The Calibration and Performance of a Non-homothetic CDE Demand System for CGE Models

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BY Y.-H. HENRY CHEN[a]

In computable general equilibrium modeling, whether the simulation results are consistent with a set of valid own-price and income demand elasticities that are observed empirically remains a key challenge in many modeling exercises, since functional forms that are not fully flexible can only allow a limited subset of elasticities. While not fully flexible, the Constant Difference of Elasticities (CDE) demand system has enough free parameters to match own-price and income elasticities in some cases, leading to its adoption by some models since the 1990s. However, perhaps due to complexities of the system, the applications of CDE demand in other models are less common. Furthermore, how well the system can represent the given elasticities is rarely discussed or examined in the existing literature. This study aims to fill this gap by revisiting calibration strategies for the CDE demand system and exploring conditions where the calibrated elasticities of the system can better match a set of valid target elasticities. Results show that the calibrated elasticities can be matched to the target ones more precisely if the sectoral expenditure shares are lower, the target own-price demand elasticities are lower, and target income demand elasticities are relatively higher. The study also incorporates a CDE demand into the GTAPinGAMS model and verifies that for the revised model with a CDE demand system, the model can successfully replicate the calibrated elasticities under various price and income shocks.

JEL codes: C6, C8, D5, R1

Keywords: Computable general equilibrium modeling; Constant difference of elasticities; Demand system flexibility; Calibration.

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1. Introduction

In Computable General Equilibrium (CGE) modeling, price and income elasticities of demand are crucial in determining the sectoral growth pattern and economic impacts of various policies (Hertel, 2012). This suggests the widely used Constant Elasticity of Substitution (CES) utility function (Sancho, 2009; Annabi *et al.*, 2006; Elsenburg, 2003), is likely unsatisfactory due to its unitary income elasticities of demand, and relative price inflexibility. Indeed, in a single-nest CES setting, after applying the Cournot aggregation, the sectoral expenditure shares will fully determine the variation in own-price elasticities of demand.

To capture the observed non-homothetic preferences with income elasticities of demand diverging from unity, one approach is to use the Linear Expenditure System (LES) derived from Stone-Geary preferences (Geary, 1950; Stone, 1954). The LES system can be calibrated to income elasticities of demand compatible to a valid demand system, although it only allows for calibration to a single price elasticity of demand. In addition, with a special multi-nest structure, the calibrated own-price elasticities of demand can be matched perfectly to any valid elasticities (Perroni and Rutherford, 1995).[1] The shortcoming of LES, however, is that due to constant marginal budget shares with respect to income, the limit property of LES is still homotheticity, and therefore the underlying income elasticities of demand will approach one as income grows and subsistence expenditures dwindle.

An alternative option to model non-homotheticity is to utilize the Constant Difference of Elasticities (CDE) demand system proposed by Hanoch (1975). With implicit additivity, a N-commodity CDE demand system has N expansion parameters and N substitution parameters to achieve a more general functional form than the single nest CES case. The N expansion parameters make it possible to incorporate various income elasticities of demand across commodities/sectors, and the income elasticities will remain at their given levels as income changes ("commodity" and "sector" are used interchangeably in this study). On the other hand, compared to a single-nest CES setting, the N substitution parameters allow modelers to come up with a somewhat better match with target own-price demand elasticities. This led Hertel et al. (1991) to propose the use of the CDE functional form in CGE models as a means of bridging the gap between fully flexible forms and the restrictive, LES/CES functions.

One caveat of CDE applications, paradoxically, comes from the relative stability of each income elasticity regardless of income levels. Specifically, if a good is a luxury it remains a luxury (Yu et al., 2003). While this limitation might not severely contradict empirical evidence for developed countries, existing studies have found that, for instance, income elasticities of some food items in developing countries

[1] While Perroni and Rutherford (1995) focuses on homothetic preferences, it points out that the multi-nest strategy achieving a perfect match in own-price elasticities calibration also works for non-homothetic preferences.

tend to decrease as income grows (Haque, 2005; Chern *et al*., 2003). In some cases, economic growth may turn luxury goods into necessities (Zhou *et al.*, 2012). One strategy to address this limitation is to recalibrate CDE parameters over time so that target income demand elasticities could be adjusted based on a given projection for economic development (Anderson and Strutt (2012); Woltjer et al. (2014)). With this treatment within-period welfare comparison can be done when the CDE parameters remain unchanged. However, between-period welfare comparison is no longer possible, since changing CDE parameters means changing preferences, and in that case equivalent variation will not be well-defined (Chen et al., 2016). To overcome this, Rimmer and Powell (1996) propose an implicit directly additive demand system (AIDADS) that allows income elasticities of demand to vary logistically as the marginal budget shares at subsistence income and very high income are separately estimated. Nevertheless, because of the implicit additivity assumption, AIDADS only allows a limited range of substitution possibilities across goods, and due to theoretical and computational reasons, AIDADS applications have thus far been limited to 10 commodities/sectors (Reimer and Hertel, 2004). As a result, CGE applications with AIDADS are less common and more project-specific. In contrast, despite some limitations, the CDE demand system seems to be more broadly applicable as a generic setting for modeling non-homothetic preferences with variation in the price-responsiveness of demand. For instance, with the CDE demand, modelers have more degrees of freedom in choosing the desired sectoral aggregation level that fits their research purposes.

While CGE models such as GTAP (Hertel and Tsigas, 1997), MAGNET (Woltjer and Kuiper, 2014), GTEM (ABARE/DFAT, 1995; ABARE, 1996), and ENVISAGE (van der Mensbrugghe, 2008) have been using the CDE demand system in modeling final consumption behaviors, perhaps due to the complexities in both calibration and implementation, other CDE applications are less common so far. More importantly, when studying the responses of CGE models with non-homothetic preferences, besides examining the implications of income elasticities of demand on future projection, the roles of own-price elasticities of demand are crucial as well since they can also influence projections if relative prices or income levels change. Existing literature also points out that to ensure the regularity of a well-behaved demand function, calibrating a CDE demand system to the target elasticities that are valid might be infeasible (Hertel, 2012; Huff *et al.*, 1997). How well the system can match those elasticities is beyond the discussion of most existing literature. One exception is Liu *et al.* (1998), which presents the differences between target and calibrated elasticities. Nevertheless, exploring sources of differences between calibrated and target elasticities is beyond the scope of that study.

Before studying how well the calibrated elasticities of a demand system can match a set of target elasticities, one needs to ensure that under a given expenditure share structure, the target elasticities are valid, i.e., they are conformable to aggregation conditions and a negative semi-definite Slutsky matrix. Therefore, the demand system under consideration will only be calibrated to a set of valid target elasticities. With that in mind, the study will answer the question both analytically and numerically: given a set of valid target own-price demand elasticities, income demand elasticities and expenditure shares, under what conditions will the calibrated elasticities of a CDE demand system most closely approximate the target values? The findings of this study can help modelers who implement a CDE demand system in explaining how well the target elasticities are represented in their models, and provide information for choosing an appropriate sectoral aggregation so that, if possible, at least target elasticities of interesting sectors can be better matched.

Following Chen (2015) this paper also presents strategies for putting the CDE demand system into GTAPinGAMS (Rutherford, 2012; Lanz and Rutherford, 2016), a global CGE model which is written in GAMS and MPSGE and which employs the GTAP database (Narayanan et al., 2012; Aguiar et al., 2016). MPSGE is a subsystem of GAMS (Rutherford, 1999), and earlier it was sometimes thought that despite being a powerful tool that handles the calibration of CES functions automatically, MPSGE can only be applied to models with CES or LES utility functions (Konovalchuk, 2006; Hertel *et al.*, 1991). Perhaps the misconception is because, until recently, CGE models built by MPSGE were largely characterized by either CES or LES preferences. This study provides an example of extending the application of MPSGE beyond the CES or LES preferences. The revised GTAPinGAMS with a CDE demand system is tested with income and price shocks to verify the model response is consistent to the calibrated elasticities. The programs for the CDE calibration and the revised GTAPinGAMS with a CDE demand system are provided in the Appendix, so readers can use them for verification or research purposes.

The rest of the paper is organized as follows: Section 2 briefly reviews the theories and settings of the CDE demand system; Section 3 presents the calibration, performance, and implementation of the CDE demand system; and Section 4 provides a conclusion.

2. Theoretical Background

To understand what constitutes a regular (i.e., valid) demand response, the section will briefly review the economic considerations for a regular demand system. In the subsequent analysis, this paper will focus on the following question: how can one evaluate the performance of a regular demand system in representing a set of valid own-price and income demand elasticity targets? To explore this, the section will discuss a demand system's flexibilities in own-price and income demand elasticities calibration, introduce the settings of CDE demand system, and finally examine the implications of CDE regularity conditions on the calibration performance of the system.

2.1 Regularity and Flexibility of a Demand System

Let us denote a cost (or expenditure) function by $C(p,u)$ where p is a N-dimensional price vector and u is the utility. For C to be considered as well-behaved, $\partial C/\partial p$, which is the Hicksian demand vector $q(p,u)$, is nonnegative and homogeneous of degree zero in p, and $\left[\partial^2 C/\partial p_i \partial p_j\right]_{N\times N}$, which is the Slutsky matrix, is negative semi-definite (NSD).[2] The intuition of a NSD Slutsky matrix is: for a given utility level u, when a good becomes more expensive, it will be replaced by other cheaper alternatives; as a result, the cost increase with the new consumption bundle after the price increase will never exceed the cost increase when the bundle cannot be altered.

The Slutsky matrix $\left[\partial^2 C/\partial p_i \partial p_j\right]_{N\times N}$, or equivalently $[\partial q/\partial p]_{N\times N}$, is symmetric and each term of the matrix is:

$$\frac{\partial q_i(p,u)}{\partial p_j} = \frac{\partial x_i(p,w)}{\partial p_j} + \frac{\partial x_i(p,w)}{\partial w} x_j(p,w) \tag{1}$$

Equation (1) is the Slutsky equation, which decomposes the impacts of a price change on the uncompensated demand $x_i(p,w)$ into the income effect and substitution effect, where w is the income (or expenditure) level. With some algebra, the Slutsky equation can also be expressed as

$$\sigma_{ij}^c = \sigma_{ij}^m + \eta_i \theta_j \tag{2}$$

where σ_{ij}^c, σ_{ij}^m,η_i, and θ_j are compensated demand elasticity of commodity i with respect to the price of commodity j, uncompensated demand elasticity of i with respect to the price of j, income demand elasticity of i, and expenditure share of j, respectively. If both sides of (2) are divided by θ_j, one can come up with a Slutsky matrix $\left[\sigma_{ij}\right]_{N\times N}$ in the form of Allen-Uzawa elasticity of substitution (AUES) (Allen and Hicks, 1934; Uzawa, 1962) with

$$\sigma_{ij} = \sigma_{ij}^m/\theta_j + \eta_i \tag{3}$$

It can be shown that $\left[\sigma_{ij}\right]_{N\times N}$ is also symmetric, and the matrix is NSD if and only if $[\partial q/\partial p]_{N\times N}$ is NSD. Therefore, a demand system is regular means 1) the Slutsky/AUES matrix $\left[\sigma_{ij}\right]_{N\times N}$ is NSD; and 2) the Hicksian demand q is non-negative. For CGE modeling, it is necessary to ensure that the demand system is globally regular (i.e., it should remain regular everywhere in the domain of price). This is because the algorithm of the solver for finding equilibria may begin from an initial point of price and quantity combination that is far from the equilibrium levels, and in the process of solving the model, the algorithm might fail if the

[2] For example, see p.59 and p.933 in Mas-Colell *et al.* (1995).

demand system is not globally regular, even the system is locally regular at the equilibrium points (Perroni and Rutherford, 1998).

Perroni and Rutherford (1995) defined a regular-flexible demand system as the one that is globally regular and can locally represent any valid configuration of compensated demands and the AUES matrix $\left[\sigma_{ij}\right]_{N\times N}$. Based on an inductive argument, Perroni and Rutherford proved that a demand system derived from a special version of the non-separable n-stage CES function is regular-flexible. Nevertheless, in general, testing whether other demand systems are regular-flexible would need to identify the domain of a regular flexible demand system first, which is beyond the scope of the current research. Instead of matching the entire AUES matrix under a given expenditure share structure, this study will focus on the ability of a demand system to match a valid combination of own-price demand elasticities, income demand elasticities, and expenditure shares.

Own-price and income demand elasticities are usually of first-order importance in characterizing the model responses to exogenous policy or productivity shocks, and are also the most common behavioral parameters available for calibrating a demand system. In particular, this study will examine whether a global regular demand system under consideration is own-price and income flexible, or equivalently, if the system can be calibrated to a valid combination of $(\sigma_{ii}^{m}, \eta_i, \theta_i)$ — the combination that is consistent to any well-behaved cost function (i.e., the aggregation conditions are satisfied, and the AUES matrix is NSD).[3] Ideally, the functional form of a demand system used in a CGE model should not become a constraint in matching any valid combination of $(\sigma_{ii}^{m}, \eta_i, \theta_i)$. However, usually that is not the case. Note that each of the three components of $(\sigma_{ii}^{m}, \eta_i, \theta_i)$ is a N-dimensional vector, and these components (σ_{ii}^{m}, η_i, and θ_i) are interdependent. For instance, Pigou's Law states that under certain assumptions on preferences, when the sectoral expenditure share θ_i is negligible, there is a proportional relationship between the income and uncompensated own-price demand elasticities (Pigou, 1910; Snow and Warren, 2015). Because of this interdependency, identifying the domain of $(\sigma_{ii}^{m}, \eta_i, \theta_i)$ is numerically challenging, unless one is willing to consider very few sectors under a given distribution of $\{\theta_i\}$, such as examples shown in Perroni and Rutherford (1998). Therefore, in the numerical examples for the CDE calibration provided later,

[3] Based on the Slutsky equation (see Equation (2)), if the following information is given: 1) expenditure share θ_i, 2) target income demand elasticity η_i^{t}, and 3) any one of the uncompensated own-price demand elasticity target σ_{ii}^{mt}, compensated own-price demand elasticity target σ_{ii}^{ct}, or compensated own-price demand elasticity target in AUES form σ_{ii}^{t}, then targeting any of the aforementioned three versions of own-price demand elasticities is equivalent to targeting another. What matters is to identify the form of the target elasticities clearly when doing the calibration job. For example, if one calibrates the compensated elasticities to the uncompensated elasticity targets, that would be incorrect.

rather than identifying the full space of valid target elasticities and sectoral shares, this study will begin by checking whether the target elasticities aggregated from the GTAP database with various sectoral resolutions and expenditure share structures actually constitute a theoretically valid demand configuration.

