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Competition with Private Capacity Information*

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Non-Linear Pricing Competition with Private Capacity Information

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We analyze the equilibrium of an incomplete information game consisting of two capacity-constrained suppliers and a single retailer. The capacity of each supplier is her private information. Conditioned on their capacities, the suppliers simultaneously and non-cooperatively offer quantity-price schedules to the retailer. Then, the retailer decides on the quantities to purchase from each supplier in order to maximize his own utility. We prove the existence of a (pure strategy) Nash equilibrium for this game. We show that at the equilibrium each (infinitesimal) unit of the supply is assigned a marginal price which is independent of the capacities and depends only on the valuation function of the retailer and the distribution of the capacities. In addition, the supplier with the larger capacity sells all her supply.

1. Introduction

Internet marketplaces facilitate matching of buyers and sellers. For example, *alibaba.com*¹ provides such a platform for a wide range of industries including apparel, furniture, energy, electronics. In this platform, buyers (e.g., retailers) post “buying leads”, which are descriptions of certain products and also the required quantities. Then, suppliers respond by sending price quotes; these quotes are only observed by the buyers, but not by the other suppliers. Subsequently, buyers select the best set of quotes.

In order to better understand competition in such environments, in this paper, we study a setting that consists of two suppliers who compete over a substitutable good that they are selling to a single retailer (buyer). The suppliers are capacity-constrained and the capacity of each supplier is her *private information*. The suppliers, conditioned on their capacities, simultaneously and non-cooperatively offer quantity-price schedules to the retailer. Then, the retailer maximizes his utility by deciding how much to purchase from each supplier. The goal of each supplier is to design price schedules that maximizes her expected revenue at the equilibrium.

This model serves as an abstraction for competition in traditional industries as well as blooming Internet markets. In the aforementioned example, the capacity each supplier can dedicate to each

¹ The Alibaba group has more than 60 million registered users across the globe and its annual revenue was more than 5 billion dollars in 2010.

product is her private information. Another example comes from the context of display advertising, where advertisers (retailers in our setting) purchase impressions (advertisement space) from online publishers (suppliers). The online publishers are constrained by the number of people viewing their websites and cannot always satisfy the large number of impressions demanded by a big advertising campaign (i.e., they are capacity-constrained). Hence, an advertiser sometimes works with more than one publisher. Yet for another example consider secondary markets for selling remnant inventories; for instance, hotels and airlines sell their excess capacities to a secondary market via websites such as hotwire.com and orbitz.com (which corresponds to the retailer in our setting); SpotCloud offers a similar service in the context of cloud computing where data centers can sell their excess capacity (see Economist (2011)). As the final example consider the electricity market. In particular, due to the emergence of renewable resources such as wind, the uncertainty about the production capacities has increased in this market (see the discussion on supply function equilibria in Section 1.1). In all the aforementioned models, the suppliers are capacity constrained. Their exact capacities are their private information.

Contributions

We present an incomplete information game for non-linear pricing competition between two suppliers where the capacity of each supplier is her private information. In this game, the strategy of each supplier is a function of her private capacity and corresponds to a quantity-price schedule offered by the supplier to the retailer.

In this game, finding the equilibrium strategies corresponds to solving an Equilibrium Problem with Equilibrium Constraints (cf. Luo et al. (1996)). These problems are notoriously challenging. Namely, the problem of finding a best response strategy for each supplier is defined by a bi-level mathematical program: each supplier chooses a pricing schedule to maximize her expected revenue — the expectation is taken with respect to the realizations of the capacity of the other supplier — and given the price schedules, the retailer optimizes over the quantities to be purchased from each supplier (the lower level program).

To the extent of our knowledge, this is the first work on competition in a supply chain context that considers a setting where the leaders of the game are privately informed.

Existence of Equilibrium: We present pricing strategies where each (infinitesimal) unit of the supply is assigned a marginal price which is independent of the capacities and depends only on the valuation function of the retailer and the distribution of the capacities. Using these pricing strategies, our main result, Theorem 1, establishes the existence of a symmetric pure strategy Nash equilibrium, under certain assumptions on the valuation function of the retailer and the distributions of the private information of the suppliers.

We provide examples of the equilibrium pricing strategies for commonly used valuation functions of the retailer that satisfy our assumptions. Examples include environments in which the retailer sells goods to a mass of consumers represented by a linear or constant-elasticity demand curves.

Properties of the Equilibrium: We show that at the equilibrium, the retailer purchases supply from both suppliers. He buys all the available quantity from the supplier with the larger capacity. The amount of supply purchased by the retailer from the supplier with the smaller capacity is decreasing in the capacity of the other supplier as well as in the production cost. In addition, we show that the revenue of each supplier is increasing in her capacity.

Furthermore, we observe that the equilibrium may not be efficient — an equilibrium is efficient if it maximizes the (combined) social welfare of the suppliers and the retailer. Namely, even though the retailer has strictly positive valuations for (at least a portion of) the remaining supply of the supplier with the smaller capacity, he may not purchase it because of the high price. This inefficiency is attributed to the asymmetry of information among suppliers; see Section 6.1.

Methodology: We simplify the problem of finding best-response strategies using a mapping from pricing strategies to allocation functions (where a supplier decides how much supply to allocate to the retailer based on the capacity and the pricing strategy of the other supplier). Our techniques for finding best response strategies can be viewed as a reduction of the equilibrium analysis, for games with incomplete information, to a mechanism design problem.

Organization After a brief overview of the related literature in Section 1.1, we formally present our model and discuss our assumptions in Section 2. The main results of the paper are presented in Section 3, followed by examples of settings that satisfy our assumptions. In Section 4, we discuss the best response strategies. The proof of Theorem 1 is presented in Section 5. In Section 6, we discuss some of the aspects of our problem in more details; namely, the efficiency of the equilibrium, the effects of marginal production cost on the pricing strategies, and our methodology. The proofs of the lemmas and propositions, unless stated otherwise, are relegated to the appendix.

1.1. Related Work

A related line of work to ours is the literature of supply function equilibria; see Grossman (1981), Hart (1985), Klemperer and Meyer (1989), Anderson and Hu (2008), Johari and Tsitsiklis (2011). In a supply function competition, competing firms sell a substitutable good to a market consisting of a mass of consumers. A supply function determines the quantity of the good that would be produced by the firm given the (per-unit) market price. In a supply function equilibrium, each firm announces a supply function, then the demand (usually stochastic) is realized and a market clearing price is determined, i.e., supply function $s_i(p)$ implies that if the market clearing price is determined to be p , then firm i would produce amount $s_i(p)$. See also Baldick and Hogan (2001), Wilson

(2008), Holmberg (2008), Genca and Reynoldsb (2011) for capacity constrained supply-function equilibria mainly motivated by applications in electricity markets. To the extent of our knowledge, the only work in this context with privately informed firms is by Vives (2011) where each supplier receives a private signal which conveys information about her own and other suppliers' costs. Vives (2011) argues that private cost information with supply function competition may provide a better explanation for patterns observed in the electricity markets compared to other theories. The main difference between the supply function equilibrium and the solution concept studied in this paper is that in supply function equilibrium the firms sell to a mass of consumers where in our model the firms sell to a single buyer and are able to charge non-linear prices for consumption. Furthermore, the market may not clear. This complicates the problem even further since the buyer solves an optimization problem to determine how much to purchase from each supplier.

Our model can be thought of as a (delegated) common agency setting (Bernheim and Whinston (1986), Laussel and Le Breton (2001), Martimort and Stole (2002), Calzolari and Scarpa (2008)), where the suppliers are the principals and the retailer is the agent. The closest work to ours in this context is by Martimort and Stole (2009) who consider a model where the retailer (agent) has private information regarding his valuation function, and there are two suppliers (principals with no private information). The suppliers simultaneously offer menus of price-quantity contracts to the retailer who subsequently chooses the best set of contracts. What distinguishes our paper is that in the setting we consider the principals, not the agent, have private information and the contracts and the equilibrium depends on their private information.

In our setting, dual-sourcing is an implication of the capacity constraints. Several recent works study dual-sourcing motivated by disruption and risk management; see Tomlin and Wang (2005), Babich et al. (2007), Yang et al. (2012), Gümüs et al. (2010). In these works, usually the capacity of each supplier is modeled with a Bernoulli random variable, with some probability the supplier fails to produce any supply and with the remaining probability yields a certain quantity.

2. Model

There are two suppliers, \mathcal{S}_1 and \mathcal{S}_2 , of a substitutable good that they are selling to a retailer. The valuation function of the retailer is represented by $v: \mathbb{R}^+ \rightarrow \mathbb{R}^+$, i.e., the retailer get value $v(q)$ from obtaining quantity q of the good. The capacity of the retailer is denoted by $\bar{\kappa} \geq 1$. This valuation function is known by the suppliers.

The amount of capacity of supplier \mathcal{S}_i is denoted by κ_i and is distributed with c.d.f. $F: [0, 1] \rightarrow [0, 1]$ (and p.d.f. f). Each supplier \mathcal{S}_i knows her own capacity but only the distribution of the other supplier's capacity. Note that the maximum capacity of each supplier is (normalized to be) at most 1.

We consider the following incomplete information game between suppliers: The suppliers lead the game. First, each supplier \mathcal{S}_i chooses a pricing strategy $p_i : \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$ where $p_i(q, \kappa_i)$ denotes how much the retailer should pay to supplier i for quantity q of supply, when the capacity of \mathcal{S}_i is equal to κ_i . Then, each supplier \mathcal{S}_i (privately) learns her own capacity, κ_i , which is (independently) drawn from distribution F . Simultaneously, each supplier \mathcal{S}_i offers to the retailer price schedule $p_i(\cdot, \kappa_i)$, where as mentioned before $p_i(q, \kappa_i)$ represents how much the retailer should pay for q amount of capacity.

