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# Temperature distribution and heat radiation of patterned surfaces at short wavelengths

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We analyze the equilibrium spatial distribution of surface temperatures of patterned surfaces. The surface is exposed to a constant external heat flux and has a fixed internal temperature that is coupled to the outside heat fluxes by finite heat conductivity across the surface. It is assumed that the temperatures are sufficiently high so that the thermal wavelength (a few microns at room temperature) is short compared to all geometric length scales of the surface patterns. Hence the radiosity method can be employed. A recursive multiple scattering method is developed that enables rapid convergence to equilibrium temperatures. While the temperature distributions show distinct dependence on the detailed surface shapes (cuboids and cylinder are studied), we demonstrate robust universal relations between the mean and the standard deviation of the temperature distributions and quantities that characterize overall geometric features of the surface shape.

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## I. INTRODUCTION

Planck's law describes the intensity of radiation of a blackbody with temperature  $T$  at a given wavelength [1]. Integration over all wavelengths yields the Stefan–Boltzmann law [2] for the total power  $P$  emitted by the blackbody:

$$P = \sigma AT^4, \quad (1)$$

where  $A$  is the surface area of the body, and  $\sigma = \pi^2 k_B^4 / (60 \hbar^3 c^2)$ . For real materials Eq. (1) is modified by multiplying  $\sigma$  by the emissivity of the material. However, recently various modifications of the radiation law due to size and shape of the body have been explored and new general approaches based on scattering theory have been developed [3]. In general, the (effective) emissivity of an object depends on its size and shape due to self-scattering of the emitted radiation. Recent scattering approaches, however, assume that the bodies' surface has a spatially constant temperature. In general, this is not strictly justified due to self-absorption of heat emitted by a body with a nonplanar surface.

Information about the temperature distribution on patterned objects and the resulting transport of energy by heat radiation [4] is important to many science and engineering applications: radiating microstructured surfaces [5], transfer in combustion chambers and heat exchangers [6], climate phenomena such as the spatial variation of land surface temperatures [7], solar energy utilization [8], and the design of sustainable buildings [9]. Modeling heat radiation and radiative heat transfer in large-scale, complex geometries consisting of many shapes, objects, and materials presents enormous challenges due to the long-range wave nature of electromagnetic radiation. The most precise solution requires numerical solution of the electromagnetic wave equation to obtain the scattering of electromagnetic waves at all surfaces. However, for large complex geometries, the computing time and lack of precision of this methods increases [10]. Hence, it is desirable to identify universal scaling laws that can predict how shape and geometry influences spatial variation of temperatures and heat radiation.

This work attempts to propose a step in this direction by considering surfaces with various geometric patterns.

We assume that the thermal wavelength  $\lambda_T = \hbar c / (k_B T)$  is short compared with all geometric length scales of the surface patterns. In this limit, geometric optics can describe heat radiation leading to the so-called radiosity method that is widely used for heat phenomena and visual rendering [11]. It assumes diffuse reflections at the surfaces and hence is an alternate method to ray tracing. The surface is decomposed into patches that are coupled via a so-called view factor matrix that measures the fraction of radiation that travels from one surface patch to another. Similar methods can be applied to interactive sound propagation in complex environments (urban or indoor environments such as auditoriums) [12].

## II. THE MODEL

We consider a geometrically structured two-dimensional surface that is decomposed into small surface “patches” given by  $N$  mutually joining polygons  $P_j$ ,  $j = 1, \dots, N$ , defined over a planar base plane ( $xy$  plane); see Fig. 1 for examples. The polygons are oriented so that their surface normals  $\mathbf{n}_j$  are pointing all into the same half-space, the “outside” (say the positive  $z$  direction), which contains the source of the incoming external heat flux. For simplicity, we assume further that the polygon surface normals are either normal or parallel to the base plane. Each polygon is further characterized by an emissivity  $\epsilon_j$ , surface thickness  $d_j$ , and thermal conductivity  $\kappa_j$ . On the “inside” (negative  $z$  direction) of the surface a local equilibrium inside temperature  $T_j^{\text{int}}$  is imposed for each polygon. We assume that the surface receives a homogeneous radiant flux  $L$  from the outside half-space or “sky.” The goal is to compute the equilibrium temperatures  $T_j$  on the outside surfaces of the polygons by assuming that they are insulated against each other. These temperatures are determined by equating the internal and external net flux densities for each polygon. The internal net flux is obtained from the stationary heat conduction equation  $q_j^{\text{int}} = -\kappa \partial_n T_j$  integrated across the surface thickness  $d_j$  yielding  $q_j^{\text{int}} = (T_j - T_j^{\text{int}}) \kappa_j / d_j$ . The

