# VALUATION OF METAL RESERVES UNDER PRICE UNCERTAINTY BY USING OPTION PRICING MODEL

by

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## ABSTRACT

This thesis is an empirical testing of the option method on valuation of metal mine under the price uncertainty and comparison with the conventional valuation, DCF.

The methodology utilized was contingent claim analysis developed by Professor Robert S. Pindyck.

The manager of metal mine has some leeway of decision depending on the volatile metal price change. This flexibility option endows manager with the opportunity of stopping loss when metal price is lower than the operating cost and consequently enhancing whole mine value.

Boundary conditions, smoothing the flexibility option, are constructed for finding the optimal switching rules. The results of model suggest that the manager of mine can maximize the project value by switching the operation mode according to the optimal decision rules, depending on the volatility of metal price.

Thesis Supervisor: Robert S. Pindyck

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# 1. Introduction

This thesis has explored the practical application of option pricing analysis and limits of using conventional discounted cash flow method, represented by NPV method, for valuation of metal mine, whose revenue and profit fluctuates widely due to exogenous metal price variation.

Discounted cash flow methods can be poorly suited for project evaluation in the presence of flexibility of managerial decision on operation mode. Options occur commonly in the context of investment timing decisions, shutdown, or abandonment decisions.

Metal mine project usually spans a long-term horizon of 20 to 30 years. It is difficult to forecast accurately the certain path of the metal price for such a long period. Furthermore, historically the metal prices have followed random walk with the volatility of 15-30 percent. Conventional discounted cash flow methods will fail when future cash flows must be conditional to future managerial decisions under an uncertain exogenous economic environment. The risk-adjusted capital cost by CAPM does not fully reflect the managerial flexibility, which can postpone the investment timing or stop the loss by abandoning or suspending the operation for the time being during the adverse economic environment.

The option approach recognizes the financial value of flexibility, which affects the profitability of the project under uncertainty. Consequently, the value of the mine by the option method looks larger than the value by the discounted cash flow method. That difference is just the value of managerial flexibility, the option which a manager can exercise to limit the contingent loss when the economic environment deteriorates negatively (Value of Mine = Conventional Value (NPV) + Option Value). The option method also provides a guide for the optimal timing of the decision rule (thresholds) on when to invest, mothball, or abandon the mine under a dynamic economic environment.

It is most important to incorporate options analysis into project valuation when the ultimate best operating mode in the future is difficult to forecast today; uncertainty is sufficiently great that differences in profits across possible operating modes are substantial.

Among several real option evaluation methods, this thesis used the Professor Robert S. Pindyck's contingent claim analysis model for evaluating the value of the Tintaya copper mine in Peru retrospectively, which was privatized by an international auction tender on October 6th, 1994. The option value results are compared with the actual bidding values.

With the assumption of the manager's perfectly free flexibility, the value of the Tintaya mine becomes 3.2 time the tender award amount. But practical limitations on the flexibility may decrease the option value and consequantely, the value of the mine will decrease toward the value by conventional NPV method. For instance, union, the covenants of project financing, or government interference shall be the limitations on manager's flexible operating mode decisons.

### 1.1. Capital Investment

Most capital investment decisions share three important characteristics in varying degrees. First, the investment is partially or completely irreversible. Second, there is uncertainty over the future rewards from investment. The best we can do is to assess the probabilities of the alternative outcomes that can mean greater or smaller profit (or loss) for the investment. Third, we have some leeway about the timing of the investment.

These three characteristics interact to determine the optimal decisions of investors.

### 1.2. The Objective of Thesis

The objective of this thesis is a valuation of metal reserves under price uncertainty and formation of decision rules (thresholds) to signal optimal time to exercise the flexibility option through contingent claim analysis method.

The valuation of metal reserves is an important issue in itself. Firms perform valuations as inputs to their bidding decisions. The government or private owner of metal reserves uses valuations to establish presale reserves prices. Recently there have been many privatization tenders of government owned metal reserves in South America and former Eastern Europe countries.

The Peruvian Government privatized Tintaya Copper Mine, a state owned copper mine, on October 6, 1994. The bids received varied from US$ 243 million to US$ 73 million, which seemed to be caused by different price forecasting among bidders. I was impressed by this result and interested in the proper objective way of valuing the metal reserves under price uncertainty by option method.

This thesis is organized as follows. Chapter 2 develops the basic model of contingent claim analysis method for valuation of typical metal reserves, the copper mine. Chapter 3 presents empirical results drawn from a sample operating copper mine. Chapter 4 summarizes of this paper.

### 1.3. Net Present Value

The most popular technique used for capital investment decision today is the discounted cash flow (DCF) method. The net present value of the mine is calculated by discounting expected future free cash flows using risk - adjusted discount rate. The discount rate is determined according to the given cash flow's undiversifiable risk using the well-known capital asset pricing model (CAPM). Then it is determined whether the NPV is greater than zero. If it is, it would be fine to go ahead and invest.

Performing these calculations correctly, however, is very difficult, especially in the case of metal mines whose project life usually span 30 to 40 years. Furthermore the DCF approach as applied has the following weakness that inhibits correct project value determination.

