Peak-load Pricing for Transmission in a Deregulated Electric Utility Industry

by

Benoît Lecinq

Submitted to the Department of Civil and Environmental Engineering
in partial fulfillment of the requirements for the degree of

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Abstract

The deregulation of the electric utility industry, introducing competition among the generators of electricity and unbundling generation, transmission and distribution services, will require new pricing mechanisms to replace current internal technical decisions made by the vertically integrated utilities. This thesis provides an analysis of such access rates, based on the technical characteristics of the system and on the objective of economic efficiency measured by social welfare maximization.

Short Run Marginal Cost (SRMC) pricing, by the use of nodal spot prices, is known to lead to a static optimum. The wheeling rates based on SRMC would efficiently cover two components of the cost entailed by power wheeling: transmission loss compensation and out-of-merit generation to accommodate line congestion.

In an attempt to compute prices that also reflect the third cost component of power transmission, i.e. transmission network capital cost, and to convey suitable incentives for network expansion, a peak-load approach of social welfare maximization is used. The cost of investment is explicitly taken into account in the objective function, to model the trade-off between network expansion and use of cheap generation. It is concluded that Long Run Marginal Cost (LRMC) pricing leads to a dynamic social optimum, over a planning time horizon of several years or decades.

Two important results are proven: Firstly, the problem of network expansion decouples into a problem of investment planning and a series of economic dispatches based on SRMC. Secondly, *under optimal investment*, SRMC equals LRMC, and the congestion rents exactly compensate for fixed capital cost.

Finally, these results, as well as a qualitative analysis of regulatory and environmental aspects of the electric power industry, are used to formulate policy recommendations and to propose a two part tariff mechanism for access rates.

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Chapter 1

Why deregulate the electric utility industry ?

1.1 Background

Electric power is an essential basic good, whose availability and quality is an important condition of the good functioning of the economy and of modern residential life. Hence, public interest and economic development require universal and reliable access to electricity, at the lowest possible price.

Moreover, like many other network industries, the electricity industry shows patterns of economies of scale. That is why, since the early ages of the development of electricity, the organization around a central power plant has been favored on local generation facilities close to each demand location. Indeed, larger power plants serving a great number of users enabled better thermal efficiency, or using technologies such as nuclear power suitable only for large scale generation.

Even more striking are the economies of scales possible at the transmission level. The multiplication of competing transmission networks between generating units and final users would obviously lead to inefficiencies, because of redundant investments as well as other technical or environmental considerations such as transmission losses, system reliability, multiplication of transmission lines in the landscape and the like. The only examples of competitive ownership and operation of transmission networks

date back to the early ages of the electricity history, e.g. in New-York City at the beginning of the century (see [Hyma83]). Since that time, it was considered necessary to grant to one single entity the right to build and operate the electric transmission network, over a given area. In other words, the transmission level of the electric industry evolved into a natural monopoly.

The resulting industry structure has often been a vertically integrated electric utility, responsible for generation, transmission and distribution of electricity over its area. However, this organization of the electric industry, segmented horizontally in local monopolies, raises many ideological debates about the potential abuse of monopoly position, public interest, managerial efficiency and incentives for technical innovation. Depending on the historical and political context, different institutional responses have been brought across the world to this issue, ranging from public ownership to publicly regulated investor-owned utilities (see for example [Fros91, Hann82]).

1.2 Current move towards deregulation

More than the ideological debates, technical innovations and the present situation of overall excess generation capacity in Western Europe and in the United States are the origins of the current evolution in the electric utility industry. This evolution was pioneered by the privatization of the Central Electricity Generating Board (CEGB) in England and Whales in 1987-88.

1.2.1 Changes in the technical environment

Whereas most of the potential for economies of scale have been exploited with large units using oil, coal or nuclear fuel, the recent developments of electricity generation favor small or medium capacity units: gas turbine with combined cycles, notably used for cogeneration, renewable energies, etc. This questions the rationale of economies of scale for centralized non competitive generation. Moreover, progress in information technology, telemeasuring devices and automation allow real-time information of the electricity users, to enable them to modify their consumption decision in response to

the cost of electricity supply.

Finally, the investments in generation capacity in the 1970's and 1980's were based on demand growth projections that turned out to be over-estimated by far. This has lead to the current situation of overall excess capacity. Simultaneously, large industrial users and independent distribution companies or municipalities have become increasingly more sensitive to the important discrepancies in the price of energy across American or European utilities. These customers, that were captive until now, are more and more reluctant to pay their electricity at a high price when they could find a much cheaper supply somewhere else.

This has lead to a considerable growth in energy exchanges among utilities, often extending beyond the typical economic transfers between adjacent companies.

1.2.2 Regulatory evolution

The exchanges of power among utilities, or the creation of power pools developed often on the free initiative of companies, conscious of the potential economic benefits and reliability improvements. However, the pressure of large customers and a liberal trend in the European Union and in the United States have raised the idea among legislators that competitive market for electricity generation could be a better organization than the existing regulated utilities or monopolies.

This explains the current thrust towards deregulation in the electric utility industry, notably through the organization of a competitive market-based generation level and by ensuring open access of independent generators to the transmission infrastructure. The main objective of regulators is to induce economic efficiency, by means of optimal use of generation across the interconnected system, and to substitute competition for regulation.

In the Western European interconnection, deregulation stems from the Single European Act of 1987, that aims at creating a continent-wide common market "without internal frontiers, in which the free movement of goods, persons, services and capital is

ensured[1]." In the United States, the Public Utility Regulatory Policy Act (PURPA) passed in 1978 encouraged competition in energy generation by favoring the development of cogeneration and independent power producers (IPPs). Moreover, the Energy Policy Act of 1992 (EPACT) removed some constraints on ownership of electric generation facilities and encouraged competition in the wholesale electric power business.

To go further than the current wheeling performed by utilities on a voluntary basis, the concept of mandatory access to the network for energy wholesalers is under discussion both in Europe and in the United States [Rose92, EIA95, Stal95]. A mandatory access to the transmission network was just ordered by the Federal Energy Regulatory Commission in April 1996. The result in the organization of the industry is an increased interest in performance based ratemaking and retail wheeling as well as cuts in common dividends and employee layoffs that all point to an increasingly competitive electric power industry.

1.3 Deregulated electricity industry

The current evolution of regulatory requirements toward competition in generation and open access to the network leads to important changes in the operations and planning of power systems. Former utilities may loose part of their native load, but at the same time keep the obligation of service[2]. Although this is very unlikely for small users, the pattern of demand may evolve towards a greater volatility, if consumers actively explore the best deals for power supply, and this could question the current methods of load forecasting for operations planning.

More important, the increasing exchanges among utilities imply a more intensive use of strong transmission lines interconnecting them. Whereas utilities or pools used to be organized to be more or less self-sufficient, tie-lines designed for reliability and voluntary exchanges will be used for bulk wheeling transactions, and transmission

[1]Single European Act, Article 8a.

[2]The obligation of service may evolve into obligation to deliver. Indeed, under competition, the utilities can not be held responsible for meeting unconditionally the load.

congestion may result from this evolution into a new way of using the network.

Finally, the separation, at least in accounting terms, of the three levels of vertically integrated utilities, namely generation, transmission and distribution, has important consequences on the economics of network expansion. In vertically integrated utilities, the enhancement of the transmission network used to be planned in coordination with new generation investments, and the cost of new lines was paid for by saving in generation cost. With the vertical dis-integration, generation profits go to the company owning the units but not the transmission lines. Hence, the network operator has no direct incentives in investing in new transmission lines.

To solve this problem, the former technical coordination between generation expansion and network expansion planning needs to be translated into economic price signals. Hence, a coherent mechanism of access rates reflecting the operating conditions of the network must be put in place. This is a two part task: it is necessary to first define the overall costs, and then to allocate them among users of the transmission services in a fair and unbiased way. This latter is far from obvious, as operation of the transmission network currently entails many "common actions" performed centrally for the common benefit of all users, the distinction of the effect of one given transaction being often impossible.

1.4 Outline

The purpose of this thesis is to provide policy recommendations about a central requirement of a deregulated electric power industry: the transmission pricing mechanism. These policy recommendations will be based on a technical and economic model of current and future practice of system operations and planning.

- Chapter 2 describes more precisely the political and historical factors that condition the possible organization of a deregulated electric power industry, notably in Western continental Europe.

- Chapter 3 and 4 present the main technical characteristics of power system operations. A wheeling transaction is defined, and its technical implementation is explained. The requirements of a good system of access rates are also summarized.

- Chapter 5 presents the spot market for electric power generation currently in place in England and Whales, and the theory of optimal spot prices based on an engineering model of the transmission network. The optimal spot prices enable to define rationally wheeling rates, and the corresponding static pricing mechanism is reviewed in Chapter 6

- In Chapters 7 and 8, adopting the point of view of a welfare maximizer regulator, we derive an economical model of network operation and planning that takes the investment cost explicitly into account. This leads us to define short run and long run optimal spot prices and wheeling rates.

- Finally, in Chapter 9, we draw qualitative conclusions proven using our model, and we formulate the corresponding policy recommendations.

The policy recommendations deal with industrial organization, regulation and economic issues. They notably lay the emphasis on the trade-off between economic efficiency, implementability and transparency of the proposed pricing mechanism.

Chapter 2

Industry structure in a deregulated environment

Since the early ages of electricity, about a century ago, the electric power industry has undergone an extraordinary development. The social and economic importance of this sector explains why the governments were particularly involved in shaping the structure of the electric utility sector. Hence, depending on national political context, on historical contingencies, as well as on economic traditions, the organizations of electric industries differ greatly across continents and nations.

Electric utilities may be characterized by their ownership, level of vertical integration, size and the level of fragmentation of the interconnected system, etc. Note that the four major functions of the electric power industry are generation, transmission, distribution, and imports or exports of energy. The latter is often forgotten, but is one of the key issues in the European deregulation.

2.1 Current structure and evolutions

2.1.1 The American electric utilities

As can be seen in Table 2.1, the American electric utility industry is quite fragmented, and mostly privately owned. The numerous publicly owned utilities corre-

spond to municipally owned non-regulated utilities. The cooperatives are common in rural areas, and may depend on the Department of Agriculture.

This industry structure is mostly inherited from the Great Depression and the New Deal policy. The electric industry was concentrated in huge electric utility holding companies in the late 1920's. But the Depression had disastrous consequences on these holdings, due to hazardous promotion of securities, questionable transactions and financial leverage effects. Several of them collapsed, leading to governmental intervention. The private electricity industry that had competed until 1930 only with municipal utilities was threatened by F. D. Roosevelt New Deal policy: Federal agencies were established to exploit important hydro-electric resources, such as the Tennessee Valley Authority (TVA) in the Middle West and the Bonneville Power Administration (BPA) in the North-West [Hyma83].

Moreover, the Federal Trade Commission began a long investigation in 1928, that lead to the Public Utility Holding Company Act of 1935. The PUHCA broke up non-contiguous holding companies, and placed interstate holding companies under the jurisdiction of the Security and Exchange Commission. The breakup of the holdings lead to the current structure of the American electric industry, i.e. the coexistence of municipal or private utilities and federal agencies, and horizontal segmentation in regional monopolies. The private utilities are tightly regulated in each state by the Public Utility Commission and interstate exchanges are overseen by the Federal Energy Regulatory Commission (FERC).

A significant evolution of that structure began with the Public Utility Regulatory

Item	Investor owned Utilities	Publicly owned Utilities	Cooperatively owned Utilities	Federally owned Utilities	Total
Number of utilities	254	2007	941	10	3212
Sales (billion kWh)	2187	407	221	46	2861
Sales (%)	76	14	8	2	100
Retail price (c/kWh)	7.2	6.1	7.0	2.8	6.9
Wholesale price (c/kWh)	3.4	3.8	4.1	3.3	3.6

Table 2.1: Ownership of American electric utilities. Source: [EIA95].

Policy Act (PURPA) passed in 1978 to encourage competition in energy generation by favoring the development of cogeneration and independent power producers (IPPs). To develop competition in the generation level and in wholesale of electricity, the Energy Policy Act of 1992 (EPACT) removed some constraints on ownership of electric generation facilities and extended the jurisdiction of the FERC over transmission utilities.

The debate over open access to the transmission grid and market-based electricity supply has become very intense in the last two years, since the California Public Utility Commission issued its Blue Book Orders and the Federal Energy Regulatory Commission issued its Notices on Proposed Rulemakings, *Promoting Wholesale Competition Through Open Access Non Discriminatory Transmission Service by Public Utilities*, and *Recovery of Stranded Costs by Public Utilities and Transmitting Utilities* [Stal95].

For the regulating authority, one of the critical issues of the deregulation of the American electric industry is the governance system that will guarantee reliability of the interconnected system. Indeed, since the great Northeast blackout of 1965, that deprived more than 30 million people from electricity for three days, industry cooperation for system reliability formalized in a self-regulation body, the nine Regional Reliability Councils (RRCs) and a coordinating agency, the North American Electric Reliability Council (NERC). This self-regulation produced rules to ensure reliable and efficient operation of the network, through voluntary cooperation, and now enjoys a great legitimacy. However, it will probably lack authority against the economic interests of profit-driven players in a deregulated industry.

2.1.2 The European electric power industry

The organization of the electric industries of Western European countries is very diverse, and is often a result of history. Some countries exhibit a state-owned vertically integrated monopoly coexisting with a few auto-producers and municipal utilities, such as in France, Italy and Great-Britain until 1987. The establishment of Electricité de France (EDF) by the nationalization law of May 1946, or the nationalization of British privately owned utilities to form the Central Electricity Generating Board

(CEGB) in April 1948 resulted from the need for state intervention in the post-war reconstruction, and of political alternance: the Communist Party was very influent in France between the Liberation and 1947, and the Labour Party wan the 1945 elections in Great-Britain (see [Fros91, Hann82, Samu87]). Electricity service in Italy is mostly carried out by a vertically integrated monopoly, the ENEL, created in 1962 by the nationalization law, and replaced in 1992 by a joint-stock company, ENEL S.p.A [Covi93].

In other countries, the electric industry is less centralized and partially or completely privately owned. For example, the German electric industry is characterized by its decentralized and pluralistic structure: nine privately owned "interconnected utilities" generate power and supply about 55 "regional" and "local utilities." They are interconnected by the national grid that is publicly owned [Kasp93].

In England, Wales and Scandinavia, the electric industry has undergone recently dramatic changes, leading to the creation of partially competitive deregulated organizations. The privatization program of Margaret Thatcher hit in 1987 the national electric power industry of England and Wales: The CEGB was divided into three generating companies, PowerGen, National Power and the publicly owned Nuclear Power, and one transmission company, National Grid [Oliv92]. The former Area Boards have been privatized into 12 regional electricity companies (RECs) regulated by a new regulatory agency, the Office of Electricity Regulation (OFFER). The industry is organized as a spot market power pool: the power generators bid every day for the next day, and the pool purchases power according to its forecast needs, at a unique spot price that equals supply and demand. The generators can also pass long term financial contracts with the RECs, to hedge long term uncertainties. The Scandinavian electric industry is completely privatized as well, and organized with bilateral contracts between power producers and distribution companies.

Under the Single Act of 1987, no internal frontiers should be opposed to the free movement of goods in the European Union. Electricity and natural gas are considered as classical goods falling within the jurisdiction of the Single Act. Consequently, the European Commission has been working since 1986 to create the legal and regu-

latory framework that would open the European internal frontiers to electric power exchanges, and to remove the obstacles to oblige utilities to compete for electric power supply. Two directives have been issued in 1990: the directive on price transparency[1] and the Transit directive[2] [Cler93, Covi93, Kind93]. The application of the directive on price transparency reveals that prices still differ substantially between member states, as well as between consumer groups (see Appendix A). The Transit directive requires utilities to accept transit across their territory, except when this is impossible for technical reasons. In other terms, it establishes the right to export and import electricity (and gas) from another system through an intervening system at equitable conditions. But it does not represent a system of common carriage, also known as third party access (TPA).

A further project was to separate the accounts of generation and transmission and enable third party access to the network. However, this implies a serious modification of current operating principles, and it was realized that missing or insufficient physical transmission capacities may seriously hinder cross-boundary transit. Therefore, the current proposal of the Commission is to adopt "a gradual approach to enable the industry to adjust to its new environment in a flexible and ordered manner" [Kind93]. After the first stage consisting of an actual implementation of the Transit directive, the second stage would require:

- the abolition of exclusive rights regarding electricity generation and the building of transmission lines,

- the unbundling, i.e. the separation of management and accounting of production, transmission and distribution activities in vertically integrated companies,

- the introduction of a limited TPA, by which transmission and distribution companies are obliged to offer access to their network to certain eligible entities, at reasonable rates, within the available transmission and distribution capacity.

[1] Directive 90/377 of 29 June 1990.
[2] Directive 90/547 of 29 October 1990.

The criteria for eligibility limit access to the network to large industrial consumers whose annual consumption exceeds 100 GWh (about 400 to 500 industrial consumers eligible in the Union), and distribution companies who supply at least 3% of the electricity consumed in their member state (distributors can join to meet this threshold, and about 100 distributors will be eligible as a whole).

Until now, the issue of system reliability has been handled on a cooperative basis in the Western Europe interconnection: a number of rules and recommendations for the synchronized operation of interconnected networks have been formulated by the Union for Coordination of Production and Transmission of Electricity (UCPTE). This entity functions like the NERC in Northern America, and focuses on reliability, quality of service, mutual help, control of frequency and reduction of consequences of failures.

The European Commission wishes to avoid excessive regulation, and each member state will remain free to set the public service obligations, in particular the obligation of supply. However, to prevent a loss of reliability with deregulation, rather detailed rules for the rights and obligations of system operators are being laid down.

2.1.3 Price variation across utilities

After presenting the current regulatory evolution in the United States and in Europe, we must emphasize that the purpose of the regulatory authorities is in both cases to enable final consumers to be served by the most efficient producers of electricity. The variations of electricity prices across utilities are considerable, and reflect the fact that some generators are much more efficient than others, or benefit from a better regulatory or physical environment.

Appendix A summarizes some data on the average price per kWh in the United States and in European countries, for both domestic use (retail price) and industrial use (wholesale price). The fluctuations are as large as 50% in the United States, and 20% to 30% in Europe. For example, California could make huge savings on its electricity bill by purchasing power from Washington or Oregon. Similarly, New England prices are nearly twice as large as the prices in the Midwest (Tennessee,

Kentucky, Illinois).

It is generally considered that the generation cost accounts for 80% of the final price of electricity, and that the transmission and distribution costs are approximately 20%. These latter entail capital cost of transmission lines, administrative cost notably billing, operation and maintenance of the transmission network, as will be described later. Hence, most of the price discrepancies are caused by differences in generation cost. The empirical data recalled here make the point that establishing competition at the generation level can have very significant effect on prices paid by end users.

However, it should be noted here that many factors explain such significant price variations across states and utilities. Managerial efficiency accounts for some of the variation, but other external factors are imposed on utilities, and may lead to unfair competition in case of deregulation:

Legal environment Some utilities are constrained by strict environment regulations. For example the California utilities have to produce a certain amount of electricity from renewable energies, such as wind, solar energy and geothermal. These sources are still significantly more expensive than classical fuels [EIA95]. In Germany and in Great-Britain, the power producers were compelled to sign high price long term coal contracts with domestic coal producer. In Great-Britain again, generators using classical fuels pay a "levy" to subsidize nuclear power [Oliv92].

Physical conditions Less dependent on local politics, the physical availability of primary energy also explains variation in generation cost. The electric utilities in New England and Mid-Atlantic (Pennsylvania, New-Jersey, New-York) rely more heavily on oil, that is more expensive than natural gas readily available in the Midwest [EIA95]. The Western utilities, e.g. Colorado and Arizona, can use more hydro-electricity, that has virtually no fuel cost.

Type of ownership Federal utilities have the cheapest price of electricity: on average for all sectors 3.96 cents/kWh for the TVA and 1.93 cents/kWh for the BPA. This is mostly because they use large scale hydro-electric facilities. Moreover,

similar to any publicly owned utilities, they can borrow money at lower rates, because the loans are guaranteed by the federal or the municipal governments and lenders require less risk premium. Sometimes, the electricity prices are also used as a public subsidy to local industries.

Finally, the variation of prices between the different sectors served by electric utilities, domestic, commercial or industrial result from differences in costs: providing base-load power at a medium or high voltage is more economic than supplying a domestic fluctuating demand at low voltage. But the price differences may also involve cross-subsidies, sometimes required by the government, for example to support local industries. The variation in the proportions of different sectors can also have significant effects on the price structure of a given utility.

As a conclusion, one must be very careful when performing cross country or cross utility comparisons. But in the current situation of overall excess capacity, reducing the segmentation of the electric generation industry will bring significant efficiency and savings.

2.2 Deregulated industry structure

As recalled in the previous section, the main goals of restructuring the electric utility industry are to enable competition among the power generators. Therefore, the end users, or their distributors should not be compelled to buy electricity from a local utility. This means the end of captive consumers, as well as of vertical integration of electric utilities, at least in accounting terms, if not in ownership. Thus, new business units will be created, and new tools to measure their performance *ex post* should be required. This increased accounting transparency should also facilitate the role of a regulator.

One of the major issues resulting from a partial loss of captive consumers, is the stranded investment: electric utilities have invested in generation facilities by planning a certain level of demand for the next 15 to 30 years, and if a share of this expected demand disappears, the remaining users may have to bear an increased

financial burden for reinbursing past investments.

This thesis does not investigate any further such institutional issues. However, a model of industry structure must be considered to characterize the operating conditions of the power system, and the modalities of access to the network.

2.2.1 Proposed frameworks

Several models have been proposed to accommodate the unbundling between the three levels of the electric utility industry. The generation level is generally thought of as a competitive industry. The distribution of domestic users should remain a natural monopoly, over a certain geographic area, if one does not want to lay new wires between distribution centers and homes. Thus, the only possible type of competition at the distribution level would be by ranking distributors according to performance measurements. It is not a market-based competition, but rather a performance-based competition. This is currently done in England and Wales to evaluate the 12 RECs and seems to bring interesting results in terms of performance incentives.

Finally, between generation and distribution, the transmission level is generally considered as a natural monopoly, and one single system operator should have sufficient authority to ensure system reliability. The greatest debate about the suitable deregulated industry structure deals with single buyer vs. third party access to the network, in Europe, or PoolCo model vs. bilateral model in the United-States [Kind93, Hoga92, Cara95].

In the PoolCo or single buyer or buy-sell model, the pool owns and operates a transmission grid. It calls for bids from the generators for a scheduled amount of supply, purchases electric energy in the order of increasing price, and sells it to the distributors. All the generators are paid the same price, called the spot price, that equals supply and demand. All the distributors pay the same price to the pool, equal to the generation spot price plus a given overhead to cover transmission costs. The spot price varies from hour to hour, according to the demand conditions, as in the current implementation in England and Wales.

To make huge capital investments into new generating units, generators need more

stability and guarantees than provided by a spot market. Therefore, they want to sign long term bilateral contracts with distributors or large industrial users, and to obtain guaranteed transmission rights. Moreover, such bilateral contracts are considered as an insurance against a potential inefficiency of the pool. Therefore, a second framework proposed for a deregulated industry is to rely exclusively on bilateral contracts among generators and distributors.

A coordinator would be responsible for system reliability and takes care of the technical requirements of transmission. It is often called CoorCo or GridCo [Klei95]. In both cases, the financial flows link directly generators and distributors. The difference is that the CoorCo would not own the grid, and is just responsible for operations, whereas the GridCo owns and operates the transmission network, and is also responsible for transmission expansion.

It is widely recognized that both the PoolCo and the bilateral models lead theoretically to the same static efficiency. However, the main issues in the debate involve transaction costs, reform of current operating practices, system support services, transmission rights allocation and transmission investment. In particular, the proponents of the CoorCo model propose that transmission rights could be traded on primary and secondary markets. In the PoolCo and GridCo models, the transmission company is the sole seller of transmission capacity. Many authors agree that neither extreme is necessary, and support a hybrid model, that would be a flexible GridCo or flexible PoolCo.

2.2.2 Model considered for Western Europe

In addition to all the difficulties of organization of a PoolCo or a bilateral model, the European industry has to tackle other problems. As explained earlier, the difference in national legal environment concerning the use of primary energy and the construction of new generating units would lead to a damaging unfair competition among generators. Moreover, the scarcity or inexistence of cross-boundary tie-lines could not accommodate an important reorientation of power flows without congestions. Finally, the centralized organizations of several countries can not be unbundled

overnight without huge social problems, but these national utilities have potentially a considerable market power.

Therefore, the European Commission favors a gradual approach, and has adopted an intermediary step with the concept of eligible consumers. This is in fact a hybrid model, where the former vertically integrated utilities would continue to serve most of their native demand. They would have to clarify accounting rules, and accommodate bilateral "by-pass" transactions at a regulated price.

We have seen that the eligible consumers are large distributors and industrial users. This would represent a fringe of the current vertically integrated activity of utilities. The exact share of bilateral transactions remains to be determined empirically, but should not exceed 10% of the total energy transfers. Moreover, the pattern of consumption of large industrial users (steel or aluminum makers, construction materials manufacturers, and so on) is typically very stable. The pattern of consumption of large distributors is fluctuating according to the behavior of domestic users, and this should not be modified by deregulation, as domestic demand is often considered totally inelastic. Hence, the demand patterns of eligible consumers are either very stable, or periodic, and in any case, could be forecast as in current practice.

Therefore, nearly all the bilateral contracts would consist in scheduled transactions. The contracts may be long term (years) or short term contracts (hours, days), but they will always agree in advance on a given power transfer at a given time, between one injection point or area and one removal point or area. In the following, this thesis focuses on the transmission access rates suitable to accommodate these fringe bilateral transactions while maintaining operations' optimality and taking provisions for transmission network expansion.

Chapter 3

Technical characteristics of power system operations

In the previous chapter, we presented a possible organization of the electric utility industry to accommodate competition at the generation level. It is emphasized that the structure of a deregulated industry greatly depends on historical and national characteristics of the present organization: centralized or decentralized mode of operations, type of ownership, political pressure and so forth.

However, institutional factors are only one aspect of the process conditioning the future of the electric utility industry. Market forces, and technological characteristics of the system are two other factors that can not be underestimated in shaping the new environment. For example, electric power utilities have greatly increased the level of energy exchanges over the recent years, driven by the economic advantages of such exchanges.

The purpose of this thesis is to provide some policy recommendations about the organization and the operating methodology in a deregulated industry. These recommendations will be based on the fundamental technical characteristics of electric power systems that impose strong constraints on the practical implementation of classical economic precepts. Indeed, it is usually considered that competitive markets lead to a social optimum, and one of the goals of the current evolution is to extend this to the formerly regulated electric utilities.

However, we must recall here that these economic theories rely on strong hypotheses, the most important of which being certainly that all the actors of a competitive market are atomistic and that no network externality should be taken into account. In other words the decisions and actions of competitive players do not depend on the behavior of the competitors and have no effect on the global outcome of the market. In the present chapter, we present several technical features of power systems that seriously question that hypothesis: the electric network turns out to require a high degree of coordination between the different actors, notably the generators.

Important specific properties of electric power systems are

- The problem of parallel flows: current follows the path of least resistance, according to the Kirchhoff's laws.

- The time dependency of demand, with daily, weekly and yearly cycles. As the balance between demand and supply must be met constantly, these fluctuations impose control actions on the generation side at different time frames: automatic generation control, operations planning (unit commitment), maintenance and refueling scheduling and the like.

