Riser Dynamic Analysis Using WKB-Based Dynamic Stiffness Method

Proceedings of the ASME 2012 31st International Conference on Ocean, Offshore and Arctic Engineering
OMAE2012
July 1-6, 2012, Rio de Janeiro, Brazil

OMAE2012-84142

RISER DYNAMIC ANALYSIS USING WKB-BASED DYNAMIC STIFFNESS METHOD

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ABSTRACT

Risers are fluid conduits from subsea equipment to surface floating production platforms. The integrity of a riser system plays a very important role in deepwater developments. Riser dynamic analysis is an important part to the system design. This paper investigates riser dynamic analysis using the WKB-Based dynamic stiffness method.

This paper first presents a theoretical formulation of the dynamic stiffness method. It then combines the dynamic stiffness method with the WKB theory, which assumes that the coefficients in the differential equation of motion are slowly varying. The WKB-based dynamic stiffness method is derived and a frequency dependent shape function is expressed implicitly. The Wittrick and Williams (W-W) algorithm is further extended to solve eigen value problem for a general non-uniform marine riser.

Examples of non-uniform riser are analyzed and the results show the efficiency of this method. In addition, a pipe-in-pipe riser system is analyzed for natural frequencies and mode shapes using the WKB-based dynamic stiffness method with the W-W algorithm. The characteristic of the mode shapes is described for such a riser system.

INTRODUCTION

A riser is a fluid conduit from subsea equipment to the surface floating production system such as a Spar or TLP. It is a key component in a deepwater drilling and production system. Its dynamic design and analysis is very important in the deepwater applications.

Historically, Kolousek first presented the idea of Dynamic Stiffness Method (DSM) in the early 1940s [1], and gave an elaborate formulation of this method in 1950 [2]. Since then the DSM has been widely used in the vibration analysis of beam structures. Improvements on calculating natural frequencies have been made by the Williams and Wittrick (W-W) algorithm [3, 4].

The DSM has a great appeal for an exact dynamic analysis of a uniform beam structure, as it is based on the exact dynamic stiffness matrix derived from the free vibration analysis. The DSM performs free and forced vibration analysis within the differential equation theory of beams, thus avoiding assumed modes and lumped masses. This method enables one to analyze an infinite number of natural frequencies and modes accurately by means of fewer degrees of freedom, compared with a traditional finite element method by using a polynomial shape function.

This paper develops the WKB-based dynamic stiffness formulation for a riser system, which assumes that the coefficients in the differential equation of motion are slowly varying. The WKB-based dynamic stiffness method is derived and a frequency dependent shape function is expressed implicitly.

The Wittrick and Williams (W-W) algorithm is extended to solve eigen value problem for a general non-uniform marine riser. Examples are analyzed and the results show the efficiency of this method. In addition, a pipe-in-pipe riser system is analyzed for natural frequencies and mode shapes using the WKB-based dynamic stiffness method with the W-W algorithm.

The characteristic of the mode shapes is described for such a riser system.

THEORETICAL FORMULATION OF A RISER SYSTEM

WKB-based dynamic stiffness matrix

A great deal of research has focused on vibration analysis of a beam structure. For a uniform Euler beam under a constant axial load, the effect of the axial load on the natural frequencies has been found by considering the natural frequencies to be functions of a non-dimensional load parameter and boundary conditions. Using a power series expansion, Dareing and Huang [5] found the natural frequencies of a uniform marine drilling riser.

A dynamic riser model is needed which is able to account for non-uniform properties such as mass density, bending rigidity and tension distribution, and discontinuities such as intermediate supports. A closed form solution to such a system is not generally possible. An approximation to the vibration analysis of such a riser may be accomplished by replacing the variable parameters with constant ones. For example, a variable axial load is often approximated by a tension that is constant over each element. However, many degrees of freedom in the approximation are required in order to obtain accurate results.

This paper investigates the vibration analysis of marine risers by combining the dynamic stiffness method with the WKB theory, which assumes that the coefficients in the differential equation of motion are slowly varying.

A general marine riser is a long slender beam system with variable tension distribution, bending rigidity and mass density. The mass/length changes are often discontinuous. Such a riser can be discretized into elements having continuously varying properties within the elements and allowing discontinuities to occur between elements.

The equation of motion of a riser is written as:

$$\frac{\partial^2}{\partial x^2}[EI(x)\frac{\partial^2 w}{\partial x^2}]-\frac{\partial}{\partial x}[T(x)\frac{\partial w}{\partial x}]+m(x)\frac{\partial^2 w}{\partial t^2}=f(x,t) \qquad (1)$$

where w is the transverse displacement of riser, E is the Young's modulus of the material, $I(x)$ is the area moment inertia of the beam, $T(x)$ is the tension of the riser, $m(x)$ is mass per unit length, and $f(x,t)$ is external force per unit length.