1. The proper timing of investment is not transparent.

2. Different companies, as well as the government, may have different assessments of future statistical distributions of project product price, and thus expected paths of revenue for many years into the future, none of which need conform to the aggregate expectations held by capital markets.

3. The process of choosing the correct set of risk-adjusted discount rates in the presence of the complex statistical structure of the cash

flows is a difficult task, which is also subject to a great deal of subjectivity and error.

These problems lead to valuations that are divergent between companies, the government, and the capital markets.[1]

One good example is the privatization tender of Tintaya Copper Mine in Peru. The dispersion of bids is assumed to be from the wide difference of price forecasting among bidders. This indicates that the conventional DCF method has the limits on valuation of the project when the project outcome shall vary enormously by the exogenous random walking product price.

The DCF method is based on some implicit assumptions that are often overlooked. Most importantly, it assumes that either the investment is reversible, that is, it can somehow be undone and the expenditures recovered should market conditions turn out to be worse that anticipated, or, if the investment is irreversible, it is a now or never proposition, that is, if the firm does not undertake the investment now, it will not be able to do so in the future. So conventional economical analysis based on discounted cash flow methods fail to capture the strategic impact of mines. In particular, the DCF method ignores the operating flexibility that gives mine manager the option to revise decision in response to changing exogenous economical conditions. The importance of such operating option becomes

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[1] Paddock, Siegel & Smith (1988)

critical when the environment is highly volatile, thus permitting managerial intervention at little cost.[2] While price uncertainty is unimportant in applications for which the relevant prices are reasonably predictable, it is of paramount importance in many natural resource industries, where price swings of 25% - 40% per year are not uncommon.[3] Under such conditions the practice of replacing distributions of future prices by their expected value is likely to cause errors in the calculation both of expected cash flows and of appropriate discount rates. This would thereby lead to a suboptimal investment decision.

### 1.4. Option Methods

It is now well known that when an investment is irreversible and future market conditions are uncertain, an investment decision must not be based solely on the usual net present value rule.

Irreversibility and the possibility of delay are very important characteristics of most investments in reality. The flexibility afforded by these decision possibilities may contribute significantly to the value of the project. What's more, the degree of managerial discretion in making future operating decisions will tend to affect the risk of the project under consideration. So the ability to delay an irreversible investment expenditure

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[2] Nalin Kulatilaka (1993)
[3] Brennan & Schwartz (1985)

can profoundly effect the decision to invest. That is because a firm with an opportunity to invest is holding an "option" analogous to a financial call option. When a firm makes an irreversible investment expenditure, it exercises, or "kills", its option to invest. This lost option value is an opportunity cost that must be included as part of the investment. The decision rule must be to invest when the value of the mine exceeds the mine development cost, by an amount equal to the value of keeping the investment option alive.

The recent trend in solving these contingent decision problems is to use option pricing techniques originally developed by Black and Scholes (1973) and Merton(1973) to analyze the pricing of options on common stocks. This is because of compelling similarities between a manager's decision-making process, contingent upon future information about mine uncertainties, and a financial option holder's decision-making process, contingent upon future stock price which is uncertain at present.

A capital investment on metal reserves (mines) is one of these cases with future uncertainties and management flexibilities.

When the value of flexibility is added, the real value of mine is around two to four times of the value calculated by the NPV method. Also this flexibility value lets the mine keep operating even when the spot price is lower than the mining cost.

# 2. Modeling

## 2.1. Valuation of Copper Mine under Price Uncertainty

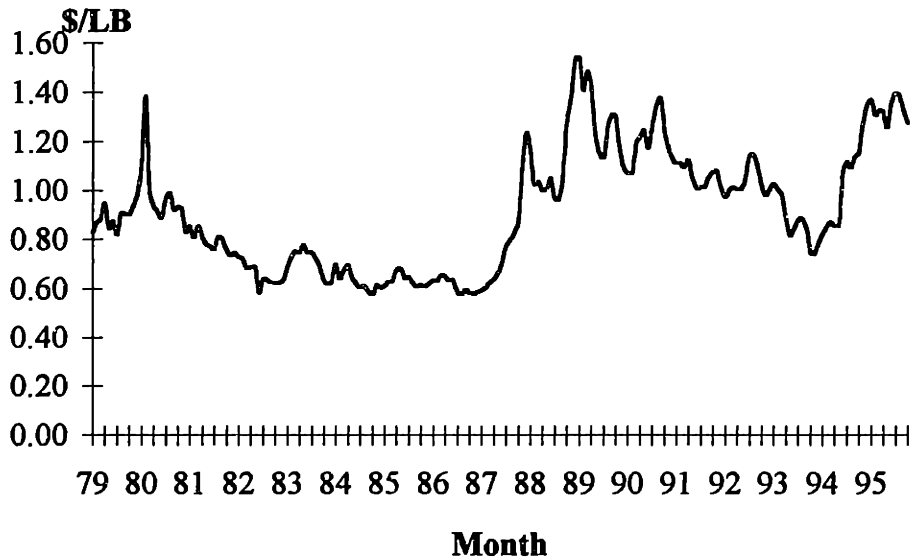
### 2.1.1. Random walking copper price

Most physical copper transactions are priced on the spot price or the future price which is quoted on the metal exchange. Standardized cash and future contracts are actively traded through out the year in the London Metal Exchange (LME) and future contracts in Commodity Exchange in New York (COMEX). LME official spot settlement price and three month future price are commonly used as a base price in most international contracts. COMEX future price is used as a base of most transactions in the U.S. Both COMEX and LME prices move together simultaneously by arbitrage trading.

