Computer Science and Artificial Intelligence Laboratory
Technical Report

MIT-CSAIL-TR-2017-009 May 16, 2017

Autonomous COLREGS Modes and Velocity Functions

Michael R. Benjamin

massachusetts institute of technology, cambridge, ma 02139 usa — www.csail.mit.edu

Autonomous COLREGS Modes and Velocity Functions

Michael R. Benjamin
Mechanical Engineering, MIT
Computer Science and Artificial Intelligence Lab
Cambridge MA 02139
Email: mikerb@mit.edu

May 15, 2017

Abstract

This paper concerns an implementation of an autonomy system for unmanned surface vessels operating in accordance with the Coast Guard Collision Regulations (COLREGS). The autonomy system is implemented by associating a dedicated ownship behavior module for each contact for collision avoidance. For each behavior, a mode determination is made based on the COLREGS rules, ownship position and trajectory, and the contact position and trajectory. Based on the mode, a velocity function is generated, over the set of possible ownship maneuvers, to bias the vehicle in accordance with the COLREGS. The focus on this paper is solely on (a) the mode determination algorithms, (b) the requisite ownship and contact terms regarding position, trajectory and relative position utilized in the mode determination algorithms, and (c) the form and equations comprising the velocity function associated with each mode.

Contents

1 Introduction

This paper describes key components of an autonomy system for unmanned surface vehicles (USVs) operating in accordance with the Coast Guard Collision Regulations (COLREGS), [3]. This autonomy system is behavior based, with certain behaviors responsible for primary mission objectives such as transiting and surveying, and additional behaviors dedicated to COLREGS collision avoidance. For each contact a new collision avoidance behavior is spawned, and lives until the contact is resolved. The helm makes a decision several times per second, each time producing a heading and speed decision. On each decision the helm proceeds by first invoking the `onRunState()` function on each behavior, which returns a velocity function, essentially an objective function ranking the utility of all possible heading-speed maneuvers. The helm collects each velocity function and performs a multi-objective optimization search for the optimal heading-speed decision based on the weighted combination of objective functions. The objective functions in this case are piece-wise linearly defined. Interval programming (IvP) is a method for representing such functions and a search algorithm for the decision optimization stage, [1].

This scope of this paper is limited to the logic of COLREGS mode determination with a full distillation and definition of participating terms, and the construction of velocity functions associated with each mode. The COLREGS project is a multi-year effort in progress, funded by ONR, Dr. Bob Brizzolara, Grant N00014-15-1-2213, with the optimization algorithms funded by ONR, Dr. Behzad Kamgar-Parsi and Dr. Don Wagner, Grant N00014-15-1-2227. This report reflects a snapshot and progress status summary. It also serves to open the mode logic for wider review and feedback from collaborators. The COLREGS code is expected to be released later in 2017, wrapped in as part of the MOOS-IvP open source marine autonomy project [2].

2 Helm and COLREGS Behavior Overview

The COLREGS collision avoidance behavior is one of perhaps several behaviors instantiated within the helm at any given time. For each behavior, the helm invokes a particular function `onRunState()`. This function is implemented differently for different behaviors, but each behavior return an objective function defined over the helm decision space. For surface vehicles this objective function is typically defined over heading and speed, and in this case they are also referred to as velocity functions. The high-level structure of the COLREGS behavior's `onRunState()` function is given below. The focus of this paper is on lines 4 and 6.

Algorithm 1: Top Level *onRunState()* Function

```
1: procedure onRunState( )                      ▷ the top-level entry for all behaviors
2:     update ownship position and pose
3:     update contact position and pose
4:     set the collision avoidance mode
5:     determine the behavior priority weight
6:     build the IvP function
7:     return the IvP function with set priority weight
8: end procedure
```

3 Vehicle Position, Pose, and Relative Geometry Relationships

The collision avoidance algorithms presented in this paper make use of certain common terms and definitions first laid out here. To begin with, in multi-vehicle situations there is often one vehicle regarded as being under our control, *ownship*, and one or more vehicles not under our control, a *contact*. This holds even if all vehicles may be running the same algorithms.

3.1 Vehicle Position and Pose

The position of each vehicle is given in local x-y coordinates, in meters. This is typically relative to a pre-agreed upon position in latitude-longitude space, a *datum*, representing the $(0, 0)$ position in local coordinates. The heading is given in degrees with zero being North, and the vehicle speed is given in meters per second. The terms used are:

- current ownship position: x_{os}, y_{os}
- current ownship heading and speed: θ_{os}, v_{os}
- current contact position: x_{cn}, y_{cn}
- current contact heading and speed: θ_{cn}, v_{cn}
- choice of next ownship heading and speed: θ, v

These terms are put into context in Figure 1.

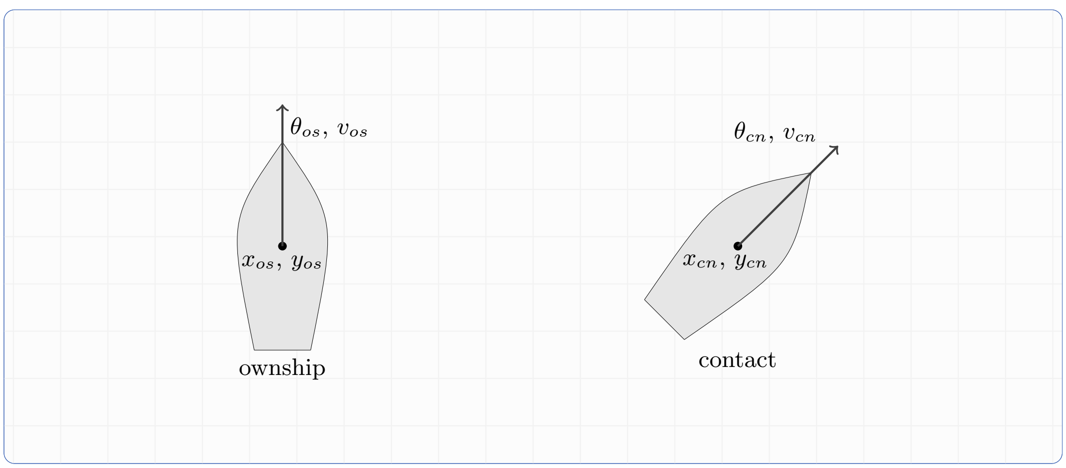


Figure 1: **Terms for vehicle position and pose:** the term *ownship* is the regarded as the vehicle being controlled and the term *contact* refers to the vehicle being avoided in a collision avoidance scenario.

In the vehicle experiments reported in this paper, each vehicle has continuous access to a GPS system for deriving x, y, and v, and θ is obtained from an on-board compass. The heading is the direction the vehicle is pointing, and not necessarily the direction the vehicle is moving. These two values may be different in wind or current situations. Ownship also has a continuous, e.g. about 1Hz, access to the contact position and pose terms, and vice versa. This is typically accomplished by each vehicle packaging its position and pose information and timestamp into a report message, which is then shared over a wireless network. Thus there is a bit of latency in reports received. This is similar to an AIS system in commercial ships.

3.1.1 A Few Useful Operators on Heading Values

In most cases, a vehicle heading is in the range of $[0, 360)$. In some cases a vehicle heading may be the result of one or more mathematical operations putting the heading outside this domain. The $[\theta]^{360}$ operator returns a heading value to this domain. For example $[405]^{360} = 45$. The definition is given by:

$$[\theta]^{360} = \begin{cases} \theta - \lfloor \frac{\theta}{360} \rfloor \cdot 360 & (\theta \geq 0) \\ \theta + (\lfloor \frac{-\theta}{360} \rfloor + 1) \cdot 360 & otherwise \end{cases} \tag{1}$$

Likewise it is sometimes convenient to represent a heading angle in the range of $[-180, 180)$. The $[\theta]^{180}$ operator modifies a heading value to this domain. For example $[315]^{180} = -45$. The definition is given by:

$$[\theta]^{180} = \begin{cases} \theta - \lfloor \frac{\theta+180}{360} \rfloor \cdot 360 & (\theta \geq 0) \\ \theta + \lfloor \frac{-\theta+180}{360} \rfloor \cdot 360 & otherwise \end{cases} \tag{2}$$

In reasoning about the difference of two headings, it is often useful to explicitly use the absolute value of their difference in the smallest of the two possible directions. For example, given the difference between heading 350 and 10, we may want be clear that we're talking about 20 degree difference, not 340. The $\delta(\theta_1, \theta_2)$ operator makes this clear. For example, $\delta(350, 10) = 20$. The *heading deviation* is given by:

$$\delta(\theta_1, \theta_2) = \left| [\theta_1 - \theta_2]^{180} \right| \tag{3}$$

Note that $cos(\theta_1 - \theta_2)$ is equal to $cos(\delta(\theta_1, \theta_2))$, e.g., $cos(340) = cos(20)$. So, in these cases we will just use the simpler notation of $cos(\theta_1 - \theta_2)$.

3.2 Range, Absolute Bearing and Relative Bearing

Three key terms used for reasoning about collision avoidance between two vehicles are *range*, *absolute bearing* and *relative bearing*. The absolute bearing is primarily useful as an intermediate step for calculating relative bearing.

- range between ownship and contact: r^{os}_{cn}
- absolute bearing from ownship to contact: bng^{os}_{cn}
- relative bearing from ownship to contact: $relbng^{os}_{cn}$, or simply β
- absolute bearing from contact to ownship: bng^{cn}_{os}
- relative bearing from contact to ownship: $relbng^{cn}_{os}$, or simply α

The range is simply the linear distance between vehicles at the current instant of time, as calculated by the Pythagorean theorem. Range is given in meters. The absolute bearing is the angle of the contact to ownship position, regardless of ownship orientation. This value is given in degrees, in the range of $[0, 360)$ with North being 0 degrees and East being 90 degrees. The relative bearing is similar to absolute bearing but it does factor ownship heading, in a sense rotating the world as if ownship heading is North. These terms are put into context in Figure 2.

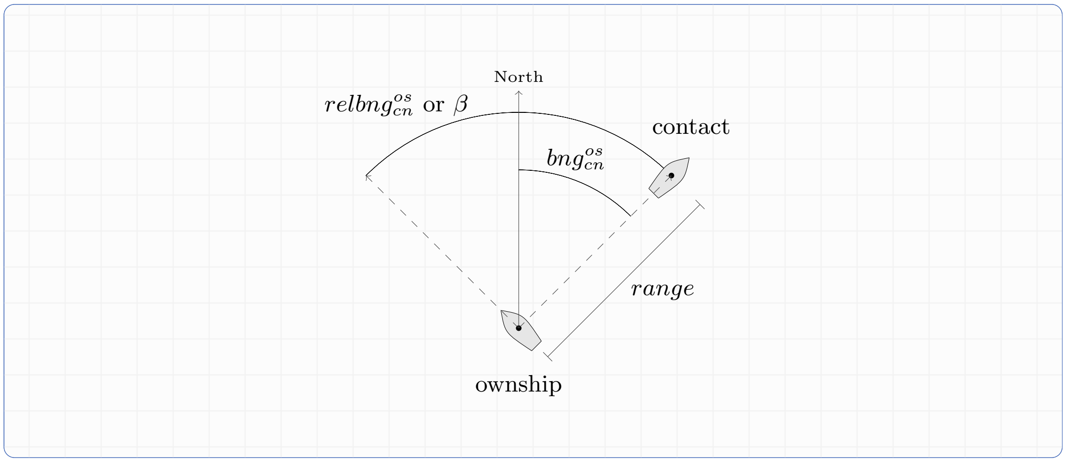


Figure 2: Core inter-vehicle relative measurements including range, absolute bearing and relative bearing. In this example the absolute bearing of the contact is 45 degrees and the relative bearing is 90 degrees.

The range between two vehicles is directly from Pythagorean theorem:

$$\texttt{range} = r_{cn}^{os} = \sqrt{(x_{os} - x_{cn})^2 + (y_{os} - y_{cn})^2} \tag{4}$$

The absolute bearing can be calculated from the below, taking care that the units are in degrees with zero pointing North:

$$bng_{cn}^{os} = \begin{cases} 0 & (x_{os} = x_{cn}) \texttt{ and } (y_{os} \leq y_{cn}) \\ 180 & (x_{os} = x_{cn}) \texttt{ and } (y_{os} > y_{cn}) \\ tan^{-1}(\frac{|y_{os}-y_{cn}|}{|x_{os}-x_{cn}|}) * \frac{180}{\pi} + 90 & (x_{os} < x_{cn}) \texttt{ and } (y_{os} \leq y_{cn}) \\ 90 - tan^{-1}(\frac{|y_{os}-y_{cn}|}{|x_{os}-x_{cn}|}) * \frac{180}{\pi} & (x_{os} < x_{cn}) \texttt{ and } (y_{os} > y_{cn}) \\ tan^{-1}(\frac{|y_{os}-y_{cn}|}{|x_{os}-x_{cn}|}) * \frac{180}{\pi} + 270 & (x_{os} > x_{cn}) \texttt{ and } (y_{os} \leq y_{cn}) \\ 270 - tan^{-1}(\frac{|y_{os}-y_{cn}|}{|x_{os}-x_{cn}|}) * \frac{180}{\pi} & (x_{os} > x_{cn}) \texttt{ and } (y_{os} > y_{cn}) \end{cases} \tag{5}$$

The *relative bearing* can then be calculated from the bearing and ownship heading:

$$relbng_{cn}^{os} = \beta = [bng_{cn}^{os} - \theta_{os}]^{360} \tag{6}$$

In the opposite direction, the bearing and relative bearing from contact to ownship is similarly calculated. The bearing may be denoted with bng_{os}^{cn} and the relative bearing with $relbng_{os}^{cn}$. A short notation for the latter term is simply the symbol α. The short vernacular term for the relative bearing from the contact to ownship is the *contact angle*. The short vernacular term for relative bearing from ownship to contact is simply the *relative bearing*.

3.3 Range Rate and Bearing Rate

The terms introduced thus far have only involved ownship and contact position and heading. A few further terms are useful that also involve the vehicle speed. The first is the *range rate* and the second is the *bearing rate*.

The *range rate* is the rate at which the range of two vehicles is changing. The range rate is negative as they get closer and positive after they pass each other with the range rate exactly at zero at their closest point of approach. The calculation of range rate is the sum of their individual speeds in the direction of the other vehicle:

$$\texttt{range rate} = \dot{r} = v_{cn}^{os} + v_{os}^{cn} \tag{7}$$

This idea is put into context in Figure 3.

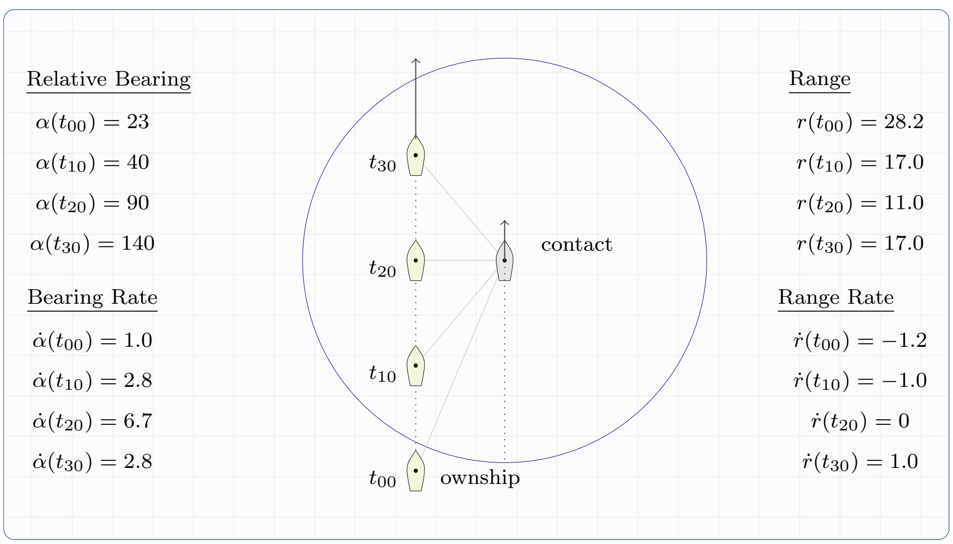


Figure 3: **Bearing rate and range rate**: Note the bearing monotonically increases as ownship passes the contact. The bearing rate peaks at the closest point of approach(CPA). The range is at a minimum at CPA, and the range rate is maximum at CPA.

The velocity of ownship in the direction of contact's current position, v_{cn}^{os}, and the velocity of the contact in direction of ownship's current position, v_{os}^{cn}, are given by:

$$v_{cn}^{os} = cos(\alpha) \cdot v_{os} \tag{8}$$
$$v_{os}^{cn} = cos(\beta) \cdot v_{cn} \tag{9}$$

We prefer to think of the range rate as the sum of these two terms because each term in isolation is useful in other considerations discussed later.

