Tip Clearance Effects on Multistage Axial Compressor Performance and Flow Structure for Small Core Application

by

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Submitted to the Department of Aeronautics and Astronautics in partial fulfillment of the requirements for the degree of Master of Science in Aeronautics and Astronautics at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Abstract

This thesis describes the effect of increasing multistage axial compressor rotor blade tip clearance on embedded stage performance and flow structure for clearance-to-span ratios ranging from 1.4% to 5.6% using steady and unsteady three-dimensional viscous flow multistage computations. Embedded stage efficiency displays decreased sensitivity as rotor tip clearance increases with two flow regimes. For clearance-to-span ratios less than 3.6%, a nearly linear decrease in stage efficiency of 1.6 points per 1% increase in clearance-to-span is identified, in agreement with published literature. For clearance-to-span ratios greater than 3.6%, the computed stage efficiency decreases at a rate of 0.5 points per 1% increase in clearance-to-span. A parameter is developed that correlates with rotor tip section loss generation over a range of rotor tip clearance-to-span ratios and flow coefficients. The blade row relative streamwise tip section blockage increases in both rotor and stator passages and follows trends in rotor and stator tip section loss generation with rotor tip clearance. The tip section velocity deficit into the stator increases with tip clearance resulting in stator suction side corner flow separation, creating a challenge to design a high efficiency stage with larger tip clearance.

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<tr>
<td>A</td>
<td>Area</td>
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<tr>
<td>$\eta$</td>
<td>Efficiency</td>
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<tr>
<td>$\epsilon$</td>
<td>Tip gap</td>
</tr>
<tr>
<td>s</td>
<td>Span, passage height</td>
</tr>
<tr>
<td>$\frac{\epsilon}{s}$</td>
<td>Rotor tip clearance-to-span ratio</td>
</tr>
<tr>
<td>p</td>
<td>Blade pitch</td>
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<tr>
<td>c</td>
<td>Blade chord</td>
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<tr>
<td>U</td>
<td>Blade rotational speed</td>
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<tr>
<td>$\beta$</td>
<td>Relative flow angle</td>
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<tr>
<td>$\gamma$</td>
<td>Stagger angle</td>
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<tr>
<td>$\phi$</td>
<td>Flow coefficient, $\frac{V_e}{U_{tip}}$</td>
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<tr>
<td>$\psi$</td>
<td>Pressure rise coefficient, $\frac{\Delta P}{\frac{1}{2} \rho U_{tip}^2}$</td>
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<td>$\psi'$</td>
<td>Work coefficient, $\frac{\Delta h_{tip}}{\frac{1}{2} \rho U_{tip}^2}$</td>
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<tr>
<td>$\sigma$</td>
<td>Solidity, $\frac{c}{p}$</td>
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<td>m</td>
<td>Mass flow</td>
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<tr>
<td>s</td>
<td>Specific entropy</td>
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<tr>
<td>h</td>
<td>Specific enthalpy</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
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Velocity $V$

Blade row relative velocity $w$

Length $L$

Radius $r$

Thickness $t$

$V$

Number of blades $N_{blade}$

Circulation $\Gamma$

Specific heat ratio $\gamma$

Gas constant $R$

Dynamic viscosity $\mu$

Kinematic viscosity $\nu$

Density $\rho$

Shear stress $\tau$

Viscous dissipation function $\Phi$

Lost work $W_{lost}$

Displacement thickness $\delta$

Kinetic energy thickness $\theta^*$
Subscript

1  inlet
2  exit
x, y, z  cartesian coordinates
x  axial direction
r  radial direction
θ  circumferential direction
s  streamwise component
c  crossflow component
E  deficit edge
cas  casing
axial  axial component
tan  tangential component
tip  at the blade tip
ref  reference
rel  relative reference frame
abs  absolute reference frame
visc  due to viscous effects
comp  compressor
ad  adiabatic
<table>
<thead>
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<th>Abbreviation</th>
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<tr>
<td>t</td>
<td>stagnation conditions</td>
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<tr>
<td>stage</td>
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<tr>
<td>rotor</td>
<td>across rotor</td>
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<tr>
<td>R3</td>
<td>third rotor</td>
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<tr>
<td>S3</td>
<td>third stator</td>
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<tr>
<td>MP</td>
<td>mixing plane</td>
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**Abbreviations**

- CFD: Computational fluid dynamics
- RANS: Reynolds Averaged Navier Stokes
- URANS: Unsteady Reynolds Averaged Navier Stokes
- TKE: Turbulent kinetic energy
- LSAC: Low-Speed Axial Compressor
- OPR: Overall pressure ratio
- BPR: Bypass ratio
- LE: Leading edge
- TE: Trailing edge
- PS: Pressure side
- SS: Suction side
- AR: Aspect ratio
- AP: Average passage
SM  Stall margin
FFT  Fast-Fourier Transform
Chapter 1

Introduction

Minimizing fuel consumption through aircraft and propulsion system modification is a high priority in the aviation industry. Increasing bypass ratio and overall pressure ratio can reduce fuel burn, but have the consequence of decreasing aircraft engine core size and thus increasing rotor tip clearance-to-span ratio in axial compressor rear stages. This thesis examines, using steady and unsteady three-dimensional viscous computational fluid dynamic simulations, the behavior of compressor flows over a wide range of compressor rotor tip clearance-to-span ratios (1.4% to 5.6% of span) with focus on rotor and stator fluid dynamic features causing efficiency decrease as a function of tip clearance.

1.1 Small core engines

The core of a jet engine is comprised of a compressor, combustor and turbine. The engine core generates high temperature, high pressure gas used to drive a turbine that drives a bypass fan. Since 1972, core size has shrunk by a factor of 10 and it will continue to do so as propulsion system design progresses [32].

Figure 1-1 depicts engines with increased overall pressure ratio, decreased core size, increased bypass ratio and decreased fan pressure ratio for improving propulsive and thermal efficiency. Compared to current in-service engines with bypass ratios of around 5, engines
entering service in 2017 to 2019 are estimated to have bypass ratios of 12 and 15% decreased fuel burn with evolution to bypass ratios of 15 to 18+ and 20% to 30% decreased fuel burn [29]. A recent example of research on advanced civil aircraft [15], showed an aircraft conceptual design in the 2035 time frame with an overall pressure ratio of 50, bypass ratio of 20, and core size of 1.5 lbm/s [14].

Figure 1-1: Commercial aircraft engine propulsion system trend towards larger fan, higher overall pressure ratio, and smaller core to increase propulsive and thermal efficiency [29]

Figure 1-2 compares current engine core size to future targets for propulsion systems that achieve higher overall pressure ratio. The size of the core is defined by high pressure compressor (HPC) exit corrected flow; current single-aisle aircraft engines have HPC exit corrected flow of approximately 5 lbm/s.
1.2 Motivation for investigating effects of large rotor tip clearance on multistage axial compressor performance

The MIT NASA N+3 research program estimated that for a small core compressor with HPC exit corrected flow of 1.5 lbm/s, the design rotor tip clearance-to-span ratio might be as high as 4.5% [14]. Tip clearance losses have been investigated extensively, but most research has focused on clearance-to-span ratios less than 3%; for which, the efficiency penalty associated with increasing rotor tip clearance is nearly linear, decreasing 1 to 2 efficiency points per 1% increase in clearance-to-span (Freeman [16], Tschirner [43], Wisler [49]). The effect of tip clearance on compressor efficiency, pressure rise, and stall margin for a low speed multistage compressor is shown in Figure 1-3; an increase in tip clearance-to-span from 1.4% to 2.8% reduced peak efficiency by 1.5 points, peak pressure rise by 9.7 points, and stalling flow range by 11% [49].
Computational and experimental research evaluating the effect of large tip clearance, beyond 3% clearance expected in small core applications, on multistage axial compressor performance is limited. Sakulkaew [38] computationally found that for clearance greater than 3.4%, rotor efficiency displayed decreased sensitivity to clearance. She hypothesized that for large tip clearance the leakage flow formation shifts downstream and does not mix out within the rotor passage, thus, the loss potential is not fully realized. Figure 1-4 shows Sakulkaew’s isolated stage unsteady computational results demonstrating decreased rotor efficiency sensitivity to rotor tip clearance compared to Denton’s clearance model where flow is assumed to mix out fully.
This thesis presents analysis of computational results for a multistage axial compressor geometry with rotor tip clearance-to-span ratios ranging from 1.4% to 5.6%.

1.3 Status of compressor tip clearance flows

Compressor tip clearance flows have been a research interest for nearly 80 years and are known to have a detrimental impact on compressor performance, including efficiency, pressure rise capability, and operating range (Cumptsy [9]). Considerable research has been carried out evaluating clearance flow loss and compressor performance sensitivity to clearance variation. The status of compressor tip clearance flows research can be summarized as follows:

- For tip clearance less than about 3% span, the decrease in compressor efficiency has been documented for various compressors (Wisler [49] and Freeman [16]), decreasing nearly linearly beyond a small optimum clearance (Cumptsy [9], Wennerstrom [46]) at a rate of 1 to 2 efficiency points per 1% increase in clearance-to-span. For these
clearance-to-span ratios, as much as 30% of overall compressor losses are attributed to tip leakage flow effects (Storer and Cumpsty [41]). Detailed understanding of clearance flow effects on compressor performance at larger clearance-to-span ratios (beyond 3% span) is not well known.

- Most research efforts, both computations and experiments, have focused primarily on single-staged machines and isolated rotors to analyze tip clearance flows (Moore [31], Inoue [19], McDougall [30], etc.) for clearance less than approximately 3% span. Over this range of clearance, endwall flows are qualitatively described and losses are estimated by quantifying the mixing effects of tip leakage and passage flow. Several research efforts (Rains [33], Dawes [11], Denton [13], Storer and Cumpsty [42]) have used approximations of the tip leakage flow to capture the effect on passage flow structure and estimate loss generated. Figures 1-5 and 1-6 show simplified models used by Rains, and Storer and Cumpsty to approximate tip clearance flow. Other research has utilized high resolution CFD to describe tip vortex structure and breakdown in the passage (Inoue [23] and [25], Furukawa [17]).

Figure 1-5: Ideal clearance flow model. From [33]
- Multistage axial compressor flow simulations of large rotor tip clearance-to-span ratios of small core compressor rear stages are not available. Many multistage compressor experiments attempt to describe the clearance flow structure in terms of unsteady flow phenomena of tip leakage flow, but do not document change in overall performance. Tschirner et al [43] and Berdanier [7] conducted multistage experiments for limited clearance values up to 4.0% span to measure compressor performance change with clearance.

- Although computations and experiments on a linear cascade (Williams [47] and [48]) attempted to demonstrate the effects of large clearance, no explanation describing the change in rotor or stator loss leading to decreased stage efficiency sensitivity is available. Williams observed no loss increase for tip clearance greater than 4% span and rotor loading and vortex formation shift aft as clearance increases. Sakulkaew [38] identified three regimes of rotor efficiency variation with tip clearance: small (less than 0.8% span), medium (0.8% to 3.4% span), and large (greater than 3.4% span). She

Figure 1-6: Storer and Cumpsty modeling of tip leakage flow. From [42]
hypothesized that the decreased sensitivity at large clearance is due to reduced opportunity for mixing out tip leakage flow in the rotor passage because vortex formation shifts aft.

- State-of-the-art compressor design practice attempts to minimize the deleterious effects of large tip clearance by incorporating active clearance control, pioneered by Redinger [34] for aircraft engine turbines, and/or increasing blade span with lower hub-to-tip radius ratio. Controlling behavior of compressor tip clearance flow effects through fluidic actuators (Bae et al. [6]) was researched and reduced clearance-related blockage.

1.4 Contributions

The contributions of this thesis include:

1. Documented multistage axial compressor overall pressure rise and efficiency, and embedded rotor and stage efficiency as a function of rotor tip clearance for clearance-to-span ratios from 1.4% to 5.6%, determined from steady average-passage and URANS computations.

2. Identification of two regimes associated with tip clearance flows based on efficiency sensitivity to clearance variation confirmed by steady average-passage and URANS computations.

3. Tip section velocity deficit characterization for rotors and stators in their respective reference frames that follows losses in rotors and stators for clearances from 1.4% to 5.6% and flow coefficients from 0.36 to 0.47 to explain the flow phenomena for observed repeating stages of multistage axial compressors.

4. Definition of the dominant rotor loss mechanism leading to the two regimes and a single non-dimensional parameter that quantifies rotor performance change for clearances from 1.4% to 5.6% and flow coefficients from 0.36 to 0.47.
5. Definition of the dominant stator loss mechanism causing stator performance change as a function of tip clearance.

6. Identification of flow and loss in the axial gap between rotor and stator as the main difference between steady average-passage and URANS computations of multistage axial compressor flow field for clearance from 1.4% to 5.6%.

7. Development of a framework for analyzing hub and tip sections of steady and unsteady computations for multistage axial compressors.

1.5 Thesis outline

Chapter 2 describes the compressor geometry and provides background on the steady and unsteady computational codes used in simulations. In Chapter 3, the effects of increasing rotor tip clearance on multistage compressor efficiency and embedded rotor and stage efficiency are documented to quantify performance penalties for rotor tip clearance-to-span from 1.4% to 5.6%. In Chapter 4, the approach used to analyze the results is presented, explaining performance metric calculations, method for identifying rotor and stator loss sources, and appropriate flow blockage definition that explains the observed repeating stage. Chapter 5 details the effect of tip clearance on embedded rotor performance and flow structure. The flow mechanism causing the efficiency penalty quantified in Chapter 3 is explained and a parameter is presented that captures the effect of tip clearance on rotor loss generation for clearances from 1.4% to 5.6% and flow coefficients from 0.36 to 0.47. Chapter 6 details loss generated in the axial gap between the rotor and stator and the downstream stator performance decrease as a function of tip clearance. The tip section velocity deficit is described in the stator absolute reference frame to quantify mixing loss in the axial gap and the effect of the velocity deficit on the stator. Chapter 7 presents the URANS simulation results that are in good accord with the steady results presented in Chapters 3 thru 6. Chapter 8 concludes with a summary and discussion of potential future work.
Chapter 2

Computational Approach

The geometry used in the assessment and analysis of the effect of rotor tip clearance-to-span on compressor performance is that of NASA Lewis Research Center Low-Speed Axial Compressor (LSAC). Details on the facility and compressor can be found in Wasserbauer et al [44] and Wellborn [45]. The LSAC, design parameters and geometry, and the computational methods, are summarized in subsequent sections.

2.1 Compressor geometry overview

Figure 2-1 gives a sketch of of the NASA LSAC. Figure 2-1a depicts the LSAC facility, Figure 2-1b shows the third stage, and Figure 2-1c illustrates the compressor flow path. The LSAC incorporates design features to achieve a low-speed simulation of a high-speed multistage axial compressor middle to rear stages. Figure 2-2 summarizes the design parameters for the LSAC, representative of middle to rear compressor stage blocks. The hub-to-tip radius ratio is 0.8 and the design flow coefficient is 0.44 (axial velocity normalized by mean blade speed) or 0.4 normalized by tip speed.

To simulate the middle to rear stages of a high-speed multistage compressor, the NASA LSAC features (1) a long inlet duct to develop thick endwall boundary layers typical of embedded rear block stages and (2) inlet guide vanes to produce flow angles in accord with
embedded rear block stators. The first two stages setup the repeating third stage, which is used to study the fluid dynamic features associated with varying rotor tip clearance.

Figure 2-1: NASA LSAC summary [45]
Casing radius | 60.96 cm  
Hub radius | 48.8 cm  
Hub-to-tip ratio | 0.80  
Blade span | 12.19 cm  
Rotational speed | 958 rpm  
Rotor tip speed (based on casing radius) | 61.15 m/s  
Mass flow | 12.3 kg/s  
Axial velocity | 24.4 m/s  
Pressure ratio | 1.042  
Temperature ratio | 1.013  
Flow coefficient, φ | 0.400  
Average pressure rise coefficient, ψ'/4 | 0.500  
Average work coefficient, ψ/4 | 0.550  
Nominal axial gap | 2.54 cm  
Number of blades |  
Rotor | 39  
Stator and IGV | 52  
Midspan aerodynamic chord |  
Rotor | 10.2 cm  
Stator | 9.4 cm  
Midspan blade setting angle |  
Rotor | 43°  
Stator | 42°  
Clearances |  
Rotor tip | 1.7 mm (1.4% span)  
Stator shroud labyrinth seal | 0.85 mm (0.70% span)

Figure 2-2: NASA LSAC design parameters [45]

The LSAC blading design is based on the Rotor B / Stator B geometry designed by General Electric for the Energy Efficient Engine. Modifications to the original blade geometry were incorporated into the NASA blade design because of the differences in hub-to-tip radius ratio in the two facilities. Figure 2-3 compares the LSAC rotor blade design to the GE design. Airfoil sections at 10%, 50% and 90% span are shown in Figure 2-4.
Figure 2-3: Comparison of NASA and GE rotor blade design [45]
Figure 2-4: NASA LSAC rotor airfoil sections at 10%, 50%, and 90% [45]

Figure 2-5 compares the LSAC stator vane design to the GE design. Airfoil sections at 10%, 50% and 90% span are shown in Figure 2-7. The NASA geometry employed a linear twist from hub to tip. NASA incorporated a 3.5° leading edge overcambering to match the GE design suction surface velocity distribution.
(a) Radial distribution of stator blade ratios, stator solidity ($\sigma$) and aspect ratio (AR) [45]

(b) Radial distribution of stator leading edge, trailing edge, and stagger angle

Figure 2-5: Comparison of NASA and GE stator vane design [45]
2.2 Steady state computations

APNASA, a three-dimensional, time-averaged Navier-Stokes code, was used to simulate the LSAC flow field at rotor tip clearance-to-span ratios from 1.4% to 5.6%. The average-passage formulation of the Navier Stokes equations solved by APNASA is derived by utilizing three averaging operators (1) ensemble average, (2) time average, and (3) passage average. The average-passage equations represent a time and passage averaged version of the Reynolds-averaged Navier Stokes (RANS) equations. APNASA utilizes body forces, energy sources and sinks, and temporal and spatial correlations to account for the effects of neighboring upstream and downstream blade rows on the blade row of interest [8]. The
correlations represent the nonlinear convection terms in the governing equations. The body force is a result of the time-averaged pressure and shear force imposed by the rotor on the fluid. The energy source (sink) is associated with the work input due to the body force and the total enthalpy flux accounts for transfer of total enthalpy between deterministic and time-averaged flow fields in the closure models [1]. To ensure the average-passage equation is continuous at all locations, only the non-singular axisymmetric component of the body force and energy source (sink) is retained in the formulation. Detailed descriptions on APNASA and average-passage derivation can be found in Adamczyk [1], and [4].