The problem of loop flow and the time varying conditions are the source of major difficulties when attempting to define meaningful access rates for power networks. However, this is somehow compensated by the fact that, contrary to telecommunication networks or transportation networks where messages or vehicles must actually be transferred from their origin to their destination, electric energy is a unique commodity, and electrons do not need to travel from the injection point to the removal point in a transmission transaction. Instead, power flows are balanced at each node, as a result of the superposition of all the transactions.

Moreover, contrary to a common idea, a real electric power network is not a linear electric circuit. One can not apply simple superposition rules to see the effect resulting from individual transactions on the transmission network. The non-linearities in the physical laws that describe the behavior of power system arise partly from the non-linear AC load flow equations, although the DC load flow equations pro-

vide a good linear approximation. But the main causes of non-linearities are the transmission losses and the various operating constraints, notably the transmission capacity constraints that may result in congestion of the network. The consequence of these non-linearities is that the electric network imposes some coupling among the individual actors, and requires to some extent a centralized intervention of a system operator.

3.1 Notations and assumptions

The notations used throughout this thesis as well as the modeling assumptions for a typical power system are introduced here. We consider a network consisting of n nodes or busses, each of them being both a generator and/or a load.

- $i, j \in \{1, \ldots, n\}$ denote the bus index.
- $l \in \{1, \ldots, n_l\}$ denotes the line index. We use indifferently l or the double index ij designating the line between the pair of busses i and j.
- P_{Gi}, Q_{Gi} are the real and reactive power generated at bus i (respectively in MW and MVAR).
- P_{Di}, Q_{Di} are the real and reactive power demanded by the load of bus i.
- $P_i = P_{Gi} - P_{Di}$, $Q_i = Q_{Gi} - Q_{Di}$ denote the net real and reactive power injected in the network at bus i. They can be either positive or negative.
- $\hat{S}_i = P_i + jQ_i$ where $j = \sqrt{-1}$ is the complex valued representation of the power injected in the network at bus i.
- $\mathbf{P}$, $\mathbf{Q}$, $\hat{\mathbf{S}}$ denote the n-vectors of the injections at each bus.
- $\hat{V}_i = V_i \exp(j\theta_i) = V_i \angle \theta_i$ is the voltage phasor at bus i, where V_i is the magnitude, and θ_i is the phase angle of the voltage relative to some synchronous (50 or 60Hz) reference frame.

- $\hat{\mathbf{V}}$ denotes the n-vector of the voltage phasor at each bus.

- If a transmission line joins bus i to bus j, its complex admittance is denoted $\hat{y}_{ij} = g_{ij} - jb_{ij}$, $g_{ij} \geq 0$, $b_{ij} \geq 0$. If there is no such line, $\hat{y}_{ij} = 0$. If the line exists $\hat{z}_{ij} = \hat{y}_{ij}^{-1} = r_{ij} + jx_{ij}$ denotes its complex impedance.

- T_{ij} is the real power flowing from bus i to bus j. Once an orientation has been chosen on the transmission line, T_{ij} can be either positive or negative, depending on the direction of the power flow.

- $L_{ij} = L_{ji}$ denotes the real loss over line i, j.

Most of the time, we consider only the net power injected into the network at each bus, by netting out the effects of simultaneous generation and consumption at the same node. As we will explain in the next chapter, the economic behavior of the generator at node i is entirely described by a generation cost function $G_i(P_{Gi})$ and the behavior of the demand by a downward sloping demand curve $P_{Di}(p)$ or an inverse demand function $p_i(P_D)$, where p_i is the price of power

The procedure of handling in a symmetrical manner generation and demand assumes that loads are manageable in the same way as generators, and does not consider power demand like an externally imposed parameter. Hence, this slightly differs from the current obligation of supply for electric utilities. However, our main interest here is transmission network access for wheeling and bilateral transactions where the contracting parties can be seen as "price takers" looking at the transmission rate to base their decisions. In this case, generation and consumption are actually symmetrical, and the netting approach is justified.

In the following, we neglect the transient response of generators to system perturbations and consider only the steady state power flow. Finally, note that all the variables presented here can be function of a time index t if several steady states are considered. This time dependency will be omitted in static analyses, when only one time period is considered.

3.2 Generation constraints

The physical characteristics of generating units, such as turbine physical limits or rotor heating tolerances, limit the feasible range of their real and reactive power output. Those limits are specific to each unit and may imply trade-off between real and reactive power, such as the maximum apparent power rating $P_G^2 + Q_G^2 \leq S_{max}^2$. Each unit has generally a minimum power output level when it is turned on. The power output is zero only if the unit is turned off. If the unit i is turned on, the generation constraints are modeled as:

$$\begin{aligned} P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \\ Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{Gi}^{max} \end{aligned} \tag{3.1}$$

Under normal conditions, to preserve some production slack to respond to sudden changes in system voltages, the units are operated far away from the constraints on reactive power, with a power factor $P_G/\sqrt{P_G^2 + Q_G^2}$ close to unity. Hence, the constraints on Q_{Gi} are usually less restrictive.

It may seem that the generation constraints presented here are only the concern of the owner of the unit, and have nothing to do with network coupling. However, the reliability of the power system involves all the actors, and more conservative generating constraints arise from reliability considerations: generators can be operated away from their limits to keep generation reserve margins. This will be evoked subsequently as part of the system ancillary services.

3.3 Voltage constraints

The voltage magnitudes at each bus of a large interconnected power system have nominal values. As will be seen below, in order to reduce transmission losses, the power network is organized in several classes of voltages, with transformers between each corresponding sub-network. Hence, the nominal voltage magnitudes depend on the type of bus considered, and range from residential low voltage such as 110 V or

220 V, to extremely high voltage such as 225 kV or 400 kV on the EHV network.

The actual voltage magnitude at each node is forced to remain within a small range around the nominal values. This range may vary according to the operating conditions, and is typically 5% for normal conditions, and 10% for critical conditions. Hence, the voltage constraints at each bus take the form

$$V_i^{min} \leq V_i \leq V_i^{max} \quad \forall i \tag{3.2}$$

The following analysis does not consider all the classes of voltages, as residential or small industrial demands are aggregated locally, or regionally into large loads. Moreover, we usually express the voltage magnitude as a ratio of the actual value over the nominal value. This is called the per unit analysis (p.u.), and it cancels out the distinction between different voltage classes. Typically, for normal operating conditions, one has $V_i^{min} = 0.95\, p.u.$ and $V_i^{max} = 1.05\, p.u.$

Finally, each bus is assumed to be a PV bus. This relies on the decoupling assumption between real power and frequency (or voltage angle) on the one hand, and reactive power and voltage magnitude on the other hand. At the time frame of interest, the reactive power Q_i is supposed to be automatically adjusted so that the voltage magnitude V_i remains constant. Hence, the bus behavior is completely specified by giving the voltage magnitude and the real power injection.

3.4 Load flow equations and transmission losses

The real and reactive power flowing over transmission line $i\,j$ are given by the AC load flow equations in Appendix B. For the complete network, the AC load flow equation takes the matrix form:

$$\hat{\mathbf{S}} = \mathbf{diag}(\hat{\mathbf{V}})\hat{\mathbf{Y}}^{\star}\hat{\mathbf{V}}^{\star} \tag{3.3}$$

where $\hat{\mathbf{Y}}$ is the network complex admittance matrix, $\mathbf{diag}(\hat{\mathbf{V}})$ is the diagonal matrix formed with the entries of vector $\hat{\mathbf{V}}$ and the symbol * denotes the complex conjugate. Equation (3.3) show the quadratic dependency of power flows on complex valued bus

voltages. The real power loss over the line $i\,j$ can be derived as

$$L_{ij} = P_{ij} + P_{ji} = g_{ij}[V_i^2 + V_j^2 - 2V_iV_j\cos(\theta_i - \theta_j)] \tag{3.4}$$

and $L = \sum_{i,j} L_{ij}$ denotes the total transmission losses over the network.

These AC load flow equations show the non linearities arising from the exact expression of power flows and transmission losses. However, as we are mostly interested in real power flows, and we often make the assumption that node voltage magnitudes are maintained close enough to their nominal values to be approximated by $1\,p.u.$ and that voltage angle differences across lines are small. Then, AC load flow equations are approximated by the DC load flow equations, and the real power flow pattern across the network can be expressed as a linear function of power injections:

$$\mathbf{T} = \mathbf{HP} \tag{3.5}$$

where $\mathbf{H}$ is the $n_l \times n$ transfer admittance matrix (see Appendix B). The real power loss on line $i\,j$ takes the form

$$L_{ij} = r_{ij}T_{ij}^2 \tag{3.6}$$

and the total losses are given as a function of real power transfers or real power bus injections:

$$L = \mathbf{T'diag(r)T} = \mathbf{P'BP} \tag{3.7}$$

where $\mathbf{T'}$ is the transpose of $\mathbf{T}$, $\mathbf{diag(r)}$ is the diagonal matrix whose elements are the line resistances r_{ij} and $\mathbf{B} = \mathbf{H'diag(r)H}$.

The transmission losses on the network are a quadratic function of the bus power injections. These transmission losses are part of the cost of power transportation: a wheeling transaction for transfering $100\,MW$ from one bus to another of the network will require generating more than $100\,MW$. This difference should be compensated for, either in money, or in power, the generator agreeing on producing the required amount to compensate for his losses.

The issue of assessing the amount of transmission losses caused by a given trans-

action is complicated by the non linearity of (3.7). Indeed, the transmission loss will not be independent from the operating conditions prevailing prior to the transaction. Less losses will be incurred to the wheeling utility if the prevailing power flows are lower. And the prices charged to different transactions for transmission losses compensation depend on the order in which the transactions are handled. As we will explain in Chapter 9, rather than using a "First In, First Served" approach, this may require to put all transactions on equal footing, by computing the prices one day in advance for scheduled transactions.

3.5 Transmission constraints

The real power transfer along a line is limited for several reasons. Firstly, the line losses produce heat according to $r_{ij}T_{ij}^2$. To limit line over-heating damage, the real power transfer is maintained within the quasi-static bound

$$|T_{ij}| \leq T_{ij}^{max,therm}$$

The heating process is cumulative over time, and the thermal limit may depend on the weather conditions, but this is not modeled here. Moreover, from the DC load flow approximation, we get

$$T_{ij} \approx b_{ij}V_iV_jsin(\theta_i - \theta_j) \tag{3.8}$$

As a result, for given voltage magnitudes, the real power transfer along a line depends on the voltage angle difference across the line, and the theoretical maximum occurs roughly at 90°. This theoretical maximum corresponds to the famous "maximum power transfer theorem." However this is not a stable operating point, because the feedback linkages between load and frequency lead to system instability when the voltage angle difference is over 90°. The voltages at the end user are also too high with such operating conditions.

The two ways to avoid this instability problem are to operate the system within

the steady-state stability limits or to enhance the transmission lines. The stability limit corresponds to small voltage angle differences on any line, typically less than 20°, and sometimes also requires maintaining the system voltage angle, i.e. the largest angle between any pair of generators on the system, less than 90°. On systems with many generators, this latter requirement may necessitate all of the individual line angles be quite small, e.g. less than $5° - 10°$.

Instability can also be alleviated by increasing the power transfer capacity of critical lines, notably by compensating the intrinsic line reactance with shunt capacitors. Using (3.8) it can be seen that reducing the line reactance, i.e. increasing b_{ij} leads to a smaller angle difference for the same power transfer.

Hence, for a given line, we can express these constraints on the phase angles in terms of power transfers, and the stability limit can be modeled as:

$$|T_{ij}| \leq T_{ij}^{max,stab}$$

As $T_{ij}^{max,stab}$ depends on the line characteristics, a shorter line will have a higher stability limit than a long line. The thermal limit is more likely to constrain a short line, whereas a long line will be constrained by the stability limit. Moreover, the physical constraint on a line flow in not necessarily a *hard constraint*, because a small violation will not instantaneously cause the line to burst into flames or the system to be de-stabilized. Instead, overly high flows cause loss of line life and increase the probability of a line failure immediately or in the future, hence they increase the maintenance cost and decrease the *quality of supply* of transmission network to the customers. These effects can be captured in a cost function that gets large rapidly as the line flows approach or exceed some level. One can adapt the barrier functions used in interior point optimization algorithms [Boyd93], and use for example a penalty function of the form

$$N_{QS}(T_{ij}) = -\alpha \log \frac{0.9|T_{ij}| - T_{ij}^{max}}{T_{ij}^{max}}$$

It is obvious that this function allows for no more than 10% constraint violation. This cost function is a part of the total transmission cost, and yields *soft constraints* on

transmission.

In the following, we favor the hard constraint approach over the quality of supply cost function, and will simply retain a transmission constraint of the form

$$|T_{ij}| \leq T_{ij}^{max} \quad \forall i, j \tag{3.9}$$

where

$$T_{ij}^{max} = \min\{T_{ij}^{max,therm}, T_{ij}^{max,stab}\}$$

We recall here that, although the transmission constraint (3.9) is expressed line by line, the network capacity on a given line should not be related only to the gap between $|T_{ij}|$ and T_{ij}^{max}. This capacity should rather be derived from a sensitivity analysis of the whole network. Indeed, the loop flows resulting from the Kirchhoff laws relate the power transfer on one line to the network-wide flow pattern, and a binding transmission capacity limit on one line may prevent additional transfers on non congested lines. Hence, congestion needs to be solved by re-dispatching generation on the whole network. This can be done in two different ways: either by performing a complete constrained economic dispatch, to obtain an optimal dispatch of generation that meets all the operating constraints, or by merely changing the output of a limited number of generating units. The later way is usually suboptimal, but easier to implement. Consequently, a trade-off between economic efficiency and difficulty of implementation must be found.

3.6 Interconnected operations (ancillary) services

The inequalities (3.1), (3.2) and (3.9) represent the operating constraints on generation output, voltage magnitude and power transfers. Our approach emphasized these constraints with the point of view of a single line or a single generating unit. On a real electric power network, several functions must be assured at the system level in addition to the economic dispatch: load following and frequency regulation, voltage regulation, loss compensation and contingency analysis. These functions are usually

referred to as *ancillary services* by FERC or *interconnected operations services* by NERC.

3.6.1 Load following and frequency regulation

Without getting into too many details, these two functions involve simultaneously decentralized actions at each generation unit and regional controls, see [Ilic96]. For frequency regulation and load following in response to small random deviations from the anticipated values, generally only a handful of generators throughout the system is participating in closed-loop regional control. These units are known as Automatic Generation Control units (AGC). They maintain the system frequency to its nominal value (50 or 60 Hz) and the tie-line flows to a scheduled value.

The feedback used in each area for this regional control is the Area Control Error (ACE), which is an aggregate of system frequency and net power interchange of the control area with its neighboring regions:

$$ACE = b(\omega - \omega^{ref}) - F^{net,sched} + \sum_{tie-lines} F_l \tag{3.10}$$

where b is called the area frequency bias, and represents the effect of 1 Hz frequency deviation on the power balance. The term $F^{net,sched}$ is the net scheduled power exchange of the control area with all its neighboring utilities. It is an aggregate of all the scheduled bilateral exchanges.

It must be recognized here that, whereas each utility regulates its frequency, and the aggregate net tie-line flow, it does not regulate each tie-line flow individually. However, in a large interconnected system, if *each* utility managed to regulate its aggregate net tie-line flow, the resulting bilateral exchanges between two adjacent utilities would always match the schedule[1]. But in real life, as can be seen from the definition of the ACE, a shift in frequency may be compensated for at the same time as a deviation in net tie-line flows, and the time response of the controller is

[1]However, if several tie-lines link the two utilities, the distributions of the exchange over these lines can not be regulated.

not very fast. Hence, the ACE may fluctuate over time in a strip around zero, and *Inadvertent Energy Exchanges* (IEE) among utilities may result. These are deep technological properties of interconnected power systems, that seriously limit what kind of wheeling transactions are possible while maintaining system frequency close to its normal value.

Finally, it must be noted that no economics are taken into account for the frequency regulation function: the AGC units are given plants equipped with the suitable hardware, and they act as swing busses whatever the "system lambda," in other words, for any system marginal cost. In France, most of the nuclear power plants and classical units participate in the load following: all the country is encompassed in a single control area, and all the units respond to the control signals proportionally to their power output variation range.

3.6.2 Voltage regulation

The voltage regulation to meet (3.2) is obtained by decentralized control of the excitation at each generator level, as well as line compensation by means of distributed devices such as switching capacitors distributed throughout a transmission network. In some utilities, such as Electricité de France, this is completed by an automated reactive power generation scheduling at the regional level, to regulate the voltage magnitude of *pilot nodes*. The generating units participating in voltage regulation, when it is automated, are known as Automatic Voltage Control units (AVC).

Although voltage regulation may involve centralized actions on the network and a central scheduling of reactive generation, this is less of a policy or economic issue, as reactive power involves no direct fuel cost. The objective of reactive power scheduling is usually to reduce transmission losses by optimizing the voltage profile over the whole network, and to avoid voltage collapse by maintaining the reactive output of generating units away from their limits. However, in a deregulated environment, charging suitably for transmission losses seems far more important than for voltage compensation.

3.6.3 Loss compensation

The issue of loss compensation stems from the fact that it is not possible to anticipate directly the transmission losses when computing the power injections required to meet a load pattern. Mathematically, this can be seen in the fact that the $2n$ real equations of the AC load flow (3.3) is not an independent system. The resolution is usually carried out by assuming that the power injection at one generator, referred to as the *swing bus* or *slack bus*, is not specified. The swing bus serves as a reference for voltage angles, and its power output serves to meet the power balance at the system level:

$$\sum_{i=1}^{n} P_i - L = 0 \tag{3.11}$$

In real power systems, the generation dispatch is computed for a scheduled load, one day in advance, and the resulting transmission losses can be distributed among the scheduled units. In real time, the departure from scheduled operating conditions, and the resulting losses, are compensated by the AGC units. Hence, the function of swing bus is distributed among several units.

As we have explained in Section 3.4, the transmission prices on power systems need contain a component related to transmission losses. These losses are not linear, hence they depend on the operating conditions of the network. This difficulty is confirmed here by the role of the swing bus: it is not possible to determine *a priori* the exact amount of transmission losses, as long as the exact operating point is unknown. Therefore, loss compensation should be considered as a central issue of transmission pricing, that can hardly be solved at a decentralized level.

3.6.4 System reliability and contingency analysis

Contingency analysis aims at protecting the system in the event of major outages or transmission line cuts. To anticipate these problems, the generation is scheduled in such a way that the system would still operate in normal conditions during at least 10 minutes after any single contingency. This is often referred to as the N-1 security requirement.

Contingency analysis is performed off-line, and the result will be management decisions about how much generation and line transmission capacity to set aside as reserves for unplanned disturbances, to avoid chain outages and blackouts. As a result, the theoretical generation and transmission constraints (3.1) and (3.9) may be substantially enhanced to have enough readily available spinning reserve ($P_{Gi}^{max} - P_{Gi}$) and transmission capacity ($T_{ij}^{max} - |T_{ij}|$).

System reliability and operating solutions raise many debates. The way these services are provided highly depends on technical regulation, in particular on the choice of performance objectives or requirements. The N-1 security requirement is known to be very conservative, but is currently imposed by the North American Reliability Council (NERC) to American utilities, and also followed by most European utilities. Moreover, considerable judgment is involved in the decisions resulting from contingency analysis, and there will often be more than one way to deal with a potential problem.

Consequently, although it is unambiguously necessary to set some transmission reserves aside to maintain system's reliability, there is often no unambiguous or obvious reason for why a particular capacity constraint is set on a transmission line. This type of limits can create the appearance that a component is being under-utilized, and should be made available to those seeking access to the system. Thus capacity reserves for network security are a potential source of disagreement regarding when a transmission service is or is not feasible, and at what cost. As long as they represented only a marginal amount of generated power, IPP's could rely on the reserve set aside by the bulk supplier, i.e. the electric utility. But in an environment with a high proportion of competitive generators, provision must be taken for who will bear the cost of spinning reserve, i.e. the cost of capacity under-utilization.

3.7 Conclusion: technology impacts on policy

The technological constraints on power system, as well as the need for ancillary services prevent the organization of a fully competitive electric utility industry. They act

as a coupling among the actors and require some degree of centralized management of the system. The compliance with operating rules, and the authority of a central system operator become a burning issue because of the financial aspects involved:

- Transmission constraints are handled by generation redispatching, and may even prevent the implementation of certain transactions. Generation redispatching requires to dispatch *out of merit* generation, i.e. the use of expensive generation to accommodate transmission congestions. This involves fuel and operating costs.

- Frequency regulation and loss compensation imply some hardware capital cost, and mostly fuel and operating costs.

- Voltage regulation by transmission line compensation implies mostly capital cost. Reactive power generation implies no direct fuel or operating costs, but it somehow reduces the availability of participating units for real power generation. Reversely, non optimal voltage profile slightly increases the transmission losses.

- Finally, reliability expressed in terms of spinning reserve or transmission capacity reserve incurs opportunity costs of missing revenues from capacity underutilization and refused transmission services.

These economic issues arise with open access, if the generating units and transmission network are owned by distinct entities. Indeed, the trade-off between out of merit generation and transmission network improvement is also a conflict between short run and long run objectives: out of merit generation may reveal cheaper than transmission investment in the short run, depending on the cumulative incentives to upgrade the lines. All these technological aspects must be suitably translated into a fair transmission network access pricing.

Chapter 4

Wheeling transactions and transmission pricing

Chapter 2 has described the current trend toward deregulation and open access to the transmission network, and Chapter 3 has reviewed the most significant technical characteristics of a power system, that require a certain degree of coordination in the system operation.

Without making any definitive assumption regarding the exact shape of a deregulated electric utility industry, it is obvious that the main feature of the new environment is the generalized exchanges of power, among utilities and generators or large users nested within these utilities. This will create a partly competitive generating level in the electric power industry. In Europe, these increasing exchanges will take place between countries, utilities and individual actors (generators or users) that are not necessarily direct neighbors. The same is being encouraged in the United States by federal and state regulators.

However, in any given zone, responsibility for network planning and operations remains in the hands of a monopolistic entity, that will be called indifferently "system operator" or "utility" thereafter. Moreover, at least during a transition period, this monopoly is likely to own a significant share of the generation capacity, and to be responsible for serving its native demand. To ensure that this monopoly does not constitute an economic obstacle to competition, the price of transit services must be

correctly set and regulated.

This chapter firstly defines a wheeling transaction and examines the costs incurred. Then, the criteria that should be met by a pricing mechanism are presented, to introduce existing proposals or practice of transmission pricing.

4.1 Wheeling transactions

As said earlier, following [Ilic96, Zobi96], we consider the case of a *nested* interconnected system: a continental power system is structured horizontally into several control areas or utilities. The word utility used here as a synonym of system operator differs slightly from the currently existing vertically integrated companies. In each area, the utility owns and operates the transmission network as well as a significant percentage of the generating capacity, and must serve the greatest share of its native load.

In parallel with this nearly monopsonist and monopolist system operator, independent generators or autonomous large users, e.g. industrial users or distribution companies that do not purchase from the utility, are nested within the utilities, and make their own decisions according to the economic opportunities that they see on the generation market and through direct interaction with other utilities or independent actors.

The interactions between independent actors and utilities lead to wheeling transactions, i.e. the transmission of electrical energy from a seller S to a buyer B, through transmission lines owned by one or several third parties K. These transactions can be of four types:

- Utility S to utility B, through the transmission network of one or more interconnected wheeling utilities.
- Utility S to private user, where the private user may or may not be located within the wheeling utility.
- Private generator to utility B, where the private generator may or may not be

located within the wheeling utility.

- Private generator to private user, where both, one or none of the wheeling parties are within the utility.

The wheeling utility can or must accommodate these transactions in parallel with its basic mission of serving its native load. The interconnected operations (ancillary) services must also be provided, to keep the power system together. Without further specification of the institutional and technical environment, we assume here that the system operator takes care of these ancillary services, notably frequency and voltage regulation, reliability analysis, transmission loss compensation and congestion accommodation. Although it is very likely that, in the near future in Europe, the system operator will perform these services himself instead of paying some service provider to do it, this does not really matter. The cost of these services must be recovered anyway.

Therefore, it is important to dispose of agreed upon rules for the operation of the transmission network, and over all, of a method to compute transmission prices that reflect the actual cost of transmission, and convey the right economic signals to potential users.

4.2 Technical implementation and costs incurred by a wheeling transaction

Consider a wheeling transaction of W MW taking place between a seller S and a buyer B. They are non-contiguous and several parallel pathways through different utilities connect them. One of the connecting utilities is the wheeling utility K. Of course, electrical flows cannot be directed from S to B, and rather, all generators inject into the transmission network and all customers remove energy from the grid.

Hence, wheeling process consist of one extra injection at node S, and one opposite extra removal at node B. How this transaction can actually take place is directly conditioned by the technical operating principles, in particular the area load following

and frequency control.

4.2.1 Selling and buying utilities

As explained in Chapter 3, each utility (or power pool) regulates its frequency and the aggregated tie-line exchanges with its neighbors by responding in real time to its ACE. Hence, if the system operator of the area containing S or B is not informed of the transaction, the ACE feedback will prevent any change in the aggregate tie-line flows. The extra power injection or removal will be contained in the utility containing S or B, so the wheeling transaction will be nothing more than a "paper transaction": the extra power injected by S is absorbed by reducing the output of other units in the control area containing S, and graver, the extra power received by B would come from increasing the output of other units in the control area of B, but no one would pay for this excess generation, and certainly not B !

Therefore, in the actual process, the system operators of the control areas containing S or B must be informed of the transaction. Then, the utility containing S will increase consequently its net scheduled interchange $F^{net,sched}$ by W, so that the automatic generation control does take into account this additional power export. Similarly, the system operator corresponding to B will decrease its net scheduled interchange by W to take into account the additional power import.

4.2.2 Time scales relevant for wheeling

We just said that no power transfer would take place, unless scheduled in the AGC regulation. More exactly, the response time of the ACE regulator is in the order of a few minutes[1]. Hence, if the AGC regulation is not rescheduled, exact compensation for the extra power injected or removed within the control area takes place after an initial transient period of a few minutes. In the steady state, it can be said that the

[1]As explained in [Ilic96], the load following and frequency regulation is carried out with the objective that "ACE goes through zero at least once every 10 minutes." No real response time can be defined for this controller, and the current design seems to be a trade-off between the fuel cost of command actions and the time performance of regulation, hence the order of magnitude of a pseudo response time.

wheeling would not take place at all. What exactly happens in the transient period depends for example on the relative reaction speeds of S increasing its output and other AGC units reducing theirs. If S is faster, some extra power will be exported from the control area containing S during the transient period, but, once again, this happens just by chance and soon disappears.

Hence, any transaction taking place faster than the AGC regulation response time, and especially *finishing* before the steady state, can not be implemented neither reliably, nor economically. Control engineers would warn here that such transaction just plays upon the imperfections of the controller. Indeed, such transaction actually means the following: ideally, a 5 min wheeling of W MW would require that the contracting parties inform their respective system operators, and these operators modify consequently, at the agreed hour and for exactly 5 min, their scheduled net tie-line flows $F_B^{net,sched}$ and $F_S^{net,sched}$. Practically, the controllers do not respond fast enough, and nothing significant happens. Because they can not be technically implemented, unless a very high transaction cost is paid in strengthening the AGC regulators, these short transactions should not be allowed.

