

AN ELECTRICAL METHOD FOR SOLVING THE EQUATIONS  
OF LONGITUDINAL STABILITY OF THE AIRPLANE

by

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1936

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February 20, 1936.

Professor A. L. Merrill, Secretary of the Faculty  
Massachusetts Institute of Technology  
Cambridge, Mass.

Dear Sir:

I hereby submit a thesis entitled  
An Electrical Method For Solving the Equations of Longitud-  
inal Stability of the Airplane in partial fulfillment of the  
requirement for the degree of Doctor of Science.

Very truly yours,

Signature redacted

Robert K. Mueller

Massachusetts Institute of Technology  
Cambridge, Mass.

## ACKNOWLEDGEMENT

The carrying through of the experimental part of this work has been made possible by the kindness of the Department of Electrical Engineering in extending the use of their facilities. Thanks are especially due to Professor O. C. Koppen, who supervised its aeronautical aspects, to William F. Millikan, who supplied most of the data for chapter VIII as well as encouraging interest, and to my brother, Richard A. Mueller, who assisted in the details of organizing this report.

## ABSTRACT

The similarity of mathematical form of laws relating certain mechanical and electrical forces and displacements is well known, as is the possibility of using information obtained from certain systems of one kind to predict the behavior of mathematically similar ones. This is the principle underlying all calculating machines, of which the M. I. T. Differential Analyzer designed by Dr. V. Bush is probably the most distinguished example. The action of the latter is based upon kinematical relations between rolling and sliding solids which may be made similar to those governing a wide variety of physical phenomena. Somewhat less powerful is the electrical machine developed in this thesis, for it can be used to solve only those problems which can be represented by a set of linear, total differential equations with constant coefficients and containing, moreover, only the dependent variables with their first and second derivatives. Its value lies largely in its relative simplicity especially where several dependent variables are involved.

Most of the development has been concerned with representation of "coupling" terms, that is, those describing the effect of one dependent variable upon another, and provision for adjustment to allow the treatment of all problems whose coefficients lie within certain ranges. The former requirement is such as to carry the problem beyond the range of classical passive networks, for in these the

coefficient matrix of the governing equations is always symmetrical about the principal diagonal. This is, of course, not generally true of arbitrary sets of equations and in particular is not true of the Equations of Longitudinal Stability for the solution of which the machine was primarily developed. Grid-controlled vacuum tubes were used in the construction of coupling units which were capable of representing each cross term separately because of their elimination of reaction from the controlled to the controlling circuit. Flexibility is in general expensive to obtain although not theoretically difficult. The airplane stability equations contain a preponderance of first derivative cross terms and fortunately the variation of these could be taken care of by sliding tap potentiometers through which flowed the currents corresponding to the dependent variables. Capacitative and inductive voltage dividers may be used in other cases.

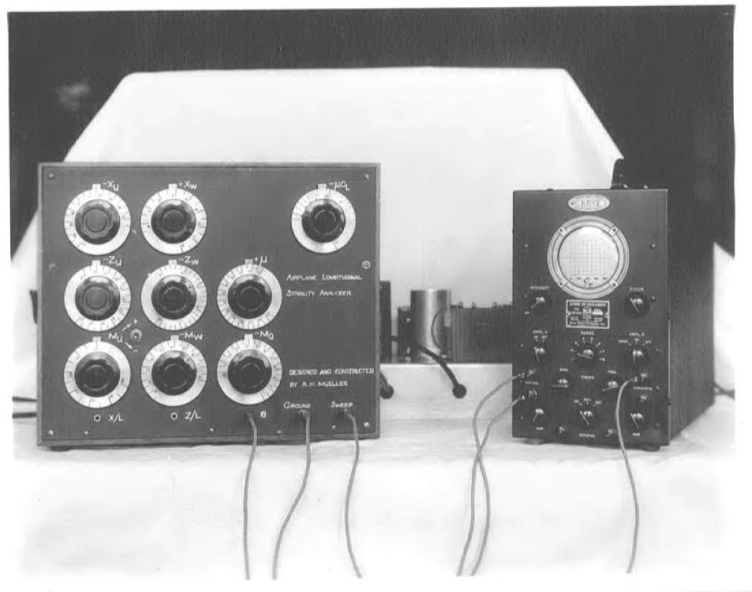
To make use of this electrical analogue as a calculator some means must be available of measuring the progress of the charge displacements with time in the circuits corresponding to the dependent variables. Also, electromotive forces corresponding to arbitrary functions of the independent variable such as usually appear on the right hand side of the equations must be applied to these circuits. So far the latter have been restricted to impulses and the resultant motion has therefore been made up of "modes" satisfying the complementary equations as these are



of principal interest to the airplane designer. Measurements of current may be made with a d'Arsonval type galvanometer provided the natural period of this is somewhat shorter than that of the electrical motion. Practically, this demands an oscillograph element whose low resistance must be matched to the circuit to be measured with a transformer for good sensitivity. This unfortunately modifies the characteristics of the circuit into which it is connected and it is simpler to employ an intermediate vacuum tube stage. The control voltage can then be taken across a capacitance, resistance, or inductance provided these are present in the circuit, and the resulting deflection of the galvanometer represents charge displacement, current, or rate of change of current respectively. If a cathode ray tube is substituted for the galvanometer, its deflections will be corresponding. For convenience the latter has been employed in this work. A rotating switch operating a linear sweep circuit and repeating the applied impulse at frequent and equal intervals allows a description of the motion to be repeated on the oscillograph screen and thus be rendered capable of visual observation.

Although it can make much less claim to precision than the Differential Analyzer, this electrical device has done very well in furnishing solutions for the longitudinal period and damping for several airplanes. In fact, the equations upon which it was designed to operate are not sufficiently close to reality to merit a much more refined treatment. Its accuracy may be conservatively

estimated at 10% which figure is probably somewhat high for the period, reasonable for the damping constant.



General View

# I

## INTRODUCTION

Two things are said to be analagous each to the other when they exhibit a certain kind of similarity. If the difference is merely one of scale, the two are said to be in the same mode or models of each other. The resemblance involved between two analogues is usually a more subtle one. As used in physics the term applies to phenomena which, though superfically quite different, receive the same form of mathematical expression. The subject of this thesis is an application of a striking relation of this sort between the domains of mechanics and electricity. A more remarkable analogy than this one can hardly be found in nature. Before it could be perceived or even suspected generations of investigators had to weave an immense fabric of measurement and mathematical manipulation, and the process is still going on.

Simple mechanical systems and their electrical analogues are often described in physics texts. The use which has been made of these in studying one through the other has not, however, been very extended. Before the principle can be of any practical use there must be some aspect of one system which renders it more tractable than the other. It may allow measurements to be more readily made, or be cheaper to construct, or be more easily modified. Although mechanical apparatus is sometimes used to convey electrical

ideas to students, it would almost never be used to study electrical circuits, for it fails to qualify in any of the above respects. Many measurements upon it are more difficult to make; it is generally more expensive to construct; variable resistances, inductances, and capacitances are common electrical elements but variable masses, springs and dampers are considerably more complicated. Cheapness and flexibility have led to the design of an electrical analogue of the automobile.<sup>1</sup> To construct an automobile with variable mass, springs etc. would be a fantastic problem. The practical expedient of spotting the field of study with representative specimens is expensive. By the use of the analogue this problem is quite easily and cheaply solved. It is, of course, not true that the information found in the electrical way is as trustworthy as that found by testing the exact object of inquiry. But it allows a manifold infinity of possibilities to be narrowed down to relatively small regions of probable interest. Empirical development reaches these regions often only by accident.

It has been suggested that the results of analogous study must be regarded critically. Although the basic laws upon which the analogue is founded may be so nearly parallel that we can detect no difference, other influences may enter which, while subordinate, definitely affect the precision of a result which leaves them out of

1 M. S. Thesis. M. I. T. 1933. S. Alexander.

account. Hooke's Law of elastic force does not apply over the whole range over which springs may be used in practice. Friction may and usually does act in a way which defies mathematical expression. There are, in fact, two places where error may enter. The analogue may be considered as built around a set of equations. These are a more or less rough description of how some physical system will act. They will also be a more or less rough description of how its analogue will act. The discrepancies between the prediction of the equations and the actual behavior in one system will not in general be the same as those between it and the other. If the analogue is being used as a substitute for the mathematical solution of these equations, their failure to comply with the conditions they were developed to describe cannot fairly be laid against it, as even an exact mathematical solution is in error because of this. The aim of this work is to design an electrical system which will be governed as nearly as possible by a set of equations already given. Its merit is to be based entirely upon the closeness of its approach to this.

If any system of masses, springs, and dampers is built up and the equations governing its motion set down, it will be found that these always exhibit a certain kind of symmetry. The coupling forces will be all contributed by the springs or dampers and will be such that the coefficient relating a force in one component of freedom to a displacement or its time derivative in another will be equal in magnitude

and sign to that relating the force and displacement in the reverse order. If the equations are set down so that the variables are arranged in separate columns in the same order as the equation rows representing respective forces along them, the above equalities cause the coefficient matrix to be symmetrical about its principal diagonal, the coefficients upon this representing the "self" or direct forces.

		<u>Variables</u>			
		$x_1$	$x_2$	$x_3$	etc.
<u>Forces</u>	$X_1$	$a_{11}(D)x_1$	$+ a_{12}(D)x_2$	$+ a_{13}(D)x_3$	- - = 0
	$X_2$	$a_{21}(D)x_1$	$+ a_{22}(D)x_2$	$+ a_{23}(D)x_3$	- - = 0
	$X_3$	$a_{31}(D)x_1$	$+ a_{32}(D)x_2$	$+ a_{33}(D)x_3$	- - = 0
	etc.	- - - - - = 0			

where force  $X_n$  is along direction  $x_n$ .

$$a_{mn} = a_{nm}$$

$$(a_{mn})^2 < a_{mm}a_{nn}$$

D is operator  $d/dt$

$a_{mn}(D)$  is a quadratic or lower function of D.

Figure 1 - 1.

This property is, in fact, common to all mechanical systems, the stresses in which are linear functions of the strains, or rates of strain. In addition it will be true that the product

of the two members of a symmetrical pair will always be less than the product of the coefficients forming the other corners of the square determined by these. References to thorough treatments of these facts will be found in the bibliography. Such systems are said to be passive as they will never spontaneously reach a state of motion.

The equations governing a large group of electrical systems are, however, similarly restricted. This group includes all combinations of resistors, inductors, and capacitors where the electromotive forces are linear functions of charge displacements and their first and second time derivatives. Slight departures are introduced by iron, changes in current distribution with acceleration, i.e. "skin effect", and other causes, but this group includes practically all systems of fixed conductors excluding gas and vacuum tubes. If we allow motion of conductors and include electrical and mechanical displacements in the total list of variables, certain pairs of coefficients symmetrically placed with respect to the principal diagonal will have opposite signs, but the magnitudes of all such pairs must be in the same ratio, this depending upon the choice of units. Examples of this are dynamos and loudspeakers, and a peculiar consequence of the equation form is that a lowering of mechanical impedance in one place results in an increase in electrical impedance in another.

The equations of longitudinal stability of the airplane do not exhibit either of these kinds of symmetry.



It is therefore futile to attempt to produce another system governed by them from combinations only of elements which are included in the broad classifications above. We require that any coefficient of the governing equations shall be capable of independent determination as regards both magnitude and sign. Although it comes out that only limited ranges are practically necessary, still the method must be an unrestricted one to conform to the necessary conditions. The invention which makes this possible with fixed conductors is the vacuum tube in which the current is no longer confined to a conductor in the usual sense. In the following chapters the manner of applying this will be developed.



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## II

### PARALLELS BETWEEN ELECTRICAL AND MECHANICAL LAWS OF MOTION

In this chapter we shall deal with the general laws governing motion in the electrical and mechanical domains and point out the aspects of similarity in these which offer the possibility of the construction of analogues. The units of matter in these two are the point charge and mass respectively, and we must first consider certain kinematical differences in the behavior of aggregates of these.

Liquids, gases, and solids are the three mass aggregates encountered in nature. Although airplane motion involves the latter two of these, the atmosphere and the rigid craft, since the longitudinal stability equations are concerned primarily with the motion of the airplane and only incidentally with that of the supporting air, the effect of this is introduced as external and the problem becomes entirely one in rigid mechanics. The character of fixed particle relationship of a solid is quite noteworthy when one considers that no such grouping of electric charges moving in conductors is found in nature. Electricity is primarily a fluid and its motion in fixed conductors is according to a definite flow pattern characteristic of the conducting system. The analogy with liquids is quite close except that the geometry of the flow patterns will not in general be the same. If the conductor forms a single circuit of length large compared

to cross-section, the system is predominantly a one-component one, that is, a knowledge of the displacement of a single particle is sufficient to measure that of all others. This is true of a solid only when somewhat constrained. The complete characterization of its displacement when entirely free, as is the airplane, requires six quantities and it therefore forms a six-component system. It is thus apparent from kinematical considerations that the complete electrical analogue will require six dependent, one-component electrical systems.

In speaking of the degree of freedom above we should have noted that this is really infinite, for solids may undergo an infinite variety of strains, and charge density at every point in a conductor may vary infinitely. Under certain conditions the elasticity of solids plays a negligible part in their motion and this is, in fact, true of the usual motion of the airplane. Likewise if the distributed inductance and capacitance of the conducting circuit and the charge acceleration are sufficiently small, the current loses all but one of its components of freedom. Our analogue will be constructed so as to satisfy this condition. Forces of frequency high enough to prohibit this simplification always exist and the problem of design is to assure its validity over the operation range.

The above differences between solids and electric currents give rise to others in the consideration of influences upon the motion. These influences are customarily

designated as forces. The idea of the "total force acting" is so intimately bound up with the development of mechanics that it seems almost as the natural way to view the phenomenon. This is largely because mechanics was at first concerned almost entirely with solids and fluids at rest where the interesting results were expressed in terms of stress surface integrals rather than body forces themselves. For purposes of comparison between mechanics and electricity, however, it is better to consider the unintegrated body force as the starting point. This is the one acting upon the ultimate particle. The total force acting upon the material in a certain region over the boundary of this region is, then, the integral of this. In many mechanical problems the integration is so obvious as to be commonly unperceived as, for example, forces due to weight and linear acceleration.

Electrical theory is likewise built around the idea of force but the total force acting upon an aggregate is not so useful a quantity. In what follows the moving charge will be considered positive and that fixed in the mass particle negative according to the usual convention. Forces influencing the charge drift directly are thus always upon positive and those producing conductor reaction upon the negative charge. We shall be concerned almost entirely with the moving positive. If the current is steady the charge density at every point will remain constant. This will still be nearly true for unsteady motion provided the conductor is long compared to its cross-section and sufficiently

removed from other conductors. The motion everywhere will be such that the forces are always in balance. In the steady case these may be electrostatic, arising from predominance of one kind of charge over another in certain regions, principally at the conductor surface, or electromagnetic due to relative velocities between charges, or resistive due to molecular disturbance. The above kind of electromagnetic forces are always normal to the current direction and have no influence upon its motion. When the motion becomes unsteady a second kind of electromagnetic force comes into action along the current direction and has an effect upon it depending upon the configuration of the conductor and the presence of magnetic material. Now these point forces are something like those acting throughout a solid if it is in a gravitational field or accelerating, and it is convenient to perform some kind of integration as we usually do with solids to serve the immediate purpose better. The proper kind comes out to be a "work" or more accurately a negative line integral of the force field. This integral is known as the electrical potential and for long, slender conductors is practically constant over a cross-section. It is then possible to take advantage of the fact that current distribution is constant and relate total current to potential difference directly. We have only simplified the problem to fit our needs overlooking irrelevant aspects. As a matter of fact the electrical potential is a good deal like mechanical stress only far simpler as the stress requires six quantities

at every point for its complete specification instead of only one. Both are space integrals of a body force. However, in dealing with solids the stress itself is commonly integrated over a surface to find the total force and it is this which appears in the equations of motion. This further integration is never made in electricity. The fundamental laws connecting the dimensions we have discussed and upon which our analog depends can be stated quite briefly. It is natural to arrange them in pairs each pair consisting of the electrical and mechanical laws which receive the same mathematical expression.

According to Coulomb, in a system of charges at rest the electrical potential at any point is  $\sum^n q_n/r_n$  where  $r_n$  is the distance between the point and the charge  $q_n$ . It can be shown as a consequence of this that when charge is transferred from one conductor to another the change in potential difference between the two is proportional to the amount transferred. The reciprocal of the constant of proportionality is called the capacitance of the two conductors and the relation between potential difference and charge is written

$$E - E_0 = q/C$$

In mechanics, forces proportional to displacements usually arise from the (slight) straining of matter. These are normal to any surface through liquids or gases at rest but in strained solids are of more complicated character. They are all usually considered to be governed by Hooke's Law although Hooke

never envisaged such generality when he formulated it. The longitudinal motion of the airplane is unaccompanied by any such strain, but an apparent displacement force still exists. It is a force along the longitudinal axis caused by a change in attitude. If one views the motion from axes fixed in the airplane, a pitching rotation appears to cause the direction of gravity to shift and if the displacement is sufficiently small the induced forces are all proportional to it. This is more fully explained elsewhere. (See bibliography)

Ohm observed that the current through a wire was proportional to the potential difference between any two sections of it, and the generalization of this states that the current density at every point in an isotropic material is proportional to the potential gradient at that point. The resistive elements in most electrical equipment are usually so connected by thin wires that the first, more special form of the law is sufficiently accurate and this is usually stated

$$I = E/R$$

Shearing forces in moving fluids are approximately linear functions of velocity gradients, but these are not the kind encountered in airplane motion. In this the forces depending upon velocities are definitely not linear ones and can be considered so only over small ranges using as the law the first order terms of their Taylor's Expansions. The accuracy



of stability theory is seriously impaired by this restriction.

Faraday's Law states that if a coil is threaded by a magnetic field which is changing with time, a potential difference proportional to the rate of change appears between the two ends. The general statement of the electrodynamic effect of currents is somewhat more complicated than this, but if the coil is wound upon an iron core, substantially all the flux passes through all its turns. Since this flux will be proportional to the current in the coil, the electromotive force due to the motion of this may be stated

$$E = L \, dI/dt$$

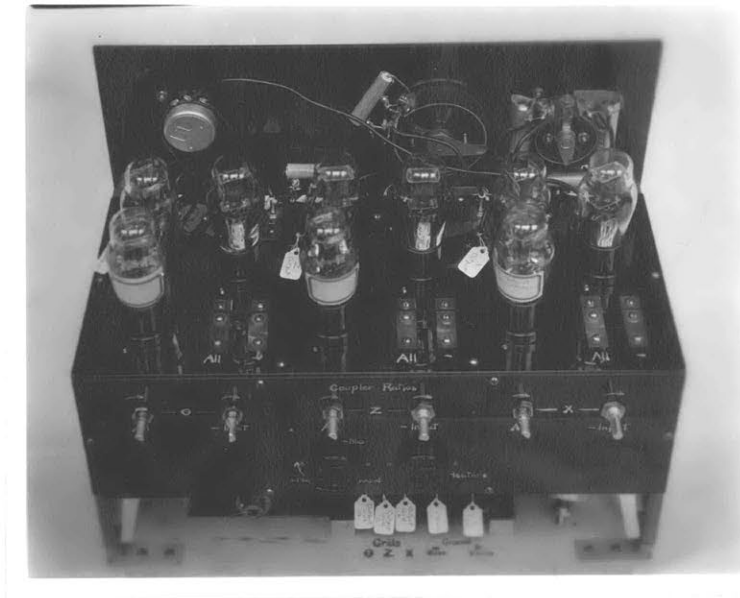
Correspondingly in mechanics an accelerated mass produces a force, usually called a reaction, which was found by Newton to be expressed by

$$\vec{F} = m \, \vec{d^2s/dt^2}$$

Here it happens that Newton's Law is a closer approximation than Faraday's if a magnetic material is used in the presence of the electric current, for this has a peculiar kind of resistance known as hysteresis while departures from Newton's Law are quite insignificant for most purposes.