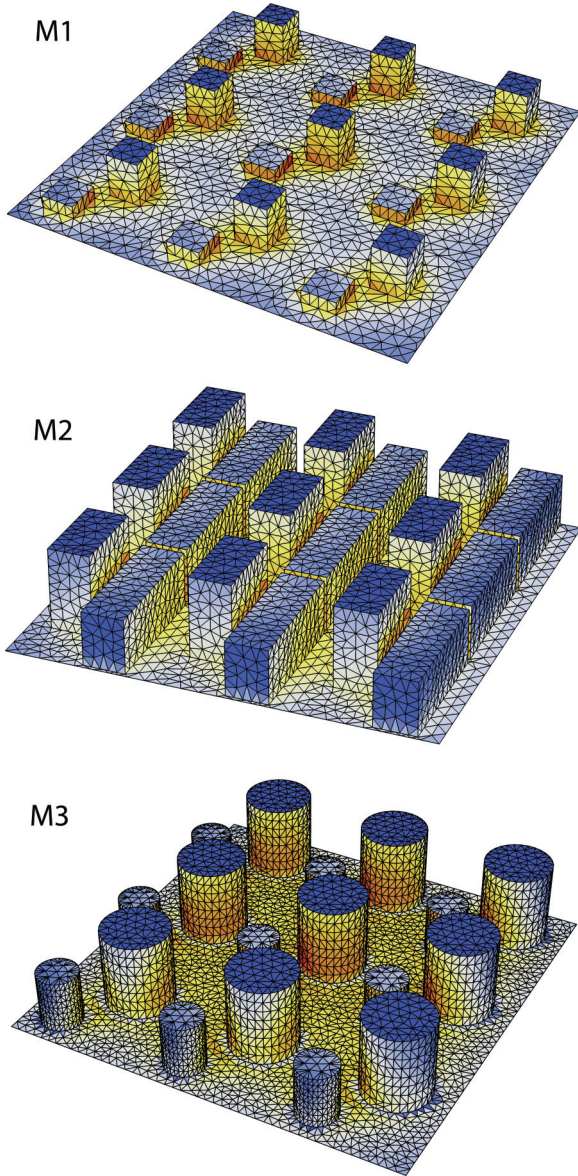


FIG. 1. Surface patch temperature distribution for models M1, M2, and M3. Colors represent temperature changes from minimum (blue) to maximum (red) temperature. For values see histograms in Figs. 2 to 4 and Table I.

external net flux  $q_j^{\text{ext}}$  is obtained as the sum of the incoming fluxes from the sky ( $L$ ) and those scattered from all other visible polygons and the heat flux  $\sigma\epsilon_j T_j^4$  radiated by the surface  $j$  where  $\sigma$  is the Stefan–Boltzmann constant.

For the simple case of a single planar surface ( $j = N = 1$ ), the condition  $q_1^{\text{ext}} = q_1^{\text{int}}$  yields

$$(T_{\text{flat}} - T^{\text{int}}) \frac{\kappa}{d} = \epsilon (L - \sigma T_{\text{flat}}^4), \quad (2)$$

which determines the outside surface temperature  $T_{\text{flat}}$  of the flat surface as function of known parameters.

For a general structured surface one has to consider multiple reflections between surface patches that contribute to the net external fluxes. To describe this effect, it is assumed that the surface patches are gray diffusive emitters, i.e., the

emissivity is frequency independent and the radiation density is constant across the surface patches and emitted independent of direction. We expect this to be a good approximation for thermal wavelengths that are small compared to the geometric structure of the surface and hence the size of the patches. Then we can apply the radiosity concept to obtain the external fluxes  $q_j^{\text{ext}}$  [4]. For a given surface patch  $j$ , the outgoing radiant flux is given by the sum of emitted thermal radiation and the reflected incoming radiation,

$$J_j = \sigma\epsilon_j T_j^4 + (1 - \epsilon_j) E_j, \quad (3)$$

where we used that the reflectivity equals  $1 - \alpha_j$  for an opaque surface where  $\alpha_j = \epsilon_j$  is the absorptivity. How much energy two surface patches exchange via heat transfer depends on their size, distance, and relative orientation, which are encoded in the so-called view factor  $F_{ij}$  between patches  $i$  and  $j$ .  $F_{ij}$  is a purely geometric quantity and does not depend on the wavelength due to the above assumption of diffusive surfaces. It is defined by the surface integrals

$$F_{ij} = \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi A_i |\mathbf{r}_{ij}|^2} dA_i dA_j, \quad (4)$$

where  $\theta_i$  is the angle between the surface patch's normal vector  $\mathbf{n}_i$  and the distance vector  $\mathbf{r}_{ij}$  which connects a point on patch  $i$  to a point on patch  $j$ , and  $A_i$  is the surface area of patch  $i$ . The view factor matrix obeys the important reciprocity relation  $A_j F_{ji} = A_i F_{ij}$  and additivity rule  $\sum_j F_{ij} = 1$ . With this geometric quantity, the radiative flux received by surface patch  $j$  from all other surface patches can be expressed as  $E_j = \sum_i F_{ji} J_i$ , and one can solve Eq. (3) for the vector of outgoing fluxes, yielding

$$\mathbf{J} = [\mathbf{1} - (\mathbf{1} - \epsilon)\mathbf{F}]^{-1} \mathbf{J}_0, \quad (5)$$

where we combined the fluxes  $J_j$  from all patches into a vector  $\mathbf{J}$  and the radiation  $\sigma\epsilon_j T_j^4$  into a vector  $\mathbf{J}_0$  to use a matrix notation. Here  $\mathbf{1}$  is the identity matrix and  $\epsilon$  is the diagonal matrix with elements  $\epsilon_j$ . To compute the surface temperatures  $T_j$  we need to compute the net heat transfer to surface patch  $j$  which is given by the incident radiation  $E_j$  minus the outgoing flux  $J_j$ , leading to the net flux  $q_j^{\text{ext}} = \sum_i F_{ji} J_i - J_j$ . In vector notation this net flux becomes

$$\mathbf{q}^{\text{ext}} = (\mathbf{F} - \mathbf{1})[\mathbf{1} - (\mathbf{1} - \epsilon)\mathbf{F}]^{-1} \mathbf{J}_0. \quad (6)$$