THE RETAILER'S PROBLEM: Given price schedules $p_1(\cdot, \kappa_1)$ and $p_2(\cdot, \kappa_2)$, the retailer chooses q_1 and q_2 , the amount of the supply he buys from each supplier, such that they maximize his utility:

$$(q_1, q_2) \in \operatorname{argmax}_{y_1, y_2} \{v(y_1 + y_2) - p_1(y_1, \kappa_1) - p_2(y_2, \kappa_2)\}. \quad (1)$$

For the simplicity of the presentation, we assume $p_i(0, \kappa_i) = 0$ and $p_i(q_i, \kappa_i) = \infty$, if $x > \kappa_i$, for all κ_i .² In addition, we normalize the marginal production cost to be 0. In Section 6.2, we explain how the pricing strategies change when the marginal production cost is positive.

Each supplier chooses a strategy to maximize her own (expected) revenue.

DEFINITION 1 (BEST-RESPONSE STRATEGY OF A SUPPLIER). Pricing strategy p_1 is a best response strategy for supplier \mathcal{S}_1 to the pricing strategy of supplier \mathcal{S}_2 , given by p_2 , if it maximizes the expected revenue of \mathcal{S}_1 , for any realization of her capacity, where the expectation is taken with respect to the capacity of \mathcal{S}_2 .

When the strategy of \mathcal{S}_2 is given by price schedule p_2 and the capacity of \mathcal{S}_1 is equal to κ_1 , the *best response* of \mathcal{S}_1 is to offer a price schedule that is a solution of the optimization problem below:

$$\begin{aligned} \text{MAXIMIZE}_{p_1(\cdot, \kappa_1): [0, \kappa_1] \rightarrow \mathbb{R}^+} \quad & \text{Rev}_1(\kappa_1) = E[p_1(q_1(\kappa_1, \kappa_2), \kappa_1)] = \int_0^1 p_1(q_1(\kappa_1, \kappa), \kappa_1) f(\kappa) d\kappa \quad (2) \\ \text{SUBJECT TO:} \quad & q_1(\kappa_1, \kappa) = \operatorname{argmax}_{y_1 \leq \kappa_1, y_2 \leq \kappa} \left\{ v(y_1 + y_2) - p_1(y_1, \kappa_1) - p_2(y_2, \kappa) \right\} \end{aligned}$$

In the above optimization problem, κ represents the capacity of \mathcal{S}_2 and $q_1(\kappa_1, \kappa)$ corresponds to the amount of supply the retailer purchases from \mathcal{S}_1 ; namely, in the lower level program, q_1 is the solution of the retailer's problem given by Eq. (1).³ The goal of supplier \mathcal{S}_1 is to design a pricing function that maximizes her expected revenue equal to $E[p_1(q_1(\kappa_1, \kappa_2), \kappa_1)]$, where the expectation is taken with respect to the capacity of \mathcal{S}_2 .

DEFINITION 2 (EQUILIBRIUM). Strategies p_1 and p_2 define a Nash equilibrium if for all $\kappa_i \in [0, 1]$, $p_i(\cdot, \kappa_i)$ is a best response to the strategy of the other supplier.

² Note that the supplier would not be able to deliver quantities beyond her capacity, unless she incurs a large cost or penalty (by negating on the contract). Therefore, we assume that suppliers do not offer price schedules for quantities larger than their capacities since the retailer may call them upon that.

³ To be more precise, when there exist multiple solutions to the retailer's problem, let $q_1(\kappa_1, \kappa) = \inf \left\{ q_1 \mid \exists (q_1, q_2) \in \operatorname{argmax}_{y_1 \leq \kappa_1, y_2 \leq \kappa} \left\{ v(y_1 + y_2) - p_1(y_1, \kappa_1) - p_2(y_2, \kappa) \right\} \right\}$.

2.1. Assumptions

In this section, we present and discuss the assumptions that are made *throughout* the paper:

ASSUMPTION 1. The valuation function of the retailer, $v : \mathbb{R}^+ \rightarrow \mathbb{R}^+$, is twice differentiable, concave, and strictly increasing in $[0, \bar{\kappa}]$ where $\bar{\kappa} \geq 1$ represents the capacity of the retailer; $v(0) = 0$ and $v(q) = v(\bar{\kappa}), q > \bar{\kappa}$.

Note that $v(q) = v(\bar{\kappa}), q > \bar{\kappa}$, corresponds to the standard free-disposal assumption.⁴ The assumption that the capacity of the retailer is at least equal to 1 captures the large, *possibly infinite*, capacity of the retailer so that he would purchase from both suppliers.

An example of valuation functions that satisfies Assumption 1 is $v(q) = q - \frac{1}{2}q^2$ with $\bar{\kappa} = 1$. It corresponds to a setting where the retailer sells goods to a mass of consumers represented by a linear demand curve. See Section 3.1 for details and other examples.

ASSUMPTION 2. The distribution of capacity, $F : [0, 1] \rightarrow [0, 1]$, is differentiable (with density f), strictly increasing, and log-concave.

Log-concavity implies that $\frac{f(x)}{F(x)}$ is decreasing and is a standard assumption in the context of mechanism design and revenue management, cf. Bagnoli and Bergstrom (2005). Assumption 2 is satisfied by many distributions including uniform, many of the beta family distributions, as well as the exponential and Normal distributions truncated and scaled to finite intervals. We mainly use this assumption to prove Lemma 6 in Section 3.

ASSUMPTION 3. For all $\kappa \in [0, 1]$, function $h(y) = v(\kappa + y) + \frac{F(\kappa)}{f(\kappa)}v'(\kappa + y)$ is quasi-concave⁵ in $y \in [0, 1]$.⁶

The quasi-concavity property implies that function h is unimodal. Note that every concave function is quasi-concave (the reverse is not true). The function is strictly quasi-concave if the inequality holds strictly. The assumption above, as it will become clearer later, corresponds to a regularity condition similar to the monotonicity of virtual values in auction design (cf. Myerson (1981)). For the aforementioned example of $v(q) = q - \frac{1}{2}q^2$, the assumption above is satisfied since $h''(y) = -1$ (i.e., h is concave). It is easy to see that the assumption above is satisfied if v satisfies Assumption 1 and the 3-rd derivative of v is negative.

We now define α as $\operatorname{argmax}_y \{h(y)\}$ defined above in Assumption 2.

DEFINITION 3 (α). Define $\alpha(\kappa) = \min_z \left\{ z \in \operatorname{argmax}_{0 \leq y \leq \min\{1, \bar{\kappa} - \kappa\}} \left\{ v(\kappa + y) + \frac{F(\kappa)}{f(\kappa)}v'(\kappa + y) \right\} \right\}$.

⁴ The assumption implies that having more supply cannot decrease the retailer's utility because the retailer can discard "extra" supply at no cost.

⁵ Function $h : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ is quasi-concave if for any $x, y \in \mathbb{R}$ and $\lambda \in [0, 1]$ we have $h(\lambda x + (1 - \lambda)y) \geq \min\{h(x), h(y)\}$.

⁶ To simplify the presentation, with slight abuse of notation, we have dropped the dependence of h on κ .

By Assumption 3, $h(y)$ is increasing in $y \leq \alpha(\kappa)$ and decreasing in $y \geq \alpha(\kappa)$.

In a setting where $v(q) = q - \frac{1}{2}q^2$ with $\bar{\kappa} = 1$, and F is the uniform distribution, we have $\alpha(\kappa) = 1 - 2\kappa$, $\kappa \in [0, \frac{1}{2}]$, and $\alpha(\kappa) = 0$, $\kappa \in [\frac{1}{2}, 1]$.

We need a couple of more definitions before presenting out next, and final, assumption. Let $\underline{\alpha} = \max_{\kappa} \{\kappa | \alpha(\kappa) = \alpha(0)\}$. Observe that since v is strictly increasing (Assumption 1), we have $\alpha(0) = 1$. In addition, in Appendix A.1, using Assumption 2, we show that α is a decreasing function. Furthermore, we denote the inverse of function α by α^{-1} and let $\tilde{\alpha}$ to be the fixed point of α , i.e., $\tilde{\alpha}$ is the solution of $\alpha(\kappa) = \kappa$; see Appendix A.1 for details.

Quasi-concavity is a common assumption in the literature on the equilibrium analysis (cf. Arrow and Enthoven (1961), Greenberg and Pierskalla (1971), Ginsberg (1973)) and incomplete information games (cf. Athey (2001), Milgrom and Shannon (1994)).

ASSUMPTION 4. For $\kappa \in [0, 1]$, pricing function $p(q, \kappa)$ is strictly increasing in $q \in (0, \kappa)$. In addition, function $v(q + y) - p(q, \kappa)$ is quasi-concave in $q \in (\underline{\alpha}, \kappa)$, for $y \in [\underline{\alpha}, 1]$.⁷

In the assumption above, we consider the range in $[\underline{\alpha}, 1]$ because, for $\kappa \leq \underline{\alpha}$, we have $\alpha(\kappa) = 1$. This follows from definition of $\bar{\alpha}$ and Lemma 6 that shows α is a decreasing function. In addition, we point out that the assumption above would be imposed only on the pricing strategy p^* defined in the following section (see Eq. (4)). In other words, we do not limit the strategy space of the suppliers to lie within the pricing functions defined by the assumption above.

3. Equilibrium Pricing Strategies

In this section, we present our main result. We start with defining $\gamma : [0, 1] \rightarrow [0, 1]$ as follows

$$\gamma(y) = \begin{cases} v'(y + \alpha^{-1}(y)) & 0 \leq y \leq \tilde{\alpha} \\ v'(y + \alpha(y)) - \frac{f(y)}{F^2(y)} \int_{\tilde{\alpha}}^y F(z)(1 - \alpha'(z))v'(z + \alpha(z))dz & \tilde{\alpha} < y \leq 1 \end{cases} \quad (3)$$

Below, we present the main result of this paper. The following theorem shows that at the equilibrium each (infinitesimal) unit dy of the supply is assigned a marginal price $\gamma(y)$ (which is *independent* of the capacity).