We now have the necessary information to set up an electrical circuit whose electromotive forces will depend upon the current in the same way that mechanical forces at a point depend upon motion at that point. Forces produced by this elsewhere will require the use of coupling

of which one kind will be developed later. If the foregoing has seemed unnecessarily involved, it might be well to recall that simplifications and approximations unrecognized as such are a prolific source of trouble. Many statements have been conditioned upon such limitations as that the conductor shall be long compared to its cross-section, sufficiently removed from other conductors, etc., but it is only when these conditions are satisfied that the degree of freedom of the system is finite. All network theory is subject to such limitations.



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### III

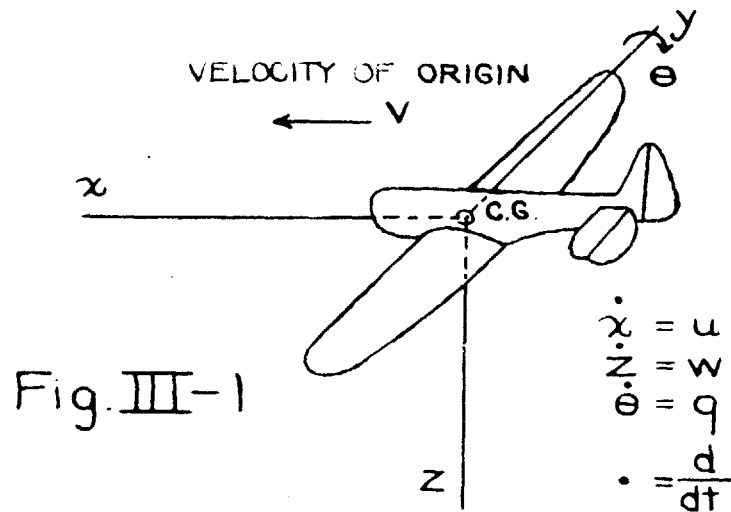
#### SKELETON FORM OF THE ELECTRICAL ANALOGUE

The equations of longitudinal stability are the result of a straightforward application of first-order vibration theory to the airplane. This theory is very much older than the airplane and applies to all motions in which the forces are linear functions of displacements or their derivatives. Newtonian forces proportional to accelerations are always present in mechanical motions and in our particular problem are found together with forces arising from the atmosphere which are usually linear only over small ranges. However, the general character of the motion under these limited conditions is only slightly departed from when they are considerably exceeded, and in practice these departures are usually not seriously large. There is no need here to derive in detail the equations and the technique of arriving at their coefficients by calculation, or test on similar models, as all this is well known. In the bibliography are to be found references to the mathematical background of the subject and the more specific aeronautical application. In the process of building up the electrical analogue, however, we shall naturally be led to rederive effectively a good deal of the theory.

By the use of vectors the equilibrium of forces can be expressed by two equations. The Newtonian

forces, for example, are proportional to accelerations and of opposite sense and can be combined into a single force and torque vector. It is also possible to concentrate the entire effect of the air into a force and a torque vector acting at a chosen point. But the behavior of electric charge in space could probably never be made similar to that of the airplane. Electrical phenomena are most readily controlled and measured when the current is confined to a path so long compared to its cross-section that planes of charge normal to the flow direction remain essentially so throughout the motion or at least in the vicinity of junctions. The high conductivity of metals as compared to air and other insulators makes it very easy to limit charge to such a one-dimensional world. This restriction would be a necessary one if we were to represent the flow and pressure in a long water pipe by current and potential in a similar electrical conductor, but in a problem where more than a one-dimensional flow is involved, a directed current can never entirely represent the condition. It can be represented, however, by a sufficient number of such currents properly related. There will be necessary one for each quantity required to specify the configuration of the mechanical system. This number is called the degree of freedom and is the same regardless of the framework or axis system to which the quantities apply. To specify the displacement of a solid the Eulerian system is usually used and the number of quantities involved six. The airplane, being near enough to an inelastic

solid when accelerations are as small as is usual in flight, is located with respect to a given system of axes in this way.



These axes originate at the center of gravity and point as shown so as to divide the airplane symmetrically. They are usually taken fixed in the airplane as the derivation of the equations is then a little clearer. Lining up the axes to the plane of symmetry causes the six equations of force corresponding to the components of freedom to break into two independent sets of three each, and it is with only one of these, called the longitudinal set, that we are here concerned.

In the last chapter the correspondence between mechanical force and displacement on the one hand and electromotive force and charge displacement on the other was treated. There we found that the sum of an equilibrium of forces at a point could be represented by potential

variations around a closed circuit. The problem of the analogue is to relate electromotive forces to charge displacements in the same way as mechanical forces are related to mechanical displacements. We showed how an inductance gives rise to a potential drop (negative) and a mass to a force opposing the motion, when the accelerations are in the direction of motion. In fact everything we need to know in relating the systems has been mentioned, but so far nothing of the method. For the present we shall omit all consideration of scale factors and nondimensional coefficients since these do not alter the form of the electrical circuits.

The equations for rotating axes are

$$\begin{aligned} [-mD^2 + X_u D] x + [ \quad + X_w D ] z + [ \quad -mgD^0 ] e &= 0 \\ [ \quad + Z_u D ] x + [-mD^2 + Z_w D] z + [ \quad +mVD ] e &= 0 \\ [ \quad + M_u D ] x + [ \quad + M_w D ] z + [-mbI^2 D^2 + M_q D] e &= 0 \end{aligned}$$

$$\text{where } b = I_y / ml^2$$

Let us take three circuits and inset<sup>r</sup> in them one at a time electrical elements which we have shown to give electromotive forces of the same kind as the forces in these equations. Where one circuit acts upon another we shall indicate this by the symbol shown in Fig. III-3.

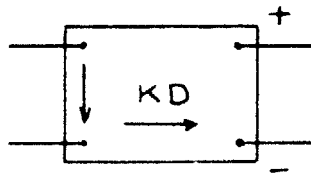


Fig. III-3

This is to be taken to mean that a current passing through the two left hand terminals produces a potential difference between the right hand two equal to  $k$  times its first time derivative. If the input current flows downward and  $k$  is positive, the upper right hand terminal will be positive with respect to the lower by  $k \times I$ . We shall assume that no potential difference except this one exists between the two terminals of each vertical pair and that no current can flow between two terminals in different pairs. The circuits corresponding to  $\underline{x}$ ,  $\underline{z}$ , and  $\underline{\theta}$  will then appear as in Fig. III-4. In order to represent  $X_u$ ,  $Z_w$ , and  $M_q$  as resistances we had to know that they would always be negative. The input ends of the coupling elements are connected where indicated so that when the current is clockwise it flows down through them. This is the barest outline of the solution of our problem, but before it is expanded the means of realizing the above-symbolized coupler will be treated.



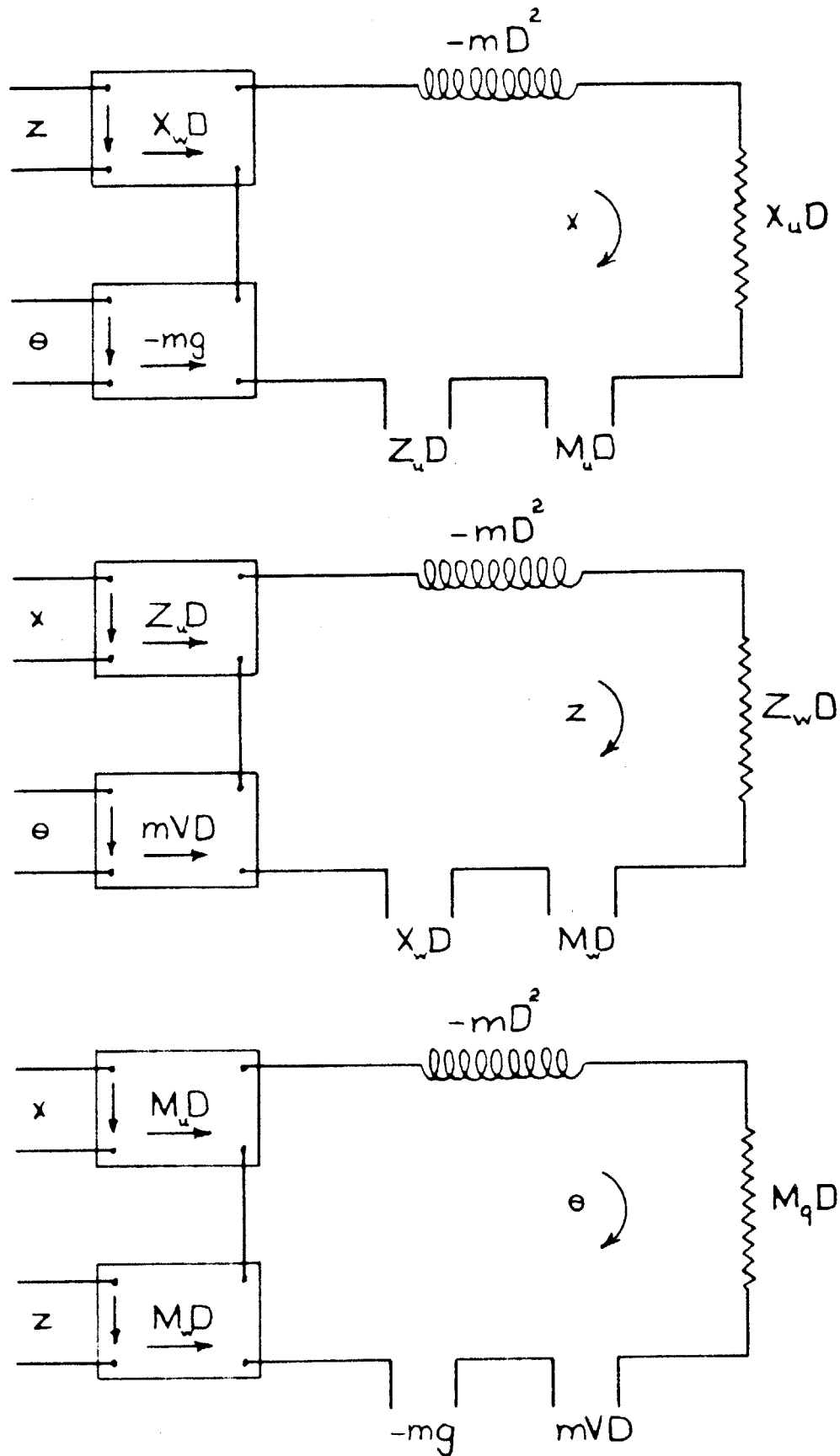
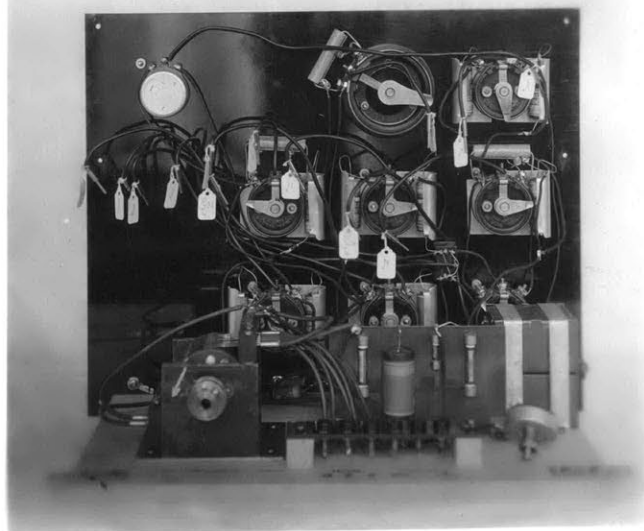


Fig. III - 4



Back View, Tube Assembly Removed

#### IV

### THE UNIDIRECTIONAL VACUUM TUBE COUPLER

In t he development of the last chapter considerable use was made of a unit which would relate potential<sup>a</sup> difference between two terminals to an unresisted current through another two. Our immediate problem is to realize this by some automatic electrical system. The solution here presented is somewhat indirect since other potential differences arise from it, but fortunately these can be compensated for by changes in the circuits. An essential aspect of the coupler is that no energy be allowed to pass through it. Energy changes will be involved with the driven circuit, but the source must be within the coupler itself and not the driving circuit. Very few means of control are known in which reaction from driven to driving element does not exist. A gate valve in a water pipe interchanges with the stream a rather small proportion of its energy as it is manipulated. The same is true of a carbon microphone. It should be emphasized that we are speaking here not of all the energy actually involved in the process of control including friction of moving parts, but of that part passing between the controlled and controlling elements. By far the most important example of such a device known today and, indeed, the one which practically requires the construction of our analogue in the electrical realm, is the electrostatic space current control, or less accurately the vacuum tube.

An explanation of this device except for the conditions governing its use is outside the scope of this work, but in view of its importance to our application and unfamiliarity to aeronautical engineers something about its action seems in place. If a group of metallic objects at different electrical potentials is arranged in free space, a field of force will act upon charges anywhere in this space so as to impel them toward conductors of opposite polarity. If charges are introduced in a swarm, they interact and so modify the force field that a steady condition of flow is reached regardless of how much extra charge is available at the source. A change in potential of any object will modify this steady flow pattern throughout in a way depending upon the configuration of the conductors. In the thermionic triode vacuum tube a space charge of negative electrons is introduced at the surface of a heated cathode, the heat serving to break the bond which normally holds the stream within the conductor. A surrounding plate of high positive potential (about 150 volts above the cathode) attracts these dislodged electrons and a third porous conductor is interposed to modify the electric field and so affect the equilibrium value of the space current. If this control grid is sufficiently negative with respect to the cathode, no part of the current will be diverted to it, but its effect upon the cathode-plate current will still be present. A potential change with no current flow is thus able to act upon the current in another circuit. Actually there is a small grid

current due to insulation leakage, capacitance existing among the electrodes and other effects, but the energy involved in these is small compared to that under control. The capacitance, however, though small leads to a definite reaction and at high frequencies may cause a serious transfer of energy. A set of characteristic curves for the 27 triode, the only type used in this work, is shown below. This chart gives all the information necessary for the present application.

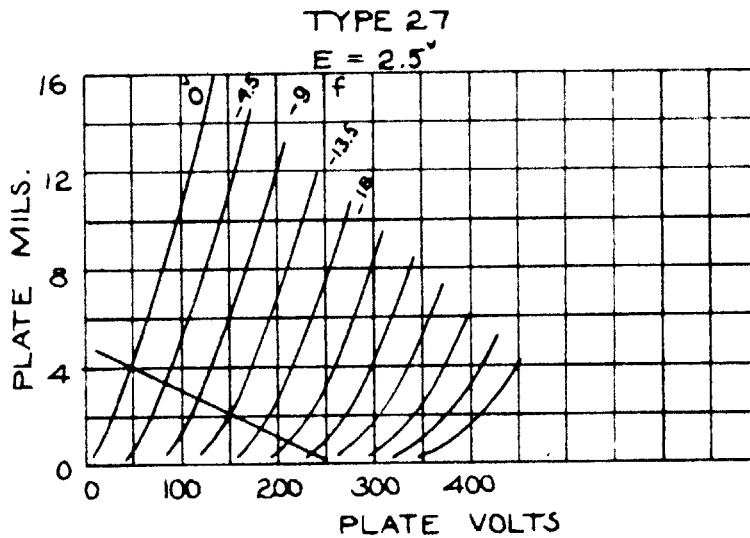


Fig. IV-1

We have shown how a change in potential unaccompanied by current can produce a current change in another circuit, but our coupler is required to do just the opposite, namely, to produce a potential difference controlled by a current. Suppose that instead of holding the plate of the tube at a fixed potential we connect it first through a resistance and then to the + terminal of a battery. The characteristic of the resistance is as explained in Chapter II. Since the sum of the potential drops across tube and

resistance is equal to the battery potential, these can be conveniently represented graphically by locating the point on the abscissa of the tube curves corresponding to battery potential and drawing a line with slope the negative of the resistance line. The current flowing can be read opposite the intersection of this line with the tube curve for the particular potential at which the grid is held. The potential opposite this point on the abscissa will be that of the plate. As the grid bias is changed, the plate potential opposite the intersection of tube and resistance curves is also seen to change. When the intersections of the resistance curve with those of the tube for equal bias increments are equally spaced, the two potential changes are linearly related. We shall always use tubes within the range where this is true.

The use of a resistance as a "load" as described above allows one potential to control another, but we require that the control be supplied by a current. This is very simply solved by inserting in the controlling circuit an element which sets up a potential difference across its terminals related to the current in the same way as the potential difference to be introduced into the driven circuit is related to it. A coupler of the type shown in Fig. IV-2a can be formed by allowing the current to pass through a resistance whose potential difference is impressed between grid and cathode of a vacuum tube as in b. For a coupler controlled by a charge displacement this element would be a condenser, while for one controlled by the second derivative

of a charge displacement, it would be an inductor. For the present we shall not consider sign as this can be adjusted within the coupler by proper design.

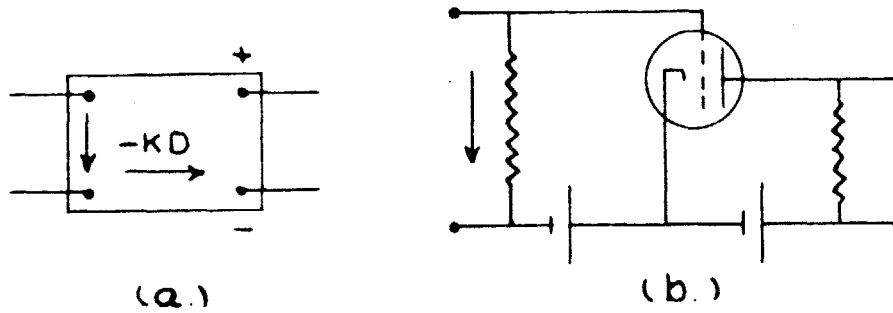


Fig. IV-2

We now have a potential difference proportional to a charge displacement or its first or second derivative, but it exists across a plate load resistor and has not yet been used to drive a second circuit. Suppose an arm of any sort is connected from plate to cathode of the vacuum tube.

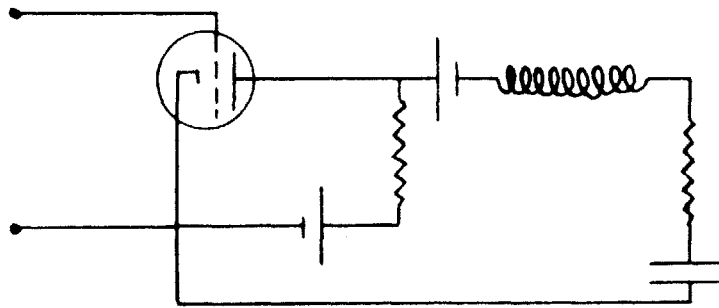


Fig. IV-3

The battery is provided to block a steady flow of current in this path when the grid is at rest, although when a condenser is present this is unnecessary. To understand the action now it is very convenient to make use of the equivalent to the plate-cathode path through the tube shown in Fig. IV-4a, where the alternator represents a source of electromotive force of any sort provided the amplitude is limited. This can be justified by referring to the tube characteristic curves. If the plate current is held constant and the grid potential changed, the successive states will lie along a horizontal line. Over the range where the curves are equally spaced the change of grid and plate potentials will be related as

$$\Delta E_p = -\mu \Delta E_g$$

If the grid potential is fixed and the plate current varied, the successive states will lie along one of the characteristic curves. Over the region where these curves are straight and parallel, the tube will act like a pure resistance whose magnitude is the reciprocal slope of the group and is designated as  $R_p$ . When the plate current and grid potential are both varying, the plate potential variation will equal

$$\Delta E_p = -\mu \Delta E_g + R_p \Delta I$$

Using this equivalent combination of elements, the tube and driven circuits appear as in Fig. IV-4b. The part enclosed in dotted lines can be still further reduced to a single



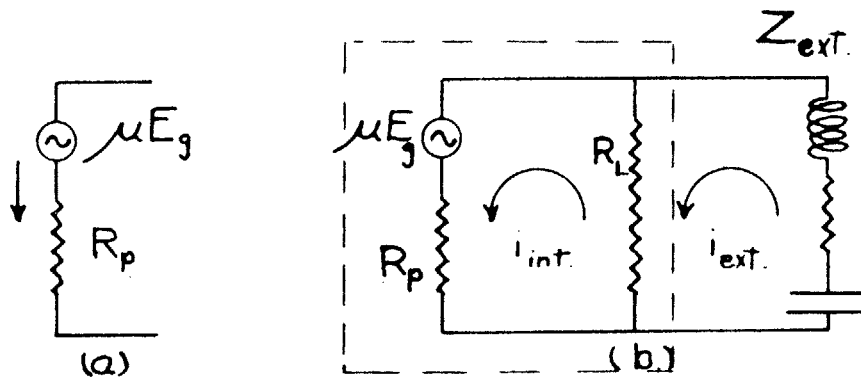


Fig. IV-4

resistance equal to  $R_p R_L / (R_p + R_L)$  in series with a source of e. m. f. of amplitude  $-R_L / (R_p + R_L) \mu \Delta E_g$ . This is a consequence of a theorem in network theory attributed to Thévenin, but can be proved for this special case as follows. Take the loop currents  $i_{int.}$  and  $i_{ext.}$  as shown. By Kirchoff's Law the sum of potential variations about both of these loops is zero. Proceeding in the direction of positive displacement

$$\begin{array}{l} \text{INTERNAL LOOP} \quad [R_L + R_p] i_{int.} + [-R_L] i_{ext.} - \mu \Delta E_g = 0 \\ \text{EXTERNAL LOOP} \quad [-R_L] i_{int.} + [R_L + Z_{EXT}] i_{ext.} = 0 \end{array}$$

$$i_{int.} = \frac{\mu \Delta E_g + R_L i_{ext.}}{R_L + R_p}$$

SUBSTITUTING

$$- \frac{\mu \Delta E_g R_L}{R_L + R_p} + \left[ -\frac{R_L^2}{R_L + R_p} + R_L + Z_{EXT} \right] i_{ext.} = 0$$

$$\left[ \frac{\mu R_L}{R_L + R_p} \right] \Delta E_g = \left[ \frac{R_L R_p}{R_L + R_p} + Z_{EXT} \right] i_{ext.}$$

The resistance with its arbitrary external impedance and the applied e. m. f. appear in this equation as indicated.