This recommendation is not necessary in Europe, where the bilateral transactions would take place between large entities such as a municipal distribution company or a steel mill and an IPP. Hence they would involve large quantities, in the order of 10 or 100 MW, and time scales in the order of hours. But some American reformers champion a very active market, with time scale of transactions in the order of minutes. Such proposals are totally incompatible with the current technology of the system, and the transaction costs involved to enable the implementation of such ideas is enormous relative to the benefit that any buyer may get from the quick transactions. Moreover, who needs substantial amounts of power for very short amounts of time ?

Note that, for technological reasons, we are discouraging *short lived* transactions here, but not stable transactions of a couple of hours *on short notice*. The problem of handling short notice transactions results from practical implementation difficulties, notably the information of the system operators, and the billing process. But this may be better formalized in the future.

4.2.3 Costs incurred by wheeling utilities

The re-scheduling in utilities containing B and S changes the flows throughout the interconnected power system, notably through all the other interconnected utilities, that are referred to as wheeling utilities K in the following (see Figure4-1). The tie-line flow change through any wheeling utility K is determined by the Kirchhoff's laws and is such that the resulting net tie-line flow of utility K is not modified. Hence, utility K does not need to reschedule its ACE.

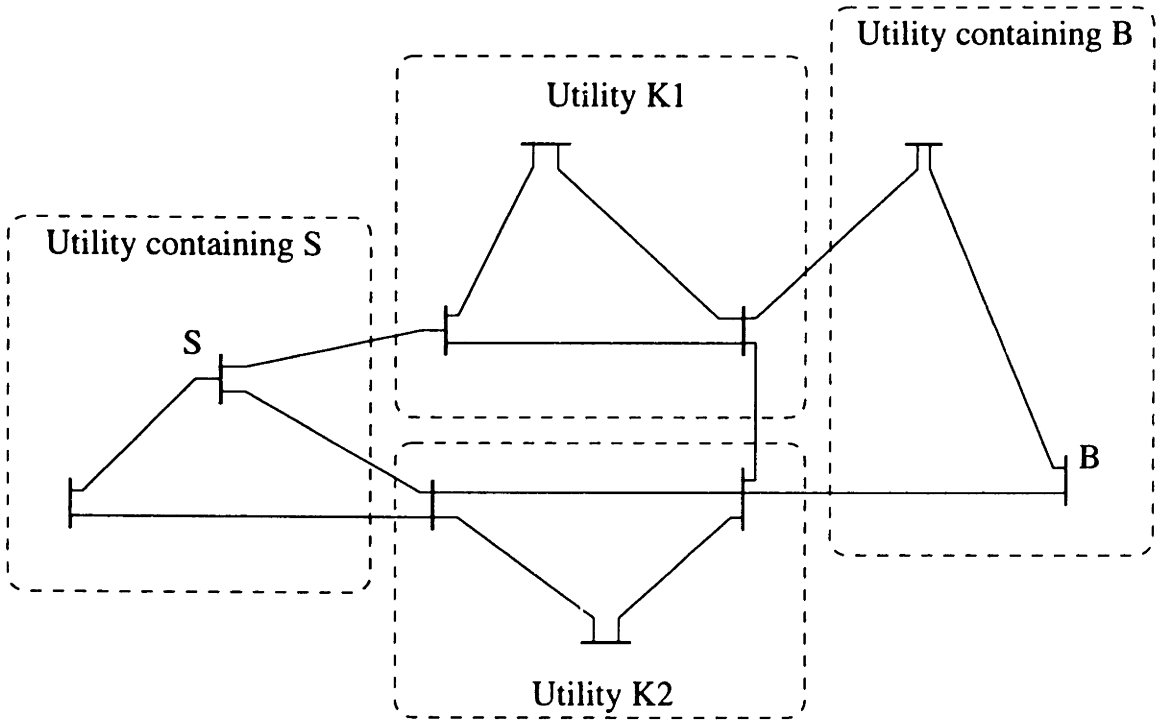


Figure 4-1: Wheeling in an interconnected network.

However, because the power network is a non linear system, utility K's costs are affected. We can separate the costs incurred by the wheeling transaction for utility K into five categories, based on technical considerations[2]:

- The cost of providing a path which is mostly the capital cost of the wires and support transmission facilities, plus the maintenance of the system. Once the

[2]Note that, apart from the fact that they reschedule their net tie-line flow, the utilities containing S and B are also wheeling utilities, and hence have the same costs.

transmission lines are in place, the cost of capital hardly depends on the transit through the utility.

- The cost of compensating the additional transmission losses incurred by the transaction, through an increase in power transfers over the wheeling utility. Note that, if the additional power transit is a *counterflow*, the increase in power flows can be negative on certain lines, hence inducing a loss reduction.

- The cost of redispatching, and notably of "out of merit" generation dispatch, to accommodate transmission congestion that may be caused by the wheeling transaction, or to respect the N-1 security requirement. The re-dispatch of generation in the wheeling utility and the use of more expensive out of merit generation must be compensated by the parties involved in the wheeling transaction causing the congestion.

- The cost of certain ancillary services such frequency regulation, voltage support and reactive losses compensation, to keep the system together.

- In the worst cases, the network may prove inadequate to ensure secure supply to certain customers, or to accommodate an agreed upon transaction. The cost of unsupplied energy, or non compliance with the contracts must then be taken into account.

As can be seen, the transmission activity is not limited to the provision of infrastructure, and it also interlinks with the generation activity, thus giving rise to variable operating costs. These variable costs are the major hardship in the determination of transmission costs, because they are highly fluctuating and dependent on the level of use of the network.

Moreover, we have said earlier that the power system network is not a linear network. Therefore, superposition rules do not apply and prevent the use of a recurrence reasoning to analyze the impact of one particular transmission service at the margin of pre-existing operating conditions. Indeed, if several transactions take place simultaneously, the order of transaction accommodations and the interactions between

transactions greatly matter. More generally, a transaction may be accommodated through complete redispatch, to reach a new optimum, or through marginal correction at a limited number of generators. Then, allocating the cost of redispatching among the various individual services provided becomes nearly impossible, as no specific transaction can be identified for triggering the need for a particular modification in the generation plan.

To solve all these issues, the most commonly proposed approach is to distinguish between two categories of users of the network: on the one hand the native load and generation associated with the transactions scheduled enough in advance, for example one day, and on the other hand, the transactions on short notice, for example less than one hour. The former category, that may be called the scheduled operating conditions, enables to run economic dispatch algorithm, and a social welfare maximization approach is still possible.

In contrary, the transactions on short notice are more uncertain, and their implementation may be refused by the system operator for technical reasons. This uncertainty certainly limits their value for potential users, and our impression is that such transactions will be very limited in the close future in the European environment. Therefore, they may be handled by a marginal analysis, based on the pre-existing operating conditions.

4.3 Orders of magnitude

The previous listing of the costs incurred by wheeling transaction is fairly exhaustive and based on a technical viewpoint. However, to reach policy recommendations, and simply to develop a suitable quantitative model of the operating conditions, it is necessary to make simplifying assumptions. Following a scientific approach, we want to base these assumptions on the relative orders of magnitude of the different components under consideration. At this point, we made no distinction between the national environment of the power system considered. However, the problems to be solved differ greatly whether we consider the Western European system, the New

England or Eastern American area, or the California situation.

To provide some insight in the economic importance of the different components of cost incurred by wheeling, we use here some estimations relevant for Electricité de France. Table 4.1 provides the cost of transportation computed as the difference in the cost of delivered energy through a transmission network and the cost of delivered energy if one assumes that all the generation and the loads are bundled in a single geographical point. These costs are the discounted results the forecast at horizon 2010, with two scenarios on the development of the transmission network: the unconstrained development scenario enables to expand the network to suppress completely transmission congestion, and therefore, the redispatching cost becomes negligible.

Cost component	Unconstrained development		Constrained development	
	cents94/kWh	%	cents94/kWh	%
Transmission investment	0.190	70	0.150	45
Transmission losses	0.085	30	0.100	30
Redispatching cost	-	-	0.080	25
Total	0.275	100	0.330	100

Table 4.1: Breakdown of overall transmission costs. Source: [EDF94].

In the constrained development scenario, environmental pressure prevents the building of transmission lines in new rights of way, therefore inducing substantial redispatching costs, but reducing the investment cost. In any case, the costs of ancillary services and unserved power seem to be negligible. These numbers should be compared with the average price of delivered energy.

As can be seen in appendix A, the wholesale price of electricity in France, if considered equal to the price to industrial users, is 6.50 ECU/100kWh, i.e. about 8 cents/kWh. This means that in both scenarios, the transmission costs represent 3% to 4% of the wholesale price of energy. Note that, in terms of fuel costs, the proportion is higher, as the transmission losses represented 29 TWh in 1994, i.e. 7.5% of the total energy production [EDF94]. This confirms the fact that generation fuel and investment cost is the greatest component of total electricity price, and the rationale

for deregulation given in Chapter 2.

These numbers should be considered with great caution, as they do not give any precision on the breakdown in fuel cost, wages, interest payments and so forth, and they are very likely to fluctuate much. Moreover, they indicate that the transmission grid of EDF is well meshed. It is not clear at all that the orders of magnitude provided here would be relevant for all the European utilities. However, they provide us with orders of magnitude and guidelines for further analysis of transmission pricing. A first conclusion to this point is that only three components should be considered in transmission services rates:

1. The capital cost of the infrastructure (including line compensation),
2. The fuel cost of active transmission losses compensation,
3. Possibly the operating cost of redispatch.

Voltage constraints and associated reactive power generation are prevalent problems as well. However, they do not directly involve fuel costs for VAR generation, but only capital cost for line compensation devices. This is included in the cost of transmission infrastructure. For a closer look at wheeling rates of reactive power flows, see for example [Li94].

4.4 Criteria for transmission pricing mechanism

The transmission pricing mechanisms required by the current increase in power exchanges among utilities, and by the likely future access of large users to the transmission network will be the main medium of communication and signaling between system operator and parties requesting transit services. But this will also be the most important tool of the regulator to limit the risk of abuse associated with the dominant position of the monopoly operating the network.

Hence, a good pricing mechanism must respect a number of criteria, that are sometimes conflicting. Following [Cler93, Cara86], we list here the most important of these criteria:

Budgetary constraint: The pricing mechanism must enable the system operator to recover its costs. We have described above the three main components of these costs: capital investment, loss compensation and, possibly, out of merit dispatch. Moreover, the pricing mechanism should provide incentives for the utilities to strengthen their transmission systems when mutually beneficial to the whole society.

Economic efficiency incentives: While allowing for as much decentralized decision making as possible, the pricing system should guide the behavior of buyers and sellers towards a collective optimum. This economic efficiency is generally measured in terms of social welfare maximization. This means that the transmission rates must increase along with the costs incurred by the wheeling utility to provide a transmission service. These price signals may vary according to numerous aspects of the service: location, time, duration, guarantee, etc.

Equity: This concept is difficult to define, even more to verify, and politically debatable. It deals with charging a price in accordance to the actual cost incurred, and avoiding cross-subsidies between customer categories. However, we recall here that the total absence of cross subsidies may go against political choices of national redistribution. For example, the alignment of tariffs for all captive customers, with urban customers subsidizing rural ones is a frequently accepted practice. At least, the pricing mechanism should ensure equal treatment for all the companies involved, with the possible exception of the native users.

Stability over time: This criterion is even more questionable, as it conflicts to some extent with the efficiency criterion. It can be argued that customers react slowly to price changes, or that industrial users need price stability over a long time to base their long term investments.

Conversely, as overall demand patterns show peaks and valleys over days, weeks or years, part of the actual cost fluctuates accordingly and economic efficiency is conditioned in the price signal conveying that time-dependent information

to potential users. Of course, the prices should be fixed in advance so that customers can adapt their demand.

Price transparency and implementability: Ruling authorities do not possess all the data required to check that the system operator is not taking unfair advantage of his dominant position. In this context of information asymmetry, transmission prices must be clear and simple, to ensure that they are understood by both the regulator and the customers using the transmission services, and to reduce as much as possible the opportunity for arbitrary or political decisions on wheeling transactions and rates. This simplicity relates to price stability over time.

Moreover, the pricing mechanism must be easy to implement, without overwhelming administrative and billing costs. This condition may not seem important, as electric energy is a commodity expensive enough to generate for the administrative costs to be negligible. However, it its interesting to think here about the example of local telephone communications, where the administrative cost of billing represents nearly half of the the overall cost of the service.

The objectives of conveying meaningful economic signal and price stability or simplicity seem obviously to conflict. A trade-off must be found, that will depend on the amplitude of variable operating costs, on the type of transactions allowed, etc.

4.5 Current proposals

The principle of transmission pricing currently advocated by the Federal Energy Regulatory Commission in the United States is the *postage stamp pricing*. It is a fixed transmission fee for any transit, whatever location, distance, direction of flows and time of use. The fee is computed to recover overall costs, and allocates them among all users according to a usage sensitive key. This coarse pricing scheme has obvious advantages, as far as simplicity, stability over time and implementation goes. However, it does not send any economic incentive to use the system during off-peak hours,

and relieve possible congestion.

In their wheeling agreements, electric utilities often use the principle of *contract path*. The wheeling rate is computed by assuming that the wheeled energy flows over a certain path, by arbitrary using the "most probable transmission lines." This system tries to recognize the geographical distance between injection and removal points but obviously relies on a fiction, as electrons will certainly not follow the contract path. Loop flows are ignored, as well as system loading conditions and direction of flows [Cara95].

A more refined pricing scheme, called *MW-mile* or *MW-km*, recognizes the parallel paths followed by energy across the network. This is obtained by looking at the power flows resulting of the transaction on an "empty" network, and does not lead to assuming any path between the injection and removal busses. However, this system is not sensitive to direction of flows, time and system loading. It notably does not recognize counterflows.

We present in the next chapter a method that considers all the operating constraints of the optimal operation of power systems and is based on spot prices of electricity.

Chapter 5

Optimal spot prices of electricity

The subject of optimal spot prices of electricity, and of a spot price based energy marketplace has been developed and discussed through a number of different reports, theses and papers, and was finally integrated into a coherent theory by Schweppe, Caramanis, Tabors and Bohn in [Schw88].

Spot prices of electricity, in cents/kWh, reflect the operating cost of generating, transmitting and distributing electric energy. They vary hour after hour, and from place to place. In fact, they represent the shadow prices of an optimal power flow, in other words, of an economic dispatch of generation under all the technical constraints described in Chapter 3.

There is nothing really new in the use of spot prices on the generation side, where utilities have been computing the generation short run marginal cost, called "system lambda," for a long time when they optimize the generation dispatch to minimize the total cost of generation.

However, [Schw88] considers the creation of an "energy marketplace that involves a variety of utility-customer transactions." At the date of release of this book, the debate on the deregulation of the electric utility industry was not as advanced as today, and this model of interactions between customers and their utility was a radical departure from common practice. In theoretic terms, this change results in the maximization of social welfare, rather than the mere minimization of the production costs for a given demand. Along with the change of objective function, this leads to

consider symmetrically generation and demand, by using the net power injection at each node of the system, as seen in Chapter 3.

This evolution was probably initiated by the development of demand side management and concepts such as the smart use of energy. Indeed, one of the important benefits of the use of spot prices is certainly the consumer response to price fluctuation, that may be used as a additional operating tool for system operators.

5.1 Optimum power flow

5.1.1 Simplifying assumptions

The energy marketplace could be imagined to eventually be a live market similar to the New-York stock exchange, where generators and wholesale customers interact through bids and calls, the spot price being just the settling price of these market interactions. This corresponds to the current practice in Wales and England, but is not so far from the practice in continental Europe, where power exchange among utilities rely already on a spot price based energy market. However, the theory of spot prices is in fact derived from the optimal power flow (OPF), currently used by vertically integrated utilities under centralized management. We will use this problem formulation to derive optimal spot prices. In principal, short-run equilibrium in a competitive market should not differ much from the prices and associated power flows obtained by economic dispatch.

The OPF, or constrained economic dispatch, is formulated as an optimization problem constrained by the technical requirements presented in Chapter 3. The problem could be formulated in its greatest generality, using AC load flow equations and all the operating constraints. However, this would prevent us from getting interpretable results. Moreover, as we have seen in Chapter 4, quantitative data and a comparison of the orders of magnitude of the phenomenon under considerations allow us to make important simplifying assumptions.

In the present case, our purpose is to derive a pricing mechanism for transmission

services. We have seen that the main three components of the cost of these services are transmission capital cost, active loss compensation and generation redispatch to relieve congestion. In contrary, system ancillary services such as frequency regulation, voltage support and reactive power generation are not economically important in transmission pricing.

Consequently, we will carry most of the following derivations using the DC load flow equations, under the assumption that voltage magnitudes do not depart much from their nominal value 1 $p.u.$ Consequently, only the generation constraints on active power will be considered. Active power losses are taken into account in this chapter, but will be neglected thereafter.

5.1.2 Objective function

In its classical formulation, the economic dispatch problem aims at minimizing the *total cost of generation* to meet the total load given as an external parameter. Under centralized management in a vertically integrated utility, the system operator has authority on all the generators, and knows the cost of generation at each bus i, $G_i(P_{Gi})$ as well as the maximum power output P_{Gi}^{max}.

In the deregulated environment envisioned here, a nearly monopsonist system operator serves its native load by using its own units and/or by buying power from independent generators. The cost of generation can be obtained from bids by the generators. These bids can have a variety of forms, such as a given amount of power for a given price, or a constant unitary price up to a maximum output. But in the best case, when generators do not have any market power, the bids yield a cost function as well, eventually obtained after several iterations[1].

Under the classical assumption of increasing marginal cost, the cost function G_i is convex. As for any optimization problem, the solution of the economic dispatch does not depend on the fixed costs. Hence we can consider only the variable cost of

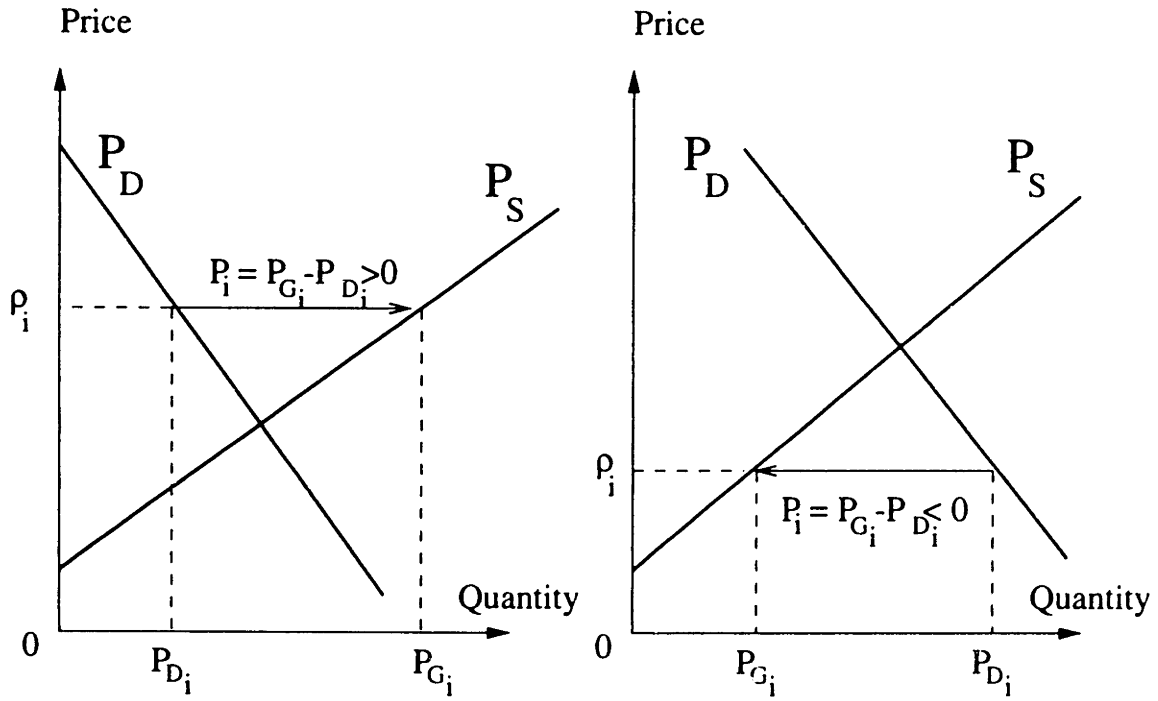
[1]The generators will tend to bid their long run marginal cost, which may not vary much with the power generated by the unit, between the bounds on possible output level. Hence, the cost curve can generally be approximated fairly well by an upward sloping segment of line.

generation, i.e. mostly fuel and operating cost, and assume that $G_i(0) = 0$, leaving aside the fixed capital cost.

Another formulation of optimal operations is to maximize the *social welfare*, as measured by the benefit of energy consumption minus the cost of generation. The behavior of the consumer at bus i is modeled by the consumer surplus B_i that is related to the inverse demand function by the equation

$$B_i(P_{Di}) = \int_0^{P_{Di}} p_i(y)dy \tag{5.1}$$

The function B_i is concave, and such that $B_i(0) = 0$.



(a) Net generation node (b) Net consumption node

Figure 5-1: Demand and supply curves at a given node.

Figure 5-1 shows the configuration of the demand and supply curves at a given bus, i.e. represent the marginal benefit (cost) as a function of the power consumed (generated). The net impact of bus i on the network only depends on the net power

injection $P_i = P_{Gi} - P_{Di}$. By netting out the effect of simultaneous generation and consumption at the same node, the social welfare contribution at node i is the opposite of the social cost:

$$C_i(P_{Gi}, P_{Di}) = G_i(P_{Gi}) - B_i(P_{Di}) \tag{5.2}$$

It can be seen from the curves on figure 5-1 that, whatever the price of power at node i, the power generated P_{Gi} and demanded P_{Di} are determined unambiguously as a function of the net power injection P_i. Hence, the social cost is a function of the power injection only. We can still assume that $C_i(0) = 0$, as the economic dispatch solution does not depend on a fixed social cost.

The social marginal cost of power generation and consumption at bus i is the sum of the marginal cost of generation and of the marginal benefit, and is positive whatever the sign of the net power injection:

$$\begin{aligned} \frac{\partial C_i}{\partial P_i} &= \frac{\partial G_i}{\partial P_i} - \frac{\partial B_i}{\partial P_i} \\ &= \frac{\partial G_i}{\partial P_{Gi}} + \frac{\partial B_i}{\partial P_{Di}} \\ &\geq 0 \end{aligned} \tag{5.3}$$

Moreover, it can be shown that the convexity of G_i and the concavity of B_i implies that $C_i(P_{Pi})$ is a convex function, as represented on figure 5-2. Finally, the definition set of C_i has an upper bound $P_i^{max} = P_{Gi}^{max}$, where the cost of generation becomes infinite. This enables us to model implicitly the generation constraints.

5.1.3 Tie-line busses

The system operator has also some control or the interactions with neighboring areas, or at least agrees with the neighbors for scheduling tie-line exchanges. Hence, the interchange with neighboring areas are modeled here by fictive busses, called the *tie-line busses*, at the intersection of the control area boundary and of the tie-lines. The tie-lie flows are the net power injected at these busses, into the control area. A cost function models the price paid to neighboring utilities. Hence, the tie-line flow busses

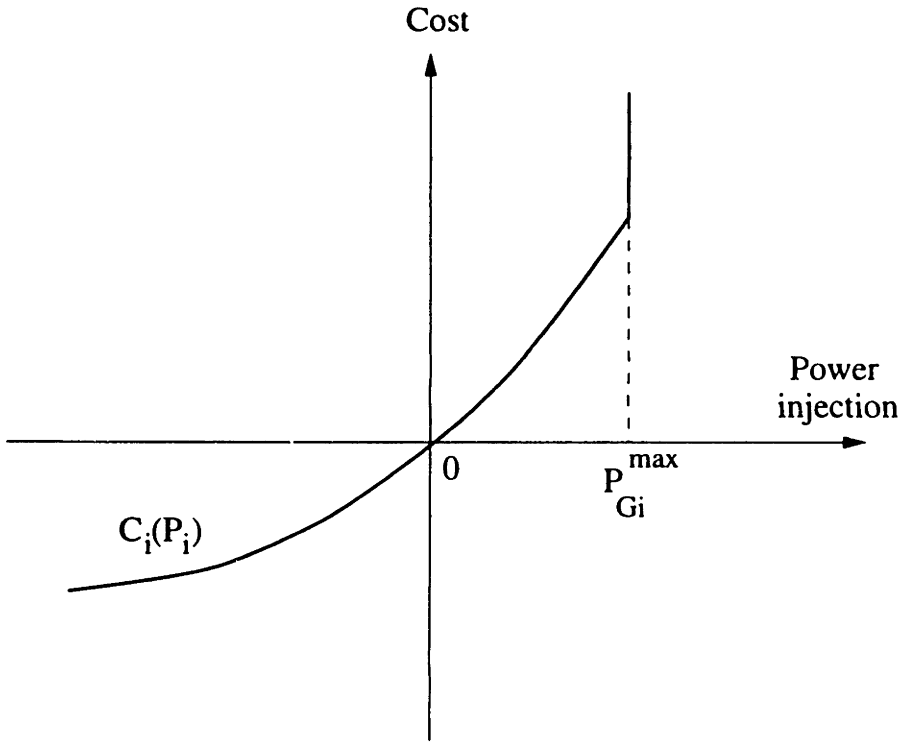


Figure 5-2: Net social cost at a given node.

behave exactly in the same way as any node of the control area.

5.1.4 Problem formulation

Mathematically speaking, the problem of economic dispatch can be formulated very concisely. At a given hour t, the control variables are the net power injections at each bus P_i and the voltage magnitude V_i, the dependent variables are the voltage angles θ_i and the reactive power injections Q_i. The problem of social welfare maximization formulates as follows:

$$\max_{\mathbf{P},\mathbf{V}} \ \Omega(\mathbf{P}) = -\sum_{i=1}^{n} C_i(P_i) \tag{5.4}$$

subject to

$$\hat{\mathbf{S}} = \mathbf{diag}(\hat{\mathbf{V}})\hat{\mathbf{Y}}^{*}\hat{\mathbf{V}}^{*} \tag{5.5}$$

$$|T_{ij}| \leq T_{ij}^{max} \quad \forall i, j \tag{5.6}$$

$$V_i^{min} \leq V_i \leq V_i^{max} \quad \forall i \tag{5.7}$$

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \quad \forall i \tag{5.8}$$

$$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{Gi}^{max} \quad \forall i \tag{5.9}$$

This general formulation corresponds to the Optimal Power Flow problem (OPF) in its exact AC formulation, using all the constraints presented in Chapter 3. In particular, the power transfer T_{ij} are obtained from the AC load flow equations. As said earlier, we are not going to use that general formulation. The proposals for transmission pricing can be based, without any significant loss of precision, on a DC formulation of the load flow equations, i.e. a decoupled formulation of the economic dispatch problem.

Assuming that an appropriate reactive power generation scheduling maintains the node voltages within the admissible limits, we can omit the voltage and reactive power constraints. Hence, the only control variables are the power injections P_i, and the dependent variables are the voltage angles θ_i as well as the real power transfers T_{ij}.

Moreover, the generation constraints are generally included in the generation cost functions G_i or C_i, as described in section 5.1.2. The load flow equations at each node are replaced by the single energy balance constraint, taking the losses into account. This modification is viable because the optimization is carried out with respect to the power injections only, and the voltage angle do not appear in the resolution. Instead, we can use the shortcuts $\mathbf{T} = \mathbf{HP}$ and $L = \mathbf{P'BP}$ presented in Chapter 3 to express the transmission constraints and losses.