The dimensionless parameters are defined as follows:

$$s=x/l, \quad \omega_0=\sqrt{\frac{E_0 I_0}{m_0 l^4}}, \quad \tau=\omega_0 t, \quad Y=\frac{w}{D_0},$$

Where the subscript '0' represents the values at a reference cross section, l is length of a riser element and D_0 is a reference diameter for the riser.

Eq. (1) is thus written into the following non-dimensional form:

$$\frac{\partial^2}{\partial s^2}[P(s)\frac{\partial^2 Y}{\partial s^2}]-\frac{\partial}{\partial s}[Q(s)\frac{\partial Y}{\partial s}]+U(s)\frac{\partial^2 Y}{\partial \tau^2}=f(s,\tau) \qquad (2)$$

Assuming $Y(s,\tau)=R(s)e^{i\Lambda\tau}$ and substituting it into Eq. (2) result in the following equation of motion of a free vibration:

$$\frac{d^2}{ds^2}[P(s)\frac{d^2 R}{ds^2}]-\frac{d}{ds}[Q(s)\frac{dR}{ds}]-U(s)\Lambda^2 R=0, \qquad (3)$$

where Λ is a dimensionless frequency, $\Lambda=\omega/\omega_0$.

Assuming that $P(s)$, $Q(s)$ and $U(s)$ in Eq. (3) vary slowly with respect to s, compared with variations of $R(s)$, $R'(s)$ and $R''(s)$, rewrite Eq. (3) as:

$$\varepsilon^4 P(z)\frac{d^4 R}{dz^4}+2\varepsilon^4 P'(z)\frac{d^3 R}{dz^3}+[\varepsilon^4 P''(z)-\varepsilon^2 Q(z)]\frac{d^2 R}{dz^2}-\varepsilon^2 Q'(z)\frac{dR}{dz}-U(z)\Lambda^2 R=0 \qquad (4)$$

Where $z\equiv\varepsilon s$, ε is a small parameter.

The formal WKB expansion is written as:

$$R(z)\approx\exp[\tfrac{1}{\delta}\sum_{n=0}^{\infty}\delta^n S_n(z)], \qquad \delta\to 0. \qquad (5)$$

The following asymptotic solution can be found by substituting Eq. (5) into (4), identifying the same order terms, truncating the series and selecting $\delta=\varepsilon$.

$$\begin{aligned}R(s)&=T_2(s)[C_1\sin(\int_0^s h_2(\xi)d\xi)+C_2\cos(\int_0^s h_2(\xi)d\xi)]\\&+T_1(s)[C_3\sinh(\int_0^s h_1(\xi)d\xi)+C_4\cosh(\int_0^s h_1(\xi)d\xi)]\end{aligned} \qquad (6)$$

Where, $C_i\ (i=1\ to\ 4)$ are constants of integrations; $T_i(s), h_i(s)\quad (i=1\ to\ 2)$ are functions of $P(s)$, $Q(s)$, $U(s)$ and Λ.

Note that: $\frac{dw}{dx}=\frac{D_0}{l}\frac{dY}{ds}$, $\frac{d^2w}{dx^2}=\frac{D_0}{l^2}\frac{d^2Y}{ds^2}$, and $\frac{d^3w}{dx^3}=\frac{D_0}{l^3}\frac{d^3Y}{ds^3}$. Neglecting higher order terms, then the element nodal displacement vector, $\mathbf{V_e}$, can be formulated in the following matrix form:

$$\begin{pmatrix} v_{1y} \\ \theta_1 \\ v_{2y} \\ \theta_2 \end{pmatrix} = D_0 \begin{bmatrix} 0 & T_2(0) & 0 & T_1(0) \\ \frac{T_2(0)h_2(0)}{l} & 0 & \frac{T_1(0)h_1(0)}{l} & 0 \\ B_1(1)T_2(1) & B_2(1)T_2(1) & B_3(1)T_1(1) & B_4(1)T_1(1) \\ \frac{B_2(1)T_2(1)h_2(1)}{l} & -\frac{B_1(1)T_2(1)h_2(1)}{l} & \frac{B_4(1)T_1(1)h_1(1)}{l} & \frac{B_3(1)T_1(1)h_1(1)}{l} \end{bmatrix} \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} \quad (7)$$

In which, $B_1(s) = \sin\int_0^s h_2(\xi)d\xi$, $B_2(s) = \cos\int_0^s h_2(\xi)d\xi$, $B_3(s) = \sinh\int_0^s h_1(\xi)d\xi$, $B_4(s) = \cosh\int_0^s h_1(\xi)d\xi$.

