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Pareto Optimality and Nash Equilibrium for Building Stable Systems

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***Abstract*— This paper introduces a design approach based on system analysis and game theory for the identification of architectural equilibrium which guarantees the stability of the system being designed and its environment after the integration. We introduce multi-objective optimization and game theory, and their links with systems engineering through mathematical models. While Pareto optimality is used to select best architectures and to support independent decisions, Nash equilibrium is used to find out architectural equilibrium and to support interdependent decisions. This approach was illustrated previously in a case study.**

Keywords—Architectural equilibrium; Nash equilibrium; Stable systems.

I. INTRODUCTION

The integration of a new system in an environment can disrupt the stability of this environment and trigger adverse or unexpected events, which in turn can affect the success of this integration. This integration becomes more complex due to the complexity of the system of interest (SOI) and its environment.

To address this problem, we propose in this paper a design approach based on system analysis and game theory for the identification of architectural equilibrium which guarantees the stability of the SOI and its environment after the integration. We have demonstrated this assertion using a practical example related to electric vehicles [1][2]. We rely on a system analysis to understanding the system environment and the stakeholders involved during the SOI`s life cycle. The objective is to find global balances between external systems (stakeholders) that would meet their needs (would satisfy their strategies), taking into account feasibility constraints (economic, technological, regulatory, societal, etc.) This balance (equilibrium) guarantees a better integration of the SOI and the stability of its environment. In order to find this equilibrium, we create a game model as defined in game theory. Solving this game and finding equilibrium allow to anticipate changes in an uncertain environment (departure or arrival of a stakeholder, new or changing needs or constraints, etc.)

A game as defined in game theory is a mathematical function which matches for each possible combination of strategies played by N players, an outcome of the game. This outcome is often represented - especially in numerical examples - by the gain of each player [3].

II. BACKGROUND

Games could be used to solve many multi-objective optimization (MOO) problems. MOO in the context of engineering and design of complex systems is very useful to support tradeoff analysis and decision making. It allows considering multidisciplinary objectives such as performance, cost, schedule and risk (cf. Maier and Rechtin, 2000 [4]). A quick overview of MOO and the most popular methods are presented in [4].

A MOO problem can be formalized mathematically as follows [5]:

We look at Minimizing $F(x)=[F_1(x), F_2(x),\ldots,F_k(x)]^T$

while $g_j(x) \leq 0, j=1,2,\ldots,m.$ and $h_i(x) = 0, i=1,2, \ldots, e$

where
- k is the number of objective functions
- m is the number of inequality constraints
- e is the number of equality constraints
- $x \in E^n$ is the vector of design variables, where n is the number of independent variables x_i
- $F(x) \in E^k$ is the vector of objective functions $F_i(x) : E^n \rightarrow E^1$
- $F_i(x)$. x_i^* is the point minimizing the objective function $F_i(x)$

The feasible decision space X is defined as a set $\{x|g_j(x) \leq 0, j=1,2,\ldots,m$; and $h_i(x)=0, i=1,2,\ldots,e\}$.
The feasible objective space Z is defined as a set $\{F(x)|x \in X\}$.

The notion of preference of decision-makers and priority between optimization objectives influence the choice of methods for solving an MOO problem. The author [4], for example, class examples into two categories: methods with expression of preferences in advance (a priori preferences) and the Pareto methods (a posteriori preferences). The reader may refer to the studies on MOO methods in [6], [7] and [8] for further references.

Indeed, in an MOO problem, there is no single and unique global solution, it is often necessary to identify a set of optimal points. The predominant concept in defining an optimal point is Pareto optimality [5]. This is the mathematical definition of a Pareto point:

A point $x^* \in X$ is a Pareto optimum if, and only if, there is not another point $x \in X$ such as $F(x) \leq F(x^*)$ and $F_i(x) < F_i(x^*)$ for at least one function. The Pareto optimal points represent the boundary of the space of feasible objectives Z which is called the Pareto frontier.

The solutions which are not Pareto optimal while they meet other constraints are called Weakly Pareto Optimal. A point $x^* \in X$ is a Weakly Pareto optimum if, and only if, there is not another point $x \in X$ such as $F(x) \leq F(x^*)$.

In other words, a point is a Weakly Pareto Optimal if there is not another point that enhances all objective functions simultaneously. A point is a Pareto optimum if there is not another point that improves at least one objective function at the expense of another objective function.

However, the preferences of decision makers are difficult to quantify and depend on several parameters within the organizations. Some decision problems in the context of systems engineering can be seen as decision problems of a single agent (one independent decision maker), but, in practice, many interdependent stakeholders around the SOI (architects, engineers, managers, project managers, program managers. etc.) may have different preferences difficult to quantify and often difficult to converge [9]. In addition, when designing a system where many external stakeholders are involved, and are expecting some added value, the decisions become interdependent. Thus, we discuss in the following paragraphs the contribution of game theory and Nash equilibrium in addressing such problems.

