like materials, whose density is 1100 kg/m^3, Young's modulus is 50 Mpa, and Poisson' ratio is 0.49. The thickness of the membrane is 0.5 mm. Support frames are not modeled for simplicity, but can be easily incorporated. In other words, support frames in this study are assumed to be completely rigid and infinitely small. Consideration of the frame supports is expected to only shift the resonance frequency slightly and will not alter the conclusion of this paper. The background medium is air, whose density is 1.25 kg/m^3 and speed of sound is 343 m/s.

It is noted that, Eq.1 is valid for 3D cases, therefore the resonance frequency for a 2D problem needs to be numerically determined. This can be simply done by analyzing the dispersion and the zero density occurs at the frequency where the Bloch wave vector is zero [15]. An alternative approach is to find the frequency where the pressure (or the phase) undergoes almost no change across the metamaterial, which is used in this study. In either case, it is much more convenient to estimate the resonance frequency for a single line of this 2D metamaterial, which is shown in Fig.2(a). In the simulation, a plane wave with an amplitude of 1 Pa enters the boundary on the left, and leaves the boundary on the right without any reflection. The simulation is implemented by COMSOL, the frequency-domain acoustic-structure interaction modulus. The boundary condition is set such that "edge load is defined as force/length". The resonance frequency is then determined to be around 745 Hz. Figure 2(b) illustrates the subtle phase change inside this 1D metamaterial along a length of 0.4m (corresponds to almost one wavelength in the air) at 745 Hz, from which the phase velocity can be estimated to be around 35000 m/s. Additional simulations show that from 740 to 750 Hz, the effective phase velocity has shown to be above around 15000 m/s, therefore this structure is expected to have near-zero-index in a narrow frequency-band, i.e., 740-750Hz. Again, unlike the 2D ZIM, the resonance frequency for a 3D ZIM has been analytically acquired [5, 14], which makes the design possible and convenient.

It has been shown that for a 2D structure consisting of this 1D waveguide, the resonance frequency and effective density are essentially about the same [8]. Note that the dimensions of the 2D metamaterial are slightly larger than one wavelength at 745 Hz in air. Additional simulations show that this is a minimum allowable size to achieve reasonably good angular filtering.

To ensure that the resonance frequency remains the same for the 2D structure, Fig.3 shows the phase distribution inside the metamaterial when a plane wave at 745 Hz enters from the left side with a normal incidence. It can be seen that even though not as good as in the 1D case, in overall the phase undergoes relatively small changes in the metamaterial compared with otherwise in the air, implying a near-zero index. To gain more insight on this 2D structure, we also calculate the eigenstate of a unit cell by applying the periodic boundary condition. Figure 4 illustrates the pressure distribution in the air and the corresponding displacement of the membranes. At the resonant frequency (745 Hz), the phase of the pressure field at the corresponding boundaries (left-right or upper-lower) are maintained, indicating there is only little phase change at this frequency in a macroscopic view, and the index of the structure is effectively zero. The displacement of the membrane shows the longitudinal mode of the first resonance.

Next, we test this metamaterial by sending plane waves towards it with various incident angles up to 45 degree, because small angles are of most interest in this study. Figure 5 shows a polar diagram of the normalized pressure square at a point that is 55 cm away from the left boundary of this metamaterial (the location of this point does not change the result significantly). Results at two different frequencies are shown. At 745 Hz, which is the resonance frequency, a strong amplitude is found at the normal incident angle, while the amplitudes at other angles quickly drop and become negligible. At a frequency slightly off the resonance frequency,i.e., 770 Hz, the polar diagram becomes less directive compared with the one at 745 Hz, as the index progressively deviates from infinity as frequency increases.

Finally, we demonstrate yet another interesting phenomenon of ZIMs, i.e., if there is a point source inside a ZIM, the shape of the wavefronts leaving the ZIM is entirely determined by the shape of the exit surfaces of the ZIM, which can be employed for the design of phase patterns [12]. To show this, a point source with an acoustic power of 1W is imbedded at the center of the metamaterial, and Fig. 6 presents the radiation pattern (acoustic pressure) in the surrounding air. As can be seen in Fig. 6(a), the radiation pattern, even though essentially resulted from a point source, strikingly resembling the radiation pattern from a line source (Fig. 6(b)) which appears to locate on the left boundary of this metamaterial. This is due to the reason that every point in this metamaterial experiences a quasi-uniform acoustic phase. Additional simulations show that placing the source off the center does not change the result significantly.