The Eq. (7) can be written in abbreviated form as:

$\mathbf{V_e}$ = D_0 **G C**. (8)

The nodal forces, $\mathbf{F}_e$, for an element with changing properties are formulated as:

$$m_1 = -EI(x)D_0 \frac{d^2R}{dx^2}\Big|_{x=0} = -\frac{EI(s)D_0}{l^2}\frac{d^2R}{ds^2}\Big|_{s=0}$$
$$m_2 = EI(x)D_0 \frac{d^2R}{dx^2}\Big|_{x=l} = \frac{EI(s)D_0}{l^2}\frac{d^2R}{ds^2}\Big|_{s=1}$$
$$s_{1y} = -EI(x)D_0 \frac{d^3R}{dx^3}\Big|_{x=0} - T(x)D_0\frac{dR}{dx}\Big|_{x=0} = -\frac{EI(s)D_0}{l^3}\frac{d^3R}{ds^3}\Big|_{s=0} - \frac{T(s)D_0}{l}\frac{dR}{ds}\Big|_{s=0}$$
$$s_{2y} = -EI(x)D_0 \frac{d^3R}{dx^3}\Big|_{x=l} + T(x)D_0\frac{dR}{dx}\Big|_{x=l} = -\frac{EI(s)D_0}{l^3}\frac{d^3R}{ds^3}\Big|_{s=1} + \frac{T(s)D_0}{l}\frac{dR}{ds}\Big|_{s=1}$$
(9)

Substituting R(s) from Eq. (6), Eqs. (9) can be written as the following matrix form:

$\mathbf{F_e}$ =D_0 **H C**. (10)

The relationship between the element nodal forces and displacements can be established by combining Eq. (8) with Eq. (10):

$\mathbf{F}_e = \mathbf{K}_e(\omega)\mathbf{V}_e$, (11)

In which $\mathbf{K}_e(\omega) = \mathbf{H}\,\mathbf{G}^{-1}$, is the WKB-based dynamic element stiffness matrix, whose elements were derived by using Maple V.

Frequency Dependent Shape Function

In order to derive the frequency dependent shape function, rewrite Eq. (6) as:

$$R(s) = \begin{bmatrix} T_2(s)\sin\int_0^s h_2(\xi)d\xi \\ T_2(s)\cos\int_0^s h_2(\xi)d\xi \\ T_1(s)\sinh\int_0^s h_1(\xi)d\xi \\ T_2(s)\cosh\int_0^s h_1(\xi)d\xi \end{bmatrix}^T \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix}. \quad (12)$$

The constants of integrations C_i ($i = 1$ *to* 4) are solved from Eq. (8) as:

C = 1/D_0 $\mathbf{G}^{-1}$ $\mathbf{V_e}$. (13)

Substituting Eq. (13) into Eq. (12) results in:

R(s) = **Φ** $\mathbf{V_e}$, (14)

Where **Φ** is the frequency dependent shape function.

Global Dynamic Stiffness Matrix Formulation

The dynamic stiffness formulation of a riser system is obtained by establishing a weak form of the equation of motion using the Galerkin procedure. Integrating over the domain of interest s and transforming to lower the order of the derivatives and incorporate the boundary conditions as forcing terms gives the variational equations to be discretized by finite element interpolations.

The formulation of the spectrum element method for a riser system is thus developed by following the procedure of the conventional finite element method [8], in which local elements are cast into a global form by coordinate transformations. The equation of motion of free vibration in the restrained global dynamic stiffness form can be written as:

$\mathbf{K_G}(\omega)\,\mathbf{X} = \mathbf{0}$. (15)

The elements of global stiffness matrix, $\mathbf{K_G}$ (**ω**), are generally transcendental functions of circular frequency **ω**. Natural frequencies can be found by equating to zero the determinant of the global dynamic stiffness matrix, $\mathbf{K_G}$ (**ω**). The eigenvalues, or the natural frequencies, are obtained by plotting det ($\mathbf{K_G}$ (**ω**)) and finding the roots.

For a uniform beam member, Wittrick and Williams (W-W) [3] presented an automatic computation of natural frequencies. For a tapered beam whose section properties vary regularly, Banerjee and Williams [4] gave a procedure to calculate natural frequencies.

However, a typical marine riser has non-uniform properties including mass distribution, bending rigidity and tension. The procedures in [3, 4] can't be directly used for the application in this paper. The W-W algorithm is thus extended to a general non-uniform riser system for an automatic computation of natural frequencies [6].

Once the natural frequencies are found, one can use Eq. (15) to solve for a specific mode shape. An effective way is to use a triangular decomposition.

Similarly, the equation of motion for a riser system under a forced excitation can be formulated as:

$$\mathbf{K_G}\,(\omega)\,\mathbf{X} = \mathbf{F}\,(\omega)\ . \qquad (16)$$

The riser frequency response can be solved by using the algorithms based on Gauss elimination [7]. The skyline reduction method is used in the computer implementation of the Gauss elimination.

A computational program has been developed to solve for the natural frequencies and mode shapes, and the frequency response for a generally riser system.