III. WHY GAME THEORY

Game theory is a rational decision theory of strategically interdependent agents, that is to say, which influence each other and are aware of these reciprocal influences. The games are interactive decision situations in which utility (welfare) of each individual depends on the decisions of other individuals [10].

A. *Definitions*

Some definitions in this section are adapted from [3]. The ingredients of a game are a list of n individuals called players, aiming to maximize an objective function or quantified gains, given the information they have (rationality conditions of the players, a rational player is the one who wants maximum gains). We then have n sets, one set by a player, whose elements are called strategies. In addition, we have a mathematical function which matches to each of the possible combinations of strategies available to N players an outcome of the game. This outcome is often represented - especially in the numerical examples – by the gain of each player. Indeed, a solution of the game is a combination of strategies, and the associated gains for each player accordingly.

Formally, we have a game with n players. Each player i has a set S_i of strategies. $s=(s_1,\ldots,s_n)$ is a combination of strategic choices of n players where s_1 is the strategic choice of the player 1, s_2 is the strategic choice of the player 2 and so on. $\Pi = (\Pi_1,\ldots,\Pi_n)$ is the result of the game where $\Pi_i(s_1,\ldots,s_n)$ is the gain of the player i when s is selected, where i can be any player $i = 1,2,\ldots,n$.

The strategies can be pure strategies (no probability) or mixed strategies (for a given pure strategy s_1, we associate a probability p_1). The solution of the game with mixed strategies may not be in the real world, we can imagine a real number (float) to quantify the gain (even if it does not exist in reality). The criterion for comparing the gains in the various possible alternatives is that of expected utility, which is to take the expected gains of pure-strategies obtained by weighting them by the probabilities chosen by the players (their mixed strategies).

We can have a complete information game (each player knows all the possible outcomes of the game and the strategies of the other players) and an incomplete information game (players assign a priori probabilities on the outcomes of the game and the strategies of the other players). The game can be zero-sum when the sum of gains of all players is zero. A non-zero sum game can be reduced to a zero-sum game by creating a fictitious player.

Unlike a non-cooperative game, in a cooperative game the players can create coalitions (agreements on the choice of strategies to play.) A dominant strategy of a player exists if he has a strategy that gives him the maximum gains whatever the strategies of the other players.

Finally, we talk about an equilibrium (a point of rest, which may be the end point of a process) when the players announce their choices and no one regrets his choice given the choices of the other players. The Nash equilibrium is an example. If all strategies is finite (bounded) and the game can include mixed strategies, then the game has at least one Nash equilibrium solution.

B. *Nash Equilibrium*

Nash equilibrium is a very important concept in game theory (name of John F. Nash, Nobel Prize in Economics in 1994. He introduced the concept of equilibrium in 1951). Nash equilibrium describes an outcome of the game in which no player has an incentive to change his strategy given the strategies of the other players. The formal description of Nash equilibrium can be given as follows [11]:

Let`s define a no-cooperative game with n player and $s_*=(s_{*1},\ldots,s_{*n})$ a combination of strategic choices of these n players where s_{*1} is the strategic choice of the player 1, s_{*2} is the strategic choice of the player 2 and so on. In addition, let $\Pi_i(s_{*1},\ldots,s_{*n})$ the gain of the player i when s_* is selected, where i can be any player $i = 1,2,\ldots,n$.

Nash Equilibrium is formally defined as follows:

A combination of strategic choices $s_*=(s_{*1},\ldots,s_{*n})$ is a Nash equilibrium if

$\Pi_i(s_{*1},\ldots,s_{*i},\ldots,s_{*n}) \geq \Pi_i(s_{*1},\ldots,s_i,\ldots,s_{*n})$ for each s_i in S_i and each player i. S_i is the set of strategies of the player i.

In summary, $\boldsymbol{s}_{*i}$ is the best strategy of player $\boldsymbol{i}$ when all the other players choose their strategies $\boldsymbol{s}_{*}$.

In this work, we are interested in studying non-cooperative games (the players cannot talk to each other and sign preferential contracts (particularly because of anti-trust laws prohibiting businesses to communicate with each other to enter into agreements, etc. [11]))

C. Link with systems engineering

As explained previously, one of the first stages of a systems engineering approach is the modeling of the SOI`s environment. The purpose of this step is to define the interactions between the SOI with the external systems with which it interacts in order to anticipate the evolution of an external system (or several) that could affect other systems. This step clarifies the external interfaces of the SOI before diving in optimizing its internal architectures. Note that the environment modeling phase is critical. Forgetting an external system could jeopardize the design of the SOI. Indeed, each external system has its own expectations, needs or constraints, throughout the life cycle of the system, the designer should anticipate and satisfy. However, these needs are often interrelated and sometimes antagonistic. In order to maximize the profits related to the SOI, each stakeholder has strategies. Thus, we arrive at a situation where the designer must look for equilibrium for the purpose of stability. This architectural equilibrium should more or less satisfy all these stakeholders and ensure a stable environment throughout the SOI's life cycle.