We now have a skeleton form of coupler which will produce a driving potential difference in one circuit proportional to a charge displacement or its first or second derivative in another with negligible backward transfer of energy.

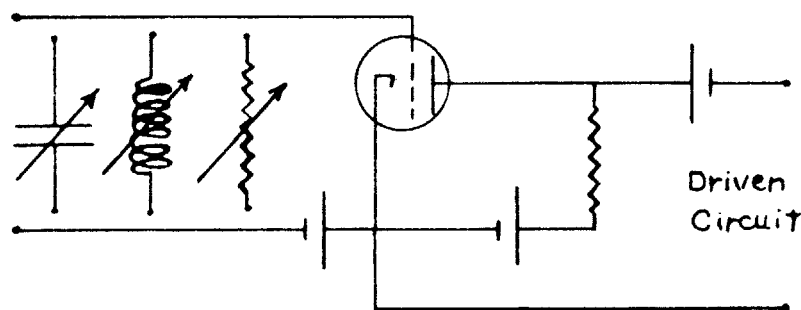


Fig. IV-5

The coupling coefficient may be adjusted by varying the input impedance. This device introduces extraneous potential differences in both circuits; in the driver across the input impedance and in the driven across the tube and load resistances in parallel. In case these circuits contain elements of this sort anyway, the total amount may be divided so as to introduce no error. In chapter VI all this will be considered in detail.

The electrical analogue to the equations of longitudinal stability requires the use of six coupling elements, one for each term not on the principal diagonal of

the coefficient matrix. Care must be used in connecting these together to avoid undesired currents, for although the potential differences between pairs of terminals at the two ends of the coupler will always be properly related, there is nothing to prevent a current from flowing between the two bottom terminals. Unless all cathodes are tied together this will most certainly result. One way to block such a flow would be to interpose an ideal transformer. This is one where the magnetizing current, i.e. with secondary open, is small compared to the operating current. It would be connected as in Fig. IV-6.

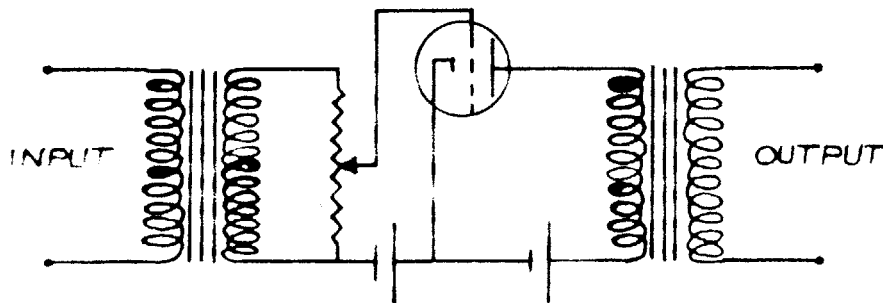


Fig. IV-6

Such a transformer simply multiplies the potential difference across the load and allows it to exist at any desired level. Since there is no conducting path between input and output, the former may be at any level desired at the point of connection. It is also possible to put such a potential block at the output and run all tubes from the same power source with cathodes tied together. The plate load resistor can then be

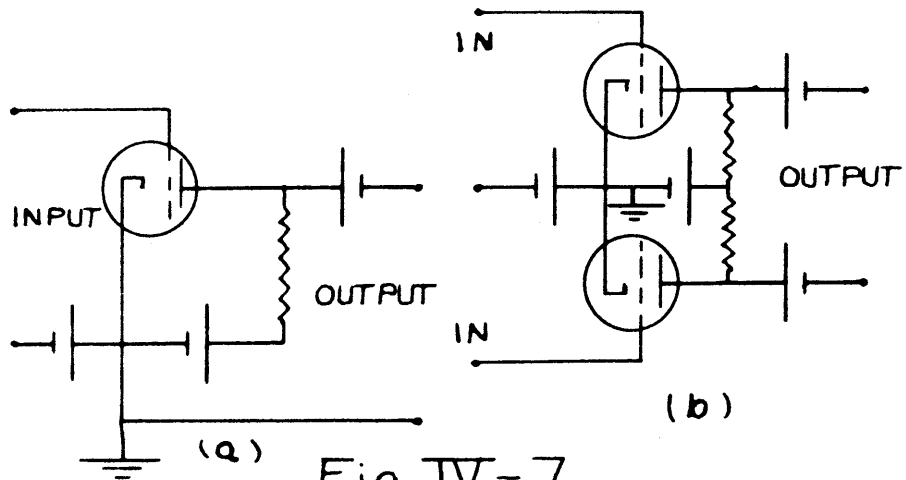


Fig. IV-7

omitted since the plate current has a closed path regardless of the type of the outside impedance.

The coupler above would seem to be the ideal of simplicity. It may be connected between any two points regardless of their potential levels. Its ratio may be varied by simply adjusting a tap on the grid load or changing the load itself. The sign of the control can be determined by the relative directions of positive current through the two external windings. Transformers, however, offer their difficulties. Unless they are carefully designed, the frequency limitations upon their use may be irksome. At low frequencies the magnetizing current is no longer relatively small and at high the leakage inductance of the windings and other causes lead to a marked departure from a simple potential and current multiplier. They were rejected in this work largely because of expense as compared to an alternative solution.

It was pointed out above that, unless transformers were used, the tube cathodes would all have to be held at the same potential to avoid undesired cross currents. Can a design be worked out with this restriction? Clearly each circuit can be driven only by either a single tube or a pair working in opposition. A pair or more in series aiding would be excluded. The opposed pair overcomes one difficulty of the single tube relating to the control of sign. Without transformers there is no way of doing this with a single stage except by reversing the positive direction of either input or output currents through the coupler.

If the opposed pair is used, the sign will be opposite for the two grids, a potential change of a certain sign in one case producing a positive, in another a negative electromotive force. In the early part of this work an attempt was made to use such an arrangement, but the absence of any point of fixed potential in the driven circuits except the cathodes led to difficulty in impressing potential differences occurring here across the tube control grids. The input circuit shown in Fig. IV-8 was tried.

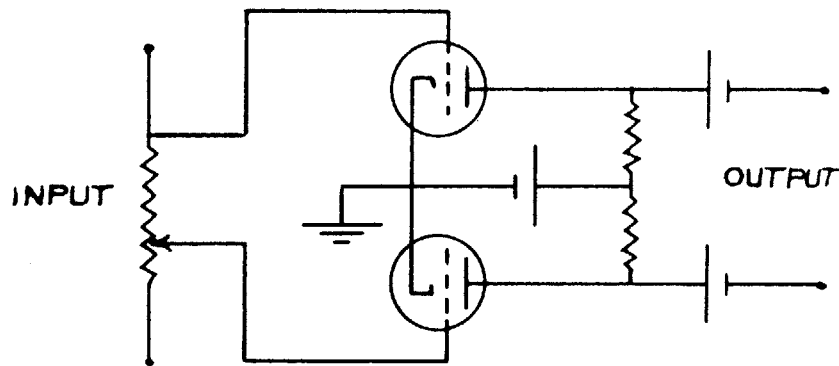


Fig. IV-8

Its sign could be changed by simply changing the input connections. Such an arrangement in any complicated system is almost certain to be unstable, and so the attempt was abandoned. The grid potential swing was also excessive.

The opposed arrangement of tubes was an attempt to control sign. This may be accomplished in another way. As has been pointed out, when a tube with resistance load has a potential variation impressed upon its grid, a variation of opposite sign equal to  $-R_1/(R_1 + R_p) \mu \Delta E_g$  appears at

the plate. If this plate variation is now impressed upon the grid of another tube after its level has been reduced to a proper negative grid bias, the variation appearing at the second plate will be of the same sign as at the first grid due to the double reversal.

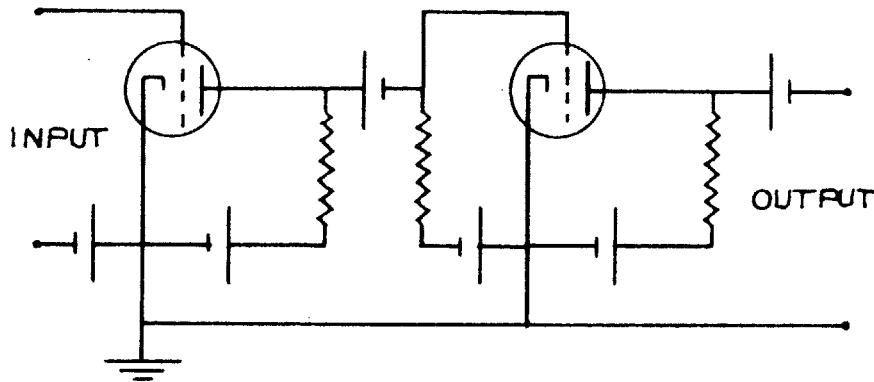


Fig. IV-9

This is the method used in the present design. The input is introduced before an even or odd number of tubes depending upon whether the sign of the output appearing at the last plate is to be the same or different from the input.

This brings us to the problem of adding several inputs to a single cascade of tubes which is intended to deliver an output potential which is a linear function of them. Dispensing with transformers and the opposed pair of tubes has confined us to a single driver tube per circuit to avoid stray currents through the couplers from input to output; so some addition of this kind is necessary. It is quite easy to construct an electrical net giving a potential which is any linear function of a group of potentials as long

as the coefficients are positive and real. The simplest is a group of resistances leading from a common point out to all the levels it is desired to combine.

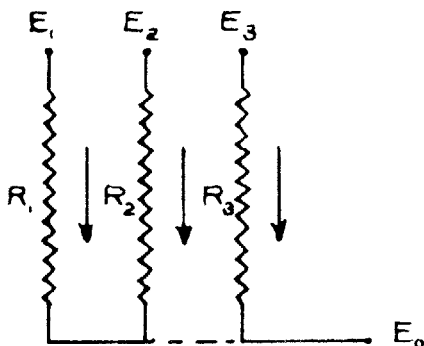


Fig. IV-10

If the positive current directions are taken toward the common connection, the sum of the currents will clearly be zero or

$$\frac{E_1 - E_0}{R_1} + \frac{E_2 - E_0}{R_2} + \dots + \frac{E_n - E_0}{R_n} + \dots = 0$$

which reduces to

$$E_0 \left( \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} + \dots \right) = \frac{E_1}{R_1} + \frac{E_2}{R_2} + \dots + \frac{E_n}{R_n} + \dots$$

The coefficient of  $E_0$  is the reciprocal of the parallel resistance of the whole group, and the coefficient of each  $E$  in the function is this resistance divided by that of the corresponding branch. Now if such a parasitic network is strung among the main circuits, it cannot help but modify the characteristics of all. However, provided these resistances



are made sufficiently large compared to the others, their disturbance can always be made negligible. The capacity of the vacuum tube for being controlled without appreciable expenditure of power from the controlling circuit here comes to the rescue, for it allows the arms of this combining network to be made with resistances large enough to satisfy the above condition. "Signals" may be impressed anywhere along the tube chain by diverting the output up to that point into one high resistance branch and the fresh input into another as in Fig. IV-11.

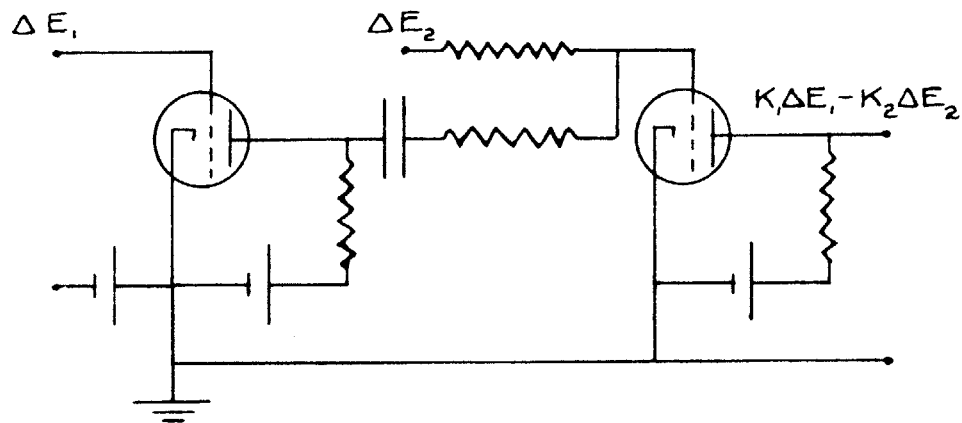
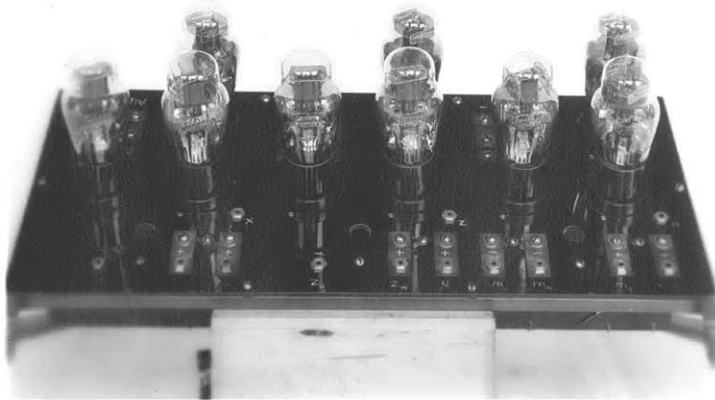


Fig. IV-11

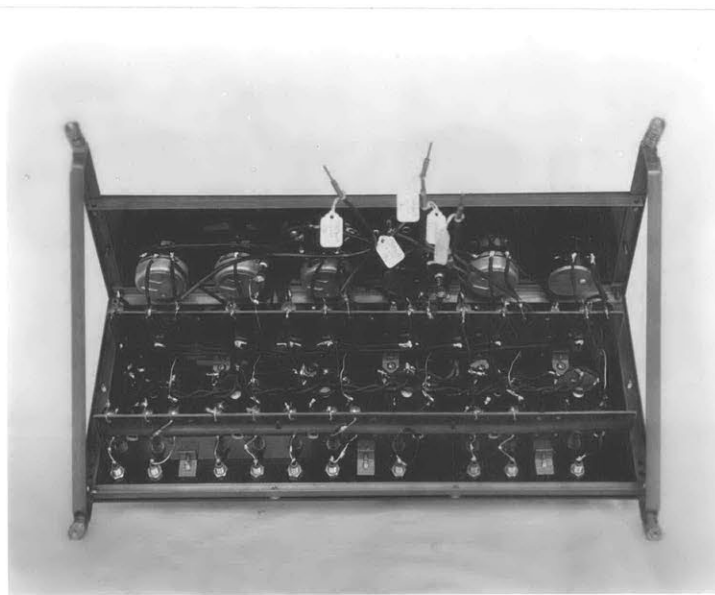
$\Delta E_1$  experiences a double sign reversal and  $\Delta E_2$  a single. This is essentially the plan followed in this work. The output drives a circuit containing electrical elements capable of producing potential differences corresponding to forces in each component of the motion due to motion of the airplane in a single one. Coupling is accomplished by connecting high resistances from suitable taps in these circuits to various grids.

The previous diagrams except Fig. IV-11 have made use of several batteries in the grid and output leads to lower the potential level to proper grid bias or cathode potential respectively. Such an arrangement is good because it eliminates low frequency cut off, but the number of batteries required together with the rather accurate potential control which must exist with at least the grid lead ones, makes it more convenient to substitute condensers where possible. These must be chosen so that the error introduced by them within the working frequency range is sufficiently small. When they are used to block the plate potential from a following grid, it is necessary to connect the latter to its bias battery through a high resistance or leak. This prevents a slow accumulation of charge on the grid and adjacent wiring and allows condenser leakage to be carried off. At the lowest frequency to be transmitted the quantity  $1/R\omega C$  for the grid condenser and leak should not be over .05. The battery in the output can be similarly replaced but no leak is necessary and the capacitance will generally need to be much larger.

Further details can best be treated in the actual design where several restrictions limit what can be done. As the reason for these restrictions cannot be evident here, we shall leave them for chapter VI.



Tube Assembly, Top View



Tube Assembly, Bottom View

DIMENSIONLESS FORM OF THE STABILITY  
EQUATIONS WITH SCALE FACTORS

In outlining the form of our analogue in chapter III we made use of certain aerodynamic coefficients for the displacements and their derivatives, but implied that these were simply slopes read from experimental curves (of flow) for each individual airplane. As a matter of fact the pattern around geometrically similar airplanes is almost the same, and this fact can be made use of to relate force coefficients between such airplanes. If it were not for this the small scale wind tunnel would be of no value in finding these quantities. It is a consequence of the close approach of air to a frictionless, massive fluid under conditions of flight, although it is more commonly taken as a direct result of aeronautical experiment than a necessary consequence of Newton's Laws of Motion. It can be stated as

$$\frac{F}{\rho/2 SV^2} = \text{constant.}$$

where F is the integrated force upon similar areas.  
 $\rho$  is the air density.  
 S is the wing area or the square of any linear dimension.  
 V is the velocity with respect to the distant stream.

Any of the forces or moments can be stated in essentially the same way. A moment will be expressed

$$\frac{M}{\rho/2 SV^2 l} = \text{constant.}$$

where  $l$  is usually taken to be the length from the C. G. to the tail post.

All this is explained by Glauert and others, but its consequences in altering the equations of motion will be treated here rather in detail to clarify the process of fitting scale factors and proportioning the analogue well to the desired range of variation, the real problem of design.

The force coefficients are generally written

$$\begin{array}{lll} X_u = x_u \rho/2 SV & X_w = x_w \rho/2 SV & X_q \text{ negligible} \\ Z_u = z_u \rho/2 SV & Z_w = z_w \rho/2 SV & Z_q \text{ negligible} \\ M_u = bm_u \rho/2 SV & M_w = bm_w \rho/2 SV & M_q = bm_q \rho/2 SV^2 \end{array}$$

$b = I/ml^2$  where  $I$  is the moment of inertia of the airplane about the lateral axis through the C. G.