The *bearing rate* is the rate of change (in degrees per second) in the relative bearing from ownship to the contact. It is given by:

$$\texttt{bearing rate} = \dot{\beta} = -(v^{os}_{tn} + v^{cn}_{tn})\frac{360}{2r\pi} \tag{10}$$

where v^{os}_{tn} is the ownship speed in the direction of the *tangent heading* and v^{cn}_{tn} is the contact speed in the direction of the tangent heading. The *tangent heading* is the heading given the current ownship to contact absolute bearing, plus 90 degrees:

$$\theta_{tn} = [bng^{os}_{cn} + 90]^{360}$$

The speed of ownship and contact in the tangent heading is given by:

$$v^{os}_{tn} = cos(\theta^{os}_{tn}) \cdot v_{os}, \quad \text{where} \;\; \theta^{os}_{tn} = \delta(\theta_{os}, \theta_{tn})$$
$$v^{cn}_{tn} = cos(\theta^{cn}_{tn}) \cdot v_{cn}, \quad \text{where} \;\; \theta^{cn}_{tn} = \delta(\theta_{cn}, \theta_{tn})$$

Figure 4 below puts this into context for a given ownship and contact position and trajectory.

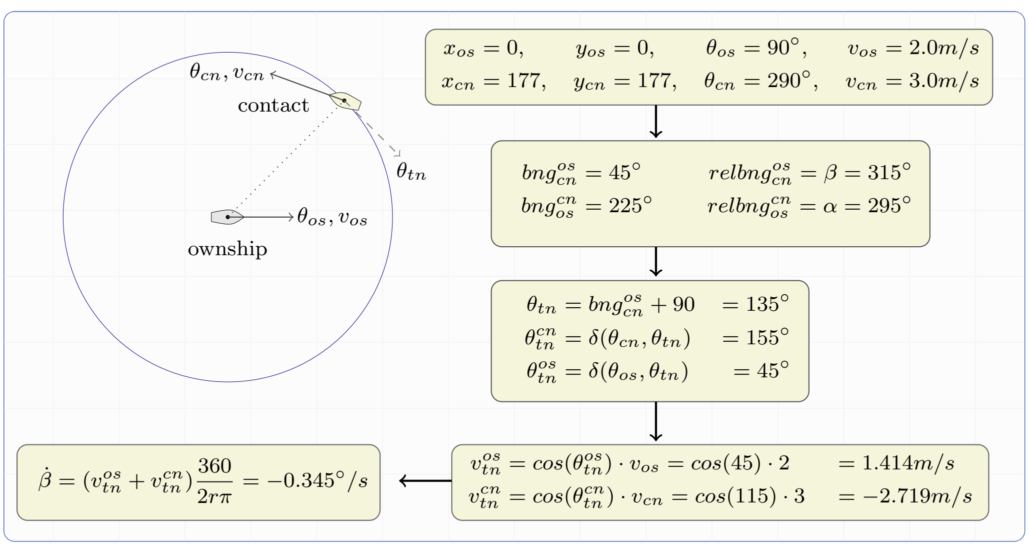


Figure 4: **An example calculation of bearing rate,** $\dot{\beta}$: The bearing rate calculation for a particular ownship and contact position and trajectory, with intermediate terms evaluated.

3.4 Fore and Aft Relative Positions

In later collision avoidance considerations, a few other terms and Boolean relationships will be handy. A Boolean term is one that is set to either a true or false value as opposed to a numerical value, like range or bearing. At times it may be useful to know, for example, whether a point or another ship is either *fore* or *aft* of another ship. Since ships are not point masses, but instead have actual width and length, the center point of a ship is used for making a fore or aft determination. These terms are put into context in Figure 5.

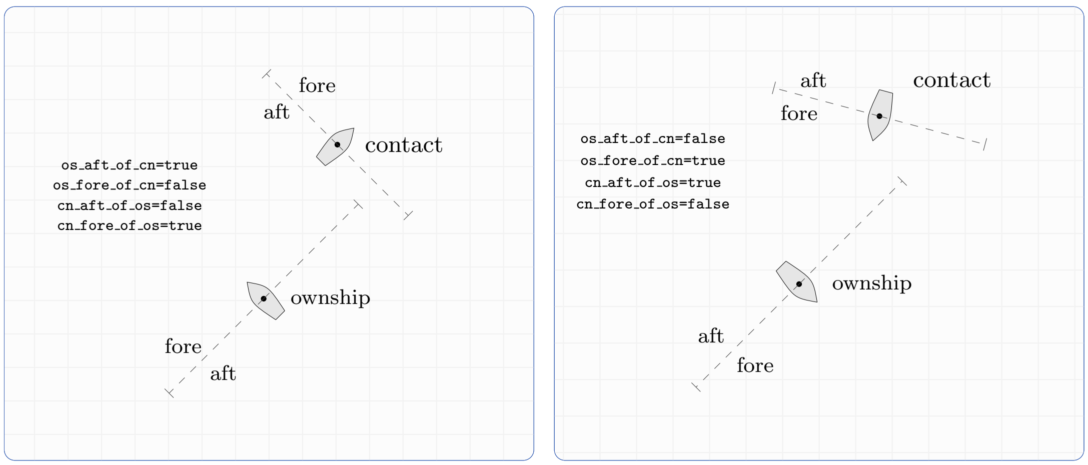


Figure 5: **Fore and aft**: are Boolean terms indicating if an object or vessel is in front of or behind a given vessel.

The Boolean values may be calculated based on relative bearing (β), and contact relative bearing (α) as follows:

$$\texttt{fore}_{cn}^{os} = \begin{cases} false & 90 < \beta < 270 \\ true & otherwise \end{cases} \tag{11}$$

$$\texttt{aft}_{cn}^{os} = \begin{cases} true & 90 \le \beta \le 270 \\ false & otherwise \end{cases} \tag{12}$$

$$\texttt{fore}_{os}^{cn} = \begin{cases} false & 90 < \alpha < 270 \\ true & otherwise \end{cases} \tag{13}$$

$$\texttt{aft}_{os}^{cn} = \begin{cases} true & 90 \le \alpha \le 270 \\ false & otherwise \end{cases} \tag{14}$$

Note that if the relative bearing of the contact is exactly 90 or 270 degrees to ownship, i.e., ”on ownship’s beam”, then the contact is technically regarded as being both aft and fore of ownship. This is true vice versa for the contact.

3.5 Port and Starboard Relative Positions

It is also often useful to reason about whether one ship is to the right or left of another. Objects to the left of ownship are *port* of ownship or ”on ownship’s port side”. Objects to the right are *starboard* of ownship, or ”on ownship’s starboard side”. Again the centerline or *bow-stern line* of the ship is used in making this determination. These terms are put into context in Figure 6.

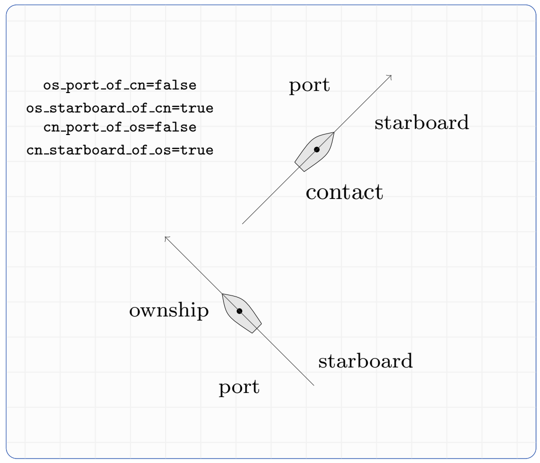


Figure 6: **Port and starboard**: refer to the common notion of the left and right side of a vehicle respectively.

The Boolean values may be calculated based on relative bearing (β), and contact relative bearing (α) as follows:

$$\texttt{port}_{cn}^{os} = \begin{cases} false & 0 \leq \alpha \leq 180 \\ true & otherwise \end{cases} \tag{15}$$

$$\texttt{star}_{cn}^{os} = \begin{cases} true & 0 \leq \alpha \leq 180 \\ false & otherwise \end{cases} \tag{16}$$

$$\texttt{port}_{os}^{cn} = \begin{cases} false & 0 \leq \beta \leq 180 \\ true & otherwise \end{cases} \tag{17}$$

$$\texttt{star}_{os}^{cn} = \begin{cases} true & 0 \leq \beta \leq 180 \\ false & otherwise \end{cases} \tag{18}$$

Note that if the relative bearing of the contact is exactly 0 or 180 degrees to ownship, i.e., ”on ownship’s bow-stern line”, then the contact is technically regarded as being on both the port and starboard side of ownship. This is true vice versa for the contact.

3.6 Boolean Crossing Relationships

Crossing relationships refer to candidate ownship maneuvers, i.e., choice of ownship heading and speed, given a contact position and trajectory. Three Boolean relationships are provided below indicating (a) whether ownship crosses the contact's bow-stern line, regardless of where it crosses, (b) whether ownship crosses contact's bow, and (c) whether ownship crosses contact's stern. First a few intermediate terms for later use.

The *ownship gamma heading*, θ_{γ}^{os} is the direction from ownship making a 90 degree angle to the contact's bow-stern line, as shown in Figure 7.

$$\theta_{\gamma}^{os} = \begin{cases} [\theta_{cn} + 90]^{360} & \texttt{port}_{cn}^{os} \\ [\theta_{cn} - 90]^{360} & otherwise \end{cases} \tag{19}$$

The *ownship gamma velocity*, v_{γ}^{os} is the velocity at which ownship is closing on the contact's bow-stern line. It is negative if the crossing occurred in the past:

$$v_{\gamma}^{os} = cos(\theta_{os} - \theta_{\gamma}^{os}) \cdot v_{os} \tag{20}$$

The *ownship gamma range*, r_{γ}, is the current range between ownship and the contact's bow-stern line. This depends on the range between the two vehicles, r, and the ownship gamma heading in Equation 19. Unlike the gamma velocity which becomes negative after a crossing, the gamma range is always non-negative.

$$r_{\gamma} = r \cdot cos(\theta_{\gamma}^{os} - relbng_{cn}^{os}) \tag{21}$$

These terms are put into context with a particular example in the Figure 7.

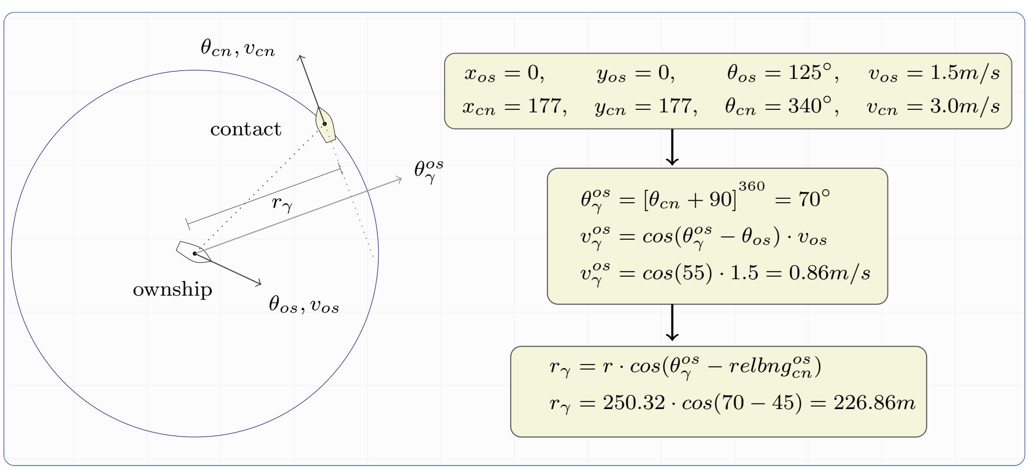


Figure 7: **Ownship speed in the sirection of crossing the contact**: This speed is used for determining whether and when ownship will cross the contact's bow-stern line.

3.6.1 Boolean Crossing Relationships - Ownship Crossing Contact

Armed with the ability to express whether ownship is on a trajectory to cross a contact's bow-stern line, and knowledge of bearing rate, a few key "crossing" relationships can be defined:

The term $\texttt{cross}^{os}_{xcn}$ is true if and only if ownship crosses the contact's bow-stern line, regardless of whether aft or fore, at some point in time either now or in the future, given the current position and linear trajectory of the two vehicles.

$$\texttt{cross}^{os}_{xcn} = \begin{cases} true & (\alpha = 0) \text{ or } (\alpha = 180) \\ true & (v^{os}_{\gamma} > 0) \\ false & otherwise \end{cases} \tag{22}$$

The term $\texttt{cross}^{os}_{xcnb}$ is true if and only if ownship crosses the contact's bow-stern line, and is fore of the contact at the moment of the crossing, at some point in time either now or in the future, given the current position and linear trajectory of the two vehicles.

$$\texttt{cross}^{os}_{xcnb} = \begin{cases} true & (\alpha = 0) \\ true & (v^{os}_{\gamma} > 0) \text{ and } (\texttt{port}^{os}_{cn}) \text{ and } (\dot{\beta} > 0) \\ true & (v^{os}_{\gamma} > 0) \text{ and } (\texttt{star}^{os}_{cn}) \text{ and } (\dot{\beta} < 0) \\ false & otherwise \end{cases} \tag{23}$$

The term $\texttt{cross}^{os}_{xcns}$ is true if and only if ownship crosses the contact's bow-stern line, and is aft of the contact at the moment of the crossing, at some point in time either now or in the future, given the current position and linear trajectory of the two vehicles.

$$\texttt{cross}^{os}_{xcns} = \begin{cases} true & (\alpha = 180) \\ true & (v^{os}_{\gamma} > 0) \text{ and } (\texttt{port}^{os}_{cn}) \text{ and } (\dot{\beta} < 0) \\ true & (v^{os}_{\gamma} > 0) \text{ and } (\texttt{star}^{os}_{cn}) \text{ and } (\dot{\beta} > 0) \\ false & otherwise \end{cases} \tag{24}$$

3.6.2 Boolean Crossing Relationships - Contact Crossing Ownship

The complementary terms involving the contact crossing ownship's bow-stern-line are given by:

- $\texttt{cross}^{cn}_{xos}$ - true if and only if the contact crosses the ownship's bow-stern line, regardless of whether aft or fore, at some point in time either now or in the future, given the current position and linear trajectory of the two vehicles.
- $\texttt{cross}^{cn}_{xosb}$ - true if and only if the contact crosses ownship's bow-stern line, and is fore of ownship at the moment of the crossing, at some point in time either now or in the future, given the current position and linear trajectory of the two vehicles.

- $\texttt{cross}^{cn}_{xoss}$ - true if and only if the contact crosses ownship's bow-stern line, and is aft of ownship at the moment of the crossing, at some point in time either now or in the future, given the current position and linear trajectory of the two vehicles.

The full formulation is not given here for sake of brevity, but may be obtained simply by swapping the ownship and contact terms in Equations 22 - 24.

3.7 Boolean Passing Relationships

Passing relationships, like crossing relationships, refer to a property of a ownship heading and speed, given the position of ownship and the position and trajectory of the contact. With crossing relationships the key line is the contact bow-stern line. In passing relationships, the key line is the contact *beam*, the line perpendicular to the bow-stern line also going through the center of the contact. Three Boolean relationships are provided below indicating (a) whether ownship passes the contact's beam, regardless of whether it passes the port or starboard beam, (b) whether ownship crosses contact's port beam, and (c) whether ownship passes the contact's starboard beam. First a few intermediate terms are provided.

The *ownship epsilon heading*, θ^{os}_{ϵ}, is the ownship heading perpendicular and toward the contact's beam:

$$\theta^{os}_{\epsilon} = \begin{cases} [\theta_{cn} + 180]^{360} & \texttt{fore}^{os}_{cn} \\ \theta_{cn} & otherwise \end{cases} \tag{25}$$

The *ownship contact velocity*, v^{os}_{cnh}, is the speed of ownship in the direction of contact's heading:

$$v^{os}_{cnh} = cos(\theta_{os} - \theta^{os}_{\epsilon}) \cdot v_{os} \tag{26}$$

The *ownship epsilon velocity*, v^{os}_{ϵ}, is the speed at which ownship is closing on the contact's beam. This is positive prior to passing the contact's beam, and negative after passing.

$$v^{os}_{\epsilon} = \begin{cases} v_{cn} - v^{os}_{cnh} & \texttt{fore}^{os}_{cn} \\ v^{os}_{cnh} - v_{cn} & otherwise \end{cases} \tag{27}$$

The *ownship epsilon range*, r_{ϵ}, is the current range between ownship and the contact's beam. This value is obtained from the range between the two vehicles, r, and the ownship epsilon heading from Equation 25:

$$r_{\epsilon} = r \cdot cos(\theta^{os}_{\epsilon} - relbng^{os}_{cn}) \tag{28}$$

These terms are put into context with a particular example in the Figure 8.