In addition, the above WKB-based dynamic stiffness formulation has been extended to a coupled pipe-in-pipe riser system [9].

RESULTS

<u>Uniform Drilling Riser under Linearly Varying Tension [5]</u>

The parameters of a simply supported riser are:

Length $L = 500$ feet;

Outer diameter $d_0 = 24$ inches;

Wall Thickness $t = 0.625$ inches;

Young's modulus $E = 30 \times 10^6$ lbf/in^2;

Mass per unit length $m = 20.8$ slugs/ft (includes mass of drilling mud and sea water);

Tension at the bottom ball joint T_0 =286,000 lbs; and

Net weight of riser per unit length in sea water $w = 214$ lb/ft (including 38 lb/ft for choke and kill lines).

Figure 1 shows the determinant of the dynamic stiffness matrix of the 500-ft riser versus frequency.

Table 1 lists the first five natural frequencies found from Figure 1. In order to verify the results, a finite element procedure which assumed constant tension over each beam element was developed. Converged values for natural frequencies were found employing 60 elements in the FEM. The approximation result [5] obtained by means of a power series expansion is also included for comparison. It is observed from Table 1 that the natural frequencies acquired by the WKB-based dynamic stiffness method using only five elements are accurate. Figure 2 depicts the first three mode shapes.

Table 1 indicates that the natural frequencies obtained by Dareing and Huang [5] are also accurate, compared with those obtained by using the FEM and the WKB-based dynamic stiffness method. However, their finding of ``points of inflection" in mode shapes is not correct.

In addition, only one element is needed to obtain accurate natural frequencies by using the W-W algorithm.

Table 1 Comparison of Circular Natural Frequencies

Order	Dareing and Huang [5]	FEM (60 elements)	WKB-DSM (5 elements)
1	0.8150	0.8150	0.8150
2	1.8036	1.8038	1.8037
3	3.0876	3.0879	3.0875
4	4.7375	4.7377	4.7375
5	6.7890	6.7896	6.7890

<u>Non-Uniform Riser under Linearly Varying Tension</u>

Due to attachments such as buoyancy modules, a typical marine riser is a system with variable properties including tension and mass density. Such a riser system, simply supported, has the following properties:

Length $L = 689.29$ m;

Outer steel diameter $d_o = 21.00$ inches;

Wall Thickness $t = 0.625$ inches; and

Buoyancy diameter $d_b = 44.50$ inches.

Figures 3 and 4 show the variations of the mass and tension at the measured points which are marked respectively. The position is measured from the bottom. These figures demonstrate that the mass density does not change continuously, and tension does not vary linearly.

There are eleven (11) segments in Figures 3 and 4, each of which has continuous variation of mass and tension. Figure 5 shows the first 20 natural frequencies found by using the WKB-based dynamic stiffness analysis with 11 elements. The approximate results using Shear7, which assumed the riser to be an equivalent uniform beam with an average linearly varying tension along the riser, are included for comparison. The Shear7 results are accurate only for lower order natural frequencies.

Figure 6 depicts the 20th mode shape, slope and curvature. The modal information is important to predict VIV fatigue damages to the riser. The locations of the antinodes are not evenly spaced. Therefore, the mode differs from trigonometric ones.

It is found that 282 elements are needed for the standard finite element method to obtain a good 20th mode shape and a converged natural frequency of 0.6952 Hz. This is close to 0.6955 Hz by the WKB-based dynamic stiffness method with only 11 elements. Very few elements are necessary if they are chosen wisely. Within each element, properties must vary slowly so as to satisfy the WKB assumptions. Discontinuities should occur at the junctions of elements. In this example, the mass/length changes abruptly ten times requiring a total of eleven elements to adequately model the system.

Coupled Pipe-In-Pipe Riser System

A coupled pipe-in-pipe riser system, shown in Figure 7, is used for demonstrating the applications. Both the external and internal casings are simply supported and their specifications are as follows:

Outer diameter of external pipe =13.375 inches;

Wall thickness of external pipe =0.380 inches;

Added mass coefficient for external pipe=1.0;

Outer diameter of internal pipe =9.75 inches;

Wall thickness of internal pipe =0.2975 inches;

Young's modulus E =30000 ksi;

Length of both cylinders L =1944 ft;

Minimum tension on external pipe $T_{10} = 2.5\times10^4$ lbs;

Tension varying factor of external pipe = 47.00 lbf/ft;

Minimum tension on internal pipe $T_{20} = 1.0\times10^4$ lbs;

Tension varying factor of internal pipe =30.63 lbf/ft;

Number of evenly distributed identical centralizers=19; and

The distance between centralizers l = 97.2 ft.

Each riser is discretized into 20 evenly distributed elements. The centralizers are evenly distributed along the risers. The following dimensionless stiffness k^* is used to describe the relative stiffness of centralizers.

$$k^* = k_n l^3 / 48 E_0 I_0 ,$$

where the subscript '0' denotes the standard reference values for the pipe and l is the distance between centralizers.