In the stationary state, the surface patch temperatures are then determined by the condition that the net external flux equals the net internal flux,  $\mathbf{q}^{\text{ext}} = \mathbf{q}^{\text{int}}$ , where  $\mathbf{q}^{\text{int}}$  defines the vector with elements  $(T_j - T_j^{\text{int}})\kappa_j/d_j$  due to heat conduction across the surface (see above). This condition uniquely fixes the temperatures  $T_j$  when all other parameters including the external (sky) flux  $L$  are known. In the following, technically we include the sky as an additional surface so that we have now  $N + 1$  surface patches. The corresponding additional matrix elements for the view factor matrix  $\mathbf{F}$  follow from reciprocity and additivity rules, and we include the downward radiation  $L$  as the  $(N + 1)$ st component in  $\mathbf{J}_0$ .

Knowing the surface temperatures, a number of interesting observables can be obtained. An *effective emissivity* of the total surface can be defined as the ratio  $\epsilon_{\text{eff}} = Q/Q_{\text{bb}}$ , where  $Q = [\mathbf{F}[\mathbf{1} - (\mathbf{1} - \epsilon)\mathbf{F}]^{-1} \mathbf{J}_0]_{\text{sky}}$  is the net flux towards the

sky, and  $Q_{\text{bb}} = [\mathbf{F}\mathbf{J}_{\text{bb}}]_{j=\text{sky}}$  is again a net flux to the sky but assuming that all surface patches radiate as ideal blackbodies, corresponding to  $\mathbf{J}_{\text{bb}} = \sigma[T_1, \dots, T_N, 0]$ . An *effective temperature*  $T_{\text{eff}}$ , as observed from the sky, can now be defined as were all surfaces blackbodies at their local temperature, so that  $\sigma T_{\text{eff}}^4 = Q_{\text{bb}}$  and  $Q = \sigma \epsilon_{\text{eff}} T_{\text{eff}}^4$ . We also define the difference  $\Delta T = T_{\text{eff}} - T_{\text{flat}}$ .

### III. NUMERICAL IMPLEMENTATION

The numerical implementation of the model described above follows these steps:

(1) The surface is decomposed into oriented patches which is done here by triangularization so that the entire surface is composed of planar triangular surface elements; see Fig. 1 with their surface normal vector pointing to the “outside” of the surface, i.e., pointing towards the sky. For later analysis, these elements are grouped into three different classes: horizontal “base” patches ( $b$ ) that are located within the base plane  $z = 0$ , horizontal “top” patches ( $t$ ) that are located above the base plane, and “vertical” patches ( $v$ ) that are perpendicular the base plane and connect the patches in classes  $b$  and  $t$ .

(2) Determine for all pairs of patches if the view between them is blocked by other patches. This is done by testing for potential intersections of the ray connecting the two centroids of a pair of patches and all other surface patches. It is sufficient to perform this visibility test for pairs of patches of the type  $(v,b)$ ,  $(v,t)$ , and  $(v,v)$  where the first (second) letter denotes the class of the first (second) patch. For all these combinations potential blocking patches must be in class  $v$ .

(3) If the view between a pair  $(i, j)$  of patches is not blocked and the first patch can “see” the outside of the second, the view factor  $F_{ij}$  is computed by using the exact closed-form expression described in Ref. [13]. This is done for all patch class combinations  $(v,b)$ ,  $(v,t)$ , and  $(v,v)$  with the restriction  $i < j$  for  $(v,v)$  since the view factors for  $i > j$  follow from reciprocity.

(4) Construct the total view factor matrix  $\mathbf{F}$  for all patches of classes  $v, b$ , and  $t$  and the single enclosing surface describing the sky. This is done by using reciprocity to obtain the matrix elements for the patch class combinations  $(b,v)$  and  $(t,v)$ . The patches of classes  $b$  and  $t$  cannot see each other so that the view factor submatrix for these classes vanishes. To obtain the view factor for the transfer from a surface patch  $i$  towards the sky we use the sum rule  $\sum_j F_{ij} = 1$ , i.e.,  $F_{i \text{ sky}} = 1 - \sum_{j \in \{b,t,v\}} F_{ij}$ . The view factor for the transfer from the sky to a patch  $i$  follows from reciprocity because  $F_{\text{sky } i} = \frac{A_i}{A} F_{i \text{ sky}}$  where  $A$  is the total area of the surface.

(5) The inverse matrix of Eq. (6) can be computed as a truncated geometric series since the emissivities are sufficiently close to unity and the view factors  $F_{ij} < 1$  with most of them in fact much smaller than unity. Hence the inverse kernel is given by  $\mathbf{K}^{-1} \equiv [\mathbf{1} - (\mathbf{1} - \epsilon)\mathbf{F}]^{-1} = \sum_{n=0}^{n_c} \mathbf{M}^n$  with  $\mathbf{M} = (\mathbf{1} - \epsilon)\mathbf{F}$ . We find that  $n_c = 6$  is sufficiently accurate approximation for the parameters used below.