THEOREM 1. Define $p^* : \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$ as:

$$p^*(q, \kappa) = \begin{cases} \int_0^q \gamma(y)dy & q \leq \kappa \\ \infty & q > \kappa \end{cases} \quad (4)$$

⁷ For instance, we observe that the above assumption holds for the wholesale price contracts — defined as $p(q, \kappa) = q\mu(\kappa)$ where μ is a function of the capacity — as long as v is strictly concave. Another example is quadratic valuations and quantity-discount price contracts. Quantity-discount price contracts can be defined as $p(q, \kappa) = q - q^2\mu(\kappa)$ where μ is a function of the capacities, cf. Cachon and Kök (2010); see Section 3.1.

Suppose Assumptions 1, 2, and 3 hold and p^* satisfies Assumption 4. The pricing strategies $p_1 = p_2 = p^*$ define a symmetric equilibrium such that the retailer purchases quantities $q_1^*(\kappa_1, \kappa_2)$ and $q_2^*(\kappa_1, \kappa_2)$ from suppliers \mathcal{S}_1 and \mathcal{S}_2 , respectively, where

$$\begin{aligned} q_1^*(\kappa_1, \kappa_2) &= \begin{cases} \min\{\alpha(\kappa_2), \kappa_1\} & \kappa_1 \leq \kappa_2 \\ \kappa_1 & \kappa_1 > \kappa_2 \end{cases} \\ q_2^*(\kappa_1, \kappa_2) &= \begin{cases} \kappa_2 & \kappa_1 \leq \kappa_2 \\ \min\{\alpha(\kappa_1), \kappa_2\} & \kappa_1 > \kappa_2 \end{cases} \end{aligned} \quad (5)$$

In other words, $q_1 = q_1^*(\kappa_1, \kappa_2)$ and $q_2 = q_2^*(\kappa_1, \kappa_2)$ maximize the utility of the retailer $v(q_1 + q_2) - p^*(q_1) - p^*(q_2)$; see Eq. (1). Note that each supplier either sells all her supply to the retailer or a small (less than $\tilde{\alpha}$) part of it. By Eq. (5) and since α is decreasing (Lemma 6), we obtain the following results.

COROLLARY 1. *At the equilibrium, the supplier with the larger capacity sells all her supply. In addition, the quantity that the retailer purchases from the other supplier is decreasing in the capacity of the supplier with the larger capacity.*

COROLLARY 2. *The amount of good the retailer purchases from a supplier and the revenue of the supplier are increasing in her capacity.*

Although the result above seems intuitive, it may not always be the case, for instance, Talluri and Martinez de Albéniz (2011) present a (full information) model for dynamic pricing competition and show that having a larger capacity may not be favorable. In their model, the seller with the lower capacity would sell her supply first. Our results suggest that the predictions of these models may change if the capacities of the sellers are private information.

The proof of Theorem 1 is given in Section 5. We divide the proof into two parts. First, we show that if p^* satisfies Assumption 4, then allocation q^* will be implemented at equilibrium. Then, we show that p^* is a best response to itself. To do so, first in Section 4, we discuss the best-response strategies in our model and show how they can be simplified under our assumptions.

3.1. Examples

In this section, we present examples of valuation functions and distributions that satisfy the assumptions of Theorem 1. The proofs of the results in this section are given in Appendix ??.

Quadratic Valuation: We call the valuation function of the retailer *quadratic* if for a constant $\rho > 0$, we have $v(q) = q - \rho q^2$, $\bar{\kappa} = \frac{1}{2\rho}$. This valuation function models a retailer who sells to a mass of consumer represented by a linear demand curve. To observe this, consider a linear demand curve defined with a parameter a . Namely, when the price is equal to p , mass $q = a - p$ of the consumers

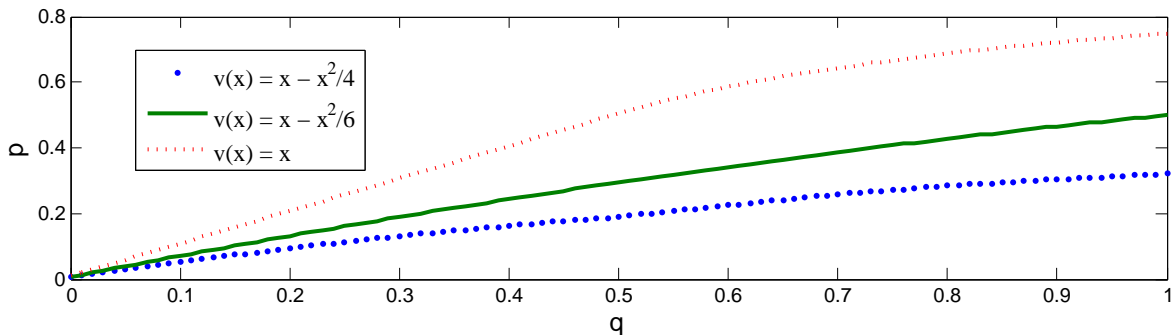


Figure 1 Price p^* (y -axis) for linear and quadratic valuations, respectively for $\rho = \frac{1}{6}$ and $\frac{1}{4}$.

would buy the good. Hence, if the retailer has quantity $q \leq \frac{a}{2}$ of the good, the price would be equal to $p = a - q$ (the retailer does not obtain additional value from the good beyond quantity $a/2$). Therefore, the retailer's valuation for quantity $q \leq \frac{a}{2}$ of the good is equal to $v(q) = q(a - q) = a(q - \frac{1}{a}q^2)$. The claim follows by letting $\rho = \frac{1}{a}$.

Note that for quadratic valuations and $\rho \leq \frac{1}{2}$, Assumption 1 holds since v is increasing in $q \in [0, 1]$. Also, as we mentioned earlier, Assumption 3 is satisfied because $v(\kappa + y) + \frac{F(\kappa)}{f(\kappa)}v'(\kappa + y)$ is concave in y .

PROPOSITION 1. *Suppose the valuation of the retailer is quadratic and F is the uniform distribution. For $0 < \rho \leq \frac{1}{6}$, we have $\alpha(\kappa) = 1$ and $\gamma(y) = v'(y + 1) - v'(y)$. For $\frac{1}{6} \leq \rho \leq \frac{1}{4}$, we have*

$$\alpha(\kappa) = \begin{cases} 1 & 0 \leq \kappa \leq \frac{1-2\rho}{4\rho} \\ \frac{1-4\rho\kappa}{2\rho} & \frac{1-2\rho}{4\rho} \leq \kappa \leq \frac{1}{4\rho} \\ 0 & \frac{1}{4\rho} \leq \kappa \leq 1 \end{cases}$$

$$\gamma(y) = \begin{cases} 1 - 2\rho - 2\rho y & 0 \leq y \leq \frac{1-2\rho}{4\rho} \\ \frac{1}{2} - \rho y & \frac{1-2\rho}{4\rho} \leq y \leq \frac{1}{6\rho} \\ \frac{1}{108\rho^2 y^2} & \min\{\frac{1}{6\rho}, 1\} \leq y \leq 1 \end{cases} \quad (6)$$

The corresponding pricing strategy p^* satisfies Assumption 4 and, using Theorem 1, implements equilibrium q^* .

For $\rho \leq \frac{1}{6}$, since $\alpha(\kappa_i) \geq \kappa_i$, both suppliers always sell all their supply at price equal to $p^*(\kappa_i, \kappa_i) = v(\kappa + 1) - v(1)$. See Figure 1 for a depiction of γ and p^* for ρ equal to $\frac{1}{6}$ and $\frac{1}{4}$. Note that since the (marginal) valuation of the retailer is decreasing in ρ , the suppliers are able to charge higher prices for $\rho = \frac{1}{6}$ compared to those with $\rho = \frac{1}{4}$.

Linear Valuations: Linear valuations refer to the setting where $v(q) = q$ in $[0, \bar{\kappa}]$. An example for such valuations comes from display advertising (cf. Feldman et al. (2009)). It is rather easy to see that Assumption 1 holds and it yields Assumption 3.

For $\bar{\kappa} \geq 2$, observe that the retailer could buy all the supply of both suppliers and the problem decomposes into two separate problems each with only one supplier. We now consider the case where $\bar{\kappa} < 2$.

PROPOSITION 2. *Suppose $\bar{\kappa} < 2$ and F satisfies Assumption 2 and $F(\frac{\bar{\kappa}}{2}) \geq \frac{1}{2}$. For linear valuations we have $\alpha(\kappa) = \bar{\kappa} - \kappa$ and*

$$\gamma(y) = \begin{cases} 1 & 0 \leq y \leq \frac{\bar{\kappa}}{2} \\ 1 - \frac{2f(y)}{F^2(y)} \int_{\bar{\alpha}}^y F(z) dz & \frac{\bar{\kappa}}{2} < y \leq 1 \end{cases}$$

$$p^*(q) = \begin{cases} q & 0 \leq q \leq \frac{\bar{\kappa}}{2} \\ 1 - q + \frac{2 \int_{\bar{\alpha}}^q F(z) dz}{F^2(q)} & \frac{\bar{\kappa}}{2} < q \leq 1 \end{cases}$$

The corresponding pricing strategy p^* satisfies Assumption 4 and, by Theorem 1, it defines an equilibrium.