The historical copper price of LME has followed the typical stochastic random walk as shown from Figure 1. From January 1979 till September 1994, copper price went up to US$ 1.54/lb at the highest and fell down to US$ 0.58/lb at the lowest.

The forecasting of future copper prices, which is a key factor in estimating future cash flow from mine operations, is really difficult in an efficient market.

**Figure 1. LME Official Copper Price**




Source: London Metal Exchange Official Price

Even the scenario estimations can not meet the full copper price volatility. The value of the operating copper mine is calculated by the cash flow that depends on a stochastic state variable, the copper price, and that can be modified by future operating decisions. The major uncertainty in valuation of operating copper mine is the spot price of copper at the time of extraction.

### *2.1.2.* Derive equations with Wiener process

Contingent claims analysis assumes that stochastic changes in value of mine ($V$) can be spanned by existing assets in the economy. With the spanning, we can determine the investment rule that maximizes the firm's market value without making any assumptions about risk preference or discount rate. All we need is some combination or portfolio of traded assets that will exactly replicate (span) the pattern of returns from our investment mine, at every future date and in every future uncertain eventuality.

Investment on mine is defined by a stream of costs and benefits that vary through time and depend on the unfolding of uncertain events. The firm that owns the right to an investment opportunity, or to the stream of operating profits from a completed mine, owns an asset that has a value. Though it is not directly traded, we can compute an implicit value for it by relating it to other assets that are traded in the market. In the case of copper mine investments, the relevant asset whose value is known will be a portfolio consisting of borrowing at a riskless rate and either the copper metal that is to be produced by the mine.

If the production costs are known, they can be discounted at the riskless interest rate. The present value of future copper production is equal to the appropriate current futures price discounted at the riskless interest rate (to reflect the fact that payment is deferred). This means that future copper output can be evaluated at the current spot price without any discounting. Given the current copper price, the uncertainty of spot copper price at a future time can be described by the variance (or standard deviation). The valuation of the operating mine is thus dependent upon these two variables; the spot price of copper at the time of valuation, and the standard deviation of the spot price at extraction from the current spot price.

The principal benefits of the inventory are production cost savings made possible by avoidance of the interruptions in production which would be inevitable in the absence of an inventory and the ability to take advantage of unforeseen local increases in the demand for the metal. Collectively these benefits, the net of the costs of storage of the inventory, are known as the "convenience yield"[4] of an inventory of the metal. It is the marginal convenience yield which must be taken into account in valuing future units of production. It turns out that the present value of future copper production is equal to the current spot price discounted by the marginal convenience yield. Now we can avoid simultaneously the twin problems of assessing the expected future spot price at which copper will be sold, and of assigning a discount rate appropriate to the risk of these revenues through convenience yield approach.

We can construct the requisite replicating self - financing portfolio on the assumption that convenience yield on the output copper metal can be written as a function of the copper price alone and that the interest rate is non-stochastic. These assumptions suffice to yield a deterministic relation between the current and futures spot price of copper, and the cash flows

[4] The convenience yield is the flow of services that accrues to an owner of the physical commodity but not to the owner of a contract for future delivery of the commodity.

from the mine can then be replicated by a self - financing portfolio of riskless bills and future contracts of copper.[5]

The copper mine profit flow depends on the copper price $P$ change. Since we will be dealing with proportional rates of return, it is convenient to assume that $P$ follows a geometric Brownian motion of equation.

$$dP = a(P, t)dt + b(P, t)dz \qquad (1)$$

where $dz$ is the increment of a Wiener process. $a(P, t)$ is called the drift rate, and $b^2(P, t)$ is the variance rate[6]. The drift and variance coefficients are function of the current state and time. The continuous - time stochastic process $P(t)$ represented by equation (1) is called an Ito - Process.

By substituting $a(P, t)$ with $\alpha P$, and $b(P, t)$ with $\sigma P$, where $\alpha$ and $\sigma$ are constant, equation (1) becomes

$$dP = \alpha P dt + \sigma P dz \qquad (2)$$

Over any time interval $\Delta t$, the change in $P$, denoted by $\Delta P$, is normally distributed, and will have an expected value $E(\Delta P) = \alpha \Delta t$ and variance $V(\Delta P) = \sigma^2 \Delta t$.