If these are substituted in the former set of equations

we have

$$\begin{aligned} [-mD^2 + x_u \frac{\rho}{2} SVD] x + [ + x_w \frac{\rho}{2} SVD] z + [ - mg \quad D ] \theta &= 0 \\ [ + z_u \frac{\rho}{2} SVD] x + [-mD^2 + z_w \frac{\rho}{2} SVD] z + [ + mV \quad D ] \theta &= 0 \\ [ + bm_u \frac{\rho}{2} SVID] x + [ + bm_w \frac{\rho}{2} SVID] z + [-bml^2 D^2 + bm_q \frac{\rho}{2} SVI^2 D] \theta &= 0 \end{aligned}$$

Since it is desirable to have the displacements nondimensional,  $x$  and  $z$  can be divided by  $l$  and  $l$ 's multiplied into their coefficients. If this is done and the  $X$  and  $Z$  equations

divided through by  $\rho/2 SV$ , and the  $\theta$  equation by  $\rho/2 SV^2$   
the result is

$$\begin{aligned} \left[ -\frac{mI}{\rho SV} D^2 + x_u D \right] \frac{x}{t} + \left[ \quad + x_w D \right] \frac{z}{t} + \left[ \quad - \frac{mg}{\rho SV} D \right] \theta &= 0 \\ \left[ \quad + z_u D \right] \frac{x}{t} + \left[ -\frac{mI}{\rho SV} D^2 + z_w D \right] \frac{z}{t} + \left[ \quad + \frac{mV}{\rho SV} D \right] \theta &= 0 \\ \left[ \quad + m_u D \right] \frac{x}{t} + \left[ \quad + m_w D \right] \frac{z}{t} + \left[ -\frac{mbI^2 D^2}{\rho b SV^2} + m_q D \right] \theta &= 0 \end{aligned}$$

The time  $t$  may be multiplied and divided by  $c\tau$ , so far  
arbitrary, when  $\frac{dt}{c\tau}$  becomes

$$dt = c\tau d\left(\frac{t}{c\tau}\right)$$

If the differential operator is considered to derive with  
respect to an independent variable  $(t/c\tau)$  and  $\tau$  is taken  
equal to  $m/(\rho/2 SV)$ , the above set of equations becomes

$$\begin{aligned} \left[ -\frac{D^2}{c^2\tau} + \frac{x_u}{c\tau} D \right] x' + \left[ \quad + \frac{x_w}{c\tau} D \right] z' + \left[ \quad - \frac{mg}{\rho SV} D \right] \theta &= 0 \\ \left[ \quad + \frac{z_u}{c\tau} D \right] x' + \left[ -\frac{D^2}{c^2\tau} + \frac{z_w}{c\tau} D \right] z' + \left[ \quad + \frac{m}{\rho SV c\tau} D \right] \theta &= 0 \\ \left[ \quad + \frac{m_u}{c\tau} D \right] x' + \left[ \quad + \frac{m_w}{c\tau} D \right] z' + \left[ -\frac{D^2}{c^2\tau} + \frac{m_q}{c\tau} D \right] \theta &= 0 \end{aligned}$$

Multiplying through by  $\underline{c\tau}$  and calling  $m/(\rho/2 S l) = \mu$   
we have

$$\begin{aligned} \left[ -\frac{D^2}{c} + x_u D \right] x' + \left[ \quad + x_w D \right] z' + \left[ \quad - c\mu c_1 D \right] \theta &= 0 \\ \left[ \quad + z_u D \right] x' + \left[ -\frac{D^2}{c} + z_w D \right] z' + \left[ \quad + \mu D \right] \theta &= 0 \\ \left[ \quad + m_u D \right] x' + \left[ \quad + m_w D \right] z' + \left[ -\frac{D^2}{c} + m_q D \right] \theta &= 0 \end{aligned}$$

If the equations are set up in terms of the two dimensionless velocities  $(u/V)$  and  $(w/V)$ , the two  $\mu$ 's will be associated with  $m_u$  and  $m_w$  but there is no essential difference. Setting them up in terms of displacements seems a little more fundamental.  $mg/(\rho/2 SV^2)$  is equal to  $c_1$  in level flight and practically so at small departures from this. The  $\underline{c}$  is included in the new time unit  $\underline{c\tau}$  to allow the time scale of the electrical analogue to be arbitrarily related to the true one for the airplane. This remains fixed once the design is worked out.

It may still not be convenient to proceed directly from here to design circuit and coupling elements to fit the desired ranges of variation of the airplane coefficients. As they stand these are of various orders of magnitude.  $\mu$ , for example, may be numerically as high as 100 and  $\mu c_1$  about the same.  $m_u$  and  $x_w$  on the other hand may be .05 and .5 respectively. Here is an extreme ratio between  $\underline{\mu}$  and  $m_u$  of about 2000. Both are coupled in about

the same way by impressing a potential difference across resistances in their circuits upon vacuum tube grids. If this load could be more evenly distributed it seems - and later good reasons will appear - as if the design would be more compact and perhaps less sensitive to stray interferences. Here we shall show how this may be done, leaving for the next chapter the determination of exact change ratios.

Clearly all the coefficients in any equation may be multiplied by the same constant without altering the solution. It can also be shown, but not quite so easily, that coefficient columns may also be multiplied by arbitrary constants with a resultant change only in amplitude ratios of the three dependent variables. To prove this most readily, use will be made of determinant theory. If the equations are to have solutions other than zero, it is necessary that the determinant of the coefficients vanish. This will happen for certain values of  $\underline{D}$ , four in all, which are the roots of the quartic equation formed from the determinant. Now, any row or column of a vanishing determinant may be multiplied by a constant without altering the roots of its equation. Hence the coefficients may be multiplied as indicated without changing  $\underline{D}$ . The complete solutions will be of the form

$$\begin{aligned} x' &= A_1 e^{r_1 t} + A_2 e^{r_2 t} + A_3 e^{r_3 t} + A_4 e^{r_4 t} \\ z' &= B_1 e^{r_1 t} + B_2 e^{r_2 t} + B_3 e^{r_3 t} + B_4 e^{r_4 t} \\ \theta &= C_1 e^{r_1 t} + C_2 e^{r_2 t} + C_3 e^{r_3 t} + C_4 e^{r_4 t} \end{aligned}$$

where  $r_1, r_2, r_3, r_4$  are the roots of the quartic in  $\underline{D}$ .



The ratios  $A_1 : B_1 : C_1, A_2 : B_2 : C_2$  etc. will be as the cofactors or minors of the elements of any row of the determinant for the appropriate  $r$ , for if multiples of these cofactors are inserted in place of the variables in all the equations, all will be seen to sum to zero, one because the the determinant vanishes and the others because if a row is multiplied by the cofactors of corresponding elements of another row, the result is zero. Now suppose we multiply the rows of our determinant by  $a_1, a_2, a_3$ , and divide the columns by  $b_1, b_2, b_3$  respectively. The multipliers when grouped in order appear as

$$\begin{array}{ccc} a_1/b_1 & a_1/b_2 & a_1/b_3 \\ a_2/b_1 & a_2/b_2 & a_2/b_3 \\ a_3/b_1 & a_3/b_2 & a_3/b_3 \end{array}$$

The cofactors of the elements of the first row will now be multiplied respectively by  $a_2 a_3 / b_2 b_3, a_2 a_3 / b_3 b_1, a_2 a_3 / b_1 b_2$ , these factors being in the ratio

$$1/b_2 b_3 : 1/b_3 b_1 : 1/b_1 b_2$$

which multiplied by  $b_1 b_2 b_3$  gives  $b_1 : b_2 : b_3$ .

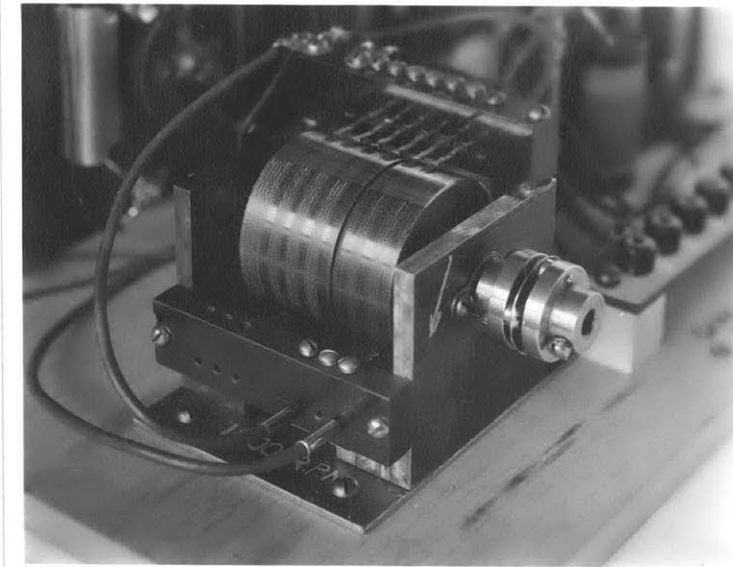
Thus the relative amplitudes of the dependent variables are modified in this ratio when the coefficient columns are divided by these constants. As long as  $b_1, b_2, b_3$  are real there will be no change in phase and the solution of the unmodified equations can easily be found by applying the reciprocal of the above ratio. This proof has been carried

out under the assumption that no external force is applied. As we may want to subject the system to arbitrary disturbances in flight, it should be pointed out that the amplitudes are here similarly distorted and nothing further. The proof follows along as before except that the determinant need not vanish and all values of  $\underline{D}$  are allowed. Its evaluation is greater than with no scale factors by the factor  $a_1 a_2 a_3 / b_1 b_2 b_3$  for all values of  $\underline{D}$ . Instead of only four terms to each solution there are now an infinite number covering the entire spectrum of  $\underline{D}$  in the complex plane and appearing as a Laplacian Integral. However, the solutions are still distorted in the ratio  $b_1 : b_2 : b_3$  the same as with no applied force.

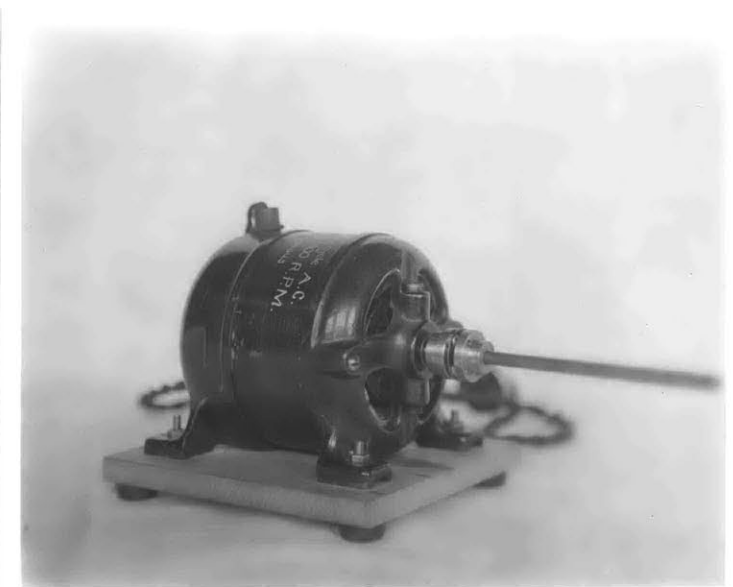
If the dimensionless coefficient matrix is now written with the new multipliers, it appears

$$\begin{array}{lll}
 \frac{a_1}{b_1} \left[ -\frac{D^2}{c} + x_1 D \right] & \frac{a_1}{b_2} \left[ \quad + x_2 D \right] & \frac{a_1}{b_3} \left[ \quad - c \mu c_1 D \right] \\
 \frac{a_2}{b_1} \left[ \quad + x_1 D \right] & \frac{a_2}{b_2} \left[ -\frac{D^2}{c} + z_2 D \right] & \frac{a_2}{b_3} \left[ \quad + \mu D \right] \\
 \frac{a_3}{b_1} \left[ \quad + m_1 D \right] & \frac{a_3}{b_2} \left[ \quad + m_2 D \right] & \frac{a_3}{b_3} \left[ -\frac{D^2}{c} + m_3 D \right]
 \end{array}$$

Values will be determined later.



Commutator



Synchronous Motor

VI

DETAILED DESIGN OF THE ANALOGUE

We are now in a position to connect together electrical elements as indicated in the modified equations of longitudinal stability. An examination of the principal diagonal of their coefficient matrix shows that each circuit will contain inductance and resistance. It has already been pointed out that the driver with its load resistor interposes a certain amount of resistance. This will be the minimum possible if these terms are represented by passive electrical elements. Later we shall have to consider this further, but now all three circuits can be represented as

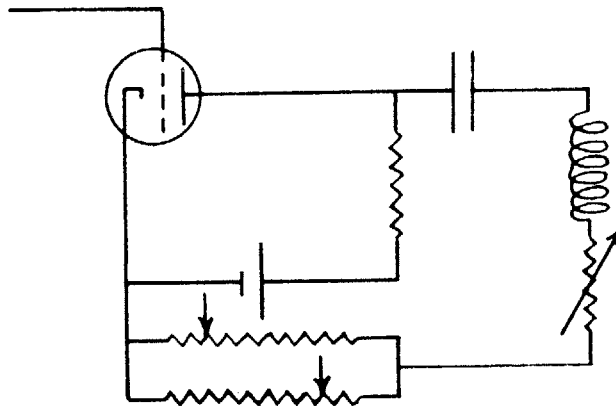


Fig. VI-1

where the condenser must be large as explained in chapter IV. The terms off the principal diagonal are all first derivative coefficients except the one in the upper right hand corner which multiplies a displacement. We shall first consider only the former which are all treated essentially alike.

The potentiometers are all connected one end to the grounded cathode. In this way the potential variation of a single tap is proportional to the potential drop across the resistor. This tap is connected through a high resistance to the grid of an even or odd tube counted from the driven circuit outward, depending upon whether the sign of the coupling term is plus or minus. When one circuit controls two others as do  $\underline{x}'$  and  $\underline{z}'$ , it is convenient to connect two control potentiometers in parallel as shown. These unfortunately still further increase the minimum value of self resistance in the circuit but are necessary to exercise the required control.

The task of representing the upper right hand term of the matrix  $-a_1/b_3 \mu c_1$  which represents the force in the  $\underline{x}$  direction produced by a displacement in  $\theta$  introduces a serious difficulty. This term would ordinarily be represented by a condenser with one end grounded and the other connected to a grid as in Fig. VI-2,

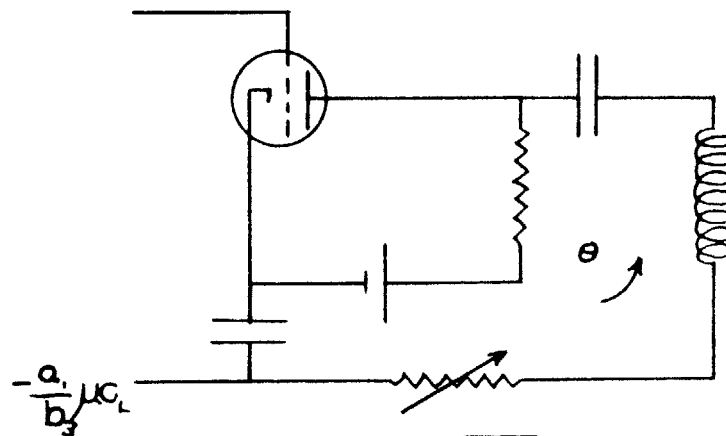


Fig. VI-2

but the  $\theta$  circuit should contain no e. m. f. proportional to its displacement, and yet if a condenser is included such will exist. We are thus faced with the dilemma of requiring the potential difference across a condenser for coupling purposes but being obliged to exclude it from the controlling circuit. It is possible further to manipulate the equations to bring a direct displacement force into one or more of the components of motion, and this would allow corresponding condensers in the analogue. In fact the equations set up for non-rotating axes have such a term. They are written (dimensional form)

$$\begin{aligned} [-mD^2 + X_u D] x + [ \quad + X_w D ] z + [ \quad + 0 ] \theta &= 0 \\ [ \quad + Z_u D ] x + [-mD^2 + Z_w D] z + [ \quad + Z_\theta \dot{D} ] \theta &= 0 \\ [ \quad + M_u D ] x + [ \quad + M_w D ] z + [-mbI^2 D^2 + M_q D + M_\theta \dot{D}] \theta &= 0 \end{aligned}$$

$$\begin{aligned} \text{WHERE } Z_\theta &= \sqrt{Z_w} \\ M_\theta &= \sqrt{M_w} \end{aligned}$$

This would seem to be the logical dodge around the difficulty. Capacitance couplings, however, are not easy to realize. Unless the controlling condenser has a rather small capacitance this cannot be made continuously variable. The same result, however, can be obtained by shunting it with a high resistance potentiometer and varying a tap on this. There is one difficulty encountered here. If the condenser controls the same grid as one or more other elements, all may be connected

by a high resistance net as explained in chapter IV.

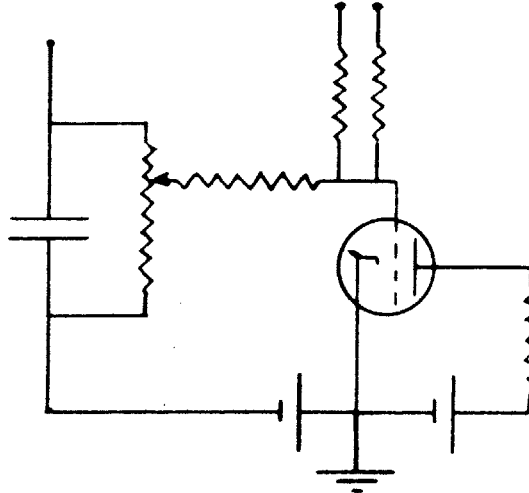


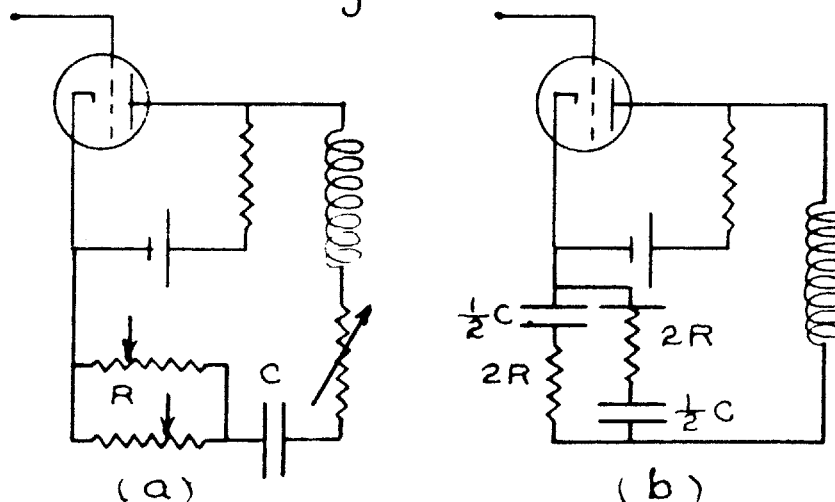
Fig. VI-3

The potential at the grid will not now be the same function of the initial potentials at the points of connection of its network as indicated in that chapter. There we assumed that the parasitic circuit did not appreciably modify the conditions at the points of connection, but this is clearly not true where a connection is made to a point on a potentiometer of the same order of magnitude as the resistances comprising the net. While it will still be possible to get any desired proportioning of effects upon the grid, the various settings at the ends will not be independent. Also difficult is the adjustment of the direct e. m. f. set up by the condenser in its own circuit. This may be done by feeding its potential back to somewhere in the driving system for that component of the motion, depending upon the direction in which it is desired to vary the e. m. f.