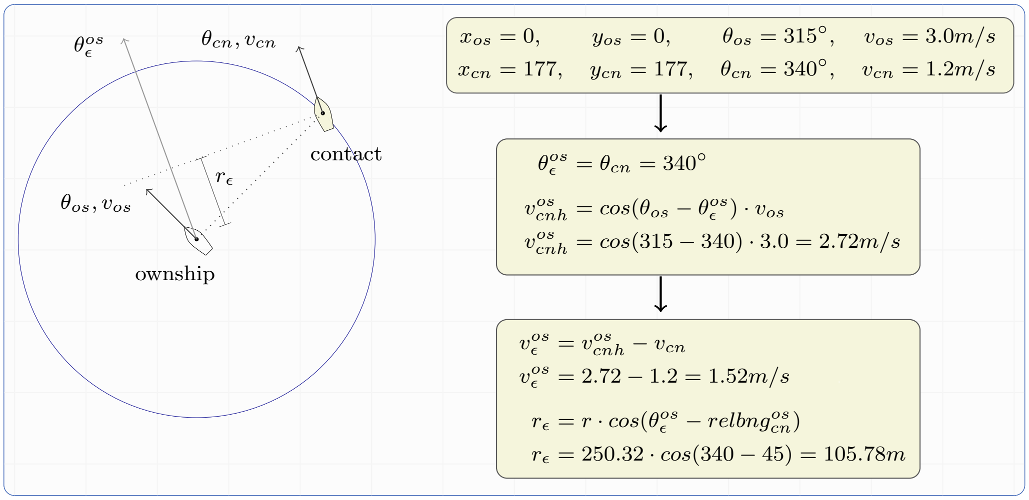


Figure 8: **Ownship speed in the direction of passing the contact**: This speed is used for determining whether and when ownship will cross the contact's beam.

3.7.1 Boolean Passing Relationships - Ownship Passing Contact

Armed with the ability to express ownship speed in the direction of contact heading, and knowledge of bearing rate, the key Boolean passing terms can be defined.

The term $\texttt{pass}_{cn}^{os}$ is true if and only if ownship crosses the contact's beam, regardless of port or starboard, at some point in time either now or in the future, given the current linear trajectory of the two vehicles.

$$\texttt{pass}_{cn}^{os} = \begin{cases} true & (v_\epsilon^{os} > 0) \\ false & otherwise \end{cases} \tag{29}$$

The term $\texttt{pass}_{cnp}^{os}$ is true if and only if ownship passes the contact's port beam at some point in time either now or in the future, given the current linear trajectory of the two vehicles.

$$\texttt{pass}_{cnp}^{os} = \begin{cases} true & \texttt{aft}_{cn}^{os} \text{ and } \texttt{pass}_{cn}^{os} \text{ and } (\dot{\beta} > 0) \\ true & \texttt{fore}_{cn}^{os} \text{ and } \texttt{pass}_{cn}^{os} \text{ and } (\dot{\beta} < 0) \\ false & otherwise \end{cases} \tag{30}$$

The term $\texttt{pass}_{cns}^{os}$ is true if and only if ownship passes the contact's starboard beam at some point in time either now or in the future, given the current linear trajectory of the two vehicles.

$$\texttt{pass}^{os}_{cns} = \begin{cases} true & \texttt{aft}^{os}_{cn} \text{ and } \texttt{pass}^{os}_{cn} \text{ and } (\dot{\beta} < 0) \\ true & \texttt{fore}^{os}_{cn} \text{ and } \texttt{pass}^{os}_{cn} \text{ and } (\dot{\beta} > 0) \\ false & otherwise \end{cases} \tag{31}$$

3.7.2 Boolean Passing Relationships - Contact Passing Ownship

The complementary terms involving the contact crossing ownship's beam are given by:

- $\texttt{pass}^{cn}_{os}$ - true if contact crosses ownship's beam at some point in time either now or in the future, given the current linear trajectory of the two vehicles.
- $\texttt{pass}^{cn}_{osp}$ - true if contact crosses ownship's port beam at some point in time either now or in the future, given the current linear trajectory of the two vehicles.
- $\texttt{pass}^{cn}_{oss}$ - true if contact crosses ownship's starboard beam at some point in time either now or in the future, given the current linear trajectory of the two vehicles.

The full formulation is not given here for sake of brevity, but may be obtained simply by swapping the ownship and contact terms in Equations 29 - 31.

3.8 Numerical Crossing Relationships

In addition to knowing *whether* ownship will cross a contact it is also useful to know the range between vehicles at the point when ownship crosses the bow-stern line of the contact. The range when ownship crosses contact's stern is given by the term r^{os}_{xcns}, and the range when ownship crosses contact's bow is given by r^{os}_{xcnb}. First a few intermediate terms for later use.

The *ownship crossing time*, t^{os}_{γ}, is the number of seconds into the future at which ownship will cross the contact. It is based on ownship gamma velocity and range, v^{os}_{γ} and r_{gamma}, defined previously in Equations 20 and 21.

$$t^{os}_{\gamma} = \frac{r_{\gamma}}{v^{os}_{\gamma}} \tag{32}$$

The ownship crossing time has a negative value if the crossing event occurred in the past ($v^{os}_{\gamma} < 0$). It has zero value if ownship is on the contact bow-stern line, i.e., $r_{\gamma} = 0$. It is undefined if $r_{\gamma} > 0$ and the gamma velocity is zero, i.e., when ownship is traveling parallel to the contact or ownship is stationary.

The next two terms in Equations 33 and 34 represent the distance that contact travels in the time between now and up to the time ownship crosses contact's bow-stern line, r^{xcn}_{ϵ}, and the distance that ownship travels in this same time, r^{xos}_{ϵ}, in direction of the contact's heading.

$$r^{xcn}_{\epsilon} = t^{os}_{\gamma} \cdot v_{cn} \tag{33}$$

$$r^{xos}_{\epsilon} = t^{os}_{\gamma} \cdot v^{os}_{cnh} \tag{34}$$

These two values are only useful in calculating range between ownship and contact at the moment of crossing. They will be negative if the crossing occurred in the past, and undefined if the crossing never occurs at all, due to either parallel trajectories or stationary ownship. The scenario in Figure 9 provides some context by way of a working example.

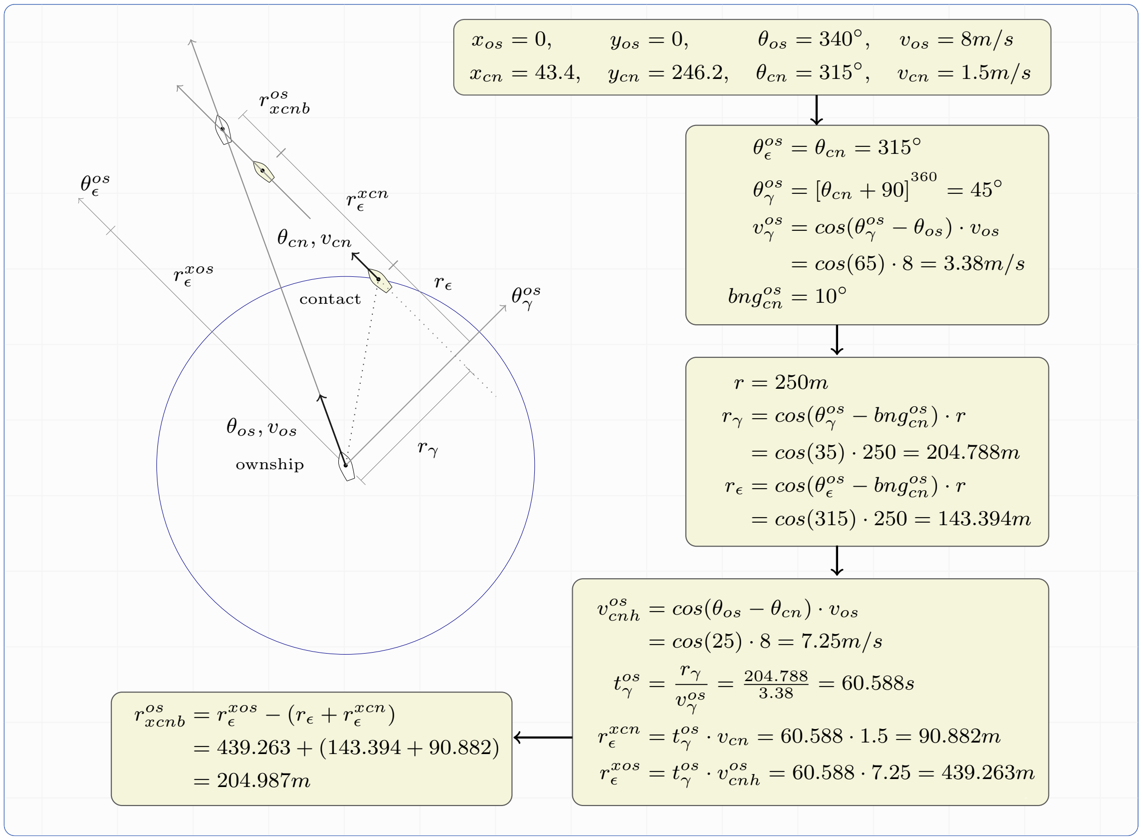


Figure 9: **Example calculation of range at time ownship crosses contact's bow**: The range calculation proceeds directly from current contact position and trajectory and ownship position and trajectory.

Armed with ownship epsilon range, Equation 28, and Equations 33 and 34, the following two useful terms are defined.

The *ownship crossing bow range*, r^{os}_{xcnb}, is the range when ownship crosses contact's bow. It is set to -1 if ownship never crosses the contact, crosses contact's stern instead, or if crossing the bow occurred in the past:

$$r^{os}_{xcnb} = \begin{cases} r^{xos}_{\epsilon} - (r_{\epsilon} - r^{xcn}_{\epsilon}) & \texttt{cross}^{os}_{xcnb} \\ -1 & otherwise \end{cases} \tag{35}$$

The *ownship crossing stern range*, r^{os}_{xcns}, is the range when ownship crosses contact's stern. It is set to −1 if ownship never crosses the contact, crosses contact's bow instead, or if crossing the stern occurred in the past:

$$r^{os}_{xcns} = \begin{cases} (r_\epsilon - r^{xcn}_\epsilon) - r^{xos}_\epsilon & \texttt{cross}^{os}_{xcns} \\ -1 & otherwise \end{cases} \tag{36}$$

The complementary terms involving the contact crossing ownship's bow-stern-line are given by:

- $\mathtt{r}^{cn}_{xosb}$ - The *contact crossing bow range* is the range when contact crosses ownship's bow. It is set to −1 if the contact never crosses ownship, crosses ownship's stern instead, or if crossing the bow occurred in the past.
- $\mathtt{r}^{cn}_{xosb}$ - The *contact crossing stern range* is the range when contact crosses ownship's stern. It is set to −1 if the contact never crosses ownship, crosses ownship's bow instead, or if crossing the stern occurred in the past.

Since the latter pair of terms are not directly used in the logic of algorithms described in this paper, the full formulation is not given here for sake of brevity.

3.9 Numerical Passing Relationships

In addition to knowing *whether* ownship will pass a contact it is also useful to know the range between vehicles at the point when ownship crosses the contact beam. The range when ownship passes contact's port beam is given by the term r^{os}_{pcnp}, and the range when ownship passes contact's starboard beam is given by r^{os}_{pcns}. First a few intermediate terms for later use.

The *ownship passing time*, t^{os}_ϵ, is the number of seconds into the future at which ownship will pass the contact. It is based on ownship epsilon velocity and range, v^{os}_ϵ and $r_{epsilon}$, defined in Equation 28 and 27.

$$t^{os}_\epsilon = \frac{r_\epsilon}{v^{os}_\epsilon} \tag{37}$$

The ownship passing time has a negative value if the passing event occurred in the past ($v^{os}_\epsilon < 0$). It has zero value if ownship is on the contact beam, i.e., $r_\epsilon = 0$. It is undefined if $r_\epsilon > 0$ and the epsilon velocity is zero, i.e., when ownship is traveling perpendicular to the contact or ownship is stationary.

The next term refers to the distance traveled by ownship, in the direction of θ^{os}_γ, perpendicular to the contact's bow-stern line, in the time interval between now and the moment ownship passes the contact beam.

$$r^{pos}_\gamma = \begin{cases} r^{pos}_\gamma = t^{os}_\epsilon \ \cdot \ v^{os}_\epsilon \end{cases} \tag{38}$$

The value r^{pos}_γ is only useful in calculating range between ownship and contact at the moment of passing. It will be negative if the passing occurred in the past, and undefined if the passing never occurs at all, due to either perpendicular trajectories or stationary ownship. The scenario in Figure 8 provides some context by way of a working example.

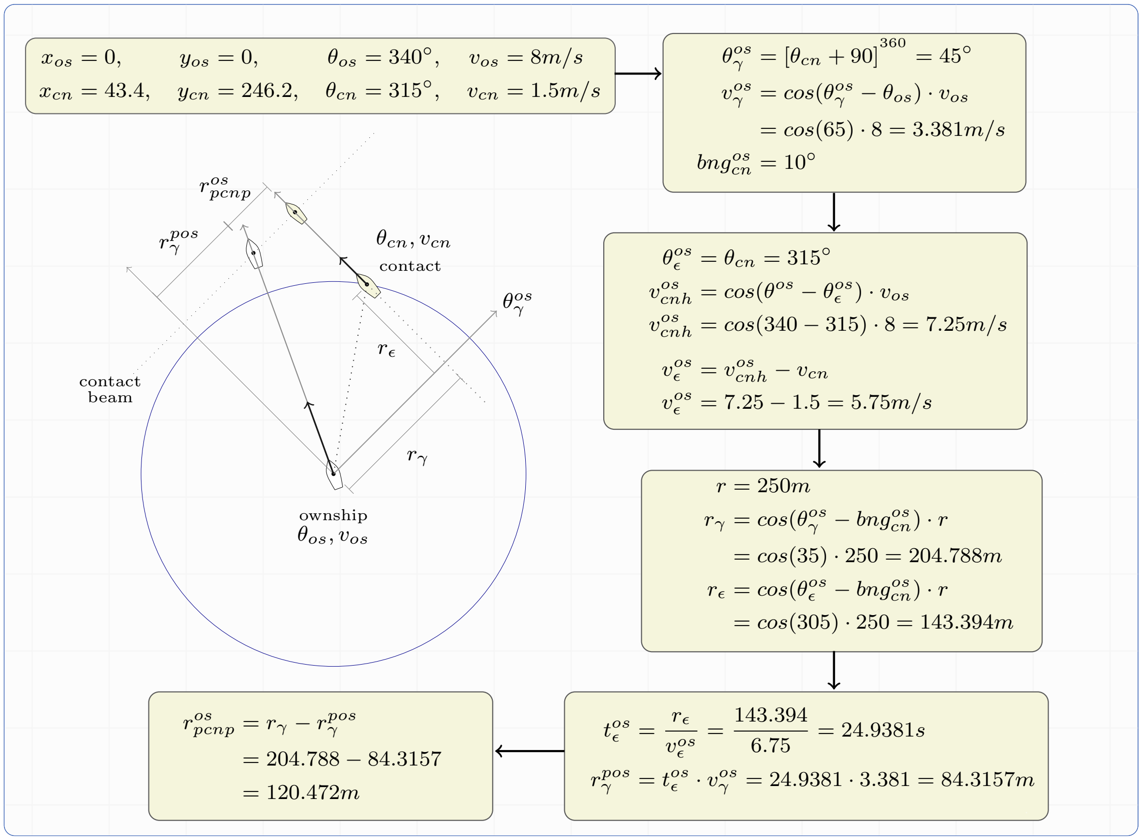


Figure 10: **Example calculation of range at time ownship passes contact's port beam,** r_{pcnp}^{os}: The range calculation proceeds directly from current contact position and trajectory and ownship position and trajectory.

Armed with Equations 15, 16, 21, and 38, the following two target terms, r_{pcnp}^{os} and r_{pcns}^{os} may be defined.

The *ownship passing port range*, r_{pcnp}^{os}, is the range when ownship passes contact's port beam. It is set to -1 if ownship never passes the contact, passes contact's starboard beam instead, or if passing the port beam occurred in the past:

$$r_{pcnp}^{os} = \begin{cases} r_\gamma - r_\gamma^{pos} & \texttt{port}_{cn}^{os} \textit{ and } (r_\gamma^{pos} \leq r_\gamma) \\ r_\gamma^{pos} - r_\gamma & \texttt{star}_{cn}^{os} \textit{ and } (r_\gamma > r_\gamma^{pos}) \\ -1 & \textit{otherwise} \end{cases} \tag{39}$$

The *ownship passing starboard range*, r_{pcns}^{os}, is the range when ownship passes contact's starboard beam. It is set to -1 if ownship never passes the contact, passes contact's port beam instead, or if passing the starboard beam occurred in the past:

$$r_{pcns}^{os} = \begin{cases} r_\gamma - r_\gamma^{pos} & \texttt{star}_{cn}^{os} \; and \; (r_\gamma^{pos} \le r_\gamma) \\ r_\gamma^{pos} - r_\gamma & \texttt{port}_{cn}^{os} \; and \; (r_\gamma > r_\gamma^{pos}) \\ -1 & otherwise \end{cases} \tag{40}$$

The complementary terms involving the contact passing ownship's beam are given by:

- $\mathtt{r}_{posp}^{cn}$ - The *contact passing port range* is the range when contact passes ownship's port beam. It is set to -1 if the contact never passes ownship, passes ownship's starboard beam instead, or if passing the port beam occurred in the past.
- $\mathtt{r}_{poss}^{cn}$ - The *contact crossing starboard range* is the range when contact passes ownship's starboard beam. It is set to -1 if the contact never passes ownship, passes ownship's port beam instead, or if crossing the starboard beam occurred in the past.