This section describes the unsteady flow features modeled in the average-passage formulation. In Chapter 7, results obtained by APNASA (steady) and URANS calculations are compared to evaluate embedded stage efficiency sensitivity to tip clearance.

2.2.1 APNASA code

A block diagram of the APNASA algorithm is shown in Figure 2-7 [28]. The average-passage flow model describes the time-average flow within an embedded blade row in a multistage machine resulting in a flow field that is periodic over the pitch of the blade row of interest. APNASA accounts for the effects of the unsteady flow environment on the average-passage flow field through deterministic stresses carried through the momentum and energy equation.

The average-passage stress and total enthalpy flux that appear in the average-passage form of the momentum and energy equations are associated with flow features that impact axial flow multistage compressor performance. The closure terms associated with the average-passage equation system are the body force, energy source (sink), average-passage stress and total enthalpy flux, and additional terms used to estimate Reynolds stress and total enthalpy flux associated with the nondeterministic flow.
The average-passage stress and average-passage total enthalpy flux are composed of terms due to (1) nondeterministic flow field, (2) unsteady deterministic flow field, and (3) average-passage flow field of blade rows other than the one of interest. The first term is related to diffusion of total enthalpy and momentum attributed to unsteady flow processes not linked to shaft rotational speed, such as turbulence.

The unsteady deterministic flow processes modeled by APNASA are (1) spanwise transport of wake fluid particles, (2) circumferential transport of wake fluid particles, and (3) straining of wakes. These flow processes are responsible for wake recovery and spanwise redistribution of momentum, and contribute to flow blockage. APNASA uses closure mod-
els to evaluate the impact of these unsteady deterministic processes and also insure the
time-averaged vorticity field and time-averaged momentum flux entering a blade row are
consistent with the time-averaged velocity field [1].

2.2.2 APNASA results validation for NASA LSAC

This section compares multistage compressor and embedded third stage performance
from APNASA simulations to test data presented in Wellborn [45]. The data corresponds
to the LSAC design clearance-to-span ratio of 1.4%.

Figures 2-8 shows compressor (a) work coefficient, (b) total pressure rise coefficient,
and (c) four-stage efficiency. For unstalled flow operation corresponding to flow coefficients
greater than approximately 0.36, the APNASA computed work coefficient is within 1.69%,
pressure rise coefficient is within 2.47%, and compressor efficiency within 0.52 points of the
experimental data.

Figure 2-9 shows the third stage total pressure rise as a function of flow coefficient from
APNASA and test data. Figures 2-10 to 2-12 demonstrate APNASA capability to capture
the third rotor and stator axial velocity and flow angle. Figure 2-13 compares radial dis-
tribution of the stage total pressure rise coefficient (ψ), work coefficient (ψ′), and efficiency
(η) from APNASA to test data. The radial distributions shown are at peak efficiency oper-
ation corresponding to an IGV inlet flow coefficient of approximately 0.4 (\(\frac{\psi_{\text{nom}}}{U_{\text{nom}}}\)). The total
pressure rise coefficient, work coefficient, and efficiency as a function of radius computed
by APNASA are in good agreement with the experimental data. Additional comparisons
between APNASA computed results and test data are given in Appendix C.
Figure 2-8: Comparison of APNASA computed NASA LSAC compressor performance metrics to test data
Figure 2-9: Third stage total pressure rise coefficient determined using APNASA and experiment for design clearance-to-span ratio of 1.4%

Figure 2-10: Third rotor inlet and exit axial velocity as a function of radius
Figure 2-11: Third rotor inlet and exit relative flow angle as a function of radius

Figure 2-12: Third stage stator exit axial velocity and flow angle as a function of radius
Figure 2-13: Third stage performance parameters as a function of radius; (1) total pressure rise ($\psi'$), (2) work coefficient ($\psi$), (3) and efficiency ($\eta$)

2.2.3 Computations executed for rotor tip clearance study

The APNASA computations for this study were provided by NASA for rotor tip clearance-to-span ratios of 1.4%, 1.8%, 2.2%, 2.8%, 3.6%, 4.2%, 4.8%, and 5.6%. The computations were run to specified domain exit flow coefficients from 0.32 to 0.47 ($\frac{\psi}{\psi_{tip}}$). Figure 2-14 presents total pressure rise characteristics for three rotor tip clearance-to-span ratios identifying each flow point obtained from APNASA computations. Cubic Hermite spline interpolation are fitted to create the pressure rise versus flow characteristic.
Figure 2-14: Total pressure rise as a function of flow coefficient from APNASA computations for 1.4%, 3.6%, and 5.6% clearance-to-span.

Figures 2-15 show the convergence history of the APNASA calculations. Tip clearances greater than 3.6% require approximately twice the iterations relative to 1.4% clearance to achieved full convergence. Convergence is defined by an unchanged mean value of the flow quantity over an iteration range corresponding to a rotor revolution. The bottom right chart in Figure 2-15 illustrates that all tip clearance simulations converged to sufficiently small residuals of approximately $10^{-6}$, below the 3-order-magnitude standard convergence criterion [22]. The convergence data shown is from calculations at peak efficiency for clearance-to-span ratios from 1.4% to 5.6%.
Figure 2-15: Convergence history for APNASA calculations at peak efficiency for clearances from 1.4% to 5.6%
2.3 Steady and unsteady RANS calculations

This section details steady and unsteady calculations executed as part of this study using a Pratt & Whitney proprietary code. The calculations were conducted for LSAC geometry at clearance-to-span ratios from 1.4% to 5.6%. The intent of the URANS calculations is to evaluate the role of unsteadiness in the effect tip clearance on compressor performance.

Challenges associated with executing steady and unsteady computations for large rotor tip clearance are also discussed. Increasing rotor tip clearance alters the tip section flow structure and the peak efficiency point moves closer to the peak of the pressure rise characteristic requiring modification of boundary conditions for convergence and increased mesh refinement to capture tip clearance flow features.

2.3.1 Mesh overview

As tip clearance increases, spanwise mesh refinement is needed at radius corresponding to the tip gap and in the vicinity blade tip to capture tip clearance flow features including the tip vortex structure and shear layer between tip leakage flow and mainstream flow. The mesh refinement implemented holds the number of points per 1% clearance-to-span constant in the tip gap while maintaining compatibility with three levels of multigrid by having \( N - 1 \) grid points divisible by \( 2^3 \), where \( N \) is the total number of points. Figure 2-16 shows the spanwise grid points for calculations with rotor tip clearances from 1.4% to 5.6% demonstrating the increased refinement. Mesh studies were conducted using the steady mixing plane calculations to quantify the efficiency sensitivity to spanwise grid resolution for 2.8% and 5.6% clearance; negligible efficiency sensitivity to spanwise mesh was determined at 1.4% clearance. Figure 2-17 shows the increase in overall compressor efficiency for 2.8% and 5.6% clearance from the spanwise mesh adequate for 1.4% clearance (denoted by *) to the spanwise mesh implemented shown in Figure 2-16 (represented by \( \times \)), increasing 0.7 points and 1.15 points for 2.8% and 5.6% clearance, respectively.
Figure 2-16: Comparison of spanwise mesh refinement as a function tip clearance
2.3.2 Unsteady computations

To determine the role of unsteadiness in tip clearance effects on compressor performance, calculations for the four-stage LSAC were executed for rotor tip clearance-to-span ratios of 1.4%, 2.8%, 3.6%, 4.2%, 4.8%, and 5.6%. The URANS computations were conducted for $1/13^{th}$ of the full wheel corresponding to 3 rotor and 4 stator blades. Figure B-5 illustrates a radial slice of an LSAC stage for the URANS calculations.
All URANS calculations were initialized with a flow field obtained from steady computations. For clearances less than 3.6%, fully converged steady mixing plane computations were used to initialize the URANS peak efficiency calculation. For clearances of 3.6% and larger, frozen rotor steady calculations run for 1.5 rotor revolutions were used to initialize the URANS calculations because of non-physical flow features identified in the mixing plane results. The boundary conditions specifications used for the URANS computations were (1) specified inlet total pressure and exit static pressure referred to as pressure control or (2) specified inlet total pressure and exit corrected flow referred to as mass flow control. Appendix B documents the mixing plane simulation results for tip clearance from 1.4% to
5.6% compared to APNASA and URANS results, and describes the challenges faced at large clearance.

Figures 2-19 and 2-20 depict acceptable convergence of (a) domain inlet flow coefficient, (b) domain exit corrected flow, (c) four-stage compressor efficiency computed using total pressure and temperature, and (d) average momentum equation residual for 1.4% and 2.8% clearance-to-span respectively. The convergence figures demonstrate that the computed efficiency oscillations are less than ± 0.05 points and residuals are below the 3-order-magnitude standard convergence criterion [22] demonstrating full convergence. For 1.4% and 2.8% clearances, URANS calculations were run with inlet total and exit static pressure specified from the steady peak efficiency point. Identification of the peak efficiency point is further explained in section 2.3.4. It was confirmed that performance computed from pressure control and mass flow control URANS calculations are the same for 1.4% and 2.8% clearance.
(a) Domain inlet flow coefficient upstream of IGV
(b) Domain exit corrected flow downstream of 4th stator
(c) Four-stage compressor efficiency
(d) Momentum equation residual with values as log10()

Figure 2-19: Convergence history for URANS calculation at 1.4% clearance
(a) Domain inlet flow coefficient upstream of IGV

(b) Domain exit corrected flow downstream of 4th stator

(c) Four-stage compressor efficiency

(d) Momentum equation residual with values as $\log_{10}(\cdot)$

Figure 2-20: Convergence history for URANS calculation at 2.8% clearance
For clearance-to-span ratios of greater than or equal to 3.6%, URANS calculations for flow coefficients less than 0.42 require mass flow control rather than pressure control to achieve a stable flow solution because the peak efficiency flow point is near the peak of the pressure rise characteristic. Figure 2-21 illustrates the URANS calculation diverging due to unstable flow using pressure control. Increased exit static pressure decreases flow as the computation iterates towards a solution. During the iteration process, flow decreases steadily until approximately 1.5 rotor revolutions then proceeds to decrease rapidly as the calculation diverges.

![Figure 2-21: Divergence of URANS calculation demonstrated by loss of flow due to unstable flow phenomena](image)

Figure 2-22 illustrates the challenge associated with achieving a converged solution using pressure control at large rotor tip clearance near peak efficiency corresponding to the peak of the pressure rise characteristic. For 1.4% clearance, the flow coefficient of approximately 0.405 is well away from the peak of the characteristic and specification of the exit static pressure is appropriate for the URANS calculations. For 5.6% clearance, the peak efficiency
flow point occurs near the peak of the pressure rise characteristic. The horizontal green line represents a specified exit static pressure (pressure control). Two solutions are possible when exit static pressure is specified, a unstalled solution to the right of the peak and a separated solution to the left of the peak. To attain a converged unsteady solution at approximately 0.415 flow coefficient for larger clearances, specifying exit corrected flow is needed. The red vertical line illustrates the solution for a specified corrected flow at the domain exit plane (mass flow control).

Figure 2-22: Difficulty in achieving stable flow unsteady solutions near the peak of the pressure rise characteristic

Figures 2-23 and 2-24 depict the acceptable convergence of (a) domain inlet flow coefficient, (b) domain exit corrected flow, (c) four-stage compressor efficiency computed using total pressure and temperature, and (d) average momentum equation residual for 4.8% and 5.6% clearance. The convergence history demonstrate that the computed efficiency oscillations are less than ± 0.25 points and residuals are below the 3-order-magnitude standard
convergence check demonstrating full convergence. These calculation results are obtained specifying the corrected flow at the domain exit plane. The figures show the calculations are fully converged and the desired inlet compressor flow coefficient in the range of 0.412 to 0.4175 has been achieved.

The oscillations in flow metrics increase with tip clearance because the role of unsteadiness is larger. Section 2.3.3 will give arguments that the oscillations are a result of physical unsteady compressor phenomena.

![Figure 2-23: Convergence history for URANS calculation at 4.8% clearance](image)

(a) Domain inlet flow coefficient upstream of IGV 
(b) Domain exit static pressure downstream of 4th stator
(c) Four-stage compressor efficiency
(d) Momentum equation residual with values as \( \log_{10}() \)
2.3.3 Source of the observed oscillation

To determine if the oscillations are due to unsteady compressor flow phenomena, oscillation frequencies due to different sources have been computed. The exit domain efficiency computed over 2 rotor revolutions based on work-averaged stagnation pressure and mass-averaged temperature is used as the signal. An FFT was performed on the efficiency obtained from both a pressure control (2.8% clearance) and mass flow control (5.6% clearance) URANS
calculation. Figures 2-25 and 2-26 depict the frequencies identified for the 2.8% and 5.6% clearance simulations, and the efficiency signals analyzed. The frequencies identified are all related to unsteady compressor flow phenomena.

The distinct oscillation frequencies, identified by spikes in the efficiency signal power spectrum, are compared to unsteady compressor time-scales below. Figure 2-25 and 2-26 show the frequencies corresponding to time-scales as follows:

- Power spectrum analysis of four-stage compressor efficiency with rotor tip clearance of 2.8% from URANS simulations with specified exit static pressure and inlet total pressure (Figure 2-25)

  1. Rotor flow through time-scale: $\frac{c_{rotor}}{V_{rel,rotor\ in}}$
  2. Acoustic wave time-scale: $\frac{L_{comp}}{\sqrt{\gamma R T_{1,\ out}}}$
  3. Stator passing time-scale: $\frac{2\pi}{N_{blade,rotor}U_{rotor,tip}}$
  4. Stator vortex shedding time-scale: $\frac{\lambda_{TE,rotor}}{L_{abs,rotor\ out}}$
  5. Rotor vortex shedding time-scale: $\frac{\lambda_{TE,rotor}}{V_{rel,rotor\ out}}$

- Power spectrum analysis of four-stage compressor efficiency with rotor tip clearance of 5.6% from URANS simulations with specified exit corrected mass flow and inlet total pressure (Figure 2-26)

  1. Compressor flow through time-scale: $\frac{L_{comp}}{V_{abs,IGV\ in}}$
  2. Rotor flow through time-scale: $\frac{c_{rotor}}{V_{rel,rotor\ in}}$
  3. Longitudinal acoustic wave time-scale: $\frac{L_{comp}}{\sqrt{\gamma R T_{1,\ out}}}$
  4. Stator passing time-scale: $\frac{2\pi}{N_{blade,rotor}U_{rotor,tip}}$
  5. Radial acoustic wave time-scale: $\frac{s}{\sqrt{\gamma R T_{1,\ out}}}$
  6. Rotor vortex shedding time-scale: $\frac{\lambda_{TE,rotor}}{V_{rel,rotor\ out}}$
Both Figures 2-25 and 2-26 show that the oscillations observed in the flow metrics for both pressure control and mass flow control URANS simulations are due to unsteady flow phenomena. Capturing the high frequency time-scale associated with vortex shedding from the rotor and stator suggests that the grid definition at the rotor and stator trailing edges is adequate.

(a) Power spectrum obtained by using an FFT on the compressor efficiency for 2.8% clearance pressure control calculations
(b) Compressor efficiency signal for 2.8% clearance over 2 rotor revolutions

Figure 2-25: Oscillation in computed compressor efficiency from 2.8% clearance pressure control URANS calculation linked to unsteady flow phenomena
2.3.4 Peak efficiency point identification for URANS calculations

To analyze the effect of tip clearance at peak efficiency, calculations were run to identify the peak efficiency flow coefficient for the four-stage compressor and third stage for each tip clearance. For clearances of 1.4% and 2.8%, the peak efficiency point was identified from steady mixing plane calculations. Figures 2-27 and 2-28 show the peak efficiency points for clearances of 1.4% and 2.8%.

For tip clearances of 3.6% and larger, the peak efficiency point was identified by using mass flow control URANS simulations to achieve desired flow coefficient based on knowledge of the peak efficiency flow coefficient from APNASA. Figures 2-29 and 2-30 show the peak efficiency point identified for 4.2% and 5.6% clearance, respectively.
Figure 2-27: Peak efficiency point identification for 1.4% clearance

Figure 2-28: Peak efficiency point identification for 2.8% clearance
Figure 2-29: Peak efficiency point identification for 4.2% clearance

Figure 2-30: Peak efficiency identification point for 5.6% clearance
2.3.5 Comparison of URANS and steady APNASA 1.4% clearance simulation results

To assess the URANS calculations, the 1.4% clearance simulation results are compared to the APNASA calculations. Figures 2-31 to 2-34 show a comparison of the radial distribution of spanwise efficiency, total pressure rise, absolute velocity, and flow angle for the third rotor and stage.

Figure 2-31: Comparison of URANS and APNASA calculation computed spanwise rotor efficiency and total pressure rise profiles for 1.4% clearance
Figure 2-32: Comparison of URANS and APNASA calculation computed rotor exit spanwise profile of absolute velocity and flow angle profiles for 1.4% clearance

Figure 2-33: Comparison of URANS and APNASA calculation computed spanwise stage efficiency and total pressure rise profiles for 1.4% clearance
Figure 2-34: Comparison of URANS and APNASA calculation computed stage exit spanwise profile of absolute velocity and flow angle profiles for 1.4% clearance
Chapter 3

Effects of Rotor Tip Clearance On Multistage Axial Compressor Performance

This chapter illustrates the decrease in four-stage compressor and embedded stage performance due to increasing rotor tip clearance-to-span ratio, as obtained from steady AP-NASA calculations. The performance decrease is established for clearance-to-span ratios from 1.4% to 5.6%. The effect of clearance given by the APNASA calculations is bench-marked against published compressor data available for rotor tip clearance up to approximately 3% span.

3.1 Four-stage compressor performance

This section documents the effect of tip clearance on the four-stage compressor performance. The decrease in compressor efficiency and total pressure rise are rationalized. Two flow regimes are identified based on decreased efficiency sensitivity to rotor tip clearance, and the efficiency penalty for each regime is quantified using a linear regression model.

Increasing rotor tip clearance decrease efficiency and pressure rise. Study of rotor and
stator loss mechanisms enables mitigation of deleterious effects of large rotor tip clearance on compressor efficiency.

3.1.1 Effect of tip clearance on compressor efficiency

Figure 3-1 shows the computed four-stage compressor peak efficiency as a function of tip clearance from 1.4\% to 5.6\% for the NASA LSAC. The efficiency presented is computed using Equation 4.2 with mass-averaged enthalpy and specific entropy upstream of the IGV and downstream the 4\textsuperscript{th} stator [10]. Two regimes are identified as rotor tip clearance increases demonstrating decreased efficiency sensitivity to tip clearance. The regime labeled "conventional range" represents the current operating range of axial compressors. The regime of large clearance represents the range of tip clearance applicable to small core compressors.