The pipe-in-pipe riser system is coupled by centralizers and ideal fluid in the annulus. Table 2 lists the natural frequencies and includes those coupling cases for springs and fluid only for comparison. This table demonstrates that the fluid lowers the natural frequencies. The case coupled by fluid only generates the lowest natural frequencies while that coupled by centralizers only generates the highest natural frequencies. The natural frequencies for a general case coupled by fluid and centralizers lie in between the two other cases.

Figure 8 illustrates the first 8 mode shapes $\phi_i, (i = 1,8)$ of the coupled riser system. It indicates that when the system is weakly coupled by centralizers, its mode shapes are either in-phase or out-of-phase. The difference in the first two modes, ϕ_1 and ϕ_2, is that the deformation of the internal riser is larger in mode one, ϕ_1, while that of the external riser is larger in mode 2, ϕ_2.

The natural frequencies of the coupled system increase with the stiffness of centralizers.

Table 2 Natural frequencies (Hz) of a coupled riser system

Order	Springs only	Fluid only	Spring/Fluid
1	0.0344	0.0144	0.0294
2	0.0692	0.0290	0.0368
3	0.0778	0.0335	0.0408
4	0.0982	0.0439	0.0525
5	0.1050	0.0591	0.0658
6	0.1259	0.0672	0.0681
7	0.1417	0.0748	0.0804
8	0.1576	0.0912	0.0960
9	0.1796	0.1020	0.1023
10	0.1923	0.1082	0.1125

CONCLUSIONS

This paper investigates the riser dynamic analysis using WKB-based dynamic stiffness method. The conclusions that can be drawn from the work in this paper are:

(1) The theoretical formulation for a general non-uniform riser system is constructed by using a spectrum element method and WKB-based frequency-dependent shape functions. The theoretical formulation can be extended to a coupled pipe-in-pipe riser system.

(2) The W-W algorithm is extended to the WKB-based dynamic stiffness method for an automatic computation of natural frequencies for a non-uniform riser. The minimum elements are needed to accurately compute natural frequencies and modal information. The advantage of this approach is evident for solving high order natural frequencies.

(3) The fluid/riser coupling can be designed to suppress the vibration of an external casing caused by VIV [9]. The coupling can be optimized to provide damping to the external casing.

ACKNOWLEDGEMENT

This paper represents part of the research work when the first author studied at MIT for his Ph.D. with the second author as his supervisor. Thanks to SHEAR7 JIP members who supported the work.

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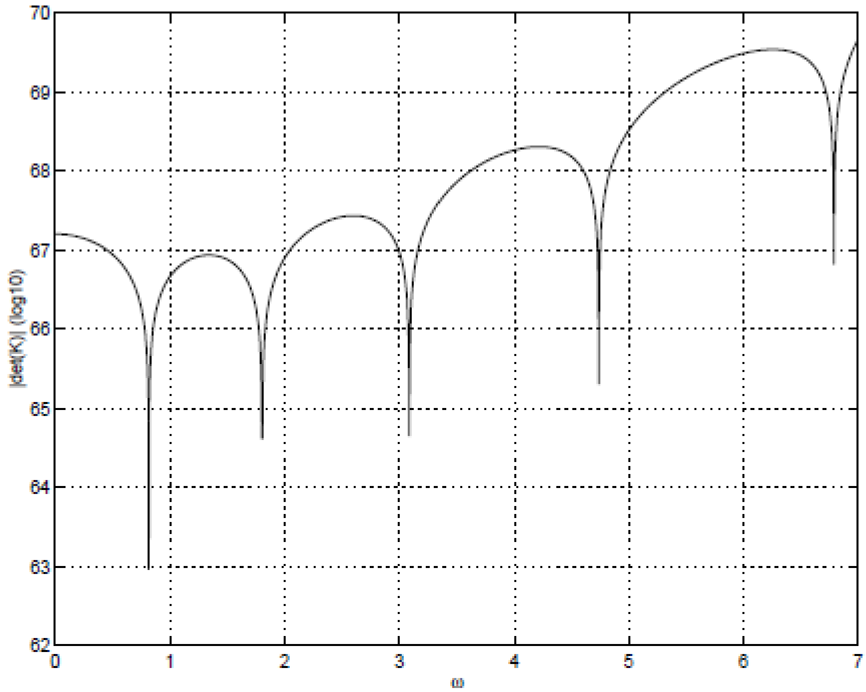