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[5] Brennan and Schwartz (1985)
[6] Dixit and Pindyck (page 71)

This result for the expectation of a geometric Brownian motion can be used to calculate the expected present discounted value of $P(t)$ over some period of time

$$E\left[\int_0^\infty x(t)e^{-rt}dt\right] = \int_0^\infty x_0 e^{-(r-\alpha)t}dt = P_0/(r-\alpha), \tag{3}$$

provided the discount rate $r$ exceeds the growth rate $\alpha$.

We can describe the value of an option to invest in a copper mine as a function of the price of copper, which in turn might be represented by a geometric Brownian motion[7].

Like any asset, the copper is held by investors only if it provides a sufficiently high return. Part of this return comes in the form of the expected price appreciation, $\alpha$. Another part may also come in the form of a dividend, directly or indirectly (convenience yield). Let total expected rate of return from holding assets of copper be $\mu = \alpha + \delta$, when $\delta$ denotes the convenience yield of copper holding.

Let $P$ be the price of copper perfectly correlated with the mine value $V$, and denote by $\rho_{Pm}$ the correlation of $P$ with the market portfolio $M$.

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[7] Though the prices of raw commodities such as copper or oil follows random walk in short run, they might be drawn back towards the marginal cost of producing the commodities, which is called as a mean - reverting process. Though copper price shall follow geometric Brownian motion, it is difficult to statistically distinguish between a random walk and a mean - reverting process, using 30 or so years of data. Dixit and Pindyck (page 78)

Since $P$ is perfectly correlated with $V$, $\rho_{Pm} = \rho_{Vm}$. We will assume that this asset pays no dividends, so its entire return is from capital gains. The $P$ evolves according to

$$dP = \mu P dt + \sigma P dz \tag{4}$$

where $\mu$, the drift rate, is the expected rate of return from holding the asset.

The riskless rate of return $r$ is exogenously specified. Then the fundamental condition of equilibrium from capital asset pricing model (CAPM) says that the risk - adjusted expected rate of return that investors would require if they are to own the mine

$\mu = r + \phi\sigma\rho_{Pm}$, where $\phi$ is market price of risk that is exogenous, and $\rho_{Pm}$ is the coefficient of correlation between returns on the particular assets $P$ and the whole market portfolio $M$. We will assume that $\alpha$, the expected percentage rate of change of $V$, is less than this risk - adjusted return $\mu$. (the firm would never invest if this were not the case. No matter what the current level of $V$, the firm would always be better off waiting and simply holding on to its option to invest). We will let $\delta$ denote the difference between $\mu$ and $\alpha$, that is, $\delta = \mu - \alpha$.. If $\delta > 0$, the expected rate of capital gain on the mine is less than $\mu$. Hence $\delta$ is an opportunity cost of delaying construction of the mine, and instead keeping the option to invest alive. If $\delta$

were zero, there would be no opportunity cost to keeping the option alive, and no one would ever invest, no matter how high the NPV of the mine. Thus we are assuming $\delta > 0$.

At what point is it optimal to pay a sunk cost $I$ in return for a mine whose value is $V$, given that $V$ evolves according to the following Brownian motion:

$$dV = \alpha V dt + \sigma V dz \tag{5}$$

Equation (5) implies that the current value of the mine is known, but future values are lognormally distributed with a variance that grows linearly with the time horizon.

We will denote the value of the investment opportunity (that is, the value of the option to invest) by $F(V)$. Since the payoff from investing at time $t$ is $V_t - I$, we want to maximize its expected present value:

$$F(V) = \max E[(V_T - I)e^{-\rho T}] \tag{6}$$

where $E$ denotes the expectation, $T$ is the (unknown) future time that the investment is made, $\rho$ is the discount rate, and the maximization is subject to equation (5) for $V$.

In the case of a call option, the price of the stock underlying the option is assumed to follow an exogenously specified stochastic process, usually a geometric Brownian motion. In our model of real investment, the corresponding state variable was the value of the mine, $V$, for which we stipulated an exogenous stochastic process. The price of the spot copper price $P$ is exogenous, and determines the value $V$ of the mine, and the value $F$ of the option to invest, in terms of the stipulated stochastic process for $P$.

We assume the idealistic unit mine producing 1 unit of copper per year perpetually with a production cost of $C$, after construction with investment cost $I$. The mine will generate a flow of operating profit equal to $(P - C) \times (1-T)$ per period where $T$ is the effective income tax rate.

We will determine the value of the option to invest $F(P)$ by constructing a risk - free portfolio, determining its expected rate of return, and equating that expected rate of return to the risk - free rate of interest.

Hold the option to invest, which is worth $F(V)$, and go short $n = F'(V)$ units of the mine ( or equivalently, of the asset or portfolio that is perfectly correlated with $V$).

The value of this portfolio is $\phi = F - F'(V)V$. As $V$ changes, $F'(V)$ may change from one short interval of time to the next, so that the composition of the portfolio will be changed. However, over each short interval of length $dt$, we hold $n$ fixed.