Figure 3-1: Four stage NASA LSAC peak efficiency as a function of tip clearance displaying decreased efficiency sensitivity to tip clearance
A linear regression model is used to quantify the efficiency penalty for each regimes. For tip clearance-to-span from 1.4% to 3.6%, APNASA indicates compressor efficiency decreases at a rate of 1.35 points per 1% increase in clearance-to-span ratio. For clearance ranging from 3.6% to 5.6%, an efficiency penalty of 0.60 points per 1% increase in clearance-to-span is computed.

3.1.2 Effect of tip clearance on compressor total pressure rise

An increase in rotor tip clearance decreases efficiency (section 3.1.1), pressure rise, and work capability (section 5.1). Figure 3-2 depicts APNASA computed total pressure rise characteristics for tip clearances from 1.4% to 5.6% of span. Relative to 1.4% tip clearance, increasing clearance to 3.6% and 5.6% gives a 14.2% and 20.4% decrease in peak total pressure rise, respectively. NASA LSAC pressure rise characteristics for intermediate clearance-to-span ratios from 1.4% to 5.6% are shown in Appendix C.

Figure 3-3 shows the decrease in peak total pressure rise and total pressure rise at peak efficiency operation as a function of rotor tip clearance-to-span ratio. The two curves coincide at a tip clearance of 4.2%, where the peak efficiency point is at the peak of the APNASA total pressure rise characteristic. At peak efficiency, the total pressure rise displays decreased sensitivity similar to similar to the efficiency trend in Figure 3-1. Peak efficiency operation is of primary interest because it represents the potential that can be achieved at a specific tip clearance-to-span ratio.
Figure 3-2: Computed NASA LSAC speedlines with peak efficiency point labeled as ● for 1.4%, 2.8%, 3.6%, and 5.6% clearance.

Figure 3-3: Peak total pressure rise coefficient and total pressure rise coefficient at peak efficiency as a function of tip clearance.
3.2 Embedded stage performance

It is useful to examine embedded stage performance defined by unchanging flow velocity profile from stage inlet to exit. The concept of ultimate steady flow, referred as an embedded stage, was first described by Howell [21] and later by Smith [39] who demonstrated that flow through a compressor does not continually deteriorate, with velocity distributions leaving the stage identical to those entering. The NASA LSAC third stage (described in Chapter 2) is a repeating stage resembling a middle to rear block. Appendix A documents the effect of varying rotor tip clearance on the NASA LSAC first stage performance, a non-repeating stage for which stage inlet flow field is unchanged as rotor tip clearance is varied, to demonstrate that decreased efficiency sensitivity to tip clearance is a general finding applicable to all compressor stages near peak efficiency operation.

3.2.1 Effect of tip clearance on embedded stage efficiency

The efficiency penalty for the third rotor and stage are characterized using a linear regression model to quantify the efficiency fall-off rate for each configuration regime. AP-NASA calculations indicate that the third stage satisfies the repeating stage condition for all clearances from 1.4% to 5.6%, as verified in Figure 3-4. The axial velocity profiles for all clearance-to-span ratios are shown in Appendix D.

Figure 3-5 shows that rotor and stage efficiency display a decreased rate of fall-off at clearances greater than 3.6% of span, as summarized below. Efficiency fall-off rates computed for each regime are characterized using linear curve fit with $R^2$ of at least 0.97.

- Clearance < 3.6%
  
  Rotor efficiency penalty of 1.5 points per 1% clearance-to-span increase

  Stage efficiency penalty of 1.6 points per 1% clearance-to-span increase

- Clearance > 3.6%
  
  Rotor efficiency penalty of 0.45 points per 1% clearance-to-span increase
Stage efficiency penalty of 0.5 points per 1% clearance-to-span increase

Figure 3-4: Comparison of third rotor inlet (solid line) and third stage exit (dashed line) axial velocity as a function of radius for 1.4% and 5.6% clearance to verify repeating stage condition
Figure 3-5: Embedded rotor and stage efficiency as a function of tip clearance at peak efficiency

3.2.2 Third rotor and stator contribution to stage efficiency debit

Rotor and stator contribution to stage inefficiency are examined in Figure 3-6. The majority of efficiency debit \((1 - \eta)\) is due to entropy produced in the rotor, accounting for 65% to 70% of the stage efficiency debit. The stator contribution to stage efficiency debit is 25% to 30% while rotor-stator axial gap loss accounts for approximately 5%. The rotor efficiency debit displays decreased sensitivity to clearance for clearance-to-span larger than 3.6%, whereas the stator and gap loss increase with tip clearance.
Figure 3-6: Rotor, stator, and rotor-stator axial gap contribution to stage efficiency debit
Chapter 4

Research Approach

In this research non-dimensional entropy as a measure of loss and streamwise blockage to characterize the tip section velocity deficit are used. This chapter describes the details and utility of each.

4.1 Entropy and efficiency

Loss (normalized entropy) and stage loading (work coefficient) as a function of clearance are analyzed because efficiency is a function of both, as summarized in Equations 4.1 through 4.4. The lost work is directly related to the irreversibility.

\[ \eta_{ad} = 1 - \frac{\text{Lost work}}{\text{Ideal work}} \quad (4.1) \]

\[ \epsilon = 1 - \eta_{ad} = \frac{\dot{W}_{\text{lost}}}{\Delta h_{t,\text{stage}}} \simeq \frac{T_{t,\text{out}} \Delta s}{\Delta h_{t,\text{stage}}} \quad (4.2) \]

Normalized entropy \[ = \frac{T_{t,\text{out}} \Delta s}{\frac{1}{2} U_{t,\text{tip}}^2} \], Work coefficient \[ = \frac{\Delta h_{t,\text{stage}}}{\frac{1}{2} U_{t,\text{tip}}^2} \quad (4.3) \]

\[ \eta_{ad} = 1 - \frac{\text{Lost work}}{\text{Ideal work}} = 1 - \frac{\text{Normalized entropy}}{2 \times \text{Work coefficient}} \quad (4.4) \]
4.2 Entropy as a measure of loss

The loss generated due to rotor tip clearance is calculated by integrating the rate of local entropy production per unit volume over a volume. The entropy production rate for a fluid particle is:

$$\frac{D_s}{Dt} = \frac{\Phi}{T} + \frac{k}{T} \nabla^2 T \quad (4.5)$$

For the NASA LSAC with IGV inlet mach number of 0.075, the loss due to heat transfer is small in relation to viscous dissipation [20], as seen in Figure 4-1. This figure presents the ratio between local entropy production rate due to heat transfer ($\frac{k}{T} \nabla^2 T$) and viscous dissipation ($\frac{\Phi}{T}$) as a function of axial chord in the third rotor passage. The viscous dissipation function is given in Equation 4.6. In Equation 4.6, the term with divergence of velocity vector is small compared to the other terms LSAC flow field because of the flow Mach number. The dissipation function integrated over a volume (Equation 4.7) represents the entropy rise across the defined region. The volumetric entropy generation rate is used throughout the thesis to identify viscous loss generation rather than low total pressure because it is not a convected quantity but rather a direction measure of local rate of entropy production. Changes in total pressure cannot be traced to the flow mechanisms responsible.

$$\Phi = \tau_{ij} \frac{\partial u_i}{\partial x_j}$$

$$= 2 \left[ \left( \frac{\partial u_x}{\partial x} \right)^2 + \left( \frac{\partial u_y}{\partial y} \right)^2 + \left( \frac{\partial u_z}{\partial z} \right)^2 \right]$$

$$+ \left( \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right)^2 + \left( \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} \right)^2 + \left( \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right)^2$$

$$- \frac{2}{3} (\nabla \cdot \vec{u})^2 \quad (4.6)$$

$$\Delta s = \int_V \frac{1}{T} \Phi \, dV \quad (4.7)$$
4.3 Characterization of tip section velocity deficit

A primary effect of rotor tip clearance is flow blockage produced. To define such blockage, the appropriate velocity component must be identified for both rotating and stationary reference frames. The conventional blockage definition utilizes the axial velocity component and is independent of reference frame. A limitation of using axial velocity is seen in discussion of blockage increase across the rotor and stator blade blade rows individually for a repeating stage. If a compressor reaches an essentially unchanging stage flow field, the flow profile at the rotor inlet and the stator exit are equal and blockage is the same at the stage inlet and exit. Since the flow in the rotor causes an increase in blockage defined with axial velocity, a blockage decrease through the stator is then required. Decreasing blockage
across the stator passage seems inconsistent because the stator has adverse pressure gradient similar to that in the rotor passage.

This can be reconciled if blockage is defined in the blade row relative reference frame, and use the streamwise direction based on the mainstream flow. Blockage defined this way increases across both the rotor and stator blade passages. In fact, the repeating stage flow field of multistage axial compressors occurs because of the rotating to stationary and stationary to rotating reference frame transformations.

The importance in defining blockage in the blade row relative reference frame is to enable investigation of losses for each blade row. Since a stator diffuses the working fluid analogous to a diffuser [27], blockage in the stator reference frame should increase across the passage similar to axial blockage in an axial diffuser. Figure 4-2 shows diffuser axial blockage increase for three area ratios (AR) from experimental data [50] for various flow coefficients. Streamwise blockage in the absolute reference frame (defined in Section 4.4) increases in the stator like axial blockage in an axial diffuser.

![Figure 4-2: Increase in axial displacement thickness across for annular diffuser. Adapted from [50]](image)
Figure 4-3 shows the calculated tip section blockage through the third stage using axial velocity and streamwise blade row relative velocity. The streamwise blockage increases through both rotor and stator passage. The tip section streamwise relative blockage increase due to the adverse pressure gradient in both blade rows, and decreases through the axial gap as the tip section flow mixes out. In the figure, the streamwise blockage upstream and within the rotor passage is computed in the rotor relative reference frame. The frame transformation is incorporated at the rotor trailing edge, a normalized chord of 1 in Figure 4-3, with the streamwise blockage for the axial gap between rotor and stator and stator passage computed in the absolute reference frame. For the axial velocity defined blockage, the blockage produced by the tip leakage flow effects decreases in the gap and stator passage.

Figure 4-3: Blockage through the stage based on axial (left) and streamwise (right) velocity. Rotor, axial gap, and stator are labeled. The frame transformation is applied at the rotor trailing edge.
4.4 Streamwise blockage in a blade row relative reference frame

The streamwise blockage increase across both the rotor and stator passage for all rotor tip clearances were shown in the previous section. This section defines streamwise blockage and the calculation procedure.

4.4.1 Streamwise velocity component

The streamwise relative velocity is defined in Figure 4-4. The figure depicts a velocity vectors representative of flow in the velocity deficit (colored black) and the velocity vectors at the deficit edge (colored red), from which the streamwise flow direction is defined. These velocity vectors illustrate the contribution of blade row relative tangential velocity to streamwise velocity. From Figure 4-4, although axial velocity decrease can be significant in the tip section, the blade row relative tangential velocity limits the decrease in streamwise velocity. For example, high streamwise velocity can result with near zero axial velocity and high blade row relative tangential velocity.
Figure 4-4: Velocity vectors representative of the deficit edge and tip section flow to demonstrate the streamwise velocity component

Figure 4-5a displays the relative flow angle variation with radius for 1.4%, 3.6% and 5.6% rotor tip clearance-to-span. The streamwise direction used in the computation of streamwise blade row relative blockage is demonstrated for 3.6% clearance in Figure 4-5b, illustrating manner in which the streamwise direction for the calculation is defined by the deficit edge.
4.4.2 Velocity deficit edge

The edge of the velocity deficit is identified using Khalid's gradient magnitude cutoff criterion [26]. A gradient-based approach is used because there is a substantial increase in magnitude of the velocity gradient in a deficit region. To identify the deficit region in a three-dimensional flow, the magnitude of $\nabla[\rho V_w]$ is calculated from the gradients which lie in the plane for which the blockage is obtained. The gradient magnitude, given in equation 4.8, depends on the mainstream flow region with the edge definition decreasing in radius as non-uniformity increases.

Figures 4-6 and 4-7 demonstrate the velocity deficit edge defined using the cutoff value suggested by Khalid, $\frac{|\nabla[\rho V_w]|}{\rho_{avg} V_w/c} = 2$. For 1.4% and 5.6% tip clearance the velocity deficit edge is determined to be 83% span and 75% span, respectively. Based on Figure 4-8, Khalid's
suggested cutoff value of 2 is a reasonable choice to distinguish the tip section velocity deficit.

\[ |\nabla [\rho V_s]_{r,\theta} = \sqrt{(\nabla_r [\rho V_s])^2 + (\nabla_\theta [\rho V_s])^2} \]

(4.8)

Figure 4-6: Rotor trailing edge tip section edge definition for 1.4% clearance at 83% span (red line)
Figure 4-7: Rotor trailing edge tip section edge definition for 5.6% clearance at 75% span (red line)
4.4.3 Calculation of blockage

The axial blockage and streamwise relative blockage are calculated using Equations (4.9) and (4.10).

\[
\delta_{axial}^* = \frac{1}{r_{cas}} \int_{r_e}^{r_{cas}} \left( 1 - \frac{v_x}{v_{x,ref}} \right) r dr 
\]  

(4.9)

\[
\delta_s^* = \frac{1}{r_{cas}} \int_{r_E}^{r_{cas}} \left( 1 - \frac{w \cos(\beta - \beta_E)}{w_E} \right) r dr 
\]  

(4.10)

The procedure to obtain the streamwise relative blockage is summarized as follows:

1. Extract blade row relative velocity profile and corresponding flow angle on axial planes
from mid axial gap upstream of the rotor to mid axial gap downstream of the stator

2. Define the velocity defect edge with the cutoff value for the gradient magnitude \( \frac{\|\nabla(p_{rel})_r, \theta}{\rho_{avg} V_x/c} = 2.0 \) using Equation 4.8.

3. Use Equation 4.10 to obtain the normalized streamwise velocity deficit in the blade row relative frame at each axial location

### 4.5 Comparison of axial and streamwise blockage

The tip section (defined at 75% of span or greater) streamwise blockage and axial blockage as a function of tip clearance at the rotor and stator inlet and exit plane are shown in Figure 4-9. The effect of frame transformation on streamwise blockage is seen by comparing the two graphs. As flow becomes more tangential (at rotor inlet and exit, and stator inlet), difference between streamwise blockage and axial blockage increases.

![Figure 4-9: Tip section axial and streamwise blockage at rotor inlet and exit and stator inlet and exit for the third stage as a function of tip clearance](image-url)
4.5.1 Rotor inlet and exit

At the rotor inlet in Figure 4-9, blockage defined by streamwise velocity is approximately 70% less than that of axial velocity because the flow entering the rotor is highly tangential flow in the relative frame. The area-averaged relative flow angle entering the third rotor is 60.7° for 1.4% and 5.6% clearance, respectively.

The velocity and flow angle at the rotor inlet for 1.4% and 5.6% clearance are compared in Figure 4-10. As clearance increases, the axial velocity in the tip section decreases and the relative tangential velocity increases. The high relative tangential velocity results in streamwise blockage that is smaller than the axial blockage. Figure 4-11 shows the flow field at the rotor trailing edge. As tip clearance increases, the rotor exit flow is increasingly tangential and, the effect of relative tangential velocity becomes more pronounced, as shown on the right-most chart in Figure 4-9.
Figure 4-10: Rotor inlet axial velocity, relative tangential velocity profiles, and relative flow angle for 1.4% and 5.6% clearance
4.5.2 Stator inlet and exit

Radial profiles at the stator inlet are shown in Figure 4-12. The absolute tangential velocity affects streamwise blockage in the same manner as in the rotor and the streamwise blockage at the stator inlet is less than the axial blockage and the rotor exit streamwise relative blockage (Figure 4-9). Figure 4-13 shows the radial profile of velocity and flow angle at the stator exit where the flow is close to axial and the streamwise blockage is similar to axial blockage.
Figure 4-12: Stator inlet axial velocity, absolute tangential velocity profiles, and absolute flow angle for 1.4% and 5.6% clearance
Figure 4-13: Stator exit axial velocity, absolute tangential velocity profiles, and absolute flow angle for 1.4% and 5.6% clearance

4.6 Streamwise blockage growth and efficiency response to increasing rotor tip clearance

Figures 4-14 and 4-15 show the tip section streamwise blockage growth in the rotor and stator blade rows. In the rotor (Figure 4-14), the streamwise relative blockage displays two regimes as a function of tip clearance as did rotor efficiency. The streamwise blockage increases with tip clearance for clearances less than 3.6%, but is nearly constant at clearances greater than 3.6%. In the stator (Figure 4-15), the absolute streamwise blockage increases with clearance for all clearance but at a lower rate for clearance larger than 3.6%.
Figure 4-16 compares the entropy (left) and the streamwise blockage increase (right) from passage inlet to exit for both the rotor and stator. The streamwise blockage follows the trend in loss generated for the rotor and stator and the trend is similar to the relative contribution to stage inefficiency of each. To utilize streamwise blockage as an analysis tool to understand entropy production in the rotor and stator, the link between changes in streamwise velocity and loss generated in the blade row needs to be established.

![Graph showing clearance-to-span versus streamwise blockage increase](image)

Figure 4-14: Rotor relative streamwise blockage increase ($\frac{\delta_{s,\text{out}}^* - \delta_{s,\text{in}}^*}{\delta_s}$) as a function of tip clearance
Figure 4-15: Change in stator absolute tip section streamwise blockage as tip clearance increases
4.7 Streamwise blockage relation to blade passage loss generation

The streamwise relative blockage (Equation 4.10) can be interpreted analogous to displacement thickness. The kinetic energy thickness as defined in Equation 4.11 measures the defect of kinetic energy in the boundary layer flow, and characterizes losses in an internal flow device. The velocity component dominating loss generation will be the one for which the kinetic energy defect is largest; further addressed in Chapter 5. The kinetic energy thickness defined for rotor relative reference frame is given in Equation 4.11, and the kinetic energy thickness of streamwise blade row relative velocity is given in Equation 4.12.
\[ \theta^* = \frac{1}{r_{cas}} \int_{r_E}^{r_{cas}} \left( 1 - \frac{w^2}{w_E^2} \right) \frac{w}{w_E} r dr \]  
(4.11)

\[ \theta^*_s = \frac{1}{r_{cas}} \int_{r_E}^{r_{cas}} \left( 1 - \left( \frac{w \cos(\beta - \beta_E)}{w_E} \right)^2 \right) \frac{w \cos(\beta - \beta_E)}{w_E} r dr \]  
(4.12)

Comparison of \( \theta^* \) and \( \theta^*_s \) quantifies the contribution of streamwise kinetic energy defect and total kinetic energy defect of the flow. This comparison is shown in Figure 4-17 for the rotor and stator. The decrease in streamwise kinetic energy in the rotor and stator passage accounts for the majority of loss generated and provides the link between streamwise blockage and loss generated. Thus, the loss mechanisms to be discussed in latter chapters are the sources of streamwise blockage.