COROLLARY 3. *If the valuation of the retailer is linear, $\bar{\kappa} = 1$, and F is the uniform distribution, then*

$$\gamma(y) = \begin{cases} 1 & 0 \leq y \leq \frac{1}{2} \\ \frac{1}{4y^2} & \frac{1}{2} < y \leq 1 \end{cases} \quad p^*(q) = \begin{cases} q & 0 \leq q \leq \frac{1}{2} \\ 1 - \frac{1}{4q} & \frac{1}{2} < q \leq 1 \end{cases}$$

See Figure 1 for a depiction of γ and p^* . Note that for $y \leq \frac{1}{2}$, the marginal price $\gamma(y)$ is equal to the marginal valuation of the retailer. Suppose $\kappa_1 \geq \kappa_2$. If $\kappa_1 \leq \frac{1}{2}$, supplier \mathcal{S}_1 does not have to give any discount to the retailer because even if she has the larger capacity, still there is more demand than supply. But, for $\kappa_1 \geq \frac{1}{2}$, the supplier should give larger discounts to “win” the competition and sell all her capacity to the retailer. As $\kappa_1 \geq \frac{1}{2}$ becomes larger, \mathcal{S}_1 decreases the marginal price.

Monomial Valuations: We call the valuation function of the retailer monomial if $v(q) = q^\rho$, for a constant $\rho \in (0, 1)$, and $\bar{\kappa} = \infty$ (i.e., the retailer does not have capacity constraints). This valuation function represents a retailer who sells to a mass of consumer with *constant price elasticity*.⁸

PROPOSITION 3. *Suppose F is the uniform distribution. Then $p^*(y, \kappa) = v(y+1) - v(1)$, $0 \leq y \leq \kappa$, satisfies Assumption 4. As implied by Theorem 1, pricing strategies $p_1 = p_2 = p^*$ implement an equilibrium.*

Note that in this case, both suppliers always sell all their supply.

⁸ The constant elasticity demand function is given by $q = ap^\epsilon$ (equivalently $p = (\frac{1}{a}q)^{\frac{1}{\epsilon}}$), where a is a constant and ϵ is the elasticity parameter. Let $v(q) = (\frac{1}{a})^{\frac{1}{\epsilon}} q^{(1+\frac{1}{\epsilon})}$ and $\rho = 1 + \frac{1}{\epsilon}$. For $\epsilon < -1$, we have $0 \leq \rho \leq 1$.

4. Best-Response Strategies

An important step in analyzing the equilibrium is computing the best-response strategies defined by the bi-level mathematical program in Eq. (2). In this section, Proposition 4 presents a rather simple condition that identifies whether a pricing strategy of a supplier is a best-response to the strategy of the other supplier. We obtain this best response condition by simplifying the bi-level program (2). To do so, we use Lemma 1 which provides a mapping from allocation q_1 to payment p_1 . In order to simplify the presentation, we assume that supplier \mathcal{S}_2 uses pricing strategy $p_2 = p^*$; however, our results can be extended to a broad class of pricing strategies.

Let $W(q, \kappa_2)$ be the value that the retailer obtains from the allocation of quantity q from supplier \mathcal{S}_1 when the capacity of supplier \mathcal{S}_2 is equal to κ_2 . Note that if \mathcal{S}_1 allocates capacity q to the retailer, he can buy additional capacity from \mathcal{S}_2 . Namely,

$$W(q, \kappa_2) = \max_{y \leq \kappa_2} \left\{ v(q + y) - p^*(y) \right\} \quad (7)$$

where p^* is defined in Eq. (4).

The following lemma establishes a mapping from allocations to payments via the envelope theorem; see Section 6.3 for more details.

LEMMA 1 (Mapping Payments to Allocations). *Suppose supplier \mathcal{S}_2 uses pricing strategy $p_2 = p^*$ and consider $q_1 : [0, 1] \times [0, 1] \rightarrow [0, 1]$, defined in Eq. (2), that is implemented at equilibrium. Then, for the pricing strategy of supplier \mathcal{S}_1 we have*

$$p_1(q_1(\kappa_1, \kappa_2), \kappa_1) = W(q_1(\kappa_1, \kappa_2), \kappa_2) + \int_{\kappa_2}^1 \frac{\partial W(q, z)}{\partial z} \Big|_{x=q_1(\kappa_1, z)} dz$$

In addition, $p(q_1(\kappa_1, 1), \kappa_1) = W(q(\kappa_1, 1), 1)$.

Note that W is decreasing in κ_2 . Therefore, $p_1(q_1(\kappa_1, \kappa_2), \kappa_1) \leq W(q_1(\kappa_1, \kappa_2), \kappa_2)$. Now let us define ψ as follows:

$$\psi(q, \kappa) = W(q, \kappa) + \frac{F(\kappa)}{f(\kappa)} \frac{\partial W(q, \kappa)}{\partial \kappa} \quad (8)$$

We are now ready to define the best-response condition.

PROPOSITION 4 (The Best-Response Condition). *Allocation function q_1 and corresponding pricing strategy p_1 , defined in Lemma 1, are a best-response to pricing strategy $p_2 = p^*$ if*

$$q_1(\kappa_1, \kappa_2) \in \operatorname{argmax}_{q \leq \kappa_1} \left\{ \psi(q, \kappa_2) \right\}$$

Proof: Using Lemma 1, we obtain the following expression for the expected revenue of \mathcal{S}_1 , denoted by $Rev_1(\kappa_1)$.

$$\begin{aligned}
Rev_1(\kappa_1) &= \int_0^1 p_1(q_1(\kappa_1, y), \kappa_1) f(y) dy \\
&= \int_0^1 \left(W(q_1(\kappa_1, y), y) + \int_y^1 \frac{\partial W(x, z)}{\partial z} \Big|_{x=q_1(\kappa_1, z)} dz \right) f(y) dy \\
&= \int_0^1 \left(W(q_1(\kappa_1, y), y) + \frac{F(y)}{f(y)} \frac{\partial W(x, y)}{\partial y} \Big|_{x=q_1(\kappa_1, y)} \right) f(y) dy \\
&= \int_0^1 \psi(q_1(\kappa_1, y), y) f(y) dy \tag{9}
\end{aligned}$$

The claim follows since supplier \mathcal{S}_1 is maximizing her expected revenue. \square

As we discuss later in Section 6.3, function ψ corresponds to the virtual value (or marginal revenue) of allocation of quantity q of supply to the retailer when the capacity of the other supplier is equal to κ .

5. Proof of Theorem 1

In this section, we prove Theorem 1 when Assumptions 1, 2, 3 hold and p^* satisfies Assumption 4. First, using Lemma 3, we show that if both suppliers use pricing strategy p^* , then allocation q^* , Eq. (5), is implemented at the equilibrium. In other words, $q_1(\kappa_1, \kappa_2)$ and $q_2(\kappa_1, \kappa_2)$ are optimal solutions to the retailer's problem (Eq. (1)). In the second part, using Lemma 5, we show that this allocation function satisfies the best-response strategy conditions given by Proposition 4.

Part I: Allocation at the Equilibrium. Suppose supplier \mathcal{S}_2 uses pricing strategy p^* . To simplify the presentation, we use $p^*(q)$ instead of $p^*(q, \kappa_1 = 1)$. We start with the retailer's problem. Define β as follows:

$$\beta(q) = \operatorname{argmax}_{0 \leq y \leq \min\{1, \bar{\kappa} - q\}} \left\{ v(q + y) - p^*(y) \right\}. \tag{10}$$

Note that using the first order conditions, if $0 < \beta(q) < \min\{1, \bar{\kappa} - q\}$, we have:

$$v'(q + \beta(q)) - \gamma(\beta(q)) = 0 \tag{11}$$

In the appendix, we prove the following properties of β .

LEMMA 2. *For $q \leq \bar{\alpha}$, function $v(q + y) - p^*(y)$ is increasing in $y \in (0, 1)$ and we have $\beta(q) = 1$. For $q \in (\underline{\alpha}, \bar{\alpha})$, we have $\beta(q) > q$. For $q \in (\bar{\alpha}, \bar{\alpha})$, we have $\beta(q) = \alpha(q)$.*

We now present the solution to the retailer's problem in terms of function β . The following is the main technical lemma of this section. The proof is given in the appendix.

LEMMA 3 (Equilibrium Allocation). *Suppose pricing strategy p^* defines a symmetric equilibrium. At the equilibrium, the following allocation is optimal for the retailer:*

$$\begin{aligned} q_1(\kappa_1, \kappa_2) &= \begin{cases} \min\{\beta(\kappa_2), \kappa_1\} & \kappa_1 \leq \kappa_2 \\ \kappa_1 & \kappa_1 > \kappa_2 \end{cases} \\ q_2(\kappa_1, \kappa_2) &= \begin{cases} \kappa_2 & \kappa_1 \leq \kappa_2 \\ \min\{\beta(\kappa_1), \kappa_2\} & \kappa_1 > \kappa_2 \end{cases} \end{aligned} \quad (12)$$

In the appendix, for $\kappa_1 \leq \kappa_2$, we show that $\min\{\beta(\kappa_2), \kappa_1\} = \min\{\alpha(\kappa_2), \kappa_1\}$ which gives rise to the following lemma that corresponds to the allocation part of Theorem 1.

LEMMA 4 (Retailer's Problem). *If both suppliers \mathcal{S}_1 and \mathcal{S}_2 use pricing strategy p^* and p^* satisfies Assumption 4, then allocation q^* will be implemented at the equilibrium.*

Part II: Satisfying the Best-Response Conditions. We now consider the optimality conditions for a best response strategy given by Proposition 4. As the first step, using Assumption 4, we obtain the following:

$$W(q, \kappa_2) = \begin{cases} v(q + \kappa_2) - p^*(\kappa_2) & \kappa_2 < \beta(q) \\ v(q + \beta(q)) - p^*(\beta(q)) & \kappa_2 \geq \beta(q) \end{cases} \quad (13)$$

and

$$\psi(q, \kappa_2) = \begin{cases} v(q + \kappa_2) - p^*(\kappa_2) + \frac{F(\kappa_2)}{f(\kappa_2)} (v'(q + \kappa_2) - \gamma(\kappa_2)) & \kappa_2 < \beta(q) \\ v(q + \beta(q)) - p^*(\beta(q)) & \kappa_2 \geq \beta(q) \end{cases} \quad (14)$$

LEMMA 5 (The Best-Response Condition). *For all capacities κ_1 and κ_2 , under the assumptions of Theorem 1, we have that given κ_2*

$$q_1^*(\kappa_1, \kappa_2) \in \operatorname{argmax}_{q \leq \kappa_1} \left\{ \psi(q, \kappa_2) \right\} \quad (15)$$

Note that due to the symmetry, we get $q_2^*(\kappa_1, \kappa_2) \in \operatorname{argmax}_{q \leq \kappa_2} \left\{ \psi(q, \kappa_1) \right\}$. In the rest of this section, we prove the above lemma.