The last modification of the equations of motion was not made in this work. Instead, condensers were

inserted where needed for coupling purposes, and the potentials generated by them in their own circuits balanced out by feeding them back through the appropriate vacuum tube couplers. We have pointed out how the condenser was needed in the  $\theta$  circuit to control  $\underline{x}$  through the coupling term  $\mu c_1 \theta$ , but unless these are provided for the other variables it will not be convenient to observe their displacements. Currents, which correspond to velocities can be measured without them, but for the sake of uniformity of design and convenience of measurement these were included for all three variables. The size is unimportant as long as its potential difference is balanced by an e. m. f. in the driving vacuum tube. There is a disadvantage of the above arrangement which can be simply remedied. Since neither end of the condenser is connected to a point of fixed potential, it appears that both ends will somehow need to be active in the balancing operation. Suppose instead of this that two equal arms containing resistances of  $\underline{2R}$  and condensers of

Fig. VI-4





$1/2 C$  are connected in parallel and substituted as shown in Fig. VI-4. The impedance of this combination is

$$\frac{1}{2} \left( 2R + \frac{2}{j\omega C} \right) = R + \frac{1}{j\omega C}$$

This is the same as for the original elements, and so evidently the rest of the circuit is not aware that there has been any change. Now both a resistance and a capacitance have an end held fixed and their potentials at the other ends are all that are needed for coupling. The condensers block out all direct current from the external circuit but one end of the controlling one is still at plate potential level. This may be avoided by splitting it into two parts such as to give equal impedance. If the parts are to be equal, the new capacitances will be twice the old giving

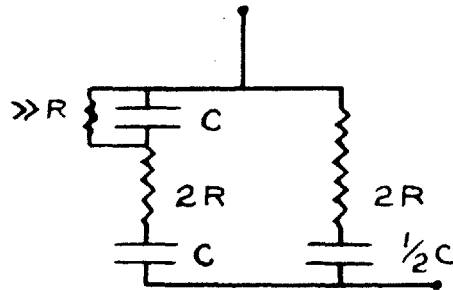


Fig. VI-5

the impedance of the whole still remaining  $R + 1/j\omega C$ . The controlling condenser is now under no steady potential difference if shunted with a high resistance to carry off any leakage through its adjacent blocking condenser.

If the direct effect of these condensers is to be eliminated, they must direct control into the coupling

system at the right point. It is not difficult to see that this should be such as to cause the plate potential of the output tube to be varied in the same direction as that of the ungrounded condenser plate and by twice the amount. The two enters because of the equal condensers in series. Any drop in potential set up by these is thus restored and the balance is complete. It has been pointed out that two tubes are necessary to accomplish this. The connections might be

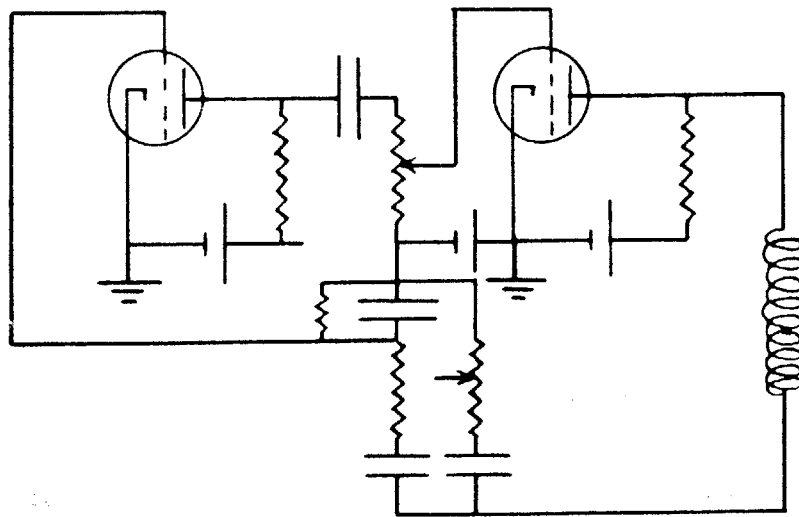


Fig. VI-6

Up to now it has been assumed that control of coupling can be exercised only at the input end. The condenser balancing operation, however, requires that the coupler itself have a fixed amplification, in this case 2. A fraction of the amount developed at the plate of the first tube may be impressed upon the grid of the second by tapping the grid resistor. This is one type of volume control used in audio frequency amplifiers. It allows us here to make the adjustment

necessary for balancing out the condenser effect.

There still exists the problem of knowing when the balance is reached. One way would be to short circuit the inductance and impress an alternating potential upon the coupler. This could also be applied simultaneously to one set of plates of a cathode ray tube. If the potential difference existing across a resistance in the driven circuit is impressed across the other pair of plates, we have a means of observing the phase relation of the two. A low frequency should be used so that the condenser reactance is about of the same magnitude as the resistance. Under these conditions the vacuum tube chain has no phase angle, that is, the e. m. f. applied at the input is exactly in phase with the output. Therefore, if the balance is correct, a straight sloping line should appear in the oscillograph, since resistance alone produces no phase shift.

The above method is good but inconvenient. As a check of this kind should be made quite often to correct for errors arising from aging of tubes and other causes, a simpler one is desirable. An attempt was made to make use of the instability of the system when overbalanced as a means of detecting when balance was just reached. It is easy to show that if a circuit contains negative capacitance no matter how small or in combination with what other elements the current will diverge. Unfortunately at the small rates of divergence which should result from slight overbalance the couplers respond poorly and moreover produce a phase lead,

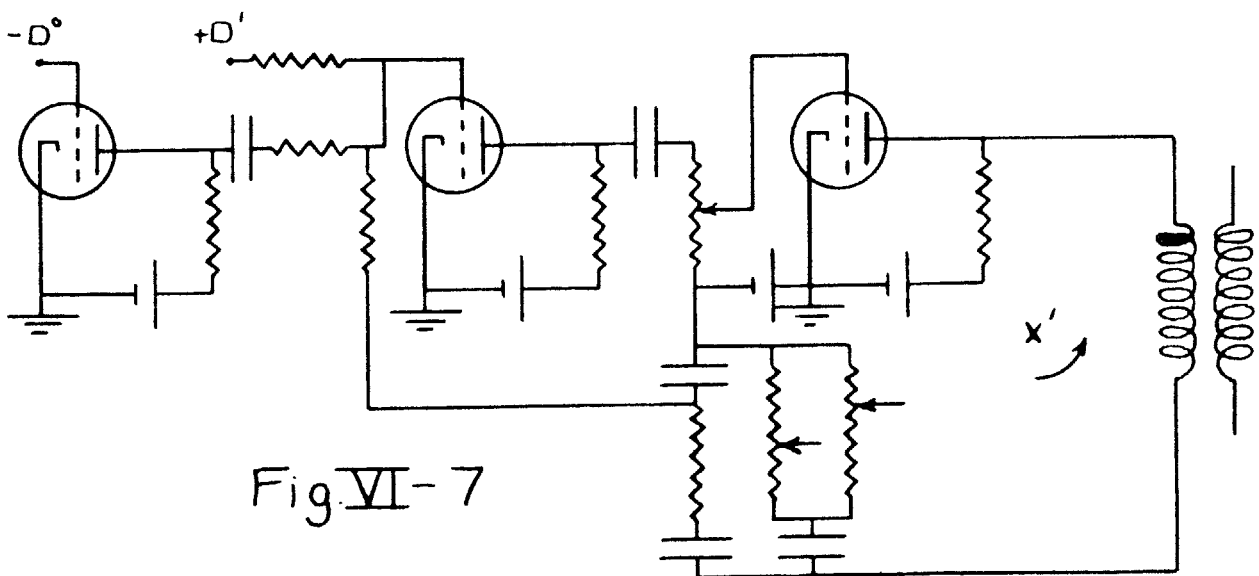


Fig. VI-7

both of which tend to delay the point of incipient instability. The net result was that by this method the coupler ratios were always set too high. It was abandoned for a direct method which will be described later.

We can now return to the modified equations of motion and proceed to build an analogue around the foregoing details. For simplicity we can write an abbreviated sort of coefficient or operator matrix which will indicate the type of electromotive forces everywhere involved. All the terms except  $\underline{m}_1$  will keep the same sign over the range here covered; so we can indicate this in the abbreviation.

$$\begin{array}{ccc} (-D^2 & -D^1) & (+D^1) & (-D^0) \\ (-D^1) & & (-D^2 & -D^1) & (+D^1) \\ (+D^1) & & (-D^1) & & (-D^2 & -D^1) \end{array}$$

where  $-D^2$  indicates an inductance.

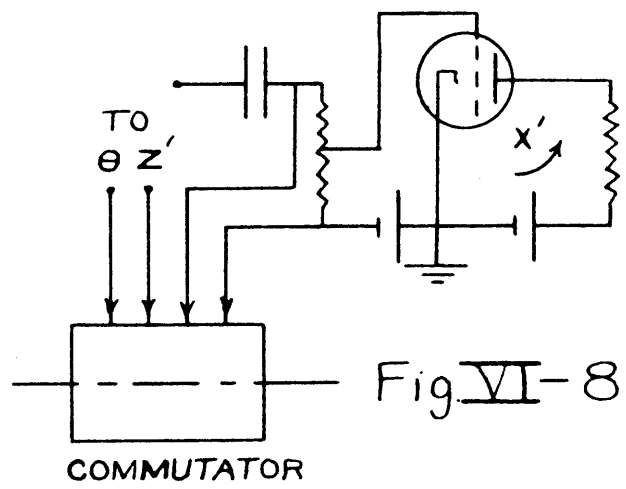
$-D^1$  indicates a resistance.

$-D^0$  indicates a capacitance.

First we shall treat the circuit  $\underline{x}'$  whose potential variations are summed in equation one. This circuit is controlled by a resistance in circuit 2 and a condenser in circuit 3. Its form with driving coupler should be as in Fig. VI-7.  $-D^0$  must be applied before a third tube instead of the first because the balancing operation modifies any network at this point. Three branches are provided in the  $\underline{x}'$  circuit because this controls two others besides providing the means of balancing out its own capacitance. The other circuits

are built up similarly. Beyond this it is difficult to go without making numerical calculations. These will be deferred for the moment to give place to a few details which must be considered before they can be made.

So far nothing has been said about the manner in which disturbances are to be applied and measurements made upon this analogue. A natural way would be to give a single disturbance and record, say, the potential existing at the balance control condensers in each circuit for any desired length of time thereafter. This can be done by passing a light sensitive film at constant velocity across an oscillograph beam for each component of freedom. The D'Arsonval type of oscillograph is more satisfactory for this than the cathode ray, particularly if the frequency is high enough so that afterglow in the latter tends to fog the film. The resulting curves represent displacements plotted against time. There are several disadvantages to such a method. First, there is no way of knowing anything about the type of motion without exposing and developing the film. Also, the D'Arsonval type of instrument requires considerable skill to operate and a relatively large amount of power since there must be a powerful light source, a motor to turn the film drum, and excitation current for the galvanometer field. What we would like is a visual means of observing the electrical phenomena so that the tendency of the solution of the equations for various changes in the coefficients could be immediately apparent without the expense and delay of photographic



operations. Such a method is available.

Although the eye cannot follow the path of a light spot across a screen, if the spot repeats the same cycle in quick enough succession and is of sufficient intensity, its path will appear as a continuous line. Therefore, if instead of giving our analogue a single disturbance we repeat a similar one at equal, close intervals, taking care that the initial conditions are in all cases the same and if, moreover, we so control the oscillograph light spot that it repeats its time sweep at intervals of the same spacing, such a visible line of light should result. As a matter of fact, this is the method actually used in this work. To produce such a repeated motion provision must be made to allow the system to come completely to rest before each successive impulse. In case it is unstable this cannot be accomplished by simply allowing sufficient time to elapse, but must be assured and if possible hastened in all cases by some circuit modification. There are various ways this might be done. The most direct seems to be to shut off all couplers during the period of recovery. This was accomplished here by short circuiting the three grid potentiometers of the output tubes. No potentials can then be impressed upon their grids, and the three component circuits become disconnected and quickly come to rest. A rotating commutator with a single sector and four brushes was used to accomplish this, connected as in Fig. VI-8. The number of degrees occupied by the conducting sector will depend upon the



proportions of time devoted to activity and recovery, while the speed of rotation will determine the frequency of repetition.

We have said nothing about the kind of disturbance to which this analogue is to be subjected. It should allow any kind which the airplane can ever experience, but we were obliged to make a temporary choice which would be fairly typical and if possible straightforward to apply. The only limitation is that during the period under observation the airplane controls must be fixed. Moving controls would require changing coefficients and these are not possible in the present machine. Disturbances arising from roughness in the atmosphere are interesting but difficult to imitate very accurately. The choice made is by no means a final one but happens to be easy to apply and fairly close to nature. It consists in suddenly endowing the  $x'$  circuit with an impulsive current. This corresponds to the situation when an airplane comes into a region of air moving relatively faster toward or away from it in its direction of motion. A vertical gust of wind is probably more common and was first tried electrically, but was less successful in bringing out the long period mode of oscillation which is of principal interest to designers. There are several ways to realize a sudden electric current in a circuit containing inductance, all depending upon diverting magnetic energy into it. The best is to have another winding on a common core with the circuit inductance. While the system is resting up for the next impulse a steady current may be attained in this secondary and, if this is broken at the instant the grid short-

circuited brushes leave their commutator segment, an initial condition of zero displacement in all circuits and zero current in two with an impulsive current in the third will exist. It happens that the same segment as that for shorting the grids may be used with an additional brush directly in line with the others. In this way the secondary is opened just when the grids are released as desired.

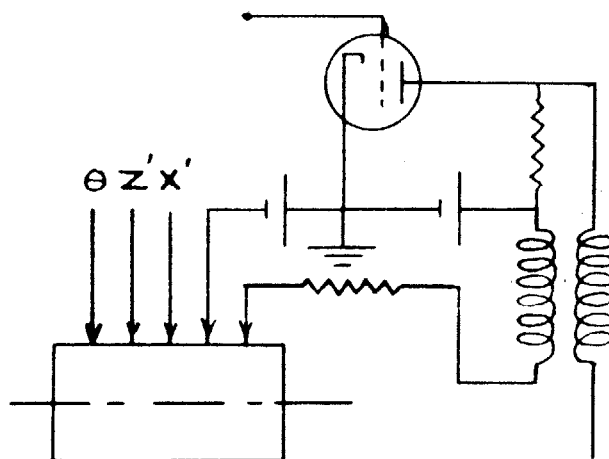


Fig. VI-9

A means of controlling the horizontal or time displacement of the cathode beam is really not a part of our apparatus as it is usually provided in the instrument. The means so provided, called a sweep circuit, causes the displacement with time to vary in a saw tooth form as in Fig. VI-10,



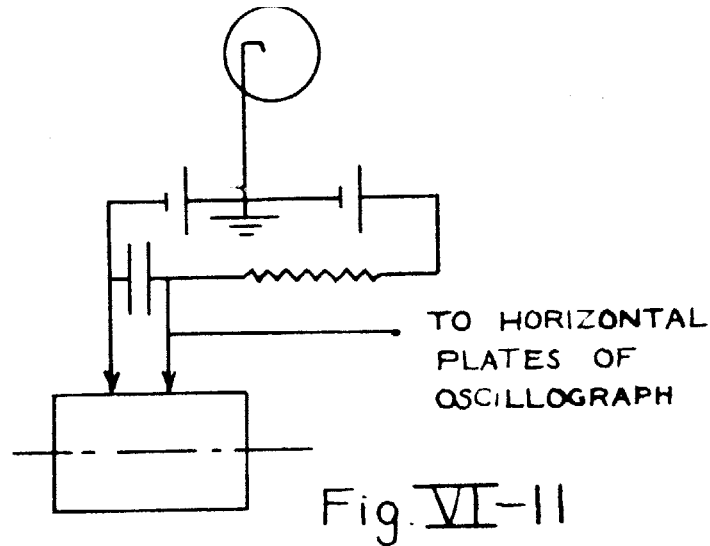
Fig. VI-10

that is, the light spot travels from left to right of the

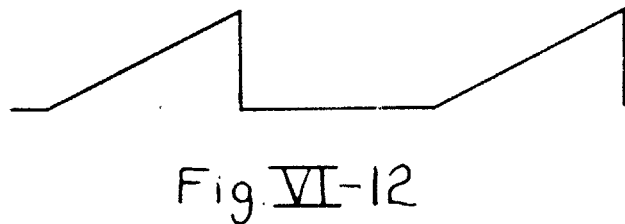
screen with constant velocity and returns almost instantly. The frequency is adjustable. This might be synchronized with the commutator of the analogue and the beam compelled to retrace its path exactly, but there is one disadvantage. It will be remembered that a good proportion of the time the system is occupied in the uninteresting process of preparing itself for the next shock. The above form of time sweep will include this period in the record. Since the screen area is usually none too large on cathode ray tubes, the image could be rendered somewhat easier to examine if the whole screen could be devoted to recording the interesting phase of the motion excluding the other entirely. Also the internal synchronizer of the oscillograph is likely to break the useful record into two parts, one at each end of the scale, instead of holding it entire at the center. Both of these disadvantages can be very neatly avoided by the use of a special sweep circuit.

To cause the cathode beam to move with constant velocity it is necessary that the potential difference between the acting pair of plates should increase uniformly with time. Such a potential difference can be easily obtained by connecting a condenser and resistance in series across a source of constant e. m. f. During the initial charging of the condenser the current flowing is determined almost entirely by the resistance and is therefore constant. The potential difference across the condenser,  $\frac{1}{C} \int i dt = (E/RC)t$ , varies uniformly with time and may be used to control the

cathode beam. When it is desired to return this to the beginning of its path, the condenser must simply be discharged. Suppose this discharge is accomplished by the same commutator which introduces the impulsive current as in Fig. VI-11.



Provided the condenser is never allowed to reach more than about 5% of the plate battery potential, its potential difference will increase nearly linearly with time while the brushes are not in contact. During the period when they are, no charge at all can be built up. The form of the potential is then



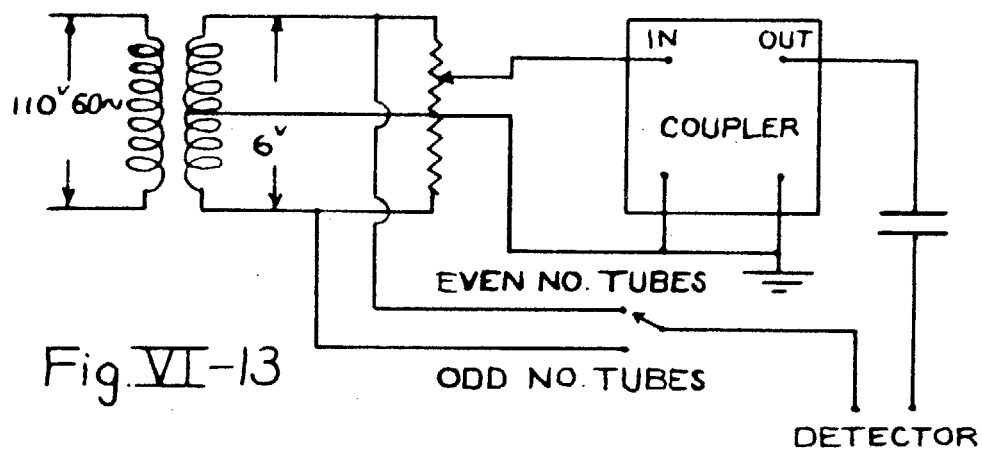
where the cut off is dependent upon the segment angle of the commutator. Thus, during the recovery period of the system the oscillograph beam will oscillate in a vertical path at the left end of the screen. It can then stretch a

record of the part of interest all the way across and, moreover, hold it absolutely in synchronism and perfectly centered.

All the commutator operations referred to thus far can be accomplished by a single wide segment with one row of six brushes. During the period of brush contact the grids will be short circuited, the sweep circuit condenser held discharged, and a steady current allowed to build up in the inductance in the  $x'$  circuit. The rest of the time the grids will be free, the condenser charging, and the inductance connected only into its proper circuit. It will be necessary if this is done to return the currents drawn from the plate battery by the sweep and impulse circuits to the cathode through the bias battery, since the grids must be short circuited to the negative terminal of this. The commutator was first constructed in this way but the segment later split into two sections, one at ground potential, to free the bias battery from the necessity of conducting a direct current.

Beside keeping the amplification ratios of the couplers at their correct values to balance out capacitance, they must also be correctly related to the scales on the control potentiometers if a definite meaning is to be attached to these scales. The numerical design has been carried out on the basis of certain fixed values for these ratios and our problem is now to find a convenient means of checking them. The one in use is a null method, that is, it works by causing the difference between two potentials to vanish.

Essentially it consists of a calibrated attenuator and a source of alternating e. m. f. The attenuator reduces the amplitude of the e. m. f. in known ratios in terms of which it is calibrated. If the amplification ratio of the coupler to be adjusted should be  $\frac{4}{1}$ , the attenuator is set to reduce by  $\frac{1}{4}$  and its output is impressed into the coupler input. The ratio of the latter is then varied until its own output is just equal to the original electromotive force. The equality can be detected by an oscillograph or other high impedance alternating potential meter connected between the two points. When balance is reached no current is drawn from the coupler output and thus its true amplification is indicated. The connections for this are shown in Fig. VI-13.