![Figure 4-17: Comparison of contribution of streamwise velocity to blade row relative velocity kinetic energy defect in rotor (left) and stator (right) tip section](image)

Figure 4-17: Comparison of contribution of streamwise velocity to blade row relative velocity kinetic energy defect in rotor (left) and stator (right) tip section
Going forward, the tip section streamwise velocity and blockage are used to identify flow mechanisms causing loss generation in the rotor and stator as a function of clearance. Figure 4-19 compares the behavior of the rotor and stator entropy rise (loss generated) and streamwise blockage increase across the blade passage as a function of tip clearance. In the figure, both entropy rise and streamwise blockage increase are normalized using equation 4.13 in order to plot them on the same scale from 0 to 1 and analyze the behavior of each as a function of clearance. Figure 4-19 demonstrates that the streamwise blockage increase as a function of clearance follows the rotor and stator entropy rise.

\[
\text{Normalized Parameter} = \frac{\text{Parameter} - \text{Parameter}_{1.4\%}}{\text{Parameter}_{5.6\%} - \text{Parameter}_{1.4\%}} \tag{4.13}
\]

Figure 4-18: Comparison of trend in rotor tip section loss and streamwise blockage growth
Figure 4-19: Comparison of trend in stator tip section loss and streamwise blockage growth

4.8 Flow field changes due to increased rotor tip clearance

Figure 4-20 depicts the absolute velocity, and flow angles exiting the rotor for three tip clearances of 1.4%, 3.6%, and 5.6%. In the top chart, the velocity deficit in the tip section increases with rotor tip clearance. The primary effect of increased rotor tip clearance is an increasing tip section streamwise blockage produced in the rotor and passed to the stator in the absolute reference frame (discussed further in Chapter 6). The increase in tip section velocity deficit results in spanwise flow redistribution, increasing velocity at the hub and midspan. The flow redistribution effect on loss generated in the rotor is negligible compared to the loss in the tip section as seen in Figure 4-21. The tip section flow becomes increasing tangential exiting the rotor resulting in positive incidence at the stator inlet, as shown in the bottom figures.
Figure 4-20: Velocity field exiting rotor as tip clearance increases
Figure 4-21 emphasizes the importance of defining blockage using streamwise velocity to characterize the tip section flow field to examine the effect of tip clearance on compressor performance. Figure 4-21a compares hub and tip section streamwise blockage growth across the rotor passage. Figure 4-21b shows the rotor entropy rise as a function of radius to illustrate the primary region of loss generation. Variation in tip section loss generated is the cause for performance degradation due to tip clearance. Figures 4-21a and 4-21b show the tip section flow behavior is responsible for decreased compressor performance with at large clearance, and hub flow redistribution effects are negligible in comparison.

A further feature in the rotor is that, as tip clearance increases, the tip vortex structure forms aft in the rotor passage and the position of the vortex core shifts towards the suction side. The impact on flow field and entropy production is discussed in Chapter 5. Figure 4-22
shows that the radial position of the vortex core is near the blade tip radius for 1.4% and 5.6% clearance. The pitchwise location of the vortex core shifts towards blade stagger angle as tip clearance increases resulting in a vortex aligned more towards the blade suction side.

![Diagram](image)

(a) Pressure coefficient \( \left( \frac{\Delta P}{\frac{1}{2}\rho V_{c}^{2}} \right) \) of 1.4% clearance at 12% chord  
(b) Pressure coefficient \( \left( \frac{\Delta P}{\frac{1}{2}\rho V_{c}^{2}} \right) \) of 5.6% clearance at 12% chord

Figure 4-22: Tip vortex core radial position is near blade tip radius

Figure 4-23 defines the tip vortex angle from the suction side. The tip vortex core is identified by following the low pressure trough which coincides with the highest region of vorticity, shown in Figure 4-23, confirming the location of the vortex core. The vortex core identification methodology is in agreement with work by Furukawa et al [18] and Inoue [23]. Figure 4-24 shows the vortex angle shift towards the blade stagger with increasing clearance and increasing flow coefficient (to be further discussed in Chapter 5).
Low static pressure (designated by blue region) region aligns with vortex core (designated by red circle).

Figure 4-23: Definition of tip vortex angle off the rotor suction side identified using low pressure trough and confirmed with vorticity.
Figure 4-24: Tip vortex angle as a function of tip clearance at peak efficiency ($\phi = 0.405 - 0.411$) and at high flow condition ($\phi = \sim 0.46$)

Figure 4-25 depicts the effect of tip clearance on stator inlet streamwise tip section blockage in the absolute frame, which increases with clearance. The stator inlet streamwise blockage increases with clearance despite nearly constant rotor exit streamwise blockage in the rotor relative reference frame (addressed further in Chapter 6).
Figure 4-25: Streamwise blockage at the stator inlet in the absolute frame as a function of clearance for peak efficiency ($\phi = 0.405$ to 0.411) and high flow ($\phi = \sim 0.46$)
Chapter 5

Rotor Performance and Flow Field

In this chapter the mechanisms causing the decrease in sensitivity for large clearances are examined, including the effect of tip clearance on rotor loss and work, and the alteration in rotor loss mechanism with tip clearance. The two flow regimes seen at small and large clearance are analyzed, and a parameter to characterize the effect of tip clearance on rotor loss is defined.

5.1 Two regimes characterizing rotor performance

The decrease in rotor performance with clearance is a result increased flow blockage which decreases rotor work and increased entropy generation, i.e. rotor loss generated. Figures 5-1 and 5-2 show the increased rotor loss and decreased work capability as a function of tip clearance. The loss increase and the work decrease both change at a decreased rate as clearance increases.

Figure 5-2 shows percent change in rotor entropy generated and work coefficient relative to 1.4% clearance illustrating that loss increase is the dominant cause of lowered rotor efficiency due to large tip clearance. The rotor efficiency debit at 1.4% clearance, defined as $\frac{T_{\text{out}} \Delta s_{\text{rotor}}}{\Delta h_{\text{rotor}}}$, is 6.4 points. Increasing clearance from 1.4% to 5.6% results in an rotor efficiency debit increase of 4.8 points. Approximately 4.0 points are a result of the 63% increase in
loss compared to 0.8 points from the 6.5% decrease in work coefficient. The efficiency fall-off rate decrease observed for clearance-to-span greater 3.6% is predominantly due to reduced rotor loss generation.

Figure 5-2: Relative effect of increasing tip clearance on rotor loss and work
5.2 Dominant region of rotor loss generation

Section 4.8 identifies the tip section as the region of additional entropy generation when clearance is increased (Figure 4-21). Figure 5-3 confirms that the majority of rotor loss is generated in the tip section, accounting for 50% at 1.4% clearance and 65% at 5.6% clearance. Loss generated at the hub decreases with clearance because of flow redistribution due to increasing rotor tip section blockage.

![Figure 5-3: Tip section (yellow), midspan (green), and hub section (blue) contribution to loss generated in rotor passage](image)

5.2.1 Identification of rotor loss sources

Rotor tip section loss sources identified using the computed local entropy generation rate (given in Equation 4.6) are summarized in Figure 5-4 at an axial chord corresponding to peak aerodynamic loading for 1.4% and 5.6% clearance-to-span, 18% and 47% axial chord respectively. To analyze the loss sources, the rotor passage is viewed as three regions: hub,
midspan, and tip region. The regions are defined using Khalid’s gradient cutoff (shown previously in Figure 4-8). For example, at 5.6% clearance, the hub region is defined from radius of 0% to 10% span, midspan region is from 10% to 75% span, and tip region from 75% to the casing.

In Figure 5-4 the hub and midspan region do not vary significantly with tip clearance. Hub loss is due to endwall boundary layer entropy production (labeled as 6). Midspan entropy production is due to blade pressure side (PS) and suction side (SS) airfoil profile loss (labeled as 5 and 7). Tip section entropy production is a result of mixing between tip leakage and mainstream flow near the suction side (labeled as 1), the casing endwall boundary layer (labeled as 2), and mixing of low streamwise fluid with mainstream near mid pitch (labeled as 3). As tip clearance increases, the tip section loss sources affect a greater spanwise portion of the passage.

Figure 5-4: Delineation of loss sources in the rotor passage at 1.4% and 5.6% clearance using entropy production rate
The dominant rotor loss mechanism is mixing between the tip leakage flow and mainstream flow near the suction side associated with the tip vortex in Figure 5-4. Subsequent sections explain how this mechanism varies with clearance leading to decreased rotor efficiency sensitivity.

The radial position of the vortex core is near the blade tip radius (shown previously in Figure 4-22) and there is radially outward velocities between the blade suction side and vortex core and radially inward velocities near midpitch. These radial velocities enhance mixing and entropy production. The loss source labeled as 1 in Figure 5-4 is a result of mixing of tip leakage flow and mainstream flow near the suction side due to the radial velocities.

Figure 5-5 shows the crossflow velocity vectors at an axial chord corresponding to the peak aerodynamic loading (47 % chord) for 5.6% clearance to demonstrate the radial velocities associated with the tip vortex that cause mixing. Of primary interest is the mixing between the leakage flow over the blade tip and the mainstream flow convected radially outward. The connection between the tip vortex and radially outward velocity near the suction side will be described in section 5.3.2.
5.2.2 Quantifying loss generation sources

Comparison of the loss generated in the tip section due to mixing caused by radially outward velocities near the suction side, radially inward velocities at midpitch, and flow navigating the blade tip originating on pressure side is illustrated in Figure 5-6 corresponding to regions labeled 1, 2, and 3, respectively at one axial plane. Region 1 is defined from the blade suction side to the vortex core (zone of radially outward velocity). Region 2 is defined starting from vortex core to the location where radially inward velocity cease (zone of radially inward velocity). Region 3 is defined for flow navigating the blade tip from the pressure side.

The left two charts in Figure 5-6 show the total entropy generated in each region defined for 1.4% and 5.6% clearance. The increase in loss generation from small to large clearance occurs in zone of radially outward velocity because of enhanced mixing between tip leakage.
flow and mainstream flow. This figure confirms that such mixing is the dominant entropy production mechanism in the rotor tip section for clearance from 1.4% to 5.6%.

Figure 5-6: Identification of dominant tip section loss region applicable to small (1.4%) and large (5.6%) tip clearance. Region 1 (solid line), region 2 (dash-dot line), region 3 (dashed line)

Figures 5-7 shows the shear layer in the tip section between tip leakage and mainstream flow at the blade tip radius for 1.4% (radial plane denoted A-A) and 5.6% clearance (radial plane denoted B-B). The entropy production occurs in the zone of radially outward velocity near the suction side. Compared to 1.4% clearance, at 5.6% clearance the shear layer affects a smaller pitchwise region because of the vortex angle more towards blade stagger (shown previously in Figure 4-24) and entropy production due to mixing near the suction side is enhanced through the passage.
Figure 5-7: Comparison of mixing shear layer due to radially outward velocity near the suction side

5.2.3 Velocity gradients and tip section entropy production

In order to identify the velocity gradients dominating local entropy production, $\dot{S}_{\text{visc}}$ is expressed in rotated cartesian coordinates ($s',c',r$) to represent polar streamline coordinates ($s,c,r$) shown in Equation 5.1 [51]. Equation 5.1 gives the volume rate of dissipation expressed in local Cartesian coordinates ($s', c', r'$) to approximate the polar coordinate system defined by streamwise, crossflow, and radial directions ($s, c, r$) [51].

$$\dot{S}_{\text{visc}}^{'''} = \frac{\mu_{\text{eff}}}{T} \left\{ 2 \left[ \left( \frac{\partial V_{s'}}{\partial s'} \right)^2 + \left( \frac{\partial V_{c'}}{\partial c'} \right)^2 + \left( \frac{\partial V_{r'}}{\partial r'} \right)^2 \right] + \left( \frac{\partial V_{s'}}{\partial c'} + \frac{\partial V_{c'}}{\partial s'} \right)^2 + \left( \frac{\partial V_{s'}}{\partial r'} + \frac{\partial V_{r'}}{\partial s'} \right)^2 + \left( \frac{\partial V_{c'}}{\partial r'} + \frac{\partial V_{r'}}{\partial c'} \right)^2 \right\}$$

Equations 5.2 gives a linear transformation to obtain streamwise and crossflow velocity
from axial and circumferential velocity (transformation from (s, c, r)-space from (x, θ, r)-space). Local Cartesian coordinates, (s', c', r')-space, representing streamwise and crossflow directions are obtained from (x, y, z)-space in Equation 5.3. Since the LSAC hub-to-tip ratio is 0.80 this is a good approximation.

\[
\begin{bmatrix}
V_s \\
V_c \\
V_r \\
\end{bmatrix} =
\begin{bmatrix}
\cos β & -\sin β & 0 \\
\sin β & \cos β & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
V_x \\
V_θ \\
V_r \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
V_{s'} \\
V_{c'} \\
V_{r'} \\
\end{bmatrix} =
\begin{bmatrix}
\cos β' & -\sin β' & 0 \\
\sin β' & \cos β' & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
V_x \\
V_y \\
V_z \\
\end{bmatrix}
\]

Figure 5-8 (top) compares the ratio of \( \frac{\nu_{teff}}{T} \left\{ 2 \left[ \left( \frac{\partial V_{s'}}{\partial s'} \right)^2 + \left( \frac{\partial V_{c'}}{\partial c'} \right)^2 + \left( \frac{\partial V_{r'}}{\partial r'} \right)^2 \right] \right\} \), referred to as term 1, and \( \frac{\nu_{teff}}{T} \left\{ \left( \frac{\partial V_{s'}}{\partial c'} + \frac{\partial V_{c'}}{\partial s'} \right)^2 + \left( \frac{\partial V_{c'}}{\partial r'} + \frac{\partial V_{r'}}{\partial c'} \right)^2 + \left( \frac{\partial V_{r'}}{\partial s'} + \frac{\partial V_{s'}}{\partial r'} \right)^2 \right\} \), referred to as term 2, to total entropy production rate \( \dot{S}_{tot} \). The entropy production due to term 1 is small compared to that of term 2.

Figure 5-8 (bottom) evaluates the contribution of streamwise and crossflow gradients to the total entropy production from term 2; the contribution of the radial gradient in streamwise velocity, \( \frac{\partial V_s'}{\partial r'} \), on the left and contribution of adial gradient in cross flow velocity, \( \frac{\partial V_c'}{\partial r'} \), on the right. The entropy production due to mixing of tip leakage and mainstream flow is primarily due to the radial gradient in streamwise velocity, accounting for over 50% (1.4% clearance) and over 70% (5.6% clearance) of the entropy production rate.

Figure 5-9 shows that loss generated in the rotor tip section corresponds to the region where radially outward velocity near suction side provides a high rate of mixing between mainstream flow and tip leakage flow. Entropy production due to terms associated with the radial gradient in streamwise velocity is a consequence of such mixing.
(a) Comparison of term 1 and term 2 from Equation 5.1

$$\frac{1}{p} \left\{ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial z} \right)^2 \right\} \text{ (term 2)}$$

$$\frac{1}{p} \left\{ 2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right\} \text{ (term 1)}$$

(b) Figure 5-8: Evaluation of velocity gradients contributing to rotor entropy production
Figure 5-9: Local entropy production at 94.75% span for 5.6% clearance case. The approximate pitchwise location of the vortex core is given by red dashed line

5.3 A tip vortex model for flow mixing augmentation and tip section entropy production

This section describes radial velocity variation with tip clearance using a vortex flow model. The mechanism causing radial velocities is also rationalized.
5.3.1 Generation of radial velocity near the suction side

The two mechanisms that cause radial velocity near the suction side are the tip vortex structure and turbulent jet entrainment. The former refers to the roll-up of the tip leakage shear layer and the latter refers to entrainment in the tip leakage jet. The radial velocities that give rise to mixing of mainstream and tip leakage flow were demonstrated in Figures 5-5 and 5-9 for the 5.6% clearance case.

Figure 5-10 shows the area-averaged radial velocity, from rotor blade suction side to the vortex core, varies as a function of tip clearance relative to 1.4% at various chordwise locations in the rotor passage. As tip clearance increases, the rate of increase of the radial velocity is lower, reflecting the two regimes identified in rotor efficiency as a function of tip clearance (Figure 3-5).

Figure 5-10: Effect of increasing tip clearance on radially outward velocity near the suction side
As demonstrated in Chapter 4 (Figure 4-22), increasing rotor tip clearance results in the vortex core at lower radius and at a pitchwise distance nearer to the rotor suction side (Figure 4-24). The radial velocity determines the level of mainstream flow convected to the mixing layer. To examine the decreased rotor loss sensitivity to clearance, a simple vortex model is used to assess the radial velocity variation with clearance. The vortex model described in section 5.3.2 shows that casing proximity limits radial velocity for the small clearance regime (less than 3.6% span), with a small effects on radial velocity for the large clearances (greater than 3.6% span).

5.3.2 Vortex model

A potential flow representation of a vortex in a corner is used to estimate radial velocity as a function of tip clearance. The model schematic is displayed in Figure 5-11. \( \zeta_{\text{cas}} \) and \( \zeta_{\text{SS}} \) represent the distance of the vortex core from the casing endwall and rotor blade suction side, respectively. The non-dimensional distance vortex position in the model is \( \frac{\zeta_{\text{cas}}}{\zeta_{\text{SS}}} \). \( V_{SS} \) is defined as the radial velocity near the suction side given by the model. \( V_{SS} \) is extracted at the boundary surface, a distance of \( \zeta_{SS} \) from the vortex core, because it corresponds to the maximum radial velocity in the APNASA computed flow field (as shown in the right chart of Figure 5-9). \( \Gamma \) is the circulation of the vortex.

The velocity potential for the potential flow solution of a vortex in a corner simulating the casing and blade surface can be found using the method of images. The tip vortex plus three image vortices are required to create a flow field with zero normal velocity on the boundary walls; Equation 5.4 gives the complex potential of the velocity field. The circulation of the vortices is set equal to that extracted from the APNASA flow field at each tip clearance. Figure 5-12 depicts the potential flow result, showing the velocity vectors at vortex core radial position along with the streamlines.