(6) Finally, we compute the surface patch temperatures  $T_j$  by an iterative solution of the equilibrium condition  $\mathbf{q}^{\text{ext}} = \mathbf{q}^{\text{int}}$  [see Eq. (6)] for given surface emissivities  $\epsilon_j$ , downward radiation  $L$ , interior temperatures  $T_j^{\text{int}}$ , and effective thermal conductivities  $\kappa_j/d_j$ . The iteration steps are as follows:

- (i) Choose initial patch temperatures  $T_j^{(v=0)}$ .
- (ii) Compute the external flux  $\mathbf{q}^{\text{ext}(v=0)} = (\mathbf{F} - \mathbf{1})\mathbf{K}^{-1}\mathbf{J}_0^{(v=0)}$  with the  $N + 1$  dimensional initial vector  $\mathbf{J}_0^{(v=0)} = [L, \sigma \epsilon_1 T_1^{(v=0)4}, \dots, \sigma \epsilon_N T_N^{(v=0)4}]$ .
- (iii) Compute the updated patch temperatures  $T_j^{(v=1)}$  from the equation  $q_j^{\text{ext}(v=0)} = (T_j^{(v=1)} - T_j^{\text{int}})\kappa_j/d_j$  for  $j = 1, \dots, N$ .
- (iv) Continue with step (i) to start the next iteration step, i.e.,  $\mathbf{q}^{\text{ext}(v=1)} = (\mathbf{F} - \mathbf{1})\mathbf{K}^{-1}\mathbf{J}_0^{(v=1)}$  with the vector  $\mathbf{J}_0^{(v=1)} = \{L, \sigma \epsilon_1 [(T_1^{(v=1)} + T_1^{\text{int}})/2]^4, \dots, \sigma \epsilon_N [(T_N^{(v=1)} + T_N^{\text{int}})/2]^4\}$ .

In step (iv) and all following iteration steps it is useful to use the average of the last two iterations for the patch temperatures, as indicated here, to obtain rapid convergence. Typically, for the models and parameters used below, after about 20 iterations a stable solution for the patch temperatures had been reached (within a relative accuracy of  $10^{-4}$ ).

### IV. RESULTS

To study the influence of the density and shape of surface patterns on the temperature distribution, we consider three different surface structures that are all periodic in both spatial directions; see Fig. 1. For all surfaces, the dimension of a unit cell are given by  $L_x \times L_y = 20 \times 20$  (in arbitrary units). It is assumed that all spatial dimensions, however, are large compared to the thermal wavelengths  $\lambda_T = \hbar c / (k_B T)$  of the surface temperatures which is in the range of a few microns for the temperatures considered below. In the following we consider model parameters that are realistic choices for an application of our model of urban climate. The downward radiant flux from the sky is set to  $L = 300$  W per unit surface area, the interior surface temperatures are all set to the temperature  $T_j^{\text{int}} \equiv T^{\text{int}} = 293.15$  °K (room temperature), and all surface thicknesses  $d_j$  and thermal conductivities  $\kappa_j$  are chosen such that ratio  $\kappa_j/d_j = 5.0$  W/K per unit surface area (typical urban building material parameters). We consider two different homogenous emissivities across all surface patches which are  $\epsilon = 0.5$  and  $\epsilon = 0.9$  (which is again the typical range for urban materials).

The resulting surface temperature distributions for the three different geometric patterns are shown in Fig. 1. The geometric characteristics of the models are as follows: each model is composed of nine unit cells. Model M1’s unit cell consists of two rectangular cuboids with dimensions  $5 \times 5 \times 2$  and  $5 \times 5 \times 7$ , respectively. Model M2’s unit cell is composed of two rectangular cuboids with dimensions  $6 \times 19 \times 8$  and  $6 \times 12 \times 12$ , respectively. Finally, the unit cell of model M3 is composed of two cylinders of radii  $r_j$  with dimensions  $r_1 = 3 \times 8$  and  $r_2 = 5 \times 12$ , respectively. The corresponding area  $A_g$  (per unit cell) of the base plane that is covered by these elements (cuboids, cylinders) and the area  $A_v$  (per unit cell) of their vertical surfaces are summarized in Table I. In that table the total number of surface patches is also indicated. As we shall see below, other important geometric quantities are certain averaged view factors: the average sky view  $\bar{F}_{\text{all} \rightarrow \text{sky}} = \sum_{j \in \{b,t,v\}} F_{j \text{ sky}}/N$  from all surface patches, and the average sky view  $\bar{F}_{b \rightarrow \text{sky}} = \sum_{j \in \{b\}} F_{j \text{ sky}}/N_b$  from



TABLE I. Geometric parameters and surface temperature characteristics for the three surface models. All temperatures and their standard deviations are given in kelvin.

Model	$A_g$	$A_v$	$\bar{F}_{\text{all} \rightarrow \text{sky}}$	$\bar{F}_{\text{b} \rightarrow \text{sky}}$	Patches	$T_{\text{flat}}$	$\bar{T}$	$\bar{T}_v$	$\bar{T}_t$	$\bar{T}_b$	$\sigma$	$\sigma_v$	$\sigma_t$	$\sigma_b$	$T_{\text{eff}}$	$\epsilon_{\text{eff}}$	$\Delta T$
M1	50	180	0.7175	0.8037	4140												
$\epsilon = 0.5$						285.49	286.45	287.20	285.61	286.20	0.64	0.34	0.13	0.39	292.74	0.5631	7.26
$\epsilon = 0.9$						282.31	284.13	285.55	282.54	283.65	1.20	0.60	0.24	0.72	290.12	0.9193	7.81
M2	186	832	0.3441	0.3171	9018												
$\epsilon = 0.5$						285.49	289.04	289.77	285.79	289.29	1.83	1.30	0.28	0.93	315.76	0.6113	30.27
$\epsilon = 0.9$						282.31	288.04	289.20	282.85	288.42	2.79	1.82	0.49	1.26	313.32	0.9293	31.00
M3	106.81	527.79	0.4780	0.5751	14211												
$\epsilon = 0.5$						285.49	287.63	288.07	285.59	287.30	0.83	0.48	0.16	0.40	315.13	0.5964	29.64
$\epsilon = 0.9$						282.31	286.12	286.91	282.51	285.52	1.47	0.82	0.29	0.69	312.62	0.9277	30.31

patches of the base plane only, where  $N_b$  is the number of base-plane patches. These averages were restricted to the central unit cell to avoid boundary effects, and they are also given in Table I.