2.2 The CDE Demand System

Let us consider the expenditure function C with a price vector p and a Hicksian demand vector q, i.e., $c_0 = C(p_0, u) \equiv \{\min p_0 q_0 : f(q_0) \geq u\}$ where the subscript 0 denotes the benchmark condition. If the function is normalized by c_0, it becomes $C(p_0/c_0, u) \equiv 1$. With this normalization, Hanoch (1975) proposes the expenditure function of a CDE demand system as follows:

$$C\left(\frac{p}{c_0}, u\right) = \sum_i \beta_i u^{e_i(1-\alpha_i)} \left(\frac{p_i}{c_0}\right)^{1-\alpha_i} \equiv 1 \qquad (4)$$

where α_i and e_i are the substitution parameter and expansion parameter, respectively. In this setting, the utility u is only implicitly defined, and in general there is no reduced form representation for u. The Hicksian demand for commodity i based on this setting is:

$$q_i = \frac{\left[\beta_i u^{e_i(1-\alpha_i)}(1-\alpha_i)\left(\frac{p_i}{c_0}\right)^{-\alpha_i}\right]}{\sum_j \beta_j u^{e_j(1-\alpha_j)}(1-\alpha_j)\left(\frac{p_j}{c_0}\right)^{1-\alpha_j}} \qquad (5)$$

For the CDE demand system, the substitution elasticity σ_{ij} in AUES form is presented in Equation (6), where the expenditure share is denoted by θ_i, and $\delta_{ij} = 1$ if $i = j$, otherwise $\delta_{ij} = 0$. The income elasticity of demand η_i is presented in Equation (7):

$$\sigma_{ij} = \alpha_i + \alpha_j - \sum_k \theta_k \alpha_k - \frac{\delta_{ij}\alpha_i}{\theta_i} \qquad (6)$$

$$\eta_i = \left(\sum_k \theta_k e_k\right)^{-1}\left[e_i(1-\alpha_i) + \sum_k \theta_k e_k \alpha_k\right] + \left(\alpha_i - \sum_k \theta_k \alpha_k\right) \qquad (7)$$

It can be shown that both Cournot aggregation and Engel aggregation conditions hold for these elasticities, i.e., $\sum_i \theta_i \sigma_{ij} = 0$ and $\sum_i \theta_i \eta_i = 1$. Note that for each off-diagonal term, the difference between the substitution elasticities, $\sigma_{ij} - \sigma_{ik} = \alpha_j - \alpha_k$, is invariant to i . Hence the name CDE since the demand system has a constant difference of (substitution) elasticities. The regularity condition for the system presented in Hanoch (1975) includes: $\beta_i > 0$; $e_i > 0$; $0 < \alpha_i < 1$ or $\alpha_i \geq 1 \; \forall \, i$ and $\alpha_I > 1$ for some $I \in i$. It is worth noting that with the regularity condition, each own-price elasticity of demand σ_{ii}^c is always negative. This is because from Equation (6) and $\sigma_{ii}^c = \sigma_{ii}\theta_i$, we have

$$\sigma_{ii}^c = -\alpha_i(1-\theta_i)^2 - \theta_i \sum_{k|k\neq i} \theta_k \alpha_k \qquad (8)$$

For a given vector of budget shares, θ_i, the requirement that all α_is should lie on the same side of one imposes a constraint in choosing the vector of α_i such that σ_{ii}^c can match the target own-price demand elasticity. For instance, some sectors may have a very small expenditure share ($\theta_i \to 0$) and so for those sectors $\sigma_{ii}^c \to -\alpha_i$. However, for those sectors, if some target own-price elasticities do not lie on the same side of one, it would be impossible to match every single σ_{ii}^c with the target elasticity value no matter what regulatory condition on α_i is chosen. Therefore, the CDE demand system is not own-price flexible. Further, the requirement of $e_i \geq 0$ also suggests that some compromise has to be made in calibrating income elasticities of demand.

3. Calibration, Performance, and Implementation

Two CDE calibration methods have been presented. The first is the three-step sequential approach documented in Hertel et al. (1991) and Huff et al. (1997). In this approach, own-price demand elasticities are calibrated to target levels first. Taking parameters determined in the first step as given, income elasticities of demand are calibrated to target levels next, and scale parameters of the system are specified last. The second method is the maximum entropy approach presented by Surry (1997) and Liu et al. (1998). Rather than calibrating the system sequentially, the idea of this approach is finding all parameters simultaneously by maximizing an objective function that considers matching both own-price and income elasticities of demand. This section will analytically examine the performance of CDE calibration (i.e., how well target elasticities can be matched by their calibrated counterparts), and then provide numerical examples based on both calibration methods. It will also demonstrate how to put the CDE demand system into GTAPinGAMS and verify the model response is consistent to the calibrated elasticities.

3.1 Calibration: sequential approach

- *Step 1: Calibrating the own-price elasticity of demand.*

Let us denote the target comopensated own-price elasticity of demand by σ_{ii}^{ct}. The purpose of this step is to choose α_i so that the objective function $g(\alpha_i) = -\sum_i \sigma_{ii}^c(\alpha_i)[\ln(\sigma_{ii}^c(\alpha_i)/\sigma_{ii}^{ct}) - 1]$ is minimized. Note that $g(\alpha_i)$ is convex in α_i, and the function achieves its minimal value when $\sigma_{ii}^c(\alpha_i) = \sigma_{ii}^{ct}$ for every i.[4] The problem can be formulated as:

$$\min_{\alpha_i} g(\alpha_i)\ s.t.\ \alpha_i \in (0,1)\ or\ \alpha_i \geq 1\ \forall\ i\ and\ \alpha_I > 1\ for\ some\ I \in i \tag{9}$$

where $\sigma_{ii}^c(\alpha_i) = -\alpha_i(1-\theta_i)^2 - \theta_i \sum_{k|k\neq i} \theta_k \alpha_k$ (see Equation (8)).

- *Step 2: Calibrating the income elasticity of demand.*

[4] $\frac{\partial g}{\partial \alpha_i} = \frac{\partial g}{\partial \sigma_{ii}^c} \cdot \frac{\partial \sigma_{ii}^c}{\partial \alpha_i} = (-\ln \sigma_{ii}^c + \ln \sigma_{ii}^{ct}) \cdot (-(1-\theta)^2)$; $\frac{\partial^2 g}{\partial \alpha_i^2} = -\frac{(1-\theta_i)^2}{\sigma_{ii}^c} > 0$.

Let us denote the target income elasticity of demand by η_i^t (η_i^t must satisfy the Engel aggregation). Given α_i determined in the previous step, by choosing e_i, the goal is to calibrate η_i to η_i^t , if possible. Similar to the idea of Step 1, the following problem is solved:

$$\min_{e_i|\alpha_i} \sum_i \theta_i \left(\eta_i - \eta_i^t\right)^2 \; s.t. \sum_i \theta_i \eta_i = 1 \; ; \; (\eta_i - 1)\left(\eta_i^t - 1\right) > 0 \; \& \; e_i > 0 \; \forall i \qquad (10)$$

The condition $\sum_i \theta_i \eta_i = 1$ is to ensure the calibrated elasticities satisfy the Engel aggregation, and as noted in Huff *et al.* (1997), the second condition is to ensure the calibrated elasticities lie on the same side of one as the target values.

- *Step 3: Calibrating scale coefficients holding the utility level equals one.*

With the calibrated α_i and e_i, and the normalization $u = 1$, $p_{0i} = 1$, and $q_{0i} = \theta_i$ (since $c_0 = \sum_i p_{0i} q_{0i} = 1$), the N scale parameters β_i can be solved by using (4) and (5):

$$\beta_i = \frac{q_{0i}}{1-\alpha_i} / \sum_k \frac{q_{0k}}{1-\alpha_k} \qquad (11)$$

Because the calibration is done sequentially, how well the income elasticities of demand can be matched to target levels is also affected by the calibration of own-price elasticities of demand. Appendix A provides the program for the sequential approach. The program is written in GAMS, and each minimization problem in the program is formulated as a nonlinear programming (NLP) problem.

3.2 Calibration: maximum entropy approach

Following the notation used before, in this approach the substitution parameters α_i and the expansion parameters e_i are chosen simultaneously by maximizing the objective function, which is the entropy relative to the unknown parameters of the CDE demand system. As in the sequential approach, the scale parameters β_i can be calculated once α_i and e_i are determined (see Equation (11)). To provide more details, let us denote the cross entropy of the substitution parameter and the cross entropy of the expansion parameter as α_{etp} and e_{etp}, respectively. The two entropy measures are defined as:

$$\alpha_{etp} = -\sum_k \theta_k \left(\alpha_k \ln \frac{\alpha_k}{\bar{\alpha}} + (1 - \alpha_k) \ln \frac{1-\alpha_k}{1-\bar{\alpha}}\right) \qquad (12)$$

$$e_{etp} = -\sum_k \theta_k e_k \ln e_k \qquad (13)$$

Since the calibrated elasticities may deviate from target levels, let us define the penalty for errors in the substitution parameters and that for errors in the expansions parameters as α_{pnt} and e_{pnt}, respectively:

$$\alpha_{pnt} = \sum_k \theta_k \cdot (\sigma_{kk}^m - \sigma_{kk}^{mt})^2 \qquad (14)$$

$$e_{pnt} = \sum_k \theta_k \cdot (\eta_k - \eta_k^t)^2 \qquad (15)$$

The maximum entropy approach is to solve the following problem:

$$\max_{\alpha_i, e_i}\{(e_{etp} + \alpha_{etp}) - (e_{pnt} + \alpha_{pnt})\} \;\; s.t. \;\; \sum_k \theta_k e_k = 1 \quad (16)$$

Interested readers may also refer to Hertel et al. (2014) for details of this approach. Note that while results from both calibration methods will be presented later, comparing one method with another or assessing which one is preferable is beyond the scope of this study, which seeks instead to simply provide numerical examples for a set of propositions aimed to guide those seeking to assess the performance of CDE calibrations (see Section 3.3). Appendix B presents the GAMS program that implements the maximum entropy approach, which is also formulated as a nonlinear programming (NLP) problem.[5]

3.3 Performance

Before putting the system into a CGE model, two interesting questions arise: under what circumstances does the calibration become more accurate, and how well are those target elasticities represented? The following analysis seek to shed light on these questions.

Proposition 3.3.1:

The lower the expenditure share, the larger the influence of the own-sector substitution parameter in determining the calibrated own-price elasticity of demand. On the other hand, the higher the expenditure share, the greater the influence of other sectors' substitution parameters in determining the calibrated elasticity.

Proof:

$\sigma_{ii}^c = -\alpha_i(1-\theta_i)^2 - \theta_i \sum_{k|k\neq i} \theta_k \alpha_k$; note that $(1-\theta_i)^2$ is decreasing but θ_i is increasing on $\theta_i \in (0,1)$, respectively. Also, note that: $\lim_{\theta_i \to 0} \sigma_{ii}^c = -\alpha_i$ and $\lim_{\theta_i \to 1} \sigma_{ii}^c = -\sum_{k|k\neq i} \theta_k \alpha_k$ ■

The compensated own-price elasticities of demand presented in GTAP 8 (Narayanan et al., 2012), from which this study, and many others, draw their data, lie between -1 and 0. Therefore, based on the discussion above, it appears that the regularity condition with $\alpha_i \in (0,1)$ produces more accurate calibration results for sectors with smaller expenditure shares. With a higher sectoral resolution, more commodities/sectors will have smaller expenditure shares, and thus having $\alpha_i \in (0,1)$ will make it possible for producing a better match between calibrated and target levels for each individual sector.

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[5] The author is grateful to Erwin Corong and Thomas Hertel from the Center for Global Trade Analysis at the Department of Agricultural Economics in Purdue University for sharing the CDE calibration code that implements the maximum entropy approach. The code presented in Appendix B follows exactly the same setting as their code, except for the fact that some variable names are changed, which makes it easier to read data from GTAPinGAMS.

Proposition 3.3.2:
When $\alpha_i \in (0, 1)$, calibrating the income elasticity of demand to a larger value is less likely to violate $e_i > 0$, which is part of the CDE regularity conditions. On the other hand, when $\alpha_i > 1$, calibrating the elasticity to a lower level is less likely to violate $e_i > 0$.
Proof:
From Equation (7), $e_i = \frac{\{\sum_k \theta_k e_k [\eta_i - (\alpha_i - \sum_k \theta_k \alpha_k)] - \sum_k \theta_k e_k \alpha_k\}}{1-\alpha_i}$. Since $\frac{\partial e_i}{\partial \eta_i} = \frac{\sum_k \theta_k e_k}{1-\alpha_i}$, $\therefore \frac{\partial e_i}{\partial \eta_i} > 0$ if $\alpha_i \in (0, 1)$, and $\frac{\partial e_i}{\partial \eta_i} < 0$ if $\alpha_i > 1$ ∎

If one considers $\alpha_i \in (0, 1)$, the second proposition suggests that matching the target income elasticities for the demand of agricultural products in developed countries might be trickier, since in general these products tend to have lower income elasticity values; as a result, the calibrated income demand elasticities for these products might end up with levels higher than the target numbers. Nevertheless, the values of α_i may also affect how well the target income elasticities of demand are met, as will be explored in the next proposition.

Proposition 3.3.3:
When $\alpha_i \in (0, 1)$, calibrating the income elasticity of demand to a target level is less likely to violate $e_i > 0$ with a smaller α_i. On the other hand, when $\alpha_i > 1$, calibrating the elasticity to the target level is less likely to violate $e_i > 0$ with a larger α_i.
Proof:
$e_i = \{\sum_k \theta_k e_k [\eta_i - (\alpha_i - \sum_k \theta_k \alpha_k)] - \sum_k \theta_k e_k \alpha_k\}/(1-\alpha_i)$. Since $\frac{\partial e_i}{\partial \alpha_i} = \frac{1}{1-\alpha_i}[(-1 + \theta_i) \sum_k \theta_k e_k - \theta_i e_i]$, $\therefore \frac{\partial e_i}{\partial \alpha_i} < 0$ if $\alpha_i \in (0, 1)$, and $\frac{\partial e_i}{\partial \alpha_i} > 0$ if $\alpha_i > 1$ ∎

Continuing our previous example for commodities with low income elasticities of demand and with $\alpha_i \in (0, 1)$, while Proposition 3.3.2 says that for given values of α_i, it is harder to calibrate the income elasticity of demand to a lower value, Proposition 3.3.3 suggests that if the calibrated α_i is small enough, it is still possible to calibrate the income elasticity of demand to a lower level.

Proposition 3.3.4:
Commodities with substitution parameters α_i close to one will have similar calibrated income elasticities of demand.
Proof:
From Equation (7), $\lim_{\alpha_i \to 1} \eta_i = \sum_k \theta_k e_k \alpha_k / \sum_k \theta_k e_k + 1 - \sum_k \theta_k \alpha_k = \lim_{\alpha_j \to 1} \eta_j$ ∎

Proposition 3.3.4 shows that the calibrated α_i may work against the calibration of income elasticities of demand. For instance, if there are two commodities with α_i and α_j both approaching unity, according to the proposition, the calibrated income elasticities of demand η_i and η_j will be very close to each other, even if their target values η_i^t and η_j^t are quite different. The four propositions presented above are summarized in Table 1.

Table 1. Summary of Propositions relating to CDE Calibration.

Proposition	Argument
3.3.1	The lower the expenditure share, the larger the influence of own-sector substitution parameter in determining the calibrated own-price elasticity of demand. On the other hand, the higher the expenditure share, the greater the influence of other sectors' substitution parameters in determining the calibrated elasticity.
3.3.2	When $\alpha_i \in (0, 1)$, calibrating the income elasticity of demand to a larger value is less likely to violate $e_i > 0$, which is part of the CDE regularity condition. On the other hand, when $\alpha_i > 1$, calibrating the elasticity to a lower level is less likely to violate $e_i < 0$.
3.3.3	When $\alpha_i \in (0, 1)$, calibrating the income elasticity of demand to a target level is less likely to violate $e_i > 0$ with a smaller α_i. On the other hand, when $\alpha_i > 1$, calibrating the elasticity to the target level is less likely to violate $e_i > 0$ with a larger α_i.
3.3.4	Commodities with substitution parameters α_i close to one will have similar calibrated income elasticities of demand.

Source: The author's summary for the propositions presented in Section 3.3.