The model calculation procedure is summarized as follows:

1. Find vortex circulation using the CFD computation

2. Evaluate the complex potential (Equation 5.4) and compute the complex velocity, \( \frac{dw(z)}{dz} \).
to determine $V_{ss}$ at the suction side blade tip (distance of $\zeta_{ss}$ from the vortex core) shown in Figure 5-12

3. Repeat steps over the range of vortex core positions

$$w(z) = \frac{i\Gamma}{2\pi} \log \left[ \frac{(z - z_{A,1})}{(z - z_{B,1})} \right] - \frac{i\Gamma}{2\pi} \log \left[ \frac{(z - z_{A,2})}{(z - z_{B,2})} \right]$$  \hspace{1cm} (5.4)

Figure 5-11: Potential flow model schematic
Figure 5-12: Potential flow calculation of a vortex in a corner

Figure 5-13 compares the radial velocities extracted from APNASA solutions at 1.4%, 3.6% and 5.6% clearance at various chordwise locations to the potential flow model. The range of non-dimensional vortex position corresponding to each tip clearance-to-span ratio are also indicated. The velocity from the potential flow model agrees with the APNASA results, illustrating the link between the radial velocity near the suction side and the tip vortex.

The vortex model demonstrates three things: (1) radial velocity (outward) between the suction side and vortex core are associated with the tip vortex, (2) proximity to the casing endwall acts to limit radial velocity and (3) the radial velocity does not continue to increase linearly with distance between the vortex core and casing.
5.3.3 Jet entrainment and radial velocity near suction side

Now, the contribution of turbulent entrainment to radial velocity is assessed by modeling the tip leakage flow as a two-dimensional plane jet, shown in Figure 5-14. In the figure, $\epsilon$ is the tip gap, $V_{c,\text{max}}$ is the maximum entrainment velocity, and $d$ represents the distance of the vortex core from the suction side. To estimate the contribution of jet entrainment to radial velocity enhancing mixing, the maximum entrainment velocity occurring at the tip gap exit plane on the suction side is compared to the APNASA computed flow field. The entrainment velocity is estimated using Goertler’s solution [5] for vertical velocity of a plane jet given by:
\[
\frac{v}{V_m} = \frac{1}{\alpha} \left[ \frac{\alpha y}{x} - \frac{\alpha y}{x} \tanh^2 \left( \frac{\alpha y}{x} \right) - 0.5 \tanh \left( \frac{\alpha y}{x} \right) \right]
\] (5.5)

where \( \alpha = 7.67 \) and \( v \) is the vertical velocity normalized by the jet centerline velocity, \( V_m \).

The maximum vertical velocity (depicted as \( V_{e,max} \) in Figure 5.5) at the suction surface edge of the jet is \( 0.065V_m \) computed from Equation 5.5. Thus, the maximum entrainment velocity estimated by the solution is 6.5% of the jet velocity (entrainment coefficient of 0.065). The entrainment coefficient computed using Goertler’s solution is in agreement with published literature for pure jets ranging from 0.06 to 0.07 [35].

Figure 5-14: Schematic of two-dimensional turbulent jet applied to a rotor blade row

The centerline velocity, \( V_m \) in Equation 5.5, is determined from the APNASA computed tip leakage flow velocity exiting the tip gap on the suction side. Figure 5-15 (Left) illustrates the tip leakage jet for 1.4% and 5.6% clearance at the tip gap exit plane at peak loading.
To compute the entrainment velocity, it is assumed that the jet is uniform with centerline velocity equal to the area-average tip leakage jet velocity from APNASA results. The area-average tip leakage jet velocity is $0.8U_{tip}$ and $0.7U_{tip}$ for 1.4% and 5.6% clearance, respectively.

With the computed entrainment coefficient of 0.065, the estimated maximum entrainment velocities for 1.4% and 5.6% clearance are $0.05U_{tip}$ and $0.045U_{tip}$, respectively. Figure 5-15 (Right) shows the radial velocity as a function of pitch at the blade tip radius between the vortex core position and blade surface suction side at peak loading axial chord because it corresponds to the location of highest entropy production rate [41]. The maximum radial velocity from APNASA between the suction side and vortex core for 1.4% and 5.6% clearance are 0.15 and 0.3 normalized by rotor tip speed; 3.3 and 6.0 times larger than the estimated maximum entrainment velocities. The radial velocity variation as a function of tip clearance are thus primarily due to the tip vortex structure, rather than entrainment. This provides a link between the tip vortex structure and radial velocities generating entropy in the rotor.

![Figure 5-15: Tip leakage flow normal to the blade at the tip gap exit plane (left); radial velocity as a function of pitch from suction side blade surface to vortex core (right) from the APNASA results](image-url)
5.4 Variation in tip section loss generated with clearance

In this section, a parameter is defined to characterize the total entropy produced in the rotor passage due to mixing. The parameter correlates with rotor tip section loss (rotor passage entropy rise) for tip clearance from 1.4% to 5.6% span, and flow coefficients along the compressor characteristic from high flow to near stall ($\phi = 0.36$ to 0.475).

5.4.1 Definition and calculation of radial mass flow parameter

Figure 5-16 illustrates the radially outward flow to the tip section shear layer, and the axial planes for calculation of radial mass flow. The radial velocities at 20% axial chord plane are illustrated in Figure 5-16 to as an example. Radially outward velocities initiate downstream of the leading edge at a axial chord location denoted as $x_{c,1}$, where the tip vortex structure is first identifiable. The zone of radially outward velocity extends to the trailing edge.

The radial mass flow parameter, $\frac{m_{r,\text{mainstream}}}{m_{\text{in, rotor}}}$, is the radially outward mass flow of mainstream flow between the suction side blade tip and vortex core position normalized by inlet rotor passage mass flow. The radial mass flow is calculated at the radius of the vortex core centerline. This non-dimensional parameter represents the amount of mainstream flow convected to the tip section shear layer.

The calculation procedure to determine the radial mass flow parameter is summarized in the steps below corresponding to the illustration in Figure 5-16.

1. Extract the radial velocity as a function of pitch at the vortex core centerline radius
2. Determine the axial chord location where radially outward velocities near suction side initiate, $x_{c,1}$
3. Compute the area integral of radial velocity from axial plane $x_{c,1}$ to $x_{c,2}$ to determine normalized radial mass flow of mainstream flow, $m_{\text{mainstream}}$, by passage inlet mass flow, $m_{\text{in, rotor}}$
\[
\frac{m_{r,\text{mainstream}}}{m_{\text{in,\text{rotor}}}} = \frac{1}{m_{\text{in,\text{rotor}}}} \int_{x_{c,1}}^{x_{c,2}} \int_{y_{ss}}^{y_{p,1}} \rho v_r \, dA
\]

where, \( y_{ss} \) is the pitchwise coordinate at the blade suction side and \( y_{p,1} \) is the pitchwise coordinate of the vortex core.

\[ (5.6) \]

Figure 5-16: Definition and calculation schematic for radial mass flow near suction side that characterizes mixing induced by vortex. Left chart shows the calculation region label in red. Right chart shows the pitchwise zone over which radial velocity integrated throughout the rotor passage at each axial plane.

### 5.4.2 Correlation of rotor radial mass flow parameter and tip section entropy generated

Figure 5-17 presents the correlation of rotor tip section loss and radial mass flow parameter for tip clearances ranging 1.4% to 5.6% and flow coefficients ranging from approximately 0.37 to 0.475 (normalized by rotor tip speed). Rotor tip section loss is defined as the total
entropy generated in the rotor passage tip section from the deficit edge radius to the casing (defined previously in Section 4.4). The relation is a linear correlation with an $R^2$ of 0.96 implying that the radial mass flow is a metric to characterize the dominant rotor loss flow mechanism, mixing of mainstream and tip leakage flow exiting the gap.

![Graph showing peak efficiency and normalized tip section loss against radial mass flow parameter](image)

Figure 5-17: Radial mass flow parameter versus total rotor tip section entropy generated for tip clearance from 1.4% to 5.6% and uninstalled flow coefficients

Further examination of the radial mass flow parameter can be used to explain the decreased rotor loss sensitivity to tip clearance. The radial mass flow is a function of the radial velocity and area over which the velocity is outward. The velocity and area are determined by (1) the radial distance between the vortex core and casing, (2) distance of the vortex core from the suction blade surface, and (3) the distance between leading edge and tip vortex formation.

The distances of the vortex core from the suction side and casing in the model are a
function of tip vortex rollup and clearance-to-span ratio. The shear layer between tip leakage flow and passage through flow rolls up into a vortex by its induced velocity [24]. The onset of tip vortex formation is a result of the blade tip loading (pressure difference between pressure side and suction side), which is driving force for tip leakage flow.

Storer and Cumptsy [41] showed for clearance up to 2.8%, increasing clearance shifts tip section blade aerodynamic loading aft with small effect on midspan loading and results in distinct low pressure trough on the suction side that increases with clearance. Figure 5-18 shows the aft shift in tip section loading from APNASA results with clearance in accord with the published literature. The decrease in blade tip pressure difference towards the leading edge delays the tip leakage flow and tip vortex rollup.

Figure 5-18: Tip section aerodynamic loading as a function of tip clearance
The pressure distribution for 1.4% and 5.6% clearance on the suction blade surface in Figure 5-19 demonstrates the increasing vortex influence and low pressure trough due to large rotor tip clearance (in accord with [41]). The lower pressure produced on the suction side increase local clearance volume flow. The blockage produce by the mixing of tip leakage and mainstream flow causes flow redistribution that leads to pressure to rise near the leading edge [41]. Thus, the vortex formation moves further downstream as clearance increases and suction side blade tip minimum pressure does likewise. Figure 5-20 compares the passage pressure distribution for 1.4% and 5.6% showing the low pressure trough due to the vortex on the suction side and high pressure region on the pressure side that increases with clearance. The vortex position at large clearance remains closer to suction side because of the angle shift towards the blade stagger due to the passage pressure field. The vortex angle affects the area over which mainstream flow is convected to the mixing layer and thus the radial mass flow parameter.

Figure 5-19: Suction surface pressure distribution. Contours of $\frac{p-p_1}{\frac{1}{2} \rho U_{tip}^2}$
Figure 5-20: Tip section passage pressure distribution. Contours of $\frac{p-p_1}{\frac{1}{2} \rho U_{tip}^2}$

(a) 1.4\% clearance-to-span

(b) 5.6\% clearance-to-span
The radial mass flow parameter implies different behavior for the two regimes of less than and greater than 3.6% clearance-to-span, similar to the rotor efficiency as a function of tip clearance. For clearances less than 3.6%, the casing effect on radial velocity decreases leading to greater non-dimensional radial mass flow. For clearance greater than 3.6%, non-dimensional radial mass flow increases at a lower rate because the radial velocity is nearly constant with clearance, since casing effects are small, and the area over which mainstream flow is convected outward decreases due to the vortex angle shift.

The loss mechanism dominating the decrease in rotor performance due to tip clearance displays decreased sensitivity determined from the near constant rotor passage entropy generation for clearance greater than 3.6% shown in Figure 5-21. Figure 5-22 compares rotor efficiency relative to 1.4% clearance from APNASA compared to Denton’s leakage mixing model [13] that assumes the tip leakage jet mixes immediately with the surrounding flow, which gives a decrease in efficiency approximately 1.1 point per 1% increase in clearance to span.

Denton’s leakage mixing mode does not capture the observed behavior for large clearance because of the assumption that all the tip leakage flow mixes with the mainstream flow and that the two streams fully mix out in the passage. As shown subsequently, for clearance-to-span greater than 3.6%, a portion of the tip leakage flow between the mixing shear layer and casing is unimpeded by mainstream flow and does not result in entropy production through mixing with mainstream flow in the rotor passage.
Figure 5-21: Rotor entropy rise as a function of tip clearance

Figure 5-22: Comparison of Denton’s leakage mixing model and APNASA efficiency decrease as a function of tip clearance
5.5 Rationalizing the decrease in rotor performance sensitivity to tip clearance

The dominant rotor loss mechanism causing the performance decrease due to tip clearance displays decreased sensitivity at 3.6% clearance. This section examines why the change in radial velocity as a function of tip clearance results in the decreased rotor loss sensitivity.

5.5.1 Low loss region

To further understand the cause of two clearance regimes, the shear layer between tip leakage and mainstream flow is examined. For clearance greater than 3.6%, the tip leakage flow between the shear layer and casing does not mix with mainstream flow in the rotor passage resulting in a region of low entropy production.

Figure 5-23: Normalized contours of local entropy generation rate defined as term 2 in section 5.2.3 for 1.4%, 3.6%, and 5.6% clearance

Figure 5-23 shows the local entropy production at 30% axial chord for 1.4%, 3.6%, and 5.6% clearance to demonstrate this portion of the tip leakage flow that does not mixing with mainstream flow observed at large clearance. The low loss jet is identified by low entropy
production above the shear layer that results from radial velocities associated with the tip vortex. At 1.4%, all tip leakage flow exiting the gap is mixed with mainstream flow.

For clearance greater than 3.6%, the region of low loss generation occurs because radial velocities are nearly constant with clearance (Figure 5-10). This region is demonstrated in Figure 5-24 using contours of entropy production rate normalized by the maximum entropy production rate in the shear layer for 5.6% clearance. The decreased sensitivity of rotor loss to clearance is linked to the low loss jet observed because the spanwise extent of the shear layer does not continue to grow with clearance for the large clearance regime.

Figure 5-25 shows that the low loss region between the mixing layer and casing occurs throughout the rotor passage and thus the loss due to mixing of this portion of tip leakage occurs downstream of the rotor passage. Figure 5-26 shows that the region above the shear layer has a low entropy production rate relative to the shear layer for clearance greater than 3.6%.

![Diagram](image)

Figure 5-24: Demonstration of low loss region between mixing shear layer and casing for 5.6% clearance
Figure 5-25: Normalized local entropy generation rate due to gradients in streamwise velocity at 5.6% clearance
Figure 5-26: Ratio of averaged local entropy production rate between the suction surface and vortex core for the mixing layer and low loss region as a function of chord for clearance of 2.8%, 3.6%, 4.2%, and 5.6%

5.5.2 Tip section rotor relative streamwise blockage

Mixing of tip leakage and mainstream flow also can be understood through examining tip section streamwise flow blockage. The rotor relative streamwise flow blockage also shows two regimes, increasing with up to 3.6% and remaining nearly constant for clearance greater than 3.6%. Figure 4-14 showed the rotor passage streamwise blockage as a function of tip clearance. The blockage behaves similar to the trend in rotor passage entropy rise.

Chapter 4 demonstrated a link between streamwise blockage and loss generation using the kinetic energy deficit in the rotor, showing that streamwise velocity component is most relevant to examine rotor entropy production. In section 5.2.3, it was identified that gradi-
ents in streamwise velocity component, primarily the radial gradient in streamwise velocity, dominate entropy production.

Figure 5-27 shows normalized streamwise velocity as a function of radius at 8% pitch and mid pitch for various axial planes at 5.6% clearance. Figure 5-27a (number 1) demonstrates that streamwise blockage (low streamwise velocity) occurs downstream of the vortex formation (downstream of 10% axial chord) in the tip section. The region of low streamwise velocity at 30% chord is due to the shear layer between mainstream and tip leakage flow, suggesting a link between the rotor loss mechanism and streamwise blockage generated in the tip section.

Comparing the suction side and mid pitch locations in Figure 5-27a, it suggested that streamwise blockage originates at the suction side where the shear layer is observed.

As tip clearance increases, the tip vortex formation occurs further downstream delaying the establishment of radially outward velocity between the suction blade surface and tip vortex core. The loss and streamwise blockage generation shift aft shown in Figure 5-28. The shift in tip vortex formation downstream in the passage is due to the aft aerodynamic loading of the rotor tip section shown in Figure 5-18. Figure 5-28 shows streamwise blockage growth through the rotor passage follows rotor loss generation because the tip vortex generates streamwise blockage in the passage at the suction side. Therefore, in the subsequent section tip section streamwise blockage is used further examine the decreased rotor efficiency sensitivity to tip clearance.
(a) Comparison of streamwise velocity deficit evolution between 8% pitch (left) and 50% pitch (right).

(b) Schematic of pitchwise location for which radial profiles are shown above to depict streamwise velocity deficit.

Figure 5-27: Tip section rotor streamwise velocity deficit
The tip leakage flow between shear layer and casing is unimpeded (corresponding to low loss region) at clearances greater than 3.6%. It thus maintains a more tangential angle and near constant relative velocity as the clearance increases from 3.6% clearance. Maintaining a high relative velocity results in the minor blockage increase with clearance seen for clearance greater than 3.6% in Figure 4-14.

The effect of the unimpeded tip leakage flow on streamwise velocity can be understood from rotor relative trailing edge tip section velocity vectors for 1.4%, 3.6%, and 5.6% clearance, shown in Figure 5-29. Increasing tip clearance from 1.4% to 3.6% decreases streamwise velocity as the rotor relative velocity decreases, due primarily to decreasing axial velocity.
The increase in streamwise velocity deficit is shown in Figure 5-30 as clearance increases from 1.4% to 3.6% clearance.

For a change in clearance from 3.6% to 5.6%, the decrease in axial velocity is offset by the relative tangential velocity above the blade tip radius. Comparison of velocity vectors for 3.6% and 5.6% clearance in Figure 5-29 shows the relative velocity and rotor streamwise velocity at spanwise locations greater than 95% is higher for 5.6% clearance than for 3.6% clearance. This results from the unimpeded flow between the shear layer and casing endwall in the low loss region.

In summary, the two clearance regimes can be understood from the velocity vectors in Figure 5-29. For tip clearances less than 3.6%, the streamwise velocity deficit grows as axial velocity decreases. For clearances greater than 3.6%, rotor streamwise blockage is nearly constant because the tip leakage flow between mixing layer and casing does not mix with mainstream flow in the rotor passage and maintains high rotor relative velocity and thus high streamwise velocity.

Figure 5-29: Tip section velocity vectors to depict the effect of unimpeded tip leakage flow at large clearance (5.6% relative to 3.6%). The velocity vectors at 75% span (black) is representative of the streamwise direction of the passage flow.
Figure 5-30: Tip section streamwise velocity deficit for top 50% span at 1.4%, 3.6%, and 5.6% clearance

5.6 Tip section flow at high flow coefficient and mitigation of effects of large tip clearance

To expand the understanding of the effect of clearance on rotor performance, high flow coefficient operation ($\phi$ up to 0.475) was analyzed using the same methodology. Although, high flow operation is associated with low efficiency due to large negative incidence and midspan pressure side separation, the tip section flow characteristics at high flow mitigate the effects of large tip clearance. Relative to peak efficiency, increasing flow by 17% decreases third stage efficiency by 5.85 and 8.34 points compared to peak efficiency at 1.4% and 5.6% tip clearance, respectively.
Figure 5-31 compares the rate of entropy production at 5.6% clearance at various axial planes in the rotor passage at peak efficiency (left) and high flow (right). Increasing flow acts to aft load the blade tip section and, for constant tip clearance, results in aft motion of the tip leakage vortex formation and entropy production occurs downstream in the passage relative to that at peak efficiency. The aft motion of vortex formation decrease the radial mass flow parameter by shifting $x_{c,1}$ downstream. The figure demonstrates that increasing flow results in a vortex angle shifted towards blade stagger. Figure 5-32 shows the rotor tip loading for 1.4%, 3.6% and 5.6% clearance.