The short position in this portfolio will require a payment of $\delta V\, F'(V)$ dollars per time period. An investor holding a long position in the mine will demand the risk - adjusted return $\mu V$, which equals the capital gain $\alpha V$ plus the dividend stream $\delta V$. Since the short position includes $F'(V)$ units of the mine, it will require paying out $\delta V\, F'(V)$. Taking this payment into account, the total return from holding the portfolio over a short time interval $dt$ is

$$dF - F'(V)dV - \delta V\, F'(V)dt. \qquad (7)$$

To obtain an expression for $dF$, use Ito's Lemma:[8]

$$dF = F'(V)dV + \frac{1}{2} F''(V)(dV)^2.$$

Hence the total return on the portfolio is

$$\frac{1}{2} F''(V)(dV)^2 - \delta V\, F'(V)dt.$$

From equation (7) for $dV$, we know that $(dV)^2 = \sigma^2 V^2 dt$ so the return on the portfolio becomes

$$\frac{1}{2} \sigma\, V^2 F''(V)dt - \delta\, VF'(V)dt. \qquad (8)$$

Note that this return is risk - free. Hence to avoid arbitrage possibilities, it must equal $r\Phi dt = r[F - F'(V)V]dt$:

[8] Dixit and Pindyck (page 151)

$$\frac{1}{2}\sigma^2 V^2 F''(V)dt - \delta VF'(V)dt = r[F - F'(V)V]dt.$$

Dividing through by $dt$ and rearranging gives the following differential equation that $F(V)$ must satisfy:

$$\frac{1}{2}\sigma^2 V^2 F''(V) + (r - \delta)VF'(V) - rF = 0. \qquad (9)$$

### *2.1.3.* Boundary conditions and assumptions

The value of the option to invest $F(V)$ must satisfy the following boundary conditions;

$$F(0) = 0 \qquad (10)$$

$$F(V^*) = V^* - I \qquad (11)$$

$$F'(V^*) = 1 \qquad (12)$$

Condition (10) arises from the observation that if $V$ goes to zero, it will stay at zero. Therefore the option to invest will be of no value when $V = 0$. $V^*$ is the price at which it is optimal to invest. So the (11) is the value - matching condition; it just says that upon investing, the firm receives a net payoff $V^* - I$. Condition (12) is the "smooth - pasting" condition. If $F(V)$ were not continuous and smooth at the critical exercise point $V^*$, one could not do better by exercising at a different point.

The value of the firm is now a function of the exogenous state variable $P$ (copper price), and of the discrete state variable that indicates whether the firm is currently idle (0) or active (1). Let $V_0(P)$ denote the value of the option to invest ( that is, the value of an idle firm), and $V_1(P)$ denote the value of an active firm. $V_1(P)$ is the sum of two components, the entitlement to the profit from operation, and the option to abandon should the price fall too far.

### 2.1.4. Solution

We begin with the idle firm. To obtain a different equation for $V_0(P)$, construct a portfolio with one unit of the option to invest, and a short position of $V_0'(P)$ units of output. The resulting equation is

$$\frac{1}{2}\sigma^2 P^2 V_0''(P) + (r-\delta)PV_0'(P) - rV_0(P) = 0 \qquad (13)$$

This has the general solution[9]

$$V_0(P) = A_1 P^{\beta 1} + A_2 P^{\beta 2}$$

where $A_1$ and $A_2$ are constant to be determined, and $\beta_1$ and $\beta_2$ are roots of the quadratic equation

[9] Dixit and Pindyck (page 217)

$$\beta_1 = \frac{1}{2} - (r-\delta)/\sigma^2 + \sqrt{\left[(r-\delta)/\sigma^2 - \frac{1}{2}\right]^2 + 2r/\sigma^2} \quad > 1,$$

and

$$\beta_2 = \frac{1}{2} - (r-\delta)/\sigma^2 - \sqrt{\left[(r-\delta)/\sigma^2 - \frac{1}{2}\right]^2 + 2r/\sigma^2} \quad < 0.$$

Since the option to invest gets very far out of the money and therefore becomes nearly worthless as $P$ goes to 0, the coefficient $A_2$ corresponding to the negative root $\beta_2$ must be zero. That leaves

$$V_0(P) = A_1 P^{\beta 1}. \tag{14}$$

This is valid over the interval $(0, P_A)$ of prices, where $P_A$ is the threshold of the activation of the mine.

As the live mine part of the portfolio pays a net cash flow

$(P - C)(1-T)dt$. Then we get

$$\frac{1}{2}\sigma^2 P^2 V_1''(P) + (r - \delta)PV_1'(P) - rV_1(P) + (P - C)(1-T) = 0 \tag{15}$$

The general solution to this equation is

$$V_1(P) = B_1 P^{\beta 1} + B_2 P^{\beta 2} + P(1-T)/\delta - C(1-T)/r.$$

The last two terms are the value of the live mine when the firm is required to keep it operating forever despite any losses, and the first two terms are the value of the option to abandon. The likelihood of abandonment in the not - too - distant future becomes extremely small as $P$ goes to $\infty$, so the value of the abandonment option should go to zero as $P$ becomes very large. Hence the coefficient $B_1$ corresponding to the positive root $\beta_1$ should be zero . This leaves[10]

$$V_1(P) = B_2 P^{\beta 2} + P(1-T)/\delta - C(1-T)/r. \tag{16}$$

This is valid for $P$ in the range $(P_i, \infty)$, where $P_i$ is threshold of idling the live mine.