Finally, replacing the absolute value by a double constraint on power flows, the economic dispatch formulation becomes:

$$\max_{\mathbf{P}} \ \Omega(\mathbf{P}) = -\sum_{i=1}^{n} C_i(P_i) \tag{5.10}$$

subject to

$$\sum_{i=1}^{n}(P_i) = L(\mathbf{P}) \tag{5.11}$$

$$-T_l^{max} \leq T_l(\mathbf{P}) \leq T_l^{max} \quad \forall l \tag{5.12}$$

5.2 Nodal spot prices

In this section, we present how the solution of the economic dispatch problem yields nodal spot prices that are optimal short run marginal costs.

5.2.1 Lagrange multipliers

The simplified economic dispatch problem is solved by forming the Lagrangian

$$\mathcal{L}(\mathbf{P}) = \Omega(\mathbf{P}) + \lambda[\sum_{i=1}^{n} P_i - L] + \sum_{l=1}^{n_l} \mu_l[T_l^{max} - T_l] + \sum_{l=1}^{n_l} \nu_l[T_l + T_l^{max}] \tag{5.13}$$

and the Kuhn-Tucker optimality conditions are:

$$\frac{\partial C}{\partial P_i} = \lambda(1 - \frac{\partial L}{\partial P_i}) - \sum_{l=1}^{n_l}(\mu_l - \nu_l)\frac{\partial T_l}{\partial P_i} \tag{5.14}$$

$$\mu_l \geq 0 \quad \text{and} \quad \mu_l[T_l^{max} - T_l] = 0 \tag{5.15}$$

$$\nu_l \geq 0 \quad \text{and} \quad \nu_l[T_l + T_l^{max}] = 0 \tag{5.16}$$

Using equations (3.5) and (3.7) to express the marginal losses and the marginal line transfers, we obtain

$$\frac{\partial L}{\partial P_i} = 2\mathbf{B}_i\mathbf{P} = 2\sum_{k=1}^{n} B_{ik}P_k \tag{5.17}$$

and

$$\frac{\partial T_l}{\partial P_i} = H_{li} \tag{5.18}$$

where $\mathbf{B}_i$ is the ith row of $\mathbf{B}$. Moreover, following [Schw88], we define the *nodal spot price* as the short run marginal cost of power injection at each bus i:

$$\rho_i \stackrel{\text{def}}{=} \frac{\partial C}{\partial P_i} \tag{5.19}$$

We have seen in the previous section that this quantity is always positive and we obtain the general expression of optimal nodal spot prices:

$$\rho_i = \lambda - 2\lambda \mathbf{B}_i \mathbf{P} - \sum_{l=1}^{n_l} H_{li}(\mu_l - \nu l) \tag{5.20}$$

Note that, whereas λ is always non zero, only the multipliers μ_l corresponding to lines where the transmission constraint (5.12) is binding will have a positive value. These lines are said to be *congested.* The spot prices thus defined reflect all the technical characteristics of the static system operations that we have modeled by inequality constraints in the problem formulation.

5.2.2 Variation of optimal spot prices

The time dependence that has been omitted until now should be reintroduced here, with the notation $\rho_i(t)$. In particular, the system lambda $\lambda(t)$ indicates the system marginal cost of generation, in other words, the cost of generation at the swing bus. As generators are optimally used in order of increasing fuel cost, the system lambda is the highest marginal cost of fuel used to meet the load during hour t. The demand pattern varies over time, as domestic and commercial users consume more power during the day than at night, and during winter (heating) and summer (air conditioning).

But the nodes across the network will not have similar bus spot prices, due to the effect of losses or congestion. The term $2\lambda \mathbf{B}_i \mathbf{P}$ in (5.20) reflects the incidence of line losses on the marginal cost of generation at bus i. It depends on all the power injections, and not only on P_i. If the swing bus is a demand bus, and we consider the spot price at a generating bus i (positive net power injection), the difference between

price at the demand bus and price ρ_i will be higher if the factors $B_{ij}P_j$ are larger.

If power injected in node i results in "overall flows going in the same direction" as flows caused by power injected in node j, than the sensitivity coefficient B_{ij} will be positive, and additional power injected in i will increase overall losses. Hence, power generated at bus i should be penalized by being purchased at a lower price from the system operator. Conversely, if B_{ij} is negative, marginal injections at nodes i and j have overall beneficial effects on transmission losses due to counterflows, and the spot price at node i is modified accordingly.

Finally, the last term in (5.20) shows the effect of congested transmission line on the node prices. If the line from bus i to j is congested, the nodal spot prices at *all* the busses are modified, and not only prices at bus i and j, hence showing the system-wide effect of a binding transmission constraint. Here again, the sensitivity coefficients H_{li} are positive or negative, if marginal injections at node i tend to relieve or increase congestion in line l.

Following the adjustment of spot prices, the power output of generating units will be set to accommodate the congestion. This is called *redispatching*, by reference to the modification of the generation dispatch that would be economically optimal in absence of transmission constraints. Redispatch leads to use out of merit generation, i.e. not the cheapest available power, hence, an opportunity cost can be assigned to the bottleneck.

This brief comments aimed at highlighting the complexity of the network-wide effects, through losses or congestion, of marginal injections or removal at a given node. Sensitivity coefficients obtained from the DC load flow model are the only rigorous way to derive economic decisions on a power network.

5.2.3 Link with a competitive spot market

We have stated that, in principle, short-run equilibrium in a competitive market, such as the British power pool, would reproduce both these prices and the associated power flows [Schw88, Hoga92]. This assumes of course that the market is perfectly competitive, and that transactions are tracked with enough precision for the spot

prices at each node to reflect all the relevant information on technical operating constraints. Of course, in a real system, the effects of losses or congestion may be small enough to be neglected or approximated, like what is currently done on the British network.

Conversely, the nodal marginal costs send the good price signals to reach short run static optimality and represent the cost at which power should be generated in bus i, or reversely, the price of consumed power at bus i. Hence, once computed, the spot prices enable decentralized optimal decision: the output of a generating unit at bus i should be set so that the power injection P_i satisfies the equation

$$\frac{\partial C}{\partial P_i} = \frac{\partial C_i}{\partial P_i}(P_i) = \rho_i(t)$$

Note that we omitted the explicit inequality (5.8) in our formulation of simplified economic dispatch, by assuming that the generation constraints were contained in the generation cost function $G_i(P_i)$. If we choose to use an explicit model of the generation constraints (5.8), than we have two additional Lagrange multipliers α_i and β_i, and the expression of optimal spot prices (5.20) becomes

$$\boxed{\rho_i(t) = \lambda(t)[1 - 2\mathbf{B}_i\mathbf{P}(t)] + \alpha_i(t) - \beta_i(t) - \textstyle\sum_{l=1}^{n_l} H_{li}[\mu_l(t) - \nu l(t)]}$$

with the complementary conditions

$$\alpha_i(t) \geq 0 \quad \text{and} \quad \alpha_i(t)[P_i(t) - P_i^{min}(t)] = 0 \quad \forall i$$
$$\beta_i(t) \geq 0 \quad \text{and} \quad \beta_i(t)[P_i^{max}(t) - P_i(t)] = 0 \quad \forall i$$

There is an indetermination on the actual price to be paid by the pool to a generator at the upper limit. Indeed, power output will be set to P_i^{max} whatever the price received by the generator, as long as it is greater than the marginal cost of generation at maximum power output. Therefore, any price between ρ_i and $\rho_i + \beta_i$ could be paid to the generator, without modifying his behavior. The authors of [Schw88] recommend to pay $\rho_i + \beta_i$, as the producer surplus serves to pay for generation investment.

Chapter 6

Spot price-based wheeling rates

We have explained in the previous chapter that optimal nodal spot prices contain all the relevant technical and economic information to reach static optimality. In [Cara86], Caramanis, Bohn and Schweppe proposed a method to compute optimal wheeling rate based on the theory of spot prices. In this chapter, we review and comment their proposal.

We consider in the following a long term wheeling transaction of W MW during an unspecified amount of time, from a seller S, located at a given node of the interconnected system, to a buyer B represented by a single bus as well. As described in Chapter 4, this transaction modifies the power flows through wheeling utilities K across the whole interconnected network, and the proposed wheeling rate enables to cover the incurred cost for each wheeling utility.

6.1 Optimal wheeling rates

Ignoring the capital cost, the optimal wheeling rate in \$/MWh during hour t is defined as the marginal impact of wheeling on utility K's cost.

$$\omega^{K,BS}(t) \stackrel{\text{def}}{=} \frac{\partial C^K}{\partial F_S^{net,\,sched}}(t) \tag{6.1}$$

where C^K is the total cost of utility K and $F_S^{net,\,sched}(t)$ is the scheduled net interchange of the utility (containing) S during hour t. This wheeling rate is defined in terms of scheduled net interchange rather than directly power injection at node S or B, to encompass the cases where S or B are a whole utility rather than just a bus. We recall here that the system operators of S and B must be informed of the wheeling anyway.

The re-scheduling of net interchange by utilities S and B have an effect on all the tie-lines flows into utility K. Although the sum of these tie-line effects is zero, power flow displacement at the boundary of utility K must be taken into account in system operation. Depending on the magnitude of the tie-line flow changes, i.e. depending on the activity of the wheeling transactions, the AGC regulation may provide sufficient response, or K might need to perform a new economic dispatch, or maybe even revise its scheduled unit commitment. In the latter cases, this would mean that K needs to be informed of the transaction as well, as economic dispatch and unit commitment are not real time operations.

We assume here that the wheeling is accommodated automatically by AGC, or optimally by completely re-dispatching the generation in utility K to minimize the total operating cost under the new tie-line flow pattern. In this context, ACG regulation or the economic dispatch determine unambiguously all the internal parameters in K as a function of tie-line flows. Notably the optimal power injections at internal busses in utility K, P_i^K are completely determined by the tie-line flows into or out of utility K.

Let F_l^K $l \in \{1, \ldots, n_f\}$ denote the n_f tie-line flows *out of* utility K. As they completely define the optimal parameters in utility K, we have

$$\omega^{K,BS}(t) = \sum_{l=1}^{n_f} \frac{\partial C^K}{\partial F_l^K} \frac{\partial F_l^K}{\partial F_S^{net,\,sched}}(t) \tag{6.2}$$

We recall that, in section 5.1, the tie-line flows were modeled as fictive busses, at the intersection of tie-line and boundary of the utility, that inject a power P_l into the network of utility K. Hence the definition (5.19) of spot prices, as the cost of power

injected in the network at a given node, also applies to tie-lines:

$$\rho_l = \frac{\partial C_l}{\partial P_l} \tag{6.3}$$

Conversely, as F_l^K is the tie-flow out of K, into tie-line bus l, we have $F_l^K = -P_l$ (if K delivers power into the tie-line, the tie-line injects the opposite power into K) and $C_l^K = -C_l$. Consequently, the tie-line spot prices are also the marginal cost incurred by K to deliver power into the tie-line l:

$$\rho_l = \frac{\partial C_l^K}{\partial F_l^K} \tag{6.4}$$

Moreover, the authors of [Cara86] define the *tie-line flow coefficient of the wheeling* by

$$D_l^{K,BS}(t) \stackrel{\text{def}}{=} \frac{\partial F_l^K}{\partial F_S^{net,\,sched}}(t) \tag{6.5}$$

These coefficient give the marginal impact of the wheeling on the tie-line flows and depend on the operating conditions in the whole power system. The sum of these coefficients must be zero. This enables us to compute the optimal wheeling rate from the spot prices at the boundary of utility K as follows:

$$\omega^{K,BS}(t) = \sum_{l=1}^{n_f} \rho_l(t) D_l^{K,BS}(t) \tag{6.6}$$

6.2 Computation of tie-line flow coefficients

We consider here only the special case where S and B are single nodes, and assume that the transfer admittance matrix **H** of the whole interconnected network is known. The DC load flow model used until now yields a very simple expression for the tie-line flow coefficients of the wheeling from S to B.

The scheduled net interchange of utility containing the seller S is

$$F_S^{net,\,sched}(t) = F_{S.0}^{net.\,sched} + W$$

where $F_{S,0}^{net,\,sched}$ is the schedule prior to the transaction. Similarly, the net interchange of the utility containing the buyer B is

$$F_B^{net,\,sched}(t) = F_{B,0}^{net,\,sched} - W$$

Hence, the impact of the rescheduling of interchanges on F_l^K will come from both sides S and B. The DC load flow equation (3.5) yields the impact of the transaction on tie-flows into K:

$$\Delta F_l^K = \mathbf{H}\Delta\mathbf{P} = W(H_{li_S} - H_{li_B}) \tag{6.7}$$

where i_S and i_B are respectively the index of the seller's node and the buyer's node. Hence, we get simply

$$D_l^{K,BS}(t) = H_{li_S} - H_{li_B} \tag{6.8}$$

Note that the tie-line flow coefficients are independent of time. The sign of the coefficient of matrix $\mathbf{H}$ depend on the orientation of the tie-lines, but recall that we have defined the orientation going *out* of utility K. Positive tie-line flow coefficients correspond to tie-lines whose flow out of K increases with the power W transferred from S to B, and are called "marginal outflow tie-lines." Reversely, tie-lines with negative tie-line coefficients are the "marginal inflow tie-lines."

The optimal wheeling rate that the contracting parties S and B must pay to the utility K per MW transferred takes the form:

$$\omega^{K,BS}(t) = \sum_{l=1}^{n_J} \rho_l(t)[H_{li_S} - H_{li_B}] \tag{6.9}$$

Let us consider the special case where the network consists of three entities: a single line going from bus S across utility K to bus B, as shown on figure 6-1. Then, the tie-line flow coefficients are simply -1 for the tie-line from K into S and $+1$ for the tie-line from K into B. Then, the wheeling rate is the difference between the spot price at the consumption and injection bus:

$$\omega^{K,BS}(t) = \rho_B(t) - \rho_S(t)$$

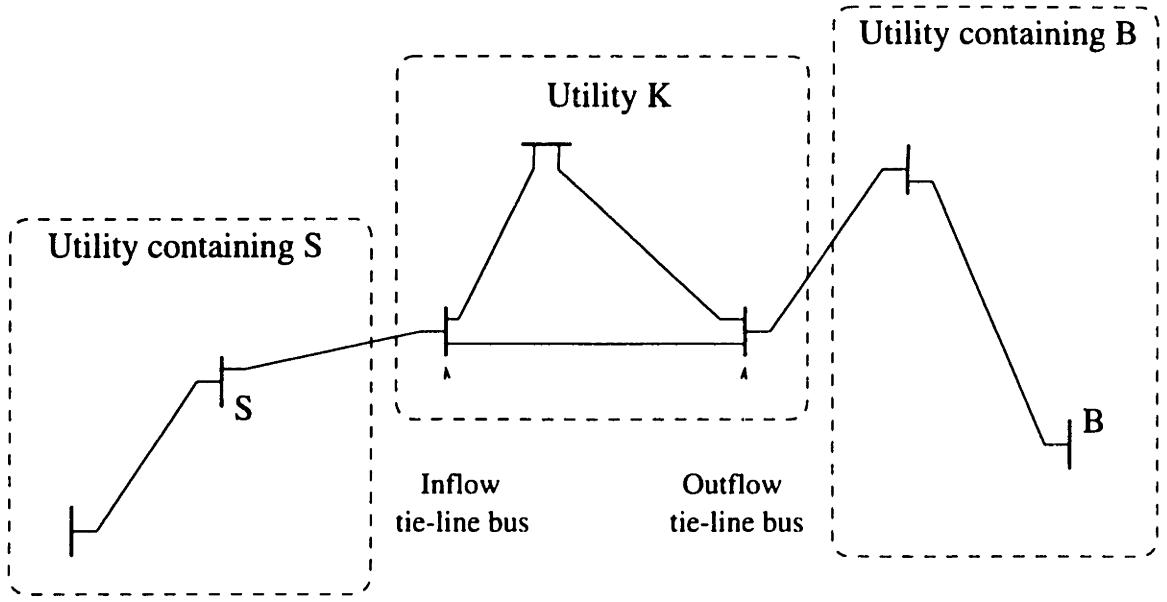


Figure 6-1: A simple case of wheeling.

This very famous result is very seldom generalized to the case of a wheeling transaction taking place in an interconnected network with more than three utilities. Then, the power transferred flows through more than one utility, and multiple wheeling rates must be paid by the contracting parties: to the utility containing S, to all the wheeling utilities K, and to the utility containing B. The theory of optimal wheeling rates presented above can compute the breakdown of all these wheeling rates, but it must be noted that they all add to the difference of spot prices between injection and removal busses.

6.3 Net revenue from wheeling

The wheeling rate defined in the previous section is a marginal cost, and is charged for each MW of power transfered from S to B. This means that the gross revenue from wheeling is simply computed by multiplying the amount of transmitted power W by the wheeling rate. Hence, the total payment from the pair B,S to utility K is

$$GR(t) = W\omega^{K,BS}(t) \tag{6.10}$$

This rate reflects on the one hand the differences in utility K's marginal losses at the marginal inflow busses and the marginal outflow busses, multiplied by system lambda, and on the other hand, the congestion caused by the wheeling that must be accommodated for by out of merit generation. Note that the gross revenue could be negative, i.e. utility K should pay the pair B,S because the wheeling transaction could reduce overall transmission losses or relieve some congestion.

The net revenue from wheeling is the difference between gross revenue and the incremental cost incurred by the wheeling, i.e. the difference in total operating costs of K with and without the wheeling transaction:

$$\begin{aligned} NR &= GR - [C^K(\mathbf{P} + wheeling) - C^K(\mathbf{P})] \\ NR &= W \frac{\partial C^K}{\partial F_S^{n,s}}\bigg|_W - \int_0^W \frac{\partial C^K}{\partial F_S^{n,s}}\bigg|_p dp \quad \geq \quad 0 \end{aligned} \tag{6.11}$$

The term in the integral is in fact the wheeling rate computed at the operating point resulting of a wheeling $W = p$. As the cost function C^K is generally convex, under the hypothesis of increasing marginal cost of generation, the integrated function is growing and the integral is less than the first term. Hence, the net revenue to utility K is always positive, even if gross revenue is negative. This is just a consequence of the fact that, whatever its sign, short run marginal cost of wheeling is always greater than incremental cost of wheeling per kWh transferred, due to the nonlinearity and convexity of cost functions [Cara95]. This result is also one of the Folk theorems recalled in [Wu94].

This means that, using the proposed marginal pricing, K has always an interest in providing the transmission service. Net revenue of S and B from the wheeling is also positive to the extent they make economic rational decisions, i.e. they are using their own instantaneous spot prices to decide whether or not to interchange.

6.4 Review of spot price-based approaches

Although it leads to economic efficiency, the adoption of a spot price mechanism for transmission access has many drawbacks. These are notably the time-fluctuations and space-fluctuations of prices, and the problems of transparency in the computation of spatial corrections of spot prices to take losses and congestion into account. These implementability issues will be discussed in Chapter 9, but we shall say now that the operation of an interconnected power system is not a simple and easy task, hence it can not be accurately reflected by a trivial pricing mechanism.

In this section, we review some other shortcomings of the spot price-based access rates, resulting from dynamic aspects of power system that are not reflected in a static approach. They involve the operations planning and the expansion planning levels.

6.4.1 Incremental costs and non superposition

We have proven in the previous section that marginal cost was greater than incremental cost, because the overall cost function of the wheeling utility is convex. One of the main explanation of this convexity is the fact that transmission losses are a quadratic function of power flows, as seen in Chapter 3.

However, our demonstration took only one transaction B,S into account. The marginal cost computed at the operating point resulting from a second transaction $\tilde{B}$,$\tilde{S}$ would not be the same if the wheeling $\tilde{B}$,$\tilde{S}$ is accommodated before or after B,S. This means that one can not use recurrent reasoning, as already pointed out.

However, if we assume that bilateral transaction will remain at the fringe of normal operations, representing no more than 10% of the total transfers, we can compute the access rates for all the transactions using the marginal costs at the final operating point. This point may be determined *ex ante* if scheduled transactions and forecast demand are accurate enough, or *ex post* otherwise. This method yields wheeling rates that do not depend on the order of transaction accommodation, as they are in fact all accommodated at the same time.

6.4.2 Dynamic aspects of operations planning

The optimal spot prices were derived from the economic dispatch problem formulation. However, economic dispatch is only a subproblem of optimal system operations, once one has decided which units will be used or turned off. This more general problem of unit commitment, also called pre-dispatch, is usually solved one day or one week in advance. It provides a hour-by-hour strategy for committing and withdrawing generating units in an economical and reliable manner and must consider many dynamic constraints on generation [Schw75, Sheb94]:

Startup and shutdown rates : technological limitations such as turbine vibrations, prevent from turning a unit on or off instantaneously, or faster than at given rate. More generally, ramp rate limits restrict the movement of a generating unit output between adjacent hours.

Minimun up and down times : once a unit is turned on, it must run at nominal power for a minimum amount of time. Once it is turned off, it can not be turned on again before a certain amount of time. The following startup cost is a increasing function of the down time, because of heat retention in the boilers.

Hydro-storage : The use of storable hydroelectric energy, including pumped storage capabilities must be coordinated on a daily or weekly basis. Storage type technologies may take on more importance in the forseeable future.

Crew constraints limit the number of units that can start at the same time in a particular plant.

More generally, the unit commitment problem depends on the yearly scheduling of maintenance operations or refueling of nuclear power plants. The absence of a large unit from the available plant mix for a amount of time that can exceed several weeks may have substantial effects on system operations.

One of the main assumptions of economic dispatch based on competitive market was that the generators do not have any market power. But all the limitations on unit availability resulting from decisions that have long consequence in time, or

simply the fact that the total generation capacity is finite, though excessive, may be used by competitors for gaming. They can base their bids on game theory and obtain significant premiums when their competitor has committed a large maintenance operation for example. This means that the assumption of competitive generation level was rather ambitious, and it is likely that a few competing generation companies will have a significant market power, similar to what is currently happening in England and Wales, where only three companies are competing.

Finally, to handle dynamic effects in economic dispatch it has been proposed to use expected values of spot price over one day, or one week, based on demand forecast and generation availability estimates [Cara81]. Operations planning would be carried out with anticipated spot prices trajectories to serve the forecasted load and to accommodate the scheduled transactions.

6.4.3 Revenue reconciliation and expansion planning

The second dynamic aspect not considered by short-run marginal prices is long term expansion and enhancement planning of the transmission network, and consequently, the issue of embedded capital cost is not solved. Indeed, as a short-term approach is used to compute optimal spot prices, the fixed capital cost does not appear in the derivations.

First of all, note that we consider only the issue of transmission capital cost recovery. The recovery of fixed generation cost is assumed to be taken care of in the bids submitted by the generators, or in the price negotiated between buyer and seller for a power wheeling. In other words, we consider that the generation function is either in the domain of increasing marginal costs greater than the average cost, or that generators sell their power at long run marginal cost, taking the investment cost into account.

In contrast to this, the transmission level of the electric industry seems still to be in the domain of decreasing marginal costs (economies of scale). Therefore, the positive net revenue from wheeling of utility K receiving the transmission rates given by (6.9) can either over or under recover the overall costs, depending on its transmission and

generation capital mix. To tackle this issue of capital cost recovery, the author of [Cara86] propose to adjust the wheeling rates (6.9), either up or down.

One of the possible methods is to perform the economic dispatch derivations under an additional constraint of yearly balanced budget. This is a form of Ramsey pricing, and yields a corrected wheeling rate of the form

$$\tilde{\omega}^{K,BS}(t) = \omega^{K,BS}(t)[1 + \mu(t)] \tag{6.12}$$

where $\omega^{K,BS}$ is the wheeling rate computed without revenue reconciliation, and $\mu(t)$ is the shadow price of the budgetary constraint. This method can be considered as a second best approach, but it does not change the fundamental static nature of short-run optimization. Consequently, no incentives for transmission network expansion are provided.

6.4.4 Outline of the remaining chapters

In the following chapters, we focus on a peak-load approach for transmission pricing. "Peak-load" means that the investment cost is explicitly taken into account, as well as the time fluctuations of demand patterns. Such an approach can be deemed dynamic in that it considers several time periods with different demand levels. However, peak load pricing can also be performed for fixed transmission capabilities, or for a transmission capacity allowed to vary over time through investment in new lines or line upgrade. This aspect makes the difference between Chapter 7 that presents a static approach, and Chapter 8 that provides a dynamic approach of optimal investment planning.

To take into consideration the spatial characteristics of a transmission grid, the network is explicitly modeled, using either a classical transportation model, or, more accurately, the DC load flow model reviewed earlier in this thesis. The innovation of our approach is to bring together the spatial representation borrowed from [Nagu93] for the transportation model, or from classical power system engineering for the DC load flow equations, and the peak-load approach formulated in [Crew79] for generation

investment (and hence without network model).

The idea of using a dynamic approach based on spot prices for transmission expansion planning is also proposed in [Cara81], but without an elaborate network representation. Finally, the results on optimal investment in transmission presented in [Kim88] are very similar to our approach using the Maximum Principle of Pontryagin. However, the authors of [Kim88] consider a regulated industry structure, and are not concerned with the issue of transmission pricing.

Chapter 7

Static peak-load pricing for transmission

This chapter proposes a new economic formulation of optimal operations for a given transmission network. According to classical economic theory, marginal cost pricing leads to economic efficiency, as measured by the social welfare maximization. This is the main justification for the spot pricing theory reviewed in the previous chapters.

However, in case of high fixed costs such as investment cost in a transmission network, the marginal cost is decreasing and its value is below the average cost (the total cost devised by the quantity processed). Hence marginal cost pricing results in deficits. According to classical economy theorists, these deficits would be optimally covered by neutral tax revenues, but this entails serious problems in managerial incentives, allocative distortions and so forth. These distortions may be more severe than the cross-subsidies that would be caused by pricing at average cost in the first place. Therefore, the classical economic theory is not practically suitable for transmission services, or at least not without taking the investment and capital cost of transmission infrastructure into account.

To take provision for capital cost recovery, several proposals of second best pricing can be found in the literature of public economics, including average cost pricing, fair rate of return regulation and welfare optimal break-even analysis. This latter approach, initiated by Boiteux and also called Ramsey pricing, is to maximize welfare

under an additional explicit break-even constraint, see [Boit56, Baum70]. This leads to departures from the marginal cost proportional to the inverse demand elasticity, hence covering the deficit by extracting more from products or users with inelastic demand. Such a pricing policy could be carried out under public control. Discussion of these proposals involve issues of managerial incentives, stockholder liability, and cross-inefficiencies.

The approach described in what follows is not a second best pricing approach, in that it does not rely on an additional accounting constraint. We instead examine extensively the nature of welfare optimal pricing, i.e. marginal cost pricing, by taking explicitly into account the fixed capital cost. This is obtained by modifying the traditional welfare function to include a cost for transmission capacity.

This approach, called peak-load pricing, is in fact the theory of long run marginal costs (LRMC). It has been well formalized by M.A. Crew and P.R. Kleindorfer in [Crew79] to take the energy generation investment cost explicitly into account in the welfare maximization. We add a model of the transmission network to apply it to the special topic of transmission pricing.

7.1 Peak load pricing approach

7.1.1 Origins

The main change from break-even analysis is that our approach handles implicitly the budget balance in the objective function rather than as an additional constraint. We, therefore, introduce new decision variables for line capacity. Other technological or economic restrictions such as the network technical requirements, explicit break-even or required profit constraint could still be included at will. However, rate of return on capital is an accounting and financial concept, that should be included in the unit cost of infrastructure used thereafter. The peak-load approach would overall result in a single tariff, rather than multi-part tariffs for network services.