![Diagram](image)

Figure 5-31: Local entropy production rate for a tip clearance of 5.6% at peak efficiency (left) and high flow of $\phi \approx 0.465$ (right)
Comparing the tip section aerodynamic loading at peak efficiency (Figure 5-18) and high flow (Figure 5-32) illustrates the decrease in pressure difference across the rotor blade tip towards the leading edge, which is the driving force for tip leakage flow. The formation of the tip vortex occurs at approximately 10% axial chord for peak efficiency compared to 20% axial chord when flow coefficient is increased by 17% as in Figure 5-31, corresponding to a shift in peak loading from approximately 45% chord to 60% chord.

![Graph showing normalized static pressure rise](image)

Figure 5-32: Tip section blade loading for high flow coefficient operation at 1.4%, 3.6%, and 5.6% clearance

Shifting tip leakage flow formation aft limits the mixing of tip leakage and mainstream flow in the rotor passage by decreasing the area over which flow is transported radially between suction blade surface and tip vortex core, thus decreasing the radial mass flow parameter and tip loss generation. Figure 5-33 shows rotor passage tip section entropy rise and radial mass flow parameter as a function of tip clearance for peak efficiency and high flow coefficient ($\phi = 0.465$). The figure illustrates the decrease in both tip section entropy
rise and radial flow as a function of clearance.

Figure 5-33: Tip section entropy rise (left) and radial mass flow parameter (right) for peak efficiency and high flow ($\phi = 0.465$) operation

Figure 5-34 shows the effect of shifting tip vortex formation aft on entropy rise and rotor passage streamwise blockage increase at peak efficiency and high flow operation. At tip clearance of 1.4%, tip leakage flow effects nearly mix out before the trailing edge so the benefit of shifting tip vortex formation aft is small. As tip clearance increases, the rotor exit flow is less mixed out and the effect of shifting vortex formation aft increases.

Thus, investigation of high flow operation demonstrates the benefit of aft loading the rotor tip section and decreasing tip section loading. A possible strategy to mitigate the effects of large clearance is a shift in loading away from the tip section to the hub and midspan and shift tip section loading downstream to move vortex formation downstream in the passage. Also, the utility of the radial mass flow parameter to study loss due to clearance is demonstrated.
Figure 5-34: Increasing flow decreases tip section loss by shifting tip vortex formation aft. Radial mass flow decrease in accord with tip section loss.
Chapter 6

Changes in Rotor-Stator Axial Gap and Stator Flow Field with Tip Clearance

This chapter describes the effect of the rotor flow field on rotor-stator axial gap loss generation and stator performance. Streamwise blockage in the absolute reference frame is used to characterize the stator tip section velocity deficit.

6.1 Rotor-stator axial gap

This section describes each of the following which relate to the rotor-stator axial gap entropy production: (1) role of the reference frame transformation from rotor relative to stator absolute reference frame in setting the rotor produced velocity deficit in the gap, (2) mixing loss in the rotor-stator axial gap, and (3) decrease in streamwise blockage in rotor-stator axial gap.

6.1.1 Velocity deficit reference frame transformation

Figure 6-1 shows the rotor exit tip section streamwise blockage in the rotor and stator reference frames as a function of rotor tip clearance. The rotor tip section streamwise blockage in the relative reference frame is nearly constant for clearances greater than 3.6%,
but the tip section streamwise blockage in the absolute reference frame increases with tip clearance over the range examined.

Figure 6-1: Tip section streamwise blockage at the rotor trailing edge in the rotor relative and stator absolute reference frame

The streamwise blockage in the absolute reference frame increases with tip clearances because translating the relative velocity to the absolute frame results in lower velocity in the reference streamwise direction. Figure 6-2 shows the rotor relative and absolute velocity for 1.4%, 3.6%, and 5.6% clearance at the rotor trailing edge. At 5.6% clearance, the rotor relative velocity is greater than that compared to 3.6% for radius greater than 92% span because of the portion of tip leakage flow that does not mix with the mainstream flow in the rotor passage (described in Chapter 5).
Figure 6-2: Spanwise profile of relative and absolute velocity at the rotor trailing edge for 1.4%, 3.6%, and 5.6% clearance
As explained in Chapter 4, streamwise velocity is defined by the main flow direction at the velocity deficit edge in the airfoil reference frame. At the rotor trailing edge, streamwise blockage is nearly constant for clearance greater than 3.6% because the increasing relative tangential velocity balances the decrease in axial velocity, as shown in Figure 6-3a and 6-3c. At 5.6% clearance, the high tip section streamwise velocity is primarily due to high tangential velocity and translating this to the absolute reference frame results in low velocity relative to that of 3.6% clearance, as shown in Figure 6-3b.

Decreased rotor efficiency clearance sensitivity results because the spanwise extent of the mixing layer does not continue to increase with clearance. The unimpeded tip leakage flow in the rotor between the mixing shear layer and casing is, however, detrimental to the stator because it results in a velocity deficit that increases with clearance in the absolute frame (Figure 6-1). The behavior in the rotor frame is summarized below. Additional velocity profiles for intermediate clearance-to-span ratios from 1.4% to 5.6% are shown in Appendix D.

- For rotor tip clearance from 1.4% to 3.6%, the tip section relative velocity decreases causing rotor relative streamwise blockage to increase

- For rotor tip clearance from 3.6% to 5.6%, tip section relative velocity decreases between the velocity deficit edge radius and 92% span and increases for radius greater than 92% span relative to 3.6% resulting in the rotor relative streamwise blockage remaining nearly constant

The increase of absolute streamwise blockage at the rotor trailing edge with tip clearance is thus linked to the transformation between rotor and stator reference frame, and the stator inlet blockage does not display decreased clearance sensitivity.
Figure 6-3: Spanwise profiles of velocity at the rotor trailing edge for 1.4%, 3.6%, and 5.6% clearance.
6.1.2 Effect of mixing in rotor-stator axial gap on streamwise blockage and loss generation

The mixing in the axial gap between the rotor and stator, causes a decrease in streamwise blockage. Figure 6-4 shows velocity triangles at midspan and 90% span at rotor trailing edge (RTE), mid rotor-stator axial gap, and stator leading edge (SLE) axial planes to illustrate the effect of mixing for 1.4% and 5.6% clearance. At the larger clearance, the flow is less mixed out than compared to 1.4% clearance at the rotor trailing edge resulting in a greater mixing effect. The mixing increases the tip section velocity and thus decreases the tip section streamwise blockage at the stator inlet.

Figure 6-4: Effect of mixing in rotor-stator axial gap on rotor exit velocity deficit for 1.4% and 5.6% tip clearance
Figure 6-5 shows the decrease in tip section streamwise blockage in the absolute reference frame due to mixing in the rotor-stator axial gap. As tip clearance increases, the decrease in tip section streamwise blockage from rotor trailing edge to stator leading edge is greater; decreasing 0.33% and 1.79% for 1.4% and 5.6% clearance, respectively, implying that mixing in the axial gap is beneficial for large clearance, despite the loss generated.

![Figure 6-5: Streamwise blockage decrease from rotor trailing edge to stator leading edge in rotor-stator axial gap for 1.4%, 3.6%, and 5.6% rotor tip clearance](image)

Figure 6-5 shows the absolute streamwise relative blockage at the rotor trailing edge and stator leading edge as a function of tip clearance. The stator inlet absolute streamwise blockage increases with rotor tip clearance increases. This is a challenge for high efficiency compressor stage design at large clearance despite the decreased rotor performance sensitivity to clearance.
The effects of mixing on performance differs at large and small clearance shown in Figure 6-5 and 6-6. At the small clearance (1.4% clearance-to-span), decreasing the axial gap from the current gap length (25% of the rotor chord) would result in decreased mixing loss and a minimal impact on stator inlet tip section streamwise blockage. For larger clearance, mixing reduces stator inlet streamwise blockage a larger amount, implying that a longer axial gap may be beneficial. For example, if the axial gap where one-half of the current length and the axial gap is increased to the current value, the stator inlet streamwise blockage would decrease by 0.15% span and 1.0% span at 1.4% and 5.6% clearance, respectively. The tradeoff between mixing loss in the axial gap and stator performance decrease due to inlet blockage should be quantified in future work.

Figure 6-7 presents the rotor-stator axial gap efficiency debit due to mixing loss as a function of rotor tip clearance. Mixing losses increase with tip clearance and the axial gap accounts for approximately 5% of the stage efficiency debit.
6.2 Stator vane row

The most important aspect of mixing in the rotor-stator axial gap is the effect on velocity deficit entering the stator. This section details the flow mechanisms that decrease stator performance as rotor tip clearance increases.

6.2.1 Stator entropy rise

Figure 6-8 shows stator vane row entropy rise at peak efficiency as a function of rotor tip clearance. Contrary to the stator inlet absolute streamwise blockage, stator entropy rise displays decreased sensitivity to clearance. This difference is due to the behavior of the tip section suction side corner flow separation, the dominant stator loss mechanism.
6.2.2 Primary stator loss mechanism

To understand stator loss generation as a function of rotor tip clearance, the rate of local entropy production, defined in Equation 5.1, is used to identify stator loss sources. Figure 6-9 compares local entropy production through the stator vane row for rotor tip clearance-to-span ratios of 1.4%, 2.8%, 4.2%, and 5.6%. The suction side tip section corner is the primary region of loss increase with rotor tip clearance due to flow separation. Figure 6-10 shows the stator inlet incidence as a function of radius for 1.4%, 3.6% and 5.6% clearance. At large clearance, the incidence in the tip section is positive causing tip section separation, whereas at 1.4% there is near constant incidence with radius. The positive incidence at larger clearance is a result of the large velocity deficit (absolute reference frame) produced by the rotor.
Figure 6-9: Rate of entropy production in the stator passage for 1.4%, 2.8%, 4.2%, and 5.6% clearance.
Figure 6-10: Spanwise profile of incidence at the stator inlet for 1.4%, 3.6%, and 5.6% clearance

6.2.3 Identification of suction side corner flow separation

Figures 6-11 shows contours of absolute velocity for 1.4%, 2.8%, 4.2%, and 5.6% clearance. The velocity deficit in the suction side tip section corner increases with tip clearance in the stator passage relative to that of 1.4% clearance indicating a growing region of flow separation. Flow redistribution occurs as a consequence causing the hub region velocity to increase with clearance as shown in Figures 6-11
Figure 6-11: Absolute velocity in the stator passage for 1.4%, 2.8%, 4.2% and 5.6% clearance

To confirm the increasing suction side corner velocity deficit (Figure 6-11) and loss generation (Figure 6-9) are due to separation, the diffusion factor and flow turning as a
function of radius are examined. Figure 6-12a shows the stator diffusion factor as a function of radius for clearances from 1.4% to 5.6%. At 1.4% clearance there is minimal flow separation confined to very near the endwall. Increasing clearance causes the diffusion factor in the tip section to exceed (the approximate separation limit) 0.6 for a greater spanwise region; radius greater than 85% span and 82% span for 4.2% and 5.6% clearance, respectively. Figure 6-12b shows the pitchwise-averaged stator exit flow direction illustrating the under-turning in the tip section at the larger tip clearances. High diffusion factor and large flow under-turning support that flow separation occurs in the tip section suction side corner.

![Spanwise radial profile of stator diffusion factor for various rotor tip clearance-to-span ratios](figure)

![Spanwise flow turning in the stator for various rotor tip clearance-to-span ratios](figure)

Figure 6-12: Flow features used to identify stator tip section flow separation

### 6.2.4 Flow dynamics features of stator loss generation

To understand stator entropy rise as a function of tip clearance (Figure 6-8), the stator passage is divided into three regions within which the loss generated is assessed as shown in Figure 6-13a: (1) suction side tip section corner, (2) pressure side tip section, and (3) hub region.
(a) Three regions used to analyze loss generation in the stator as tip clearance varies

(b) Entropy production in the stator passage for Region 1 (suction side tip section corner), Region 2 (pressure side tip section), and Region 3 (hub)

Figure 6-13: Quantifying dominant regions of loss production in stator passage

Figure 6-13b shows entropy generated in each region as a function of tip clearance. The loss generated due to suction side tip section corner separation (Region 1) increases with clearance from 1.4% to 5.6% clearance. As clearance increases, the separation grows in Region 1 and is the primary source of entropy production in the stator passage. Flow redistribution (pitchwise and spanwise) due to blockage from separation increases flow velocity in the hub region and tip section pressure side (Regions 2 and 3). For Regions 2 and 3, the entropy generated decreases with clearance causing the decreased stator loss generation sensitivity to clearance greater than 4.2% in Figure 6-8. The stator static pressure rise normalized by inlet flow dynamic head is roughly constant at peak efficiency flow conditions, as in Figure 6-14, and implying velocity deficit at the inlet is the primary cause of separation and increased loss generation with clearance.
6.2.5 Flow response to suction side corner separation

Radially outward velocities on the suction side exist as a consequence of tip section suction side corner flow separation. Figure 6-15 shows the radial velocities for 1.4% and 5.6% clearance. There is outward radial velocity in the flow separation region causing low-energy fluid to accumulate in the suction side tip section corner. The radial velocities occurring due to separation increase with clearance, resulting in more streamwise blockage, as identified in Figure 6-11. Figure 6-16 compares the radial velocity at 50% axial chord for 1.4% and 5.6% clearance.
Figure 6-15: Radial velocities in stator passage for various axial chords at 1.4% clearance (top) and 5.6% clearance (bottom)
Figure 6-16: Comparison of radial velocity at 1.4% and 5.6% clearance at 50% axial chord, demonstrating flow migration at 5.6% as a consequence of increased separation in the tip section suction side corner.

The blockage resulting from separation leads pitchwise and spanwise flow redistribution which explain the decreased loss generation with increasing tip clearance in Regions 2 and 3 (Figure 6-13a). Figure 6-17 shows streamwise velocity as a function of radius (as defined in Chapter 4) to show the effect of flow redistribution. Figure 6-17a and 6-17b show the tip section velocity near the pressure side and suction side, respectively. Tip section suction side streamwise velocity decreases with clearance as the separation grows, with the effect decreasing from clearances of 4.2% to 5.6%. The velocity for radius greater than 95% span in Figure 6-17a increases with clearance because of pitchwise flow redistribution due to blockage, leading to a decrease in loss generated as a function of tip clearance in Region 2 (Figure 6-13) because endwall deficit is energized.

Figure 6-17c and 6-17d show the hub region streamwise velocity as a function of radius for various clearances. The increased blockage in the tip section causes spanwise flow redistribution leading to a decrease in Region 3 loss generation with increasing clearance as hub endwall flow is energized.
Figure 6-17: Spanwise profiles of velocity at the rotor trailing edge for 1.4%, 3.6%, and 5.6% clearance
6.2.6 Diffuser flow behavior

Another view of the arguments about the effect of inlet absolute blockage and flow separation on stator performance is with reference to diffuser flows, as in Figure 6-18. The parameters to relate stator performance to diffuser performance are (1) inlet blockage, (2) area ratio, and (3) non-dimensional length. The inlet blockage of the stator is determined by the stator inlet streamwise blockage in the absolute reference frame. The area ratio and non-dimensional length are defined in the schematic in Figure 6-18. The LSAC stator vane row has an effective area ratio of 1.52, based on inlet and exit average flow angles, and the non-dimensional length is 2.1, determined by area-averaging the solidity and inlet flow angle as a function of radius.

\[
\text{Area Ratio} = \frac{s \cos(\beta_2)}{s \cos(\beta_1)}
\]

\[
\frac{L}{D} = \frac{c}{s \cos(\beta_1)} = \frac{\sigma}{\cos(\beta_1)}
\]

Figure 6-18: Schematic of stator diffuser analogy
Figure 6-19 shows data on maximum pressure recovery and diffuser effectiveness (the ratio of pressure recovery to ideal pressure recovery) at low Mach number ($M_m = 0.2$) as inlet blockage varies [37]. The diffuser performance as a function of inlet blockage has decreased sensitivity to blockage as in stator performance with rotor tip clearance. This suggests that as separation grows, performance decreases at a lower rate despite increasing inlet blockage quantitatively similar to the stator behavior.

Stator performance change with rotor tip clearance can be summarized as:

- As the rotor tip clearance increases from 1.4% to 4.2%; (1) stator inlet absolute streamwise blockage increases, (2) suction side tip section flow separation grows, and (3) loss generated increases.

- As rotor tip clearance increases from 4.2% to 5.6%; (1) stator inlet absolute streamwise blockage increases, (2) level of suction side tip section flow separation remains nearly constant, and (3) loss generated increases at a lower rate.

Figure 6-19: Diffuser data for maximum pressure recovery (left) and diffuser effectiveness extract from diffuser performance maps (middle) and a correlation (right) [37]
6.2.7 A suggestion for desensitizing stator performance to increasing rotor tip clearance

Figure 6-20 shows tip section streamwise blockage growth (y-axis) as a function of tip section static pressure rise (x-axis) and tip section inlet streamwise blockage in the absolute reference frame (contours). The data is from APNASA calculations for tip clearance ranging from 1.4% to 5.6% and a range of flows ($\phi = 0.36$ to 0.47). The increase of streamwise blockage across the stator passage is used as a metric to resemble tip section loss generated as a function of clearance because of the linkage described in prior sections and similar behavior to stator entropy rise with tip clearance (Figure 4-19).

![Diagram](attachment://image.png)

Figure 6-20: Stator performance as a function of inlet streamwise relative blockage and tip section static pressure rise

Figure 6-20 shows that the increase in tip section streamwise blockage decreases with tip section static pressure rise and inlet streamwise blockage. At near constant flow coeffi-
cient, stator tip section inlet streamwise blockage increases with clearance, thus, decreasing clearance reduces stator loss generated. As tip section static pressure rise decreases (due to increasing flow coefficient), less tip section loss is generated and the streamwise blockage increase across the stator passage is lower.

Since increasing tip clearance increases tip section inlet streamwise blockage, stator loss generation at large clearance could be mitigated by decreasing tip section static pressure rise. At peak efficiency, increasing clearance increases inlet tip section streamwise blockage while tip section static pressure rise is nearly constant, demonstrated in Figure 6-20 by the colored asterisks (1.4% black, 3.6% red, and 5.6% blue).