Since the latter pair of terms are not directly used in the logic of algorithms described in this paper, the full formulation is not given here for sake of brevity.

3.10 Closest Point of Approach

The closest point of approach, or CPA, is the point where ownship and contact have minimal range during the course of a maneuver. On ships, CPA is often reported with range, time and bearing components, but here we interchangeably use the terms CPA and r_{CPA}, range at the time of CPA. This value is directly calculated once the time of CPA (t_{cpa}) is known, by the following:

$$\begin{aligned} CPA &= r(t_{cpa}) \\ &= \sqrt{k_2 t_{cpa} + k_1 t_{cpa} + k_0} \end{aligned} \tag{41}$$

The time of CPA is considered to be now ($t = 0$) if the range rate (Equation 7) is not negative. This would indicate the two vehicles are opening range and the time of CPA is in the past. Time of CPA is given by:

$$t_{cpa} = \begin{cases} 0 & \dot{r} \ge 0 \\ \frac{-k1}{k2} & otherwise \end{cases} \tag{42}$$

where k_0, k_1, and k_2 are given by:

$$k_2 = \cos^2(\theta_{os})v_{os}^2 - 2\cos(\theta_{os})v_{os}\cos(\theta_{cn})v_{cn} + \cos^2(\theta_{cn})v_{cn}^2 + \sin^2(\theta_{os})v_{os}^2 - 2\sin(\theta_{os})v_{os}\sin(\theta_{cn})v_{cn} + \sin^2(\theta_{cn})v_{cn}^2$$

$$k_1 = 2\cos(\theta_{os})v_{os}y_{os} - 2\cos(\theta_{os})v_{os}y_{cn} - 2y_{os}\cos(\theta_{cn})v_{cn} + 2\cos(\theta_{cn})v_{cn}y_{cn} + 2\sin(\theta_{os})v_{os}x_{os} - 2\sin(\theta_{os})v_{os}x_{cn} - 2x_{os}\sin(\theta_{cn})v_{cn} + 2\sin(\theta_{cn})v_{cn}x_{cn} \quad (43)$$

$$k_o = {y_{os}}^2 - 2y_{os}y_{cn} + y_{cn}^2 + {x_{os}}^2 - 2x_{os}x_{cn} + x_{cn}^2$$

The primary use of CPA will be in the creation of objective functions described in Section 5 over candidate values of ownship heading and speed, (θ, v). It is also used for evaluating the present observed trajectory of ownship and contact as a criteria for declaring the COLREGS mode, as in Algorithm 7.

3.11 Summary of Terms

Listing 3.1: Inter-vehicle Terms Used in COLREGS Logic.

Term	Description
θ	The choice of next commanded ownship heading in degrees. Section 3.1
v	The choice of next commanded ownhip speed in meters per second. Section 3.1
(θ, v)	The maneuver space: all possible choices for next commanded heading and speed.
x_{os}	Position of ownship in the x-y plane on the x-axis. Section 3.1
y_{os}	Position of ownship in the x-y plane on the y-axis. Section 3.1
θ_{os}	Heading of ownship in degrees. Section 3.1
v_{os}	Speed of ownship in meters per second. Section 3.1
x_{cn}	Position of contact in the x-y plane on the x-axis. Section 3.1
y_{cn}	Position of contact in the x-y plane on the y-axis. Section 3.1
θ_{cn}	Heading of contact in degrees. Section 3.1
v_{cn}	Speed of contact in meters per second. Section 3.1
$\texttt{turn}_{port}^{os}$	True iff ownship turns to its port side.
$\texttt{turn}_{star}^{os}$	True iff ownship turns to its starboard side.
r	Range between contact and ownship in meters. Section 3.2
bng_{cn}^{os}	Bearing from ownship to contact in degrees. Section 3.2
β	Relative bearing from ownship to contact in degrees. Section 3.2
bng_{os}^{cn}	Bearing from contact to ownship in degrees. Section 3.2
α	Relative bearing from contact to ownship (contact angle) in degrees. Section 3.2
$\dot{r}$	Rate of change of range between contact and ownship. Section 3.3
$\dot{\beta}$	Rate of change in bearing from ownship to contact, per second. Section 3.3
$\dot{\alpha}$	Rate of change in bearing from contact to ownship, per second. Section 3.3

$\texttt{port}_{cn}^{os}$	True iff ownship is on the port side of contact. Section 3.5.
$\texttt{star}_{cn}^{os}$	True iff ownship is on the starboard side of contact. Section 3.5.
$\texttt{port}_{os}^{cn}$	True iff contact is on the port side of ownship. Section 3.5.
$\texttt{star}_{os}^{cn}$	True iff contact is on the starboard side of ownship. Section 3.5.
$\texttt{fore}_{cn}^{os}$	True iff ownship is fore of contact. Section 3.4.
$\texttt{aft}_{cn}^{os}$	True iff ownship is aft of contact. Section 3.4.
$\texttt{fore}_{os}^{cn}$	True iff contact is fore of ownship. Section 3.4.
$\texttt{aft}_{os}^{cn}$	True iff contact is aft of ownship. Section 3.4.
$\texttt{cross}_{xcnb}^{os}$	True iff ownship crosses contact's bow. Section 3.6
$\texttt{cross}_{xcns}^{os}$	True if ownship crosses contact's stern. Section 3.6
$\texttt{cross}_{xcn}^{os}$	True if ownship crosses contact's bow or stern. Section 3.6
$\texttt{cross}_{xosb}^{cn}$	True iff contact crosses ownship's bow. Section 3.6
$\texttt{cross}_{xoss}^{cn}$	True if contact crosses ownship's stern for Section 3.6
$\texttt{cross}_{xos}^{cn}$	True if contact crosses ownship's bow or stern. Section 3.6
$\texttt{r}_{xcnb}^{os}$	Range when ownship crosses contact's bow. Section 3.8
$\texttt{r}_{xcnb}^{os}$	Range when ownship crosses contact's stern. Section 3.8
$\texttt{r}_{xosb}^{cn}$	Range when contact crosses ownship's bow. Section 3.8
$\texttt{r}_{xosb}^{cn}$	Range when contact crosses ownship's stern. Section 3.8
$\texttt{pass}_{cnp}^{os}$	True iff ownship passes contact's port beam. Section 3.7
$\texttt{pass}_{cns}^{os}$	True if ownship passes contact's starboard beam. Section 3.7
$\texttt{pass}_{cn}^{os}$	True if ownship passes contact's beam. Section 3.7
$\texttt{pass}_{osp}^{cn}$	True iff contact passes ownship's port beam Section 3.7
$\texttt{pass}_{oss}^{cn}$	True if contact passes ownship's starboard beam. Section 3.7
$\texttt{pass}_{os}^{cn}$	True if contact passes ownship's beam. Section 3.7
$\texttt{r}_{pcnp}^{os}$	Range when ownship crosses contact's port beam. Section 3.8
$\texttt{r}_{pcns}^{os}$	Range when ownship crosses contact's starboard beam. Section 3.8
$\texttt{r}_{posp}^{cn}$	Range when contact crosses ownship's port beam. Section 3.8
$\texttt{r}_{poss}^{cn}$	Range when contact crosses ownship's starboard beam. Section 3.8
$r(t)$	Range between ownship and contact t seconds in the future. Section 3.10
t_{cpa}	Time of minimal range (CPA) between ownship and contact. Section 3.10
cpa	Closest Point of Approach between ownship and contact. Section 3.10

4 Collision Avoidance Modes

Mode determination is a critical initial step undertaken by each COLREGS behavior for each contact, on each iteration of the helm. The modes correspond to the protocol rules, and some modes also have submodes. The mode may or may not change between iterations, but an important transition is the *initial mode*. This is the behavior mode that occurs when ownship and contact first come within range close enough to warrant the generation of a collision avoidance objective function to influence the helm with an evasive maneuver.

The mode determination considers the relative bearing of ownship to the contact as the contact is first reported. This idea is conveyed on the left in Figure 11. The reverse is true too - the relative bearing of ownship to the contact is also considered. This is conveyed on the right in figure. Three values of $relbng_{cn}^{os}$ (β) are rendered, where the pair of vehicles at each position each will have different modes depending on the value of $relbng_{os}^{cn}$ (α).

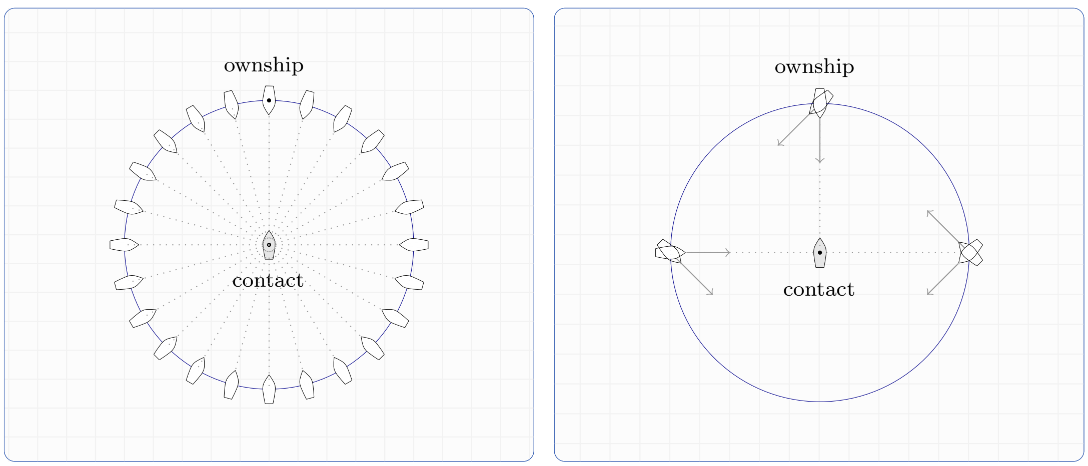


Figure 11: **Initial collision mode consideration**: As ownship and contact first come into range, the relative bearing of ownship to contact is considered (left). The relative bearing of contact to ownship further determines the collision avoidance mode (right).

To illustrate the number of possible initial relationships, consider the continuous range of $[0, 360)$, for possible relative bearings, in each direction. Consider a discretization of angles at a single whole degree, where the set now contains 360 possible integer values. Since the assumption is that the two vehicles are closing range, one vehicle must be at least tangential to the range circle, making $360 * 181 = 65,160$ distinct possible trajectories pairs. Our goal is to group these possibilities into one of six possible collision avoidance modes described next.

4.1 Collision Avoidance Major Modes

The previous section addressed terms updated on each iteration of the helm related to ownship and contact position, pose, and relationships such as relative bearing as well as fore, aft, port and starboard relative positions between platforms. These updates cover lines 2 and 3 in Algorithm 1 of the COLREGS behavior. The next step, line 4 in this algorithm, is to use this information to determine the *collision avoidance mode*. The mode plays a big role in determining the shape of the IvP objective function built in a subsequent step. Below is a list of possible modes, with the corresponding major COLREGS rules indicated.

- GiveWayOT, or GiveWay Overtaking Mode (Rules 13, 16)
- StandOnOT, or StandOn Overtaken Mode (Rule 13,17)
- HeadOn Mode (Rule 14)
- GiveWayX, or GiveWay Crossing Mode (Rule 15,16)
- StandOnX, or StandOn Crossing Mode (Rule 15,17)
- CPA, or Closest Point of Approach Mode
- Null

The null mode indicates a contact is out of range of ownship. In the null mode, the COLREGS behavior will not produce an objective function.

Algorithm 2: Determining the COLREGS Major Mode

```
procedure setAvoidMode( )                      ▷ Executed each iteration of the behavior
    if mode ∈ {Null, CPA, GiveWayOT} then
        mode ← checkModeGiveWayOT()            ▷ See Section 4.2
    end if
    if mode ∈ {Null, CPA, HeadOn} then
        mode ← checkModeHeadOn()               ▷ See Section 4.3
    end if
    if mode ∈ {Null, CPA, StandOnOT} then
        mode ← checkModeStandOnOT()            ▷ See Section 4.4
    end if
    if mode ∈ {Null, CPA, GiveWayX} then
        mode ← checkModeGiveWayX()             ▷ See Section 4.5
    end if
    if mode ∈ {Null, CPA, StandOnX} then
        mode ← checkModeStandOnX()             ▷ See Section 4.6
    end if
    if mode ∈ {Null, CPA} then
        mode ← checkModeCPA()
    end if
end procedure
```

4.2 The GiveWay Overtaking Mode (Rules 13, 16)

The giveway overtaking mode applies when ownship is approaching the contact from behind and at a speed that will result in passing the contact if continued. In short it has the responsibility to keep out of the way of the vehicle it is overtaking. The vehicle being overtaken has a responsibility to holds its course and speed.

4.2.1 The COLREGS Language for Rule 13

An overtaking vessel's responsibility and criteria for being regarded as overtaking, is prescribed in the protocol as follows:

RULE 13: Overtaking

- (a) any vessel overtaking any other shall keep out of the way of the vessel being overtaken.
- (b) A vessel shall be deemed to be overtaking when coming up with another vessel from a direction more than 22.5 degrees abaft her beam; that is, in such a position with reference to the vessel she is overtaking, that at night she would be able to see only the sternlight of that vessel but neither of her sidelights.
- (c) When a vessel is in any doubt as to whether she is overtaking another, she shall assume that this is the case and act accordingly.

4.2.2 The Algorithm for Setting the GiveWay Overtaking Mode

The algorithm for determining the giveway overtaking mode is given in Algorithm 3. To refresh, this function call is invoked from the `setAvoidMode()` provided earlier in Algorithm 2. The parent function ensures that the check for giveway overtaking is applied only if the behavior is already in the giveway overtaking mode or is presently in the Null or CPA mode.

The algorithm has four parts. The first part is an absolute release check based on the range between ownship and contact. Regardless of prior modes, or trajectories or current relative bearings, if the range is greater than the minimum priority weight range, $\check{r}_{pwt}$, then the collision avoidance mode is Null.

Part two of the algorithm handles the case where the behavior is currently in the Overtaking mode. There are two criteria for release of this mode into CPA mode. The first is based on the contact angle α. If ownship has sufficiently passed the contact in terms of relative bearing, the giveway overtaking mode is released. Likewise, if the current ownship and contact trajectories indicate that ownship is opening range to the contact, then the giveway overtaking mode is also released. In this part the giveway overtaking *submode* is also reconsidered. Without this reconsideration, the submode would never change once it is initially set. If initially set to port, the behavior would heavily penalize any candidate maneuver resulting in passing the contact to the starboard and vice versa. This commitment is useful in preventing thrashing between attempts to pass the contact to one side then another. However, an absolute commitment can be detrimental if, through other circumstances ownship finds itself on a trajectory passing the contact to a side different than initially committed to. To guard against thrashing and to allow a reasonable change of passing side, the conditions the switch-submode conditions of part two included. If the contact, initially ownship's port side, evolves to be more than five degrees on ownship's starboard side, then the submode is switched, and vice versa in the opposite direction. This creates a 10 degree buffer against thrashing if there is modest jitter in the measured relative bearing of the contact to ownship, β.

The third part of the algorithm handles the case where the behavior is not presently in the giveway overtaking mode and checks the criteria for entering. The first check is based on the contact angle, α. From the COLREGS, if ownship is "more than 22.5 degrees abaft the beam" of the contact, it is considered to be overtaking. This check is made by the equivalent criteria of whether α is in the range of $[112.5, 247.5]$. If α is outside this range, the CPA mode is returned. The CPA mode is returned, and not the Null mode, because the two vehicles do have a range less than $\check{r}_{pwt}$, requiring some form of collision avoidance to be applied. In the parent algorithm 2, a return value of CPA will allow for subsequent checks for the other major modes, but will default to the basic CPA mode if none of the other modes apply.

Algorithm 3: Checking for the COLREGS GiveWay Overtaking Mode

```
procedure checkModeGiveWayOT( )                          ▷ Called from within setAvoidMode()
    if (r > ř_pwt) then                                   ▷ Part 1: Absolute release check
        return Null
    end if
    if (mode = GiveWayOT) then                            ▷ Part 2: Check release from GiveWayOT
        if (α > 337.5) or (α < 22.5) or (ṙ > 0) then
            return CPA
        end if
        if (submode = Port) and (180 < β < 355) then      ▷ Switch submode
            return GiveWayOT, Starboard
        else if (submode = Starboard) and (β < 5) then    ▷ Switch submode
            return GiveWayOT, Port
        else
            return GiveWayOT                              ▷ Retain submode
        end if
    end if
    if (α > 247.5) or (α < 112.5) or ¬pass^os_cn then     ▷ Part 3: Check GiveWayOT mode entry
        return CPA
    end if
    if pass^os_cns then                                   ▷ Part 4: Determine entry submode
        return GiveWayOT,Starboard                        ▷ GiveWayOT entry case A
    else
        return GiveWayOT,Port                             ▷ GiveWayOT entry case B
    end if
end procedure
```

The final part of the algorithm is invoked upon initially entering the GiveWay Overtaking mode. At this point a determination is made as to whether ownship is presently on a trajectory to overtake the contact on its port side or starboard side. If the contact is on ownship's starboard side then the trajectory is to pass the contact's port side and vice versa. Examples of these cases are shown in Figure 12. The submode logic assumes that the contact is fore of ownship, which is a safe assumption given checks both the contact angle, α, and that the range is not opening.