Figure 1 Determinant of the Dynamic Stiffness Matrix

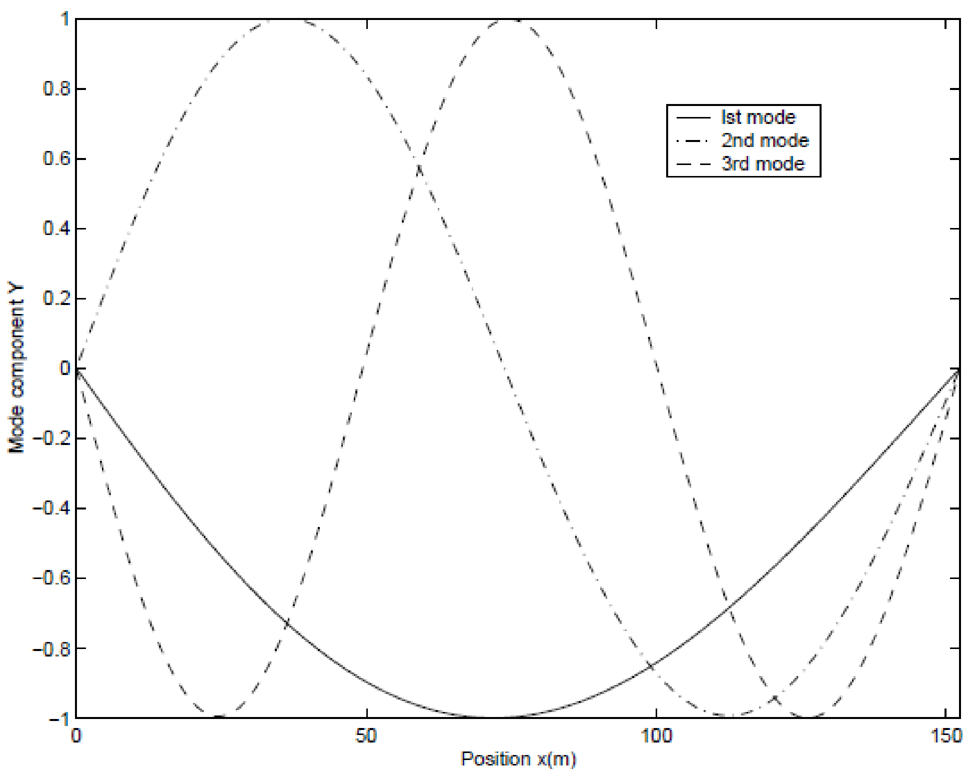

Figure 2 First Three Natural Mode Shapes

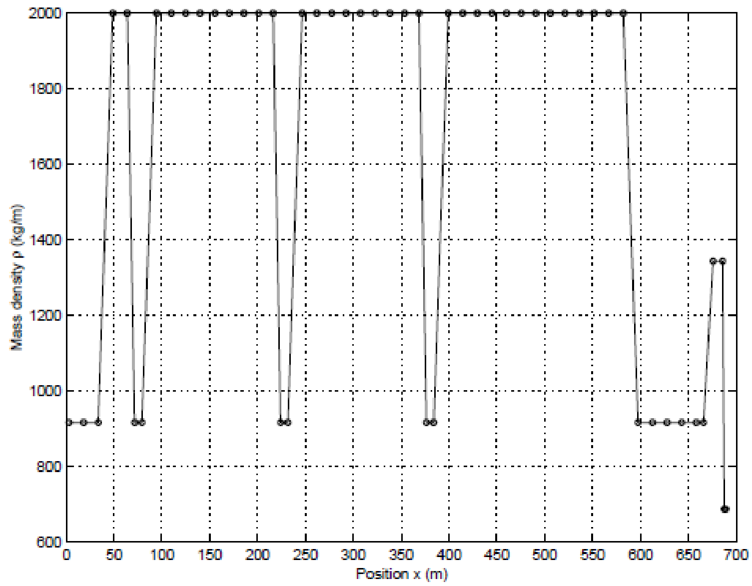

Figure 3 Mass Variation along the Riser

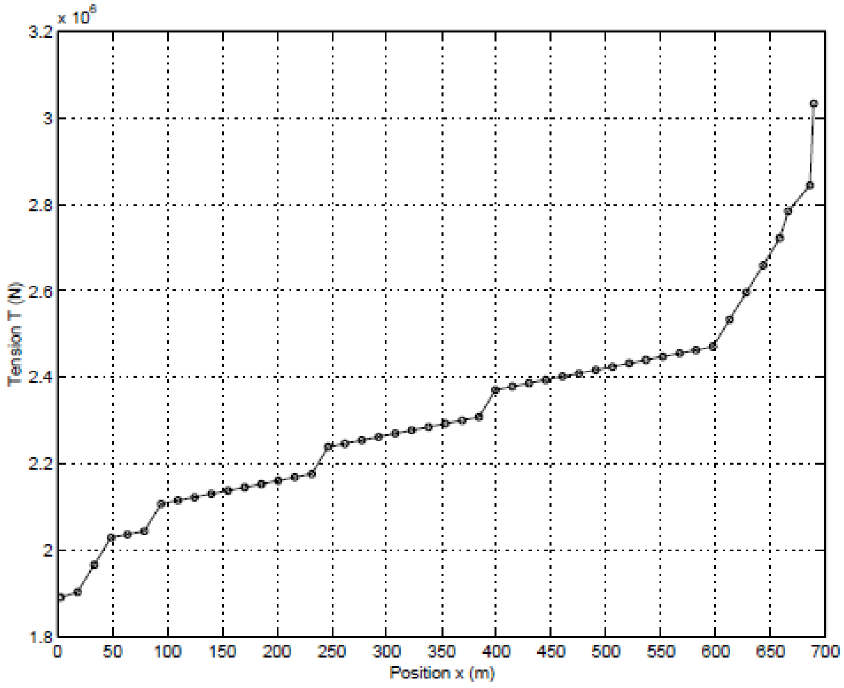