The low frequency response of the couplers is good enough so that the ubiquitous 60 cycle/second e. m. f. is quite satisfactory as a power source. To make the adjustments the apparatus need only be removed from its case and connections made into tip jacks after withdrawing their plugs temporarily.

The attenuator is calibrated in reciprocal attenuation ratios or in coupler ratios directly.

We intimated earlier that it was desirable to be able to attain a low minimum resistance in all the component circuits to allow the use of as little inductance as possible. If there is a large resistance handicap to start with arising from output tubes and control potentiometers, the necessary inductance for the  $x'$  component at least will be quite large or else the time unit  $1/c\tau$  very short, as at the low values of damping in this component the airplane has a time constant of about 50. To economize on inductance and establish the oscillation in the middle audio frequency range, it was decided to make the minimum convenient circuit resistance correspond to maximum damping coefficient values and produce the effect of lower values by a feedback connection which would partially annul the resistance. As it happens, scarcely any additional complexity is necessary for this. In place of the fixed resistors in the condenser balancing branch of the control bank another potentiometer was connected. The feedback tap was then taken off this rather than the active condenser plate, as in Fig. VI-14. It is possible to combine these functions because in both the sign is the same and the resistance drop through the potentiometer is always added to the total capacitance drop wherever the tap is located. An inherent weakness is the increasing error encountered when the difference of two things of more or less constant uncertainty decreases. The scales were calibrated on the

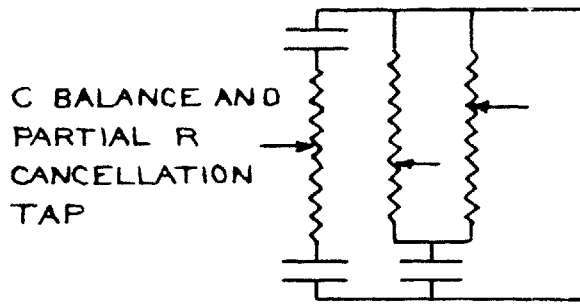


Fig. VI-14

basis of a coupler ratio of  $\frac{2}{3}$ , the amount necessary for balancing the capacitance.

In the laboratory development of the analogue an unforeseen difficulty connected with the phase characteristics of the couplers caused considerable trouble. Ordinarily a small phase change arising in an amplifier has no serious consequences as the initial effect upon the amplification ratio is scarcely detectable, and it is the latter which is of primary interest. Where a group of vacuum tube stages are interconnected in a more complicated manner, however, it turns out that a phase shift of entirely negligible proportions for straight amplification work can cause instability. A case where this may happen is a system using vacuum tube couplers and representing the equations,

$$\begin{aligned} (-D - b_{11})i_1 & \quad (+ b_{12})i_2 = 0 \\ (- b_{21})i_1 & + (-D - b_{22})i_2 = 0 \end{aligned}$$

It may be shown by simple reasoning. If an alternating current exists in circuit  $\underline{1}$ , it will cause an e. m. f. to be applied in circuit  $\underline{2}$  through the coupling term  $-b_{21}i_1$ .



The current in circuit 2 will therefore be generally opposite to that in circuit 1 and will also tend to resist that current through the coupling term  $+b_{12}i_2$ , the latter acting to pull the currents more or less into phase. The result is a rapid damping as can be shown easily mathematically. However, in case the couplers are slow about transmitting their signals, by the time an e. m. f. is coupled from circuit 1 into 2 and back again it may be in such phase as to assist 1 instead of oppose it as called for by the equations. For oscillation to exist the product  $b_{21}b_{12}$  must be somewhat larger than  $b_{11}b_{22}$ , but if the equations of longitudinal stability are inspected, it will be observed that one square of coefficients can be isolated where this is true.

$$\begin{array}{cc} (-D^2 - z_w D) & +\mu D \\ -m_w D & (-D^2 - m_q D) \end{array}$$

At large values of  $\mu$  the product  $\frac{\mu m}{z_w}$  can become a good deal larger than  $z_w m_q$ .

The presence of high frequency oscillations in the analogue led to the above theory which appeared to be verified by its disappearance when the coupler phase lag was compensated for. The lag arose in this way. Most of the tube grids are connected to ground through high impedances. Under these conditions the interelectrode capacitance and especially that between grid and ground cannot be ignored. The grid network of the output tubes is approximately as

shown in Fig. VI-15a.

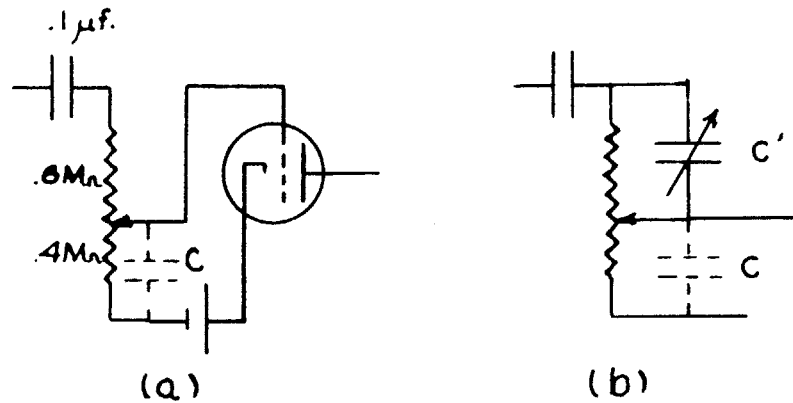
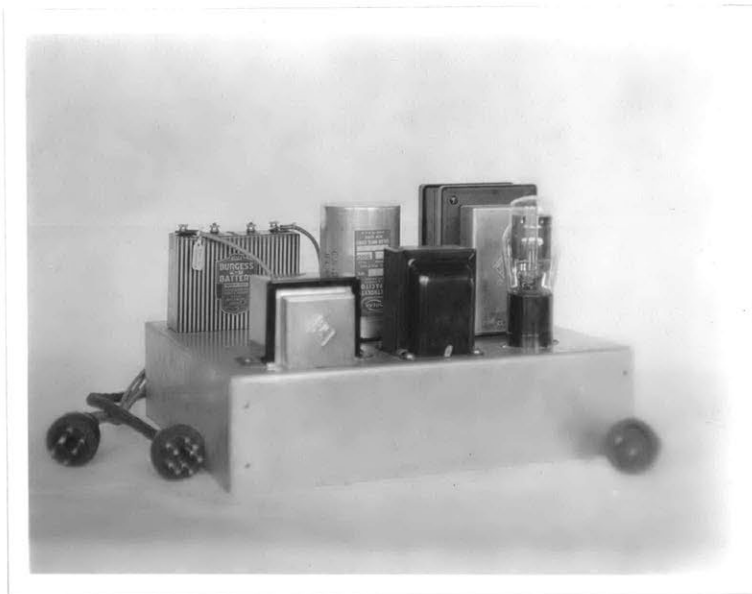


Fig. VI-15

If the shunting capacitance  $C$  is even as small as  $10\mu\mu f$ , the impedance angle of this and  $.04$  Mn in parallel is  $RC\omega = 4 \times 10^{-6}$  which at an  $\omega$  of  $10,000$  or about  $1500$  cycles per second is  $.04$ . The lag between input and output of the network will be about half of this, but it must be remembered that there are many places where additions are made in couplers with several tubes. To eliminate the tendency a small variable condenser  $C'$  of maximum capacitance about  $50\mu\mu f$  is connected as at  $b$  and at similar places throughout the analogue. This is a first order correction and works only for small phase angles but cured the tendency to oscillate very nicely.



Power Supply

## VII

### DESIGN CALCULATIONS

Enough has been said about the details of the design to allow us to proceed to the actual selection of elements. It may seem that this has been unduly delayed, but the interconnection of all parts of the analogue is such that literally a change in a single resistance may require the alteration of half the remainder together with a complete recalibration of potentiometer scales. This high degree of interdependence was very troublesome in carrying out the experimental work because the desirable changes which were inevitably suggested could be incorporated only by practically a complete rebuilding. There are of course an infinite variety of possible analogues and the dimensions of this one are necessarily rather arbitrary. We shall try to make clear the general reasons behind each selection as we proceed without a thorough consideration of all other steps which might have been taken at those points.

A logical place to start is in the selection of tubes. Of these the output tubes exert most far reaching effect because of the resistance they add to the circuits they drive, this having a distinct bearing upon the time scale of the whole analogue. Since the required amplifications will not be high, a triode can be relied upon to serve the purpose. This type also has a lower plate resistance than multi-grid tubes of the same basic size. Just how far it

is desirable to go in reducing plate resistance is not obvious. Power tubes usually have an  $R_p$  in the vicinity of 2000 ohms but may go to less than 1000. They require a good deal of power to operate, however, and if supplied with rectified alternating current, put more exacting demands upon the filter system. Moreover, their amplification factor is low, seldom more than about 3. In spite of this the original experimental set up used 45 tubes as outputs. They were rejected largely because the use of alternating current to light their filaments introduced a large fraction of 60 cycle A. C. interference. In this first attempt 27's were used in all other places because of their simplicity together with sufficient amplification.

In the final model 27 tubes were used throughout. To avoid the necessity for larger inductors than were used in the original, the time scale had to be increased somewhat and also higher resistance control potentiometers provided. In both cases the resistance of the bank of control potentiometers was made about equal to the parallel resistance of the output tube and its load resistor. Also the two condensers in the branch controlling capacitance and resistance feedback were made equal so that a feedback amplification of +2 was sufficient for balance, and an effective resistance range of from zero to a maximum equal to the resistance directly present in the circuit due to tube, load resistor, inductor and potentiometers. All this seems rather involved when stated but may be rendered clearer by Fig. VII-1.

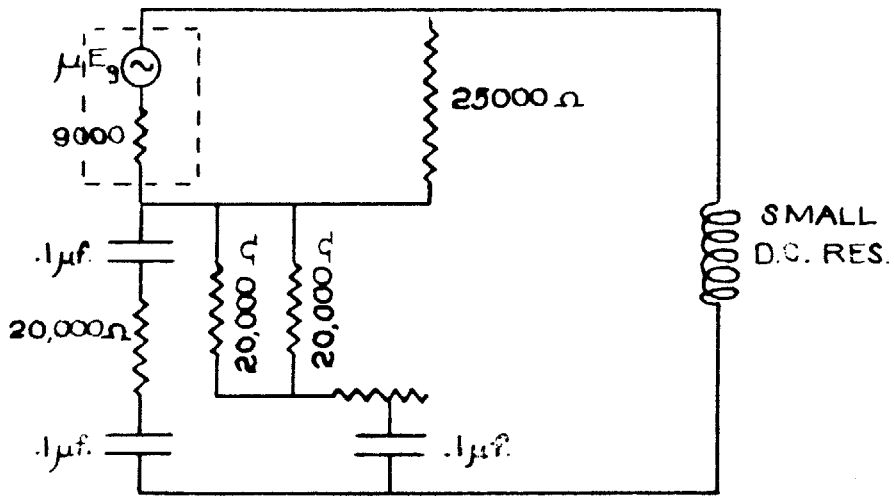


Fig. VII-1

The parallel resistance of tube and load resistor is

$$\frac{9 \times 25,000}{34} = 6620 \text{ ohms}$$

and of potentiometer bank

$$\frac{20,000}{3} = 6670 \text{ ohms}$$

The small adjustable resistance is provided to allow the RC product of the right branch to be matched to the left so that the currents will be in phase. Circuits corresponding to components z' and q are similar except that the q one contains only two potentiometers, its third controlling element being a condenser. In all cases tubes, potentiometers, and condensers are of the same size. Inductors will be different.

The necessary inductor sizes will depend upon the time scale. This is arbitrary and here depended a good deal upon such plebian considerations as the availability

of cheap radio parts which could be adapted. To put the whole history in writing sounds most unscientific but runs somewhat in this way.

1. The largest inductors readily available are filter chokes running about 30 henries at rated currents.
2. In the x component of motion of the airplane the aerodynamic force coefficient in that direction is smaller than the others and will have a maximum value of about -0.2.
3. Recalling that
  - a. the expression in the first row and column of the modified coefficient matrix is  $(-D/c - x_u D)$
  - b. the maximum self resistance corresponding to maximum  $x_u$  (-0.2) is about 13300 ohms.
4. The value of c necessary with an inductance of 30 henries is
$$1/.2c = 30/13300$$
$$c = 2250$$
5. But at zero direct current the chokes available all ran very low and so
6. A value for c of 5000 was chosen.

This is not quite the true scale as the modified time unit is  $(1/c\tau)$  where  $\tau = m/(l^2/2 SV)$  and is in the neighborhood of 1.5 to 2.0. This little story is included to illustrate the vicissitudes of design. Whenever an apparently arbitrary step is taken a similar explanation may be assumed.

It is time we decided within what limits we wish to be able to vary the airplane coefficients. These will not be rigidly adhered to but are necessary to reach

the approximate sizes of the electrical elements. Such a tentative plan follows.

$x_u$ (-0.02 to -0.2)	$x_w$ , (0 to +0.8)	$c_1$ , (0 to -100)
$z_u$ (0 to -2.0)	$z_w$ , (-3 to -5.5)	$\mu$ , (0 to +100)
$m_u$ (0 to +0.05)	$m_w$ , (0 to -10)	$m_q$ , (-5 to -12)

Now if the total direct resistance in each component circuit is 13,300 ohms as explained, and if when this is entirely unopposed the situation is to correspond to the maximum values of the coefficients  $x_u$ ,  $z_w$ ,  $m_q$  respectively, and if, moreover, the value of  $c$  is 5000, the required inductances in the three circuits will be

$$L_x / (1/5000) = 13,300 / .2 \quad ; \quad L_x = 13.3 \text{ henries.}$$

$$L_z / (1/5000) = 13,300 / 5.5 \quad ; \quad L_z = .483 \text{ h.}$$

$$L_\theta / (1/5000) = 13,300 / 12 \quad ; \quad L_\theta = .222 \text{ h.}$$

It is rather difficult to make an inductance come up to an exact specification. However, this is not necessary as the upper limits set upon the coefficients  $x_u$ ,  $z_w$ ,  $m_q$  are only approximate. The scheme followed was to bring the inductances to about the correct values and then figure back to find the corresponding coefficient values. These actually turned out to be

$L_x = 13.8 \text{ h.}$	$x_u(\text{max.}) = \frac{13,300}{13.8 \times 5000} = -.193$
$L_z = .483 \text{ h.}$	$z_w(\text{max.}) = \frac{13,300}{.483 \times 5000} = -5.5$
$L_\theta = .226 \text{ h.}$	$m_q(\text{max.}) = \frac{13,300}{.226 \times 5000} = -11.8$



$L_x$  was made up of two 30 henry chokes with a small gap in the iron circuit.  $L_z$  and  $L_\theta$  were made by inserting iron in old transformer coils.

It will be recalled that the modified form of the equations of motion with variables  $x/l, z/l, \theta, t/cT$  developed in chapter V was referred to as a dimensionless form. Although this is consistent with the usual nomenclature, it conveys a misrepresentation. The application of mathematics to the physical world can never be dimensionless, for in this, numbers in general have no meaning apart from the process of measure. What we meant was that the units of length and time were so chosen that the set up and solution of the equations of motion was of the same numerical form for all geometrically similar airplanes flying in the same attitude provided only that  $\mu = m/(\rho/2 S l)$ , often referred to as the "relative density" of the airplane, remained constant. Instead of fixing units once and for all and letting the equations take their consequent form, we chose to vary the units to keep this form always the same. But in a very real sense we still had a unit of length  $\underline{l}$  and a unit of time  $\text{cm}/(\rho/2 S V)$ , where these were true physical entities regardless of whether they were expressed as so many inches, feet, centimeters or hours, seconds, years. Now in drawing a correspondence between airplane and analogue it is necessary first to draw a correspondence between units in the two physical realms. If the equations governing motion in both are then of the same numerical form, it

follows that the dimensions of the analogue in terms of its units will be the same throughout the motion as the dimensions of the airplane in its own. There is no reason for choosing an unusual set for the former as there is no question of comparing a variety of electrical forms with aspects of similarity. Adopting the usual engineering system, the correspondence has been made as follows.

<u>Kind of unit</u>	<u>Mechanical</u>	<u>Electrical</u>
Displacement	The geometrical length $\underline{1}$	1 Coulomb
Time	The physical time $\frac{5000m}{c/2 SV}$	1 Second

Our problem is now simply to make the governing equations expressed in terms of these look the same. If this is attempted at once, it is found that the problem is not very easy if even possible using common electrical parts for the analogue. The next step is a purely mathematical one. We developed a system of multipliers to produce a new set of equations which, however, would have the same solutions as the others except for differences readily adjustable. The electrical correspondence was drawn with this set instead of the original one. Of course these steps could not be taken in exactly the order stated. Before it could be ascertained just what kind of transformation of the last sort to make, a good deal had to be known about the limitations of available parts. All the reasons for the endless shifting too and fro are too numerous to recount here. As we have said

the solution is not unique.

The thread of the design may now be resumed. We have chosen all the elements of the three component circuits - - tubes, potentiometers, condensers, and inductors. The sizes of the last were determined by the necessity of keeping the ratio of coefficients of  $\underline{D}^2$  and  $\underline{D}^1$  the same for airplane and analogue for each component circuit. Now the coefficient of  $\underline{D}^2$  in the first row, first column is 1/5000 for the airplane, 13.8 for the analogue. To convert the equations to the same numerical form, the multiplier  $a_1/b_1$  must be applied to the airplane coefficient.

$$\begin{aligned}(1/5000)(a_1/b_1) &= 13.8 \\ a_1/b_1 &= 69,000\end{aligned}$$

similarly

$$\begin{aligned}(1/5000)(a_2/b_2) &= .483 \\ a_2/b_2 &= 2415 \\ (1/5000)(a_3/b_3) &= .226 \\ a_3/b_3 &= 1130\end{aligned}$$

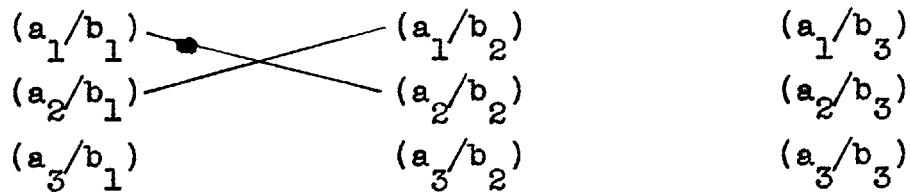
The principal diagonal is now taken care of. All three constant multipliers could be arbitrarily chosen. This is possible for only two of the rest. If  $a_1/b_2$  is next selected at will,  $a_2/b_1$  becomes

$$\frac{(a_1/b_1)(a_2/b_2)}{(a_1/b_2)} = a_2/b_1$$

Next  $a_2/b_3$  might be selected whence follows,

$$\begin{aligned}
 (a_1/b_3) &= (a_1/b_2)(a_2/b_3)/(a_2/b_2) \\
 (a_3/b_2) &= (a_2/b_2)(a_3/b_3)/(a_2/b_3) \\
 (a_3/b_1) &= (a_2/b_1)(a_3/b_2)/(a_2/b_2) \text{ or } (a_1/b_1)(a_3/b_3)/(a_1/b_3)
 \end{aligned}$$

All of these can be readily seen from the matrix. If any  $\underline{X}$  is drawn, the products of both pairs so determined will be the same.