Note that, in the first place, we give no guarantee for total cost recovery other than

taking all the components of cost into account. However, in the absence of economies of scale in network expansion, long run marginal cost will turn out to be higher than average cost, and the network operator will recover its cost. We will confirm this in Section 7.4.4 by computing the actual revenue of the network operator.

The investment cost appears in the generation and/or transmission capacity. However, as power demand fluctuates over day or seasons and electricity is mostly a non storable commodity, utilities face the non uniform utilization of capacity. Thus, Crew and Kleindorfer advise the use of a "peak-load pricing" policy, to discourage consumption in peak periods and encourage off-peak demand, hence improving the utilization of capacity. The peak-load models, as originated by Boiteux and Steiner (see [Boit49, Stei57]) and developed by Crew and Kleindorfer, deal with the trade-off between utilization gains and consumer welfare. They focus mostly on generation, and yield conclusions on the optimal pricing as well as the optimal plant mix, i.e. the economical trade-off between generation technologies using various fuel and investment cost.

7.1.2 Assumptions

As said earlier, our goal is to propose a pricing system for transmission services. Hence, we focus mostly on transmission cost, as we consider that the generation function of the electric industry is perfectly competitive, and notably that generators sell their power at long run marginal cost. All the problems of peak-load pricing of generation, as developed by Crew and Kleindorfer are now handled by competition at the generation level. Hence, they loose some of their relevance as energy generation seems not to be in the domain of decreasing costs anymore, thank to new technologies.

Note that, under the assumption that marginal transmission cost is essentially negligible, the new relevant trade-off for social welfare maximization in a deregulated environment occurs between *transmission investment cost and generation fuel cost.* In particular, we do not take any transmission operating cost into account and rather consider only investment cost of transmission. If necessary, the model could be refined to include an operating cost component in transmission services pricing, but we believe

that this simplification is completely hidden by other approximations of real life power systems.

This approach is not aimed at taking into account the possible market power of some generators: if we refer to the British example, where only two conventional generators and one company operating the nuclear power plants compete, the number of generating companies is likely to be too small for a perfectly competitive supply of electricity to take place. However, the generation cost function used here does not depend on competitors' decisions.

Moreover, we consider here a monopolist GridCo, regulated to maximize social welfare in buying electricity from generators, selling to loads and providing transmission services without discrimination. Opacity of technical constraints, economic interest of the GridCo that may have stakes in the generation level, and asymmetry of information make this assumption somewhat idealistic. However, these issues of optimal regulation are not considered here. For more information, [Laff93, Laff94] are interesting references.

7.1.3 Outline

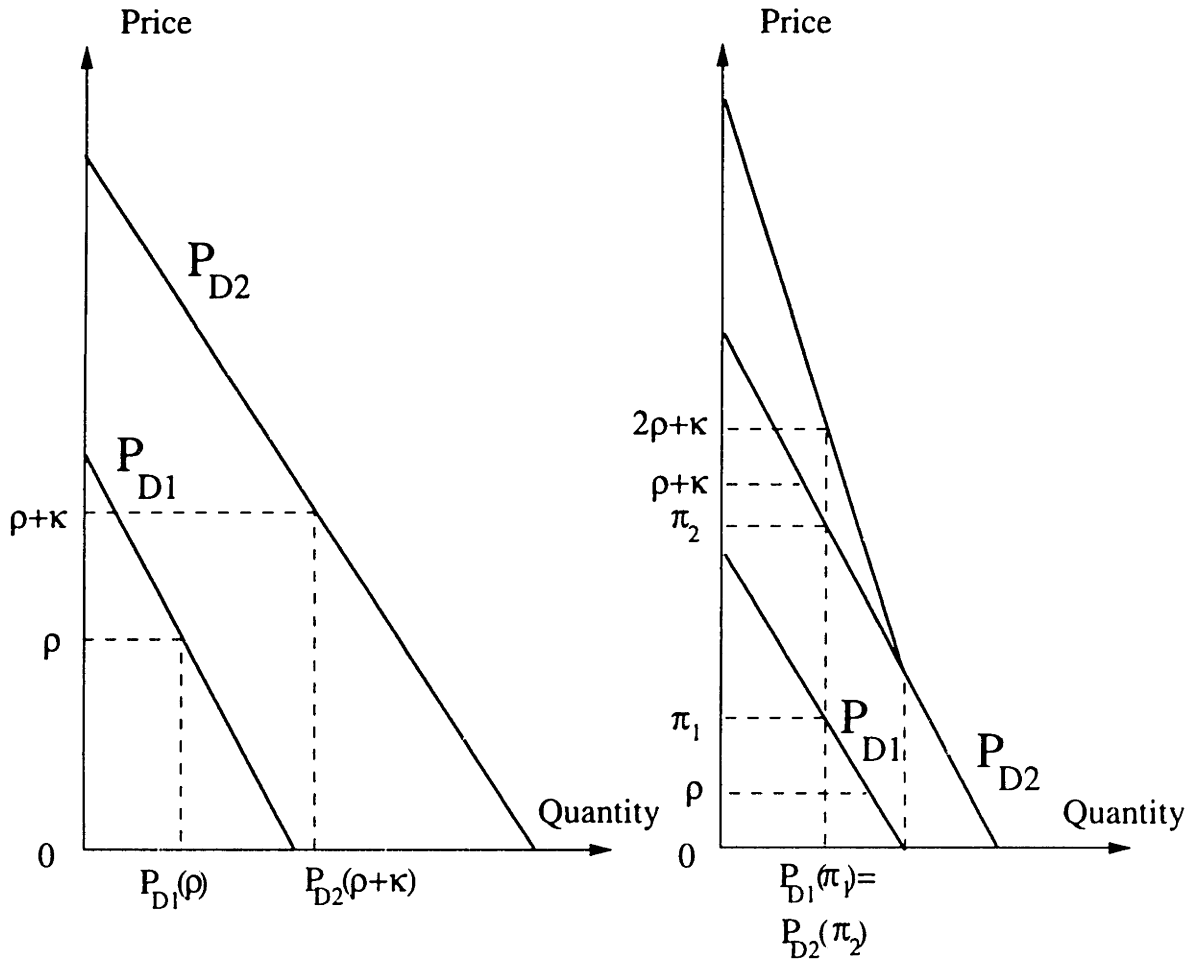
The next section gives a simple introductory two-bus example. Next, two static formulations are given, using two different models of the network: the classical transportation model and the DC load flow model. Finally, these static formulations set up the stage for a dynamic model given in the next chapter, where we consider optimal investment in a transmission network whose capacity is allowed to vary over time.

7.2 The Boiteux-Steiner model

Let us assume the simplest topology power network, i.e. a generator at bus 1 connected to a load at bus 2 by a transmission line. We assume that the demand at the load bus is known with certainty, and that the "typical day" is divided into two equal length periods. Each period is governed by its independent demand curve $P_{D1}(\pi)$ and

$P_{D2}(\pi)$ shown on Figure 7-1, one of these curves lying everywhere below the other:

$$P_{D1}(\pi) < P_{D2}(\pi) \quad \forall \pi \tag{7.1}$$



(a) Firm peak case (b) Shifting peak case

Figure 7-1: Demand curves in the two bus example.

The costs are assumed linear and we neglect transmission loss and any operating constraints other than transmission capacity. The generating cost is ρ per unit of energy (or power during the entire period), and the cost of building the line is κ per unit of capacity. Thus, a unit of power demanded in a period will cost ρ if the capacity already exists to transfer it, and $(\rho + \kappa)$ if additional capacity has to be installed. Note that a unit of installed capacity is available for meeting demand in

both periods.

If $P_{D1}(\rho) < P_{D2}(\rho+\kappa)$, the welfare maximizing solution to the two-period problem is to build a transmission capacity $K = P_{D2}(\rho + \kappa)$ and to charge prices

$$\pi_1 = \rho \text{ and } \pi_2 = \rho + \kappa \tag{7.2}$$

This is known as the *firm peak* case, where only the peak users bear the capacity cost. This results from the fact that investment in capacity is made for peak users only, as the off-peak demand is only $P_{D1}(\rho) < K$.

By contrast, if $P_{D1}(\rho) > P_{D2}(\rho + \kappa)$, then the power served in both cases is the same and equals capacity K such that

$$\pi_1(K) + \pi_2(K) = 2\rho + \kappa \tag{7.3}$$

This result is obtained by adding vertically the two demand curves, as shown on the Figure 7-1. In this case, known as the *shifting-peak* case, the capacity is fully used in both periods, and no single category of users is responsible for the investment in capacity. However peak-users pay a higher price than off-peak users. This may be seen as price discrimination, but only depends on the relative strength of the two demands, as well as on the cost κ of building new capacity.

7.3 Peak-load pricing with a transportation model

7.3.1 Problem formulation

Let us consider a power system composed of m generation nodes serving n demand nodes over one "typical day". The day is divided into T periods of equal length indexed by $t \in \{1 \dots T\}$. Note that the assumption of equal length is easy to relax, and just enables us to use the same unit for energy and capacity, hence simplifying the following notations:

- P_{Gi}^t denotes the energy produced by the generator i over period t at a constant marginal generation cost of ρ_i.
- $P_{Dj}^t(p)$ is the downward sloping demand curve of load bus j over period t. The inverse demand function is $p_j^t(P)$, and π_j^t is the equilibrium price at bus j.
- T_{ij}^t is the power transfered over line i, j if it exists. The transmission capacity of the line is denoted by K_{ij} and κ_{ij} is the unit capacity cost.

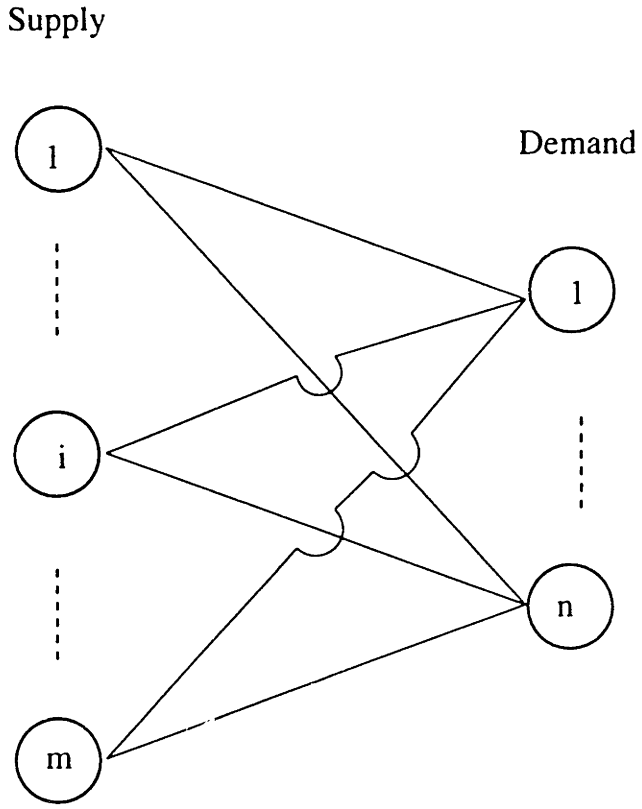


Figure 7-2: Transportation representation of a network.

The problem of peak-load pricing is to maximize the social welfare Ω defined as the sum of consumer surplus and producer surplus. As the supply curve resulting from constant generation marginal cost is flat, the generation surplus is zero here. Hence, social welfare becomes the difference between the demand function and the production cost. Moreover, as we take investment cost of transmission into account, the social welfare is now a function of power exchanges, but also of transmission

capacity. The addition of the latter in the control variable is a significant departure from conventional formulation, but is justified from the regulator's or GridCo's point of view. It can also be seen as a way to internalize the "economic externality" of transmission services in social welfare maximization.

We formulate the problem as follows:

$$\max_{P^t_{Gi},\, P^t_{Dj},\, T^t_{ij},\, K_{ij}} \quad \Omega = \sum_{t=1}^{T} \left[\sum_{j=1}^{n} \int_0^{P^t_{Dj}} p^t_j(y)dy - \sum_{i=1}^{m} \rho_i P^t_{Gi} \right] - \sum_{i=1}^{m} \sum_{j=1}^{n} \kappa_{ij} K_{ij} \tag{7.4}$$

subject to node and capacity constraints:

$$\sum_{j=1}^{n} T^t_{ij} = P^t_{Gi} \quad \forall i, t \tag{7.5}$$

$$\sum_{i=1}^{m} T^t_{ij} = P^t_{Dj} \quad \forall j, t \tag{7.6}$$

$$T^t_{ij} \leq K_{ij} \quad \forall i, j, t \tag{7.7}$$

$$0 \leq P^t_{Gi} \quad \forall i, t \tag{7.8}$$

$$0 \leq P^t_{Dj} \quad \forall j, t \tag{7.9}$$

$$0 \leq T^t_{ij} \quad \forall i, j, t \tag{7.10}$$

$$0 \leq K_{ij} \quad \forall i, j \tag{7.11}$$

These constraints merely state the balance of power at each generation or demand node. The last four sign constraints are unique to the transportation model, and state that generators have a non negative output, loads have a non negative consumption, line capacity is positive and power flows always from a generator toward a load. Indeed, in the transportation model, *separate lines* link each generation node to all the demand busses, and no superposition or parallel flow occurs, as can be seen on Figure 7-2. Hence, the transportation model is a very coarse representation of a power network.

7.3.2 Problem solution

The problem (7.4)-(7.11) is solved by forming the Lagrangian $\mathcal{L}$ as follows:

$$\begin{aligned}\mathcal{L}(P_{Gi}^t, P_{Dj}^t, T_{ij}^t, K_{ij}) &= \Omega(P_{Gi}^t, P_{Dj}^t, T_{ij}^t, K_{ij}) \\ &+ \sum_{i=1}^{m}\sum_{t=1}^{T}\gamma_i^t(P_{Gi}^t - \sum_{j=1}^{n}T_{ij}^t) + \sum_{j=1}^{n}\sum_{t=1}^{T}\lambda_j^t(\sum_{i=1}^{m}T_{ij}^t - P_{Dj}^t) \\ &+ \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T}\mu_{ij}^t(K_{ij} - T_{ij}^t) \\ &+ \sum_{i=1}^{m}\sum_{t=1}^{T}\alpha_i^t P_{Gi}^t + \sum_{j=1}^{n}\sum_{t=1}^{T}\beta_j^t P_{Dj}^t \\ &+ \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T}\tau_{ij}^t T_{ij}^t + \sum_{i=1}^{m}\sum_{j=1}^{n}\zeta_{ij}K_{ij} \end{aligned} \tag{7.12}$$

The integral term in Ω is concave, as $p_j^t(P)$ is a decreasing function. All the other terms in Ω, as well as the constraints are linear. Hence, this is a concave problem of maximization, and the Kuhn-Tucker theory states that P_{Gi}^t, P_{Dj}^t, T_{ij}^t, K_{ij} *meeting the constraints* (7.5)-(7.11) are solution of the problem (7.4) if and only if there exists *non negative* multipliers γ_i^t, λ_j^t, μ_{ij}^t, α_i^t, β_j^t, τ_{ij}^t, ζ_{ij} such that:

- the derivatives of the Lagrangian with respect to the control variables are zero.
- the multipliers associated with non binding constraints are zero.

By limiting the analysis to the cases where power is generated at all busses i, and consumed at all busses j, the simplified optimality conditions are as follows (see Appendix C.1):

$$\pi_j^t = \lambda_j^t \quad \forall j, t \tag{7.13}$$

$$\rho_i^t = \gamma_i^t \quad \forall i, t \tag{7.14}$$

$$\sum_{j=1}^{n} T_{ij}^t = P_{Gi}^t \quad \forall i, t \tag{7.15}$$

$$\sum_{i=1}^{m} T_{ij}^t = P_{Dj}^t \quad \forall j, t \tag{7.16}$$

$$0 \leq \mu_{ij}^t \quad \text{and} \quad \mu_{ij}^t(K_{ij} - T_{ij}^t) = 0 \quad \forall i, j, t \tag{7.17}$$

$$\lambda_j^t \leq \gamma_i^t + \mu_{ij}^t \quad \text{and} \quad T_{ij}^t(\lambda_j^t - \gamma_i^t - \mu_{ij}^t) = 0 \quad \forall i, j, t \tag{7.18}$$

$$\sum_{t=1}^{T} \mu_{ij}^t \leq \kappa_{ij} \quad \text{and} \quad K_{ij}(\sum_{t=1}^{T} \mu_{ij}^t - \kappa_{ij}) = 0 \quad \forall i, j \tag{7.19}$$

The interpretation of these optimality conditions is easier on an example. However, we can already recognize that (7.17) defines the Lagrange multiplier μ_{ij}^t as the shadow price for transmission capacity between nodes i and j, i.e. the cost of energy transportation between these nodes. This cost is positive only if the capacity of line ij is reached by the power transfer during period t.

Equation (7.18) is the classical transportation equilibrium condition, stating that the price of energy consumed in node j is the same whatever the origin of that energy, and is equal to the cost of generation γ_i^t or ρ_i^t plus the transportation cost μ_{ij}^t. This is true only if a transaction actually takes place between i and j, i.e. $T_{ij}^t > 0$. In the opposite case, no energy is transferred from i to j if the resulting cost $\rho_i^t + \mu_{ij}^t$ would be higher than π_j^t.

Note that in the DC load flow model, or more generally, in any model of meshed network, as presented in Chapter 6 and following [Cara86], one defines the transmission cost as the difference between the spot prices at removal and injection nodes. In this case, the transportation equality of the prices of delivered energy coming from different generators is trivial. However, it must be noted that the DC load flow equations equalize the marginal transmission losses along each path between injection node and removal node.

Finally, (7.19) relates the cost of installing the transmission capacity K_{ij} to the transportation costs at various time periods. It states that, at the social optimum, the investment cost of the line is exactly covered by the sum of transportation costs. Indeed, if the system operator purchases power from node i at price ρ_i^t and sells it to demand node j at price π_j^t, his net revenue is μ_{ij}^t. In the case of a bilateral transaction between nodes i and j, the exchange price is not known to the system operator, but the transportation fee is set to the difference between spot prices, i.e. μ_{ij}^t. Hence, (7.19) ensures capital cost recovery at the end of the day.

Generator #	1	2
Generation marginal cost ρ_i (\$/MW)	100	200
Capacity investment cost κ_{ij} (\$/MW)	150	20

Table 7.1: Marginal costs for the transportation numerical example.

It must be noted that, in this transportation model, all the transmission lines connect *individually* a pair of generator and load, therefore eliminating any problem of loop flow. Moreover, as this model does not take into account any constraint on generation, what is generated to supply any load does not interfere with the supply of other demands. Hence, this transportation model collapses into *decoupled problems for each demand bus.* A more realistic model will be derived in next section, using the DC load flow equations to consider a general network topology, with multiple *shared* connections among generators and loads.

7.3.3 An example with two generators

Let us consider a power system of two generators supplying one load, during a day divided into two equal-length periods. We assume that generator 2 has a high generation marginal cost, but is located close to the load. In contrary, generator 1 produces cheap energy, but is located further away from the load, implying a higher unit investment cost for the transmission line, as summarized in Table 7.1. Period 1 is the off-peak period, with a demand of $100\,MW$ and period 2 corresponds to the peak period, with a demand of $200\,MW$, assuming a constant totally inelastic demand throughout both periods, and a firm peak case.

By a simple process of elimination in the optimality conditions (7.13)-(7.19), we obtain the following solution

$$\pi^1 = 2\rho_1 + \kappa_1 - (\rho_2 + \kappa_2) = \lambda^1 = 130\,\$/MW$$
$$\pi^2 = \rho_2 + \kappa_2 = \lambda^2 = 220\,\$/MW$$
$$K_1 = P^1_{G1} = P^1_D = 100\,MW$$
$$P^1_{G2} = 0$$

$P^2_{G1} = P^1_{G1} = 100\,MW$

$K_2 = P^2_{G2} = P^2_D - P^2_{G1} = 100\,MW$

$\mu^1_1 = \lambda^1 - \rho_1 = 30$

$\mu^2_1 = 0 > \lambda^1 - \rho_2 = -70$

$\mu^1_2 = \lambda^2 - \rho_1 = 120$

$\mu^2_2 = \lambda^2 - \rho_2 = 20$

where we have omitted the demand bus index $j = 1$.

Several cost conditions must hold, as a result of the optimality conditions (7.13)-(7.19), for both generators to be used at optimum. Since both plants are used at optimum, and $\rho_2 > \rho_1$, we must have $\kappa_2 < \kappa_1$ otherwise energy from generator 2 would be both more expensive to produce and transport. Moreover, as the generator 2 is used to meet peak demand, the marginal cost of one peak unit of energy is cheaper from generator 2 so we must have

$$\rho_2 + \kappa_2 < \rho_1 + \kappa_1 \tag{7.20}$$

Finally, the marginal cost of one unit of energy supplied in both periods by plant 1 is less than from plant 2 so we have

$$2\rho_2 + \kappa_2 > 2\rho_1 + \kappa_1 \tag{7.21}$$

From the previous inequalities, we obtain the following bounds:

$$\rho_1 < \pi^1 < \rho_2 < \rho_2 + \kappa_2 = \pi^2 < \rho_1 + \kappa_1 \tag{7.22}$$

By comparing (7.2) and (7.22), one notes that the introduction of a more diverse technology, i.e. the supply by various generators, leads to lower peak-period prices (here, 220 instead of 250 if only generator 1 were available) and higher off-peak prices (here, 130 instead of 100). Of course, energy with cheap generating cost will always be used first. Technical variety leads to a smoother price evolution with time and demand. General cost conditions and rules of price are derived in [Crew79] for more

variety in generating technology, but the principles remain the same.

As expected, the optimal prices are set at long run marginal cost λ_j^t, and the fuel and transmission investment costs are completely covered. Indeed, one can verify on the numbers that the total cost over the two periods is equal to the total revenue of the system operator:

$$\begin{aligned} TC &= \rho_1(P_{G1}^1 + P_{G1}^2) + \rho_2 P_{G2}^2 + \kappa_1 K_1 + \kappa_2 K_2 \\ &= \pi^1 P_D^1 + \pi^2 P_D^2 \\ &= \$57,000 \end{aligned} \tag{7.23}$$

7.4 Peak-load pricing using a DC load flow model

We modify here the problem formulation of the previous section to give a more realistic model of a power system, by a meshed network. This enables to take parallel flows into account, using the DC load flow equations instead of the transportation model. Moreover, to better reflect real operating conditions we add constraints on power output at each generator, and consider a general generation cost function. Finally, to simplify the notations, we net out the effect of generation and demand a each node.

7.4.1 Net power injection at each bus

Using our usual notations, we consider a meshed network composed of n nodes, being both generators and/or loads, and linked by n_l transmission lines. We analyze one "typical day" divided into T equal-length periods.

At a given node i and during period t, the power generated is P_{Gi}^t at a generation cost $G_i(P_G)$. $P_{Dj}^t(p)$ is the demand curve and $p_i^t(P_D)$ the inverse demand function. The contribution of node i in the social welfare is the opposite of the social cost at node i:

$$-C_i^t(P_{Gi}^t, P_{Di}^t) = \int_0^{P_{Di}^t} p_i^t(y)dy - G_i(P_{Gi}^t) \tag{7.24}$$

For given operating conditions, there is a unique nodal spot price of power at bus i

denoted ρ_i^t. It is the price paid for power consumed at node i, or the price received by a generator injecting power at bus i. The effect of node i on the power flows is entirely captured in the net power injected in the network $P_i^t = P_{Gi}^t - P_{Di}^t$. The injection P_i^t is positive if node i has a generation surplus, and negative otherwise.

Moreover, as we have shown in Chapter 5, the actual power generated and consumed at node i can be inferred from the power injection, and the social cost at node i can be expressed as a function of P_i^t only:

$$C_i^t(P_i^t) = G_i[P_{Gi}(P_i^t)] - \int_0^{P_{Di}(P_i^t)} p_i^t(y)dy \tag{7.25}$$

Therefore, we can formulate the problem of social welfare maximization in terms of the net power injections only, and in both cases we have

$$\frac{\partial C_i^t}{\partial P_i^t} = \rho_i^t > 0 \tag{7.26}$$

whatever the sign of P_i^t.

7.4.2 Problem formulation

Let us first present the main technical characteristics of the power system under consideration. If $\mathbf{T}^t$ denotes the shipment pattern during period t, defined by the DC load flow equations, we have

$$\mathbf{T}^t = \mathbf{H}\mathbf{P}^t \quad \forall t \tag{7.27}$$

where $\mathbf{H}$ is the $n_l \times n$ transfer admittance matrix (see Appendix B). Without loss of generality, we denote the swing bus by index 1, and the first column of matrix $\mathbf{H}$ is in fact composed of zeros. The power balance condition of the system is

$$P_1^t + \sum_{i=2}^{n} P_i^t = 0 \quad \forall t \tag{7.28}$$

The net generation/demand constraints at each node are modeled by the inequal-

ities:

$$P_i^{min} \leq P_i^t \leq P_i^{max} \quad \forall i, t \tag{7.29}$$

If there is only generation at bus i, then the unit operates between the bounds P_i^{min} and P_i^{max} at a thermal efficiency and hence a fuel cost that remains about constant. The minimum output P_i^{min} when plant i is turned on is a strictly positive quantity for real units, and the power injection can be zero only if the plant is turned off. However, this leads to discontinuities between on and off units, that should be handled by non convex mixed-integer programming. This is far beyond the scope of the present model. Hence, we assume that $P_i^{min} \leq 0 \leq P_i^{max}$ in the following.

In our model, the transmission capacity on each line K_{ij} is considered as a control variable. However, we use the DC load flow equation (7.27) with a constant matrix $\mathbf{H}$. This means that we consider here that line capacity and line reactance are not directly related. This model is appropriate for thermal limits on transmission capacity, that can be relieved by increasing the section of a line, but not for line enhancements that would improve stability limits by reducing the line reactance or using FACTS, and hence modify the matrix $\mathbf{H}$. The dependency of $\mathbf{H}$ on capacity and the cost of modifying its entries are not modeled here, and would lead to a non linear formulation. We let κ_l denote the unit cost of transmission capacity on line $l \in \{1 \ldots n_l\}$, and K_l the transmission capacity of this line.

The problem of social welfare maximization takes on the form

$$\max_{P_i^t,\, K_l} \quad \Omega = -\sum_{t=1}^{T} \left[\sum_{i=1}^{n} C_i^t(P_i^t) \right] - \sum_{l=1}^{n_l} \kappa_l K_l \tag{7.30}$$

subject to

$$-K_l \leq T_l^t = \sum_{i=1}^{n} H_{li} P_i^t \leq K_l \quad \forall l, t \tag{7.31}$$

$$0 = P_1^t + \sum_{i=2}^{n} P_i^t \quad \forall t \tag{7.32}$$

$$P_i^{min} \leq P_i^t \leq P_i^{max} \quad \forall i, t \tag{7.33}$$

$$0 \leq K_l \quad \forall l \tag{7.34}$$

7.4.3 Problem solution

The problem (7.30)–(7.34) is solved by forming the Lagrangian $\mathcal{L}$ as follows:

$$\begin{aligned}\mathcal{L}(P_i^t, K_l) = \Omega(P_i^t, K_l) &+ \sum_{l=1}^{n_l}\sum_{t=1}^{T}\mu_l^t(K_l - \sum_{i=1}^{n} H_{li}P_i^t) \\ &+ \sum_{l=1}^{n_l}\sum_{t=1}^{T}\nu_l^t(\sum_{i=1}^{n} H_{li}P_i^t + K_l) + \sum_{t=1}^{T}\lambda^t\sum_{i=1}^{n}P_i^t \\ &+ \sum_{i=1}^{n}\sum_{t=1}^{T}\alpha_i^t(P_i^t - P_i^{min}) + \sum_{i=1}^{n}\sum_{t=1}^{T}\beta_i^t(P_i^{max} - P_i^t) \\ &+ \sum_{l=1}^{n_l}\zeta_l K_l\end{aligned} \tag{7.35}$$

where the Lagrange multipliers are all non negative.