At the investment threshold $P_A$, the firm pays the lump-sum cost $I$ to exercise its investment option, giving up this asset of value $V_0(P_A)$ to get the live mine which has the value $V_1(P_A)$.

### 2.2. Model of Entry, Mothballing, Reactivation, and Scrapping

The manager of the copper mine has other flexibilities besides permanently abandoning an operating mine when the price of copper falls. The mine could be put into a state of care maintenance base (mothballed), allowing it to be reactivated in the future at a sunk cost much less than the

[10] Dixit and Pindyck (page 218)

cost of building a new green field mine. Mothball, like permanent abandonment, requires a sunk cost, which we will denote by $E_M$. In addition, once a mine is mothballed, maintaining the capital and mine site requires a cost flow $M$. The operation can be reactivated in the future at a further sunk cost $R$. Mothballing only makes sense if the maintenance cost $M$ is less than the cost $C$ of actual operation, and if the reactivating cost $R$ is less than the cost of fresh investment $I$.

### 2.2.1. Decision rule

The decision rule we develop is then the thresholds of the spot price of copper, given a standard deviation of changing copper price. The decision rule is the following;

(1) if the spot price is above the entry threshold ($P_H$), invest in the mine with expenditure of capital investment sunk cost of development ($I$).

(2) if the spot price is below the mothball threshold ($P_M$), suspend the operation of the mine with expenditure of capital sunk cost of mothball ($E_M$) and flow maintenance expenses ($M$).

(3) if the spot price increases above the reactivating ($P_R$) threshold, reactivate the mothball mine with the expenditure of capital sunk cost of reactivating ($R$).

(4) if the spot price is decreased down below the scrapping threshold ($P_S$), abandon the operating mine with the expenditure

of capital sunk cost of abandoning $E$ or scrap the mothballed mine with expenditure of capital cost $E_S$.

### *2.2.2.* Basic assumptions

We assume that a particular copper mine under study is already explored and developed and that the quality and quantity of copper in the reserve are known with certainty. We also assume that the following cost is known with certainty:

The cost to operate the mine ($C$).

The capital sunk cost to mothball the mine ($I$).

The capital sunk cost to reactivate the mothballed mine ($R$).

The maintenance expense of the mothballed mine ($M$).

The scrapping cost of the mothballed mine ($E_S$).

The mothballing capital cost of operating the mine ($E_M$).

The abandoning cost of operating the mine ($E$).

The production quantity per year ($Q$).

We also assume that the volatility of the copper price ($\sigma$) and convenience yield ($\delta$) are fixed for the mine life as a certain figure.

These assumptions were made not to ignore the significance of technical uncertainties but to eliminate the effect of technical uncertainties and thus allow us to selectively study how significant the option for flexibility on the operation is in valuation of operating mine.

### *2.2.3.* Rule of optimal switches

Starting from a state in which it does not have any kind of capital installed, the firm will make the investment if the price rises to a threshold $P_H$. The firm will mothball an operating mine if the price falls to another threshold $P_M$. Given a mine in mothballs, the firm will reactivate it if the price rises to yet a third threshold $P_R$. Since the cost of reactivating is less than that of investing from scratch, we expect $P_R < P_H$. If instead the price falls, making reactivating a sufficiently unlikely or remote event, there is a fourth threshold, $P_S$, at which the mothballed mine will be scrapped altogether to save on maintenance cost with a sunk capital cost of $E_S$. To keep the exposition simple, we will assume that $E_M + E_S = E$, the cost of abandoning a live mine.

The mine can be left idle over the interval $(0, P_H)$ of prices. Then its value is once again given by equation (14)

$$V_0(P) = QA_1P^{\beta 1}$$

where $A_1$ is a constant to be determined. This is just the value of the option to invest. Similarly, the operating state can prevail over the interval $(P_M, \infty)$, with the value of the mine again given by equation (16)

$$V_1(P) = Q\{B_2P^{\beta 2} + P(1-T)/\delta - C(1-T)/r\}$$

where the constant $B_2$ remains to be determined. Now the term $B_2P^{\beta 2}$ is the value of the option to mothball. The other two terms give the expected present value of continuing operations forever. Of course the mothball option derives its value from further possibilities of reactivating or scrapping.

The mothballed state can continue over some range of prices ($P_S$, $P_R$). Since neither zero nor infinity is included in this range, we cannot eliminate either the positive or negative power in the option value part of the solution. Therefore the value of the mothballed mine is given by[11]

$$V_m(P) = Q\{D_1P^{\beta 1} + D_2P^{\beta 2}\} - M/r \tag{17}$$

where the constant $D_1$ and $D_2$ remain to be determined. The first term is the value of the option to scrap the mine. The last term is simply the capitalized maintenance cost, assuming the mine remains in the mothballed state forever.