By taking the vocabulary of the systems engineering and game theory, we assume that each stakeholder can be considered as a player around the SOI and he is rational. Each player has his own strategies to maximize his gains, which account for the satisfaction of his own needs. However, the designer must try to best satisfy the stakeholders (players). At the same time, the designer should find out an architectural equilibrium, meaning that it guarantees the stability of the environment throughout the cycle life of the SOI and the life cycles of all the stakeholders. Finally, we can say that the pre-design of this architectural equilibrium is a solution of a game where the stakeholders are the players. If we consider that the stakeholders can build coalitions in the sense that they combine their choice of strategies, it is called cooperative game. Otherwise, it is in a non-cooperative game.

The use of game theory in the context of systems engineering and its relation to design architectural frameworks is mentioned in [12]. Indeed, the authors propose a framework called Engineering Systems Multiple-Domain Matrix (ES-MDM). The framework is based on the use of several different areas (environmental, social, functional, technical, process) and matrices (designer / decision maker, stakeholder, objectives, functions, objects, activities). Then the authors propose to cross the matrices together and in different areas, for example, the stakeholder matrix about a SOI representing the social field, Stakeholders are human entities that contribute to the objectives of the system and control system components. The extent of their control system defines the scope of the system. For the identification of stakeholders in a given system, it is useful to ask the following questions (cf. Rechtin and Maier (2000) in [12]): Who benefits? Who pays? Who supplies? And who loses? Then, the authors propose a combination of matrices, such as that of stakeholders with those goals. Each stakeholder has his own objectives, and other stakeholders can declare their support, opposition or indifference. This is used to store information about the positions of the various stakeholders that can be used in a game model to analyze strategies and to align their interests (cf. Dixit and Skeath (2003) in [12]) on a tutorial on game theory and applications in the real world.)

D. Proposed approach

In our approach, we propose the following interdependence analysis process to identify an architectural equilibrium:

1. *Analysis and modeling of the SOI`s environment*
2. *Stakeholders identification*
3. *Macro-needs analysis and identification of strategies (for each stakeholder)*
4. *Identification of the interdependence between strategies, so between stakeholders*
5. *SOI Total Cost of Ownership calculation and life-cycle assessment*
6. *Realization of a distribution matrix - normal or extensive form of the game – according to the combination of strategies*
7. *Setting examples of distribution scenarios*
8. *Equilibrium calculation*
9. *If cooperation is possible, create coalitions of stakeholders, and go to 7*
10. *Emphasize the equilibrium solutions.*

IV. Discussion

One difficulty in finding an architectural equilibrium is that it is not easy to empirically observe the emergence of behavior consistent with a mixed strategy. Indeed, in that case, there is no strict incentive that an actor plays the equilibrium strategy, the use of random strategies implies the use of assumptions on the expected utility and assumptions of risk [10]. Another difficulty in solving games is related to the fact that the information can be complete or incomplete. It is not easy for a player to know all the strategies of other players without contracts for cooperation. In addition, sometimes, it is very difficult to formalize gains functions (we can even be in front of non-formalized gains, such as brand image where the notion of perception is important, etc.) We can also consider the emergence of new issues that brings us clearly in decisions in uncertain situations. However, building such models helps greatly in finding architectural equilibriums, or at least approaching them. The example we presented in [1] related to electric vehicles is instructive.

Indeed, to designing complex industrial system, we believe that two types of optimization models are complementary and necessary. These models take into account the types of decisions that come into play in the choice of solutions.

 Pareto Optimality for finding the best architectural solutions of an SOI given the needs and constraints of the stakeholders / external systems. These solutions correspond to different options or alternatives of the system architectures. Pareto Optimality is useful for "independent" decisions in the sense that these decisions depend only on the choices (or preferences) as defined by the SOI designer. The decision space related to these decisions is a set of endogenous factors.

 Nash equilibrium for finding the best equilibrium between the stakeholders surrounding the SOI. The results correspond to different architectural equilibrium. The equilibrium models are useful for "interdependent" decisions, where the decision space contains endogenous and exogenous factors, which depend not only on the SOI designer, but also on other stakeholders.

Finally, we can consider that architectural equilibrium of a given system has an impact on the design, therefore on the independent decisions. The architectural equilibrium gives the first orientations that are made during the design process. Some preliminary design constraints result from the choice of the architectural equilibrium.

V. Conclusion

In this paper, we present the contribution of game theory in finding architectural equilibrium, in the context of complex systems engineering. Indeed, for interdependent decisions of different stakeholders involved during the lifecycle of a given system, the concept of equilibrium is important. An architectural equilibrium of the SOI guarantees a better integration, stability of its environment and the satisfaction of all the stakeholders. This assertion is shown by a practical example related to the market of electric vehicles in [1][2]. Many factors may contribute to the success of electric vehicles, which do not necessarily depend on the car manufacturers but also on other stakeholders such as energy suppliers, local authorities, governments, etc.

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