This is a concave problem of maximization. Hence, the optimality conditions result from the Kuhn-Tucker theory and are derived in the same way as in Section 7.3.2 (see Appendix C.2). If we limit the analysis to the cases where the power injection constraint (7.33) is not binding, the Lagrange multipliers α_i and β_i are null, and we obtain:

$$\sum_{i=1}^{n} P_i^t = 0 \tag{7.36}$$

$$\mu_l^t \geq 0 \quad \text{and} \quad \mu_l^t(K_l - \sum_{i=1}^{n} H_{li}P_i^t) = 0 \quad \forall l, t \tag{7.37}$$

$$\nu_l^t \geq 0 \quad \text{and} \quad \nu_l^t(\sum_{i=1}^{n} H_{li}P_i^t + K_l) = 0 \quad \forall l, t \tag{7.38}$$

$$\lambda^t = \rho_i^t + \sum_{l=1}^{n_l} H_{li}(\mu_l^t - \nu_l^t) \quad \forall i, t \tag{7.39}$$

$$\sum_{t=1}^{T}(\mu_l^t + \nu_l^t) \leq \kappa_l \quad \text{and} \quad K_l\left[\sum_{t=1}^{T}(\mu_l^t + \nu_l^t) - \kappa_l\right] = 0 \quad \forall l \tag{7.40}$$

The multipliers μ_l^t and ν_l^t are the shadow prices of transmission capacity over line l and are positive only if the line capacity K_l or $-K_l$ is reached during period t. Equation (7.39) replaces the transportation equilibrium seen in previous section, and

relates system lambda during period t to the nodal prices over all the network and to the transmission capacity costs. Indeed, the marginal price on the system is still a function of the cost of generation ρ_i^t at each bus[1] and the costs of transmission $\mu_l^t - \nu_l^t$ over each line. However, the sensitivity coefficients H_{li} represent the actual technical network equilibrium, as a consequence of Kirchhoff laws and loop flows.

7.4.4 Investment cost recovery

The equation (7.39) has exactly the same form as (5.20) defining the nodal spot prices in an optimal power flow, if we neglect the transmission losses. So our peak-load approach of welfare maximization seems to yield the same spot prices as a classic static economic dispatch. Where is the departure from short run marginal costs ?

The difference between the present approach and the previous chapter is that μ_l^t and ν_l^t now represent capacity costs rather than transmission congestion shadow prices. Whereas the OPF problem is solved for given transmission constraints, resulting from the existing transmission capacity of each line, the peak-load formulation "sets" the optimal line capacity, so that the cost of building or upgrading new lines is exactly balanced by the social benefit of the additional capacity. This means that transmission "congestion cost" is exactly balanced by the investment cost of new infrastructure. The optimal prices are long run marginal costs, in that they take into account the fixed cost of the infrastructure. This aspect of optimal investment is somehow hidden by the static approach followed here. But the present model should be considered as an introductory step towards a more realistic dynamic representation, and the trade-off between transmission investment cost and social benefit will become even clearer in the next chapter.

However, we will show here that equation (7.40) actually states that, at the social optimum, the capacity investment cost is exactly covered by the sum of transportation revenues at the end of the day. For each transmission line l that actually exist, i.e.

[1] If the power injections constraints are binding, we already discussed that the generator should receive $\rho_i^t - \alpha_i^t + \beta_i^t$ instead of just ρ_i^t.

that has a strictly positive transfer capacity K_l, we have from (7.40)

$$\kappa_l = \sum_{t=1}^{T} (\mu_l^t + \nu_l^t) \tag{7.41}$$

The capital cost of a line is $K_l \kappa_l$, and we should check that the system operator actually collects the transmission rent $\mu_l^t + \nu_l^t$ during period t. As explained in the previous chapter, he purchases the power $P_i^t > 0$ from the generator i at the nodal spot price ρ_i^t, and sells the power $-P_j^t > 0$ to the user j at the nodal spot price ρ_j^t. Therefore the net revenue of the system operator during period t is

$$NR = -\sum_{i=1}^{n} P_i^t \rho_i^t \tag{7.42}$$

$$NR = -\sum_{i=1}^{n} P_i^t [\lambda^t - \sum_{l=1}^{n_l} H_{li} (\mu_l^t - \nu_l^t)] \tag{7.43}$$

Using the power balance (7.36), and by redistributing the terms of the double summation, we get

$$NR = \sum_{l=1}^{n_l} \left[\sum_{i=1}^{n} H_{li} P_i^t \right] (\mu_l^t - \nu_l^t) \tag{7.44}$$

Finally, using conditions (7.37) and (7.38) we obtain

$$NR = \sum_{l=1}^{n_l} K_l (\mu_l^t + \nu_l^t) \tag{7.45}$$

This proves that the net revenue of the system operator exactly covers the transmission capacity capital cost. In the case of bilateral transactions where the system operator just perceives a transmission fee, this fee should be set to the difference of spot prices at injection and reception busses, and nothing is changed in the above proof of cost recovery.

7.4.5 A three bus example

We consider the same network as in the transportation example, after addition of a transmission line between the two generators, as shown in Figure 7-3. The marginal

Transmission line #	1	2	3
From bus #	1	1	2
To bus #	2	3	3
Reactance X_l (p.u.)	10.0	10.0	5.0
Capacity investment cost κ_l (\$/MW)	variable	150.00	20.00

Table 7.2: Transmission line characteristics for the three bus example.

costs of generation are given in Table 7.1, and we consider that the generation limit P_i^{max} is never binding. The characteristics of the transmission lines are given in Table 7.2. The day is divided in two equal-length periods. During period 1, the demand at bus 3 is 100 MW and period 2 corresponds to a peak demand of 200 MW. We assume constant totally inelastic demands throughout both periods.

The demand bus 3 is considered as the swing bus, and the transfer admittance matrix takes the form

$$\mathbf{H} = \begin{bmatrix} 0.4 & -0.2 \\ 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix} \tag{7.46}$$

Table 7.3 gives the optimum prices, and power generated at bus 1 and 2, as well as the Lagrange multipliers for three values of the capacity cost κ_1 between bus 1 and 2. Column 4 corresponds to the same capacity costs as column 2, except that power is generated at bus 1 only.

These results illustrate that the trade-off between marginal cost of generation and transmission investment depends on the capacity costs. For a very high cost of capacity between bus 1 and 2, on line 1, the optimum generation is dispatched in such a way that $T_1^t = 0$ and hence $K_1 = 0$. This means that *line 1 is not used*, but it exists and its existence requires an expensive generation dispatch. We can say that line 1 is actually harmful in this case. This shows the shortcomings of our model, that decouples the capacity cost and the transfer admittance matrix $\mathbf{H}$. But this problem is related to the issues raised by conterflows and transmission pricing, and requires further investigation.

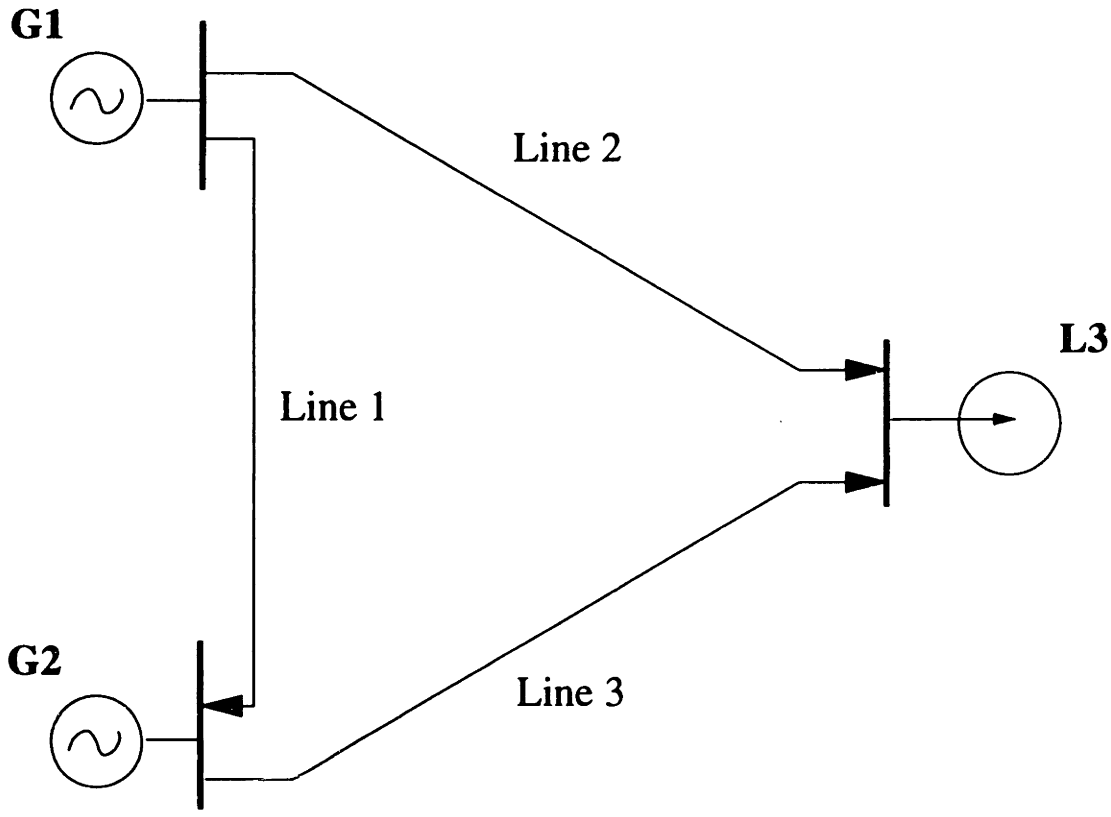


Figure 7-3: Three bus network.

Capacity cost κ_1 (\$/MW)	50.00	100.00	250.00	100.00
Social Cost C	53,600	56,800	62,667	57,600
Capacity K_1	80.00	40.00	0	80
Capacity K_2	120.00	93.33	66.67	120.00
Capacity K_3	80.00	106.67	133.33	80.00
Shadow price ζ_1 of $(K_1 \geq 0)$	0	0	3.33	0
Off-peak period: $P_D^1 = 100\,MW$				
P_{G1}^1	100.00	100.00	33.33	100.00
P_{G2}^1	0	0	66.67	0
μ_{12}^1	0	20.00	166.67	0
μ_{13}^1	0	0	0	0
μ_{23}^1	0	0	0	0
Shadow price α_2^1 of $(P_{G2}^1 \geq 0)$	100.00	88.00	0	100.00
π_3^1	100.00	108.00	166.67	100.00
Peak period: $P_D^2 = 200\,MW$				
P_{G1}^2	200.00	133.33	66.67	200.00
P_{G2}^2	0	66.67	133.33	0
μ_{12}^2	50.00	80.00	80.00	100.00
μ_{13}^2	150.00	150.00	150.00	150.00
μ_{23}^2	20.00	20.00	20.00	20.00
Shadow price α_2^2 of $(P_{G2}^2 \geq 0)$	18.00	0	0	100.00
π_3^2	218.00	230.00	230.00	238.00

Table 7.3: Sensivity analysis of the three bus example.

The second line of Table 7.3 gives the optimal total cost of generation and transmission. It can be checked that this cost is covered by the revenues. For instance, in the case $\kappa_1 = \$50.00$ we have

$$\begin{aligned} TC &= \$100.00 \times (100 + 200) + \$50.00 \times 80 + \$150.00 \times 120 + \$20 \times 80 \\ &= \$53,600 \\ TR &= \$100.00 \times 100 + \$218.00 \times 200 = \$53,600 \end{aligned}$$

and the optimal social welfare is zero in every case.

Finally, this simulation shows that, as said before with the transportation model, technical variety improves the social welfare by reducing peak prices and increasing off-peak prices, as can be seen by comparing columns 2 and 4: in the latter, only generation at bus 1 is available, and a high transmission capacity is required, whereas in the former case, peak generation at bus 2 enables to reduce the capacity cost and the total cost from \$57,600 to \$56,800 for similar generation and capacity unitary costs.

Chapter 8

Dynamic peak-load pricing under demand certainty

In the previous chapter, although we considered several time periods, the approach was static in the sense that the transmission capacity was constant throughout the typical day studied. We now derive a dynamic formulation for optimal investment in transmission expansion, that can also be used for peak-load pricing transmission services.

The power system is studied over a planning time horizon T of typically several years. The transmission capacity is now allowed to vary, depending on investment decisions made by the network owner. We use a control theory approach: the transmission capacity over each line is a state variable, the level of investment on each line and the power generated or consumed at each bus are control variables.

In the static model, we considered the investment cost of the *total* transmission capacity over a line. Hence, the capacity was allowed to vary between zero and the desired positive value, whereas the transfer admittance matrix $\mathbf{H}$ was considered constant, to obtain a linear formulation. This simplifying assumption resulted in shortcomings, as was shown in the three bus example. The fundamental reason of these shortcomings is that the problem can not be considered linear around zero. To avoid this in the following, we linearize the problem around a *non zero initial point.* In other words, we will focus on *line upgrades* and investment decisions in line

reinforcements such as FACTS that will add capacity to the existing network.

Generation technology will also evolve over the time horizon considered. However, as generation investments are the business of the power producers and as the generation level is assumed competitive here, neither do we model investment decisions in new generators, nor are we concerned with the issue of generation investment cost recovery.

This model is adapted from [Cara81] and [Crew79]. In the former paper, Caramanis focuses on the use of spot prices for expansion planning. He deals mostly with the investments in new generation facilities, and does not explicitly model the network. His approach is dynamic and takes the investment cost explicitly into account. In the latter, Crew and Kleindorfer use a control theory approach to model the dynamic decisions of investment. Their results implicitly formulate in spot prices, referred to as shadow prices in their book. However, they focus only on generation expansion planning. The innovation brought by our formulation is to model explicitly the network, in order to highlight the coupling of the system. The DC load flow model turns out to be sufficiently precise to represent accurately the coupling effects of the network, and still simple enough for interesting derivations.

Note that [Kim88] provides a quite similar formulation of the optimal transmission expansion planning. In this paper, the authors model line upgrades by increase of line admittance, and assume a linear cost function on line admittance. However, they consider neither explicit capacity constraints, nor congestion shadow price, which is precisely our goal here. Briefly said, transmission capacity varies as a function of line admittances and the values of maximum transfers K_l. A cost can be assumed for both, but to obtain a linear formulation, only one of the two parameters can be allowed to vary. Finally, whereas we model continuous capacity expansion, [Kim88] uses a discrete time approach, and capacity upgrades are rounded to discrete values. This models real life operations better, but our approach could easily be restated in a discrete formulation.

8.1 Model

We consider our usual network of n nodes that can be either generation and/or load busses, linked by n_l lines. The overall topology of the network is considered constant, i.e. we do not add or remove any right of way, and each line between two nodes exists throughout the whole study period. However the transfer capacity of these lines can vary, as lines are upgraded by the amount $k_l(t)$, starting from an initial transmission capacity K_l^0 on each line. Hence, the total transmission capacity installed on line l is a function of time of the form

$$K_l(t) = K_l^0 + k_l(t) \tag{8.1}$$

that varies according to the dynamic

$$\dot{K}_l(t) = -\delta_l K_l(t) + U_l(t) \quad \forall t \in [0,T] \ \forall l \tag{8.2}$$

and the initial condition

$$K_l(0) = K_l^0 \quad \forall l \tag{8.3}$$

The command $U_l(t)$ is the upgrade rate at time t over line l, measured in MW/sec or MW/year. The non negative number δ_l is a depreciation rate. Equation (8.2) can take the vector form:

$$\dot{\mathbf{K}}(t) = -\Delta \mathbf{K}(t) + \mathbf{U}(t) \quad \forall t \in [0,T] \tag{8.4}$$

where $\mathbf{K}(t)$ and $\mathbf{U}(t)$ are n_l-vectors and $\Delta = \mathbf{diag}(\delta_l)$ is an $n_l \times n_l$ (diagonal) matrix.

We denote $\Psi(\mathbf{U}(t), t)$ the investment cost in transmission capacity upgrade, measured in \$/hour or \$/year. This investment cost may be non linear and a function of time. Hence, the analog of the capacity cost κ_l of the static model is now the marginal cost of line upgrade

$$\mathrm{M}\Psi(\mathbf{U}(t), t) = \frac{\partial \Psi}{\partial \mathbf{U}}(\mathbf{U}(t), t)$$

Note that the transmission capacity is allowed to vary continuously here, which is a coarse approximation of discrete line upgrades[1]. The function Ψ is assumed to be twice continuously differentiable, increasing with $\mathbf{U}$ and such that $\Psi(0,t) = 0$. We assume no scale economies in each line upgrade so the marginal cost $\mathrm{M}\Psi$ is supposed to increase with $\mathbf{U}$. The long run planning analysis is carried out by using a discount rate r, chosen under accounting or financial considerations.

The cost of generation $G_i(P_{Gi}, t)$ at each bus is assumed to be a function of time and power output only. It does not directly depend on transmission capacity, neither on the power injected at other busses. This means that the generation level of the electric utility industry is considered to be perfectly competitive, as no oligopolistic interactions with other generators are modeled. Using a bidding mechanism, electric power is sold by the generators to the system operator or eligible consumers at the long run marginal cost. This LRMC, as well as the maximum output of a generator $P_i^{max}(t)$ may evolve over time, due to technical progress. However, we consider neither investments in generation, nor inter-temporal constraints such as ramping rates in unit commitment. Therefore, the cost G_i depends on the power output P_{Gi} at instant t but not on the expected values thereafter.

Finally, $P_{Di}(p,t)$ is the demand curve and $p_i(P,t)$ the inverse demand function at bus i. As before, by netting out the effect of generation and demand at bus i, we can define the net power injected in the network at instant t

$$P_i(t) = P_{Gi}(t) - P_{Di}(t)$$

that is within the segment $[P_i^{min}(t), P_i^{max}(t)]$ containing zero to avoid discontinuities. The social cost at node i is

$$C_i(P_i, t) = G_i(P_{Gi}, t) - \int_0^{P_{Di}(t)} p_i(y,t)dy$$

Consequently, the social welfare over the network at instant t, including nodal social

[1]One possible way to justify this would be to use this approach for cumulative cost recovery of discrete transmission investments.

costs and transmission investment cost, takes on the form

$$\Omega(\mathbf{P}(t), \mathbf{U}(t), t) = -\sum_{i=1}^{n} C_i(P_i, t) - \Psi(\mathbf{U}(t), t) \tag{8.5}$$

The dynamic transmission expansion problem consists of maximizing the sum of discounted social welfare over time horizon T:

$$\max_{\mathbf{P}(t),\, \mathbf{K}(t)} \quad \mathcal{J} = \int_0^T e^{-rt}\, \Omega(\mathbf{P}(t), \mathbf{U}(t), t)\, dt \tag{8.6}$$

subject to

$$\dot{\mathbf{K}}(t) = -\Delta\mathbf{K}(t) + \mathbf{U}(t) \quad \forall t \tag{8.7}$$

$$-K_l(t) \leq T_l(t) = \sum_{i=1}^{n} H_{li} P_i(t) \leq K_l(t) \quad \forall l, t \tag{8.8}$$

$$0 = \sum_{i=1}^{n} P_i(t) \quad \forall t \tag{8.9}$$

$$P_i^{min}(t) \leq P_i(t) \leq P_i^{max}(t) \quad \forall t, i \tag{8.10}$$

$$0 \leq \mathbf{U}(t) \leq \mathbf{U}^{max}(t) \quad \forall t \tag{8.11}$$

$$\mathbf{K}(0) = \mathbf{K}^0 \tag{8.12}$$

$$\mathbf{K}(T) \geq \mathbf{K}^T \tag{8.13}$$

where $\mathbf{U}^{max}(t)$ is a technical or financial upper bound on line l upgrading at time t, and $\mathbf{K}^T$ is a minimum target at the planning horizon. Note that this formulation closely reflects the modeling for dynamic peak-load pricing in [Crew79]. The difference here is that we focus on network expansion, and therefore introduce the DC load flow model.

8.2 Problem solution

This problem is solved by using the control theory that provides Kuhn-Tucker-like optimality conditions for constrained dynamic optimization problems with the Max-

imum principle of Pontryagin. We recall here that the state variable is the transmission capacity $\mathbf{K}$, and the control variables are the transmission investments $\mathbf{U}$ and the power injections $\mathbf{P}$. The system dynamics are given by (8.4) and it should be noted that the state variable $\mathbf{K}$ does not appear directly in the performance objective $\mathcal{J}$ used here.

We define the Hamiltonian function $\mathcal{H}$ as

$$\mathcal{H}(\mathbf{P}, \mathbf{U}, \mathbf{K}, \Xi, t) = \Omega(\mathbf{P}, \mathbf{U}, t) + < \Xi, -\Delta\mathbf{K}(t) + \mathbf{U}(t) > \tag{8.14}$$

and the Lagrangian function $\mathcal{L}$ as

$$\begin{aligned}
\mathcal{L}(\mathbf{P}, \mathbf{U}, \mathbf{K}, \Xi, t) &= \mathcal{H}(\mathbf{P}, \mathbf{U}, \mathbf{K}, \Xi, t) \\
&+ \sum_{l=1}^{n_l} \mu_l(t) \left[K_l(t) - \sum_{i=1}^{n} H_{li} P_i(t) \right] + \sum_{l=1}^{n_l} \nu_l(t) \left[\sum_{i=1}^{n} H_{li} P_i(t) + K_l \right] \\
&+ \lambda(t) \left[\sum_{i=1}^{n} P_i(t) \right] + \sum_{i=1}^{n} \alpha_i(t) [P_i(t) - P_i^{min}(t)] + \sum_{i=1}^{n} \beta_i(t) [P_i^{max}(t) - P_i(t)] \\
&+ \sum_{l=1}^{n_l} \zeta_l(t) U_l(t) + \sum_{l=1}^{n_l} \sigma_l(t) [U_l^{max}(t) - U_l(t)]
\end{aligned} \tag{8.15}$$

where $< , >$ denotes the scalar product in $\mathcal{R}^{n_l}$ and Ξ is the vector of adjoint variables ξ_l defined by

$$\begin{aligned}
\dot{\xi}_l(t) &= r\xi_l(t) - \frac{\partial \mathcal{L}}{\partial K_l} \\
&= (r + \delta_l)\xi_l(t) - \nu_l(t) - \mu_l(t)
\end{aligned} \tag{8.16}$$

The Maximum principle of Pontryagin gives necessary optimality conditions, see for example [Mack82, Pont62]. If the power injections $\hat{\mathbf{P}}$, the investment decisions $\hat{\mathbf{U}}$ and the resulting transmission capacity trajectories $\hat{\mathbf{K}}$ are optimum solutions of problem (8.6)–(8.13), there must be an adjoint variable Ξ solution of (8.16) along the optimal trajectories, with the terminal conditions (also called transversality conditions):

$$\xi_l(T) \geq 0 \text{ and } \xi_l(T)[K_l(T) - K_l^T] = 0 \quad \forall l \tag{8.17}$$

Moreover, at every instant $t \in [0,T]$, $\hat{\mathbf{P}}(t)$ and $\hat{\mathbf{U}}(t)$ must minimize under the constraints (8.7)–(8.13) the function $(\mathbf{P},\mathbf{U}) \mapsto \mathcal{H}(\mathbf{P},\mathbf{U},t)$. In other terms, $\hat{\mathbf{P}}$ and $\hat{\mathbf{U}}$ must minimize the Lagrangian $\mathcal{L}$, that can be considered here as a generalized Hamiltonian.

Hence, the following Kuhn-Tucker-like optimality conditions must hold at every instant $t \in [0,T]$:

$$\frac{\partial \mathcal{L}}{\partial P_i}(t) = -\rho_i(t) + \sum_{l=1}^{n_l} H_{li}[\nu_l(t) - \mu_l(t)] + \lambda(t) + \alpha_i(t) - \beta_i(t) = 0 \quad (8.18)$$

$$\frac{\partial \mathcal{L}}{\partial U_l}(t) = -\mathrm{M}\Psi_l(\hat{U}_l(t),t) + \xi_l(t) + \zeta_l(t) - \sigma_l(t) = 0 \quad \forall l \quad (8.19)$$

where $\rho_i(t)$ is the nodal spot price at bus i at instant t. The complementary conditions are:

$$\mu_l(t) \geq 0 \text{ and } \mu_l(t)\left[\hat{K}_l(t) - \sum_{i=1}^{n} H_{li}\hat{P}_i(t)\right] = 0 \quad \forall l \quad (8.20)$$

$$\nu_l(t) \geq 0 \text{ and } \nu_l(t)\left[\sum_{i=1}^{n} H_{li}\hat{P}_i(t) + \hat{K}_l(t)\right] = 0 \quad \forall l \quad (8.21)$$

$$\sum_{i=1}^{n} \hat{P}_i(t) = 0 \quad (8.22)$$

$$\alpha_i(t) \geq 0 \text{ and } \alpha_i(t)[\hat{P}_i(t) - P_i^{min}(t)] = 0 \quad \forall i \quad (8.23)$$

$$\beta_i(t) \geq 0 \text{ and } \beta_i(t)[P_i^{max}(t) - \hat{P}_i(t)] = 0 \quad \forall i \quad (8.24)$$

$$\zeta_l(t) \geq 0 \text{ and } \zeta_l(t)\hat{U}_l(t) = 0 \quad \forall l \quad (8.25)$$

$$\sigma_l(t) \geq 0 \text{ and } \sigma_l(t)[U_l^{max}(t) - \hat{U}_l(t)] = 0 \quad \forall l \quad (8.26)$$

The optimality condition (8.18) gives the nodal spot price and consequently the optimal power injected (generated of consumed) at each node. The optimality condition (8.19) yields the optimal investment in line upgrade:

$$\mathrm{M}\Psi(\hat{U}_l) = \xi_l(t) + \zeta_l(t) - \sigma_l(t) \quad \forall l, t \quad (8.27)$$

From Pontryagin theory, we can recognize that the adjoint variables are in fact the marginal variation of the optimal objective function with respect to the state variable,

i.e. the marginal social benefit of capacity upgrade:

$$\xi_l(t) = \frac{\partial J}{\partial K_l(t)}(t) \tag{8.28}$$

Hence, (8.27) simply states that the optimum upgrade rate $\hat{\mathbf{U}}(t)$ should be set to equalize the marginal cost of investment and the marginal social benefit from capacity expansion.

The optimal investment problem turns out to be a system of differential equations of dimension $2n_l$ in the augmented variable $[\mathbf{K}, \Xi]$, with the particularity of so-called two point boundary-value limit conditions: at time $t = 0$ for $\mathbf{K}$, with the initial condition (8.3) and at time $t = T$ for the adjoint variable Ξ with the transversality condition (8.17). Such constrained systems of differential equations are generally very difficult to solve, and analytical solutions for the optimal investment problem are usually impossible. The use of computer to obtain numerical solutions becomes the only resort when explicit solutions are desired. However, we will see in next sections that, in the present case, the structure of the problem enables here interesting qualitative conclusions.