To show how different sectoral aggregation levels could affect the accuracy of elasticity calibration, the study considers several different aggregation levels (Table 2). The mapping between the GTAP sector numbers shown in Table 2 and their abbreviations are presented in Appendix C. For demonstration purposes, all GTAP regions are combined into a single region using the aggregation routine of GTAPinGAMS. In particular, wherever needed, target elasticities are aggregated based on expenditure shares. It is worth noting that the 10-sector income demand elasticity estimates based on an AIDADS system were mapped to and used as the target income demand elasticities of the original GTAP database, and following Zeitsch et al. (1991), income demand elasticities are then used to compute the own-price demand elasticities of the database, as documented in Hertel et al. (2014).[6]

[6] Interested readers may refer to page 14-6, Table 14.5 and Table 14.6 in Hertel et al. (2014) for details.

Table 2. Settings for calibration exercises with various sectoral aggregation levels.

Aggregation level	Setting
1r3s2f	3 sectors: combine GTAP sector 1 (g01) to sector 14 (g14) & sector 22 (g22) to sector 26 (g26) into s01 (agriculture); g15 to g21 & g27 to g46 into s02 (manufacturing); and g47 to g57 into s03 (service)
1r4s2f	4 sectors: similar to "1r3s2f" except for the fact that the service sector in "1r3s2f" is disaggregated into trade and transport sector (g47 to g51) and service sector (g52 to g57).
1r5s2f	5 sectors: combine g01 to g17 into s01; g18 to g27 into s02; …; g48 to g57 into s05
1r8s2f	8 sectors: combine g01 to g15 into s01; g16 to g21 into s02; …; g52 to g57 into s08
1r16s2f	16 sectors: combine g01 to g12 into s01; g13 to g15 into s02; g16 to g18 into s03; …; g55 to g57 into s16
1r29s2f	29 sectors: combine g01 & g02 into s01; g03 & g04 into s02; …; g55 & g56 into s28; g57 becomes s29
1r57s2f	57 sectors: keep the original GTAP sectors (g01 to g57)

Notes: All settings have one aggregated region and two aggregated primary factors: labor and capital.

To assess the calibration performance for each type of elasticity, in addition to a one-by-one comparison between calibrated and target numbers for each commodity, it is informative to have an index for measuring how far the point of calibrated elasticities is from the point of target elasticities as follows:

$$d = \sqrt{\sum_{i=1}^{N} \theta_i \cdot \left(x_i - x_i^t\right)^2} \quad (17)$$

Depending on the type of elasticity evaluated, x_i in Equation (17) could be either the own-price elasticity of demand σ_{ii}^c or the income elasticity of demand η_i, while the superscript x_i^t denotes target value.

When the 57 GTAP sectors are aggregated into a 3-sector setting, even the smallest sectoral expenditure share, denoted by θ_{min} approximates 12%, and with this setting the largest share θ_{max} exceeds 63%. As the sectoral resolution increases, the absolute difference between θ_{max} and θ_{min} is reduced. In the most disaggregated case where all 57 GTAP sectors are kept, θ_{max} is slightly above 17% and θ_{min} is only 0.0002% (Table 3). Per compensated own-price demand elasticity targets, the range between the largest one σ_{max}^{ct} and the smallest one σ_{min}^{ct} increases as the sectoral resolution gets higher, since more disaggregation means extreme values are more likely to appear. In general, σ_{max}^{ct} becomes larger ($|\sigma_{max}^{ct}|$ becomes smaller, i.e., less elastic) and σ_{min}^{ct} becomes smaller ($|\sigma_{min}^{ct}|$ becomes larger, i.e., more elastic) as the sectoral resolution increases. The same story applies to the income demand elasticity targets—with more disaggregated sectors, the range between η_{max}^{ct} and η_{min}^{ct} increases as η_{min}^{ct} becomes smaller (less elastic) and/or η_{max}^{ct} becomes larger (more elastic).

When trying to calibrate the CDE demand system to the target own-price demand elasticities, it is important to verify if there exists an AUES matrix that is NSD and is compatible to those elasticity targets. For instance, with the 3-sector setting, based on Cournot aggregation, the three off-diagonal terms of the AUES matrix are fully determined once the own-price demand elasticities in AUES form (i.e., the diagonal terms of the matrix) are given, and hence the whole AUES matrix is identified. However, given the target own-price demand elasticities in this particular example, one cannot find an AUES matrix that is NSD, which means the target own-price demand elasticities are invalid, and one cannot claim the CDE demand system is not own-price flexible based on this setting. On the other hand, in the 4-sector, 5-sector, 8-sector, and 16-sector settings, it can be shown that under each setting, the target own-price demand elasticities are compatible to an AUES matrix that is NSD, and therefore the target elasticities are valid. More specifically, if one denotes the number of sectors/commodities by n, there will be $n \cdot (n-1)/2$ variables (cross-price demand elasticities in AUES form) that are off-diagonal terms in an AUES matrix, and after considering n constraints imposed by the Cournot aggregation, there will be $n \cdot (n-1)/2 - n$ variables that can be assigned by using random number generators, provided that the diagonal terms (compensated own-price demand elasticities in AUES form) are given. The remaining n variables can be solved based on the aforementioned n constraints. The task can be done iteratively, and if a NSD AUES matrix can be found, then the target own-price demand elasticities are valid. The MATLAB subroutine for doing this job is presented in Appendix D. For income demand elasticity targets, on the other hand, they are valid as long as the Engel aggregation is satisfied.

Since sectoralown-price demand elasticity targets are all between 0 and 1, in all cases, to calibrate the CDE demand system, similar to Huff *et al.* (1997), the study chooses $\alpha_i \in (0, 1)$, a setting that produces a more accurate own-price demand elasticity calibration when the sectoral resolution becomes higher or the sectors under consideration have smaller expenditure shares, based on Proposition 3.3.1. It is worth noting that since the CDE demand system is regular, under each sectoral aggregation setting presented in Table 2, the calibrated own-price and cross-price demand elasticities always constitute a valid AUES matrix, regardless of whether the target own-price demand elasticities can form a valid AUES matrix. For instance, while the target own-price demand elasticities under the 3-sector setting do not constitute a valid AUES matrix, one can still try to calibrate the CDE demand system to those targets, and although the calibrated own-price demand elasticities will not (and should not) match those invalid targets, they (the calibrated elasticities) will produce a valid AUES matrix (calibration results with the 3-sector setting are included in Table 3). As mentioned, what one cannot do is to use those invalid targets to assess the performance of CDE calibration.

The study finds that with the 4-sector, 5-sector, 8-sector, and 16-sector settings, although the target elasticities are valid, under both calibration methods, the calibrated own-price demand elasticities cannot match their target levels since the corresponding distance measure d_σ is nonzero for each case. Nevertheless, in general, d_σ gets smaller as the sectoral resolution increases (Table 3). Indeed, if one moves further to the 29-sector or 57-sector settings, a perfect match between the calibrated own-price demand elasticities and their target levels is possible since in both sectoral settings $d_\sigma = 0$ under the sequential approach and $d_\sigma \to 0$ under the maximum entropy approach. Also, as sectoral shares get smaller, the calibrated own-price demand elasticity σ_{ii}^c will be closer to $-\alpha_i$ (Appendix E and Appendix F). These findings can also be explained by Proposition 3.3.1.

The results also show that for both calibration methods, the calibrated income elasticities of demand are matched to their target levels more precisely than the cases for calibrated own-price demand elasticities (Table 3). According to Propositions 3.3.2 and 3.3.3, under the same α_i, a bigger η_i^t is more achievable, and under the same η_i^t, a smaller α_i makes match η_i^t easier. For both calibration methods, if one looks at the substitution parameter α_i and the income demand elasticity target η_i^t, they are strongly positively correlated (Figure 1), which means under the given data, a bigger α_i may be less of a problem since the relevant income demand elasticity target η_i^t also tends to be higher, and at the same time a smaller η_i^t also tends to be coupled with a smaller α_i, which may raise the possibility of a precise income demand elasticity match.[7] Another finding is that under both calibration methods, in general, η_i^t is matched more precisely when sectors become more disaggregated. This is mainly due to the fact that with more disaggregated sectors, the given data tend to yield smaller α_i, the substitution parameters of the CDE demand, while there is no obvious trend for the income demand elasticity targets η_i^t (Figure 1), and Proposition 3.3.3 explains why η_i^t can be matched better in this case. In fact, under the sequential approach with 29-sector and 57-sector settings, a perfect match between calibrated and target income demand elasticity can be achieved. Finally, regardless of calibration methods, when there are multiple sectors with their own α_i close to 1, the calibrated income demand elasticities will converge to the same level, despite the fact that the target

[7] The strong positive correlation between α_i and the η_i^t goes back to the fact that in the GTAP database, only the income elasticities of demand are estimated. The own-price demand elasticities are obtained from a Frisch parameter, using the assumption of additivity, as shown in Zeitsch et al. (1991). Interested readers may refer to p.14-6 in Hertel et al. (2014) for details.

elasticity levels are different (Appendix E and Appendix F). Proposition 3.3.4 provides an explanation to this observation.

Table 3. Summary statistics, calibration performance, and validity of the AUES matrix.

Setting	1r3s2f	1r4s2f	1r5s2f	1r8s2f	1r16s2f	1r29s2f	1r57s2f
Number of sectors	3	4	5	8	16	29	57
Summary statistics of targets							
Sectoral expenditure share							
θ_{max}	63.430%	39.553%	46.242%	39.553%	26.481%	20.440%	17.186%
θ_{min}	11.779%	11.779%	3.432%	2.279%	0.092%	0.017%	0.0002%
Own-price demand elasticity							
σ_{max}^{ct}	-0.4294	-0.4294	-0.2056	-0.1942	-0.1669	-0.0936	-0.0711
σ_{min}^{ct}	-0.7658	-0.7800	-0.7608	-0.7800	-0.7974	-0.7957	-0.8095
σ_{avg}^{ct}	-0.6201	-0.6542	-0.5807	-0.6022	-0.6093	-0.5331	-0.5294
σ_{std}^{ct}	0.1410	0.1363	0.2064	0.1813	0.1634	0.2269	0.2220
Income demand elasticity							
η_{max}^{t}	1.0502	1.0543	1.0513	1.0543	1.0987	1.0916	1.1190
η_{min}^{t}	0.7300	0.7300	0.5504	0.5387	0.4874	0.3382	0.2704
η_{avg}^{t}	0.9267	0.9569	0.8947	0.9181	0.9457	0.8851	0.8970
η_{std}^{t}	0.1406	0.1326	0.1920	0.1708	0.1547	0.2344	0.2272
Calibration: sequential							
Match each σ_{ii}?	no	no	no	no	no	yes	yes
d_σ	0.3526	0.1321	0.1879	0.1441	0.0406	0.0000	0.0000
Match each η_{ii}?	no	no	no	no	no	yes	yes
d_η	0.2363	0.0080	0.0041	0.0081	0.0131	0.0000	0.0000
Calibration: max entropy							
Match each σ_{ii}?	no	no	no	no	no	no	no
d_σ	0.3578	0.1322	0.1856	0.1427	0.0405	0.0024	0.0017
Match each η_{ii}?	no	no	no	no	no	no	no
d_η	0.0815	0.0382	0.0971	0.0842	0.0243	0.0070	0.0051
Validity of the AUES matrix based on target elasticities							
σ_{ii}^{ct} compatible to a NSD AUES?	no	yes	yes	yes	yes	yes	yes

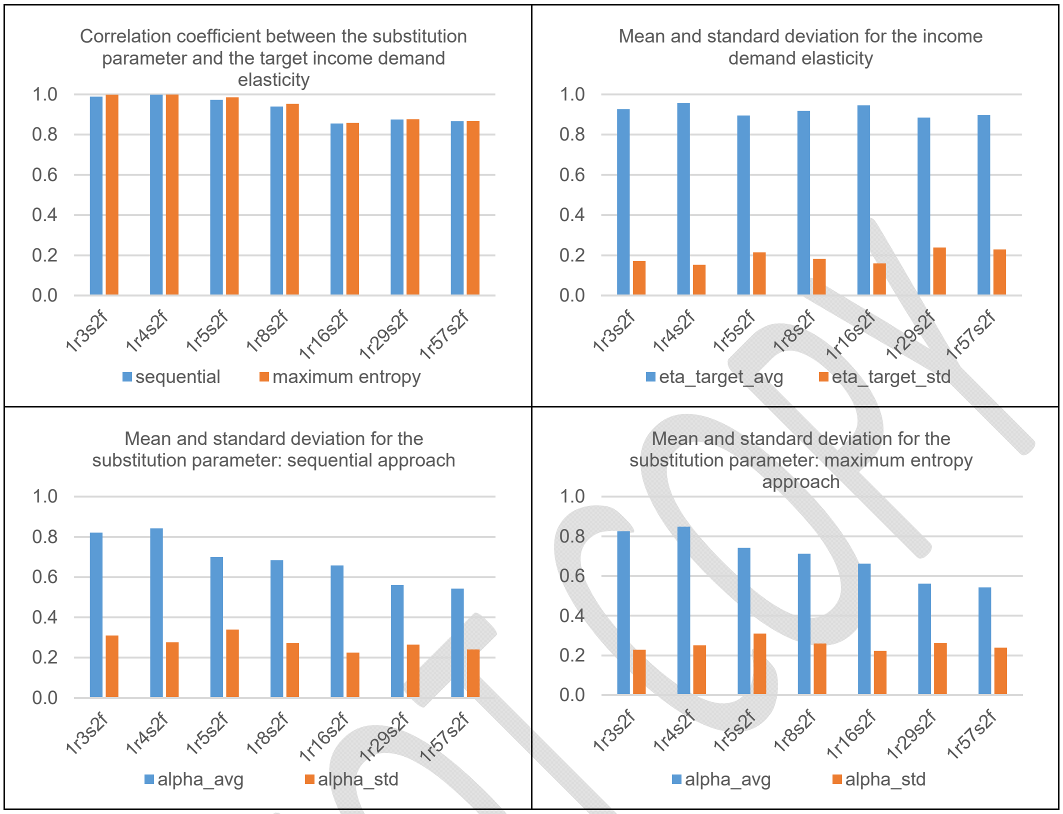

Figure 1. Summary statistics for income demand elasticity targets and substitution parameters.

Notes: "avg" and "std" stand for mean and standard deviation, respectively.

Source: The author's calculation based on the GTAP 8 database.

3.4 Implementation

With the calibrated parameters, the study demonstrates how to put the CDE demand system into the multi-region and multi-sector CGE model of GTAPinGAMS. The original CGE model is constructed based on CES technologies for both production and final consumption. It includes a series of mixed complementary problems (MCP) (Mathiesen, 1985; Rutherford, 1995; Ferris and Peng, 1997) written in MPSGE, a subsystem of GAMS (Rutherford, 1999). To implement the CDE demand system, the CES expenditure function is dropped, and by declaring auxiliary variables and equations in MPSGE to formulate relevant MCP, three sets of conditions below are incorporated into the revised model:

- *The equation for total expenditure*. The total expenditure c for purchasing one unit of utility (Equation (4)) is added into the model to form a MCP with a complementarity variable c. Note that in Equation (4), c is only implicitly defined. The purpose of this problem is to determine c jointly with other conditions. As previously mentioned, in the benchmark, both the utility level and price indices of commodities are normalized to unity.