III. CONCLUSIONS

To conclude, we have numerically verified a 2D membrane-based acoustic metamaterial that has near-zero-index. It is shown that the phase changes slowly in the metamaterial, indicating a large phase velocity. Transmission of plane waves entering this ZIM from different angles are calculated. It is demonstrated that, around the resonance frequency, only a wave of normal direction can penetrate through the ZIM, while other incident waves are reflected. Therefore this ZIM behaves as a perfect angular filter, thus can be potentially developed to be an angular sensor. This metamaterial is also shown to be able modify the radiation phase pattern according to the shape of the exit surface. In this study, we successfully manipulate a point source so that it behaves as a line source. It is worth pointing out that, better performance of this ZIM can be achieved by also reducing the compressibility to a near-zero value using branch openings or open channels, such further increasing the effect phase velocity. This, however, necessitates a full 3D simulation and will be addressed in a future study. Finally, it is noted that a recent paper reports on a different design of ZIM by coiling up space [16], where wave tunneling is studied. Nevertheless, angular filtering and radiation phase pattern manipulation of an acoustic ZIM have not been studied before to the best of our knowledge.

---

[1] N. Fang, D. Xi, J. Xu, M. Ambati, W. Srituravanich, C. Sun, and X. Zhang, Nature Mater. **5**, 452-456 (2006).

[2] S. H. Lee, C. M. Park, Y. M. Seo, Z. G. Wang, and C. K. Kim, J. Phys,:Condens. Matter **21**, 175704 (2009).

[3] S. H. Lee, C. M. Park, Y. M. Seo, Z. G. Wang, and C. K. Kim, Phys. Rev. Lett. **104**, 054301 (2010).

[4] S. H. Lee, C. M. Park, Y. M. Seo, Z. G. Wang, and C. K. Kim, Phys. Lett. A **373**, 4464-4469 (2009).

[5] S. Yao, X. Zhou and G. Hu, New Journal of Physics **12**, 103025 (2010)

[6] S. Zhang, L. Yin, and N. Fang, Phys. Rev. Lett. **102**, 194301 (2009).

[7] Z. Yang, H. M. Dai, N. H. Chan, G. C. Ma and Ping Sheng, Appl. Phys. Lett. **96**, 041906 (2010).

[8] C. M. Park, J. J. Park, S. H. Lee, Y. M. Seo, C. K. Kim, and S. H. Lee, Phys. Rev. Lett. **107**, 194301 (2011).

[9] X. Zhu, B. Liang, W. Kan, X. Zou and J. Cheng, Phys. Rev. Lett. **106**, 014301 (2011).

[10] Enoch, S., Tayeb, G., Sabouroux, P., Gurin, N. and Vincent, P. A, Phys. Rev. Lett. **89**, 213902 (2002).

[11] V. Mocella, S. Cabrini, A. S. P. Chang, P. Dardano, L. Moretti, I. Rendina, D. Olynick, B.

Harteneck, and S. Dhuey Phys. Rev. Lett. **102**, 133902 (2009).

[12] A. Alu, M. Silveirinha, P. Salandrino, N. Guerin, and P. Vincent, Phys. Rev. B **75**, 155410 (2007).

[13] Hao, J., Yan, W. and Qiu, M., Appl. Phys. Lett. **96**, 101109 (2010).

[14] F. Bongard, H. Lissek, and J. R. Mosig, Phys. Rev. B **82**, 094306 (2010).

[15] X. Zhou and G. Hu, Appl. Phys. Lett. **98**, 263510 (2011).

[16] Z. Liang and J. Li, Phys. Rev. Lett. **108**, 114301 (2012).