Figure 4 Tension Variation along the Riser

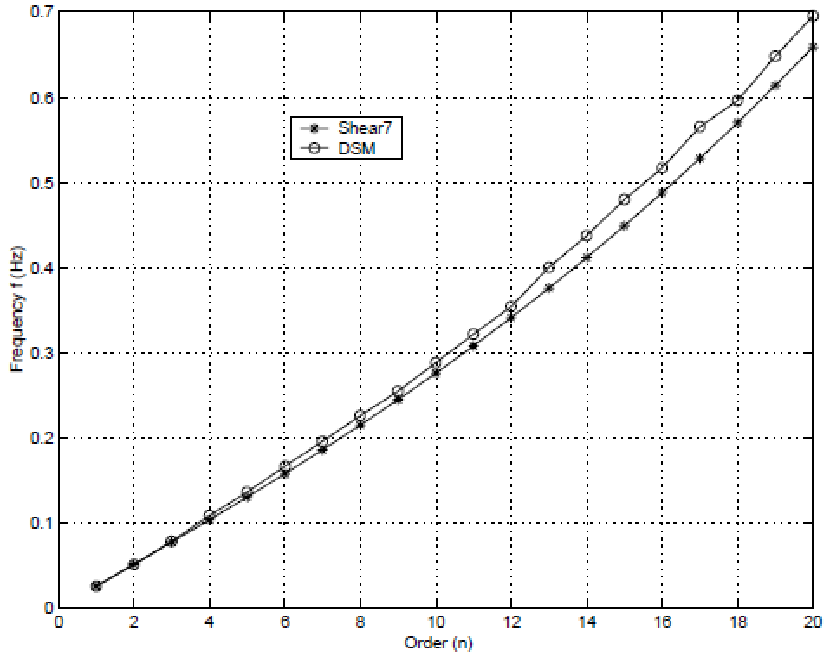

Figure 5 Natural Frequencies of a Non-Uniform Riser

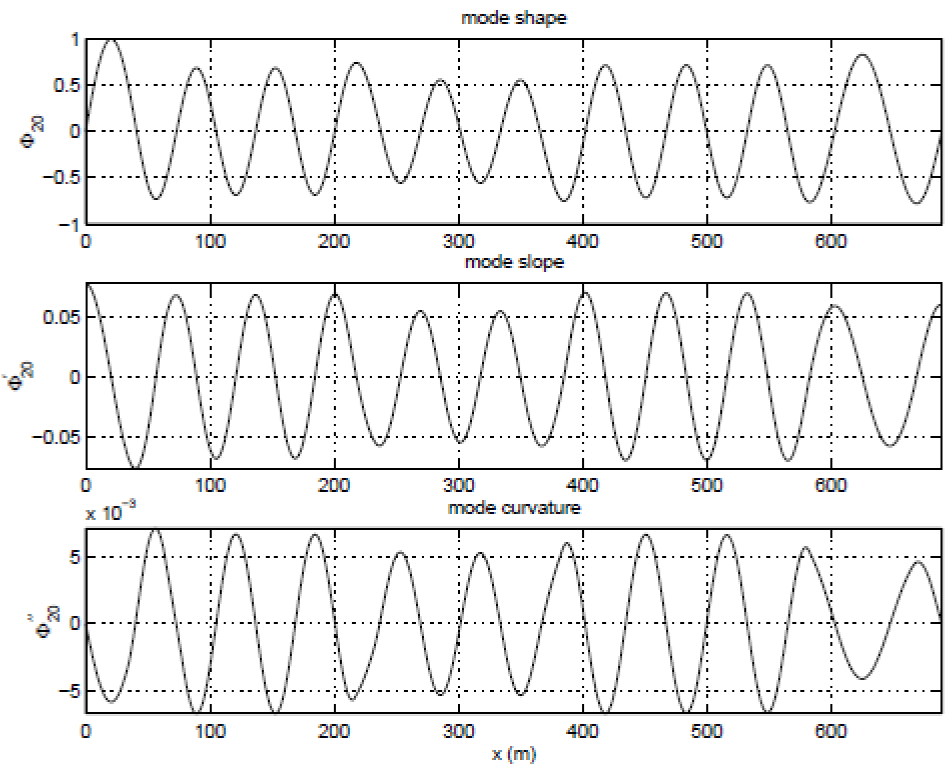

Figure 6 20th Modal Shape, Slope and Curvature of the Riser