- *The equation for final demand*. This equation (Equation (5)) is coupled with its complementarity variable, the activity level of final demand, to form a MCP. The problem is incorporated into the model to solve for the final demand of each commodity.
- *The zero profit condition for utility*. Let us denote the marginal cost and marginal revenue of utility (i.e., price of utility) by mcu and pu, respectively.[8] The zero profit condition of utility and the activity level of utility compose another MCP:

$$mcu \geq pu; u \geq 0; (mcu - pu) \cdot u = 0; mcu = \frac{c \sum_i \beta_i e_i (1-\alpha_i) u^{e_i(1-\alpha_i)-1} \left(\frac{p_i}{c}\right)^{1-\alpha_i}}{\sum_i \beta_i (1-\alpha_i) u^{e_i(1-\alpha_i)} \left(\frac{p_i}{c}\right)^{1-\alpha_i}} \tag{18}$$

Condition (18) states that in equilibrium, if the supply of utility u is positive, the marginal cost of utility mcu must equal the marginal revenue pu, and if mcu is higher than pu in equilibrium, u must be zero.

With the commodity price being a complementarity variable, the market clearing condition of each commodity is also formulated as a MCP by comparing the commodity supply (determined by its zero profit condition) with the final demand shown above plus the intermediate demand derived from a CES cost function as the original GTAPinGAMS. Similarly, with the price of utility being the complementarity variable, the supply of utility combined with the demand for utility (income/pu) make up the MCP for the market clearing condition of utility. The model code is provided in Appendix G, and interested readers may refer to Rutherford (1999) and Markusen (2013) for details of MPSGE.

For demonstration purposes, the study considers a setting with the aggregation level of two regions, four sectors, and one primary factor, and denotes this setting by "2r4s1f." The two regions are USA and the rest of the world (ROW); four sectors are agriculture (agri), manufacturing (man), trade and transport (tran), and service (serv), following the sectoral classification for the setting "1r4s2f" presented in Table 2; and the only one primary factor is the aggregation of all primary factors of GTAP8.[9] As before, prior to conduct and evaluate the CDE calibration, one

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[8] mcu in Condition (18) can be derived by taking the total derivative of Equation (4) with respect to u and c at a given commodity price vector.

[9] With a very extreme sectorial expenditure share distribution (such as the final consumption structure of 0.0007, 30.7722, 0.00002, and 191.5950 billion US$ for coal, gas, crude oil, and refined oil products for the U.S. extracted from GTAP 8), as of GAMS version 23.7.3, the MCP solvers may encounter numerical issues in solving the model.

needs to check if the target elasticities under this setting (2r4s1f) are consistent to an AUES matrix that is NSD, and it can be shown that this is indeed the case (the NSD AUES matrix can be found numerically based on the subroutine presented in Appendix D). Taking the sequential approach as an example, Table 4 presents the calibration performance for the CDE demand system under the 2-region and 4-sector setting.

Table 4. Performance of the CDE Calibration under the setting "2r4s1f"

	θ_i	α_i	e_i	σ_{ii}^{ct}	σ_{ii}^{cc}	η_i^t	η_i^c
Region: USA							
agri	0.04909	0.70623	2.00000	-0.67034	-0.68528	0.81292	0.99981
manu	0.18381	0.99999	0.00000	-0.82044	-0.81353	0.99514	1.00000
tran	0.20250	0.99999	0.00000	-0.85294	-0.79457	1.01152	1.00000
serv	0.56460	0.99999	3.37090	-0.85273	-0.42725	1.01372	1.00002
Distance					0.32082		0.04303
Region: ROW							
agri	0.14694	0.38159	2.95186	-0.39520	-0.39795	0.71822	0.71822
manu	0.27510	0.87414	3.04606	-0.62097	-0.63376	1.00104	1.00104
tran	0.25415	0.99999	3.38127	-0.70506	-0.71395	1.05431	1.07114
serv	0.32380	0.99999	14.65377	-0.72614	-0.63556	1.08436	1.07115
Distance					0.05218		0.01133

Source: The author's calculation based on the GTAP 8 database.

Let us parameterize the revised CGE model of GTAPinGAMS, based on calibrated parameters in Table 4. In the model, the aggregated primary factor along with the choice of the numeraire, which is the price for the aggregated primary factor, facilitate the identification of income effect. Now, to verify whether the CDE demand system is correctly implemented, the study will test if the outputs of the CGE model are consistent to the underlying calibrated elasticities under given price or income shocks. For example, with the shock on the price of agricultural product in the U.S., the first exercise changes the cost of final consumption for agricultural product in the U.S. exogenously to create the considered price shock.[10] The goal is to calculate the uncompensated (Marshallian) own-price arc elasticity for the demand of agricultural product based on the model response, and see if the realized elasticity from the model output is consistent to the calibrated level.

Table 4 shows that while the target own-price elasticity of demand for the agricultural product is $\sigma_{ii}^{ct} = -0.6703$, the calibrated own-price demand elasticity is $\sigma_{ii}^{cc} = -0.6853$. The target is not matched precisely due to the fact that CDE demand is not own-price flexible. Besides, since with a nontrivial price shock

[10] For instance, in the revised CGE model of GTAPinGAMS, a 10% increase in the price of agricultural product is achieved by multiplying both vdfm("agri", c, "usa") and vifm ("agri", c, "usa") in GTAPinGAMS by 1.1. Note that in GTAPinGAMS, both vdfm and vifm are redefined in a way such that both intermediate and final consumption are considered. For example, vdfm(i,g,r) means the domestically produced good i is used by g in region r, where g includes both users from industrial sectors and from final consumption.

imposed on the CGE model, it is more convenient to derive a "realized" uncompensated arc demand elasticity based on the model's output, for comparison purposes, the study will also convert the calibrated own-price demand elasticity σ_{ii}^{cc}, which is a compensated point elasticity, into an uncompensated arc demand elasticity with the same price shock so one can easily compare the realized level to the calibrated one. Note that while changes in expenditure share may change the point elasticity levels of both own-price and income elasticities of demand, for simplicity, this study uses the point elasticity level under the original expenditure share to derive the arc elasticity since changes in the structure of expenditure share are relatively moderate under the considered shocks. Besides, it is the calibrated income demand elasticity, rather than its target level, that is used in the calculation of the uncompensated own-price demand elasticity.

The calibrated uncompensated own-price demand elasticity, $\sigma_{ii}^{m} = -0.7344$ (a point elasticity), can be derived from σ_{ii}^{c}, η_i, and θ_i based on the Slutsky equation presented in Equation (2). Let us consider the quantity index $\tilde{q}_i = q_i/\theta_i$ with the benchmark level $\tilde{q}_{0i} = 1$ since $q_{oi} = \theta_i$ (see Step 3 in Section 3.1). Because the percentage change in $\tilde{q}_i$ is equivalent to the percentage change in q_i, $\tilde{q}_i$ can replace q_i in deriving the uncompensated (Marshallian) arc demand elasticity σ_{ii}^{ma}—with both price and quantity indices normalized to unity, the arc elasticity σ_{ii}^{ma} can be expressed as:[11]

$$\sigma_{ii}^{ma} = \frac{q_i - q_i^0}{(q_i + q_i^0)/2} \Big/ \frac{p_i - p_i^0}{(p_i + p_i^0)/2} = \frac{q_i - 1}{p_i - 1} \cdot \frac{p_i + 1}{q_i + 1} = \frac{p_i^{\sigma_{ii}^{m}} - 1}{p_i - 1} \cdot \frac{p_i + 1}{p_i^{\sigma_{ii}^{m}} + 1} \qquad (19)$$

where p_i is the after-shock price level. When various price shocks of agricultural product are in place, the values for σ_{ii}^{ma} (the calibrated Marshallian arc demand elasticity) and the realized arc elasticity levels σ_{ii}^{mar} (derived from the model output) are both presented in Figure 2. Note that with the exogenous price shocks in agricultural product, in the new equilibrium, one may also observe changes in prices of other commodities relative to their pre-shock levels, and this will in turn affect the equilibrium food consumption level due to the existence of cross-price elasticities of food demand. The exogenous price shock may also induce an income effect as reflected by the change in total (final) expenditure level. Therefore, to calculate σ_{ii}^{mar}, the consumption index $\tilde{q}_i$ is adjusted such that it is net of the cross-price and income effects.[12] The result in Figure 2 shows that, as expected, the larger

[11] $\sigma_{ii}^{m} = \frac{dq_i}{q_i} / \frac{dp_i}{p_i}$. Therefore $\int_{q_i^0}^{q_i} dq_i/q_i = \int_{p_i^0}^{p_i} \sigma_{ii}^{m} dp_i/p_i$, and so $q_i = p_i^{\sigma_{ii}^{m}}$.

[12] To calculate the "realized" Marshallian arc own-price demand elasticity from the CGE model's output, σ_{ii}^{mar}, the following steps are done: 1) step 1: calculate the substitution effect due to changes in all prices but the own-price based on prices change and the theoretical cross-price arc demand elasticity σ_{ij}^{ma}. The calculation is conditional on the original income level and the new (after shock) own-price level; 2) step 2: calculate the

the price shock, the more the arc elasticity deviates from the point elasticity σ_{ii}^{m}, which is the calibrated level without any price shock in the figure. Figure 2 also verifies that the uncompensated arc demand elasticity σ_{ii}^{mar} calculated from the model output replicates its calibrated counterpart σ_{ii}^{ma}.

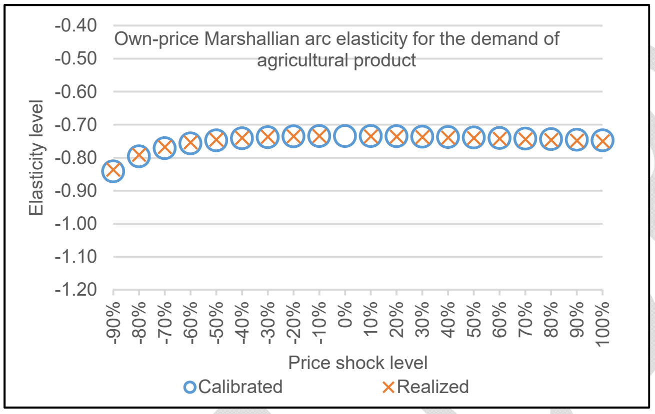

Figure 2. Own-price arc elasticity for the demand of agricultural product in the U.S.

The study continues to examine the model response under various income shocks in the U.S. The shocks are created by changing the quantity of the aggregated primary factor of the U.S., which is just the real GDP level of the U.S. Since GDP is not only spent on private consumption, to calculate the income elasticities of various commodities based on the model response, one needs to use the percentage change in the portion of income dedicated to private consumption, or equivalently, the percentage change in total expenditure on private consumption. Following the same logic as Equation (19), the income arc demand elasticity can be written as:

$$\eta_i^a = \frac{c^{\eta_i}-1}{c-1} \cdot \frac{c+1}{c^{\eta_i}+1}; \; c \text{ is the after-shock income level} \qquad (20)$$

Under various levels of income shock, Equation (20) is used to convert the calibrated point elasticity into the calibrated arc elasticity, which serves as the benchmark for the comparison between the realized arc elasticity from model outputs and the calibrated level the model is given. Finally, as the previous example, the new equilibrium with an income shock will generally accompany changes in price levels of various commodities. This means that the resulting consumption levels will be contaminated by changes in prices, although these

income effect on top of changes in all prices; based on the theoretical elasticities η_i^a and σ_{ij}^{ma}; 3) step 3: calculate the adjusted demand net of cross-price effect calculated from step 1 and income effect calculated from step 2, and then calculate σ_{ii}^{mar} accordingly.

changes are usually small. The study accounts for this price effect and removes it from the consumption levels, and then for each commodity, uses the percentage change of the adjusted consumption level as the numerator of the income demand elasticity.[13] Figure 3 demonstrates that for the final consumption of agricultural product, the realized income arc demand elasticity levels, as expected, replicate their calibrated counterparts. The two exercises presented here can be extended to other sectors and regions. For instance, with this 2-region and 4-sector setting, most of the calibrated income demand elasticities are close to one, although the target income demand elasticities can significantly deviate from one (Table 4). One exception is the income demand elasticity for the agricultural product in the rest of the world, $\eta_i^c = 0.7182$ (Table 4). For this elasticity, the calibrated elasticity and the realized numbers are matched as well (Figure 4).

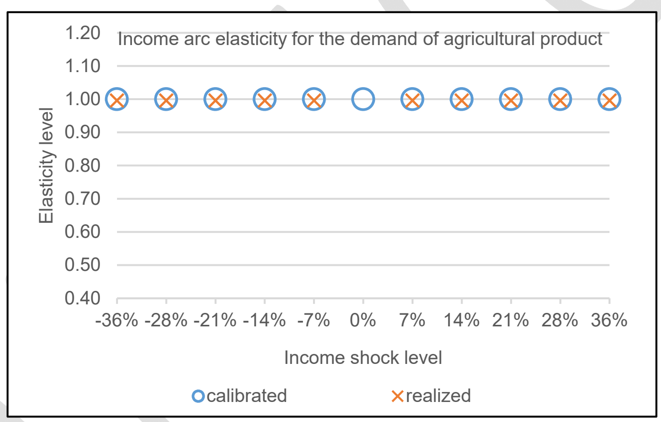

Figure 3. Income arc elasticity for the agricultural product demand in the U.S.

[13] To calculate the "realized" income arc demand elasticity from the model's output, η_i^{ar}, the following steps are done: 1) step 1: calculate the expected quantity level due to pure income effect, based on changes in income level and the theoretical income arc demand elasticity η_i^a; 2) step 2: under the expected quantity derived from the pure income effect calculated in step 1, calculate the quantity changes due to changes in all prices, based on prices changes and the theoretical own-price and cross-price arc demand elasticity σ_{ij}^{ma}; 3) step 3: subtract the quantity change due to price effect calculated in step 2 from the observed quantity of model output, and then calculate η_i^{ar} accordingly.

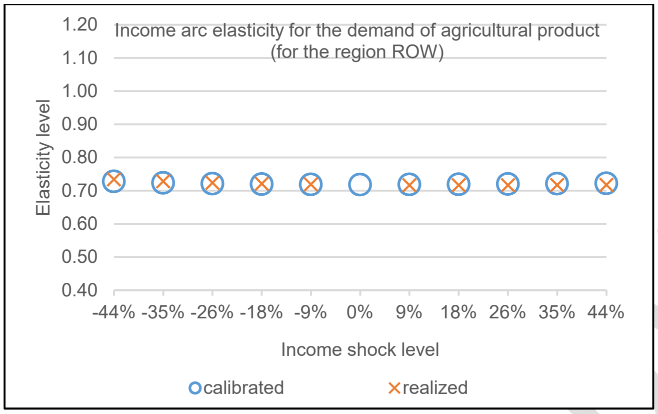

Figure 4. Income arc elasticity for the agricultural product demand in the rest of world

4. Conclusion

This paper provides the first comprehensive investigation of the circumstances under which the calibrated own-price and income elasticities of demand in a CDE demand system can be matched more accurately to their target levels. It finds that while the system is neither own-price nor income flexible, the elasticity match improves with smaller sectoral expenditure shares (i.e., higher sectoral resolution), lower target own-price demand elasticities, and higher target income demand elasticities. In any case, to understand the extent to which the elasticity targets are correctly represented in a CGE model, it is crucial to check whether the target elasticities are valid (i.e., compatible to aggregation conditions and a NSD AUES matrix), and disclose how well the calibrated elasticities match their target counterparts. Without having these inspections, when the calibrated elasticities deviate from target levels, it will not be possible to determine if that is due to targeting elasticity levels that are invalid, or if the inflexibility of the demand system is indeed the cause of the mismatch. For modelers who need to make sure the target income or own-price demand elasticities are reasonable, they can apply the program presented in Appendix D, which tests if the Engel aggregation is satisfied to ensure income demand elasticity targets are valid, and conditional on the satisfaction of Cournot aggregation, it also checks if a NSD AUES matrix can be found to verify the legitimacy of own-price demand elasticity targets.