Proof of Lemma 5: As shown by Lemma 7 in the appendix, β is a decreasing function. We consider the following cases:

1. If $\kappa_2 < \beta(q)$, then $\frac{\partial \psi(q, \kappa_2)}{\partial q} = v'(q + \kappa_2) + \frac{F(\kappa_2)}{f(\kappa_2)} v''(q + \kappa_2)$. Assumption 3 implies that $v(q + \kappa_2) + \frac{F(\kappa_2)}{f(\kappa_2)} v'(q + \kappa_2)$ is increasing in $q \leq \alpha(\kappa_2)$ and takes its maximum at $\alpha(\kappa_2)$. Therefore, $\psi(q, \kappa_2)$ is increasing in $q \in [0, \alpha(\kappa_2))$, takes its maximum at $\alpha(\kappa_2)$, and is decreasing in q afterwards as long as $\kappa_2 < \beta(q)$.

2. If $\kappa_2 > \beta(q)$, then ψ is increasing in q , i.e., $\frac{\partial \psi(q, \kappa_2)}{\partial q} \geq 0$. The reason is that for $\beta(q) < \bar{\kappa} - q$, we have

$$\frac{\partial \psi(q, \kappa_2)}{\partial q} = v'(q + \beta(q)) + \beta'(q) (v'(q + \beta(q)) - \gamma(\beta(q))) = v'(q + \beta(q)) \geq 0.$$

The equality follows from the first order conditions in Eq. (11). Also, if $\beta(q) = \bar{\kappa} - q$, then $\frac{\partial \psi(q, \kappa_2)}{\partial q} = v'(\bar{\kappa}) - (v'(\bar{\kappa}) - \gamma(\bar{\kappa})) = \gamma(\bar{\kappa}) \geq 0$.

3. By construction, for $\kappa_2 \geq \tilde{\alpha}$, we have $\psi(\alpha(\kappa_2)) = \psi(\kappa_2)$: Plugging into Eq. (14), we have

$$\begin{aligned} \psi(\alpha(\kappa_2), \kappa_2) - \psi(\kappa_2, \kappa_2) &= \left(v(\kappa_2 + \alpha(\kappa_2)) - p^*(\kappa_2) + \frac{F(\kappa_2)}{f(\kappa_2)} (v'(\alpha(\kappa_2) + \kappa_2) - \gamma(\kappa_2)) \right) \\ &\quad - \left(v(\kappa_2 + \alpha(\kappa_2)) - p^*(\alpha(\kappa_2)) \right) \\ &= p^*(\alpha(\kappa_2)) - p^*(\kappa_2) + \frac{F(\kappa_2)}{f(\kappa_2)} (v'(\alpha(\kappa_2) + \kappa_2) - \gamma(\kappa_2)) \\ \text{Eq. (3)} \Rightarrow &= -\frac{1}{F(\kappa_2)} \int_{\tilde{\alpha}}^{\kappa_2} F(y) (1 - \alpha'(y)) v'(y + \alpha(y)) dy \\ &\quad + \frac{F(\kappa_2)}{f(\kappa_2)} \left(\frac{f(\kappa_2)}{F^2(\kappa_2)} \int_{\tilde{\alpha}}^{\kappa_2} F(z) (1 - \alpha'(z)) v'(z + \alpha(z)) dz \right) \\ &= 0 \end{aligned}$$

Therefore, if $\kappa_2 > \kappa_1$, ψ is maximized at $\min\{\kappa_1, \alpha(\kappa_2)\}$; otherwise, for $\kappa_1 > \kappa_2$, the maximum is taken at κ_1 . \square

We conclude section with the following remark. The best response condition in Lemma 5 implies that for pricing strategy p^* to implement an equilibrium, we should have $\psi(\alpha(k), \kappa) = \psi(\kappa, \kappa)$. With algebraic manipulation, this equality leads to the following differential equation (the solution of which gives us pricing strategy p^*)

$$p(\kappa) + \frac{F(\kappa)}{f(\kappa)} p'(\kappa) = v(\alpha(\kappa) + \kappa) + \frac{F(\kappa)}{f(\kappa)} v'(\alpha(\kappa) + \kappa) \quad (16)$$

As we discuss later in Section 6.2, the above equation, or its extensions, can be used to find equilibrium pricing strategies in more general settings.

6. Discussions & Extensions

In this section, we first discuss the efficiency of the equilibrium corresponding to pricing strategies p^* . Then, we present an extension of our result with production costs. Finally, we discuss the connection between the best-response condition of Proposition 4 and ideas from mechanism design for optimal auctions.

6.1. Efficiency

In this section, we consider the efficiency of the equilibrium corresponding to pricing strategies p^* . An equilibrium is efficient if it maximizes the (total) social welfare that is the sum of the utilities of the suppliers and the retailer.

In our model, in an efficient equilibrium, either both suppliers sell all their supplies (i.e., $q_1(\kappa_1, \kappa_2) = \kappa_1$ and $q_2(\kappa_1, \kappa_2) = \kappa_2$) or all the demand of the retailer is satisfied (i.e., $q_1(\kappa_1, \kappa_2) + q_2(\kappa_1, \kappa_2) = \bar{\kappa}$). Therefore, by Eq. (5), we observe that the equilibrium corresponding to pricing strategies p^* is efficient if $\alpha(\kappa) = \bar{\kappa} - k$. By this observation, and plugging $\alpha(\kappa) = \bar{\kappa} - k$ into Eq. (3), we obtain the following proposition.

PROPOSITION 5 (Efficient Equilibrium). *Suppose Assumptions 1, 2, and 4 hold and in addition, $v(\kappa + y) + \frac{F(\kappa)}{f(\kappa)}v'(\kappa + y)$ is increasing in $y \in (0, \min\{1, \bar{\kappa} - \kappa\})$ for $\kappa \in [0, 1]$. Then, $\alpha(\kappa) = \bar{\kappa} - k$ and*

$$\gamma(y) = \begin{cases} v'(\bar{\kappa}) & 0 \leq y \leq \frac{\bar{\kappa}}{2} \\ v'(\bar{\kappa}) \left(1 - \frac{2f(y)}{F^2(y)} \int_{\bar{\alpha}}^y F(z) dz\right) & \frac{\bar{\kappa}}{2} < y \leq 1 \end{cases} \quad (17)$$

Furthermore, the equilibrium corresponding to pricing strategies $p_1 = p_2 = p^*$ is efficient.

Recall that Assumption 3 requires that $v(\kappa + y) + \frac{F(\kappa)}{f(\kappa)}v'(\kappa + y)$ to be quasi-concave in y . The stronger monotonicity assumption in the above proposition, by the definition of α , implies that $\alpha(\kappa) = \bar{\kappa} - k$. For instance, this assumption holds for linear valuations.

6.2. Marginal Production Cost

We now consider the case when the marginal production cost is equal to $c > 0$. More precisely, the utility of a supplier from selling q unit of supply at price p to the retailer is equal to $p - cq$. We start with the following generalizations of our previous definitions. Define

$$\alpha_c(\kappa) = \operatorname{argmax}_{0 \leq y \leq \min\{1, \bar{\kappa} - \kappa\}} \left\{ v(\kappa + y) + \frac{F(\kappa)}{f(\kappa)}v'(\kappa + y) - cy \right\} \quad (18)$$

and let marginal prices be equal to

$$\gamma_c(y) = \begin{cases} v'(y + \alpha_c^{-1}(y)) & 0 \leq y \leq \tilde{\alpha}_c \\ v'(y + \alpha_c(y)) + \frac{f(y)}{F(y)}(c(y - \alpha_c(y))) & \tilde{\alpha}_c < y \leq 1 \\ -\frac{f(y)}{F^2(y)} \left(\int_{\tilde{\alpha}_c}^y \left(F(z)(1 - \alpha'_c(z))v'(z + \alpha_c(z)) + (c(z - \alpha_c(z)))f(z) \right) dz \right) & \end{cases}$$

We obtain the following generalization of Theorem 1. The proof is omitted due to its similarity to Theorem 1.

PROPOSITION 6 (**Pricing Strategies with Production Cost**). *Define p_c^* as follows:*

$$p_c^*(q, \kappa) = \begin{cases} \int_0^x \gamma_c(y) dy & q \leq \kappa \\ \infty & q > \kappa \end{cases} \quad (19)$$

Suppose Assumptions 1, 2, and 3 hold and p_c^* satisfies Assumption 4. The pricing strategies $p_1 = p_2 = p_c^*$ define a symmetric equilibrium such that the retailer purchases quantities $q_1^*(\kappa_1, \kappa_2)$ and $q_2^*(\kappa_1, \kappa_2)$ from suppliers \mathcal{S}_1 and \mathcal{S}_2 , respectively, where

$$q_1^*(\kappa_1, \kappa_2) = \begin{cases} \min\{\alpha_c(\kappa_2), \kappa_1\} & \kappa_1 \leq \kappa_2 \\ \kappa_1 & \kappa_1 > \kappa_2 \end{cases} \quad (20)$$

$$q_2^*(\kappa_1, \kappa_2) = \begin{cases} \kappa_2 & \kappa_1 \leq \kappa_2 \\ \min\{\alpha_c(\kappa_1), \kappa_2\} & \kappa_1 > \kappa_2 \end{cases}$$

For instance, for the example discussed in Section 3.1, with linear valuations and uniform distributions, we have $\alpha_c(x) = \alpha(x) = 1 - \kappa$, for $c \leq 1$. Therefore, we get

$$\gamma_c(y) = \begin{cases} 1 & 0 \leq \frac{1}{2} \\ \frac{1-c}{4y^2} + c & \frac{1}{2} < y \leq 1 \end{cases} \quad p_c^*(q) = \begin{cases} q & 0 \leq \frac{1}{2} \\ (1-c)(1 - \frac{1}{4q}) + cq & \frac{1}{2} < q \leq 1 \end{cases}$$

Note that as marginal cost c increases, γ_c gets closer to c and the marginal profit of each supplier decreases.