At each switching point, we have appropriate value - matching and smooth - pasting conditions. For the original investment, these conditions are

$$V_0(P_H) = V_1(P_H) - I, \qquad V_0'(P_H) = V_1'(P_H).$$

[11] Dixit and Pindyck (page 232)

For mothball, the conditions are

$$V_1(P_M) = V_M(P_M) - E_M, \qquad V_1'(P_M) = V_M'(P_M);$$

for reactivating,

$$V_M(P_R) = V_1(P_R) - R, \qquad V_M'(P_R) = V_1'(P_R);$$

and finally, for scrapping,

$$V_M(P_S) = V_0(P_S) - E_S, \qquad V_M'(P_S) = V_0'(P_S).$$

Using the functional forms above, the equations at the thresholds become[12]:

between interaction of mothball and reactivating;

$$Q\{-D_1 P_R^{\beta 1} + (B_2 - D_2)\, P_R^{\beta 2} + P_R(1-T)/\delta - C(1-T)/r\} + M/r = R \qquad (18)$$

$$Q\{-\beta_1\, D_1 P_R^{\beta 1-1} + \beta_2(B_2 - D_2) P_R^{\beta 2-1}\} + (1-T)/\delta = 0 \qquad (19)$$

$$Q\{-D_1 P_M^{\beta 1} + (B_2 - D_2)\, P_M^{\beta 2} + P_M(1-T)/\delta - C(1-T)/r\} + M/r = -E_M \qquad (20)$$

$$Q\{-\beta_1\, D_1 P_M^{\beta 1-1} + \beta_2(B_2 - D_2) P_M^{\beta 2-1}\} + (1-T)/\delta = 0. \qquad (21)$$

The value - matching and smooth - pasting conditions for new investment;

$$Q\{-A_1 P_H^{\beta 1} + B_2\, P_H^{\beta 2} + P_H(1-T)/\delta - C\, (1-T)/r\} = I \qquad (22)$$

[12] Dixit and Pindyck (page 233)

$$Q\{-\beta_1 A_1 P_H^{\beta 1\text{-}1} + \beta_2 B_2 P_H^{\beta 2\text{-}1}\} + (1\text{-}T)/\delta = 0 \tag{23}$$

Those conditions at the scrapping threshold become

$$Q\{-(D_1 - A_1)P_S^{\beta 1} + D_2\, P_S^{\beta 2}\} - M/r = -\,E_S \tag{24}$$

$$Q\{-\beta_1(D_1 - A_1)P_S^{\beta 1\text{-}1} + \beta_2 D_2 P_S^{\beta 2\text{-}1}\} = 0\,. \tag{25}$$

This system is too complicated to solve analytically. But we can find a homogenous solution of thresholds $P_H$, $P_L$, $P_M$, $P_S$, and coefficient $A_1$, $B_2$, $D_1$, $D_2$ numerically by the Excel Solver program easily.

If R or $\sigma$ are both zero, then mothball is tantamount to costless suspension, and the model is changed into an entry-exit only model;

$$Q\{-A_1 P_H^{\beta 1} + B_2\, P_H^{\beta 2} + P_H(1\text{-}T\,)/\delta - C\,(1\text{-}T\,)/r\} = I \tag{26}$$

$$Q\{-\beta_1 A_1 P_H^{\beta 1\text{-}1} + \beta_2 B_2 P_H^{\beta 2\text{-}1}\,\} + 1(1\text{-}T\,)/\delta = 0 \tag{27}$$

$$Q\{-A_1 P_S^{\beta 1} + B_2\, P_S^{\beta 2} + P_S(1\text{-}T\,)/\delta - C\,(1\text{-}T\,)/r\} = -\,E \tag{28}$$

$$Q\{-\beta_1 A_1 P_S^{\beta 1\text{-}1} + \beta_2 B_2 P_S^{\beta 2\text{-}1}\} + 1(1\text{-}T\,)/\delta = 0 \tag{29}$$

As we keep on increasing R, the mothball threshold $P_M$ falls and the scrapping threshold $P_S$ rises.

# 3. Valuation of Operating Copper Mine

## 3.1. Tintaya Copper Mine in Peru

The Peruvian Government privatized Tintaya Copper Mine, a state owned copper mine, and associated exploration properties, located approximately 1,000 miles southeast of Lima by an international auction tender for 98.43% of shares (employees hold 1.57%) on October 6 1994.

Table 1. **Tender Result of Tintaya Copper Mine**

| Company | Country | Cash Payment |
|---|---|---|
| Magma Copper Co. | US | US$ 218 million |
| Metalle | Canada | US$ 214 million |
| RTZ | UK | US$ 191 million |
| BHP & | Australia | US$ 161 million |
| Antofugasta | UK | |
| Empresas Frisco | Mexico | US$ 131 million |
| Rio Algom & | U.S. | US$ 91 million |
| Minera Milpo | Peru | |
| Phelp Dodge | U.S. | US$ 73 million |

*. All bidder take assumption of Peruvian government debt US$ 55 million and committed US$ 85 million investment within 5 years

* The Mining Journal, October 14, 1994

Magma Copper Company submitted the highest bid for Tintaya with US$ 218-million in cash and US$ 55-million in Peruvian government debt paper in total, equivalent to cash US$ 243 million and a minimum investment commitment of $ 85-million for 5 years. The bidding amount of tender participants are widely varied as in Table 1.