Figure 6-21 illustrates a suggestion for desensitizing stator performance based on the stator-diffuser linkage from Section 6.2.6. The LSAC stator hub (black), midspan (red), and tip (blue) region are plotted on a diffuser performance chart with area ratio on the y-axis and non-dimensional length on the x-axis. The directional arrows point in the direction of increasing rotor tip clearance. Two optimum diffuser lines, $c_p^*$ and $c_p^{**}$, described by Sovran and Klomp [40], are shown on the figure for 1.0% and 3.0% diffuser inlet blockage.

Since $c_p^*$ represents the maximum pressure recovery for a given length, which occurs in a regime with some stall [36], the figure suggests the tip section flow is overloaded (i.e. undergoing too much diffusion or flow turning) at large clearance due to the tip section inlet velocity deficit. The hub and midspan loading is further from the separation limit (relative to small clearance) due to flow redistribution. A possible suggestion for desensitizing stator performance to large tip clearance is shifting loading from away from tip section to midspan and hub regions. For a constant non-dimensional length (i.e. compressor length restriction), this could be achieved by modifying the tip section blade to decrease area ratio, which is a function of $\beta_{out}^*$ as shown in Figure 6-21.

Since a compressor stage requires a specific static pressure rise to operate, decreasing loading across the span would not be beneficial. Therefore, investigation of stator blade design modifications to shift loading away from the tip region while maintaining the required stator static pressure rise could desensitize stator performance to tip clearance.
Figure 6-21: Stator diffuser analogy applied to the hub, midspan, and tip section
Chapter 7

Assessment of the role of unsteadiness on embedded stage tip clearance effect

This chapter compares the APNASA findings from Chapter 3 thru 6 regarding the embedded stage performance decrease with tip clearance to URANS calculation results. The flow mechanisms described are compared to the time-averaged unsteady flow field for the overall performance, rotor, axial gap, and stator.

7.1 Embedded stage performance comparison

The key findings are that rotor and stage efficiency display decreased efficiency sensitivity to rotor tip clearance, and two regimes, for both URANS and APNASA (steady) calculations are identified with comparable magnitudes in rotor and stator loss generation.

7.1.1 Third rotor and stage efficiency

Figure 7-1 and 7-2 show third rotor and stage efficiency from URANS and APNASA calculations, both depicting two regimes. The linear regression model for each regime is summarized as follow:
- Third rotor
  - Clearance < 3.6%: Decrease of 1.2 points per 1% increase in clearance-to-span
  - Clearance > 3.6%: Decrease of 0.7 points per 1% increase in clearance-to-span
- Third stage
  - Clearance < 4.2%: Decrease of 1.7 points per 1% increase in clearance-to-span
  - Clearance > 4.2%: Decrease of 1.0 points per 1% increase in clearance-to-span

![Graph](image)

Figure 7-1: Comparison of URANS (time-averaged) and APNASA (steady) computed third rotor efficiency
7.1.2 Entropy rise and stage loading

To isolate differences between URANS and APNASA the embedded stage efficiency, rotor, rotor-stator axial gap, and stator contribution to stage efficiency debit are compared in Figure 7-3 for 1.4%, 3.6%, and 5.6% tip clearance. The efficiency debit is computed using Equation 4.2. To find the efficiency debit from unsteady flow field, spatial and temporal averaging is required to analyze time-averaged effects. The averaging for flow quantities such as entropy is given by Equation 7.1.

$$\tilde{\dot{S}}_{ma,ta} = \frac{1}{\Delta t} \frac{\int_{\Delta t} \int_{A} s \rho V_x \, dA \, dt}{\int_{\Delta t} \int_{A} \rho V_x \, dA \, dt}$$

(7.1)
Figure 7-3: Comparison of URANS and APNASA computed rotor, axial gap, and stator efficiency debit

The URANS and APNASA rotor and stator efficiency debit, shown in Figure 7-3, are in general agreement. Rotor passage entropy rise and stage loading are compared in Figure 7-4. Both URANS and APNASA calculations show a decreasing rate of rotor loss increase with tip clearance. The URANS computed work coefficient decreases linearly as clearance increases, so the decreased rotor efficiency sensitivity to tip clearance (Figure 7-1) is a result of decreased rotor loss sensitivity. URANS results support the APNASA rotor loss and efficiency variation with clearance, and the two regimes quantitatively described in Chapter 3. The evaluation of the flow features described in Chapter 5 controlling rotor loss generation are described in subsequent sections.
Figure 7-4: Comparison of unsteady and APNASA rotor loss and work coefficient to further analyze the rotor efficiency trends as tip clearance varies.

Figure 7-5 shows that stage entropy rise from both URANS and APNASA results increase at a lower rate as tip clearance increases, both displaying decreased embedded stage efficiency sensitivity to clearance. The decreased stage efficiency sensitivity observed is a result of rotor loss generation as a function of tip clearance. Unsteady time-averaged stage efficiency supports the embedded stage performance and two regimes documented in Chapter 3 from the APNASA results.
Figure 7-5: Comparison of unsteady and APNASA computed third stage entropy rise

The main difference between the APNASA and URANS results is the computed mixing loss in the rotor-stator axial gap, which is greater for the unsteady calculations as shown in Figure 7-6. The URANS rotor-stator axial gap mixing loss accounts for approximately 12% of the stage efficiency debit compared to 5% in APNASA.
Figure 7-6: Comparison of unsteady and APNASA rotor-stator axial gap efficiency debit

Figure 7-7 compares the third stage accumulative efficiency debit versus axial distance from URANS and APNASA calculations at 1.4% and 5.6% clearance. URANS results are in good accord with APNASA, with some difference in rotor-stator axial gap mixing loss and in leading edge incidence loss. APNASA shows additional loss generation due to clearance occurs downstream of 10% rotor axial chord (once the tip vortex structure is formed and mixing between tip leakage and mainstream flow is established). Rotor leading edge incidence loss is nearly constant with clearance in the APNASA results.
7.2 Third rotor

Chapter 5 identifies the flow mechanisms controlling rotor loss generation as a function of tip clearance. This section evaluates the findings using the time-averaged flow field computed from URANS calculations for 1.4% and 5.6% rotor tip clearance.

The fluid dynamic features for the third rotor examined in this section are (1) additional entropy generated due increasing rotor tip clearance occurs in the tip section, (2) dominant rotor loss mechanism causing decreasing rotor efficiency as tip clearance increases is mixing of tip leakage flow and mainstream flow between the suction side and vortex core due to
radial velocities associated with the tip vortex, (3) for the large clearance regime (greater than 3.6% span) a portion of tip leakage flow occurs that does not mix with mainstream flow in the rotor passage leading to the decreased rotor entropy rise sensitivity, and (4) rotor relative streamwise blockage characterizes rotor tip section flow following the trend in tip section entropy rise.

### 7.2.1 Rotor loss mechanism

Based on the steady APNASA results, additional entropy is produced in the tip section towards the suction side as rotor tip clearance increases relative to 1.4% clearance. Figure 7-8 illustrates that URANS calculations yield decreased rotor performance for the 5.6% clearance relative to 1.4% clearance because of increased tip section loss generation. The change in loss generated in the hub and midspan do not contribute significantly to the performance decrease with tip clearance.

![Graph](image)

Figure 7-8: Spanwise rotor entropy rise for 1.4% and 5.6% rotor tip clearance computed by unsteady calculation
Viscous losses due to mixing of tip leakage flow and mainstream occurring between the suction side and vortex core is the dominant rotor loss mechanism causing the decrease in efficiency clearance sensitivity as described in Chapter 5. The URANS results also show the tip section shear layer as the dominant rotor loss mechanism.

Figure 7-9 shows local entropy production computed from the time-averaged URANS flow field for 5.6% clearance. The primary source of entropy production occurs in the tip section suction side corresponding to the shear layer between tip leakage and mainstream flow that is convected radially outward. This supports the APNASA findings. Figure 7-10 shows that radial velocities associated with the tip vortex enhanced mixing between tip leakage and mainstream flow because high turbulent kinetic energy is observed at the suction side, corresponding to a region of high dissipation.

Figure 7-9: Time-averaged entropy generation rate for 5.6% clearance (URANS computation)
Figure 7-10: Radially outward velocity associated with the tip vortex (left) correspond to a region of high dissipation (right) (5.6% clearance, URANS computation)

The URANS results shows that the radial velocity associated with the tip vortex that enhances mixing between tip leakage and mainstream flow increase with clearance. Figure 7-11 shows the radial velocity approximately 0.2% pitch away from the suction side blade surface at the blade tip radius for 1.4% and 5.6% clearance. This demonstrates that radial velocity is limited by the casing at small clearance as discussed in Chapter 5.
Figure 7-11: Radially outward velocity associated with the tip vortex (URANS computation)

URANS results also confirm the tip vortex structure and resulting radially outward velocities are established aft in the rotor passage relative to 1.4% clearance. Figure 7-12 shows the mixing layer (top) and radial velocity (bottom) at the blade tip radial plane for 1.4% and 5.6% clearance. Entropy production occurs further downstream at large clearance because vortex formation shifts aft in the rotor passage and the flow is less mixed out at the rotor trailing edge. For 1.4% clearance, there is near zero radial velocity towards the trailing edge. At 5.6% clearance, radially outward velocities initiate downstream and the mixing occurs throughout the rotor passage as radial velocity is observed at the trailing edge.
Figure 7-12: The mixing between tip leakage flow and mainstream flow due to radially outward velocities associated with the tip vortex.
Figure 7-13 shows radial profiles of rotor exit entropy to illustrate that the flow is more non-uniform as clearance increases. The mixing loss is realized in the axial gap and the velocity deficit affects the downstream stator performance.

![Figure 7-13: Rotor entropy radial profile for 1.4% and 5.6% clearance (URANS computation)](image)

### 7.2.2 Rotor relative streamwise blockage growth

Rotor relative streamwise blockage has been developed to characterize the tip section flow and analyze the rotor loss variation with tip clearance. Rotor tip section entropy production was related to rotor passage streamwise blockage growth, in Chapter 4 and 5. APNASA computed rotor relative streamwise blockage growth displays two regimes, increasing for clearance less than 3.6% and remaining nearly constant for clearance greater than 3.6%. This result is also seen in the time-averaged URANS results in Figure 7-14.
7.3 Rotor-stator axial gap

This section compares the tip section entropy generated and streamwise blockage reduction in the axial gap between the rotor and stator. Chapter 6 explained the effect of rotor-stator axial gap on the flow field exiting the rotor passage. The findings discussed are (1) tip section absolute streamwise blockage at the rotor trailing edge increases with clearance despite near constant rotor relative streamwise blockage for clearance greater than 3.6%, (2) the decrease in tip section streamwise blockage in the rotor-stator axial gap due to mixing increases with clearance, and (3) tip section stator inlet streamwise blockage increases at an increasing rate with clearance in the absolute reference frame. The primary difference between URANS and APNASA results is rotor-stator axial gap entropy rise.
7.3.1 Tip section streamwise blockage and mixing loss

Figure 7-15 shows tip section stator inlet absolute streamwise blockage increases with clearance greater than 3.6% span. The primary effect of increasing rotor tip clearance on the downstream stator is increasing tip section stator inlet absolute streamwise blockage. Figure 7-6 shows the URANS computations have additional rotor-stator axial gap mixing loss compared to APNASA. As tip clearance increases, tip section absolute streamwise blockage increases at the rotor trailing edge, as in Figure 7-15. The additional mixing loss in the axial gap predicted by URANS is due to mixing out of the rotor wake and velocity deficit. Figure 7-16 compares the decrease in tip section streamwise blockage in the axial gap computed by URANS calculations and APNASA. URANS results shows a greater decrease in tip section streamwise blockage which is consistent with greater rotor-stator axial gap entropy rise.

![Figure 7-15: URANS calculations tip section streamwise blockage computed at the rotor trailing edge and stator leading edge in the absolute reference frame](image)

Figure 7-15: URANS calculations tip section streamwise blockage computed at the rotor trailing edge and stator leading edge in the absolute reference frame
Figure 7-16: Comparison of the decrease in tip section streamwise blockage across the rotor-stator axial gap due to mixing

7.3.2 Source of rotor-stator gap loss generation

Figure 7-17 shows local entropy production rate at 10% rotor-stator axial gap axial plane for 1.4% and 5.6% clearance. The loss increase with clearance in the gap is due to mixing out of the tip section velocity deficit exiting the rotor. For APNASA, the effect of the upstream and downstream blade rows on the axial gap flow field is determined using average-passage methodology, whereas URANS simulations compute the flow field directly. Based on the difference in axial gap loss observed, further analysis is suggested to understand the role of unsteadiness in mixing of the rotor wakes and tip section velocity deficit. Additionally, the tradeoff between axial gap mixing loss and decreased stator loss from a decreased tip section stator inlet streamwise blockage (from axial gap mixing) should be quantified in future work.
Figure 7-17: Local entropy production rate in rotor-stator axial gap at 10% axial gap axial plane from URANS simulations.
7.4 Third stator

This section compares the effect of increasing rotor tip clearance on the downstream stator for APNASA and URANS calculations. In Chapter 6, the key findings are: (1) the primary effect of increasing rotor tip clearance on the downstream stator is increasing tip section stator absolute streamwise blockage, (2) the dominant stator loss mechanism is suction side tip section corner separation, and (3) the increase in stator loss generation with clearance is due to increasing tip section inlet streamwise blockage and static pressure rise.

7.4.1 Stator inlet flow field

Figure 7-18 confirms the APNASA finding that increasing rotor tip clearance increases tip section absolute streamwise blockage at the stator inlet, resulting in a greater propensity for stator passage flow separation. Figure 7-19 shows tip section rotor relative trailing edge streamwise blockage as a function of clearance differs in behavior from streamwise blockage in the absolute reference frame (Figure 7-18 at the stator inlet. The difference is due to the frame transformation translating the velocity deficit from rotor to stator reference frame, as shown in Chapter 6.
Figure 7-18: Stator inlet tip section streamwise blockage change with rotor tip clearance

Figure 7-19: Tip section rotor relative exit streamwise blockage for URANS and APNasa
7.4.2 Stator loss mechanism

Increasing stator tip section absolute streamwise blockage results in tip section suction side corner separation due to the high incidence. Figure 7-20 compares local entropy production in the stator passage for 1.4% and 5.6% rotor tip clearance. Increased rotor tip clearance increases loss generation in the suction side tip section corner due to flow separation resulting in decrease stator performance.

Figure 7-21 shows stator radial velocities at 50% axial chord for 1.4% and 5.6% rotor tip clearance indicating that the low energy fluid accumulates in the suction side tip section corner resulting in blockage and loss. The stator loss mechanism identified from time-averaged URANS results is thus in general accord with the APNASA results.
Figure 7-20: Normalized local entropy production rate in stator passage
7.4.3 Stator entropy rise

Figure 7-22 compares stator entropy rise and tip section absolute streamwise blockage increase as a function of tip clearance from URANS calculations. The results show that (1) stator loss increases at a lower rate with rotor tip clearance because tip section suction side corner flow separation is the main feature of loss generation and (2) streamwise blockage follows stator entropy rise as a function of clearance.
Stator performance is analogous to that found in diffusers as described in Chapter 6. The increase in stator entropy rise as a function of tip clearance is analogous to decreased diffuser performance due to increasing inlet boundary layer thickness. Figure 7-18 shows stator inlet tip section absolute streamwise blockage as a function of tip clearance. Thus, the stator performance variation with tip clearance is primarily due to increasing stator inlet tip section absolute streamwise blockage.
Chapter 8

Summary, Conclusions, and Future Work

Compressor efficiency for an embedded stage in a multistage environment for rotor tip clearance-to-span ranging from 1.4% to 5.6% has been computationally assessed using steady and unsteady three-dimensional viscous calculations. The rotor and stage peak efficiency sensitivity to tip clearance have been determined and the impacts of tip clearance on rotor loss, stator loss, and the effect of rotor-stator axial gap on stator inlet flow field have been identified.

8.1 Summary and Conclusions

1. Steady and unsteady computations have been carried out to define the changes in multistage compressor efficiency and fluid dynamic features for rotor tip clearance-to-span ratios from 1.4% to 5.6%, using the NASA LSAC compressor geometry. The steady calculations were carried out using APNASA. The unsteady computations were carried out using a proprietary URANS code.

2. For the compressor and range of tip clearances investigated, the embedded rotor and stage efficiency display decreased clearance sensitivity for clearances greater than 3.6% span compared to clearances less than 3.6% span. There are thus two regimes associated with tip clearance flows. For clearance-to-span ratios less than 3.6%, embedded
stage efficiency decreases 1.6 points per 1% increase in clearance-to-span in agreement with published literature. For clearance-to-span ratios greater than 3.6%, efficiency decreases 0.5 points per 1% increase in clearance-to-span.

3. The radially outward velocity associated with the tip clearance vortex is an important feature in setting the rate of mixing of the mainstream and tip leakage flow, and thus the loss generation. There are two regimes that characterize the rates. For the smaller clearances (1.4% to 3.6% span), radial velocity increases with clearance. For the larger clearances (3.6% to 5.6%), radial velocity is nearly constant. The radial velocity behavior is captured by a simple vortex model. A single parameter, was defined to quantify the amount of mainstream flow convected to the shear layer; this parameter is found to have a near linear relation tip section loss generation for all clearances investigated from high flow to near stall.

4. At clearance greater than 3.6% span, it is found that a portion of the tip leakage flow does not mix with mainstream flow in the rotor passage because radial velocity is nearly constant with clearance; resulting in the two regimes identified.

5. The tip section blockage is closely related to rotor and stator loss generation as a function of tip clearance. An average mainstream direction in the blade row reference frame is the appropriate definition for blockage in analyzing individual both blade row loss generation as a function of tip clearance and the effect of inlet flow field on blade row performance.

6. The primary effect of increased rotor tip clearance on stator performance is increased stator inlet tip section blockage and an increase in stator loss generation due to tip section suction side corner separation. Although rotor exit blockage is constant for clearance greater than 3.6% span, stator inlet blockage increases with clearance because of the reference frame transformation.

7. The URANS calculations are found to be in good accord with the APNASA (steady) results, and the decreased embedded stage efficiency sensitivity to rotor tip clearance is
confirmed by both steady and unsteady calculations. The primary difference between the computations is a greater mixing loss in the rotor-stator axial gap in the URANS computations.