Next we analyze the results for the temperature distributions because they follow from the numerical approach outlined above. As can be seen from Fig. 1, the coldest patches are those on the top of the structures (class t). Since the top patches of the highest structures do not interact with any other patches, their temperature equals the temperature  $T_{\text{flat}}$  of a planar surface which sets hence the minimum value for the temperature distribution. Highest temperatures are observed on the vertical surface patches with an increase in temperature from the top to the bottom. This pattern results from a decreased view of open space (sky) for vertical patches and reflections from the base patches close to the bottom of the elevated structures. The base patches' temperature decays away from the structures, which is clearly visible for the low structures of model M1. The noncentral unit cells show colder surface patches towards the edges of the surface due to their proximity to the boundaries which enables an increased emission of heat.

Figures 2–4 show histograms for the surface temperature distributions of the three models, indicating the number of patches at a given temperature. Different colors label the three different classes of surface patches: vertical, base, and top patches. To reduce boundary effects, the histograms show the distribution of the center unit cell. For all models, panels (a) and (b) show the entire distribution for  $\epsilon = 0.5$  and  $\epsilon = 0.9$ , respectively. Panels (c) and (d) show the distributions for the vertical patches only, again for  $\epsilon = 0.5$  and  $\epsilon = 0.9$ , respectively, with different colors labeling now equidistant height intervals over the base plane in which the patches are located. A general feature of all models is that the surface temperatures increase from top patches over base patches to vertical patches. It is interesting to note that only for model M1 there is a clear separation of base and vertical temperature ranges whereas for M2 and M3 the base temperatures fall into the mid or lower range of vertical temperatures. Another interesting observation is that the vertical temperature distribution has a single peak for models M1 and M3, particularly in the latter, and a two-peak structure for model M2. We interpret this as a consequence of the proximity of two cuboids of different height. This view is supported by the variation of the distribution of vertical temperatures with height, see Figs. 3(c) and 3(d): Only the

peak at smaller temperatures contains patches of the largest height class H4, and hence must represent mainly the taller cuboid. In general, models M1 and M2 display little overlap between the temperatures corresponding to different height intervals while model M3 shows less separated temperature ranges for the height intervals. This is presumably related to the continuous range of vertical surface patch orientations for cylinders as compared with cuboids.

Table I summarizes various characteristics of the temperature distributions. In addition to the quantities  $T_{\text{eff}}$ ,  $\Delta T$ , and  $\epsilon_{\text{eff}}$  defined above, the mean temperature  $\bar{T}$  of the full distribution and the mean temperatures  $\bar{T}_j$  of the patch classes  $j = v, t, b$  are shown. The measure the temperature variations across different surface areas, we have also computed the standard deviation  $\sigma$  for the full distribution and the standard deviations  $\sigma_j$  for the different patch classes. Generally, a surface profile with deeper “canyons” leads a trapping of radiation and hence a larger  $T_{\text{eff}}$  which measures shape effects. Similarly, the effective emissivities  $\epsilon_{\text{eff}}$  show a larger increase for profiles with narrow canyons since they render the surface more black due to the trapping of radiation. A surface with a lower bare emissivity ( $\epsilon = 0.5$ ) has a larger shape-induced increase in emissivity than an already highly emissive surface ( $\epsilon = 0.9$ ).

An important problem is the identification of geometric parameters that characterize relevant features of the surface shape and show a universal relation to certain moments of the surface temperature distributions. Universal means here that the relation, instead of depending on particular details of the surface structure, relates to simple overall features of the surface shape. Potential candidates for such geometric parameters are listed in Table I: The surface areas  $A_g$ ,  $A_v$ , and the averaged view factors  $\bar{F}_{\text{all} \rightarrow \text{sky}}$ ,  $\bar{F}_{\text{b} \rightarrow \text{sky}}$ .

According to the Stefan–Boltzmann radiation law, the total radiative power emitted by an ideal blackbody is proportional to its surface area. For nonideal bodies, the radiative power is reduced by an effective emissivity that depends in general on material, size and shape of the body. Postulating that multiple reflections of heat radiation is of subleading order for the surface models considered here, one can expect that the shape-induced increase in mean surface temperature  $\bar{T}$  is proportional to the increase in surface area due to the surface pattern. Figure 5 shows the dependence of  $\bar{T}$  of the relative increase in surface area (due to vertical patches of total area  $A_v$ ). Indeed, the data are well described by a linear scaling,