The proof of the proposition above is very similar to Theorem 1 and is omitted; instead, we present a sketch of the proof. First observe that the results in Section 4 (Lemma 1 and Proposition 4) still hold. Using Eq. (9), we obtain the expression below for the expected profit of a supplier (with production cost).

$$\int_0^1 (p_1(q_1(\kappa_1, y), \kappa_1) - cq_1(\kappa_1, y)) f(y) dy = \int_0^1 \psi_c(q_1(\kappa_1, y), y) f(y) dy \quad (21)$$

where

$$\psi_c(x, \kappa) = W(x, \kappa) + \frac{F(\kappa)}{f(\kappa)} \frac{\partial W(x, \kappa)}{\partial \kappa} - cx = \psi(x, \kappa) - cx$$

Subsequently, the definition of α (see Definition 3) is extended to Eq. (18). Similar to Eq. (16), the differential equation for the pricing strategies is given by:

$$p(\kappa) + \frac{F(\kappa)}{f(\kappa)} p'(\kappa) = v(\alpha_c(\kappa) + \kappa) + \frac{F(\kappa)}{f(\kappa)} v'(\alpha_c(\kappa) + \kappa) + c(\kappa - \alpha_c(\kappa))$$

Solving the equation above, we obtain Eq. (21).

We conclude this section with the following corollary.

COROLLARY 4. *As the production cost increases, the suppliers sell smaller quantities to the retailer.*

The result holds because α_c is decreasing in c . This follows from Assumption 3 using the first order conditions.

6.3. Revisiting Our Methodology

In this section, we explain how our techniques for finding the best response strategy can be interpreted as a reduction of the equilibrium analysis to a mechanism design problem. In other words, the problem of finding the best response strategy, under some conditions, can be solved using the ideas developed for optimal auction design.

To show this, we first consider the hypothetical case where the retailer knows the capacity of the suppliers, i.e., κ_1 and κ_2 . Fixing the strategy of \mathcal{S}_2 , supplier \mathcal{S}_1 faces the following mechanism design problem: the retailer is privately informed of κ_2 , which corresponds to the capacity of \mathcal{S}_2 . The valuation function of the retailer for the supply of \mathcal{S}_1 is given by W (Eq. (7)). The goal of \mathcal{S}_1 is to find an allocation and pricing mechanism that maximizes her expected revenue.

The revelation principle (cf. Gibbard (1973)) allows us to focus only on direct (incentive compatible) mechanisms.⁹ A mechanism \mathcal{M} is defined with an allocation rule $q: \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$ and pricing scheme $p: \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}$. The mechanism asks the retailer to report the capacity of \mathcal{S}_2 . An allocation rule $q(z, \kappa_1)$ determines how much capacity should be allocated to the retailer given that he reports z and \mathcal{S}_1 's capacity is equal to κ_1 . The pricing scheme $p(q, \kappa_1)$ determines how much should be charged for allocation q . A mechanism is called *incentive compatible* if the utility of the retailer is maximized by reporting truthfully (i.e., $z = \kappa_2$) to the mechanism. Then, using techniques similar to Myerson (1981), Milgrom and Segal (2002), one can establish results resembling Lemma 1 and Proposition 4 that map the payment of any incentive compatible mechanism to its allocations.

We now revisit the hypothetical scenario where the retailer knows the capacities and complete the reduction. Note that if the price schedule each supplier offers uniquely maps to her capacity, then the retailer learns the capacity of each supplier. Moreover, the revelation principle still holds even though \mathcal{S}_1 has private information. This follows from Maskin and Tirole (1990), because the utility of the retailer depends only on the allocation he receives (private value setting) and is quasi-linear in the payments.

7. Conclusions

Non-linear pricing competition with asymmetric or incomplete information, in the contexts of supply chains or revenue management, is not yet well-understood mainly due to challenges in analyzing their equilibria. In this paper, we studied a setting where the capacities of the suppliers are private information. We observe that quasi-concave pricing strategies emerge at the equilibrium even when no restriction is imposed on the pricing strategies.

⁹ The revelation principle implies that any mechanism can be converted to a direct mechanism.

To analyze the equilibrium, we presented an approach that allows us to think about equilibrium strategies as allocation rules rather than more complicated pricing functions. We believe that the ideas developed in this work can be extended to other problems, for instance where the cost, quality, or reliability of the suppliers are private information.

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Appendix A: Proofs

A.1. Appendix to Section 2

We start by formally defining the inverse function of α , denoted by α^{-1} . First observe that since v is strictly increasing, we have $\alpha(0) = 1$. We show that α has the following shape: it is constant in an interval, denoted by $[0, \underline{\alpha}]$ (possibly $\underline{\alpha} = 0$), then it is decreasing, and then constant again in an interval $[\bar{\alpha}, 1]$ (possibly $\bar{\alpha} = 1$). More precisely, define $\underline{\alpha} = \max_{\kappa} \{\kappa | \alpha(\kappa) = \alpha(0)\}$ and $\bar{\alpha} = \min_{\kappa} \{\kappa \in \arg\min_{\alpha \leq z \leq 1} \{\alpha(z)\}\}$. Furthermore, define $\tilde{\alpha}$ as follows: if there exists $\kappa \in [\underline{\alpha}, \bar{\alpha}]$ such that $\alpha(\kappa) = \kappa$, let $\tilde{\alpha} = \kappa$; otherwise let $\tilde{\alpha} = 1$.

LEMMA 6. $\alpha(\kappa)$ is strictly decreasing in $\kappa \in (\underline{\alpha}, \bar{\alpha})$.

Proof: Suppose $\alpha(\kappa)$, $\kappa \in (0, 1)$, is positive. Consider $0 < \delta < \alpha(\kappa)$. We show that $\alpha(\kappa + \delta) < \alpha(\kappa) - \delta$. Define $h(y, \kappa) = v(\kappa + y) + \frac{F(\kappa)}{f(\kappa)}v'(\kappa + y)$. We have

$$\begin{aligned} \left. \frac{\partial h(\kappa + \delta, y)}{\partial y} \right|_{y=\alpha(\kappa)-\delta} &= v'((\kappa + \delta) + (\alpha(\kappa) - \delta)) + \frac{F(\kappa + \delta)}{f(\kappa + \delta)}(v''(\kappa + \delta) + (\alpha(\kappa) - \delta)) \\ &= v'(\kappa + \alpha(\kappa)) + \frac{F(\kappa + \delta)}{f(\kappa + \delta)}v''(\kappa + \alpha(\kappa)) \\ &\leq v'(\kappa + \alpha(\kappa)) + \frac{F(\kappa)}{f(\kappa)}v''(\kappa + \alpha(\kappa)) = \left. \frac{\partial h(\kappa, y)}{\partial y} \right|_{y=\alpha(\kappa)} = 0 \end{aligned}$$

The third inequality follows from Assumption 2; since $F(\kappa)$ is log-concave, $\frac{F(\kappa + \delta)}{f(\kappa + \delta)} \geq \frac{F(\kappa)}{f(\kappa)}$. Since $h(\kappa + \delta, y)$ is non-increasing at $\alpha(\kappa) - \delta$, we have $\alpha(\kappa + \delta) < \alpha(\kappa) - \delta$. \square

Note that by the above lemma, the inverse of α , denoted by α^{-1} , is well-defined over $[\underline{\alpha}, \bar{\alpha}]$. With slight abuse of notation, we define $\alpha^{-1}(\kappa) = 1$, $\kappa \in [0, \underline{\alpha}]$.

A.2. Proofs from Section 3.1

Proof of Proposition 1: Note that Assumption 2 holds immediately for the uniform distribution. Also, we have

$$v(\kappa + y) + \frac{F(\kappa)}{f(\kappa)}v'(\kappa + y) = (\kappa + y) - \rho(\kappa + y)^2 + \kappa(1 - 2\rho(\kappa + y))$$

and the first and the second derivatives with respect to y are respectively equal to $1 - 2\rho y - 4\rho\kappa$ and -2ρ . Note that the function above is (strictly) concave and hence (strictly) quasi-concave. Therefore, Assumption 3 holds.

Using the first and the second order derivatives, for $\rho \leq \frac{1}{4}$, it is easy to see that $\underline{\alpha} = \frac{1-2\rho}{4\rho}$, $\bar{\alpha} = \min\left\{\frac{1}{4\rho}, 1\right\}$, and

$$\alpha(\kappa) = \begin{cases} 1 & 0 \leq \kappa \leq \frac{1-2\rho}{4\rho} \\ \frac{1-4\rho\kappa}{2\rho} & \frac{1-2\rho}{4\rho} \leq \kappa \leq \bar{\alpha} \\ 0 & \bar{\alpha} \leq \kappa \leq 1 \end{cases}$$

For $\rho \leq \frac{1}{6}$, $\alpha(\kappa) \geq \kappa$ for $\kappa \in [0, 1]$. For $\frac{1}{6} \leq \rho \leq \frac{1}{4}$, we have $\tilde{\alpha} = \frac{1}{6\rho}$. In addition, $\alpha^{-1}(\kappa) = \frac{1-2\rho\kappa}{4\rho}$, for $\kappa \in [\underline{\alpha}, \bar{\alpha}]$.