In addition, using GTAPinGAMS, the study also incorporates the CDE demand system into a global CGE model written in MPSGE. Furthermore, price and income shocks are imposed on this revised GTAPinGAMS, and the model outputs successfully replicate the calibrated elasticities of the CDE demand system. Since implementing a CDE demand could be complicated and error-prone, future studies may examine if other CGE applications with the CDE demand can produce results in line with the calibrated elasticities, or they may investigate the flexibility and calibration performance of other demand systems—these issues are rarely studied yet essential because a more flexible demand system can better represent a set of valid target elasticities observed empirically, and allows CGE models to produce results more consistent to the underlying characteristics of the economy.

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Appendix A. The CDE calibration program: sequential approach[14]

```
$title  Calibrate a CDE Demand System using GTAP data

$if not set ds $set ds g20
$if not set datadir $set datadir .\input\
$if not set wt $set wt 0
$include gtap8data_old

set     info    Information about this calibration /
        ds      "%ds%",
        datadir "%datadir%",
        workdir "%gams.workdir%"
        date    "%system.date%"
        time    "%system.time%" /;

alias(i,j,k);

set rr(r) dynamic subset of r;
rr(r) = no;

parameters
z(i,r)         normalized price
theta(i,r)     value share in final demand
vafm(i,r)      Aggregate final demand,
delta(i,j,r)   diagonal-one off-diagonal-zero
sigma(i,j,r)   Allen partial elasticity of substitution
epsilon_(i,r)  targeted own-price elasticity of demand
eta_(i,r)      targeted income elasticity of demand
p0(i,r)        benchmark price index
q0(i,r)        benchmark consumption level
c0(r)          expenditure level
mc0(r)         marginal cost when u is one
weight(i,r)    weight for the square distance
beta(i,r)      scale coefficient

;

vafm(i,r) = vdfm(i,"c",r)*(1+rtfd0(i,"c",r))+vifm(i,"c",r)*(1+rtfi0(i,"c",r));
theta(i,r) = vafm(i,r) / (vom("c",r)*(1-rto("c",r)));
abort$sum(r, round(abs(1-sum(i,theta(i,r))),5)) "Shares do not add up.";

epsilon_(i,r)           = epsilon(i,r);
eta_(i,r)               = eta(i,r);

$ontext
theta(i,r)              = data(r,i,"shr");
epsilon_(i,r)           = data(r,i,"vt");
eta_(i,r)               = data(r,i,"eta_");
$offtext

p0(i,r)                 = 1;
```

[14] To run this program, one needs: 1) the GTAP 8 data in the gdx format (created by GTAPinGAMS); 2) the subroutine "gtap8data.gms," which is also included in GTAPinGAMS, that reads data needed in the calibration program; 3) to type "gams cdecalib" under the DOS command prompt—this will use the default database "2r4s1f.gdx". The environment variable "ds" can be used to overwrite the default database setting. Similarly, to run the maximum entropy approach calibration shown in Appendix B, one can type "gams cdeetp --ds=2r4s1f".

```
q0(i,r)                  = theta(i,r)/p0(i,r);
c0(r)                    = sum(i,p0(i,r)*q0(i,r));
delta(i,j,r)             = 0;
delta(i,j,r)$sameas(i,j) = 1;
weight(i,r)              = theta(i,r)$(%wt% eq 0) + (1/card(j))$(%wt% ne 0);

*       Finish reading data
*       --------------------------------------------------------

variables
ALPHA(i,r)   substitution coefficient
V(i,r)       own-price elasticity of demand
E(i,r)       expansion coefficient
ETAV(i,r)    income elasticity of demand
OBJONE       objective value for own-price elasticity calibration
OBJTWO       objective value for income elasticity calibration
OBJTHR       objective value for the dummy
OBJFOR       objective value for the dummy
U(r)         utility
;

* The equation "e_engel" deals with the case where eta from data doesn't
* satisfy the Engel aggregation equations
e_v(i,r)        for v
e_eta(i,r)      for ETAV
e_objone        for OBJONE
e_objtwo        for OBJTWO
e_objthr        for OBJTHR
e_objfor        for OBJFOR
e_engel(r)      Engel aggregation
e_etaside(i,r)  ensure eta & eta_ lies on the same side of one
e_exp(r)        expenditure function
e_dfn(i,r)      compensated demand
;

*       Step 1: Calibrating to the own-price elasticity of demand

e_v(i,rr) ..
V(i,rr)$theta(i,rr) =e= theta(i,rr)*(2*ALPHA(i,rr)-
sum(k,theta(k,rr)*ALPHA(k,rr)))-ALPHA(i,rr);

e_objone ..
OBJONE =e= -sum((i,rr),V(i,rr)*(log(V(i,rr)/epsilon_(i,rr))-1));

model demandelas / e_v, e_objone /;

loop(r,
rr(r)          = yes;
ALPHA.L(i,rr)  = 0.5;
ALPHA.UP(i,rr) = 0.99999;
ALPHA.LO(i,rr) = 0.00001;
V.L(i,rr)      = epsilon_(i,rr);
OBJONE.L       = 0;
solve demandelas using nlp minimizing OBJONE;
sigma(i,j,r)$theta(i,r) = ALPHA.L(i,r)+ALPHA.L(j,r)-
sum(k,theta(k,r)*ALPHA.L(k,r))-delta(i,j,r)*ALPHA.L(i,r)/theta(i,r);
rr(r)          = no;
);

*       Step 2: Calibrating the income elasticity of demand

e_eta(i,rr) ..
ETAV(i,rr) =e= (1/sum(k,theta(k,rr)*E(k,rr)))*(E(i,rr)*(1-
```

```
ALPHA.L(i,rr))+sum(k,theta(k,rr)*E(k,rr)*ALPHA.L(k,rr)))
          + (ALPHA.L(i,rr)-sum(k,theta(k,rr)*ALPHA.L(k,rr)));

e_objtwo ..
OBJTWO =e= sum((i,rr),weight(i,rr)*(ETAV(i,rr)-eta_(i,rr))*(ETAV(i,rr)-
eta_(i,rr)));

e_engel(rr) ..
sum(i,theta(i,rr)*ETAV(i,rr)) =e= 1;

e_etaside(i,rr) ..
(ETAV(i,rr)-1)*(eta_(i,rr)-1) =g= 0;

model incomeelas /e_objtwo, e_engel, e_eta, e_etaside/;

loop(r,
rr(r)          = yes;
E.LO(i,rr)     = 1e-6;
E.L(i,rr)      = 1;
ETAV.L(i,rr)   = eta_(i,r);

OBJTWO.L       = 0;
solve incomeelas using nlp minimizing OBJTWO;
rr(r)          = no;
);

*      Step 3: Calibrating the scale coefficient BETA holding the utility level
*      equals one

beta(i,r) = (q0(i,r)/(1-ALPHA.L(i,r)))/sum(j,q0(j,r)/(1-ALPHA.L(j,r)));
U.FX(r)   = 1;

parameter  epsilonv00(i,r)  EPSILONV solved by the CDE calibration routine,
           etav00(i,r)      ETAV solved by the CDE calibration routine
           alpha00(i,r)     ALPHA solved by the CDE calibration routine
           e00(i,r)         E solved by the CDE calibration routine
           u00(r)           U solved by the CDE calibration routine
           beta00(i,r)      beta solved by the CDE calibration routine
           mc00(r)          Marginal cost
;

epsilonv00(i,r) = V.L(i,r);
etav00(i,r)     = ETAV.L(i,r);
alpha00(i,r)    = ALPHA.L(i,r);
e00(i,r)        = E.L(i,r);
u00(r)          = U.L(r);
beta00(i,r)     = beta(i,r);


mc00(r) =

c0(r)*sum(i,beta(i,r)*E.L(i,r)*(1-ALPHA.L(i,r))*(U.L(r)**(E.L(i,r)*(1-
ALPHA.L(i,r))-1))*(p0(i,r)/c0(r))**(1-ALPHA.L(i,r)))
      /sum(i,beta(i,r)*(1-ALPHA.L(i,r))*(U.L(r)**(E.L(i,r)*(1-
ALPHA.L(i,r))))*(p0(i,r)/c0(r))**(1-ALPHA.L(i,r)));

parameter data model output;
data(i,r,"theta")      = theta(i,r);
data(i,r,"epsilon")    = epsilon_(i,r);
data(i,r,"epsilonv00") = epsilonv00(i,r);
data(i,r,"eta")        = eta_(i,r);
data(i,r,"etav")       = ETAV.L(i,r);
data(i,r,"alpha")      = ALPHA.L(i,r);
```

```
data(i,r,"e")           = E.L(i,r);
data(i,r,"beta")        = beta(i,r);
data("mc",r,"mc")       = mc00(r);
data(i,r,"weight")      = weight(i,r);

execute_unload ".\output\cdecalib_%ds%_lnobj.gdx";
```

Appendix B. The CDE calibration program: maximum entropy approach

```
$title  Calibrate a CDE Demand System using GTAP data

$if not set ds $set ds g20
$if not set datadir $set datadir .\input\
$if not set wt $set wt 0
$include gtap8data_old

set     info    Information about this calibration /
        ds      "%ds%",
        datadir "%datadir%",
        workdir "%gams.workdir%"
        date    "%system.date%"
        time    "%system.time%" /;

alias(i,j,k);

set rr(r) dynamic subset of r;
rr(r) = no;

parameters
z(i,r)        normalized price
theta(i,r)    value share in final demand
vafm(i,r)     Aggregate final demand,
delta(i,j,r)  diagonal-one off-diagonal-zero
sigma(i,j,r)  Allen partial elasticity of substitution
epsilon_(i,r) realized compensated own-price elasticity of demand
p0(i,r)       benchmark price index
q0(i,r)       benchmark consumption level
c0(r)         expenditure level
mc0(r)        marginal cost when u is one
weight(i,r)   weight for the square distance
beta(i,r)     scale coefficient
uncelas(i,r)  targeted uncompensated own-price demand elasticity
incelas(i,r)  targeted income demand elasticity
bound         to avoid zero division
tt            to scale the part of objective function
regind        index for region
result(r)     the real objective value for each r
;

bound  = 0.000001;
tt     = 1000;

vafm(i,r)  = vdfm(i,"c",r)*(1+rtfd0(i,"c",r))+vifm(i,"c",r)*(1+rtfi0(i,"c",r));
theta(i,r) = vafm(i,r) / (vom("c",r)*(1-rto("c",r)));
abort$sum(r, round(abs(1-sum(i,theta(i,r))),5)) "Shares do not add up.";
uncelas(i,r)             = epsilon(i,r)-eta(i,r)*theta(i,r);
incelas(i,r)             = eta(i,r);

p0(i,r)                  = 1;
q0(i,r)                  = theta(i,r)/p0(i,r);
c0(r)                    = sum(i,p0(i,r)*q0(i,r));
delta(i,j,r)             = 0;
delta(i,j,r)$sameas(i,j) = 1;
weight(i,r)              = theta(i,r)$(%wt% eq 0) + (1/card(j))$(%wt% ne 0);

*      Finish reading data
*      --------------------------------------------------------

variables
ALPHA(i,r)     substitution coefficient
```

```
V(i,r)         own-price elasticity of demand
E(i,r)         expansion coefficient
ETAV(i,r)      income elasticity of demand
ALPHAETP(r)    entropy of ALPHA
EETP(r)        entropy of E
ALPHAPNT(r)    penalty for deviations in ALPHA
EPNT(r)        penalty for deviations in E
AHAT(i,r)      deviation of uncompensated own-price demand elasticity
AHAT1(i,r)     deviation of income demand elasticity
UNCELASAC(i,r) actual uncompensated price elasticity
ALPHAMEAN(r)   mean substitution coefficient
OBJ
;

equations
objective(r)
alphaetpeq(r)
eetpeq(r)
alphapnteq(r)
epnteq(r)
icmels(i,r)
ahat1eq(i,r)
epsnml(r)
uelasaceq(i,r)  actual price elasticity
ahateq(i,r)     error in elasticity
alphameaneq(r)
;

* Objective function: maximize the entropy relative to the unknown parameters
* of the cde function
OBJECTIVE(r)$(ord(r) eq regind)..
    OBJ =E= -TT*(EPNT(R) + ALPHAPNT(R)) + EETP(R) + ALPHAETP(R);

* Penalty for errors in the expansion parameter
EPNTEQ(r)$(ord(r) eq regind)..
    EPNT(r) =E= sum(i, theta(i,r)*sqr(AHAT1(i,r)));

* Deviation of income elasticity
AHAT1EQ(i,r)$(ord(r) eq regind)..
    AHAT1(i,r) =E= ETAV(i,r) - incelas(i,r);

* Income elasticity expression found in Hanoch (1975) or Hertel et al (1990)
icmels(i,r)$(ord(r) eq regind)..

ETAV(i,r) =e= (1/sum(j,theta(j,r)*E(j,r)))*(E(i,r)*(1-
ALPHA(i,r))+sum(j,theta(j,r)*E(j,r)*ALPHA(j,r)))
          + (ALPHA(i,r)-sum(j,theta(j,r)*ALPHA(j,r)));

* Penalty for errors in the substitution parameter
alphapnteq(r)$(ord(r) eq regind)..
    ALPHAPNT(r) =E= sum(i, theta(i,r)*sqr(AHAT(i,r)));

* Deviation of uncompensated own-price demand elasticity
ahateq(i,r)$(ord(r) eq regind)..
    AHAT(i,r) =E= UNCELASAC(i,r) - uncelas(i,r);

* This last constraint pertains to the uncompensated direct price elasticities
uelasaceq(i,r)$(ord(r) eq regind)..
    UNCELASAC(i,r) =E= -(1-theta(i,r))*ALPHA(i,r) - theta(i,r)*E(i,r)
                       +  theta(i,r)*(ALPHA(i,r)*E(i,r) -
SUM(j,theta(j,r)*ALPHA(j,r)*E(j,r)));

* Cross entropy of the expansion parameter
```

```
eetpeq(r)$(ord(r) eq regind)..
    EETP(r) =E= -SUM(i, theta(i,r)*E(i,r)*log(E(i,r)));

* Normalize the expansion parameter
epsnml(r)$(ord(r) eq regind)..
    SUM(i, theta(i,r)*E(i,r)) =E= 1;

* Cross entropy of the substitution parameter
alphaetpeq(r)$(ord(r) eq regind)..
    ALPHAETP(r) =E= -SUM(i, theta(i,r)*(ALPHA(i,r)*log(ALPHA(i,r)/ALPHAMEAN(r))
                                        +(1-ALPHA(i,r))*log((1-
ALPHA(i,r))/(1-ALPHAMEAN(r)))));

* Mean substitution parameter
alphameaneq(r)$(ord(r) eq regind)..
    ALPHAMEAN(r) =E= sum(i,theta(i,r)*ALPHA(i,r));

* Variable bounds

ALPHA.LO(i,r)    = bound;
ALPHA.L(i,r)     = 0.5;
ALPHA.UP(i,r)    = 1.0 - bound;
ALPHAMEAN.L(r)   = 0.5;
E.LO(i,r)        = bound;
E.L(i,r)         = 1.0;
UNCELASAC.L(i,r) = uncelas(i,r);
ETAV.L(i,r)      = incelas(i,r);
alias (r, rreg);

model cdent /all/;

loop (rreg,
      regind=ord(rreg);
      solve cdent using nlp maximizing obj;
      display obj.l;
      result(rreg) = obj.l + tt*(sum(i,theta(i,rreg)*power(ahat.l(i,rreg),2))

+sum(i,theta(i,rreg)*power(ahat1.l(i,rreg),2)));
      epsilon_(i,r) = UNCELASAC.L(i,r)+ETAV.L(i,r)*theta(i,r);
     );

execute_unload ".\output\cdeetp_%ds%.gdx";
```

Appendix C. Sectors in GTAP 8 database

Notation used in Table 2	Code	Description
g01	PDR	Paddy rice
g02	WHT	Wheat
g03	GRO	Cereal grains nec
g04	V_F	Vegetables, fruit, nuts
g05	OSD	Oil seeds
g06	C_B	Sugar cane, sugar beet
g07	PFB	Plant-based fibers
g08	OCR	Crops nec
g09	CTL	Bovine cattle, sheep and goats, horses
g10	OAP	Animal products nec
g11	RMK	Raw milk
g12	WOL	Wool, silk-worm cocoons
g13	FRS	Forestry
g14	FSH	Fishing
g15	COA	Coal
g16	OIL	Oil
g17	GAS	Gas
g18	OMN	Minerals nec
g19	CMT	Bovine meat products
g20	OMT	Meat products nec
g21	VOL	Vegetable oils and fats
g22	MIL	Dairy products
g23	PCR	Processed rice
g24	SGR	Sugar
g25	OFD	Food products nec
g26	B_T	Beverages and tobacco products
g27	TEX	Textiles
g28	WAP	Wearing apparel
g29	LEA	Leather products
g30	LUM	Wood products
g31	PPP	Paper products, publishing
g32	P_C	Petroleum, coal products
g33	CRP	Chemical, rubber, plastic products
g34	NMM	Mineral products nec
g35	I_S	Ferrous metals
g36	NFM	Metals nec
g37	FMP	Metal products
g38	MVH	Motor vehicles and parts
g39	OTN	Transport equipment nec
g40	ELE	Electronic equipment
g41	OME	Machinery and equipment nec
g42	OMF	Manufactures nec
g43	ELY	Electricity
g44	GDT	Gas manufacture, distribution
g45	WTR	Water
g46	CNS	Construction
g47	TRD	Trade
g48	OTP	Transport nec
g49	WTP	Water transport
g50	ATP	Air transport
g51	CMN	Communication
g52	OFI	Financial services nec
g53	ISR	Insurance
g54	OBS	Business services nec
g55	ROS	Recreational and other services
g56	OSG	Public Administration, Defense, Education, Health
g57	DWE	Dwellings

Source: GTAP (2015).