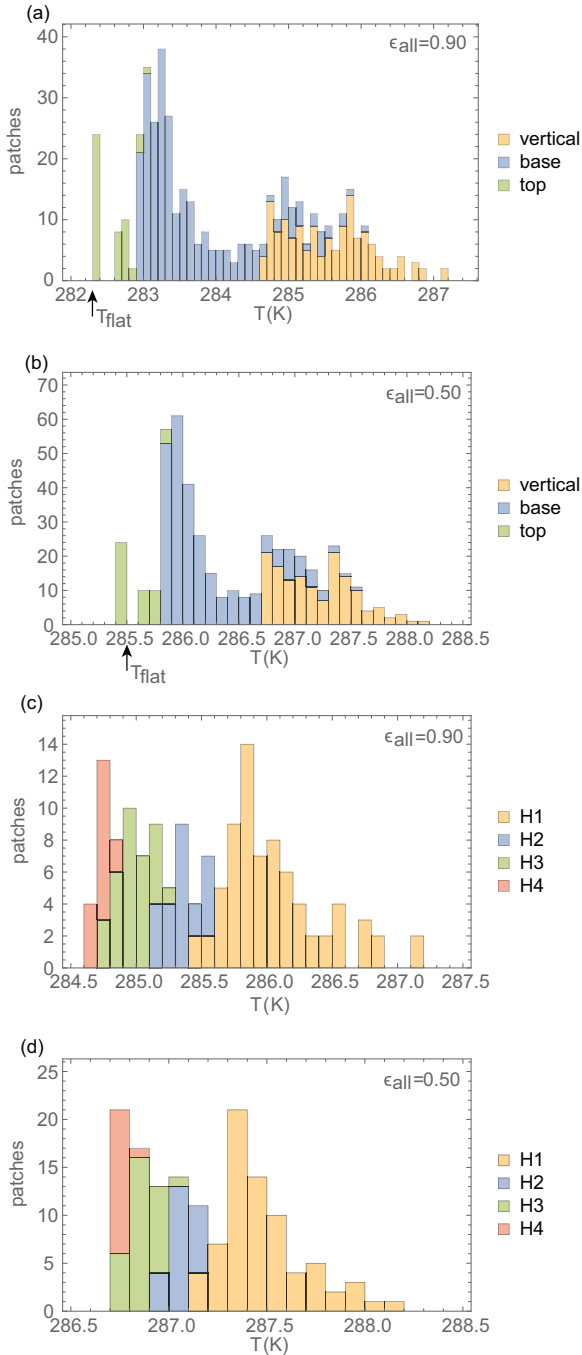


FIG. 2. Histograms for surface patch temperatures of the central unit cell of model M1: (a) temperatures for the three different patch classes vertical (*v*), base (*b*), and top (*t*) for emissivity  $\epsilon = 0.9$ , (b) same as panel (a) for emissivity  $\epsilon = 0.5$ , (c) temperatures for vertical (*v*) patches for emissivity  $\epsilon = 0.9$ , grouped into four different equidistance height classes *H1* to *H4* according to their height over the base plane, and panel (d) is the same as panel (c) for emissivity  $\epsilon = 0.5$ .

demonstrating that the detailed shape of surface structures is unimportant for the mean temperature.

Another geometric quantity that is more sensitive to shape than the overall increase in surface area is the averaged open (sky) view  $\bar{F}_{b \rightarrow sky}$  from the base plane patches. For a planar

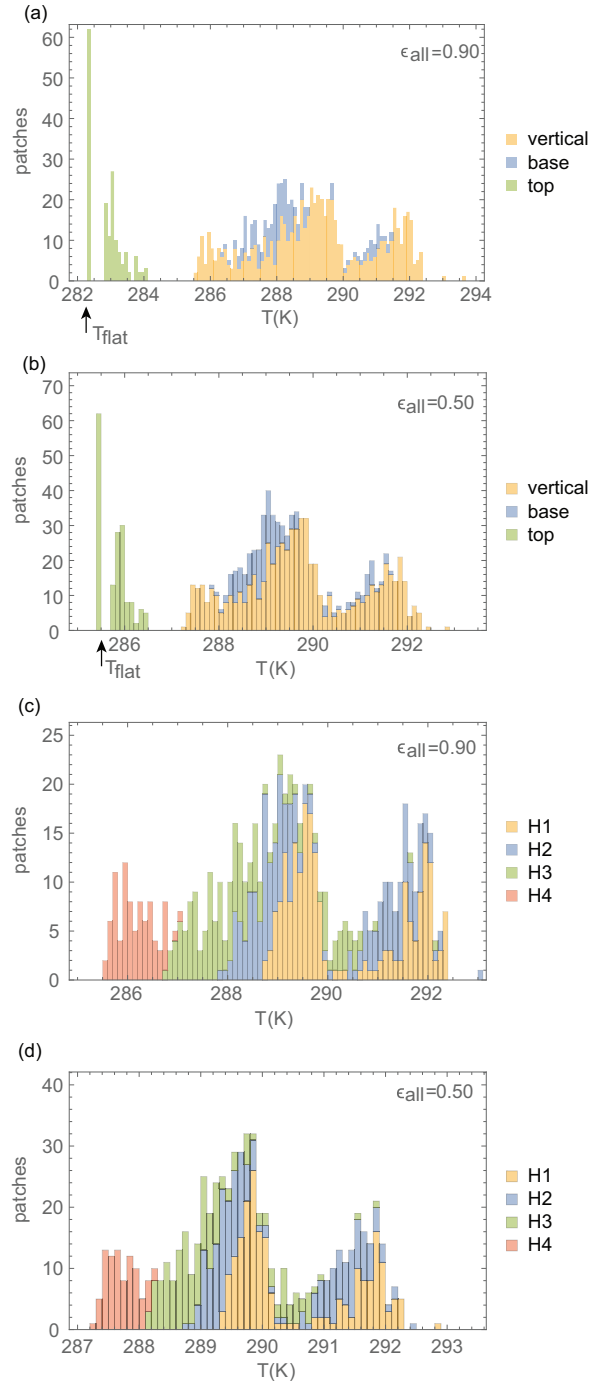


FIG. 3. Histograms for surface patch temperatures as in Fig. 2 for model M2.