There are thus five arbitrary ratios and one arbitrary constant equivalent to the six arbitrary constants  $a_1, a_2, a_3, b_1, b_2, b_3$ . The values finally selected will now be set down along with the coupler ratios necessary to represent the new equations.

$$\begin{array}{lll}
 (a_1/b_1) = 69,000 & (a_1/b_2) = 2080 & (a_1/b_3) = 180 \\
 (a_2/b_1) = 80,000 & (a_2/b_2) = 2415 & (a_2/b_3) = 209 \\
 (a_3/b_1) = 433,000 & (a_3/b_2) = 13,050 & (a_3/b_3) = 1130
 \end{array}$$

Coupler ratios.

$$\begin{array}{lll}
 -2 & +2 & -27 \\
 -24 & -2 & +4 \\
 \underline{+4} & -20 & -2
 \end{array}$$

The -2's on the principal diagonal are those necessary for the elimination of the effect of capacitance in the component circuits. A matrix of maximum coefficient values will also

be set down

-.193	+.8	-100
-2.0	-5.5	+127.5
<u>+.0615</u>	-10.2	-11.75

These are found as follows. For all the resistance controlled terms except  $x_w$  the maximum available resistance at the potentiometer bank is  $20,000/3 = 6670$  ohms. This times the coupler ratio divided by the constant multiplier for that term will be equal to the maximum value of the airplane coefficient. That is, the product of 6670 by terms of the second matrix above divided by corresponding terms in the first should give corresponding terms in the third. For  $z_u$  this gives

$$z_u (\text{max.}) = (6670)(-24)/80,000 = -2.00$$

and similarly for the others. The coefficients whose maximum values were arbitrarily set are the ones which appear integral in the matrix, that is,  $z_u (\text{max.}) = -2.00$ ;  $\mu c_1 (\text{max.}) = -100$ . Others are decimal because they were calculated from these. The apparent exception  $x_w (\text{max.}) = +.8$  is controlled by a 250 ohm potentiometer in series with a fixed resistance of 17,500 ohms instead of one of 20,000 ohms. It is desirable to have coupler ratios integral because otherwise commercial fixed resistors cannot be used in several places.

We can proceed with a complete coupler design now that all the potential ratios between input and output

are known. The one driving the  $\underline{x}'$  component circuit will first be considered. This is to have three input points with respective ratios of +2, +2, -27. The first two will be tied to taps on relatively low resistance (20,000 ohm) potentiometers since they represent first derivative terms. The last represents a displacement term and could be tied to the free end of a variable condenser or to a tap on a high resistance (1 megohm) potentiometer shunting a fixed condenser. The latter system is much simpler since a variable condenser of .1 ~~μ~~f. would be quite difficult to construct unless it varied by steps. However, it was pointed out when high resistance input networks were considered that the action of these is independent of impedances at the ends only when these are small in comparison. Such a high resistance potentiometer would not satisfy this requirement. Fortunately there is only one negative input to the  $\underline{x}'$  coupler and so the problem of addition need not be dealt with. The negative input may be fed directly to the grid of a third tube whose resistance is high even compared to the condenser-shunting potentiometer. The circuit diagram for the complete coupler is shown in Fig. Vll-2. The grid resistances at the + input points for  $x_u$  and  $x_w$  are made equal so that the amplification ratios for both will be the same. Where different amplifications are desired the resistances must be in the inverse ratio. Because of possible variations in the third or negative input stage, the coupling between this and the second is through an adjustable resistance which must be reset from time to time.

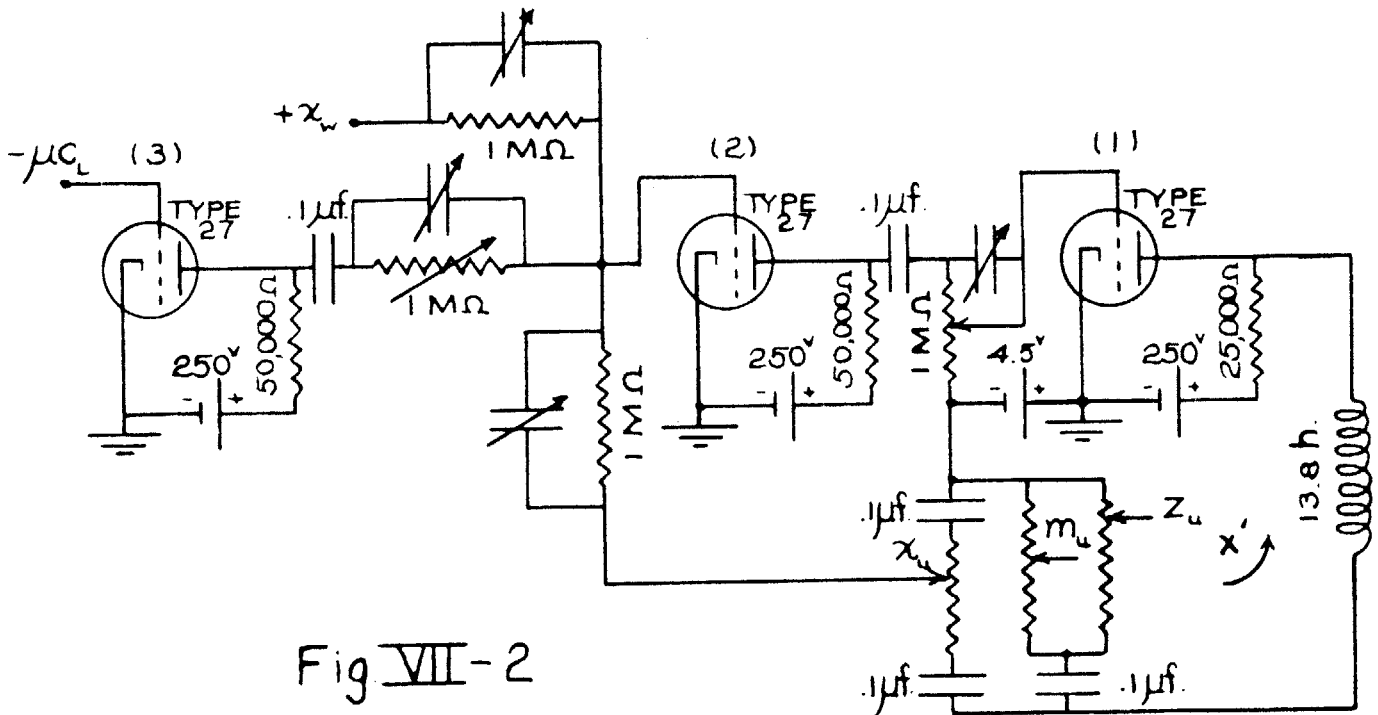


Fig VII-2

The next problem is the calibration of the potentiometers to read directly in values of the coefficients. One way would be to measure the resistance of each one individually between tap and fixed end and mark for the value of the coefficient at every point the quotient of this resistance multiplied by the coupler ratio, by three times the constant  $a_m/b_n$  for the particular coefficient. The three enters because there are three equal potentiometers in parallel. There is another way which is not affected by slight variations in the total resistance of the potentiometers or by the small resistances added to make the RC products of both branches the same. The condensers are first short circuited. If a current of 1 ampere is now passed through the bank, points at a potential of 1 volt above the fixed end will correspond to coefficient values

equal to the appropriate coupler ratios divided by the appropriate constants  $a_m/b_n$ . The coupler ratio enters because it indicates how a controlling resistance appears to the driven circuit when viewed through the coupler. It is not desirable to use a voltmeter to measure potential as this consumes some current and alters the conditions slightly. A better way is by the use of a bridge. If a calibrated potentiometer with one end at zero potential has this end tied to the fixed end of the bank in the analogue and if a current of 1 ampere is likewise passed through it, the potential in volts at any point will be numerically equal to the resistance in ohms between the movable tap of this with that of the potentiometer to be calibrated, a measure of potential is possible without altering original conditions.

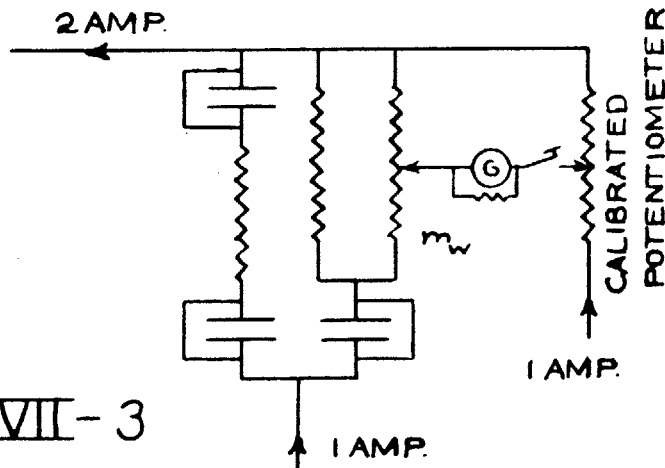


Fig. VII-3

Moreover, it is not necessary that the currents be 1 ampere but only that they be equal. To take a concrete example

$$a_3/b_2 = 13050$$

$$m_w \text{ coupler ratio} = -20$$



Therefore the point on the  $m_w$  potentiometer at the same potential as the  $1$  ohm point on the calibrated potentiometer corresponds to a value of  $m_w$  of  $-20/13050$  whatever the current flowing. Now this conversion factor between the two scales is rather inconvenient. We would like to have it either unity or some simple number. If the current through the calibrated potentiometer is made  $13,050/20$  of the other,  $1$  ohm will then correspond to  $1$  coefficient unit. To make this clearer the actual set up will be described.

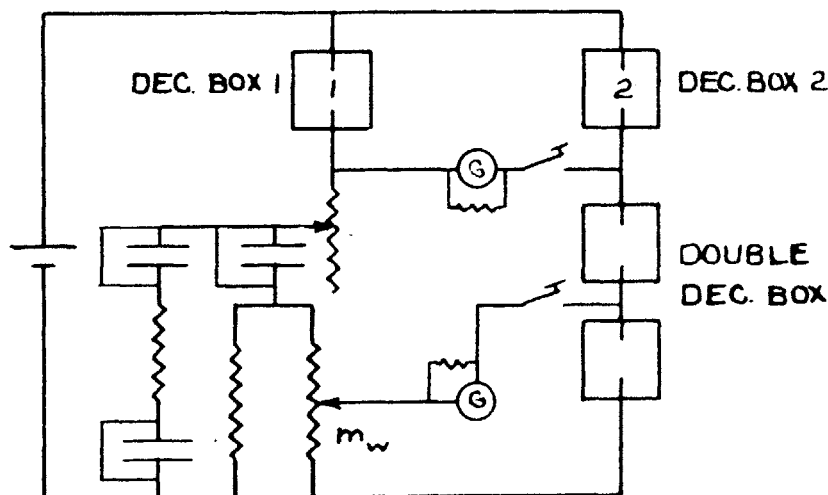


Fig. VII-4

The double decade box had a total resistance of 1111 ohms which remained the same for all settings. The other two were single and were used only for setting the current ratio through the two branches which stayed the same throughout the calibration of each potentiometer. If it is desired to make the current through  $1$  twice that through  $2$ , the

resistance of box 1 is made  $1/2$  that of 2, and when the upper galvanometer indicates a potential balance the currents are in the correct relation. The decision as to what this ratio was to be was made as follows for  $m_w$ . This coefficient has a range of about 0 to -10;  $a_3/b_2 = 13,050$ ; coupler ratio = -20. The calibrated potentiometer runs from 0 to about 1000 ohms. Now we would like to have both taps reach their upper limits at about the same time. This will be influenced by the current ratio. Since the resistance of the potentiometer bank is about 6670 ohms, its current must certainly be less than  $1000/6670$  times the other if calibration of the whole scale is to be made, for otherwise there would be points on it at a higher potential than any in the double decade box. We shall, then, endeavor to bring this current as close to  $1000/6670$  of the other as is consistent with a simple scale relationship between the two resistances. If one unit of  $m_w$  is made to correspond to 100 ohms on the double decade box, a ratio of the current through box 1 to that through box 2 of  $(100)(20)/13,050 = 1/6.52$  is required. The two single decade boxes were about 10,000 ohms maximum each. Number 2 was set at 1000 ohms and number 1 at 6520. A slider rheostat was adjusted in the left branch until the upper galvanometer indicated a balance. It was placed in this branch because in all cases (except for  $m_w$ ) the current in this was less than  $1/6.67$  of the other and required the addition of resistance. In this way

the double decade box always had a little to spare at its upper end. Then the  $m_w$  potentiometer was calibrated by balancing against the double decade box calling 100 ohms in this equivalent to 1 unit. The table of values of current ratios and decade box settings for all coefficients is given on page 80. The high resistance potentiometer controlling the capacitive coupling term for  $\theta$  acting upon  $x'$  of  $-\mu c_1$  was calibrated separately against the same double decade box by shunting it across the terminals and dividing it as shown.

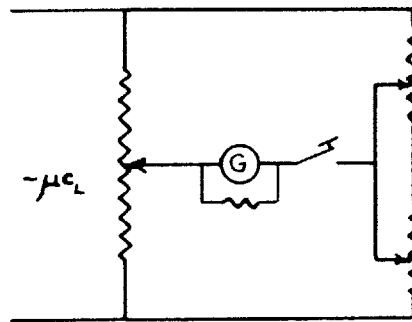


Fig. VII-5

Since the condenser size is such as to give a maximum value for this coefficient of  $-100$  exact, the potentiometer could be marked off with  $1/100$  of the decade resistance against each unit without the necessity of adjusting the current through the two branches.

The last step in adjusting the complete analogue is the alignment of the small condensers shunting high resistances in the grid input networks. As was explained in chapter VI, these serve to keep the couplers from shifting the phase of the impressed wave at high frequencies thus

Coef.	Range	Coupler Ratio $\frac{a_m}{b_n}$	No. ohms bal. against 1 unit	Ratio $\frac{i_1}{i_2}$	Single Decade Branch 1	Resistances Branch 2
$x_u$	-.02 to -.2	2/69,000	5000	$\frac{2 \times 5000}{69,000} = 1/6.9$	6900 ohms	1000 ohms
$z_u$	0 to -2.0	24/80,000	500	1/6.67	6670	1000
$m_u$	0 to $\pm 0.06$	4/433,000	10,000	1/10.8	1080	100
$x_w$	0 to +.8	2/2080	100	1/10.4	1040	100
$z_w$	-1.0 to -5.5	2/2415	100	1/12.1	1210	100
$m_w$	0 to -10	20/13050	100	1/6.52	6520	1000
$\mu$	0 to +120	4/209	10	1/10.5	1050	100
$m_q$	-2.0 to -12	2/1130	50	1/11.3	1130	100

eliminating a source of error and also oscillation. A beat frequency oscillator was used to operate a cathode ray oscillograph by connecting its output directly to one pair of plates and through the coupler to be adjusted to the other pair. Thus if the coupler produced no shift, a straight diagonal line appeared on the screen, otherwise an ellipse. Because of the amplification of the coupler an attenuator had to be connected across the oscillator output to reduce the input in the same proportion so that both inputs to the oscillograph would be about the same. This adjustment could have been made right at the oscillograph preamplifier except that this was of the high resistance grid potentiometer type and had a phase shift of its own. The oscillator attenuator was merely a tapped load resistor of 18,000 ohms in the output plate circuit. Because of its low resistance almost no phase shift was observable below 10,000 cycles per second. In making the adjustment the oscillator was connected to one of the several input points of the coupler. The shunting condensers were then adjusted to give a straight line on the oscillograph. Then the oscillator was connected to another input point and the operation repeated. It took very little time to arrive at a set of adjustments which would eliminate phase shift at all input points. This operation need never be repeated. It should be mentioned that this must be carried out with the amplification ratio adjusting potentiometers set at their approximate running values. Small deviations are unimportant.

## ADDENDUM

When the analogue was finally completed according to the preceding specifications, it became evident from the erratic character of the oscillograph record at certain sets of coefficient values that undesired interactions of some kind were present. These were found to arise from the same high frequency oscillation which was believed to have been eliminated by the phase regulating condensers. Since the real cure was found only after more than a month of continual experiment and after a description of the work had already been written out, an account of the overcoming of the difficulty will be appended here.

At first, improper phase compensation was blamed for the trouble and various other settings of the regulating condensers were tried. It was always possible to stop the oscillation in this way but upon changing the coefficient values it would always appear again somewhere else, more often than not where stability had existed before. This chase soon proved to be futile as there were always wide regions of instability with any adjustments made.

Next, suspicion was directed at the inductors which simulate mass in its three component aspects. The iron losses in these were known to be a source of error as hysteresis and eddy current effects have no counterparts in airplane motion, but there was no obvious way in which this could contribute to the ability to oscillate. Air core

coils were tried in two places with very little change. There was still the possibility that shunting capacitances existing between turns of the coil might be effective especially as the frequency of the oscillation was about 20,000 cycles per second. However, these are never completely avoidable, and if they lay at the root of the trouble a different kind of circuit equivalent to the original in certain ways should solve the problem if such could be found.

The solution turned out to be such an equivalent circuit. It will be recalled that the three circuits representing  $\underline{x}$ ,  $\underline{z}$ , and  $\Theta$ , although containing several branches, could be considered equivalent to resistance, inductance, capacitance and a source of alternating e. m. f. in series. Currents corresponded to velocities and coupling effects were contributed by the reactionary e. m. f.'s set up by certain resistors and condensers. Now if these controlling elements could be connected into some other system containing a source of e. m. f. and of such a character that this e. m. f. was related to the current through the controlling elements just as in the original, the equivalence would be sufficient to allow of our substituting this new system for the old. The problem, stated more precisely, is to find a four terminal network having a transfer impedance, when resistance and capacitance in series are connected across the output terminals, equal to the input impedance of the original series system or at least proportional to it. Within certain limits the system shown in Fig. VII-6 satisfies the requirement.

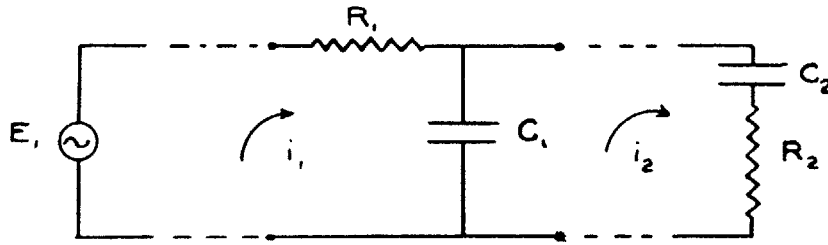


Fig. VII-6

The equations of motion for this are,

$$\begin{aligned} \left(R_1 + \frac{1}{j\omega C_1}\right)i_1 + \left(-\frac{1}{j\omega C_1}\right)i_2 &= E_1 \\ \left(-\frac{1}{j\omega C_1}\right)i_1 + \left(R_2 + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}\right)i_2 &= 0 \end{aligned}$$

Solving,

$$i_2 = E_1 A_{12} / \Delta = \frac{E_1}{j\omega C_1 R_1 R_2 + R_1 + R_2 + \frac{R_1 C_1}{C_2} + \frac{1}{j\omega C_2}}$$

The same current  $i_2$  would exist in a series circuit containing,

$$\begin{aligned} L &= C_1 R_1 R_2 \\ R &= R_1 + R_2 + \frac{R_1 C_1}{C_2} \\ C &= C_2 \\ E &= E_1 \end{aligned}$$

If  $C_1$ ,  $R_1$ ,  $R_2$ , and  $C_2$  are properly chosen, it is in many cases possible to satisfy the above equalities with  $L$ ,  $R$ , and  $C$  given. In order not to be obliged to change the



parts already calibrated it will be convenient to let  $R_2$  represent the resistance of the bank of control potentiometers and  $C_2$  the capacitance effectively in series with this. If the relations can be satisfied under these conditions it will not be necessary to make any changes in the coupler connections. Let  $R_1$  equal the driver resistance, that is, of the output tube and its load resistor in parallel. Then, knowing  $L$ , we can find  $C_1 = L/R_1R_2$ . However, the second relation has now passed out of control as  $R_1, R_2, C_1, C_2$  have already been determined. Moreover, the original  $R$  was equal to  $R_1 + R_2$  and hence the equality cannot now be satisfied because of the term  $R_1C_1/C_2$ . The difficulty can be escaped by considering that the original circuit contained resistance not equal to  $R_1 + R_2$  but to  $R_1 + R_2 + R_1C_1/C_2$ . In the new system, therefore, the maximum values of the coefficients  $x_u, z_w, m_q$  would be larger than before. This can very simply be corrected for by displacing the dials on the potentiometers representing these coefficients so that when no feedback is provided the indicated coefficient values are these new larger ones. All other readings will then be correct. In the case of circuit  $\underline{x}'$  the value of  $C_1$  found from  $L = C_1R_1R_2$  was so large as to make the term  $R_1C_1/C_2$  difficult to deal with. As  $C_2$  is here unnecessary for coupling purposes it was removed, that is, made infinite and a 4 f. blocking condenser inserted in the driver lead. The effect of the latter can be safely neglected. Measurements upon the three component circuits and calculations

were made as follows:

<u>Circuit</u>	$\underline{R}_1$	$\underline{R}_2$	$\underline{C}_2$	$\underline{L}$	$\underline{C}_1 = L/R_1 R_2$
x'	5900 ohms	7000 ohms		13.8 h.	.334 $\mu$ f.
z'	5700	6400	.15	.483	.0132
$\theta$	6200	6700	.15	.226	.00544

<u>Circuit</u>	$\underline{R}_1 C_1 / C_2$	$\underline{R}_1 + \underline{R}_2 + \underline{R}_1 C_1 / C_2 a_n / b_n$	<u>Max. coef. values</u>
x'	0 ohms	12,900 ohms	.187 ( $x_u$ )
z'	500	12,600	5.22 ( $z_w$ )
$\theta$	224	13,100	11.6 ( $\theta'$ )

Since these changes were made no trace of uncalled for oscillation has been seen. The method is inherently better in other ways as the losses peculiar to iron are entirely absent, there being no iron at all in active places. A small change is necessary to provide for an initial impulse corresponding to the suddenly applied current used before. This effect turns out to be furnished if the shunting condenser  $C_1$  is charged and connected into its proper place. Fig. VII-7 shows how this switching may be performed rapidly.