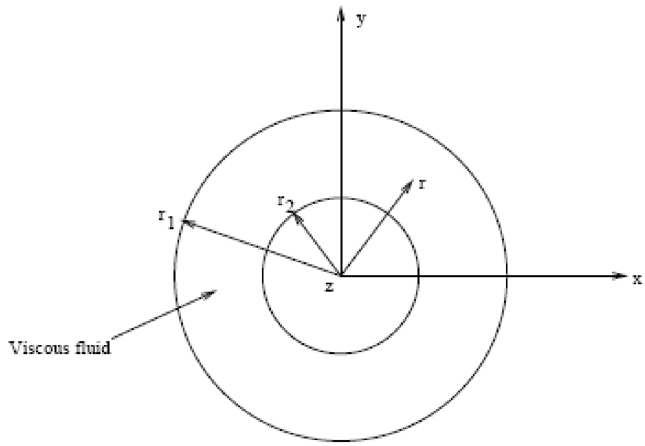

Figure 7 Schematic of Two Concentric Pipes Containing Viscous Fluid

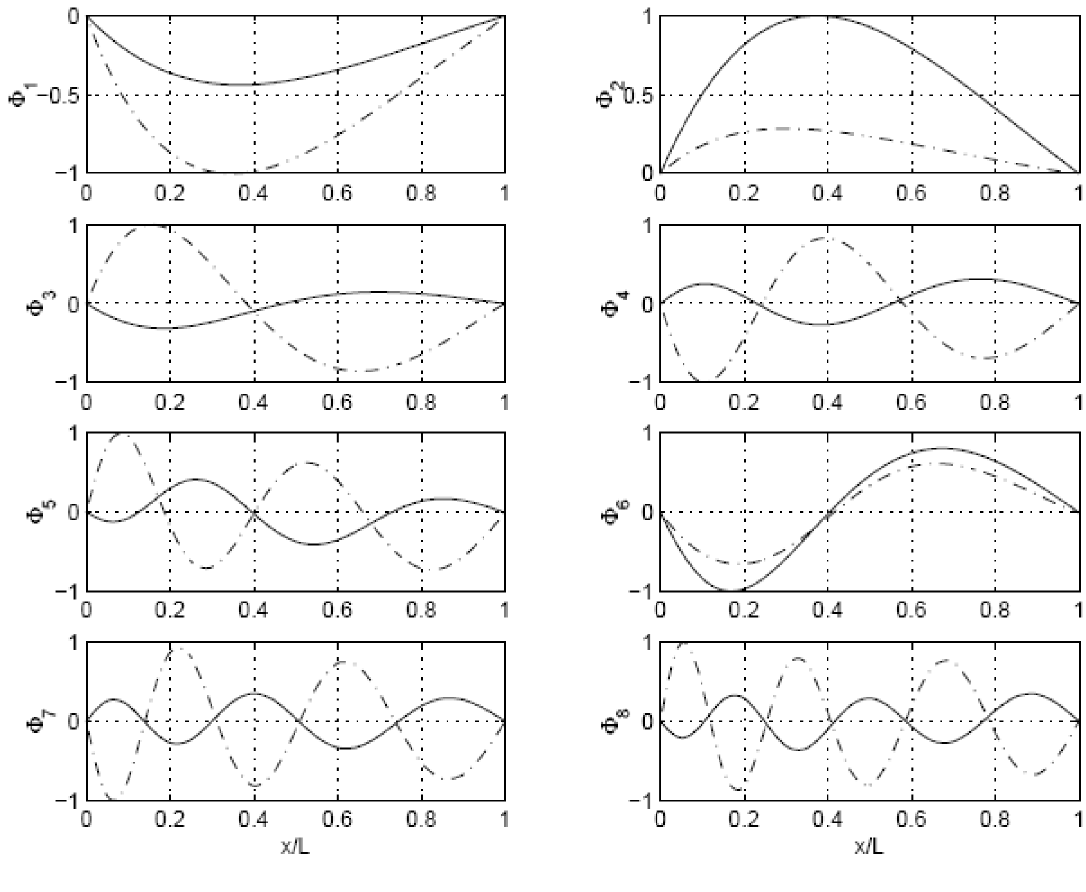

Figure 8 First 8 Mode Shapes of the Coupled Riser System

(k^*=0.02, solid line: external riser; dash-dot line: internal riser)