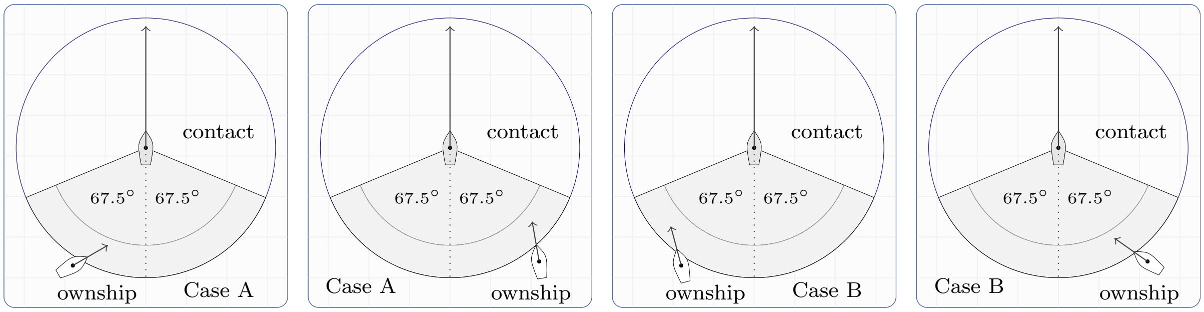


Figure 12: **Entering the GiveWay Overtaking mode**: The giveway overtaking submode depends on whether contact is to the port or starboard side of ownship at the moment ownship comes in range of the contact in an giveway overtaking mode.

A canonical giveway overtaking example is shown in Figure 13. Typically at time t_{00} the behavior would not yet have been spawned, with spawning happening at the time the contact comes within range $\check{r}_{pwt}$, around time $t10$. At this point it enters the GiveWayOT mode with the Port submode. It is release from the GiveWayOT mode some time after t_{30} when either the contact angle release criteria is met, or the range rate criteria is met for release from GiveWayOT. In this example, the contact stays on a linear track, which may be consistent with its standon obligation, but the linear track is not assumed.

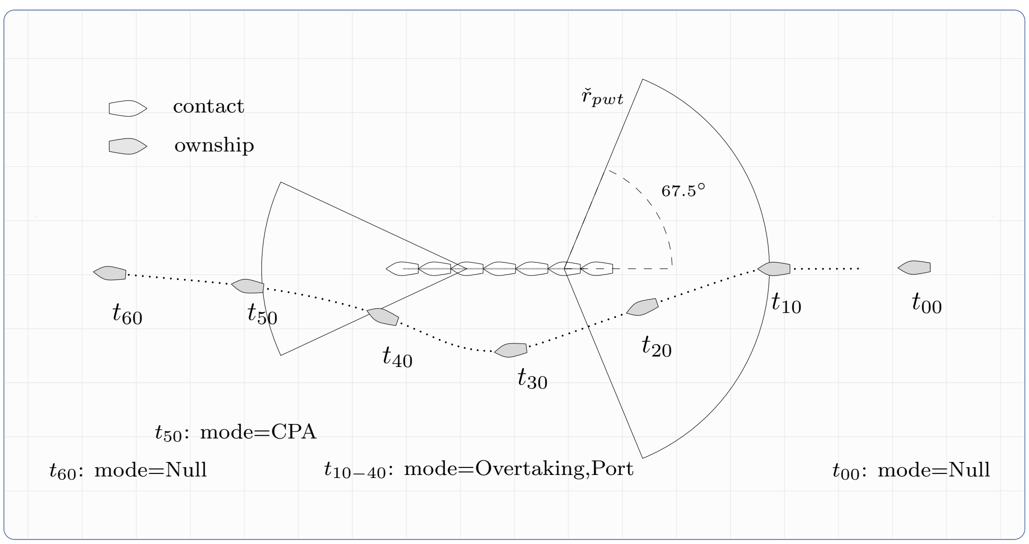


Figure 13: **Canonical giveway overtaking example from simulation**: Ownship approaches the contact and at time t_{10} comes within range $\hat{r}_{pwt}$ and with a contact angle α meeting the giveway overtaking criteria. At time just beyond t_{40}, it sufficiently passes the contact and transitions from an giveway overtaking mode to CPA mode, and is finally completely clear at time t_{60} after its range to contact once again exceeds $\hat{r}_{pwt}$.

4.3 The HeadOn Mode (Rule 14)

A head on situation applies when two vehicles are on a trajectory that risks a head on collision. Both vehicles should turn to the right (starboard) and pass with the other vehicle on its port side.

Once a head on situation has been entered, the situation is regarded as head on until there is no longer a risk of collision, even if their trajectories evolve to be no longer head on as they pass each other.

4.3.1 The COLREGS Language for Rule 14

A head on vessel's responsibility and criteria for being regarded as head on, is prescribed in the protocol as follows:

RULE 14: Head-on Situation

- (a) When two power-driven vessels are meeting on reciprocal or nearly reciprocal courses so as to involve risk of collision each shall alter her course to starboard so that each shall pass on the port side of the other.
- (b) Such a situation shall be deemed to exist when a vessel sees the other ahead or nearly ahead and by night she could see the masthead lights of the other in a line or nearly in a line and/or both sidelights and by day she observes the corresponding aspect of the other vessel.
- (c) When a vessel is in any doubt as to whether such a situation exists she shall assume that it does exist and act accordingly.

4.3.2 The Algorithm for Setting the Head-On Mode

Once the two vehicles are within a certain range, the primary criteria for entering the head-on mode is relative bearing of each vehicle to the other. The exact number is left unspecified in [3], but in our implementation we use $\phi = 12$ degrees from 0 degrees relative bearing as shown on the left in Figure 14.

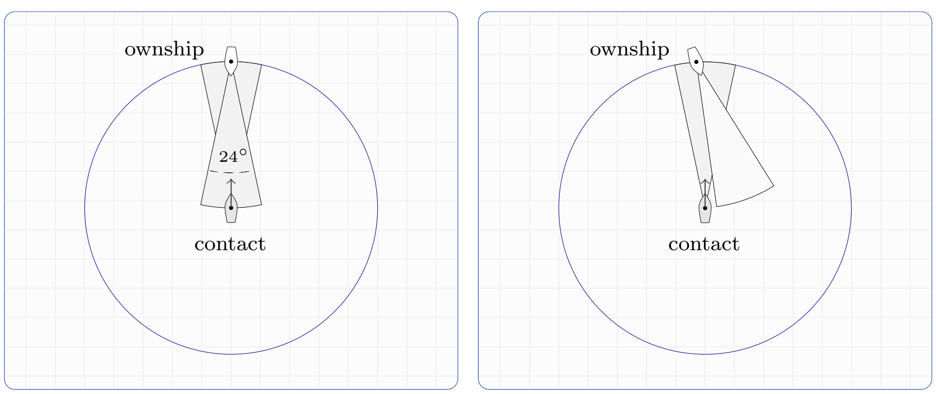


Figure 14: **Entering the Head-On mode**: (Left) Ownship crosses within the range threshold to the contact. The contact is within $\phi = 12°$ degrees of relative bearing threshold to ownship $(\left|[\beta]^{180}\right| \leq \phi)$ and vice versa for the contact angle, $(\left|[\alpha]^{180}\right| \leq \phi)$. (Right) The range criteria is met, and the contact angle threshold has been met, but the relative bearing threshold has not been met, it is not a Head-On situation.

The algorithm for determining the head-on mode is given in Algorithm 3. Like the check for giveway overtaking mode, this function call is also invoked from the `setAvoidMode()` provided earlier in

Algorithm 2. The parent function ensures that the check for head-on is applied only if the behavior is already in the head-on mode or is presently in the Null or CPA mode.

The algorithm has three parts. The first part is an absolute release check based on the range between ownship and contact. Regardless of prior modes, or trajectories or current relative bearings, if the range is greater than twice the minimum priority weight range, $\check{r}_{pwt}$, then the collision avoidance mode is Null.

Part two of the algorithm handles the case where the behavior is currently in the head-on mode. There are three criteria for release from this mode into CPA mode. The first is based on the range rate, $\dot{r}$. If range rate becomes positive, indicating the vehicles are opening, or moving away from each other, then the head-on mode is regarded as completed. Likewise if ownship is aft of the contact, $\texttt{aft}^{os}_{cn}$ is true, then the head-on maneuver is regarded as complete. Typically if one of these conditions hold, the other does as well, but both are applied for redundancy. The third release criteria deals with the special scenario where, despite the behavior having previously entered the head-on mode, the vehicle has continued on a trajectory of passing the contact on its starboard side. This could be due to other collision avoidance or mission objectives. This release check, beginning on line 10 in the algorithm is discussed further in Section 4.3.3.

Algorithm 4: Checking for the COLREGS Head-On Mode

1: **procedure** CHECKMODEHEADON( ) ▷ Called from within setAvoidMode()
2:     **if** $(r > (2 \cdot \check{r}_{pwt}))$ **then** ▷ Part 1: Absolute Release check
3:         **return** Null
4:     **end if**
5:     **if** (mode = HeadOn) **then** ▷ Part 2: Check release from HeadOn
6:         **if** $(r > \check{r}_{cpa})$ and $(\dot{r} > 0)$ **then** ▷ HeadOn completed
7:             **return** CPA
8:         **else if** $(\texttt{aft}^{os}_{cn})$ **then** ▷ Ownship passed contact
9:             **return** CPA
10:         **else if** $(\texttt{star}^{os}_{cn})$ and $(\texttt{star}^{cn}_{os})$ **then** ▷ Port-Port may be no longer advisable
11:             $\beta_\delta \leftarrow (\beta - \phi)$
12:             $\alpha_\delta \leftarrow (\alpha - \phi)$
13:             **if** $((\beta_\delta > \frac{\phi}{2})$ or $(\alpha_\delta > \frac{\phi}{2})$ or $((\beta_\delta + \alpha_\delta) > \frac{2\phi}{3}))$ **then**
14:                 **return** CPA
15:             **end if**
16:         **else**
17:             **return** HeadOn ▷ Mode unchanged
18:         **end if**
19:     **end if**
20:     **if** (mode == CPA) or (mode == Null) **then** ▷ Part 3: Check HeadOn entry criteria
21:         **if** $(\left|[\alpha]^{180}\right| \leq \phi)$ and $(\left|[\beta]^{180}\right| \leq \phi)$ **then**
22:             **return** HeadOn
23:         **else**
24:             **return** CPA
25:         **end if**
26:     **end if**

27: **end procedure**

The third part of the algorithm is invoked upon initially entering the Head-On mode. At this point a determination is made as to whether ownship and the contact are presently on reciprocal trajectories presenting a risk of head-on collision. The contact and angle, α, and relative bearing, β, each must be within $\phi = 12$ degrees of zero degrees relative bearing. The value of ϕ is a behavior configuration parameter.

A canonical head-on example is shown in Figure 15. As the vehicles approach each other, they enter the head-on mode roughly at time t_{20} when their range becomes less than $2\check{r}_{pwt}$. Each vehicle begins a collision avoidance maneuver passing to the port side of each other. Shortly after time t_{30} they begin opening range and revert back to a CPA mode allowing it to return to their initial trajectories. By the time t_{40} collision avoidance is no longer influencing the vehicle trajectory and the COLREGS mode is Null.

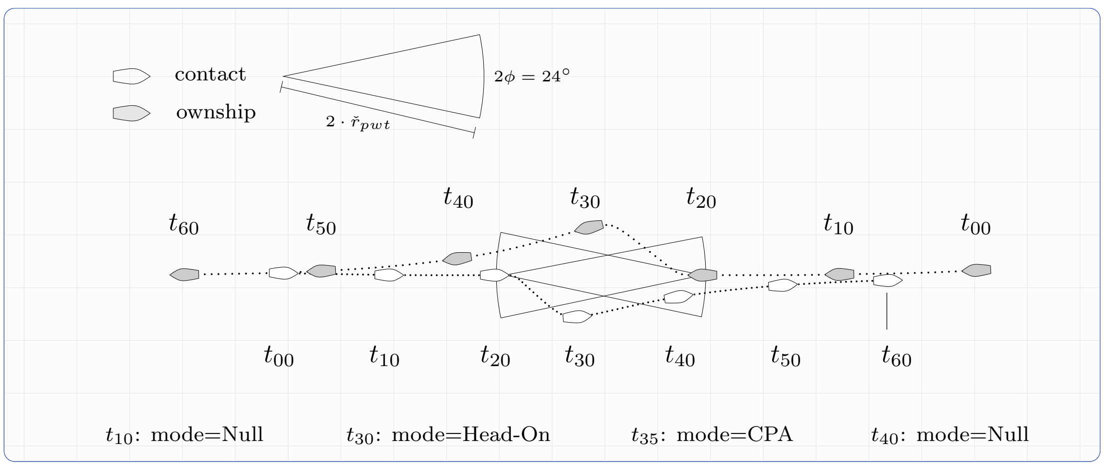


Figure 15: **Canonical HeadOn example from simulation**: Ownship approaches the contact and at time t_{20} comes within range ($r < (2\check{r}_{pwt})$) and with a contact angle α and relative bearing β meeting the head-on criteria. At time about t_{30}, it begins to open range to the contact and transitions from a head-on mode to CPA mode, and is finally completely clear just before time t_{40} after its range to contact once again exceeds $2\check{r}_{pwt}$.

4.3.3 Conditions for Abandoning a Head-On Maneuver

Here the logic in lines 10-15 of Algorithm 4 is further explained. Typically when a COLREGS behavior enters a head-on mode for a particular contact, a starboard maneuver will result shortly afterwards. Often the two vehicles are either directly head-on as in Figure 15, or they may be already to each other's port side a bit. Occasionally the vehicles may be on each other's starboard side, yet still meet the head-on criteria, as in Figure 16. In these situations, the behavior will exert a starboard maneuver instantly, but with a priority weight that grows as the contact becomes closer. If no other collision or mission objectives are involved, the vehicle will indeed execute its starboard maneuver, similar to Figure 15, but will simply take longer to execute given they started on each other's starboard side.

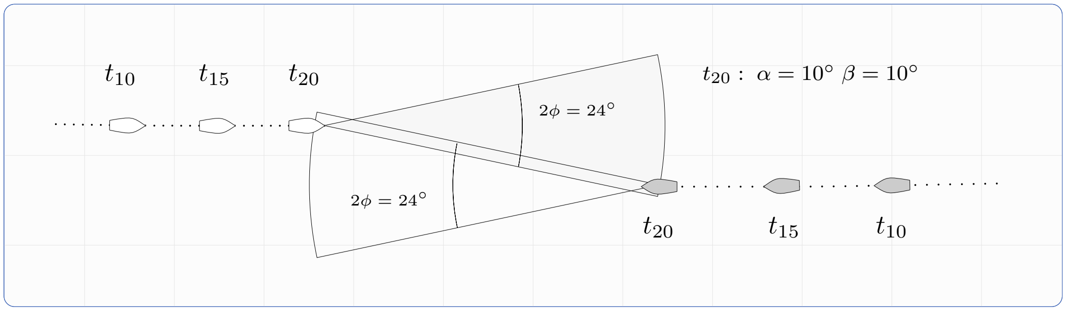


Figure 16: **Initial HeadOn situation**: Ownship and contact barely meet their relative bearing criteria for the Head-On mode.

During this initial activation of the HeadOn mode, there sometimes may be other collision avoidance or mission considerations at play that prevent the immediate commencement of the typical turn-to-starboard collision avoidance maneuver. Consider for example the scenario shown in Figure 17, continuing the scenario in the previous figure. Here there is a third contact North of both vehicles, heading directly South between the two vehicles. Both vehicles have COLREGS obligations preventing them from executing a starboard maneuver. After a bit of time, shown in the figure, they no longer satisfy the relative bearing criteria that would constitute a risk of head-on collision. The head-on mode is thereby released.

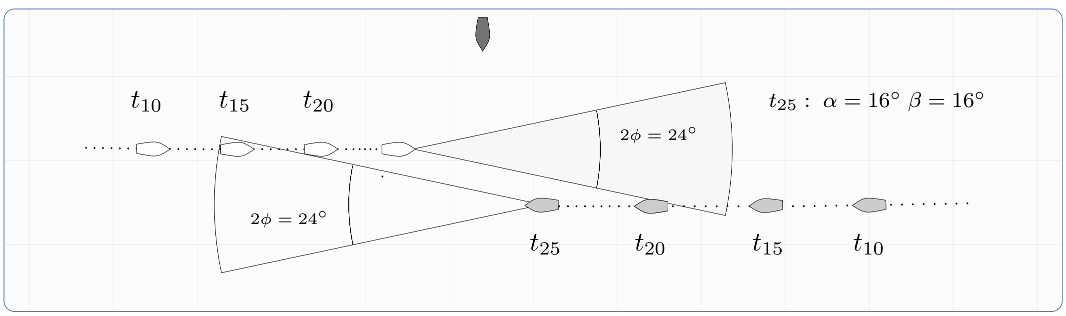


Figure 17: **HeadOn situation evolves to non HeadOn situation**: Before either vehicle can initiate a starboard turn, due to obligations related to a third contact coming from the North, the relative bearings evolve to be well outside ($\beta > (3\phi/2)$) of the original threshold for the Head-On mode.