We now consider the following cases and find γ .

- By Eq. (3), for $y \leq \frac{1-2\rho}{4\rho}$, we get $\gamma(y) = v'(y+1) = 1 - 2\rho(y+1) = 1 - 2\rho - 2\rho y$.
- For $\frac{1-2\rho}{4\rho} \leq y \leq \min\left\{\frac{1}{6\rho}, 1\right\}$, we have

$$\gamma(y) = v'\left(y + \frac{1-2\rho y}{4\rho}\right) = v'\left(\frac{1+2\rho y}{4\rho}\right) = 1 - (1+2\rho y)/2 = \frac{1}{2} - \rho y.$$

- We now consider $y \in \left[\frac{1}{6\rho}, \min\left\{\frac{1}{4\rho}, 1\right\}\right]$. We have

$$\gamma(y) = v'\left(y + \frac{1-4\rho y}{2\rho}\right) - \frac{1}{y^2} \int_{\tilde{\alpha}}^y 3zv'\left(z + \frac{1-4\rho z}{2\rho}\right) dz.$$

Note that $v'\left(y + \frac{1-4\rho y}{2\rho}\right) = v'\left(\frac{1}{2\rho} - y\right) = 1 - 2\rho\left(\frac{1}{2\rho} - y\right) = 1 - 1 + 2\rho y = 2\rho y$ and $\int_{\tilde{\alpha}}^y 3zv'\left(z + \frac{1-4\rho z}{2\rho}\right) dz = \int_{\tilde{\alpha}}^y 6\rho z^2 dz = 2\rho(y^3 - \tilde{\alpha}^3)$. Hence, for $\frac{1}{6\rho} \leq y \leq 1$, we get

$$\gamma(y) = 2\rho\left(y - \frac{1}{y^2}(y^3 - \tilde{\alpha}^3)\right) = 2\rho\left(\frac{\tilde{\alpha}^3}{y^2}\right) = \frac{1}{108\rho^2 y^2}.$$

We now verify Assumption 4. For $y \leq \frac{1-2\rho}{4\rho}$, we get $\gamma'(y) = -2\rho$ and $v''(x+y) - \gamma'(y) = 0$. For $\frac{1-2\rho}{4\rho} \leq y \leq \min\left\{\frac{1}{6\rho}, 1\right\}$, we have $\gamma'(y) = -\rho$ and $v''(x+y) - \gamma'(y) = -\rho$. Hence, $v(x+y) - p^*(y)$ is concave. We now consider $y \in \left[\frac{1}{6\rho}, \min\left\{\frac{1}{4\rho}, 1\right\}\right]$. In this case $\gamma'(y) = \frac{-1}{54\rho^2 y^3}$. We now show that $v(x+y) - p^*(y)$ is increasing in $y \in \left[\frac{1}{6\rho}, \min\left\{\frac{1}{4\rho}, 1\right\}\right]$. Note that in this interval, the derivative of $v'(x+y) - \gamma(y) = 1 - 2\rho(x+y) - \frac{1}{108\rho^2 y^2}$ with respect to y is equal to $-2 + \frac{1}{54\rho^2 y^3} \leq -2 + \frac{1}{54\rho^2} \leq -1$. Therefore, $v'(x+y) - \gamma(y)$ is (strictly) decreasing in $y \in \left[\frac{1}{6\rho}, \min\left\{\frac{1}{4\rho}, 1\right\}\right]$. Since $y \geq \frac{1}{6\rho}$ we have

$$v'(x+y) - \gamma(y) \leq v'\left(x + \frac{1}{6\rho}\right) - \gamma\left(\frac{1}{6\rho}\right) \leq 1 - 2\rho x - \frac{1}{3} - \frac{1}{108 \times \frac{1}{36}} = \frac{1}{3} - 2\rho x$$

If $v'(x+y) - p^*(y)$ is decreasing in y , at the end of interval $y \in \left[\frac{1-2\rho}{4\rho}, \frac{1}{6\rho}\right]$, then it is decreasing afterwards (and hence quasi-concave). On the other hand, if $\frac{1}{3} - 2\rho x > 0$, then we have that $v'(x+y) - p^*(y, \cdot)$ is increasing in y , for $y \in \left[\frac{1-2\rho}{4\rho}, \frac{1}{6\rho}\right]$, then it takes its maximum in $\left[\frac{1}{6\rho}, 1\right]$.

□

Proof of Proposition 2: First, we observe that for linear valuations $\alpha(\kappa) = \bar{\kappa} - \kappa$ since $v(\kappa+y) + \frac{F(\kappa)}{f(\kappa)}v'(\kappa+y) = \kappa + y + \frac{F(\kappa)}{f(\kappa)}$ is increasing in $y \in [0, \bar{\kappa} - \kappa]$.

We now show that p^* is strictly increasing or equivalently γ is positive. The claim is trivial for $x \leq \frac{\bar{\kappa}}{2}$. Now consider, $x \geq \frac{\bar{\kappa}}{2}$. If F is log-concave, for $y \leq x$, we have $\frac{f(x)F(y)}{F(x)} \leq f(y)$. Hence, we get

$$\frac{1}{F(x)} \int_{\frac{\bar{\kappa}}{2}}^x \frac{f(x)F(y)}{F(x)} dy \leq \frac{1}{F(x)} \int_{\frac{\bar{\kappa}}{2}}^x f(y) dy = \frac{F(x) - F(\frac{\bar{\kappa}}{2})}{F(x)} = 1 - \frac{F(\frac{\bar{\kappa}}{2})}{F(x)}$$

Thus, $\gamma(x) \geq 1 - 2 \left(1 - \frac{F(\frac{\bar{\kappa}}{2})}{F(x)}\right)$ which is positive if $2F(\frac{\bar{\kappa}}{2}) \geq 1$.

We now show that $v(x+z) - p(x, \kappa)$ is increasing in $x \in [0, \bar{\kappa} - z]$ which implies the quasi-concavity property. Note that for linear valuation we have $v'(x+z) - \frac{\partial p(x, \kappa)}{\partial x} = 1 - \gamma(x)$. The claim follows since $\gamma(x) \leq 1$. \square

Proof of Proposition 3: For monomial valuations, we observe that $v(\kappa + y) + \frac{F(\kappa)}{f(\kappa)}v'(\kappa + y)$ is always increasing in y if $\kappa \geq (1 - \rho)\frac{F(\kappa)}{f(\kappa)}$, $\kappa \in [0, 1]$ because $\frac{\partial}{\partial y} \left((\kappa + y)^\rho + \rho \frac{F(\kappa)}{f(\kappa)} (\kappa + y)^{\rho-1} \right) = \rho(\kappa + y)^{\rho-2} \left(y + \kappa - (1 - \rho)\frac{F(\kappa)}{f(\kappa)} \right)$. This inequality holds for the uniform distribution. Therefore, $\alpha(\kappa) = 1$, for all κ , and by definition $\underline{\alpha} = \bar{\alpha} = \tilde{\alpha} = 1$. Now by Equations (3) and (4), we get $\gamma(y) = v'(y+1)$ and $p^*(y, \cdot) = v(y+1) - v(0)$, $0 \leq y \leq 1$.

Note that p^* satisfies Assumption 4. It is concave in y . Also, we have

$$\frac{\partial^2}{\partial y^2} (v(x+y) - v(1+y)) = \rho(\rho-1) \left((x+y)^{\rho-2} - (1+y)^{\rho-2} \right).$$

The inequality holds because for $\rho \leq 1$, $(\rho-1) \leq 0$ and $(x+y)^{\rho-2} \geq (1+y)^{\rho-2}$. \square

A.3. Proofs from Section 4

Proof of Lemma 1: Fixing κ_1 , define $g(x, \kappa_2) = W(x, \kappa_2) - p_1(x, \kappa_1)$. Also, let $U_1(\kappa_2)$ denote the utility the retailer obtains from \mathcal{S}_1 (with capacity κ_1). Observe that

$$U_1(\kappa_2) = \max_{x \leq \kappa_1} \{g(x, \kappa_2)\} = g(q_1(\kappa_1, \kappa_2), \kappa_2)$$

The last equality follows from the definition of q_1 since it is the solution of the retailer's problem Eq (1).

Also, note that $W(x, \kappa_2)$ is absolutely continuous in κ_2 for all x . In addition, $\frac{\partial W(x, \kappa)}{\partial \kappa} \Big|_{\kappa=\kappa_2}$ is bounded for all x and almost all κ_2 . Hence, g and U_1 are absolutely continuous in κ_2 . Therefore, U_1 is differentiable almost everywhere and by Milgrom and Segal (2002), Theorem 2, we have

$$U_1(\kappa_2) = U_1(1) - \int_{\kappa_2}^1 \frac{\partial g(x, k)}{\partial \kappa} \Big|_{x=q_1(\kappa_1, \kappa_2)} d\kappa = U_1(1) - \int_{\kappa_2}^1 \frac{\partial W(x, k)}{\partial \kappa} \Big|_{x=q_1(\kappa_1, \kappa_2)} d\kappa \quad (22)$$

Also, by the definition of U_1 , we have $U_1(\kappa_2) = W(q_1(\kappa_1, \kappa_2), \kappa_2) - p_1(q_1(\kappa_1, \kappa_2), \kappa_1)$. Therefore,

$$p_1(q_1(\kappa_1, \kappa_2), \kappa_1) = U_1(1) + W(q_1(\kappa_1, \kappa_2), \kappa_2) + \int_{\kappa_2}^1 \frac{\partial g(x, k)}{\partial \kappa} \Big|_{x=q_1(\kappa_1, \kappa_2)} d\kappa \quad (23)$$

Finally, using contradiction, we now show that $U_1(1) = 0$, i.e., $p_1(q_1(\kappa_1, 1), \kappa_1) = W(q_1(\kappa_1, 1), 1)$. Let $\Delta = W(q_1(\kappa_1, 1), 1) - p_1(q_1(\kappa_1, 1), \kappa_1)$. Consider pricing strategy \tilde{p}_1 such that $\tilde{p}_1(x, \kappa) = p_1(x, \kappa) + \Delta$. Note that the utility of the retailer still remains non-negative. Also, observe that the first order conditions have not changed. Therefore, q_1 and q_2 remain the same. However, the revenue of the supplier has increased since $\tilde{p}_1 > p_1$. This is in contradiction with p_1 being a best-response (revenue-maximizing) strategy. This contradiction completes the proof. \square

A.4. Proofs from Section 5

We use the following lemma in the proofs of Section 5.