Appendix D. The program (in MATLAB) checking if elasticity targets are valid

```
% Read EXCEL input: share; eps_target; eps_calib; eta_target;
eta_calib
data = xlsread('.\input\elastheta.xlsx','4x4','B3:F6');

%{
data in the worksheet "4x4"

sector  share   eps_target  eps_calib   eta_target  eta_calib
s01     0.1178  -0.4294     -0.4657     0.7300      0.8442
s02     0.2479  -0.6650     -0.7201     0.9997      1.0000
s03     0.3955  -0.7800     -0.5767     1.0543      1.0289
s04     0.2388  -0.7424     -0.7445     1.0435      1.0289
%}

% Declare dimension
n = 4;

% Check Engel aggregation (variable engel = 1 must hold)
eta_target = data(1:n, 4:4);
theta = data(1:n, 1:1);
engel = theta'*eta_target;

% Create a diagonal matrix with diagonal terms being the own-
price AUES elasticities
eps_target = data(1:n, 2:2);
theta_diag = diag(theta);
aues_diag = diag(inv(theta_diag)*eps_target);

% Initialize the determinants for checking ND (sa stores values
of various determinants)
sa = zeros(n,1);
for i = 1:n-1
    sa(i) = (-1)^(i+1);
end

while sa(1)>0|sa(2)<0|sa(3)>0|abs(sa(4))>0.00000001

% Empty aues from the previous run
aues_off = zeros(n,n);

% For each row create random variables no larger than the
|diagonal term|/n
for i = 1:n-3
    offi = (-1+2*rand)*abs(aues_diag(i,i))/n;
    for j = i+1:n-1
         aues_off(i,j) = offi;
         aues_off(j,i) = aues_off(i,j);
```

```
    end
end

aues = aues_diag + aues_off;

% Create the "A" (LHS coefficient) matrix for solving the
unknowns
A = zeros(n,n);
for i = 1:n-1
    A(i,i) = theta(n,1);
    A(n,i) = theta(i,1);
end
A(n-2,n) = theta(n-1,1);
A(n-1,n) = theta(n-2,1);

% This incomplete aues matrix is suitable for finding "C" (RHS
coefficient) matrix
C = -aues*theta;

% The unknowns are in "B" and are solved by A*B = C
B = inv(A)*C;

% Assign "B" to uknowns in aues, and now all aues unknows are
found
for i = 1:n-1
    aues(i,n) = B(i,1);
end
aues(n-2,n-1) = B(n,1);

% Assign the solved AUES unknows (i,j) to their corresponding
(j,i) elements
for i = 1:n
    for j= 1:n
aues(j,i) = aues(i,j);
    end
end

% Check Cournot aggregation
cournot = aues*theta;

% Check NSD
for i = 1:n
sa(i) = det(aues(1:i, 1:i)/10);
end

end
```

Appendix E. CDE calibration results: sequential approach

1r3s2f	θ	α	σ_{ii}^{ct}	σ_{ii}^{c}	e	η_i^t	η_i
s01	0.1178	0.4631	-0.4294	-0.4643	1.0000	0.7300	0.9999
s02	0.2479	1.0000	-0.6650	-0.7364	0.0000	0.9997	1.0000
s03	0.6343	1.0000	-0.7658	-0.3256	1.3911	1.0502	1.0000
1r4s2f							
s01	0.1178	0.4297	-0.4294	-0.4365	0.7478	0.7300	0.7300
s02	0.2479	0.9396	-0.6650	-0.7012	0.2804	0.9997	0.9997
s03	0.2388	1.0000	-0.7424	-0.7416	0.0000	1.0435	1.0502
s04	0.3955	1.0000	-0.7800	-0.5720	3.9133	1.0543	1.0503
1r5s2f							
s01	0.0343	0.1896	-0.2056	-0.2073	0.6446	0.5504	0.5504
s02	0.1160	0.5659	-0.5208	-0.5385	0.7877	0.8187	0.8187
s03	0.1041	0.7476	-0.6654	-0.6852	1.4006	1.0073	1.0073
s04	0.2831	1.0000	-0.7511	-0.6873	1.0425	1.0513	1.0479
s05	0.4624	1.0000	-0.7608	-0.4893	2.4110	1.0458	1.0479
1r8s2f							
s01	0.0335	0.1781	-0.1942	-0.1951	0.3881	0.5387	0.5387
s02	0.0228	0.4921	-0.4878	-0.4894	0.5455	0.8120	0.8120
s03	0.0940	0.5632	-0.5301	-0.5386	0.4910	0.8221	0.8221
s04	0.0978	0.7363	-0.6657	-0.6769	0.8496	1.0046	1.0046
s05	0.0436	0.7548	-0.7219	-0.7267	0.9549	1.0329	1.0329
s06	0.0705	0.7497	-0.6971	-0.7050	0.9726	1.0368	1.0368
s07	0.2421	1.0000	-0.7411	-0.7253	1.0104	1.0434	1.0502
s08	0.3955	1.0000	-0.7800	-0.5513	1.3088	1.0543	1.0502
1r16s2f							
s01	0.0294	0.1513	-0.1669	-0.1671	0.2314	0.4874	0.4874
s02	0.0041	0.3869	-0.3871	-0.3872	0.5142	0.9021	0.9020
s03	0.0009	0.6656	-0.6652	-0.6652	0.6649	1.0408	1.0408
s04	0.0219	0.4834	-0.4803	-0.4806	0.3580	0.8023	0.8023
s05	0.0192	0.4157	-0.4157	-0.4159	0.3151	0.7334	0.7334
s06	0.0749	0.5855	-0.5594	-0.5607	0.3508	0.8449	0.8448
s07	0.0310	0.6529	-0.6379	-0.6384	0.5201	0.9652	0.9651
s08	0.0669	0.7200	-0.6785	-0.6798	0.6222	1.0228	1.0227
s09	0.0033	0.6498	-0.6482	-0.6483	0.6867	1.0524	1.0523
s10	0.0403	0.7559	-0.7279	-0.7288	0.6392	1.0313	1.0313
s11	0.0445	0.7445	-0.7147	-0.7156	0.6555	1.0371	1.0371
s12	0.0260	0.6810	-0.6669	-0.6674	0.6555	1.0363	1.0363
s13	0.2078	0.9988	-0.7513	-0.7582	1.4324	1.0518	1.0418
s14	0.0344	0.6991	-0.6792	-0.6799	0.5587	0.9927	0.9927
s15	0.1307	0.9359	-0.7974	-0.8010	1.2667	1.0987	1.0985
s16	0.2648	1.0000	-0.7714	-0.6927	0.0000	1.0324	1.0404
1r29s2f							
s01	0.0014	0.0927	-0.0936	-0.0936	0.4274	0.3399	0.3399
s02	0.0160	0.1052	-0.1148	-0.1148	0.4025	0.3382	0.3383
s03	0.0007	0.0984	-0.0988	-0.0988	0.6283	0.4375	0.4375
s04	0.0029	0.1315	-0.1331	-0.1331	0.4741	0.3911	0.3911
s05	0.0052	0.3090	-0.3099	-0.3099	1.3384	0.8322	0.8322
s06	0.0031	0.2742	-0.2751	-0.2751	1.3963	0.8430	0.8430
s07	0.0040	0.3858	-0.3859	-0.3859	1.4633	0.8956	0.8956
s08	0.0002	0.4175	-0.4175	-0.4175	1.9735	1.0562	1.0562
s09	0.0009	0.6658	-0.6654	-0.6654	1.9047	1.0409	1.0409
s10	0.0182	0.5188	-0.5146	-0.5146	1.0012	0.8143	0.8143
s11	0.0169	0.4667	-0.4646	-0.4646	1.0538	0.8035	0.8035
s12	0.0060	0.2107	-0.2130	-0.2130	0.6965	0.5412	0.5412
s13	0.0653	0.5766	-0.5541	-0.5541	0.9351	0.8280	0.8280
s14	0.0298	0.6400	-0.6260	-0.6260	1.4583	0.9578	0.9578

	θ	α	σ_{ii}^{ct}	σ_{ii}^{c}	e	η_i^t	η_i
s15	0.0107	0.6381	-0.6331	-0.6331	1.5840	0.9807	0.9807
s16	0.0390	0.6960	-0.6733	-0.6733	1.6876	1.0076	1.0076
s17	0.0307	0.7007	-0.6825	-0.6825	1.9223	1.0443	1.0443
s18	0.0005	0.6415	-0.6412	-0.6412	2.0950	1.0754	1.0754
s19	0.0358	0.7612	-0.7356	-0.7356	1.8487	1.0360	1.0360
s20	0.0166	0.6963	-0.6866	-0.6866	1.8167	1.0278	1.0278
s21	0.0325	0.7445	-0.7224	-0.7224	1.8542	1.0359	1.0359
s22	0.0217	0.6728	-0.6612	-0.6612	1.8849	1.0377	1.0377
s23	0.0077	0.6791	-0.6749	-0.6749	1.8503	1.0322	1.0322
s24	0.2044	0.9942	-0.7530	-0.7530	3.3150	1.0520	1.0520
s25	0.0085	0.6395	-0.6354	-0.6354	1.6713	0.9971	0.9971
s26	0.0608	0.8046	-0.7560	-0.7560	2.1086	1.0643	1.0643
s27	0.0958	0.8884	-0.7957	-0.7957	2.7073	1.0916	1.0916
s28	0.1718	0.9652	-0.7725	-0.7725	1.0188	1.0314	1.0314
s29	0.0930	0.8527	-0.7693	-0.7693	1.7658	1.0343	1.0343
1r57s2f							
s01	0.0068	0.6543	-0.6507	-0.6507	2.1406	0.9948	0.9948
s02	0.0264	0.5688	-0.5591	-0.5591	1.2538	0.8320	0.8320
s03	0.0002	0.0910	-0.0911	-0.0911	0.6386	0.3696	0.3696
s04	0.0258	0.7105	-0.6937	-0.6937	2.0241	0.9912	0.9912
s05	0.0077	0.5204	-0.5183	-0.5183	1.2649	0.8091	0.8091
s06	0.0034	0.6500	-0.6482	-0.6482	2.4491	1.0359	1.0359
s07	0.0002	0.4169	-0.4169	-0.4169	2.5926	1.0566	1.0566
s08	0.0279	0.7035	-0.6858	-0.6858	2.4960	1.0440	1.0440
s09	0.0008	0.2753	-0.2755	-0.2755	1.9500	0.8767	0.8767
s10	0.0930	0.8570	-0.7693	-0.7693	2.2324	1.0343	1.0343
s11	0.0120	0.7014	-0.6938	-0.6938	2.4665	1.0405	1.0405
s12	0.0190	0.6667	-0.6560	-0.6560	2.4651	1.0388	1.0388
s13	0.0030	0.6789	-0.6771	-0.6771	2.5176	1.0459	1.0459
s14	0.0010	0.4853	-0.4851	-0.4851	2.5983	1.0574	1.0574
s15	0.0029	0.3503	-0.3505	-0.3505	1.7234	0.8380	0.8380
s16	0.0008	0.6807	-0.6802	-0.6802	2.4340	1.0356	1.0356
s17	0.0027	0.6948	-0.6932	-0.6932	2.3817	1.0302	1.0302
s18	0.0015	0.1116	-0.1124	-0.1124	0.7905	0.4375	0.4374
s19	0.0003	0.6087	-0.6085	-0.6085	2.7937	1.0861	1.0861
s20	0.0259	0.7789	-0.7585	-0.7585	2.1602	1.0176	1.0176
s21	0.0074	0.6077	-0.6044	-0.6044	1.9147	0.9525	0.9525
s22	0.0033	0.6985	-0.6964	-0.6964	2.4886	1.0430	1.0430
s23	0.0132	0.5107	-0.5074	-0.5074	1.3500	0.8204	0.8204
s24	0.0328	0.7660	-0.7410	-0.7410	2.3762	1.0351	1.0351
s25	0.0003	0.6721	-0.6719	-0.6719	2.6726	1.0654	1.0654
s26	0.0028	0.6511	-0.6496	-0.6496	2.5371	1.0479	1.0479
s27	0.0044	0.3155	-0.3161	-0.3161	1.7155	0.8243	0.8243
s28	0.0699	0.8784	-0.8095	-0.8095	3.9681	1.1190	1.1190
s29	0.0024	0.0696	-0.0711	-0.0711	0.4082	0.2704	0.2704
s30	0.0389	0.5646	-0.5507	-0.5507	1.2271	0.8253	0.8253
s31	0.0349	0.8333	-0.8020	-0.8020	3.5802	1.1183	1.1182
s32	0.0000	0.5457	-0.5457	-0.5457	2.3675	1.0163	1.0163
s33	0.0163	0.7321	-0.7208	-0.7208	2.4110	1.0361	1.0361
s34	0.0162	0.7354	-0.7240	-0.7240	2.4057	1.0357	1.0357
s35	0.0001	0.5628	-0.5628	-0.5628	2.7215	1.0777	1.0777
s36	0.0104	0.5146	-0.5119	-0.5119	1.3286	0.8182	0.8182
s37	0.0005	0.1011	-0.1013	-0.1013	0.8754	0.4597	0.4597
s38	0.1051	0.8856	-0.7804	-0.7804	2.0727	1.0311	1.0311
s39	0.0046	0.6701	-0.6675	-0.6675	2.1155	0.9942	0.9942
s40	0.0325	0.6297	-0.6138	-0.6138	2.1947	0.9983	0.9983