surface with  $\bar{T} = T_{flat}$ , the view is unobstructed and hence  $\bar{F}_{b \rightarrow sky} = 1$ . Any surface structure reduces  $\bar{F}_{b \rightarrow sky}$  and in fact it has been observed experimentally in the context of urban climate that mean air and building surface temperatures tend to increase linearly with a decrease in the so-called sky view. To probe this relation quantitatively, we show in Fig. 6 the mean surface temperature as function of the mean open view factor  $\bar{F}_{b \rightarrow sky}$ . Our data for  $\bar{T}$  show a clear linear decrease with increasing mean sky view, with a universal slope that is independent of the particular surface patterns. The slope,

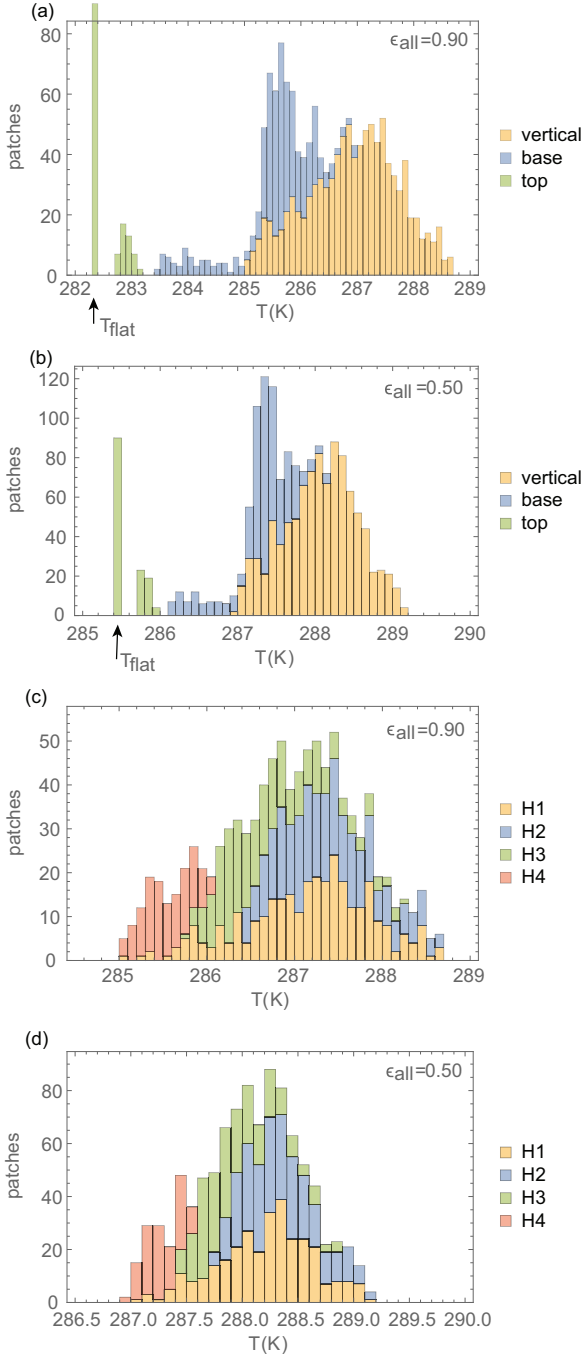


FIG. 4. Histograms for surface patch temperatures as in Fig. 2 for model M3.

however, does depend on the emissivity. The total view factor  $\bar{F}_{\text{all} \rightarrow \text{sky}}$ , averaged over all surface patches (see Table I) does not show a universal linear relation across all models.

Figure 1 shows that the temperature distributions have strong spatial variations. Hence, it is interesting to identify the key geometric parameters that determine the statistical moments of the temperature distributions. We have computed the standard deviation  $\sigma$  of the total distribution which is shown in Fig. 7, rescaled by the temperature difference  $\bar{T} - T_{\text{flat}}$ . The value of  $\sigma$  increases with the emissivity  $\epsilon$  which sets the scale for the typical surface temperatures (which are

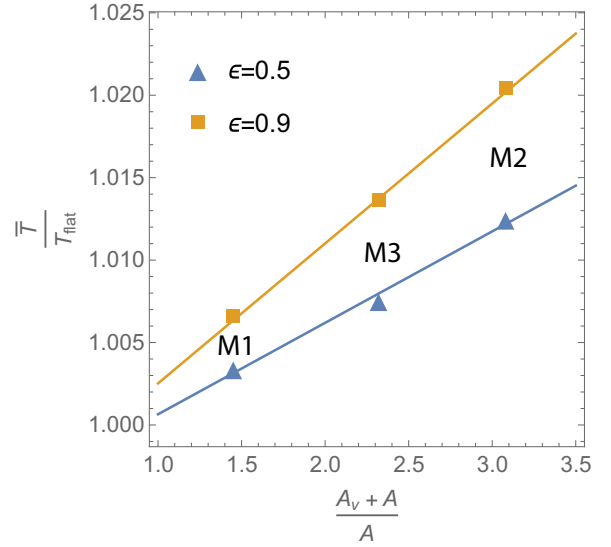


FIG. 5. Mean surface temperature (rescaled by the flat surface temperature) as a function of the relative increase  $(A_v + A)/A$  in surface area  $A = L_x L_y$  due to vertical surface patches.