LEMMA 7. *Function $\beta(q)$ is decreasing in $q \in (0, 1)$, for $\kappa \in [0, 1]$. Also, for $0 < q_1 \leq q_2 < 1$, if $q_1 \leq \beta(q_2)$, then $q_2 \leq \beta(q_1)$.*

Proof of Lemma 7: Consider $0 < q_1 < q_2 < 1$. We observe that

$$v'(q_1 + \beta(q_2)) - \gamma(\beta(q_2)) \geq v'(q_2 + \beta(q_2)) - \gamma(\beta(q_2)) \geq 0$$

The first inequality follows from the concavity of v and the second one follows from Eq. (11). Since $v(q + \cdot) - p(\cdot)$ is (strictly) quasi-concave, we have $\beta(q_2) < \beta(q_1)$.

We now prove the second part of the lemma. If $\underline{\alpha} \leq q \leq \tilde{\alpha}$, then $q_2 \leq \beta(q_2)$. Since β is decreasing, we have $q_2 \leq \beta(q_1)$. Otherwise, if $\tilde{\alpha} \leq q_2 \leq \bar{\alpha}$, by the quasi-concavity property of pricing strategies (Assumption 4), it suffices to show that $v'(q_1 + q_2) - \gamma(q_2) \geq 0$. We observe that

$$v'(q_1 + q_2) - \gamma(q_2) \geq v'(q_2 + \beta(q_2)) - \gamma(q_2) = v'(q_2 + \alpha(q_2)) - \gamma(q_2) \geq 0$$

The first inequality follows from $q_1 \leq \beta(q_2)$ and concavity of v . The second equality is implied by Lemma 2 and the last inequity, for $q_2 \geq \tilde{\alpha}$, follows from Eq. (5). \square

Proof of Lemma 2: By Eq. (5), for $q \leq \bar{\alpha}$, we get $p^*(q) = \int_0^q \gamma(y)dy = \int_0^q v'(y+1)dy = v(q+1) - v(1)$. Therefore, for any $x \leq 1$, we have $v(q+x) - p^*(x) = v(q+x) - (v(q+1) - v(1)) \geq v(q) + v(1) - v(q+1) \geq 0$. The first inequality holds since v is increasing and the second follows from concavity of v .

For $q \in (\underline{\alpha}, \tilde{\alpha})$, we have

$$\begin{aligned} & v'(q + \alpha^{-1}(q)) - \gamma(\alpha^{-1}(q)) \\ &= v'(q + \alpha^{-1}(q)) - \left(v'(\alpha^{-1}(q) + \alpha(\alpha^{-1}(q))) - \frac{f(q)}{F^2(q)} \int_{\tilde{\alpha}}^q F(y)(1 - \alpha'(y))v'(y + \alpha(y))dy \right) \\ &= \frac{f(q)}{F^2(q)} \int_{\tilde{\alpha}}^q F(y)(1 - \alpha'(y))v'(y + \alpha(y)) \geq 0 \end{aligned}$$

Hence, by Assumption 4, in this case we have $\beta(q) \geq \alpha^{-1}(q) \geq \tilde{\alpha} > q$.

We now consider $q \in (\tilde{\alpha}, \bar{\alpha})$. Since $\alpha(q) \leq \tilde{\alpha} \leq q$, we get

$$v'(q + \alpha(q)) - \gamma(\alpha(q)) = v'(q + \alpha(q)) - v'(\alpha(q) + \alpha^{-1}(\alpha(q))) = v'(q + \alpha(q)) - v'(\alpha(q) + q) = 0. \quad (24)$$

Therefore, by Assumption 4, in this case we have $\beta(q) = \alpha(q)$. \square

Proof of Lemma 3: Let q_1 and q_2 denote the amount of the good that the retailer purchases from \mathcal{S}_1 and \mathcal{S}_2 . Hence, the utility of the retailer is equal to $v(q_1 + q_2) - p^*(q_1) - p^*(q_2)$.

If $q_1 \leq \bar{\alpha}$, then by Lemma 2, we have $q_2 = \kappa_2$. By the quasi-concavity of p^* (Assumption 4), we have $q_1 = \min\{\kappa_1, \beta(\kappa_2)\}$. A similar argument holds when $q_2 \leq \bar{\alpha}$.

We now consider the case where q_1 and q_2 are at least equal to $\bar{\alpha}$.

Due to the quasi-concavity property, fixing q_1 , we observe that if $\kappa_2 \geq \beta(q_1)$, we have $q_2 = \beta(q_1)$; otherwise, we have $q_2 = \kappa_2$. Similarly, if $\kappa_1 \geq \beta(q_2)$, we have $q_1 = \beta(q_2)$; otherwise, we have $q_1 = \kappa_1$.

Without loss of generality, assume $\kappa_1 < \kappa_2$. We consider the following cases:

1. $q_1 = \kappa_1$ and $q_2 = \kappa_2$: By the above argument, this implies that $\kappa_1 < \beta(\kappa_2)$ and $\kappa_2 < \beta(\kappa_1)$.
2. $q_1 = \kappa_1$ and $q_2 = \beta(\kappa_1)$: This implies that $\kappa_1 < \beta(\kappa_2)$ and $\kappa_2 > \beta(\kappa_1)$. Note that the first two cases are exclusive.
3. $q_1 = \beta(\kappa_2)$ and $q_2 = \kappa_2$: This implies that $\kappa_1 > \beta(\kappa_2)$ and $\kappa_2 < \beta(\kappa_1)$. Recall that we assumed $\kappa_2 < \kappa_1$. If $\kappa_2 < \beta(\kappa_1)$, by Lemma 7, we have $\kappa_1 \leq \beta(\kappa_2) \leq \beta(\kappa_2)$. This contradiction eliminates this case.
4. $q_1 = \beta(q_2)$ and $q_2 = \beta(q_1)$: We consider the following cases
 - (a) If $q_1 \geq q_2$, then the utility of the retailer can be written as $v(q + \beta(q)) - p^*(q) - p^*(\beta(q))$, where $q > \beta(q)$. The derivative of the utility of the retailer with respect to q is equal to

$$v'(q + \beta(q)) - \gamma(q) + \beta'(z) \Big|_{z=y} \left(v'(q + \beta(q)) - \gamma(\beta(q)) \right) = \left(1 + \beta'(z) \Big|_{z=y} \right) \left(v'(q + \beta(q)) - \gamma(\beta(q)) \right)$$

We now prove that the expression above is equal to 0. Note that by Lemma 2, $q > \beta(q)$ implies that $q \geq \tilde{\alpha}$. Therefore, $\alpha(q) = \beta(q)$. The claim follows from Eq. (24).

Since expression (25) is non-negative, the utility of the retailer is increasing in y . Therefore, it is maximized at $y = \kappa_1$. Note that this reduces the forth case either to the first or to the second case.

- (b) Suppose $q_1 < q_2$. Note that $q_1 \leq q_2 \leq \kappa_2$ and $q_2 \leq \kappa_2 \leq \kappa_1$ — recall that we assumed $\kappa_1 \geq \kappa_2$. Therefore, the retailer could have purchased q_1 from \mathcal{S}_2 and q_2 and obtain the same utility. Therefore, this case will be reduced to the previous one.

□

Proof of Lemma 4: Note that it suffices to only consider the case when $\kappa_1 \leq \kappa_2$. The proof follows from the following observations:

- If $\alpha(\kappa_2) \leq \kappa_1 \leq \kappa_2$, then $\beta(\kappa_2) = \alpha(\kappa_2)$. The reason is $y = \alpha(\kappa)$, $\kappa > \tilde{\alpha}$, satisfies the first order conditions given by Eq. (11), i.e., $v'(\kappa + \alpha(\kappa)) = \gamma(\alpha(\kappa))$.
- If $\kappa_1 < \alpha(\kappa_2) \leq \kappa_2$, then $\beta(\kappa_2) \geq \kappa_1$ and $\alpha(\kappa_2) \geq \kappa_1$, i.e., $\min\{\beta(\kappa_2), \kappa_1\} = \min\{\alpha(\kappa_2), \kappa_1\} = \kappa_1$. Observe that with a similar argument as the previous case, $v(\kappa + y) - p^*(y)$, for $\kappa > \tilde{\alpha}$, is increasing in $y \in [0, \alpha(\kappa)]$, takes its maximum at $\alpha(\kappa)$, and is decreasing afterwards.
- If $\kappa_1 < \kappa_2 \leq \alpha(\kappa_2)$, then $\beta(\kappa_2) \geq \kappa_2$. The reason is that $v(\kappa + y) - p^*(y)$ is increasing at $y = \kappa_2$, $\kappa_2 \leq \tilde{\alpha}$. We observe that $v'(\kappa_2 + \kappa_2) - p^*(\kappa_2) = v'(\kappa_2 + \kappa_2) - v'(\kappa_2 + \alpha^{-1}(\kappa_2)) \geq 0$. The last inequality holds because v is concave and $\alpha^{-1}(\kappa_2) \geq \kappa_2$ — recall that by Lemma 6 α is decreasing.

□

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