	θ	α	σ_{ii}^{ct}	σ_{ii}^{c}	e	η_i^t	η_i
s41	0.0274	0.6612	-0.6461	-0.6461	2.1463	0.9967	0.9967
s42	0.0035	0.1042	-0.1062	-0.1062	0.6533	0.3847	0.3847
s43	0.0003	0.1290	-0.1292	-0.1292	1.1345	0.5657	0.5657
s44	0.0005	0.4196	-0.4195	-0.4195	2.1107	0.9481	0.9481
s45	0.0115	0.7466	-0.7382	-0.7382	2.3761	1.0336	1.0336
s46	0.0030	0.2750	-0.2757	-0.2757	1.8077	0.8367	0.8367
s47	0.0666	0.8177	-0.7601	-0.7601	2.2719	1.0318	1.0318
s48	0.0025	0.3636	-0.3637	-0.3637	1.3976	0.7620	0.7620
s49	0.0096	0.5996	-0.5955	-0.5955	1.9731	0.9595	0.9595
s50	0.1719	0.9858	-0.7794	-0.7794	4.1494	1.0622	1.0622
s51	0.0145	0.1070	-0.1150	-0.1150	0.4832	0.3278	0.3278
s52	0.0037	0.3115	-0.3121	-0.3121	1.4169	0.7433	0.7433
s53	0.0202	0.6512	-0.6404	-0.6404	1.8649	0.9570	0.9570
s54	0.0011	0.0839	-0.0846	-0.0846	0.4105	0.2833	0.2833
s55	0.0002	0.2653	-0.2654	-0.2654	2.1931	0.9436	0.9436
s56	0.0017	0.5765	-0.5758	-0.5758	2.2908	1.0062	1.0062
s57	0.0043	0.6986	-0.6959	-0.6959	2.3714	1.0293	1.0293

Appendix F. CDE calibration results: maximum entropy approach

1r3s2f	θ	α	σ_{ii}^{ct}	σ_{ii}^{c}	e	η_i^t	η_i
s01	0.11779	0.56735	-0.42935	-0.54290	0.00001	0.72997	0.64026
s02	0.24791	0.91148	-0.66503	-0.68939	0.00001	0.99974	0.98439
s03	0.63430	0.99999	-0.76584	-0.31946	1.57654	1.05025	1.07291
1r4s2f							
s01	0.11779	0.47688	-0.42935	-0.47267	0.30153	0.72997	0.69691
s02	0.24791	0.91785	-0.66503	-0.69034	0.05446	0.99974	0.98462
s03	0.23876	0.99999	-0.74242	-0.74165	1.49754	1.04350	1.06230
s04	0.39553	0.99999	-0.77997	-0.57204	1.50032	1.05432	1.06230
1r5s2f							
s01	0.03432	0.25533	-0.20559	-0.26916	0.22697	0.55041	0.49290
s02	0.11603	0.64208	-0.52076	-0.59909	0.12963	0.81865	0.75703
s03	0.10413	0.81541	-0.66544	-0.74073	0.34188	1.00727	0.94707
s04	0.28309	0.99999	-0.75108	-0.69247	1.26147	1.05134	1.06855
s05	0.46242	0.99999	-0.76079	-0.49766	1.26389	1.04581	1.06855
1r8s2f							
s01	0.03352	0.21697	-0.19417	-0.23191	0.27903	0.53872	0.50524
s02	0.02279	0.52754	-0.48782	-0.52354	0.38500	0.81197	0.77922
s03	0.09404	0.60461	-0.53011	-0.57362	0.28488	0.82213	0.78703
s04	0.09783	0.77552	-0.66566	-0.70984	0.55209	1.00456	0.96923
s05	0.04364	0.79015	-0.72192	-0.75957	0.66587	1.03290	0.99966
s06	0.07051	0.78699	-0.69707	-0.73803	0.68360	1.03684	1.00238
s07	0.24214	0.99999	-0.74111	-0.72872	1.29038	1.04339	1.06978
s08	0.39553	0.99999	-0.77997	-0.55687	1.29285	1.05432	1.06978
1r16s2f							
s01	0.02937	0.15807	-0.16693	-0.17349	0.33003	0.48744	0.48309
s02	0.00415	0.39278	-0.38712	-0.39301	0.75346	0.90206	0.89746
s03	0.00092	0.67097	-0.66517	-0.67050	0.96645	1.04085	1.03612
s04	0.02187	0.48901	-0.48033	-0.48602	0.51246	0.80230	0.79804
s05	0.01917	0.42144	-0.41568	-0.42141	0.45036	0.73339	0.72916
s06	0.07487	0.59118	-0.55941	-0.56565	0.49446	0.84486	0.84049
s07	0.03095	0.65831	-0.63789	-0.64360	0.74680	0.96517	0.96065
s08	0.06687	0.72558	-0.67851	-0.68480	0.89316	1.02279	1.01784
s09	0.00330	0.65520	-0.64824	-0.65365	1.00117	1.05236	1.04757
s10	0.04035	0.76113	-0.72795	-0.73366	0.91363	1.03131	1.02653
s11	0.04451	0.74982	-0.71468	-0.72051	0.94049	1.03715	1.03228
s12	0.02600	0.68642	-0.66691	-0.67260	0.94999	1.03631	1.03148
s13	0.20777	0.99654	-0.75135	-0.75725	1.04063	1.05178	1.04730
s14	0.03437	0.70443	-0.67923	-0.68492	0.80025	0.99270	0.98812
s15	0.13072	0.93584	-0.79738	-0.80115	1.66686	1.09873	1.08995
s16	0.26481	0.99999	-0.77138	-0.69316	1.05077	1.03240	1.04716
1r29s2f							
s01	0.00143	0.09410	-0.09357	-0.09498	0.22477	0.33991	0.34056
s02	0.01600	0.10662	-0.11479	-0.11612	0.21207	0.33825	0.33892
s03	0.00069	0.10003	-0.09883	-0.10044	0.32785	0.43748	0.43792
s04	0.00290	0.13281	-0.13312	-0.13438	0.24916	0.39110	0.39172
s05	0.00520	0.31026	-0.30995	-0.31123	0.69479	0.83221	0.83232
s06	0.00315	0.27565	-0.27506	-0.27646	0.72419	0.84299	0.84306
s07	0.00398	0.38691	-0.38590	-0.38704	0.76003	0.89563	0.89572
s08	0.00017	0.41870	-0.41753	-0.41870	1.02280	1.05623	1.05609
s09	0.00092	0.66650	-0.66536	-0.66601	0.99417	1.04089	1.04089
s10	0.01817	0.51938	-0.51461	-0.51517	0.52572	0.81432	0.81489
s11	0.01690	0.46753	-0.46461	-0.46536	0.55130	0.80348	0.80392
s12	0.00597	0.21189	-0.21299	-0.21418	0.36404	0.54116	0.54163
s13	0.06527	0.57708	-0.55410	-0.55441	0.49379	0.82800	0.82875
s14	0.02982	0.64072	-0.62598	-0.62656	0.76412	0.95776	0.95809

	θ	α	σ_{ii}^{ct}	σ_{ii}^{c}	e	η_i^t	η_i
s15	0.01073	0.63875	-0.63307	-0.63370	0.82849	0.98065	0.98088
s16	0.03895	0.69671	-0.67330	-0.67386	0.88450	1.00760	1.00781
s17	0.03068	0.70142	-0.68252	-0.68313	1.00500	1.04433	1.04433
s18	0.00054	0.64222	-0.64124	-0.64196	1.09070	1.07543	1.07529
s19	0.03579	0.76172	-0.73564	-0.73607	0.97206	1.03603	1.03618
s20	0.01658	0.69692	-0.68660	-0.68718	0.95065	1.02778	1.02788
s21	0.03249	0.74508	-0.72240	-0.72288	0.97329	1.03591	1.03603
s22	0.02171	0.67358	-0.66119	-0.66185	0.98430	1.03770	1.03771
s23	0.00765	0.67976	-0.67491	-0.67553	0.96691	1.03219	1.03224
s24	0.20439	0.98732	-0.75305	-0.74862	1.37647	1.05204	1.04761
s25	0.00855	0.64016	-0.63545	-0.63611	0.87329	0.99708	0.99724
s26	0.06077	0.80518	-0.75598	-0.75634	1.10962	1.06429	1.06420
s27	0.09577	0.88820	-0.79570	-0.79535	1.42731	1.09161	1.09061
s28	0.17177	0.96110	-0.77251	-0.76951	0.83040	1.03136	1.03624
s29	0.09304	0.85299	-0.76929	-0.76933	0.94566	1.03432	1.03485
1r572f							
s01	0.00681	0.65468	-0.65067	-0.65101	0.84353	0.99476	0.99522
s02	0.02640	0.56886	-0.55911	-0.55914	0.49822	0.83196	0.83292
s03	0.00017	0.09223	-0.09111	-0.09233	0.25212	0.36960	0.37035
s04	0.02582	0.71068	-0.69373	-0.69386	0.80181	0.99124	0.99191
s05	0.00775	0.52059	-0.51827	-0.51849	0.50089	0.80914	0.80998
s06	0.00337	0.65045	-0.64823	-0.64866	0.96254	1.03589	1.03616
s07	0.00017	0.41785	-0.41692	-0.41784	1.01271	1.05659	1.05665
s08	0.02792	0.70389	-0.68575	-0.68607	0.98322	1.04398	1.04428
s09	0.00078	0.27643	-0.27545	-0.27660	0.76189	0.87675	0.87696
s10	0.09304	0.85664	-0.76927	-0.76884	0.90365	1.03432	1.03544
s11	0.01203	0.70175	-0.69382	-0.69413	0.97172	1.04050	1.04082
s12	0.01903	0.66713	-0.65599	-0.65639	0.96942	1.03880	1.03907
s13	0.00301	0.67925	-0.67710	-0.67748	0.99030	1.04588	1.04614
s14	0.00105	0.48613	-0.48513	-0.48591	1.01593	1.05735	1.05744
s15	0.00293	0.35123	-0.35053	-0.35143	0.67488	0.83797	0.83832
s16	0.00081	0.68103	-0.68020	-0.68055	0.95810	1.03557	1.03589
s17	0.00273	0.69515	-0.69315	-0.69346	0.93856	1.03015	1.03052
s18	0.00152	0.11284	-0.11243	-0.11367	0.31113	0.43746	0.43811
s19	0.00026	0.60920	-0.60854	-0.60908	1.09443	1.08610	1.08616
s20	0.02590	0.77890	-0.75851	-0.75848	0.86042	1.01763	1.01839
s21	0.00739	0.60808	-0.60442	-0.60479	0.75442	0.95247	0.95300
s22	0.00334	0.69880	-0.69638	-0.69670	0.98011	1.04296	1.04326
s23	0.01320	0.51101	-0.50737	-0.50768	0.53343	0.82036	0.82110
s24	0.03278	0.76611	-0.74100	-0.74111	0.94180	1.03512	1.03564
s25	0.00028	0.67250	-0.67193	-0.67234	1.04989	1.06543	1.06559
s26	0.00276	0.65150	-0.64960	-0.65003	0.99662	1.04786	1.04807
s27	0.00442	0.31645	-0.31607	-0.31705	0.67139	0.82431	0.82463
s28	0.06985	0.87761	-0.80949	-0.80876	1.56298	1.11904	1.11816
s29	0.00239	0.07074	-0.07113	-0.07224	0.16292	0.27043	0.27139
s30	0.03887	0.56469	-0.55069	-0.55071	0.48778	0.82531	0.82628
s31	0.03495	0.83315	-0.80198	-0.80181	1.41104	1.11825	1.11783
s32	0.00000	0.54637	-0.54574	-0.54637	0.92773	1.01626	1.01647
s33	0.01632	0.73234	-0.72077	-0.72099	0.95231	1.03608	1.03649
s34	0.01617	0.73560	-0.72405	-0.72426	0.95049	1.03574	1.03616
s35	0.00012	0.56348	-0.56280	-0.56343	1.06525	1.07767	1.07774
s36	0.01042	0.51487	-0.51188	-0.51216	0.52528	0.81818	0.81895
s37	0.00052	0.10241	-0.10135	-0.10270	0.34384	0.45970	0.46028
s38	0.10513	0.88465	-0.78040	-0.77956	0.85702	1.03107	1.03276
s39	0.00456	0.67044	-0.66755	-0.66783	0.83463	0.99424	0.99475
s40	0.03252	0.63018	-0.61379	-0.61422	0.86338	0.99835	0.99872

	θ	α	σ_{ii}^{ct}	σ_{ii}^{c}	e	η_i^t	η_i
s41	0.02743	0.66158	-0.64607	-0.64640	0.84604	0.99669	0.99715
s42	0.00350	0.10541	-0.10620	-0.10736	0.25795	0.38467	0.38542
s43	0.00029	0.13040	-0.12919	-0.13055	0.44445	0.56568	0.56614
s44	0.00052	0.42040	-0.41952	-0.42036	0.82592	0.94812	0.94836
s45	0.01153	0.74676	-0.73824	-0.73841	0.93999	1.03358	1.03406
s46	0.00296	0.27616	-0.27569	-0.27680	0.70672	0.83670	0.83697
s47	0.06663	0.81758	-0.76006	-0.75991	0.90890	1.03182	1.03263
s48	0.00248	0.36437	-0.36375	-0.36448	0.54894	0.76202	0.76255
s49	0.00960	0.60002	-0.59548	-0.59589	0.77669	0.95945	0.95993
s50	0.17186	0.97988	-0.77940	-0.77534	1.49251	1.06220	1.05916
s51	0.01448	0.10798	-0.11504	-0.11600	0.19225	0.32782	0.32873
s52	0.00370	0.31240	-0.31206	-0.31294	0.55569	0.74329	0.74375
s53	0.02022	0.65146	-0.64045	-0.64067	0.73704	0.95695	0.95760
s54	0.00115	0.08493	-0.08462	-0.08562	0.16399	0.28326	0.28424
s55	0.00019	0.26653	-0.26536	-0.26658	0.85608	0.94356	0.94369
s56	0.00174	0.57705	-0.57583	-0.57639	0.89885	1.00619	1.00647
s57	0.00428	0.69889	-0.69591	-0.69620	0.93490	1.02927	1.02965

Appendix G. The CGE model with a CDE demand in GTAPinGAMS[15]

```
$title      Read GTAP8 Base data and Replicate the Benchmark in MPSGE
* To run the model, type, for example: gams mrtmge_cde --start=0.1 --end=20 --
step=0.1

* The following pre-assignment for ds will be used in a $gdxin command in
gtap8data.gms
$if not set ds $set ds 2r4s1f

$if not set wt $set wt 0

* Sets, parameters declarations and assignments are done in gtap8data.gms
$include ..\build\gtap8data

set c(g) private consumption /c/;
set e(g) exogenous consumption /g, i/;

parameters
esub(g)         Top-level elasticity in demand /C 1/
vcm(i,c,r)      Tax included Armington good i for private consumption,
data(*,*,*)     Output from cdecalib,
cde             CDE calibration,
chkd(i,r)       Check final expenditure D;

* Aggregate final demand (Armington good)
vcm(i,c,r) = vdfm(i,c,r)*(1+rtfd0(i,c,r))+vifm(i,c,r)*(1+rtfi0(i,c,r));

* Read the CDE coefficients
*execute_load ".\input\cdecalib_%ds%_wt=%wt%.gdx" data = data;
execute_load ".\input\cdecalib_%ds%_lnobj.gdx" data = data;
cde(i,r,"alpha")      = data(i,r,"alpha");
cde(i,r,"e")          = data(i,r,"e");
cde(i,r,"beta")       = data(i,r,"beta");
cde("utility",r,"u") = 1;
cde("mc",r,"mc")      = data("mc",r,"mc");

cde(i,r,"alpha")$(cde(i,r,"alpha") eq eps)              = 0;
cde(i,r,"e")$(cde(i,r,"e") eq eps)                      = 0;
cde(i,r,"beta")$(cde(i,r,"beta") eq eps)                = 0;
cde("utility",r,"u")$(cde("utility",r,"u") eq eps)      = 0;
cde("mc",r,"mc")$(cde("mc",r,"mc") eq eps)              = 0;

$ontext
$model:gtap8

$sectors:
    y(g,r)$(not c(g) and vom(g,r))        ! Supply
    m(i,r)$vim(i,r)                       ! Imports
```