8.2 Future work

8.2.1 Experimental investigation of compressor performance behavior at large rotor tip clearances

Aircraft engine core size continues to decrease leading to larger rotor tip clearance-to-span ratios. This thesis quantified the multistage compressor efficiency penalty and fluid dynamic features leading to rotor and stator losses at tip clearances from 1.4% to 5.6% based on computations. Experimental data quantifying multistage compressor efficiency and stall margin as a function of rotor tip clearance-to-span ratio over this range would be beneficial to supplement the computational results.

8.2.2 Four-stage compressor performance analysis

Chapter 3 and 7 quantify the computed overall LSAC and embedded stage efficiency as a function of rotor tip clearance obtained from steady average-passage (Chapter 3) and URANS (Chapter 7) multistage calculations. The embedded third stage efficiency fall-off rate with tip clearance decreases at the larger clearances investigated for both steady average-passage and URANS calculations, leading to the identification of two flow regimes. The embedded stage results obtained from both computational tools are in good agreement, but, the overall compressor efficiency obtained from average-passage and URANS calculations differ.

Figure 8-1 compares the four-stage compressor and embedded stage efficiency as a function of rotor tip clearance. It would be useful to investigate the cause of the difference. One source might be the differences in the processes for mixing out wakes and velocity deficits in downstream blade rows (a primary difference between the APNASA (steady) and URANS
results was mixing loss in rotor-stator axial gap). It would also be useful to investigate the mixing process downstream of an embedded stage to understand the decreased multistage compressor efficiency sensitivity to clearance. For example, at large clearance, the flow is not mixed out at the rotor exit giving potential for compressor stage design that is desensitized to large clearance.

Figure 8-1: Comparison of overall four-stage compressor efficiency and embedded stage efficiency for URANS and APNASA

8.2.3 Rotor-stator axial gap

The rotor-stator axial gap mixing loss is the primary difference between the steady average-passage and URANS computations of embedded stage efficiency variations with clearance. Compressor design practice for conventional rotor tip clearance-to-span ratios tends to minimize rotor-stator axial gap length. However, mixing in the axial gap between
the rotor and stator decreases the tip section velocity deficit with greater effect at larger clearances. It would be useful to expand on the results concerning the effect of stator inlet blockage on stator performance as a function of clearance, especially the tradeoff between mixing loss and stator inlet blockage. Considering the differences in mixing loss between the unsteady and average-passage code, further investigation into the cause of the increased loss is also suggested.

8.2.4 Compressor stage performance change with airfoils designed for the flow field at large tip clearance

For all calculations carried out in the thesis, the LSAC geometry was unchanged (i.e. the airfoils designed for 1.4% clearance). It would be useful to examine stator vanes that were modified for the inlet flow field produced by a rotor with large clearance, for example, stators designed for an inlet flow field representative of clearance-to-span ratio of 5.6% to see what benefits might be possible.
Appendix A

Effect of rotor tip clearance on the
NASA LSAC first stage performance

A contribution of this thesis is quantifying the effect rotor tip clearance on embedded stage performance in a multistage environment and identifying the fluid dynamic features controlling rotor and stator losses. As described in Chapter 2, the primary focus of this research effort was the assessment of NASA LSAC embedded third stage. But, it is also useful to quantify the effect of increasing rotor tip clearance on the first non-repeating stage of the NASA LSAC, for which the stage inlet flow field does not vary with tip clearance. The focus of this section is to show that the findings determined from in-depth analysis of the third stage, namely decreased stage efficiency fall-off rate with rotor tip clearance, is applicable to non-repeating compressor stages at peak efficiency operation.

A.1 First stage flow field

Figure A-1 and A-2 show the first stage and embedded stage inlet axial and tangential velocity for both 1.4% and 5.6% clearance. The first stage has the same inlet flow field for all rotor tip clearance calculations. The long NASA LSAC inlet duct produces thick endwall boundary layers representative of middle to rear stage blocks. Figure A-3 demonstrates that
the first stage is not a repeating stage at any rotor tip clearance-to-span ratio. This figure compares the constant stage inlet velocity to the stage exit velocity for 1.4%, 3.6% and 5.6% clearance.

(a) First stage inlet axial velocity for 1.4% and 5.6% rotor tip clearance
(b) First stage inlet tangential velocity for 1.4% and 5.6% rotor tip clearance

Figure A-1: NASA LSAC first stage inlet flow field
(a) Third stage inlet axial velocity for 1.4% and 5.6% rotor tip clearance  
(b) Third stage inlet tangential velocity for 1.4% and 5.6% rotor tip clearance

Figure A-2: NASA LSAC embedded, third stage inlet flow field

Figure A-3: NASA LSAC inlet absolute velocity for all rotor tip clearance compared to exit absolute velocity for 1.4%, 3.6%, and 5.6% tip clearance
A.2 First stage efficiency change with clearance

In Chapter 3, the third rotor and stage efficiency fall-off rate as a function of rotor tip clearance is documented. Figure A-4 illustrates that first rotor and stage efficiency displays decreased efficiency sensitivity to rotor tip clearance and two flow regimes quantitatively similar to the third stage analysis. The efficiency sensitivity decrease for the first rotor and stage occurs between 3.6% to 4.2% clearance also similar to the embedded stage. Figure A-5 demonstrates the behavior of the tip section streamwise blockage as a function of tip clearance, demonstrating its utility to captures trends in rotor and stator performance.

![Figure A-4: First rotor and stage efficiency as a function of rotor tip clearance](image_url)

Figure A-4: First rotor and stage efficiency as a function of rotor tip clearance
(a) Rotor relative streamwise blockage increase across first stage rotor passage as a function of rotor tip clearance

(b) Stator absolute streamwise blockage increase first stage stator passage as a function of rotor tip clearance

Figure A-5: Streamwise blockage increase across rotor and stator for NASA LSAC first stage
Appendix B

Mixing Plane Calculations

This Appendix describes the steady RANS mixing computations executed for clearance-to-span ratios from 1.4% to 5.6%. These calculations were intended to serve the following purposes: (1) initialize URANS computations, (3) assess spanwise grid resolution and computational setup, and (3) provide steady simulation results in addition to APNASA results to assess the role of unsteadiness.

B.1 Literature review on average-passage model compared to mixing plane approach

Results obtained from the average-passage model are expected to be similar to those derived from an unsteady model averaged over time with difference arising from the closure models used for the additional terms in the average-passage formulation [2]. The section compares steady average-passage and mixing plane simulation results to show the CFD models assessed in this study to analyze tip clearance effects.

Figure B-1 illustrates a comparison of the product of density and velocity field along with total and static pressure field at the inlet of an embedded rotor using average-passage simulation and unsteady calculation. The good agreement between the two results indicates the average-passage formulation is in accordance with the time-averaged flow conditions,
time-averaged axial impulse, and the time-averaged radial and tangential momentum flux. Further details on this comparison can be found in Adamczyk [2].

Figure B-1: Evaluation of average-passage computation compared to time-averaged unsteady computation of the inlet conditions to an embedded rotor. From [2]

Figure B-2 compares the rotor exit mass-averaged total pressure and temperature and rotor efficiency computed using unsteady and average-passage calculation. The average-passage pressure outboard of 40% span reflects more energy added to flow relative to the unsteady calculation. The spanwise distribution of efficiency computed by the average-passage simulation is in agreement with the unsteady calculation. Difference between the efficiency computed as shown for spanwise locations between 20% and 80% maybe tied to the mixing of the stator wakes as they pass through the rotor and the recovery process [2].
Figure B-2: Evaluation of average-passage computation compared to time-averaged unsteady computation of the exit conditions to an embedded rotor. From [2]

Figures B-3 and B-4 illustrate the comparison of embedded rotor inlet and exit conditions computed using mixing plane model and time-average unsteady calculation. Compared to the average-passage computations, the mixing plane displays significant difference from unsteady results. The exit conditions computed by the mixing plane also depict less agreement with the unsteady calculation than that of the average-passage results. The mixing plane does not model for transport of momentum or the unsteady pressure field due to upstream blade row which could cause the difference observed.
Figure B-3: Evaluation of mixing plane model computation compared to time-averaged unsteady computation of the inlet conditions to an embedded rotor. From [2]
Figure B-4: Evaluation of mixing plane model computation compared to time-averaged unsteady computation of the inlet conditions to an embedded rotor. From [2]

B.2 Steady mixing plane computations

This section describes the steady RANS simulations executed to establish a baseline for comparing with URANS results and used to initialize simulations. The challenges associated with mixing plane simulations at large clearance are described.

B.3 Additional loss across mixing plane as a function of clearance

The mixing plane model assumes little impact of the unsteady intrablade row flow processes on compressor performance and is valid in the limit as the axial spacing between blade rows becomes large [1]. The mixing plane averages circumferential non-uniformities
instantaneously (placed at mid rotor-stator axial gap) to translate flow field from rotating to stationary and stationary to rotating reference frames. The mixing process conserves mass, momentum, and energy but generates entropy as the real mixing process would [12].

![Graph showing third stage efficiency vs. clearance-to-span (%)](image)

Figure B-5: Third stage efficiency computed using steady mixing plane, average-passage calculations (APNASA), and URANS calculations

As tip clearance increases, increasing tip section non-uniformity produced by the rotor results in additional loss across the mixing plane and reverse flow at the mixing plane interface. Figure B-5 depicts the LSAC third stage compressor efficiency computed by the steady RANS mixing plane calculations, steady average-passage calculations (APNASA), and URANS calculations. The difference between mixing plane and the unsteady calculation computed efficiency increases with tip clearance, underestimating performance because of the entropy produced by the mixing plane averaging procedure. Enforcing circumferentially uniform flow too close to the leading and trailing edge of airfoils modifies the static
pressure distribution and blade loading, thus affects the mixing plane computed flow field at large clearance.

Figure B-6 depicts the efficiency debit across the mixing plane as a function tip clearance. The entropy rise across the mixing plane increases at an increasing rate similar to the velocity deficit produced by the rotor in the absolute reference frame shown in Figure 6-6. An increased velocity deficit requires more mixing to achieve the uniform state, resulting in increased mixing plane entropy rise. The velocity deficit exiting the rotor and rotor-stator axial gap flow field are described in Chapter 6.

Figure B-6: Efficiency debit across mixing plane as a function of tip clearance. The efficiency debit is defined as $\frac{T_{\text{out}} - T_{\text{entry}}}{T_{\text{entry}}}$.
B.4 Non-physical flow features due to large tip clearance

For rotor tip clearances of 3.6% and greater, obtaining an accurate representation of the flow field requires modification to the standard mixing plane model. The traditional mixing plane model mixes out circumferential tip section flow non-uniformities, zeroing out reverse flow that occurs at the interface. Figure B-7 and B-8 shows the mixing plane circumferentially-averaged axial velocity as a function of radius at the rotor trailing edge, 25% rotor-stator axial gap, and mid rotor-stator axial gap downstream of the mixing plane for 2.8% and 4.2% tip clearances, respectively. Reverse flow at the mixing plane is observed for clearance of 3.6% and larger; the 4.2% clearance solution is used to demonstrate the non-physical flow feature identified.

![Graph showing normalized axial velocity](image)

Figure B-7: Radial distribution of axial velocity normalized by blade tip speed for 2.8% clearance at the rotor trailing edge (dash-dot line), 25% rotor-stator axial gap (dotted line), and 50% rotor-stator axial gap downstream of the mixing plane (solid line)
For 2.8% rotor tip clearance, reverse flow is observed for locations greater than 98.9% span at the rotor trailing edge. Spanwise mixing of the high entropy tip section flow (low momentum) and low entropy mainstream flow (high momentum) in the rotor-stator axial gap increases axial velocity in the tip region, thus, there is no reverse flow at the upstream interface of the mixing plane as shown in Figure B-9.

Figure B-8: Radial profiles of axial velocity normalized by blade tip speed for 4.2% clearance at the rotor trailing edge (dash-dot line), 25% rotor-stator axial gap (dotted line), and 50% rotor-stator axial gap downstream of the mixing plane (solid line)

At 4.2% rotor tip clearance, reverse flow at the rotor trailing edge occurs for spanwise location greater 94.95% span. Due to the larger spanwise extent of the velocity deficit compared to that of 2.8% clearance, spanwise mixing upstream of the mixing plane in the rotor-stator gap does not energize the tip section flow sufficiently to remove reverse flow at the upstream interface of the mixing plane as shown in Figure B-9. Figure B-9 compares the axial velocity deficit at 4.2% clearance and 2.8% clearance upstream of the mixing plane to
show the reverse flow. To satisfy the flux conditions of the mixing plane formulation, the tip section reverse flow occurring for tip clearances greater than 3.6% (demonstrated in Figure B-8) is set to zero velocity.

This non-physical velocity profile downstream of the mixing plane is an issue with steady mixing plane simulations of a multistage compressor with large rotor tip clearance. Because of this, the steady APNASA calculations were used to analyzed the flow features associated with large rotor tip clearance to compare with the URANS simulations.

Figure B-9: Velocity field for 2.8% and 4.2% clearance at 50% rotor-stator axial gap upstream of the mixing plane
In section 2.2.2, APNASA simulations of the NASA LSAC at design rotor tip clearance-to-span ratio of 1.4% are benchmarked against experimental data from Wellborn [45] at near peak efficiency operation. Additional comparisons between NASA LSAC test data and APNASA computed third stage radial profiles are shown to demonstrate the capability of APNASA. No data at rotor tip clearance-to-span ratios other than 1.4% is available for the NASA LSAC. The APNASA computations of the NASA LSAC compressor performance and flow field at 1.4% clearance-to-span in addition to APNASA benchmarking studies published by NASA for various compressor geometries provide confidence in the APNASA computed results for all clearance-to-span ratios. This section expands on the comparisons presented in Chapter 2.
C.1 Third rotor

Figures C-1 to C-5 present comparisons of the NASA LSAC third rotor flow field computed by APNASA and experimental rig data. Equation C.1 and C.2 define diffusion factor and loss coefficient presented in Figures C-4 and C-5, respectively. APNASA adequately computes the NASA LSAC third rotor inlet and exit flow field.

\[
\text{Diffusion Factor} = 1 - \frac{V_{R3out}}{V_{R3in}} + \frac{\Delta V_{tan,R3}}{2\sigma V_{R3in}} \tag{C.1}
\]

\[
\text{Loss coefficient, } \omega = 1 - \frac{P_{t,R3in} - P_{t,R3out}}{P_{t,R3in} - P_{R3in}} \tag{C.2}
\]

Figure C-1: Third rotor inlet relative tangential velocity comparison.
Figure C-2: Third rotor exit relative tangential velocity comparison

Figure C-3: Third rotor relative turning comparison
Figure C-4: Third rotor diffusion factor comparison

Figure C-5: Third rotor loss coefficient comparison
C.2 Third stator

Figures C-6 to C-8 compare APNASA computed NASA LSAC third stator flow field to experimental data. APNASA adequately computes the NASA LSAC third stator inlet and exit flow field.

![Graph showing comparison between APNASA and Test data for absolute tangential velocity](image)

Figure C-6: Third stator inlet absolute tangential velocity comparison
Figure C-7: Third stator exit absolute tangential velocity comparison

Figure C-8: Third rotor absolute turning comparison
Appendix D

Intermediate Tip Clearance Calculation Results

In the main text, 1.4%, 3.6%, and 5.6% clearance-to-span computational results have been used to demonstrate how performance characteristics and flow quantities vary due to tip clearance. Clearance-to-span ratios of 1.4% and 5.6% are selected because they are the smallest and largest clearance-to-span ratios analyzed. 3.6% clearance-to-span is selected because it represents the boundary of the two regimes described in Chapter 3 based on APNASA computed efficiency change as a function of rotor tip clearance. This section presents APNASA computed results for all rotor tip clearances to (1) show the computational results used to analyze the effect of increasing rotor tip clearance on compressor performance and (2) demonstrate that APNASA computed results are consistent for clearance-to-span ratios from 1.4% to 5.6%.

D.1 Four-stage compressor performance analysis

Figure D-1 shows NASA LSAC total pressure rise characteristic and Figure D-2 shows computed compressor efficiency as a function of IGV inlet flow coefficient for clearance-to-span ratios from 1.4% to 5.6%, expanding on the results presented in Chapter 3. Again, for
uninstalled flow operation increasing rotor tip clearance decreases compressor efficiency, total pressure rise, and flow range.

Figure D-1: NASA LSAC total pressure rise characteristic for clearance-to-span ratios from 1.4% to 5.6%
Figure D-2: NASA LSAC efficiency as a function of IGV inlet flow coefficient for clearance-to-span ratios from 1.4% to 5.6%

D.2 Third stage performance analysis

Figure D-3 to D-5 confirm that results presented in Chapter 5 on rotor performance are consistent for all clearance-to-span ratio. Figures D-4 and D-5 show rotor loss generation and streamwise blockage increase through the rotor passage; the two regimes can be identified in these figures. For clearance from 3.6% to 5.6%, both rotor loss generation and rotor relative streamwise blockage growth increase at a decreased rate compared to that from 1.4% to 3.6% clearance.
Figure D-3: Confirmation that repeating stage condition for third stage is satisfied for all clearance-to-span ratios

Figure D-4: Normalized tip section rotor loss generation as a function of chord for all clearance-to-span ratios depicts two regime behavior
Figure D-5: Increase in rotor relative streamwise blockage as a function of chord for all clearance-to-span ratios depicts two regime behavior.

Figure D-6 shows streamwise blockage decrease as a function of rotor tip clearance in the rotor-stator axial gap. Again, increasing rotor tip clearance results in a larger rotor exit tip section velocity deficit leading to increased mixing in the rotor-stator axial gap.
Figure D-6: Decrease in streamwise blockage in rotor-stator axial gap as a function of chord for clearance-to-span ratios from 1.4% to 5.6%

Figure D-7 to D-10 confirm that results presented in Chapter 6 on stator performance are consistent for all clearance-to-span ratios. The separation identified in the suction side tip section corner is observed for all clearances and APNASA computes a consistent flow field as tip clearance varies. Figure D-7 illustrate separation in the tip section grows with tip clearance and the resulting flow redistribution improves the hub region. Figures D-8 to D-10 compare the velocity profiles through the third stage in both the absolute and relative reference frame.
(a) Radial distribution of rotor solidity and aspect ratio

(b) Radial distribution of rotor leading edge, trailing edge, and setting angles

Figure D-7: Identification of stator flow separation for clearance-to-span ratios from 1.4% to 5.6%

(a) Absolute reference frame

(b) Relative reference frame

Figure D-8: Third stage inlet velocity deficit for clearance-to-span ratios from 1.4% to 5.6%
Figure D-9: Third rotor exit velocity deficit for clearance-to-span ratios from 1.4% to 5.6%

Figure D-10: Third stage exit velocity deficit for clearance-to-span ratios from 1.4% to 5.6%
Bibliography