8.3 Investment cost recovery

As in the static case, investment cost in transmission capacity upgrade is recovered. Once again, this results from social welfare maximization: if the minimum target for transmission capacity upgrade $\mathbf{k}^T = \mathbf{K}^T - \mathbf{K}^0$ is null, $\mathbf{K} \equiv \mathbf{K}^0$ is a feasible solution of problem (8.6)–(8.13). The corresponding social welfare is zero, and is less than the optimal social welfare. Hence, the optimal social welfare is strictly positive. As all the generation fuel costs and transmission upgrade investment costs were taken into account in the objective function, a positive social welfare implies that revenues from power sold cover all these costs, and in particular the transmission investments.

This argument does not hold if the minimum target for transmission capacity at the horizon T is $\mathbf{K}^T > \mathbf{K}^0$. This means that an excessive minimum imposed by a

regulator may prevent the network owner from investing optimally. Consequently, under these conditions the transmission investment cost could be only partially recovered. Moreover, $K \equiv K^0$ is a feasible solution only if the depreciation Δ is zero. In other words, network maintenance costs resulting from a positive depreciation may sometimes not be fully recovered.

Similarly to the previous chapter, it is interesting to verify that the transmission price component represented by $\mu_l(t) - \nu_l(t)$ in the nodal spot prices actually recovers the transmission investments. The equation (8.18) is the same as (7.39) in the previous chapter. We have already seen in previous chapter that the resulting net revenue to the network operator at instant t is

$$\sum_{l=1}^{n_l} K_l(t)[\mu_l(t) + \nu_l(t)]$$

However, in the dynamic case, it is not trivial to see that this transmission rent covers the transmission capacity capital cost. We prove it here, in the simple case where the time value of money r is zero, and there is no depreciation on transmission capacity, i.e. $\Delta = 0$.

The cost of transmission line l capacity upgrade at an instant t is

$$\begin{aligned} \Psi_l(U_l(t), t) &= \int_0^{U_l(t)} \mathrm{M}\Psi_l(u, t)du \\ &\leq U_l(t)\,\mathrm{M}\Psi_l(U_l(t), t) \end{aligned} \tag{8.29}$$

as we assumed that $\mathrm{M}\Psi_l$ was increasing with U_l. Hence, the total investment in transmission line l capacity upgrade over the planning period can be bounded using equation (8.19) as follows:

$$\text{Tr. Investment}_l = \int_0^T \Psi_l(\hat{U}_l(t), t)dt \tag{8.30}$$

$$\text{Tr. Investment}_l \leq \int_0^T \hat{U}_l(t)\,\mathrm{M}\Psi_l(\hat{U}_l(t), t)dt \tag{8.31}$$

$$\text{Tr. Investment}_l \leq \int_0^T \hat{U}_l(t)[\xi_l(t) + \zeta_l(t) - \sigma_l(t)]dt \tag{8.32}$$

If we assume that transmission expansion was never constrained by the constraints on U_l, than the Lagrange multipliers $\zeta_l(t)$ and $\sigma_l(t)$ are null. Moreover, using (8.2) with $\delta_l = 0$ to express $\hat{U}_l$, a partial integration, and equation (8.16) we obtain

$$\text{Tr. Investment}_l \leq \int_0^T \dot{\hat{K}}_l(t)\xi_l(t)dt \tag{8.33}$$

$$\text{Tr. Investment}_l \leq K_l(T)\xi_l(T) - K_l(0)\xi_l(0) - \int_0^T \hat{K}(t)\dot{\xi}_l(t)dt \tag{8.34}$$

$$\text{Tr. Investment}_l \leq K_l(T)\xi_l(T) - K_l^0\xi_l(0) + \int_0^T \hat{K}(t)[\mu_l(t) + \nu_l(t)]dt \tag{8.35}$$

The last term of (8.35) is the transmission rent for line l received by the system operator. Using the transversality condition (8.17), it can be seen that the first term of (8.35) is null unless the minimum target on transmission expansion is binding. The second term of (8.35) represents the heritage of the past, i.e. stranded investment or insufficient investment. This proves that, at the optimum, the total investment in transmission capacity is covered by the transmission rent received by the network operator.

It is important to recall here all the simplifying assumptions underlying that proof of transmission investment recovery, and their physical meaning:

- $r = 0$, in other terms, the investments and revenues have the same value, whatever the time. For positive discount rates, investments do not have the same cost throughout time, but the spot prices and revenues will be adjusted as well, so this assumption is not limiting.

- $\Delta = 0$. In case of line depreciation, capacity must be maintained by a costly maintenance. The optimal spot prices do not provide any explicit revenue for this.

- $\zeta_l(t) = 0$ and $\sigma_l(t) = 0$. If the transmission expansion is constrained for environmental of technical reasons, the transmission capacity may be non optimal, and the optimal spot prices may lead to insufficient investment recovery, or to excessive recovery.

- The transversality condition (8.17) may be binding, and this has already been described as an excessive minimum target imposed by the regulator, that could lead to non optimal investment capacity.

- Finally, this model is deterministic, and optimal planning relies on exact knowledge of future generation and demand patterns at every bus. This is obviously never the case in real life, where the investment planner has to make do with imperfect forecasts or a stochastic approach. Keeping this in mind, we still can obtain interesting results from our coarse model.

Finally, even in the case of unconstrained optimal expansion the inequality (8.35) may be strict, meaning that the network operator may receive more than the exact investment cost. This is due to the increasing investment marginal cost $M\Psi$, and the fact that the rent is collected using the highest marginal cost incurred. The excess transmission rent received by the system operator is nothing else than a classical producer surplus, and is not a departure from optimal welfare. Hence, there can be different reasons for over recovery: producer surplus, and non optimal transmission expansion that results in non optimal spot pricing. Whereas the former is viewed as legitimate by economists, the latter may result in excessive recovery or insufficient recovery of capital cost. This distinction is hard to make in the real world, and may lead to many disputes among competitive players, system operator and regulator.

8.4 Decoupling of investment and operations

By inspection of problem (8.6)–(8.13), it can be noticed that the control variables $\mathbf{P}$ and $\mathbf{U}$ are neither contained in the same function, e.g. C or Ψ, nor in the same constraints. Hence, we can separate this dynamic optimization problem into two subproblems. This decouples the problem of optimal transmission investment and economic dispatch. This decoupling has very important consequences on the pricing of transmission services.

The first of these subproblems is the economic dispatch and optimal spot pricing

at instant t. It is formulated as follows:

$$\max_{\mathbf{P}(t)} \mathcal{F}(\mathbf{K},t) = -\sum_{i=1}^{n} C_i(P_i,t) \tag{8.36}$$

subject to

$$-K_l^0 - K_l(t) \leq T_l(t) = \sum_{i=1}^{n} H_{li}P_i(t) \leq K_l^0 + K_l(t) \quad \forall l \tag{8.37}$$

$$P_i^{min}(t) \leq P_i(t) \leq P_i^{max}(t) \quad \forall i \tag{8.38}$$

$$\tag{8.39}$$

This is the same as problem (5.10)-(5.12) already described in Chapter 4, if we neglect the transmission losses, and $\mathcal{F}(\mathbf{K},t)$ represents the maximum social welfare as a function of transmission capacity and nodal demand at instant t.

The second problem is the transmission expansion planning, and it is formulated as follows:

$$\max_{\mathbf{U}(t)} \mathcal{J} = \int_0^T e^{-rt}\left[\mathcal{F}(\mathbf{K}(t)t) - \Psi(\mathbf{U}(t),t)\right] dt \tag{8.40}$$

subject to

$$\dot{\mathbf{K}}(t) = -\Delta[\mathbf{K}^0 + \mathbf{K}(t)] + \mathbf{U}(t) \quad \forall t \tag{8.41}$$

$$0 \leq \mathbf{U}(t) \leq \mathbf{U}^{max}(t) \quad \forall t \tag{8.42}$$

$$\mathbf{K}(0) = 0 \tag{8.43}$$

$$\mathbf{K}(T) \geq \mathbf{K}^T \tag{8.44}$$

The proof of this decompositition is straightforward: firstly, the Lagrangian of the first problem (8.36)-(8.39) is the part of the Lagrangian (8.15) referring to the control variable $\mathbf{P}$. Hence, both problems imply the same optimization with respect to $\mathbf{P}$, and yield the same optimal solution $\hat{\mathbf{P}}(t)$. Secondly, the Hamiltonian and the Lagrangian of the second problem problem (8.40)-(8.44) are the same as the

parts of (8.14) and (8.15) referring to variable $\mathbf{U}$ only. Hence, they lead to the same dynamic optimization, the same solution $\hat{\mathbf{U}}(t)$ and the same optimal network capacity trajectory $\hat{\mathbf{K}}(t)$.

This result means that the complete problem of network expansion planning and economic dispatch decouples into a dynamic investment problem and a series of static economic dispatchs. Moreover, we have seen that, at each instant t, the first problem is the same as the static economic dispatch described in Chapter 4. That problem was solved by using decentralized generation and consumption decisions based on short run marginal costs (SRMC).

The dynamic expansion problem balances the instantaneous welfare returns from generation dispatch and the cost of transmission capacity maintenance and upgrade. Hence, the static and the dynamic approaches of peak load pricing are structurally the same, except that the dynamic approach requires prior determination of the optimal capacity by equating the marginal cost of expansion and the marginal benefit, as specified by (8.27). Moreover we have just proven that, for an optimal investment, SRMC is equal to LRMC.

This latter result is very important for policy purposes: with an optimal network, pricing at SRMC is like pricing at LRMC. Hence the use of spot prices set to SRMC as described in Chapter 4 leads to transmission investment cost recovery, as proved in the previous section. However, the previous computations show all the hardship involved in expansion planning, i.e. in computing LRMC. Moreover, we have seen several cases where optimal network expansion was not possible for regulatory, environmental or technical reasons. In these cases, *SRMC will not equal LRMC*, and additional provision will be required in transmission pricing to ensure capital cost recovery. We will see in the next chapter how this can be done.

Chapter 9

Policy recommendations: transmission pricing and regulation

It is now time to summarize our approach thus far. We sought to derive a pricing mechanism reflecting all the technical and economical characteristics of the power system, hence conveying the right economic signals to reach social optimality. Social optimality is measured in terms of social welfare, i.e. the benefit of energy consumption minus the fixed and variable costs of supply. We have reviewed in Chapter 6 a proposal for wheeling rates based on the difference in spot prices at injection and removal nodes. The optimal spot prices necessary for this approach can be computed in two different manners.

In a first step, social welfare optimization was carried out using the formulation of economic dispatch, to model the outcome of a short-run competitive energy market. This leads to the derivation of optimal nodal spot prices, as presented in [Schw88]. This short run optimization reflects optimally two of the three cost components that matter in transmission pricing: transmission losses and line congestion. But spot prices do not take into account the dynamic constraints related to unit-commitment and mid-term generation scheduling that could give market power to some generators. More importantly spot prices do not convey any relevant signals for investment in

the transmission network.

Therefore, in a second step, we used a peak-load approach, and social welfare optimization was carried out as a problem of long run network expansion optimization. This lead us to the derivation of long run marginal costs that have the same expression as spot prices, with congestion rents adjusted by the capacity investment to match exactly the investment cost. Moreover, we have proven that the problems of optimal expansion and optimal operation decouple. Once an optimal network investment plan has been found, short run marginal cost equals long run marginal cost, and the use of the previously derived spot prices should meet the third objective of embedded capital cost recovery.

However, these results depend on ambitious conclusions, and should be reviewed carefully from the policy maker's point of view, using the list of objectives that should be met by a transmission pricing mechanism presented in Chapter 4. Can we reach the state of optimal network expansion ? Does this criteria of social welfare maximization match currently announced investment policies, notably in the Western European interconnection ? Finally, to which extent is the spot price-based transmission pricing mechanism practically implementable ?

9.1 Optimal capital cost recovery

The assumptions that enabled us to prove that, at the optimum, short run marginal cost is equal to long run marginal cost are very ambitious.

9.1.1 Modeling assumptions

It is not necessary here to dwell on the modeling assumptions: we have used a linear model, with continuous expansion of capacity and demand certainty over the whole planning area.

In a real power system, the expansion would be discrete, and line impedance and capacity changing simultaneously, the problem to solve would be non linear, may be not even convex. But these are not the critical restrictions, and our problem

formulation could be easily adapted.

Similarly, the deterministic formulation presented here could be modified into a probabilistic approach, for example with a stochastic model of demand at each node over the planning horizon. Instead of maximizing social welfare, one would maximize its expectation over the time horizon, and similar results on the expected average spot prices could be obtained.

9.1.2 Environmental constraints

We have pointed out that one of the conditions to reach an optimal network was the absence of constraints to the development of new lines. This condition is likely to not be fulfilled in real life. Indeed, even though we considered expansion in existing right of ways, the environmental pressures against the construction of new lines, or the doubling of existing lines are very strong nowadays. And it is very difficult for European utilities and political authorities to argue that, although demand in energy is staggering and despite the resulting excess-capacity, new transmission lines should be built to improve social welfare.

Hence, the future expansion of the transmission network may rely heavily on the emerging technology of FACTS. The study of these new devices, and their optimal use would be the object of another complete thesis. Suffice it to say here that FACTS, like new transmission lines in environmentally constrained areas, are *expensive* investments. Moreover, it is still unclear how the use of FACTS would yield expanded transmission capacity on certain lines without increasing the risk of instability over the complete system. Hence, one should be wary of FACTS miracles, and more systematic work on the fundamental limitations to FACTS effects is needed [Ilic96].

9.1.3 Optimum transmission network

It is crucial for a policy maker to understand what the "optimal network" actually is. Firstly, we recall that, from the welfare maximizer's point of view, it is optimal to invest in additional transmission capacity up to the point where marginal benefit

from investment is equal to marginal cost. This means that, as transmission capacity becomes more expensive, due to external reasons, while the benefits from cheaper generation remain unchanged, the optimal MW capacity expansion required to reach the equilibrium diminishes. This is no surprise, and just means that the environmental cost may be included in the capacity cost function, and should not prevent us from reaching an optimal network.

However, we insist on the fact that, always from the social welfare maximizer's point of view,

AN OPTIMAL NETWORK IS A NETWORK WITH ENOUGH CONGESTION FOR THE TRANSMISSION RENTS TO EXACTLY COVER THE INVESTMENT COST.

In other words, at the equilibrium, the congestions are solved by out of merit generation. Hence, the marginal costs of generation, or the spot prices of delivered energy vary across the network, and the resulting excess revenue collected by the GridCo covers its investment cost.

For example, this can be seen on the example with two generators in Chapter 7. In the absence of generating constraints, or under a perfectly competitive generation industry, if the network were never congested, all the power would be served by generator 1, at the constant marginal cost of 100 $\$/MW$. The investment in a network capable of transferring only $100MW$ between generator 1 and the load, and $100MW$ between generator 2 and the load is optimal in that the use of more expensive but closer generation for peaking and the resulting congestion rents exactly recover the transmission investment.

This concept of optimal network somehow conflicts with the current planning practice of utilities. Very often, expansion planning is carried out firstly by determining the investment in new generation capacities, whose location may be determined by land or environmental constraints, than by searching the transmission investments that will enable to carry electric power from generators to load centers. Very often, this means that the planned network is not congested at all in the expected operating conditions. For example, we recall that, in the unconstrained expansion scenario,

EDF is planning a totally uncongested network at horizon 2010 (see the numbers in Chapter 4. Naturally, congestion may arise from an important change in the operating conditions, such as the unexpected massive import or export of electricity. The ongoing deregulation may cause this change, and the assessment of network optimality will need to be revised anyway.

Along the same ideas, the European Commissioners plan accompanying measures to implement the directive on TPA for eligible customers and the European electricity internal market. They notably recommend the construction of new cross boundary tie-lines to ease international competition [Kind93]. Whether these lines will be accepted by local population is far from granted. This authoritarian policy of transmission expansion may be good to lower the price of electricity in certain regions, but is does not result a joined optimization approach considering the generation-transmission trade-off.

9.1.4 Political choices

Political and ideological forces tend to decouple the problem of transmission expansion and economic dispatch, but not in the "optimal" way presented in the previous chapter. A bundled social welfare maximizing approach is therefore increasingly irrelevant, as technocratic decisions give way to political choices. This should not be considered as a move towards a sub-optimal situation, as technical centralized decisions implemented by most of the former regulated utilities did not follow welfare maximization anyway.

Therefore, we consider that the concept of optimal network presented here is a very interesting benchmark, but does not represent a practical basis for transmission access pricing. Consequently, we consider that one should not argue of economic principles such as short run/long run marginal pricing or peak load pricing in the context of capital cost recovery and network expansion. The policy maker should rather aim at simplicity and transparency, and the decoupling of transmission investment decisions and economic dispatch could take many original forms, none of which is dictated by strict theoretical efficiency:

- Public investment in a powerful European interconnection network, that relieves most of the tie-line congestions. This solution was used by the American Federal Government for the construction of the Interstate highway system in the late 1940's, early 1950's, and is currently advocated by some european political leaders in Europe. A policy of large infrastructure construction would include highways, railways, but could also develop stronger electric transmission lines across Europe.

- Public acquisition at depreciated value of the existing transmission equipment to solve the allocation of sunk investment costs, and creation of a public carrier.

- Creation of a common-carrier fund, privately or publicly operated, paid by access rates, to fund investment in transmission capacity upgrade.

The choice of the retained solution, as well as its actual implementation involve real political debates, that can not be solved by sheer arguments of optimality. In particular, the issue of priority of certain classes of users over others, e.g. the former native demand, and the imperatives of simplicity and transparency can strongly orient the choices.

9.2 Multi-component transmission prices

We propose here a transmission pricing mechanism taking the form of a two-part tariff.

9.2.1 Fixed entry fee

It is very likely that the current generation price discrepancies, and the relative smaller share of transmission services in the final cost of electricity will justify to invest in an over dimensioned network. Once this has been decided, the capital cost recovery involves no time-dependency, and one should not consider peak-load pricing mechanism anymore. Hence, a fixed entry fee such as postage-stamp pricing, or a simple

allocation key, such a annual MWh-miles, could be used to recover capital costs of the transmission network.

Once again, we recall that this fee should just enable network expansion, and does not include the other major two components of transmission cost, namely line losses, and possible congestion relieving by generation redispatch.

We consider that using a fixed annual fee, or transaction fee is more flexible and readable than the Ramsey pricing approach. Indeed, multiplying short-run marginal prices by a correcting factor to recover transmission investment has the disadvantage that the exact factor can only be computed *ex ante*. This approach has only the appearance of optimality, and in fact only yields a second best. And by doing this, a lot of simplicity is lost, and spot price can not be used symmetrically for sale or purchase of power.

The administrative overhead and all the operating costs should be bundled in the entry fee, notably the cost of maintenance, ancillary services except transmission losses, etc. This will preserve the symmetry of spot prices, that can either be paid by the GridCo to generators or cogenerators, or collected from power consumers. Hence, power generated or consumed at a given node would have exactly the same (wholesale) price. The allocation issue involved in transmission access rate would only be applied on this fee, and could be stated with much transparency, as will be seen below.

9.2.2 Nodal spot-price based component

As seen in Chapter 5 the short-run marginal cost of energy computed at each node reflects all the information necessary to reach short-run optimality, notably the influence of losses and transmission constraints. This is also the theoretical outcome of a competitive market of electricity. This media should be used to organize the system-wide competitive supply of energy. We have not made any assumption on the level of aggregation of our model of the network, but it is very likely that at most one generator will be present at each node. Hence, it is not reasonable to think of a competitive market at each node.

However, the sensitivity coefficients $\mathbf{B_i}$ and H_{li} that appear in equation (5.20) enable to qualify or penalize the bids of any generator, according to its electrical position on the network. The same can be done for calls for energy from industrial users or distributors. This enables to organize a competitive energy marketplace, where bids and calls would clear to yield a single spot price. This spot price represents the system lambda, or system marginal cost of generation, and (5.20) can be used to retrieve nodal spot price at each node.

Finally, the usage based component of transmission rates would be the difference of nodal spot prices. This transmission rate can be broken down into the cost payable to each wheeling utility, by using the tie-line flow coefficients as presented in Chapter 6.

9.2.3 Implementation

Two questions remain to be answered:

- Is this feasible ?
- Who does it ?

The practical feasibility relies on the availability of information on the network parameters, to compute the sensitivity coefficients. The network parameters relevant to use the DC load flow model, i.e. line resistances and impedances, are currently publicly published in the European interconnection. In the United States, the FERC has just released an order to require the publication of these data. Hence, there are no institutional obstacles to organize a unique energy marketplace in these interconnected systems.

The real issue is the identity of the market supervisor. This entity is responsible for collecting bids and calls for energy, one day in advance, and for each hour of the following day. Using these bids and the sensitivity correction, the spot price can be determined for each hour. Then, the nodal spot prices are conveyed to all the market players, who will base their decentralized decisions on the announced price...

...the process that has just been described is nothing else than a centralized economic dispatch, where all the authority over the system is in the hands of the market

supervisor, that should actually be called the central dispatch. However, system operations are more complex than just an economic dispatch, and the multiple players, i.e. public and private utilities, distribution companies, industrial users, will not accept easily such a central authority. Indeed, the system of Third Party Access has be postponed in Europe, because no member state wants a European central dispatch, because it would change too radically the current operational practice.

However, we recall that we did not make any assumption on the level of aggregation of the network in our model. Hence, it should be possible to implement a hierarchical energy marketplace, where inter-utility exchanges are planned in a coordinated manner by the central dispatch, each utility being responsible for the inter-utility economic dispatch and operations. The theoretical basis of this hierarchical practice has been lain in [Ilic96].

In short, for the central dispatch, each utility should be represented by a small number of nodes and by its tie-lines with neighboring utilities, and the central dispatch sold use the relevant aggregated network parameters. Each utility is responsible for collecting the requests of all its internal actors, and aggregates them suitably in terms of bids and calls, by applying the method of penalization described above for its internal network. At the central level, the market clearing price is computed, as well as the tie-line flows on the aggregated network, for each hour, one day in advance. Then, each utility can internally break down the aggregated system conditions, either by performing an economic dispatch taking into account the scheduled tie-line flows, or by organizing a local energy marketplace, or just by computing more refined nodal spot prices to obtain decentralized decision based on prices.

Many details of the hierarchical organization of an interconnected power system remain to be studied. However, one should note here that the purpose of this hierarchical organization is not to centralize all the technical decisions at all the possible time frame. The goal is only to allow competitive exchanges of real power among utilities to take place. These exchanges will reap much of the gains possible from the current situation of excess generation capacity, and the DC load flow formulation used here is the simplest possible relevant model of the network. Any other technical

issue, such as voltage profile, reactive power generation, spinning reserve are not economically significant and should be handled at the utility level, according to current practice.

Finally, reliability issues should rely on the current cooperative practice. Note that one of the main effects of reliability and stability considerations is the choice of the line capacity K_l on each line. Transmission capacity has been considered as a constraint in all the optimization problems solved in this thesis, assuming that K_l was known a priori. But a great deal of information and judgment is contained in these numbers, as explained in [IliA96].

9.3 Transmission capacity rights

9.3.1 Price variability

The transmission prices computed according to our proposal vary significantly, both in time and location. The time variation of spot prices reflect the fluctuation of operating conditions, due to the variability of demand. But these variations tend to be fairly periodical, as domestic consumption follows daily, weekly and yearly patterns. Hence, the system operator, as well as the regulator should be used to this variability and it should not be considered as an excessively confusing aspect of our transmission rates. Moreover, it is always possible to average these spot prices into bi-daily values: peak and off-peak, and also any coarser or more refined representation.

The space fluctuation of spot prices reflect the advantages or drawbacks of the electric location of each node, according to the publicly available sensitivity matrices $\mathbf{B}$ and $\mathbf{H}$. Once again, this variability should not pose too many problems of information asymmetry. Any bilateral contract is charged a rate that is independent of the path followed by the energy, and depends only on the injection and removal buses.

9.3.2 Contract network

Investment in new generation units are amortized over periods often exceeding 20 years. They require some guarantees in terms of potential sales, and more stability than given by spot prices. Hence, fluctuations in spot prices will be financially hedged using long term contracts between the generator and distributors or industrial users.

However, the ability to perform the transaction depends on the transmission capacity between the two nodes of interest, and this capacity is highly conditions dependent. Hence, in addition to the long term sales contracts, investors in generation need guaranteed transmission capacity rights. These transmission capacity rights can take mostly two forms, as presented in [Wu94]: Firm transmission capacity rights, and virtual transmission capacity rights. The holder of the former is entitled to transfer power between the two specified nodes, whatever the network operating conditions. This may conflict with the economic dispatch of the rest of the transactions, and in some cases, it might be socially preferable to compensate the capacity right holder rather than let him perform.

Virtual capacity rights are basically the right to transfer power between two nodes, of to be compensated for non performance. In [Hoga92] W.W. Hogan describes how these virtual transmission capacity rights could be set up, based on the theory of optimal spot prices. We recall from Chapter 5 that the transmission access rates, or wheeling rate between two nodes takes the form:

$$T_{ij}(t) = \rho_j(t) - \rho_i(t) = T_{ij}^{Loss}(t) + T_{ij}^{Cong}(t) \tag{9.1}$$

where

$$T_{ij}^{Loss}(t) = 2\lambda(t)(\mathbf{B}_j - \mathbf{B}_i)\mathbf{P}(t) \tag{9.2}$$

and

$$T_{ij}^{Cong}(t) = \sum_{l=1}^{n_l}(H_{lj} - H_{li})(\nu_l(t) - \mu l(t)) \tag{9.3}$$

and all the users of the transmission network pay this rate. In his paper, Hogan explain that the right holder "receives a 'rental' payment from the grid, equal to T_{ij}^{Cong},

the congestion charge, applied to their full capacity right." This means that the capacity right holder pays only the loss charge if he can fully exercise his right, and otherwise, if network conditions prevent full exercise of the capacity right, he is compensated by the exact amount necessary to purchase power from another generator to perform the long term transaction.

The main feature of these capacity rights is that the GridCo refunds the congestion rents. This is another serious reason for establishing a fixed fee to recover transmission investment costs. If transmission capacity rights between any pair of nodes on the network are owned other entities, the GridCo receives only the loss charge, but needs to use it for loss compensation. Hence, the spot price based component of our transmission pricing mechanism generates no net revenues for the GridCo, explaining why every transmission cost other than loss compensation and out of merit dispatch must be included in the fixed fee.

Finally, it must be noted that an investor should build a power plant if he owns enough capacity rights originating from the node of the project to evacuate all the power output. The investment is decided by making sure that the discounted net revenues *of power actually sold*, i.e. revenue of power sold through the long term contracts minus fuel costs, cover the capital cost, and the expected labor costs (that are fixed at the economic dispatch time frame). This net revenue is also the difference between spot price at the destination nodes and spot price at the supply node, once losses are compensated, i.e. T_{ij}^{Cong}. The long term sale contract may specify permanent supply, and if network conditions prevent the generator from performance, the congestion rent will enable him to purchase power from other generator to honor the contract anyway.