Without this additional logic the COLREGS behavior, once in a head-on mode, would remain committed to executing a starboard maneuver until the normal release criteria, upon passing the contact. This could result in a last-ditch effort to execute a now-dangerous starboard maneuver crossing the contact's bow. The logic in lines 10-15 prevent this from occurring.

4.4 The StandOn Overtaken Mode (Rule 13, Rule 17)

The StandOnOT mode applies to a StandOn vessel in an Overtaking Situation. A canonical example of this situation was shown previously in Figure 13. By virtue of Rule 13, the giveway overtaking vessel is obligated to keep out the way, and by virtue of Rule 17, the overtaken vessel is obligated to hold her course and speed. This mode is very similar to the StandOnX mode in a crossing situation, but has slight differences and is easier to treat as a separate mode.

4.4.1 The COLREGS Language for Rule 13 and 17

The Overtaking Situation is described in Rule 13:

RULE 13: Overtaking Situation

- (a) any vessel overtaking any other shall keep out of the way of the vessel being overtaken.

RULE 17: Action by Stand-On Vessel

- (a) (i) Where one of two vessels is to keep out of the way the other shall keep her course and speed. (ii) The latter vessel may however take action to avoid collision by her maneuver alone, as soon as it becomes apparent to her that the vessel required to keep out of the way is not taking appropriate action in compliance with these Rules.
- (b) When, from any cause, the vessel required to keep her course and speed finds herself so close that collision cannot be avoided by the action of the give-way vessel alone, she shall take such action as will best aid to avoid collision.
- (c) A power-driven vessel which takes action in a crossing situation in accordance with subparagraph (a)(ii) of this Rule to avoid collision with another power-driven vessel shall, if the circumstances of the case admit, not alter course to port for a vessel on her own port side.

4.4.2 The Algorithm for Setting the StandOnOT Mode

The algorithm for determining the StandOnOT mode is given in Algorithm 5. Like the check for the other modes described thus far, this function call is invoked from the `setAvoidMode()` provided earlier in Algorithm 2. The parent function ensures that the check for StandOnOT is applied only if the behavior is already in the StandOnOT mode or is presently in the Null or CPA mode.

The algorithm has four parts. The first part is an absolute release check based on the range between ownship and contact. Regardless of prior modes, or trajectories or current relative bearings, if the range is greater than the minimum priority weight range, $\check{r}_{pwt}$, then the collision avoidance mode is Null.

Part two of the algorithm handles the case where the behavior is currently in the StandOnOT mode. If the range rate, $\dot{r}$, becomes positive, indicating the vehicles are opening, or moving away from each other, then the StandOnOT mode is regarded as complete. Otherwise it will stay in this mode regardless of the relative bearing or contact angle.

The third part of the algorithm is invoked upon initially entering the StandOnOT mode. The overtaking vessel must be behind ownship within 67.5 degrees of relative bearing in either direction. This value is according to the COLREGS rules and is not a user configurable value. If the contact has at a relative bearing of ($\beta < 112.5$) or ($\beta > 247.5$), the StandOnOT mode will not apply. Furthermore the contact must be on a trajectory that would result in it passing ownship, $\texttt{pass}_{os}^{cn}$. This is not the same as having a negative range rate ($\dot{r} < 0$), although these two conditions are often true simultaneously.

Algorithm 5: Checking for the COLREGS StandOn Overtaken Mode

```
procedure checkModeStandOnOT( )                              ▷ Called from within setAvoidMode()
    if (r > ř_pwt) then                                      ▷ Part 1: Absolute Release check
        return Null
    end if
    if (mode = StandOnOT) then                               ▷ Part 2: Check release from StandOnOT
        if (ṙ > 0) then                                      ▷ StandOnOT completed
            return CPA
        else
            return StandOnOT
        end if
    end if
    if (β < 112.5) or (β > 247.5) or ¬pass^cn_os then        ▷ Part 3: StandOnOT entry criteria
        return CPA
    end if
    if (submode = InExtremis) then                           ▷ Part 4: Determine submode
        return StandOnOT, InExtremis
    else if (r < r̂_cpa) and (r_cpa < ř_cpa) then
        return StandOnOT, InExtremis
    else if (port^cn_os) then
        return StandOnOT, Port
    else
        return StandOnOT, Starboard
    end if
end procedure
```

The fourth part of the algorithm is invoked upon initially entering the StandOnOT mode to determine the submode: *InExtremis, Port, or Starboard.* The InExtremis mode deals with the COLREGS clause that allows a stand on vessel to "take action to avoid collision by her maneuver alone, as soon as it becomes apparent to her that the vessel required to keep out of the way is not taking appropriate action". The first condition in part four of the algorithm asserts that once a vehicle is in the InExtremis submode, it will stay in that submode for the duration of the StandOnOT major mode. The criteria for entering the InExtremis submode requires that the vessels be currently within the range of $\hat{r}_{cpa}$ and that the two vehicles, given their present position, heading and speed would result in a CPA range, r_{cpa} of less than $\check{r}_{cpa}$. If the InExtremis submode does not apply, the submode is then determined by whether the contact is on the port or starboard side of ownship. The Port and Starboard submodes are set accordingly. These submodes will determine the shape of the objective function produced by the behavior while in the StandOnOT mode.

4.5 The GiveWay Crossing Mode (Rule 15, Rule 16)

A crossing situation is one where two vessels are on a trajectory to cross paths, where there is risk of collision, and where the HeadOn and Overtaking modes don't apply. One of these vessels, the

GiveWay vessel, has the responsibility of maneuvering to avoid a collision. A COLREGS behavior for a given contact will be in the GiveWay Crossing mode during such situations.

4.5.1 The COLREGS Language for Rule 15, 16

The GiveWay Crossing Mode applies to a GiveWay vessel in a Crossing Situation. The Crossing Situation is described in Rules 15 and 16:

RULE 15: Crossing Situation

- (a) When two power-driven vessels are crossing so as to involve risk of collision, the vessel which has the other on her starboard side shall keep out of the way and shall, if the circumstances of the case admit, avoid crossing ahead of the other vessel.

RULE 16: Action by Give-way Vessel

- Every vessel which is directed to keep out of the way of another vessel shall, so far as possible, take early and substantial action to keep well clear.

4.5.2 The Algorithm for Setting the GiveWay Crossing Mode

The algorithm for determining the GiveWay Crossing mode is given in Algorithm 6. Like the check for previously discussed modes, this function call is invoked from the `setAvoidMode()` provided earlier in Algorithm 2. The parent function ensures that the check for GiveWay Crossing is applied only if the behavior is already in the GiveyWay Crossing mode or is presently in the Null or CPA mode.

The algorithm has four parts. The first part is an absolute release check based on the range between ownship and contact. Regardless of prior modes, or trajectories or current relative bearings, if the range is greater than the minimum priority weight range, $\check{r}_{pwt}$, then the collision avoidance mode is Null. Part two of the algorithm handles the case where the behavior is currently in the GiveWay Crossing mode. If range rate becomes positive, indicating the vehicles are opening, or moving away from each other, then the GiveWay Crossing mode is regarded as completed and the CPA mode is returned.

Algorithm 6: Checking for the COLREGS GiveWay Crossing Mode

procedure CHECKMODEGIVEWAYX( ) ▷ Called from within setAvoidMode()
    if ($r > \check{r}_{pwt}$) **then** ▷ Part 1: Absolute Release check
        return Null
    end if
    if (mode = GiveWayX) **then** ▷ Part 2: Check release from GiveWayX
        if ($r > \check{r}_{cpa}$) and ($\dot{r} > 0$) **then** ▷ GiveWayX completed
            return CPA
        else if ($\alpha \leq 180$) **then** ▷ ownship on contact starboard side
            return CPA
        else

11: **return** GiveWayX ▷ Mode/Submode unchanged
12: **end if**
13: **end if**
14: **if** ($\alpha < 247.5$) or ($\beta > 112.5$) **then** ▷ Part 3: GiveWayX entry criteria
15: **return** CPA
16: **end if**
17: **if** ($\texttt{cross}^{os}_{xcnb}$) and ($\mathbf{r}^{os}_{xcnb} > (\hat{r}_{cpa} + \check{r}_{cpa})/2$) **then** ▷ Part 4: Determine submode
18: **return** GiveWayX,Bow
19: **else**
20: **return** GiveWayX,Stern
21: **end if**
22: **end procedure**

The third part of the algorithm is invoked to determine initial entry to the GiveWayX mode. Both the relative bearing, β, and the contact angle, α, are considered in this determination. Figure 18 conveys the conditions on line 12. When the contact angle criteria is satisfied, but not the relative bearing criteria, the vehicles may be technically crossing paths, as for example, if the relative bearing on the left in Figure 18 were 359 degrees. But in this case the situation does not constitute a risk of collision. If ownship subsequently maneuvers to port altering its relative bearing to be in the range of $[0, 112.5]$ then there is risk of collision and the collision avoidance behavior for this contact would be in the GiveWayX mode.

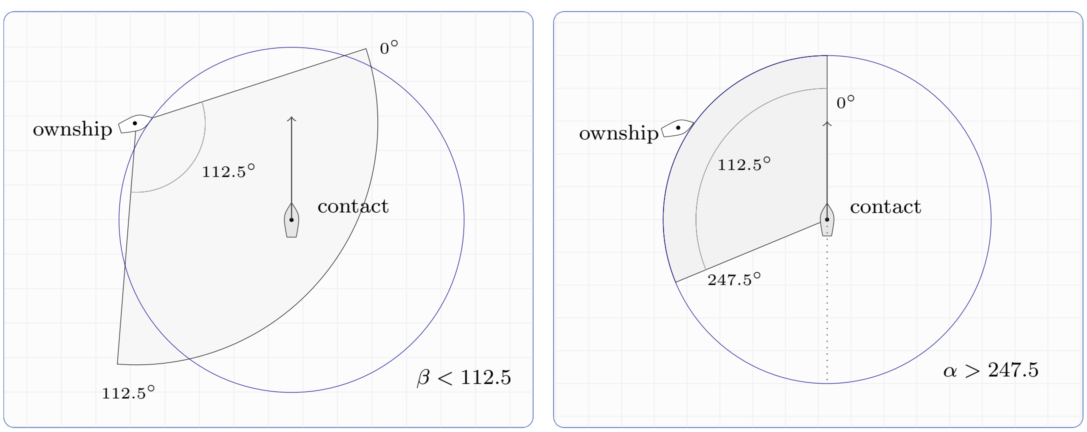


Figure 18: **Entering the GiveWay mode**: (Left) Ownship crosses within the range threshold to the contact and has the contact on her starboard side with a relative bearing ($0 \leq \beta \leq 112.5$). (Right) Ownship crosses within range of the contact and ownship has contact angle ($\alpha \geq 247.5$).

The fourth part of the algorithm is invoked upon initially entering the GiveWay Crossing mode to determine the submode: *Bow or Stern*. The required maneuver for a vehicle in a GiveWay Crossing mode is to cross behind the contact which would be indicated by the Stern submode. This is derived from the language of Rule 15: "the vessel which has the other on her starboard side shall keep out of the way and shall, if the circumstances of the case admit, *avoid crossing ahead* of the other vessel.". In some circumstances it does indeed make more sense to cross ahead of the contact indicated by the Bow submode. Algorithm 6 defines these circumstances in part four of the algorithm on line 15. Crossing ahead of the contact is allowed if (a) the current trajectory would lead to ownship crossing ahead of the contact, given by the Boolean $\texttt{cross}^{os}_{xcnb}$, and (b) the range at the instant ownship crosses ahead the contact, $\texttt{r}^{os}_{xcnb}$, is greater than the mid-range between $\hat{r}_{cpa}$ and $\check{r}_{cpa}$. The latter is the threshold beyond which there is considered to be no risk of collision.

4.6 The StandOn Crossing Mode (Rule 15, Rule 17)

A crossing situation is one where two vessels are on a trajectory to cross paths, where there is risk of collision, and where the HeadOn and Overtaking modes don't apply. One of these vessels, the StandOn vessel, has the responsibility of keeping her course and speed. A COLREGS behavior for a given contact will be in the StandOn Crossing mode during such situations.

4.6.1 The COLREGS Language for Rule 15, 17

The StandOn mode applies to the StandOn vessel in a crossing situation. The crossing situation is described in Rule 15 and the responsibility of the StandOn vessel is described in Rule 17:

RULE 15: Crossing Situation

- (a) When two power-driven vessels are crossing so as to involve risk of collision, the vessel which has the other on her starboard side shall keep out of the way and shall, if the circumstances of the case admit, avoid crossing ahead of the other vessel.

RULE 17: Action by Stand-On Vessel

- (a) (i) Where one of two vessels is to keep out of the way the other shall keep her course and speed. (ii) The latter vessel may however take action to avoid collision by her maneuver alone, as soon as it becomes apparent to her that the vessel required to keep out of the way is not taking appropriate action in compliance with these Rules.
- (b) When, from any cause, the vessel required to keep her course and speed finds herself so close that collision cannot be avoided by the action of the give-way vessel alone, she shall take such action as will best aid to avoid collision.
- (c) A power-driven vessel which takes action in a crossing situation in accordance with subparagraph (a)(ii) of this Rule to avoid collision with another power-driven vessel shall, if the circumstances of the case admit, not alter course to port for a vessel on her own port side.
- (d) This Rule does not relieve the give-way vessel of her obligation to keep out of the way.

4.6.2 The Algorithm for Setting the StandOn Crossing Mode

The algorithm for determining the StandOn Crossing mode is given in Algorithm 7. Like the check for the other modes described thus far, this function call is invoked from the `setAvoidMode()` provided earlier in Algorithm 2. The parent function ensures that the check for StandOnX is applied only if the behavior is already in the StandOnX mode or is presently in the Null or CPA mode.

The algorithm has four parts. The first part is an absolute release check based on the range between ownship and contact. Regardless of prior modes, or trajectories or current relative bearings, if the range is greater than the minimum priority weight range, $\check{r}_{pwt}$, then the collision avoidance mode is Null.

Part two of the algorithm handles the case where the behavior is currently in the StandOnX mode. If the range rate, $\dot{r}$, becomes positive, indicating the vehicles are opening, or moving away from each other, then the StandOnX mode is regarded as complete. Otherwise it will stay in this mode regardless of the relative bearing or contact angle.

Part three of the algorithm determines if the behavior should enter the StandOnX mode, based on (a) the relative bearing β, (b) the contact angle, α, and (c) the current range rate.

Algorithm 7: Checking for the COLREGS StandOn Crossing Mode

```
procedure checkModeStandOnX( )                         ▷ Called from within setAvoidMode()
    if (r > ř_pwt) then                                ▷ Part 1: Absolute Release check
        return Null
    end if
    if (mode = StandOnX) then                          ▷ Part 2: Check release from StandOnX
        if (ṙ > 0) then                                ▷ StandOnX completed
            return CPA
        else
            return StandOnX                            ▷ Remain in same mode and submode
        end if
    end if
    if (α > 112.5) or (β < 247.5) or (ṙ ≥ 0) then      ▷ Part 3: StandOnX entry criteria #1
        return CPA
    end if
    if (submode = InExtremis) then                     ▷ Part 4: Determine submode
        return StandOnX, InExtremis
    else if (r < r̂_cpa) and (r_cpa < ř_cpa) then
        return StandOnX, InExtremis
    else if (cross^cn_xosb) then
        return StandOnX, Bow
    else
        return StandOnX, Stern
    end if
end procedure
```

The two criteria for entering the StandOnX mode, based on the relative bearing, β, and contact angle, α, are shown below in Figure 19.