[15] To run this MPSGE program “mrtmge_cde.gms,” one needs to 1) place it inside the subdirectory “model” of GTAPinGAMS; 2) set either price shock or income shock within the loop; 3) set the output file name that distinguishes price shock from income shock; and 4) type, for example, “gams mrtmge_cde --start=0.1 --end=20 --step=0.1” under the DOS command prompt. With the default setting, this will produce 20 different price shocks for the agricultural product—the first shock will be created by multiplying both vdfm("agri",c,"usa") and vifm("agri",c,"usa") by 0.1, and for each following shock, the multiplicand increases by 0.1 compared to that in the previous shock.

```
    yt(j)$vtw(j)                        ! Transportation services
    ft(f,r)$(sf(f) and evom(f,r))       ! Specific factor transformation
    yc(i,c,r)$vcm(i,c,r)                ! Private consumption by commodity

$commodities:
    p(g,r)$vom(g,r)                     ! Domestic output price
    pm(j,r)$vim(j,r)                    ! Import price
    pt(j)$vtw(j)                        ! Transportation services
    pf(f,r)$evom(f,r)                   ! Primary factors rent
    ps(f,g,r)$(sf(f) and vfm(f,g,r))    ! Sector-specific primary factors
    pc(i,c,r)$vcm(i,c,r)                ! Private consumption price

$consumers:
    ra(r)                               ! Representative agent

$auxiliary:
    TC(r)                               ! Expenditure for the CDE system
    U(r)                                ! Activity level of Utility
    D(i,r)$vcm(i,"c",r)                 ! Activity level of final consumption

* Sectoral output
$prod:y(j,r)$vom(j,r)      s:esub(j)      i.tl:esubd(i)  va:esubva(j)
     o:p(j,r)       q:vom(j,r)     a:ra(r)  t:rto(j,r)
     i:p(i,r)       q:vdfm(i,j,r)  p:(1+rtfd0(i,j,r))   i.tl:  a:ra(r)
t:rtfd(i,j,r)
     i:pm(i,r)      q:vifm(i,j,r)  p:(1+rtfi0(i,j,r))   i.tl:  a:ra(r)
t:rtfi(i,j,r)
     i:ps(sf,j,r)   q:vfm(sf,j,r)  p:(1+rtf0(sf,j,r))   va:    a:ra(r)
t:rtf(sf,j,r)
     i:pf(mf,r)     q:vfm(mf,j,r)  p:(1+rtf0(mf,j,r))   va:    a:ra(r)
t:rtf(mf,j,r)

* Government consumption and investment (exogenous consumption)
$prod:y(e,r)$vom(e,r)      s:esub(e)      i.tl:esubd(i)
     o:p(e,r)       q:vom(e,r)     a:ra(r)  t:rto(e,r)
     i:p(i,r)       q:vdfm(i,e,r)  p:(1+rtfd0(i,e,r))   i.tl:  a:ra(r)
t:rtfd(i,e,r)
     i:pm(i,r)      q:vifm(i,e,r)  p:(1+rtfi0(i,e,r))   i.tl:  a:ra(r)
t:rtfi(i,e,r)

* Private consumption: new
* Level 1: Armington good of commodity i
$prod:yc(i,c,r)$vcm(i,c,r) s:esubd(i)
        o:pc(i,c,r)     q:vcm(i,c,r)                          a:ra(r)
t:rto(c,r)
        i:p(i,r)        q:vdfm(i,c,r)     p:(1+rtfd0(i,c,r))   a:ra(r)
t:rtfd(i,c,r)
        i:pm(i,r)       q:vifm(i,c,r)     p:(1+rtfi0(i,c,r))   a:ra(r)
t:rtfi(i,c,r)

* Level 2: Aggregate various goods to a single consumption good c.  This is
where we need to work on for CDE.
* Let's temporarily remove the declaration of y(c,r), and move the sources and
* sinks in this block to demand block.
* This strategy is similar to linking the top-down and bottom-up.
* Now this is moved to the representative agent block.

$prod:yt(j)$vtw(j)      s:1
     o:pt(j)        q:vtw(j)
     i:p(j,r)       q:vst(j,r)

$prod:m(i,r)$vim(i,r)      s:esubm(i)      s.tl:0
     o:pm(i,r)      q:vim(i,r)
```

```
    i:p(i,s)       q:vxmd(i,s,r)  p:pvxmd(i,s,r)       s.tl:  a:ra(s)  t:(-
rtxs(i,s,r))  a:ra(r)  t:(rtms(i,s,r)*(1-rtxs(i,s,r)))
    i:pt(j)#(s)    q:vtwr(j,i,s,r) p:pvtwr(i,s,r)       s.tl:  a:ra(r)
t:rtms(i,s,r)

$prod:ft(sf,r)$evom(sf,r)  t:etrae(sf)
    o:ps(sf,j,r)   q:vfm(sf,j,r)
    i:pf(sf,r)     q:evom(sf,r)

$demand:ra(r)
    d:p("c",r)     q:vom("c",r)
    e:p("c",rnum)  q:vb(r)
    e:p("g",r)     q:(-vom("g",r))
    e:p("i",r)     q:(-vom("i",r))
    e:pf(f,r)      q:evom(f,r)
    e:p(c,r)       q:vom(c,r)      r:U(r)
    e:pc(i,c,r)    q:(-vcm(i,c,r)) r:D(i,r)

$constraint:TC(r)
        sum(i,cde(i,r,"beta")*(cde("utility",r,"u")*U(r))**(cde(i,r,"e")*(1-
cde(i,r,"alpha")))*
        (PC(i,"c",r)/TC(r))**(1-cde(i,r,"alpha"))) =e= 1;

$constraint:U(r)
        TC(r)*sum(i,cde(i,r,"beta")*cde(i,r,"e")*(1-
cde(i,r,"alpha"))*(U(r)**(cde(i,r,"e")*(1-cde(i,r,"alpha"))-
1))*(PC(i,"c",r)/TC(r))**(1-cde(i,r,"alpha")))
         =e= data("mc",r,"mc")*P("c",r)*sum(i,cde(i,r,"beta")*(1-
cde(i,r,"alpha"))*(U(r)**(cde(i,r,"e")*(1-
cde(i,r,"alpha"))))*(PC(i,"c",r)/TC(r))**(1-cde(i,r,"alpha")));

$constraint:D(i,r)$(vcm(i,"c",r))
        vcm(i,"c",r)/vom("c",r)*D(i,r)*sum(j,cde(j,r,"beta")*(U(r)**((1-
cde(j,r,"alpha"))*cde(j,r,"e")))*(1-cde(j,r,"alpha"))*(PC(j,"c",r)/TC(r))**(1-
cde(j,r,"alpha")))
         =e= (cde(i,r,"beta")*(U(r)**((1-cde(i,r,"alpha"))*cde(i,r,"e")))*(1-
cde(i,r,"alpha"))*(pc(i,"c",r)/TC(r))**(-cde(i,r,"alpha")));


$offtext

$sysinclude mpsgeset gtap8

TC.L(r)   = 1;
TC.LO(r)  = 0.000001;
U.L(r)    = 1;
U.LO(r)   = 0.000001;
D.L(i,r)  = 1;
D.LO(i,r) = 0.000001;
PF.FX("primary","usa") = 1;

gtap8.workspace = 128;
gtap8.iterlim = 0;
$include gtap8.gen
solve gtap8 using mcp;

chkd(i,r) = vcm(i,"c",r)/vom("c",r)*D.L(i,r)
          - (cde(i,r,"beta")*(U.L(r)**((1-cde(i,r,"alpha"))*cde(i,r,"e")))*(1-
cde(i,r,"alpha"))*(PC.L(i,"c",r)/TC.L(r))**(-cde(i,r,"alpha")))
            /sum(j,cde(j,r,"beta")*(U.L(r)**((1-
cde(j,r,"alpha"))*cde(j,r,"e")))*(1-cde(j,r,"alpha"))*PC.L(j,"c",r)**(1-
cde(j,r,"alpha"))*TC.L(r)**cde(i,r,"alpha"));
```

```
execute_unload ".\output\mrtmge_cde_ref_ds=%ds%.gdx";

* The code below is for testing whether the model's realized elasticities equal
the calibrated levels it is given to

$if not set step $set step 0
set x shock level /1*%end%/
parameters
step        step of the shock level,
start       initial shock coefficient,
vdfm0       vdfm value from GTAP,
vifm0       vifm value from GTAP,
evom0       evom value from GTAP,
pfx         realized PF with shock level x,
pcx         realized PC over PF with shock level x,
dx          realized D with shock level x,
theta_i     final consumption expenditure share,
eta_i       calibrated income demand point elasticity,
priexp      total private expenditure,
priexpi     total private expenditure index,
eta_i_a     calibrated arc income demand elasticity,
sigma       calibrated AUES price demand elasticity (point elasticity),
delta(i,j,r)       diagonal-one off-diagonal-zero,
sigma_c     calibrated compensated price demand elasticity (point elasticity),
sigma_m     calibrated Marshallian price demand elasticity (point elasticity),
sigma_ma    calibrated Marshallian price demand elasticity (arc elasticity),
dxn         realized D with shock level x net of prices & income effects,
sigma_mar   realized Marshallian price elasticity (arc elasticity),
cds         change in d due to changes in other prices,
cdi         change in d due to change in income,
eqi         expected quantity due to pure income effect,
cqp         change in quantity due to change in own-price,
dxi         realized D with shock level x net of prices effects,
eta_i_ar    realized arc income demand elasticity;

alias(i,k);

* Assign start and step in the command line using environment variables
start   = %start%;
step    = %step%;

* Read the shares and calibrated elasticities
theta_i(i,r,"theta")        = data(i,r,"theta");
eta_i(i,r,"etav")           = data(i,r,"etav");

* Store the original vdfm, vifm, and evom in GTAP
vdfm0(i,c,r)                = vdfm(i,c,r);
vifm0(i,c,r)                = vifm(i,c,r);
evom0(f,r)                  = evom(f,r);

* Step 1: Calculate the Marshallian price demand elasticity (point elasticity)
delta(i,j,r)                = 0;
delta(i,j,r)$sameas(i,j)    = 1;
sigma(i,j,r)                = cde(i,r,"alpha")+cde(j,r,"alpha")-sum(k,
theta_i(k,r,"theta")*cde(k,r,"alpha"))
                      -delta(i,j,r)*cde(i,r,"alpha")/theta_i(i,r,"theta");
sigma_c(i,j,r)              = sigma(i,j,r)*theta_i(i,r,"theta");
sigma_m(i,j,r)              = sigma_c(i,j,r)-
eta_i(i,r,"etav")*theta_i(i,r,"theta");

loop(x,

* Consumer's price shock:
```

```
vdfm("agri",c,"usa") = vdfm0("agri",c,"usa")*(start+(ord(x)-1)*step);
vifm("agri",c,"usa") = vifm0("agri",c,"usa")*(start+(ord(x)-1)*step);

* Endowment shock:
*evom(f,"usa") = evom0(f,"usa")*(start+(ord(x)-1)*step);

* Avoid raise 0 by a negative number in the third auxiliary equation
PC.LO(i,"c",r) = 0.000001;

gtap8.iterlim = 50000;
$include gtap8.gen
solve gtap8 using mcp;

* Step 2: Within the loop, derive the calibrated arc elasticities associated
with the shock

** Marshallian price demand elasticity (arc elasticity)
pfx(r,x)   = PF.L("primary",r);
pcx(i,r,x) = PC.L(i,"c",r)/pfx(r,x);
sigma_ma(i,j,r,x)$(pcx(j,r,x) ne 1) = (pcx(j,r,x)**sigma_m(i,j,r)-
1)/(pcx(j,r,x)-1)*(pcx(j,r,x)+1)/(pcx(j,r,x)**sigma_m(i,j,r)+1);
sigma_ma(i,j,r,x)$(pcx(j,r,x) eq 1) = sigma_m(i,j,r);

** Income demand elasticity (arc elasticity)
priexp(r,x) = sum(i,pcx(i,r,x)*D.L(i,r)*vcm(i,"c",r));
priexpi(r,x) = priexp(r,x)/sum(i,vcm(i,"c",r));
eta_i_a(i,r,x)$(priexpi(r,x) ne 1) = (priexpi(r,x)**eta_i(i,r,"etav")-
1)/(priexpi(r,x)-1)
            *(priexpi(r,x)+1)/(priexpi(r,x)**eta_i(i,r,"etav")+1);
eta_i_a(i,r,x)$(priexpi(r,x) eq 1) = eta_i(i,r,"etav");

* Step 3-1: Calculate the substitution effect due to changes in PC-
others|original income; after shock PC-own
dx(i,r,x)  = D.L(i,r);
cds(i,r,x) = sum(j$(not sameas(i,j)), (pcx(j,r,x)-
1)/((pcx(j,r,x)+1)/2)*sigma_ma(i,j,r,x)*(1+sigma_ma(i,i,r,x)*(pcx(i,r,x)-
1)/((1+pcx(i,r,x))/2)*((1+dx(i,r,x))/2)));

* Step 3-2: Calculate the income effect on top of changes in all PC
cdi(i,r,x) = (priexpi(r,x)-
1)/((priexpi(r,x)+1)/2)*eta_i_a(i,r,x)*(1+sigma_ma(i,i,r,x)*(pcx(i,r,x)-
1)/((1+pcx(i,r,x))/2)+cds(i,r,x));

* Step 3-3: Calculate the adjusted demand net of cross-price effect and income
effect
dxn(i,r,x) = dx(i,r,x)-cds(i,r,x)-cdi(i,r,x);
sigma_mar(i,j,r,x)$(pcx(j,r,x) ne 1) = (dxn(i,r,x)-1)/((dxn(i,r,x)+1)/2) /
((pcx(j,r,x)-1)/((pcx(j,r,x)+1)/2));

* Step 3-4: Calculate expected quantity level due to pure income effect
eqi(i,r,x) = (priexpi(r,x))**eta_i(i,r,"etav");

* Step 3-5: Based on the expected quantity derived from pure income effect,
calculate the quantity changes due to changes in prices
cqp(i,r,x) = sum(j, (pcx(j,r,x)-
1)/((pcx(j,r,x)+1)/2)*sigma_ma(i,j,r,x)*eqi(i,r,x));

* Step 3-6: Substract quantity change due to price effect from the observed
quantity
dxi(i,r,x) = dx(i,r,x) - cqp(i,r,x);

* Step 3-7: Calculate the realized income demand elasticity
```

```
eta_i_ar(i,r,x) = (dxi(i,r,x)-1)/((dxi(i,r,x)+1)/2)/((priexpi(r,x)-
1)/((priexpi(r,x)+1)/2));

execute_unload ".\output\mrtmge_cde_policy_ds=%ds%_priceshock=%step%.gdx";
*execute_unload ".\output\mrtmge_cde_policy_ds=%ds%_incomeshock=%step%.gdx";

);
```