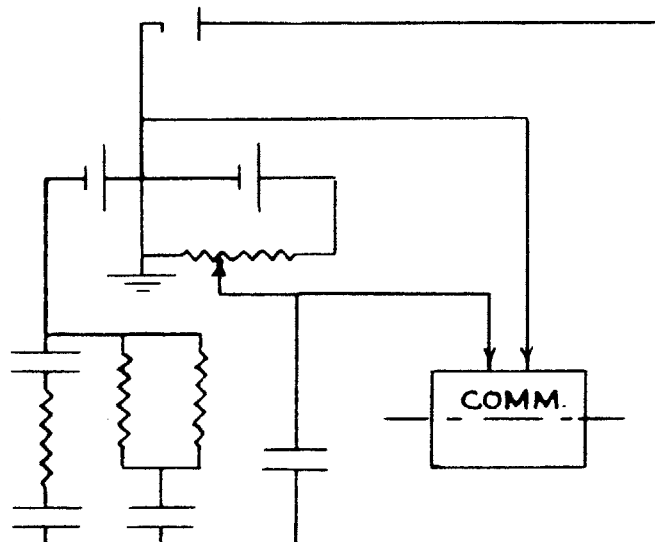


Fig. VII - 7

A disadvantage lies in the fact that the brush grounding the lower side of the shunting condenser is in contact during the progress of the motion and therefore may affect this if its contact is poor. Resistance is particularly intolerable in this place as the branch contains none at all. Careful and frequent cleaning of the commutator is necessary and constitutes the most bothersome maintenance detail at present. The use of multi-contact brushes is imperative.



Coupler Ratio Indicator

## VIII

### TESTS AND CONCLUSIONS

In an electrical system of the complexity of the stability analogue it is virtually impossible to form a dependable estimate of the degree of error which may be expected in any problem. Where the arrangement is such that a single chain exists between input and output, as in a vacuum tube amplifier, errors introduced at separate points may be found quite accurately, and the maximum possible overall error is not far from the sum of these. When more complicated cases are encountered, the direct method of submitting actual problems and checking results with answers calculated or found accurately in some other way is the only really satisfactory one. The result so found is <sup>as</sup> only good as the number of examples treated and is always subject to revision by the outcome of further tests.

Mr. W. F. Millikan of the Staff in Aeronautical Engineering has kindly supplied sets of aerodynamic coefficients with periods and dampings calculated therefrom for several different airplanes. The calculations for the latter were performed according to approximate methods, but in view of the uniformly close approach to coincidence of the results so found and those coming from the electrical analogue, there seems little reason to believe that their error is serious. On page 89 is a table showing the comparison.

<u>Subject</u>	<u>Power</u>	$\alpha$	$\mu$	$c_1$	$\frac{x_u}{x_w}$	$\frac{\mu c_1}{z_u}$	$\frac{z_u}{z_w}$	$\frac{m_u}{m_w}$	$\frac{m_u}{m_q}$			
Twin-eng. transport	1200 H.P.	-	16.9	-.214	-.089	+.126	-3.62	-.43	-4.53	-.032	-4.31	-8.90
10 pass., high speed trans.	800	-	19.4	-.21	-.071	+.115	-4.07	-.42	-4.4	-.046	-5.3	-8.72
Kendrick sailplane	0	3°	3.54	-.6	-.04	+.47	-2.12	-1.2	-5.08	0	-4.95	-8.8
Kendrick Sailplane	0	8°	3.54	-.95	-.076	+.74	-3.36	-1.9	-5.08	0	-5.17	-8.8
Millikan-2 airplane	50	2°	11.7	-.198	-.079	+.079	-2.32	-.40	-4.07	-.038	-3.58	-9.08

<u>Subject</u>	<u>Calculated by Millikan</u>		<u>Found from Machine</u>	
	<u>Period</u>	<u>Time to damp to 1/2</u>	<u>Period</u>	<u>Time to damp to 1/2</u>
Twin-eng. transport	27 (t/τ)	-	26 (t/τ)	12 (t/τ)
10 pass., high speed trans.	25.7	-	25	10
Kendrick sailplane	14.1	17.7 (t/τ)	12	12
Kendrick sailplane	8.91	8.4	8	9
Millikan-2 airplane	32.7	18.58	27	20

Table VIII-A

A part of the error in these results can be laid to the oscillograph as the beam deflection is not strictly proportional to the potential difference applied between the plates. This is caused partly by a change in beam focus for different positions making it difficult to estimate the mean center of the light spot. The most accurate way of finding the period is to divide the time corresponding to a number of oscillations by this number. It is best to omit the first one or two in forming this group as these are contaminated by the transients due to the impulsive start. The transients, being usually highly damped exponentials, quickly decay and leave only the long period motion which is the calculated one.

This assortment of examples does not illustrate very well the breadth of possible motions which may be encountered in airplanes. To show this a little better a few will be added which represent variations upon a design for a 4 passenger monoplane having a top speed of about 158 M.P.H. and a 225 H.P. engine, made by some Seniors at M. I. T. Its exact details are not important for these purposes. In the calculations at different speeds and flight attitudes the effect of propeller thrust and slipstream variations have been neglected as these, though important, tend to obscure certain basic trends of stability in flight with power off.

(1) The first example (Fig. VIII-1) shows the character of motion after a fixed disturbance at cruising speed (140 M.P.H.) and at a relatively high angle of attack

corresponding to about 67 M.P.H. The tendency of the period to shorten at the lower speed is typical of all conventional airplanes. Because of the presence of the factor  $\tau$  which is inversely proportional to speed and therefore larger at lower speed, the shortening of period is less than would appear from the electrical solution. Actually the times are about 35 seconds and 17 seconds. Data for these results as well as those of the following examples are found in Table VIII-B.

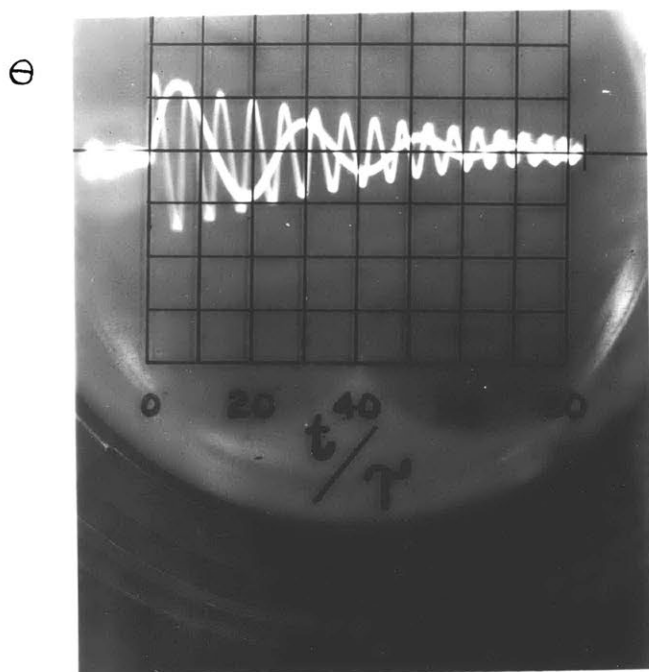


Fig. VIII-1

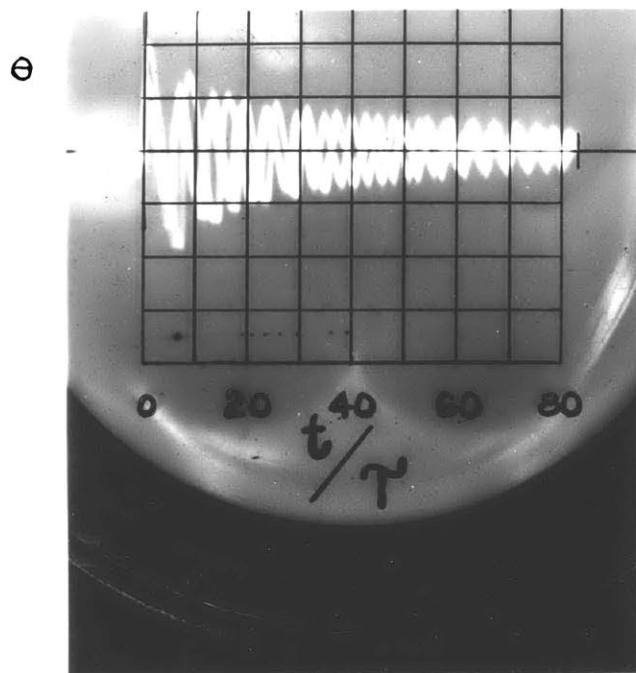


Fig. VIII-2

(2) Since airplanes of different sizes often show a remarkable tendency to look alike, the effect of size with constant wing loading is interesting. (Fig. VIII-2) Here is shown the results at the same high angle as above



of halving and doubling the size as originally given. The motions as found electrically are comparable except for amplitude, since the quantity  $\tau$  is unchanged. However, the amplitudes (angular) are not so because the disturbance is proportional to the size. By halving the amplitude for the large airplane and doubling that for the small one as in Fig. VIII-2a the results are completely comparable. This correction is necessary because the equations around which the electrical analogue has been designed are a dimensionless set.

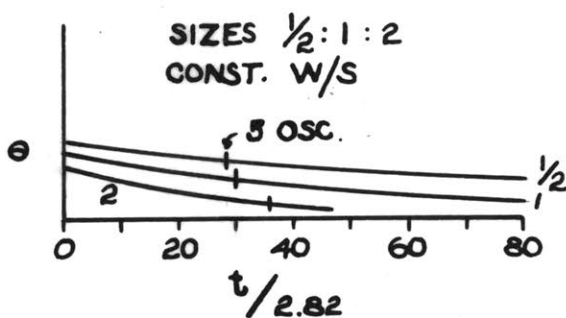


Fig. VIII-2a

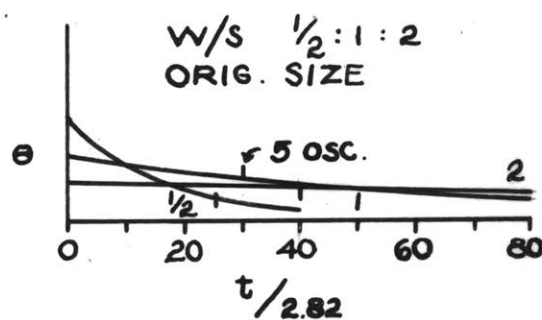


Fig. VIII-3

(3) A design tendency which has been continuing since the earliest days of flying is the increase of wing loadings. Although this has been accompanied by other changes so that the general appearance of the typical airplane has changed somewhat, it is possible roughly to indicate the general result of this trend by considering three airplanes identical in size, appearance, and weight distribution, but powered and loaded in the ratio 1:2:4. Again overlooking the variations normally introduced by

power on, structural practice, and differences in speed corresponding to the same attitude (i. e. Reynolds' Number) the respective motions at the same high angle may be found from the same set of curves as for example (2). The heavier airplane appears to be more poorly damped as was the smaller one above. When applied to this example the curves have correct amplitude but incorrect time relationship, just opposite to their previous application. When adjustment is made (Fig. VIII-3) it is found that damping of the heavier airplane is even worse than it appears at first sight and its period somewhat longer. Incidentally all effects of an increase in wing loading are encountered with a decrease in air density in the same proportion with no change in wing loading. Thus, after a given disturbance the heavy or high altitude airplane has a motion like the small one except that it pursues this less rapidly and with smaller amplitude, and the tendency in all cases is toward poorer damping.

(4) A final example shows the effect of what may be considered a more genuine design variation in the sense that it alters the appearance of the airplane. This consists in holding the ratio between the static pitching moment and damping moment coefficients constant while changing their magnitude. Such a change is produced fairly closely if the tail area alone is varied. A more precise adjustment requires a slight shift in C. G. location. Holding this ratio at 1:2, about where it lies for the original design, a set of curves is found which show very little variation

in period but considerable in damping at the high angle of attack. For a tail area about  $1/2$  the original there is no tendency to subside at all and smaller areas produce a divergence.

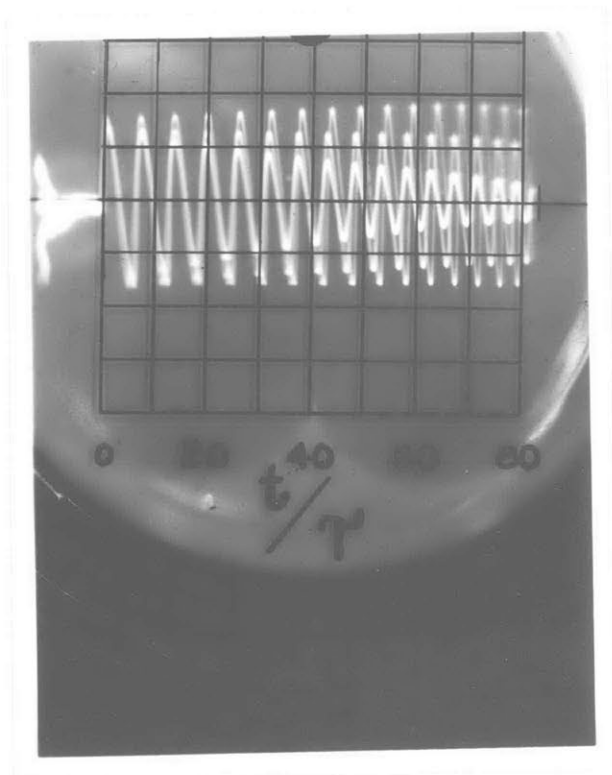


Fig. VIII-4

Before the accuracy and usefulness of this device can be fully known much more testing and checking with calculation will be necessary than can be included here. The research project as originally laid down was to have consisted in a thorough investigation of the possibility and means of constructing an electrical analogue to the airplane in its longitudinal motion, and if possible the actual carrying through of a more or less rough working

model. The present model may be so characterized for it still leaves something to be desired, but the author feels that the account of its development is complete enough to guide<sup>i</sup> any who may later make specific improvements or extend the method to other problems having differential equations of a similar kind.

<u>Example</u>	<u>Variation</u>	$\frac{c_1}{-}$	-	-	$\frac{x}{u}$	$\frac{x}{w}$	$\frac{-c_1}{-}$	$\frac{z}{u}$	$\frac{z}{w}$	$\frac{m}{u}$	$\frac{m}{w}$	$\frac{m}{q}$
1	Original	-.22	13.6	1.39	-.05	+.126	-3.00	-.44	-4.5	0	-3.72	-7
	"	-.9	13.6	2.82	-.115	+.515	-12.2	-1.8	"	"	-3.72	"
2	Twice size	-.9	6.8	2.82	"	"	-6.1	"	"	"	"	"
	Original size	"	13.6	"	"	"	-12.2	"	"	"	"	"
	Half size	"	27.2	"	"	"	-24.4	"	"	"	"	"
3	Twice weight	"	27.2	4.00	"	"	-24.4	"	"	"	"	"
	Original weight	"	13.6	2.82	"	"	-12.2	"	"	"	"	"
	Half weight	"	6.8	2.00	"	"	-6.1	"	"	"	"	"
4	Different $m_w$ 's	"	13.6	2.82	"	"	-12.2	"	"	"	2 -3.0	-6
	with constant	"	"	"	"	"	"	"	"	"	-2.0	-4
	ratio $\frac{m_w}{m_q} = 1/2$	"	"	"	"	"	"	"	"	"	-1.5	-3

Table VIII - B

## BIBLIOGRAPHY

To anyone seriously interested in this subject a list of general references should be more useful than an array of allusions to specific pages and statements. A perusal of those listed below may suggest to the reader ways in which the results of this thesis may be improved.

Chapter I. - A good treatment of vibrating systems, especially those described by a set of equations with a symmetric coefficient matrix can be found in chapter V of Rayleigh's The Theory of Sound vol. I (London 1926).

In chapter  $X_B$  is also an application to electrical systems. Guillemin's Communication Networks vol. I (New York 1931) also treats the symmetric case and in chapter V is a general treatment of the transient solution. Woods' Advanced Calculus (Boston 1932) contains a chapter (XI) on linear differential equations with constant coefficients where the matrices need not be symmetric, a condition not satisfied by the Equations of Longitudinal Stability.

Chapter II. - The simple analogues between "lumped" mechanical and electrical elements can be found outlined in any physics textbook. For anyone wishing to see more about the consistency between the relations for the electrical elements and the general electrostatic and electrodynamic laws, the reduction of "residuals" to negligible proportions

etc., such advanced treatments may be mentioned as Jeans' Mathematical Theory of Electricity and Magnetism (Cambridge University Press 1923).

Chapter III. For the early work on airplane stability see Stability in Aviation by G. H. Bryan (MacMillan 1911), also Dynamical Stability of Aeroplanes by Jerome C. Hunsaker and others (Smithsonian 2419, 1916). A good and not too complicated treatment of the motion of a rigid body with reference to axes fixed in itself can be found in chapter II of Page's Introduction to Theoretical Physics (vanNostrand 1932), also in numerous other works on dynamics among which may be mentioned Elementary Rigid Dynamics by Routh (MacMillan 1930) and The Dynamics of Particles and of Rigid, Elastic and Fluid Bodies by Webster (New York 1922).

Chapter IV. - The RCA-Cunningham Tube Manual together with a little elementary knowledge of electricity will carry the experimenter a long way. Detailed treatments of the action and use of vacuum tubes are Van Der Bijl's Thermionic Vacuum Tube (McGraw-Hill 1920) and Morecroft's Electron Tubes (John Wiley 1933) especially chapter VII of the former and X of the latter.

Chapter V. - The non-dimensional form of the stability equations used here was developed by Glauert and is described rather inadequately in R. and M. 1093 (March 1927). Enough determinant theory to follow the matrix manipulation

can be found in Fine's College Algebra (Ginn 1904). For those having a deeper curiosity about the reduction of equations to dimensionless form Bridgman's Dimensional Analysis (Yale University Press 1931) may be mentioned.

Chapters VI-VII. Previous references cover the material in these chapters. For Thévenin's Theorem see Guillemin's Communication Networks vol. II, chapter IV.



## AUTOBIOGRAPHY

I was born at Waterbury, Connecticut on December 19, 1909, and attended public school there through the first year of the technical course at Crosby High School. This year was to fulfil certain language requirements of the Hotchkiss School at Lakeville, Conn. where I entered as a Junior (first year) in September 1924, graduating four years later in June 1928. Although Hotchkiss is primarily a non-technical preparatory school, my course had some time previously been laid toward the Massachusetts Institute of Technology, and it was possible there to satisfy the entrance requirements of the Institute. During these four years I held a full scholarship of no specific designation but one of a number (10% of the student body) which had been provided for in the original charter of the school. I received upon graduating the Donald MacDonald Prize, the Harvard-Hotchkiss Prize, and certain book prizes in specific courses. Of the latter there were a few others from previous years.

I was registered in the course in Mechanical Engineering as a Freshman at M. I. T. 1928-1929. The next year this was changed to Aeronautical Engineering in which course I received the Degree of Bachelor of Science in 1932. Continuing at the Institute I received the Degree of Master of Science in 1934 and expect that of Doctor of Science about June 1936, the latter two also in Aeronautical Engineering. During this period (1928-1936) I held full tuition scholar-

ships for the first two graduate years and smaller amounts at other times.