of course also dependent on the heat flux from the interior side of the surface, characterized by the temperature  $T^{\text{int}}$  and heat conductivity of the surface patches.) However, after the rescaling by  $\bar{T} - T_{\text{flat}}$ , we observe a convincing collapse of the data for different  $\epsilon$  (see Fig. 7). Interestingly, the shape dependence of  $\sigma/(\bar{T} - T_{\text{flat}})$  is controlled by the ratio of vertical surface area  $A_v$  and base surface area  $A_b$  covered by elevated structures. This ratio measures the aspect ratio of height and width of the surface structures, and it shows a linear relation to  $\sigma/(\bar{T} - T_{\text{flat}})$ . We interpret this observation as follows: by how much the temperature actually varies within the typical range between the minimum  $T_{\text{flat}}$  and the mean  $\bar{T}$  is controlled by the homogeneity of the heat flux impinging on the surface patches. Tall and thin antenna-like structures (like

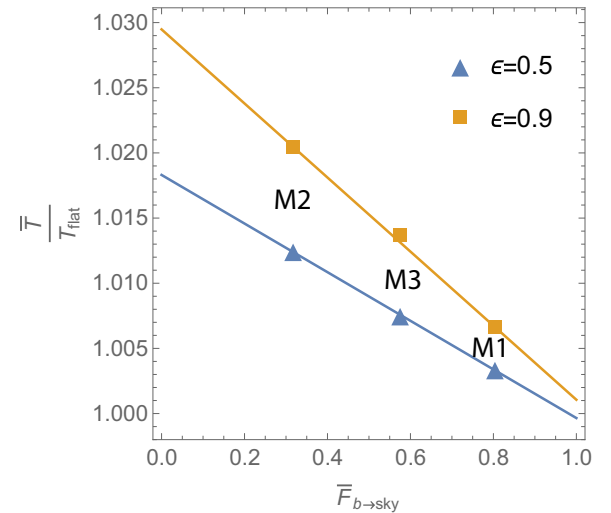


FIG. 6. Mean surface temperature (rescaled by the flat surface temperature) as a function of the mean view factor  $\bar{F}_{b \rightarrow \text{sky}}$  from base surface patches towards the sky.

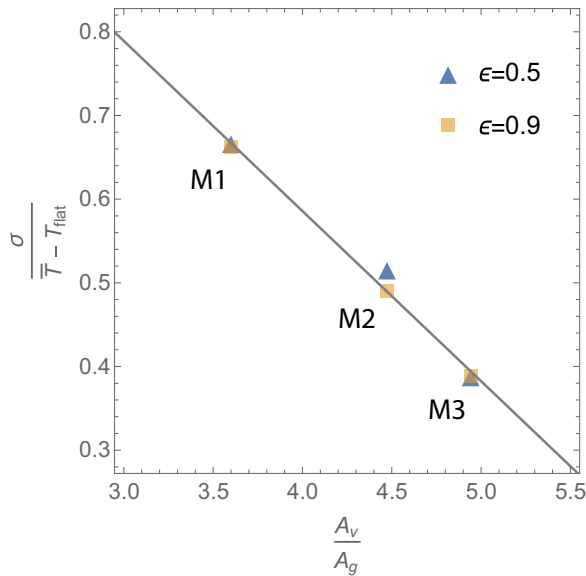


FIG. 7. Standard deviation  $\sigma$  of the surface temperature distribution, rescaled by the difference  $\bar{T} - T_{\text{flat}}$ , as a function of the ratio of vertical surface area  $A_v$  and surface area  $A_g$  covered by patterns (cuboids, cylinders). Data collapse is observed for different emissivities.

the cylinders of model M3) produce a more homogeneous heat flux (due to their increased view factors) and hence less temperature variation. This can be observed clearly from the temperature distribution on the base plane patches in Fig. 1 which shows the least variation for model M3.

## V. CONCLUSIONS

We have analyzed the influence of geometric surface patterns and emissivity on the surface temperature distribution by assuming a homogeneous internal temperature and external

radiative flux. The surface geometry is assumed to vary on scales large compared with the thermal wavelengths, i.e., the temperatures have to be sufficiently large. The details of the temperature distributions show a rich structure that is dependent on the detailed surface shape. However, we could identify parameters that measure relevant overall geometric features which obey universal relations to the mean and standard deviation of the surface temperature distributions. These geometric parameters are given by the ratio of horizontal and vertical surface areas of the surface patterns; see Figs. 5 and 7. It would be interesting to probe more geometries and a larger range of parameters to determine the range of validity of these relations. Larger periodic systems can be studied by imposing periodic boundary conditions which can be implemented in our approach by a periodic view factor matrix. Also, our study should be extended to nonperiodic patterns, and random surface profiles. There are a number of interesting conceptual extensions of the approach presented here. For lower temperatures, or shorter scale surface patterns, i.e., when the thermal wavelength  $\lambda_T$  becomes of the order of the size and/or distance of the surface structures, diffraction effects should be added to the radiosity approach. For highly reflective materials, specular reflections are expected to be important and hence should be included in the interaction (view) matrix. Surface geometry is also expected to modify convective heat transfer which influences surface temperatures. There is a plethora of possible applications of our results ranging from heat transfer between structured surfaces to the study of climate phenomena.

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