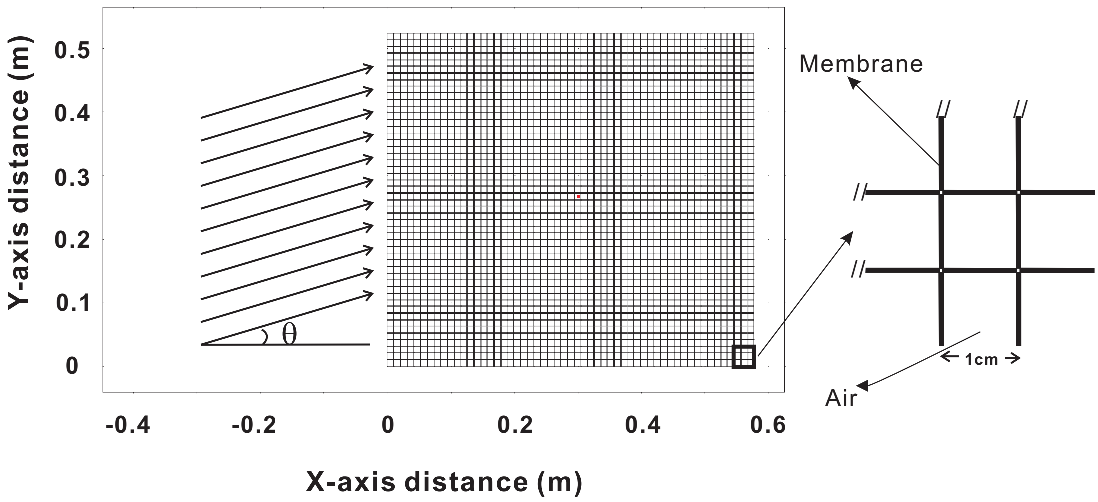

FIG. 1: Schematic illustration of the 2D membrane-based ZIM. A magnified plot of the right bottom corner of the ZIM is shown on the right side. Acoustic plane waves are entering the ZIM from the left side with an incident angle of θ.

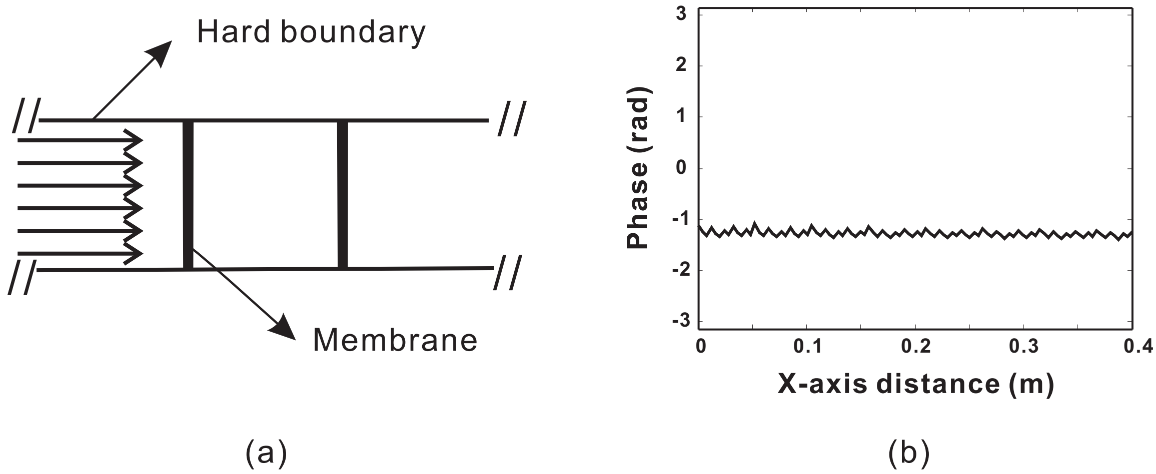

FIG. 2: (a) Schematic illustration of the 1D membrane-based ZIM. Acoustic waves are entering the ZIM from the left side. This structure is used to determine the resonance frequency. (b) The phase change inside the 1D ZIM at 745 Hz.