9.3.3 Allocation of capacity rights

The contract network definition of capacity rights requires some allocation of those rights. The issue is mostly the initial allocation, as the rights will possibly be traded on a secondary market afterwards. Another allocation issue is to compute the entry fee to cover transmission investment cost. These two allocation issues are highly

political, and involve value judgment rather than optimality considerations.

One possible initial allocation of capacity rights is to give enough rights to the existing utility that serves the native load of the area. This would be justified by the fact that deregulation should not disadvantage captive users that often have paid much of the existing network through their electricity bills. The extra capacity available could be given to Non Utility Generators, or to IPPs, for example by an auction mechanism. The same could be made with the possible added capacity from line upgrade.

The issue of allocation involved in the fixed fee is slightly more technical. The fixed fee is not aimed at driving the users towards an optimum equilibrium, so any allocation is possible. A usage sensitive mechanism is possible, such as using a MWh-mile allocation key. However, once again, if one deals with the existing investment, one should make sure that existing utility's captive customers do not pay twice an infrastructure. Similarly, why should they pay for line upgrades, if they continue to be served by their utility, and do not get any benefit from network expansion.

Chapter 10

Conclusion

10.1 Electric utility deregulation

The electric power supply industry is characterized by the importance of capital investments necessary to produce and transfer electricity. These large fixed costs generally have meant that average unit cost was below marginal cost. Moreover, economies of scale could be obtained from centralized operations. These technical and economic characteristics, combined with the social and political importance of electricity in modern societies, explain the traditional organization of electric utilities: vertically integrated monopolies serving one given area, regulated or publicly owned. The electric industry was strongly geographically segmented, any utility hardly competing with its neighbors for generation or distribution.

This horizontal segmentation resulted in a great variation of the prices of electricity across utilities or states. This has aroused the interest of industrial users and municipal distributors. Moreover, since the oil shocks of the 1970's, due to over estimation of demand growth of better energy consumption (demand side management), many utilities now have an overall excess generation capacity. Finally, new generation technologies such as combined cycle gas turbines allow relatively small facilities to reach excellent yields, and the development of renewable energies (wind, sun, geothermal) also favors smaller sized units.

Over capacity, price variations and greater interest for small scale generation are

three of the main factors that advocate for the introduction of more competition in the electricity generation industry. Such competition requires some unbundling of the three levels of the electric utility industry: generation, transmission and distribution, and advocates against monopolies of imports or exports of electricity. This unbundling can be reached through several scenarios, such as either a complete deregulation and the divestiture of electric utility assets, like in Wales and England, or the mere adoption of clearer accounting rules, and allowing bilateral transactions to use the transmission network.

In Western Europe, the gradual approach favored by the European Commission is a cautious application of the second scenario, where power exchanges would be allowed for eligible consumers, notably large industrial users and distribution companies. Note that important cross boundary energy exchanges between adjacent utilities already exist, and the introduction of these bilateral contracts, as well as power exchanges among non adjacent utilities mean that the transmission network is going to be partly used by third parties. Consequently, one of the major challenges of deregulation, and the main focus of this thesis, is the establishment of suitable access rates to the transmission network.

10.2 Transmission access rates

An access rate mechanism must respect the technical imperatives of power system operations, notably the fact that the system operator must permanently maintain power balance, and faces a time varying demand. System reliability must be insured and the physical properties of power systems, notably the loop flows that prevent anyone from "directing" power flows along given lines rather than others must be recognized. Simultaneously, an access rate mechanism must meet the objective of economic efficiency. In other words, it must reflect the actual variation of costs and enable the wheeling utility to recover all the incurred costs. The main components of transmission costs are the capital cost of infrastructure, the compensation of transmission losses and the cost of out-of-merit generation to accommodate possible con-

gestion. Finally, a transmission pricing mechanism must meet regulatory imperatives of transparency and equity.

In a first step, we have reviewed the proposal of Schweppe, Caramanis and Bohn in [Cara86, Schw88] that provides a unified engineering basis for wheeling rates based on nodal spot prices of electricity. In particular, SRMC and the settlement mechanism associated with spot markets could be organized to achieve a static optimality reflecting two of the three cost components listed above: transmission losses and redispatching costs.

To propose a mechanism that recognizes the dynamic evolution of the network, and provides the right incentives for expansion and enhancement thereof, a peak-load approach has been followed in a second step. This approach is derived from the work of Crew and Kleindorfer on peak-load pricing of power generation [Crew79]. A DC model of the transmission network was added, to take capital cost of transmission capacity explicitly into account. This lead to a dynamic formulation of optimal investment planning under spot pricing of electricity, similar to some extend to [Cara81], and the computation of LRMC. Two important results were proven:

- The problem of network expansion decouples into a problem of investment planning and a series of economic dispatches based on SRMC.
- *Under optimal investment*, SRMC equals LRMC, and the congestion rents collected by the system operator exactly compensate for fixed capital cost.

However, the difficulties of computing LRMC and optimal expansion planning under demand uncertainty and environmental constraints were also highlighted.

We consider that, in the real world, environmental constraints and political decisions are the main determinants of investment planning. For example, it is possible that European authorities decide massive investment in cross boundary tie-lines, under the *accompanying measures* of deregulation, to ensure that lack of infrastructure does not prevent real competition among generators. Therefore, it seems that the issues of capital cost recovery and optimal network expansion should be handled separately from short run social welfare maximization.

Following this analysis, we proposed a two-part tariff mechanism for transmission pricing: on the one hand, a fixed fee, based on an allocation key to be chosen democraticly, should provide capital cost recovery and pay for interconnected operations ancillary services that cannot easily be broken down. On the other hand, and a usage based variable rate would be computed using SRMC to send the relevant short-run economic signals about system operating conditions.

The organization of continental energy exchanges to reach static optimality at the interconnected system level should be based on the computation of nodal spot prices. Network parameters could be used to penalize generation fuel costs, obtained from a bidding mechanism, according to the electric location of generators and the operating conditions. We advocate an approach based on the DC load flow equations, as it should be implementable as soon as network parameters are publicly available, and is the simplest model that actually reflects the physical reality of power systems. A hierarchical approach, similar to the European concept of subsidiarity, could ease the practical implementation: coordination of inter-utility exchanges at the central level, and economic dispatch or bilateral transactions accommodation at the utility level.

10.3 Further research required

First of all, the modeling assumptions used in this thesis should be validated using empirical data. Moreover, the formulation derived here could be adapted to get a probabilistic, non linear, discrete time model. The robustness of our model to market power of generators should also be tested: a game theoretic approach could be adopted for the design of optimal regulation taking into account the problems of information asymmetry.

The complex topic of transmission capacity is at the core of this thesis and of transmission pricing issues. No well defined standards exist for defining, modeling and enhancing transmission capacity [IliA96]. Further research on that subject is performed in many places and is of fundamental value for policy makers.

Finally, we have underlined in this thesis that, while the concept of transmission capacity relies on technical decisions, the allocation of existing capacity will require real political choices, just as the allocation of capital cost among the network users to compute the fixed entry fee. One of the interests of this fixed fee is indeed to highlight the need for an allocative key, instead of hiding allocative decisions behind pseudo-optimality. Along the same ideas, transmission capacity should be considered as a *common good* whose allocation does not depend merely on technical optimality. Democratic decision making and value judgments should be involved here.

Appendix A

Electricity prices

A.1 Prices in Western European utilities

Table A.1 below gives the electricity prices for a typical residential user and a typical industrial user in Western European countries. These prices are given for January 1995 in ECU / 100 kWh (1 ECU 95 = 1.241 US $).

Country (city)	Household use	Industrial use
Belgium	14.99	7.76
Denmark	14.72	4.97
Germany (Dusseldorf)	16.72	10.85
Germany (Hamburg)	17.59	11.14
Germany (Munich)	18.01	9.35
Germany (Rostock)	15.21	9.29
Spain (Madrid)	12.26	7.31
France	13.17	6.50
Ireland (Dublin)	8.26	6.29
Italy ("National")	19.81	8.62
Luxemburg	11.30	7.65
Netherlands (Rotterdam)	10.97	6.99
Portugal (Lisboa)	13.19	7.98
England (London)	10.40	6.00
Mediane	12.71	7.15

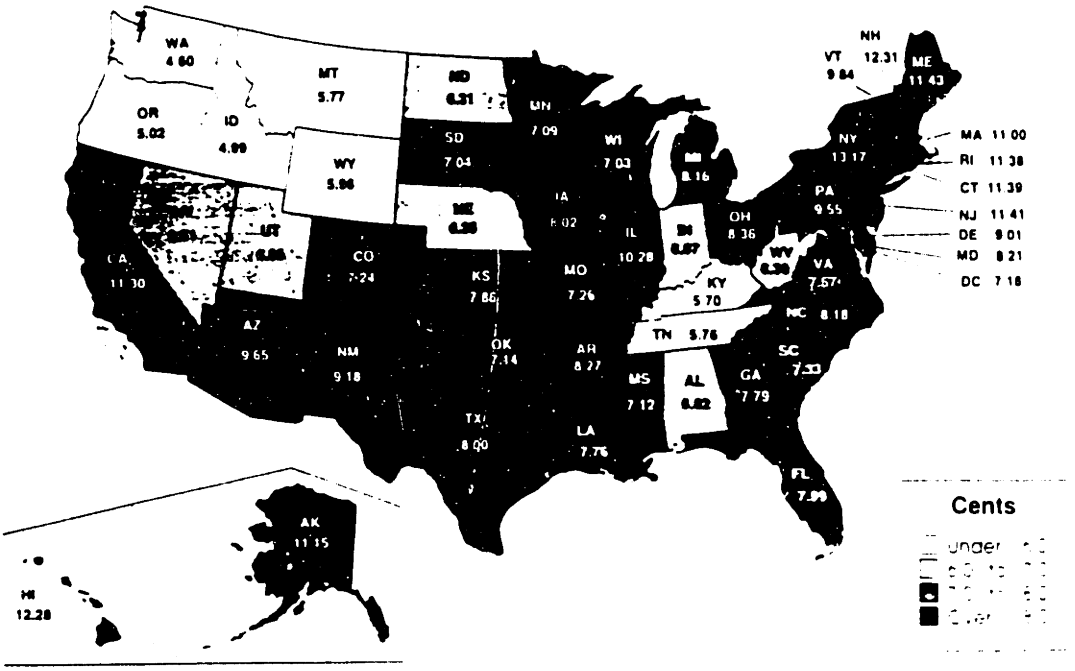
Table A.1: Electricity prices in Western Europe. Source: [Euro95]

The column "Household use" gives the electricity prices for a standard residential customer that has a annual consumption of 3500 kWh, of which 1300 kWh are consumed during night. The electricity is supplied at low voltage (110 V or 220 V) and for a subscribed demand of 4–9 kW. The given prices include value added tax (VAT). The VAT rates vary between 15% and 25% of the price excluding VAT in the majority of the countries listed, but Italy (9% for domestic use) Portugal (5%) and Luxembourg (6%).

The column "Industrial use" gives the electricity prices for a standard industrial customer that has a annual consumption of 2 million kWh. The electricity is supplied at a voltage varying between 10 and 20 kV depending on the country. The maximum demand (maximum offtake in any quarter of an hour recorded in one year) is 500 kW, and the annual load factor (number of hours to reach the annual consumption if the demand level were permanently equal to maximum demand) is 4000 hours. The industrial prices of supply are calculated for $\cos\varphi = 0.9$ and are given excluding VAT.

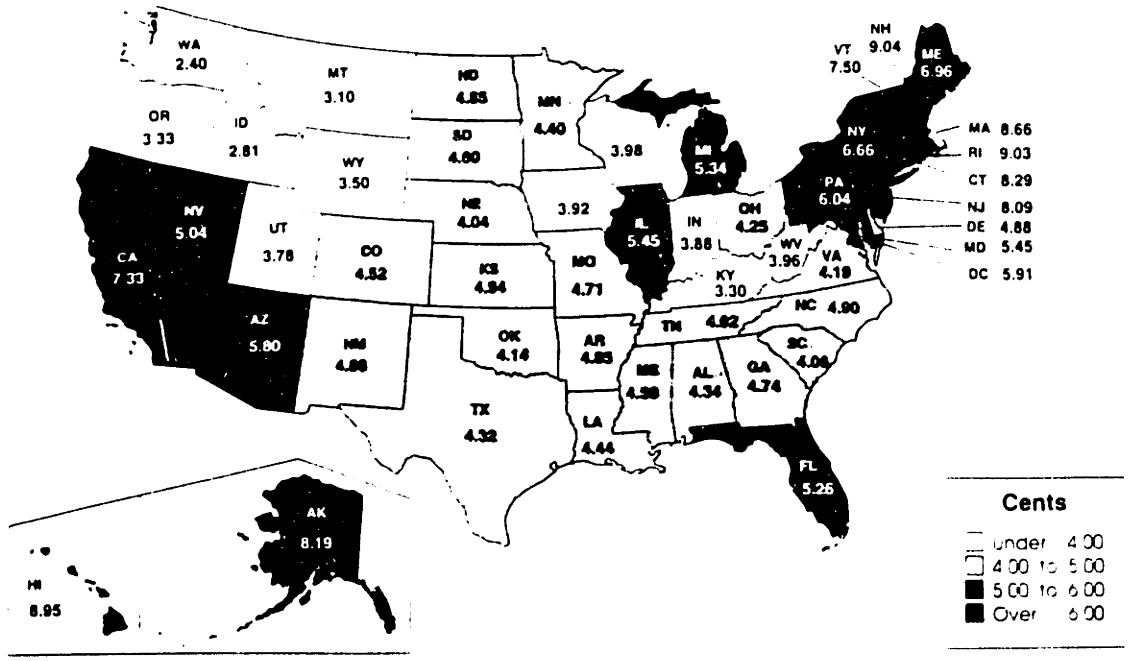
A.2 Prices in American utilities

The following two maps give the average revenue per kilowatthour for American utilities. The prices are presented by state, for 1993. For the whole Union, the residential average price is 8.32 cents/kWh and the industrial average price is 4.85 cents/kWh.



KWh = Kilowatthour
Note: The average revenue per Kilowatthour of electricity sold is calculated by dividing revenue by sales
Source: Energy Information Administration, Form EIA-861, "Annual Electric Utility Report."

Figure A-1: Average revenue per kWh for the residential sector by state.



KWh = Kilowatthour.
Note: The average revenue per Kilowatthour of electricity sold is calculated by dividing revenue by sales
Source: Energy Information Administration, Form EIA-861, "Annual Electric Utility Report."

Figure A-2: Average revenue per kWh for the industrial sector by state.

Appendix B

DC-load flow equations

This appendix recalls the basic load flow equations for a transmission line, as well as the AC and DC load flow equations for a power network. We use the notations presented in chapter 3.

B.1 Load flow equations for a transmission line

The real and reactive power sent into a line ij can be combined into the complex valued power that has following expression:

$$\hat{S}_{ij} = P_{ij} + jQ_{ij} = \hat{V}_i \hat{I}_{ij}^* = \hat{V}_i \hat{y}_{ij}^* (\hat{V}_i^* - \hat{V}_j^*) \tag{B.1}$$

where * denotes the complex conjugate and $\hat{I}_{ij}$ is the current flowing in line ij. One obtains the "trigonometric" expression of the real and reactive power sent into line ij by taking the real and imaginary part of (B.1). This yields the following expression for the real power transfer from i to j:

$$T_{ij} = P_{ij} = g_{ij}[V_i^2 - V_i V_j \cos(\theta_i - \theta_j)] + b_{ij} V_i V_j \sin(\theta_i - \theta_j) \tag{B.2}$$

For the typical transmission line, we have

$$g_{ij} \ll b_{ij} \tag{B.3}$$

Moreover, if we assume that operating conditions do not differ much from nominal operating point, in a per unit analysis, we have

$$\theta_i - \theta_j \ll 1 \tag{B.4}$$

$$V_i \simeq V_j \simeq 1\ p.u. \tag{B.5}$$

Then, (B.2) reduces to

$$T_{ij} \simeq b_{ij} V_i V_j \sin(\theta_i - \theta_j) \simeq b_{ij}(\theta_i - \theta_j) \tag{B.6}$$

which is the DC load flow equation for one line.

The real power transmission loss over line ij is the difference between the power P_{ij} sent in i and the power $-P_{ji}$ received in j. Therefore, using (B.2), L_{ij} takes the form

$$L_{ij} = P_{ij} + P_{ji} = g_{ij}[V_i^2 + V_j^2 - 2V_iV_j\cos(\theta_i - \theta_j)] \tag{B.7}$$

The DC load flow equation (B.6) corresponds to a lossless line. However, one can simplify directly (B.7) by using (B.4) and (B.5). This yields the approximation

$$L_{ij} \simeq g_{ij}(\theta_i - \theta_j)^2 \tag{B.8}$$

Using (B.6) to express the angle difference, we get

$$L_{ij} = \frac{g_{ij}}{b_{ij}^2} T_{ij}^2 = \frac{r_{ij}(r_{ij}^2 + x_{ij}^2)}{x_{ij}^2} T_{ij}^2 \tag{B.9}$$

Finally, as (B.3) is equivalent to $r_{ij} \ll x_{ij}$, we obtain the following expression for line losses as a function of power transfer in the DC approximation

$$L_{ij} = r_{ij} T_{ij}^2 \tag{B.10}$$

B.2 AC load flow equations for a network

In a general network, each line ij is conventionally oriented in the direction $i \rightarrow j$ if $i < j$. The line can also be denoted by its index $l \in \{1 \ldots n_l\}$. The power balance at each node i requires that the net power injection is equal to the sum of the powers sent into the lines leaving node i minus the sum of the powers received from lines arriving in node i.

$$\hat{S}_i = \sum_{j>i} \hat{S}_{ij} - \sum_{j<i} \hat{S}_{ji} \tag{B.11}$$

We define the network incidence matrix $\mathbf{A}$ as the $n_l \times n_b$ matrix with 0, 1, -1 elements corresponding to the network interconnections. This enables to write the vector of currents flowing in the n_l lines in the form

$$\hat{\mathbf{I}} = \mathbf{diag}(\hat{\mathbf{y}})\mathbf{A}\hat{\mathbf{V}} \tag{B.12}$$

where $\mathbf{diag}(\hat{\mathbf{y}})$ is the $n_l \times n_l$ diagonal matrix of complex line admittances. Then, combining (B.1) and (B.11), we get

$$\hat{\mathbf{S}} = \mathbf{diag}(\hat{\mathbf{V}})\mathbf{A}'\mathbf{diag}(\hat{\mathbf{y}})^*\mathbf{A}\hat{\mathbf{V}}^* = \mathbf{diag}(\hat{\mathbf{V}})\hat{\mathbf{Y}}^*\hat{\mathbf{V}}^* \tag{B.13}$$

where $\hat{\mathbf{Y}} = \mathbf{A}'\mathbf{diag}(\hat{\mathbf{y}})^*\mathbf{A}$ is the network transfer admittance matrix. This matrix has the typical form

$$\hat{Y} = \begin{bmatrix} \hat{y}_{12} + \hat{y}_{13} + \hat{y}_{14} & -\hat{y}_{12} & -\hat{y}_{13} & -\hat{y}_{14} \\ -\hat{y}_{12} & \hat{y}_{12} + \hat{y}_{23} & -\hat{y}_{23} & 0 \\ -\hat{y}_{13} & -\hat{y}_{23} & \hat{y}_{13} + \hat{y}_{23} + \hat{y}_{34} & -\hat{y}_{34} \\ -\hat{y}_{14} & 0 & -\hat{y}_{34} & \hat{y}_{14} + \hat{y}_{34} \end{bmatrix} \tag{B.14}$$

and is singular.

B.3 DC load flow equations for a network

One node of the network is defined as the swing bus; its phase angle is zero, so that it is a reference for phase angles on the whole network. We define the *reduced* network incidence matrix $\tilde{\mathbf{A}}$ as the $n_l \times (n_b - 1)$ matrix with 0, 1, -1 elements obtained by removing in $\mathbf{A}$ the column corresponding to the swing bus. Using (B.6), this enables to write the vector of real power flows in the n_l lines, in the form:

$$\mathbf{T} = \mathbf{diag(b)}\tilde{\mathbf{A}}\theta \tag{B.15}$$

The real power balance at each node yields the following expression of power injections at each node, but the swing bus:

$$\mathbf{P} = \tilde{\mathbf{A}}'\mathbf{T} \tag{B.16}$$

By combining these two equations, we obtain

$$\mathbf{P} = \tilde{\mathbf{A}}'\mathbf{diag(b)}\tilde{\mathbf{A}}\theta \tag{B.17}$$

$\tilde{\mathbf{A}}'\mathbf{diag(b)}\tilde{\mathbf{A}}$ is a $(n_b - 1) \times (n_b - 1)$ invertible matrix, and solving for the vector of bus phase angles θ and substituting into (B.15) yields

$$\mathbf{T} = \mathbf{diag(b)}\tilde{\mathbf{A}}[\tilde{\mathbf{A}}'\mathbf{diag(b)}\tilde{\mathbf{A}}]^{-1}\mathbf{P} = \mathbf{HP} \tag{B.18}$$

where $\mathbf{H} = \mathbf{diag(b)}\tilde{\mathbf{A}}[\tilde{\mathbf{A}}'\mathbf{diag(b)}\tilde{\mathbf{A}}]^{-1}$ is called the transfer admittance matrix. The dimension of $\mathbf{H}$ is $n_l \times (n_b - 1)$, but one can adjoint a column of zeros to get an $n_b \times n_b$ matrix.

B.4 Network transmission losses

From (B.10), the total transmission loss over the network takes on the form

$$L = \mathbf{T}'\mathbf{diag(r)}\mathbf{T} \tag{B.19}$$

Substituting $\mathbf{T}$ from (B.18) yields

$$L = \mathbf{P}'\mathbf{B}\mathbf{P} \tag{B.20}$$

where $\mathbf{B}$ is defined as

$$\begin{aligned} \mathbf{B} &= \mathbf{H}'\mathbf{diag}(\mathbf{r})\mathbf{H} \\ &= [\tilde{\mathbf{A}}'\mathbf{diag}(\mathbf{b})\tilde{\mathbf{A}}]^{-1}\tilde{\mathbf{A}}'\mathbf{diag}(\mathbf{b})\mathbf{diag}(\mathbf{r})\mathbf{diag}(\mathbf{b})\tilde{\mathbf{A}}[\tilde{\mathbf{A}}'\mathbf{diag}(\mathbf{b})\tilde{\mathbf{A}}]^{-1} \end{aligned} \tag{B.21}$$

Appendix C

Kuhn-Tucker optimality conditions

C.1 Transportation model

The derivatives of the Lagrangian write as follows:

$$\frac{\partial \mathcal{L}}{\partial P_{Gi}^t} = -\rho_i^t + \gamma_i^t + \alpha_i^t = 0 \quad \forall i, t \tag{C.1}$$

$$\frac{\partial \mathcal{L}}{\partial P_{Dj}^t} = \pi_j^t - \lambda_j^t + \beta_j^t = 0 \quad \forall j, t \tag{C.2}$$

$$\frac{\partial \mathcal{L}}{\partial T_{ij}^t} = -\gamma_i^t + \lambda_j^t - \mu_{ij}^t + \tau_{ij}^t = 0 \quad \forall i, j, t \tag{C.3}$$

$$\frac{\partial \mathcal{L}}{\partial K_{ij}} = -\kappa_{ij} + \sum_{t=1}^{T} \mu_{ij}^t + \zeta_{ij} = 0 \quad \forall i, j \tag{C.4}$$

The Lagrange multipliers are all non negative, and the complementary conditions are:

$$\sum_{j=1}^{n} T_{ij}^t = P_{Gi}^t \quad \forall i, t \tag{C.5}$$

$$\sum_{i=1}^{m} T_{ij}^t = P_{Dj}^t \quad \forall j, t \tag{C.6}$$

$$\mu_{ij}^t (K_{ij} - T_{ij}^t) = 0 \quad \forall i, j, t \tag{C.7}$$

$$\alpha_i^t P_{Gi}^t = 0 \quad \forall i, t \tag{C.8}$$

$$\beta_j^t P_{Dj}^t = 0 \quad \forall j, t \tag{C.9}$$

$$\tau_{ij}^t T_{ij}^t = 0 \quad \forall i, j, t \tag{C.10}$$

$$\zeta_{ij} K_{ij} = 0 \quad \forall i, j \tag{C.11}$$

By limiting the analysis to the cases where power is generated at all busses i, and consumed at all busses j, one can eliminate the multipliers α_i^t and β_j^t that are null. Moreover, using (C.3) and (C.4), one can eliminate τ_{ij}^t and ζ_{ij} that are non negative. This yields the simplified optimality conditions:

$$\pi_j^t = \lambda_j^t \quad \forall j, t \tag{C.12}$$

$$\rho_i^t = \gamma_i^t \quad \forall i, t \tag{C.13}$$

$$\sum_{j=1}^{n} T_{ij}^t = P_{Gi}^t \quad \forall i, t \tag{C.14}$$

$$\sum_{i=1}^{m} T_{ij}^t = P_{Dj}^t \quad \forall j, t \tag{C.15}$$

$$0 \leq \mu_{ij}^t \quad \text{and} \quad \mu_{ij}^t (K_{ij} - T_{ij}^t) = 0 \quad \forall i, j, t \tag{C.16}$$

$$\lambda_j^t \leq \gamma_i^t + \mu_{ij}^t \quad \text{and} \quad T_{ij}^t (\lambda_j^t - \gamma_i^t - \mu_{ij}^t) = 0 \quad \forall i, j, t \tag{C.17}$$

$$\sum_{t=1}^{T} \mu_{ij}^t \leq \kappa_{ij} \quad \text{and} \quad K_{ij} (\sum_{t=1}^{T} \mu_{ij}^t - \kappa_{ij}) = 0 \quad \forall i, j \tag{C.18}$$

C.2 DC load flow model

The derivatives of the Lagrangian write as follows:

$$\frac{\partial \mathcal{L}}{\partial P_i^t} = -\rho_i^t + \sum_{l=1}^{n_l} H_{li} (\nu_l^t - \mu_l^t) + \lambda^t + \alpha_i^t - \beta_i^t = 0 \quad \forall i, t \tag{C.19}$$

$$\frac{\partial \mathcal{L}}{\partial K_l} = -\kappa_l + \sum_{t=1}^{T} (\mu_l^t + \nu_l^t) + \zeta_l = 0 \quad \forall l \tag{C.20}$$

where α_i^t, β_i^t, μ_l^t, ν_l^t and ζ_l are non negative. The complementary conditions are:

$$\mu_l^t (K_l - \sum_{i=1}^{n} H_{li} P_i^t) = 0 \quad \forall l, t \tag{C.21}$$

$$\nu_l^t(\sum_{i=1}^{n} H_{li}P_i^t + K_l) = 0 \quad \forall l, t \tag{C.22}$$

$$\sum_{i=1}^{n} P_i^t = 0 \quad \forall t \tag{C.23}$$

$$\alpha_i^t(P_i^t - P_i^{min}) = 0 \quad \forall i, t \tag{C.24}$$

$$\beta_i^t(P_i^{max} - P_i^t) = 0 \quad \forall i, t \tag{C.25}$$

$$\zeta_l K_l = 0 \quad \forall l \tag{C.26}$$

Using (C.20) to eliminate ζ_l that is non negative, one obtains

$$\sum_{t=1}^{T}(\mu_l^t + \nu_l^t) \leq \kappa_l \text{ and } K_l\left[\sum_{t=1}^{T}(\mu_l^t + \nu_l^t) - \kappa_l\right] = 0 \quad \forall l \tag{C.27}$$

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