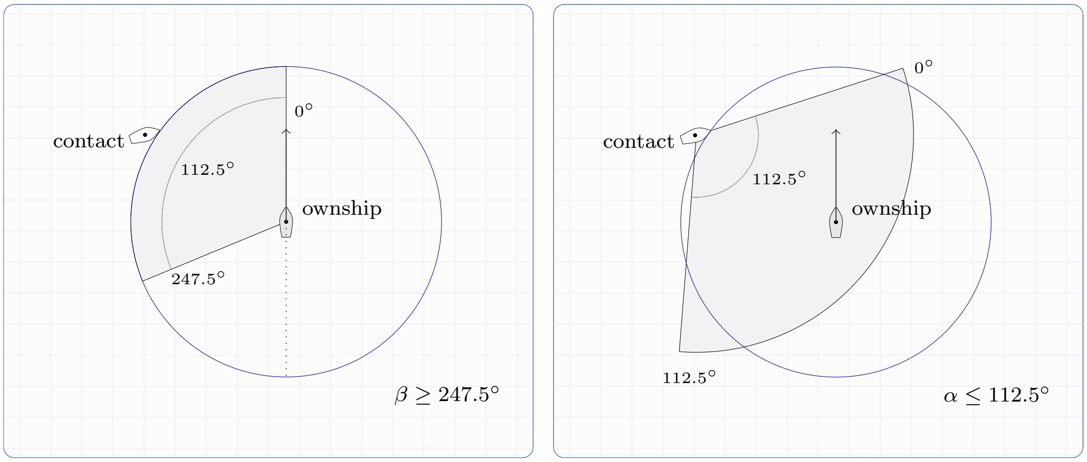


Figure 19: **Entering the StandOnX mode**: (Left) The contact must have a relative bearing to ownship, $\beta \geq 247.5$. (Right) Ownship must have a contact angle, $\alpha \leq 112.5$

5 Collision Avoidance Objective Functions

Once the COLREGS behavior mode has been determined, the behavior produces an objective function based on the mode, submode if any, and the actual relative position and trajectories between ownship and the contact. In this section the objective functions definitions are provided. In each case the objective function can be considered to be a function defined solely over the two decisions variables, i.e., the desired next heading and speed of ownship:

$$f(\theta, v)$$

for a given ownship position (x_{os}, y_{os}), contact position and trajectory $(x_{cn}, y_{cn}, \theta_{cn}, v_{cn})$.

5.1 The GiveWay Overtaking Objective Function

A *giveway overtaking* objective function is generated when the vehicle is in the giveway overtaking mode with respect to a contact. The criteria for being in this mode were discussed in Section 4.2. The giveway overtaking objective function is given by:

$$f(\theta, v) = g(cpa(\theta, v)) \;\cdot\; h(\theta, v) \tag{44}$$

This objective function has three components, (a) calculation of CPA ($cpa(\theta, v)$), (b) application of utility applied to CPA ($g(cpa(\theta, v))$), and (c) a bias to avoid changing passing sides once a side is chosen ($h(\theta, v)$). The raw CPA calculation is the first step. This calculation was discussed in Section 3.10. An example overtaking situation is shown in Figure 20 on the left. In this case the COLREGS mode is (overtaking,port). The mapping of CPA values to the maneuver space, $cpa(\theta, v)$, is shown on the right.

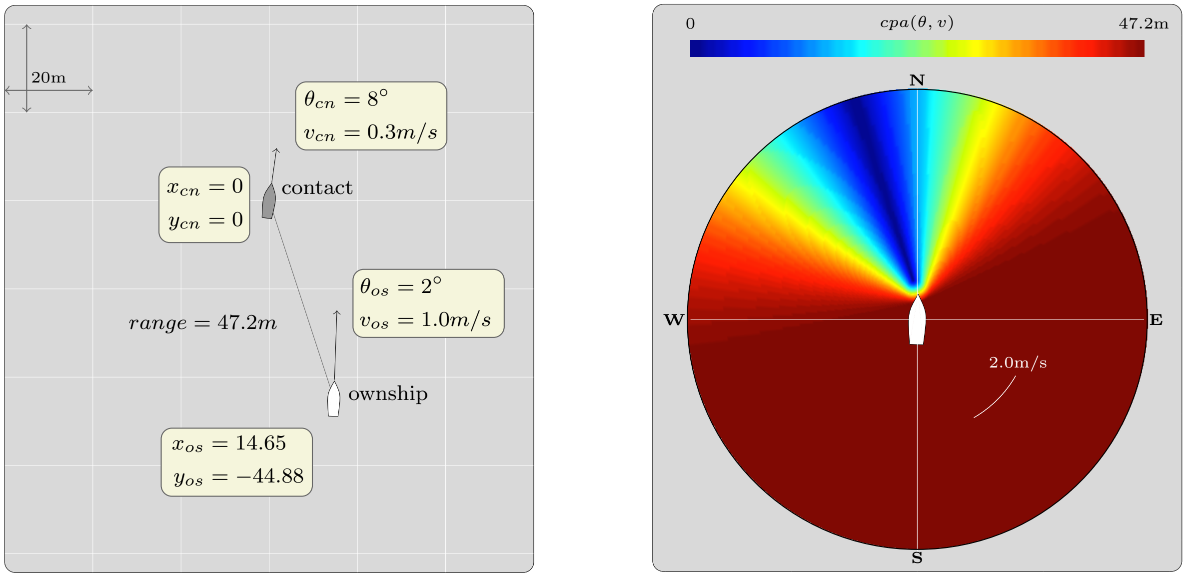


Figure 20: An example overtaking situation (left), and the evaluation of closest point of approach for all possible maneuvers (θ, v), max ownship speed 4.0 m/s.

The function $g(x)$ is applied to the CPA range to generate a utility, in the range $[0, 100]$ for each maneuver. This function is dependent on two behavior configuration parameters, $\check{r}_{cpa}$ and $\hat{r}_{cpa}$. CPA ranges less than or equal to $\check{r}_{cpa}$ are regarded as collisions, having the lowest possible utility. Values above $\hat{r}_{cpa}$ are regarded as having a separation range so safe that anything greater has no added utility. Values in between are graded linearly. The function is given by Equation 45:

$$g(x) = \begin{cases} 100 & x \geq \hat{r}_{cpa} \\ 0 & x \leq \check{r}_{cpa} \\ \frac{x - \check{r}_{cpa}}{(\hat{r}_{cpa} - \check{r}_{cpa})} & otherwise \end{cases} \tag{45}$$

When $(\check{r}_{cpa} = \hat{r}_{cpa})$, the function $g(cpa(\theta, v))$ performs as a *velocity obstacle*, essentially allowing maneuvers above the threshold and disallowing all those below.

The final component of the giveway overtaking objective functions is $h(\theta, v)$. This function penalizes maneuvers that would result in a passing of the contact on the side opposite of the current passing side. Recall that the current passing side is indicated in the *submode* set previously when the overall mode/submode determination was made as in Algorithm 3. The function is given by:

$$h(\theta, v) = \begin{cases} 1 & (submode = port) \;\; and \;\; \texttt{pass}^{os}_{cnp} \\ 1 & (submode = starboard) \;\; and \;\; \texttt{pass}^{os}_{cns} \\ 0 & otherwise \end{cases} \tag{46}$$

An example of all components comprising the final objective function for our example is shown on the right in Figure 21.

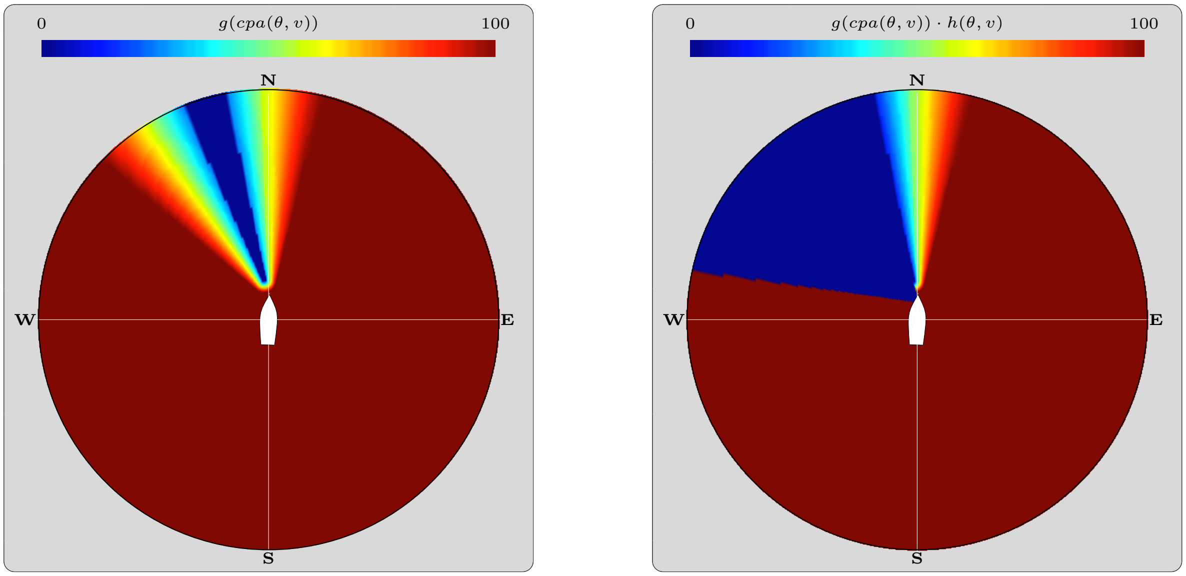


Figure 21: (left) The utility of all ownship maneuvers, (θ, v) based on a mapping of CPA to utility, Equation 45. In this case $\hat{r}_{cpa} = 20m$ and $\check{r}_{cpa} = 8m$. (right) The final utility function, $f(\theta, v)$ based on Equation 44. The final function biases against changing crossing side once a starboard crossing has begun.

5.2 The Head-On Objective Function

An *head-on* objective function is generated when the vehicle is in the head-on mode with respect to a contact. The criteria for being in this mode were discussed in Section 4.3. The head-on objective function is given by:

$$f(\theta, v) = g(cpa(\theta, v)) \ \cdot \ h(\theta, v) \tag{47}$$

This objective function has three components, (a) calculation of CPA, $cpa(\theta, v)$, (b) application of utility applied to CPA, $g(cpa(\theta, v))$, and (c) a restriction to avoid any maneuver that does not result in ownship passing the contact's port side. The raw CPA calculation is the first step. This calculation was discussed in Section 3.10. An example head-on situation is shown in Figure 22 on the left. The mapping of CPA values to the maneuver space, $cpa(\theta, v)$, is shown on the right.

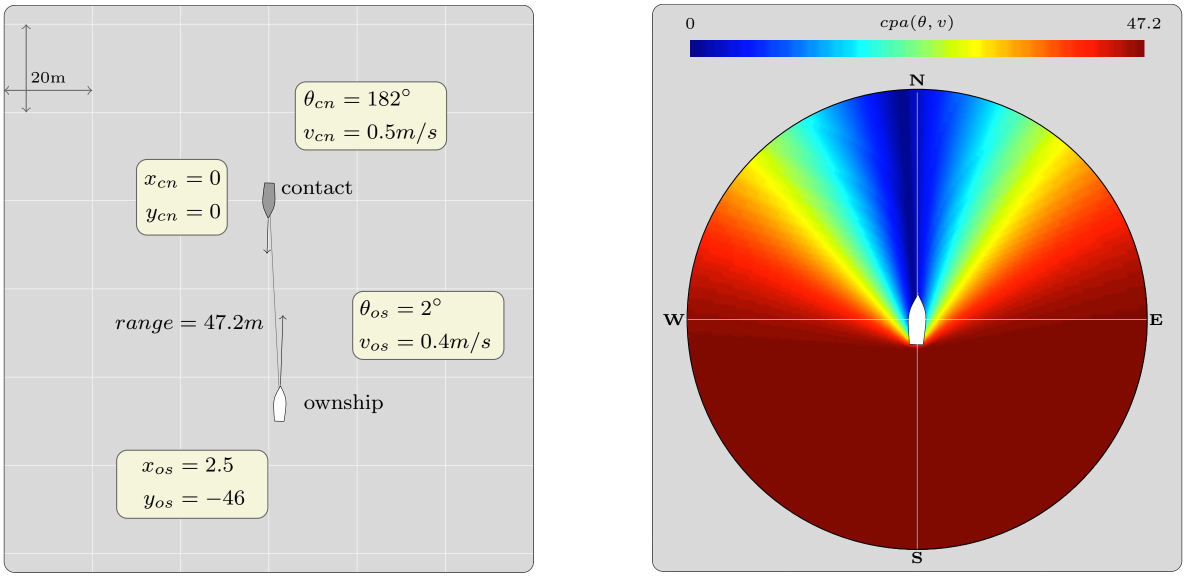


Figure 22: An example head-on situation (left). The evaluation of closest point of approach for all possible maneuvers (θ, v), max ownship speed 4.0 m/s, on the (right).

The function $g(x)$ is then applied to the CPA range to generate a utility, in the range $[0, 100]$ for each maneuver. This function is dependent on two behavior configuration parameters, $\check{r}_{cpa}$ and $\hat{r}_{cpa}$. CPA ranges less than or equal to $\check{r}_{cpa}$ are regarded as collisions, having the lowest possible utility. Values above $\hat{r}_{cpa}$ are regarded as having a separation range so safe that anything greater has no added utility. Values in between are graded linearly. This function was defined in Equation 45. An example is shown on the left in Figure 23.

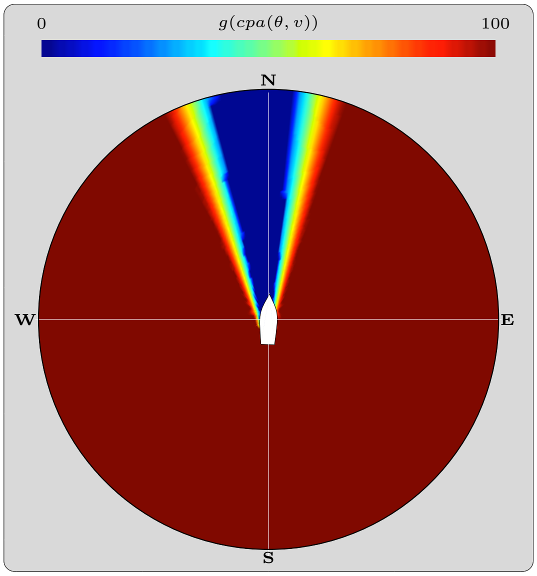
The final component of the head-on objective functions is $h(\theta, v)$. This function penalizes maneuvers that would not result in a passing of the contact on its port side, $\texttt{pass}^{os}_{cnp}$, defined in Section 3.7. The function $h(\theta, v)$ is given by:

$$h(\theta, v) = \begin{cases} 1 & \texttt{pass}^{os}_{cnp} \\ 0 & otherwise \end{cases} \tag{48}$$

An example of all components comprising the final head-on objective function for our example is shown on the right in Figure 23.

5.3 The GiveWay Crossing Objective Function

An *giveway crossing* objective function is generated when the vehicle is in the GiveWay Crossing mode with respect to a contact. The criteria for being in this mode were discussed in Section 4.5. The giveway crossing objective function is given by:



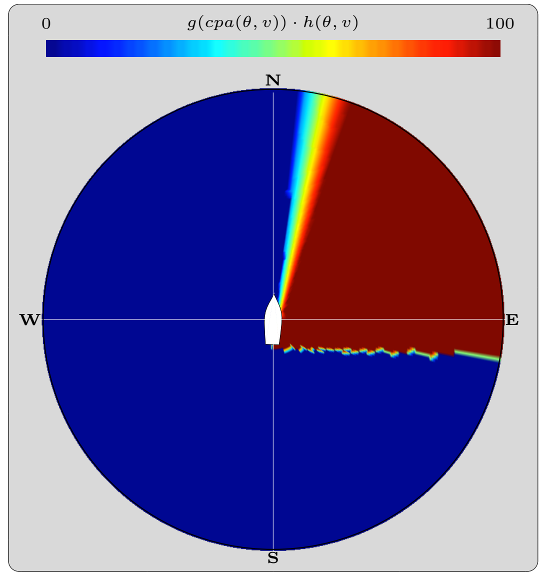


Figure 23: (left) The utility of all ownship maneuvers, (θ, v) based on a mapping of CPA to utility, Equation 45. In this case $\hat{r}_{cpa} = 20m$ and $\check{r}_{cpa} = 8m$. (right) The final utility function, $f(\theta, v)$ based on Equation 47. The final function biases against ownship maneuvers that do not result in passing the contact on its port side.

$$utility(\theta, v) = g(cpa(\theta, v)) \; \cdot \; h(\theta, v) \tag{49}$$

5.3.1 The GiveWay Crossing Objective Function in the Bow Submode

The objective function has three components, (a) calculation of CPA, $cpa(\theta, v)$, (b) application of utility applied to CPA, $g(cpa(\theta, v))$, and (c) and a further restriction on maneuvers $h(\theta, v)$ depending on whether the giveway crossing submode is giveway to the bow or stern. The raw CPA calculation is the first step. This calculation was discussed in Section 3.10. An example giveway crossing situation is shown in Figure 24 on the left. The mapping of CPA values to the maneuver space, $cpa(\theta, v)$, is shown on the right.