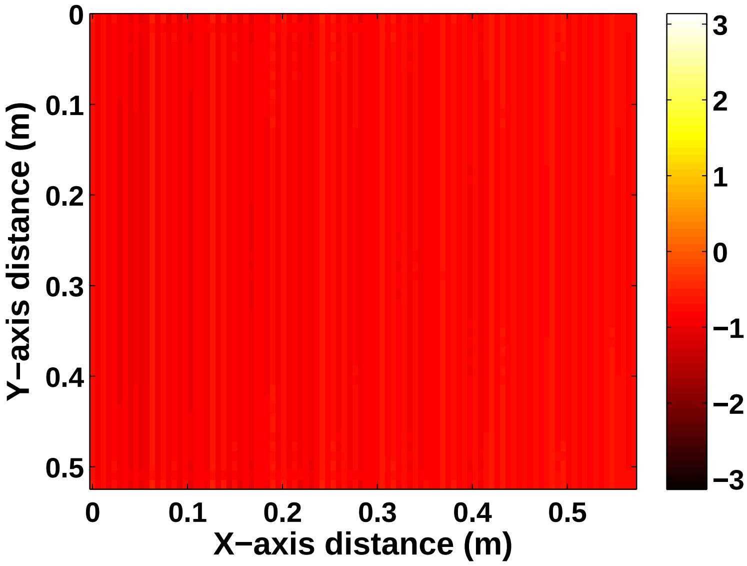

FIG. 3: (Color online) Phase distribution inside the 2D ZIM at 745 Hz as a plane wave enters from the left perpendicularly.

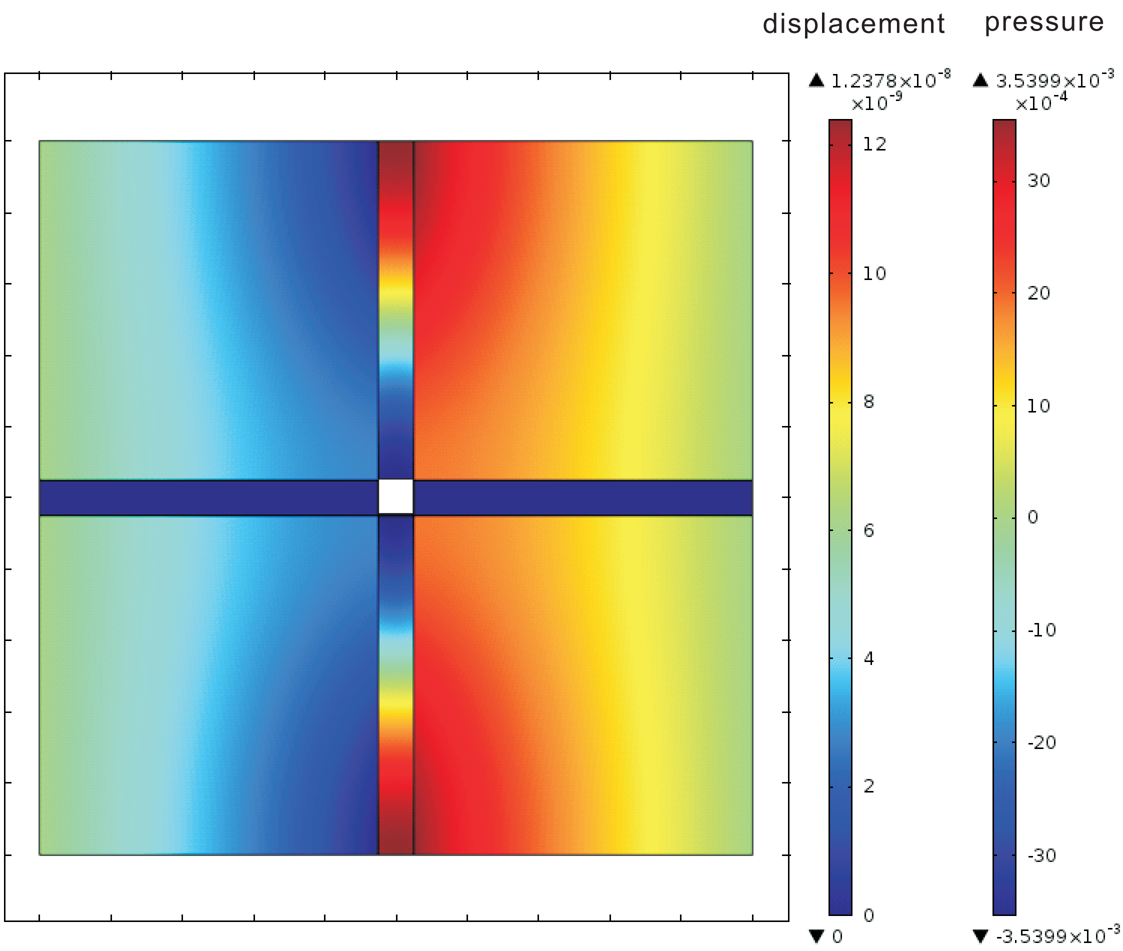

FIG. 4: (Color online) Eigenstate of a unit cell at 745 Hz.

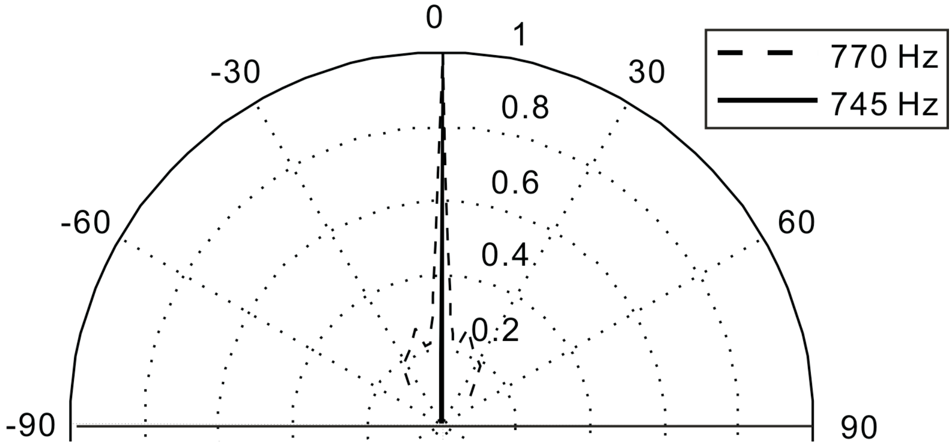

FIG. 5: Polar diagram of the pressure square in the metamaterial at various angles for 745 Hz and 770 Hz.

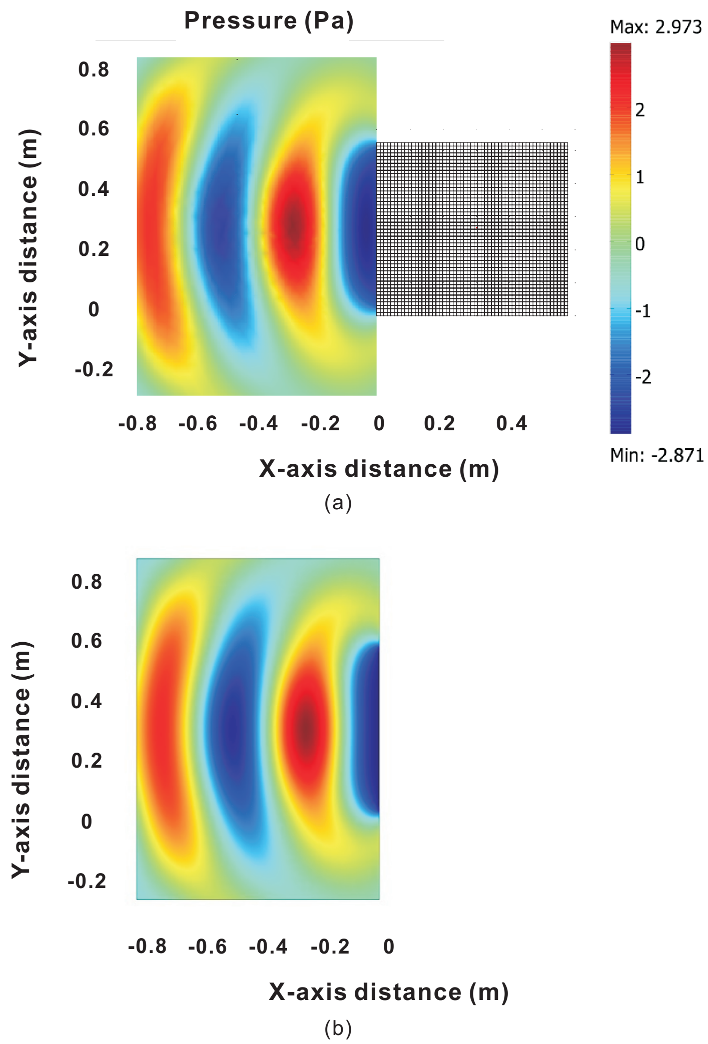

FIG. 6: (Color online) Radiation pattern in the surrounding air from (a) a point source inside the metamaterial. (b) a line source on the left boundary of the metamaterial.