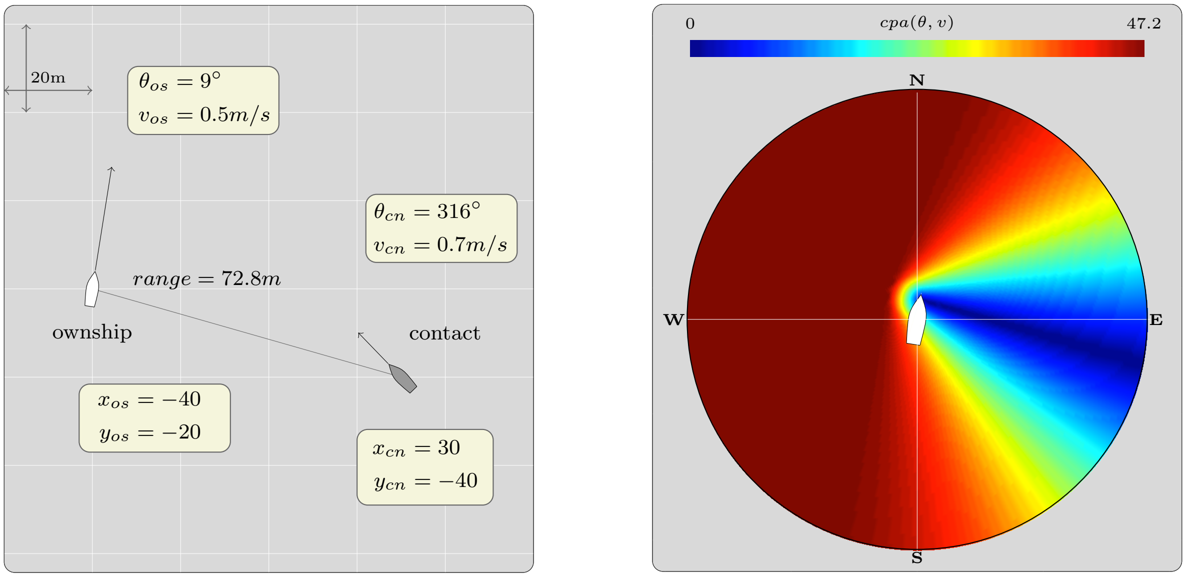


Figure 24: An example giveway crossing situation (left). The evaluation of closest point of approach for all possible maneuvers (θ, v), max ownship speed 4.0 m/s, on the (right).

The function $g(x)$ is then applied to the CPA range to generate a utility, in the range $[0, 100]$ for each maneuver. This function is dependent on two behavior configuration parameters, $\check{r}_{cpa}$ and $\hat{r}_{cpa}$. CPA ranges less than or equal to $\check{r}_{cpa}$ are regarded as collisions, having the lowest possible utility. Values above $\hat{r}_{cpa}$ are regarded as having a separation range so safe that anything greater has no added utility. Values in between are graded linearly. This function was defined in Equation 45. An example is shown on the left in Figure 25.

The final component of the giveway crossing bow objective functions is $h(\theta, v)$. This function depends on whether the giveway crossing mode has the submode of bow or stern. In the example above in Figure 24, the submode is *bow*, primarily because ownship is so far ahead of the slow-moving contact. The exact criteria for giveway crossing submodes was given in Algorithm 6. In the giveway crossing bow mode, ownship starboard turns that cross the contact stern are disallowed. The function $h(\theta, v)$ is given by:

$$h(\theta, v) = \begin{cases} 0 & \texttt{turn}^{os}_{star} \text{ and } !\texttt{cross}^{os}_{xcnb} \\ 1 & otherwise \end{cases} \tag{50}$$

An example of all components comprising the final giveway crossing objective function in the bow submode for our example is shown on the right in Figure 25.

5.3.2 The GiveWay Crossing Objective Function in the Stern Submode

The objective function in the stern submode also has the three components in Equation 49, differing only $h(\theta, v)$ component which biases ownship turns to cross the contact's stern. An example giveway

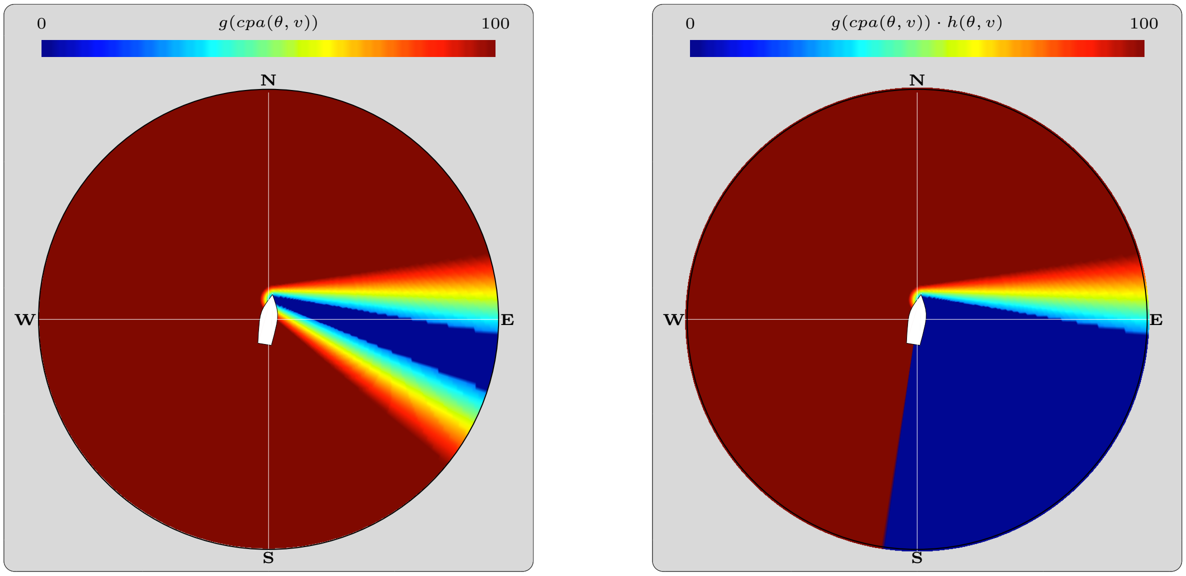


Figure 25: (left) The utility of all ownship maneuvers, (θ, v) based on a mapping of CPA to utility, Equation 45. In this case $\hat{r}_{cpa} = 20m$ and $\check{r}_{cpa} = 8m$. (right) The final utility function, $f(\theta, v)$ based on Equation 49. The final function disallows ownship starboard turns that do not cross the contact's bow.

crossing stern scenario is shown on the left in Figure 26. This scenario differs from the giveway crossing bow scenario in Figure 24 only in that the contact is moving roughly three times the speed, making a crossing behind the contact more appropriate. On the right in Figure 26, the mapping of CPA values to the maneuver space, $cpa(\theta, v)$, is shown.

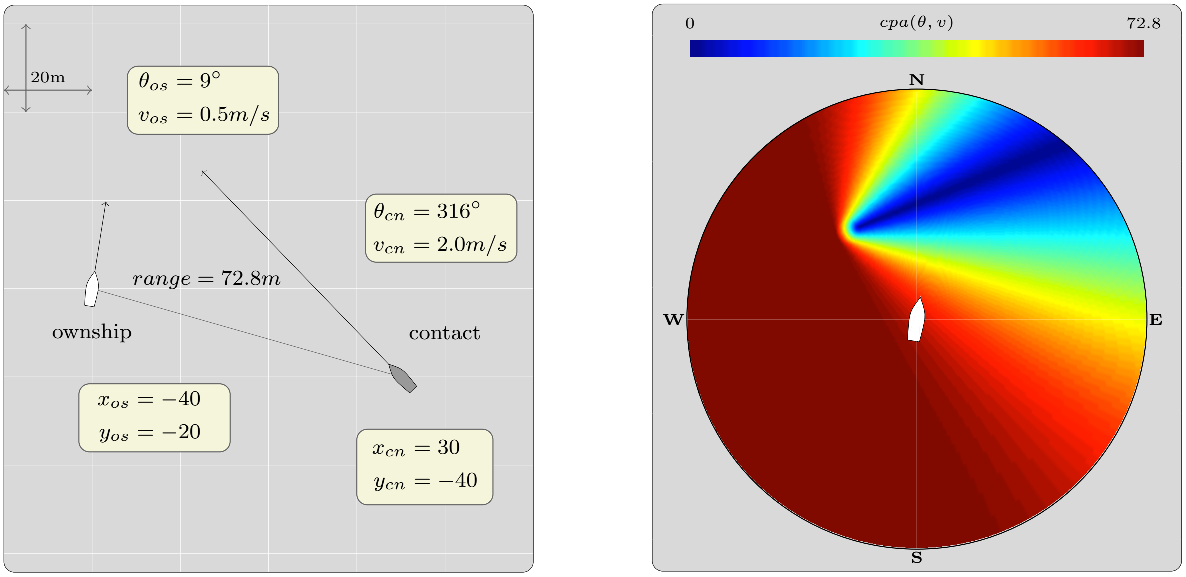


Figure 26: An example giveway crossing situation, where the giveway vehicle will cross behind the contact (left). The evaluation of closest point of approach for all possible maneuvers (θ, v), max ownship speed 4.0 m/s, on the (right).

The function $g(x)$ is then applied to the CPA range, in the same way as in the giveway crossing bow mode, to generate a utility, in the range $[0, 100]$ for each maneuver. This function is dependent on two behavior configuration parameters, $\check{r}_{cpa}$ and $\hat{r}_{cpa}$. CPA ranges less than or equal to $\check{r}_{cpa}$ are regarded as collisions, having the lowest possible utility. Values above $\hat{r}_{cpa}$ are regarded as having a separation range so safe that anything greater has no added utility. Values in between are graded linearly. This function was defined in Equation 45. An example is shown on the left in Figure 27.

The final component of the giveway crossing stern objective functions is $h(\theta, v)$. This function depends on whether the giveway crossing mode has the submode of bow or stern. In the example above in Figure 26, the submode is *stern*, primarily because the contact is moving at a speed where crossing ahead is not advisable. The exact criteria for giveway crossing submodes was given in Algorithm 6. In the giveway crossing bow mode, ownship starboard turns that cross the contact stern are disallowed. The function $h(\theta, v)$ is given by:

$$h(\theta, v) = \begin{cases} 0 & \texttt{turn}^{os}_{port} \\ 0 & \texttt{cross}^{os}_{xcnb} \\ 1 & otherwise \end{cases} \tag{51}$$

An example of all components comprising the final giveway crossing stern objective function for our example is shown on the right in Figure 27.

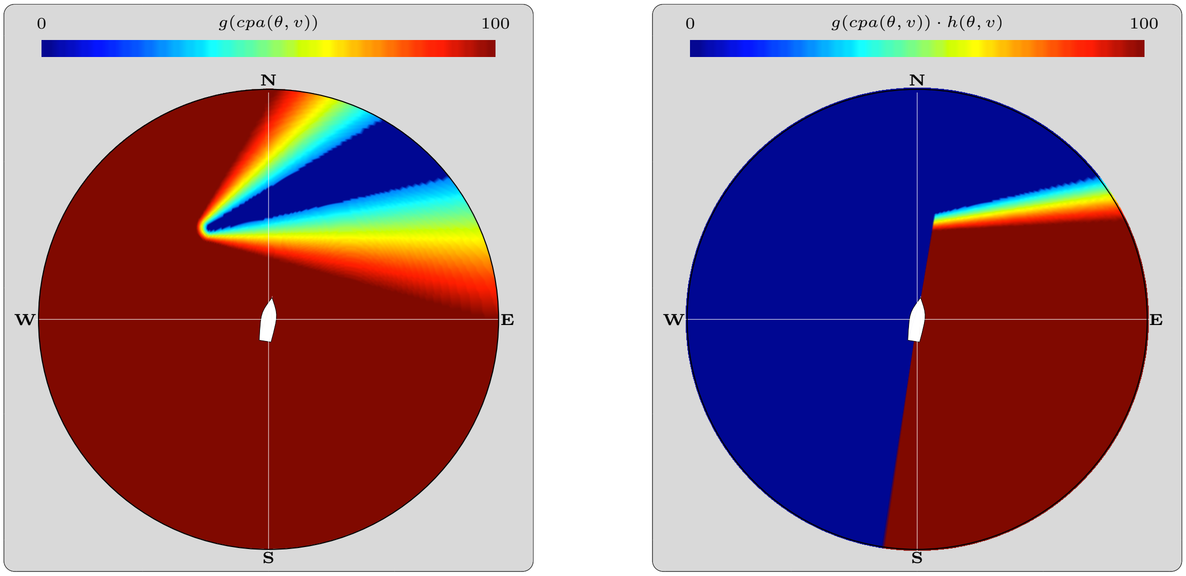


Figure 27: (left) The utility of all ownship maneuvers, (θ, v) based on a mapping of CPA to utility, Equation 45. In this case $\hat{r}_{cpa} = 20m$ and $\check{r}_{cpa} = 8m$. (right) The final utility function, $f(\theta, v)$ based on Equation 49. The final function disallows ownship port turns and maneuvers that cross the contact's bow.

5.4 The StandOnX and StandOnOT Objective Functions

The StandOn objective function is generated when the vehicle is in the StandOnX or StandOnOT mode with respect to a contact. The one type of function form described here will be later expanded to vary between the two major modes and their submodes. The criteria for being in this mode were discussed in Sections 4.6 and 4.4. The StandOn objective function is given by:

$$utility(\theta, v) = \frac{k}{2} \cdot f(\theta) + \frac{k}{2} \cdot g(v) \tag{52}$$

This objective function has two components, one for heading and one for speed. The value k sets the utility range, by default 100, as in Equation 45. Unlike the objective functions in the giveway overtaking, head-on and giveway crossing modes, this objective function treats the two decision variables independently. The heading is evaluated, $f(\theta)$, with respect to ownship's heading *at the time ownship entered the StandOnX or StandOnOT mode*, $\hat{\theta}$. The utility degrades linearly the greater the heading deviation ($\delta(\theta, \hat{\theta})$, see Equation 3) up to a heading deviation of θ_Δ, beyond which the utility is zero:

$$f(\theta) = \begin{cases} 1 - \frac{\delta(\theta,\hat{\theta})}{\theta_\Delta} = & \delta(\theta, \hat{\theta}) \leq \theta_\Delta \\ 0 & otherwise \end{cases} \tag{53}$$

The non-negative choice for speed, v, is evaluated with respect to ownship's speed *at the time ownship entered the StandOnX or StandOnOT mode*, $\hat{v}$. The utility degrades linearly the further the candidate speed deviates from $\hat{v}$, up to a delta speed of v_Δ, beyond which the utility is zero:

$$g(v) = \begin{cases} 1 - \frac{|(v-\hat{v}|}{v_\Delta} = & |v - \hat{v}| \leq v_\Delta \\ 0 & otherwise \end{cases} \tag{54}$$

An example is shown below in Figure 28, inverted from the previous give-way example in Figure 26, with ownship now the StandOn vessel. An example of all components comprising the final StandOn objective function for our example is shown on the right in the figure.

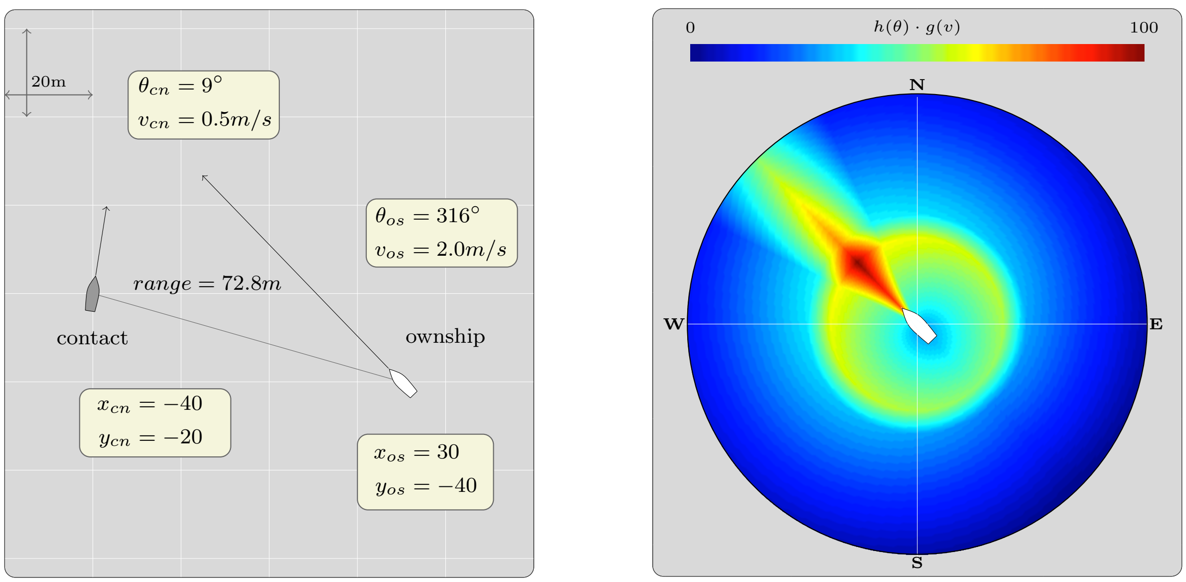


Figure 28: An example StandOn situation (left). The objective function for all possible maneuvers (θ, v), with max ownship speed 5.0 m/s, on the (right).

References

[1] Michael R. Benjamin, Henrik Schmidt, Paul M. Newman, and John J. Leonard. Nested autonomy for unmanned marine vehicles with moos-ivp. *Journal of Field Robotics*, 27(6):834–875, November/December 2010.

[2] Mike Benjamin, Henrik Schmidt, and John J. Leonard. `http://www.moos-ivp.org`.

[3] Commandant Uniteds States Coast Guard. *Navigation rules, international-inland / U.S. Department of Transportation, United States Coast Guard.* 1977-19 : CG. [Washington, DC] : The Guard : [For sale by the Supt. of Docs., U.S. G.P.O.], 1999.