Tintaya is Peru's second-largest copper mine. Tintaya is a skarn copper deposit, first discovered in the early 1900s and brought into production in February 1985 with a capital investment of US$ 325 million (in 1986 dollars).[13] The open pit mine, located high in the Andes halfway between the cities of Arequipa and Cuzco at an altitude of 4,100 meters, was producing 63,000 tons per year of copper in concentrates from its current operation based upon a sulphide deposit grading of 1.78% copper. The deposit contained 26.8 million tons of oxide ores grading an average of 1.98% copper. Of this 9.96 million ton, grading 2.0% copper, was already mined and stockpiled and 58 million tons of 1.78% copper sulfide ore with associated gold (0.6 gram/ton) and silver (13 gram/ton)[14]for about 21 years operation life. Employment peaked at 1,400 in 1989, but staffing is down to levels of 780 at tender.

Based on mine specification opened to tender participants and consultation with Tintaya employee, Morrison & Knudsen Corp. the U.S.

[13] The Mining Journal April 5, 1985 Pg. 237
[14] The Mining Journal January 14, 1994

estimated that Tintaya can produce an average of 145 million lb. copper per year till 2015 by equivalent annual full operating cost US$ 0.76/lb (in 1994 dollar) after capital investment on solvent extrusion and electro - winning facility for exploiting 26.8 million tons of oxide ore (Table 2).

Table 2. **Estimation of Production Quantity and Cost**

(Unit: million lb., US$/lb)

| Year | Production Quantity | Production Cost | Year | Production Quantity | Production Cost |
|---|---|---|---|---|---|
| 1995 | 109 | 1.01 | 2006 | 151 | 0.73 |
| 1996 | 116 | 1.01 | 2007 | 151 | 0.71 |
| 1997 | 157 | 0.71 | 2008 | 153 | 0.68 |
| 1998 | 157 | 0.68 | 2009 | 141 | 0.74 |
| 1999 | 158 | 0.74 | 2010 | 141 | 0.74 |
| 2000 | 156 | 0.74 | 2011 | 141 | 0.76 |
| 2001 | 156 | 0.65 | 2012 | 150 | 0.76 |
| 2002 | 156 | 0.71 | 2013 | 156 | 0.78 |
| 2003 | 155 | 0.80 | 2014 | 121 | 0.80 |
| 2004 | 156 | 0.66 | 2015 | 116 | 0.80 |
| 2005 | 155 | 0.87 | | | |

## 3.2. Valuation of Mine by NPV

All the bidding to tender were assumed to be calculated by the NPV method. The most important parameter is the expected future spot price at the time of copper extraction. Metal Bulletin Research forecasts the copper price in three scenarios in Copper market Update (Table 3)[15]

Table 3. **Copper Price Forecasting by Scenario**

| Period | Pessimistic | Base | Optimistic |
|---|---|---|---|
| 1995 | US$1.27/lb | US$1.37/lb | US$1.42/lb |
| 1996 | US$0.95/lb | US$1.16/lb | US$1.25/lb |
| 97-2001 | US$0.80/lb | US$0.90/lb | US$0.95/lb |
| 02-2015 | US$0.85/lb | US$0.95/lb | US$1.00/lb |

With a risk-adjusted discount rate of 12%[16] per year, the value of the mine from NPV calculation for each case are:

Pessimistic case : US$ 112.2 million

Base case : US$ 188.9 million

[15] Metal Bulletin Research, Copper Market Update, March 1995
[16] 12 % is the average WACC of Cyrus, Magma, Phelps Dodge, Asarco, Noranda, Freeport, BHP, Cominco, AAC at end of September 1994.

Optimistic case : US$ 226.4 million

These figures are congruent with bidding results. This proves that the DCF method may not fairly value the natural resources under exogenous price uncertainty.

### 3.3. Valuation of Mine by Contingent Claim Analysis

We use the equation (18), (19), (20), (21), (22), (23), (24), and (25) for valuation of Tintaya copper mine at operating status and finding the optimal decision rule (threshold prices) at the assumption that the manager has the flexibility of mothballing, reactivation, and scrapping the mine.

$$Q\{-D_1 P_R^{\beta 1} + (B_2 - D_2)\, P_R^{\beta 2} + P_R(1-T)/\delta - C(1-T)/r\} + M/r = R \quad (18)$$

$$Q\{-\beta_1\, D_1 P_R^{\beta 1-1} + \beta_2(B_2 - D_2) P_R^{\beta 2-1}\} + (1-T)/\delta = 0 \quad (19)$$

$$Q\{-D_1 P_M^{\beta 1} + (B_2 - D_2)\, P_M^{\beta 2} + P_M(1-T)/\delta - C(1-T)/r\} + M/r = -\, E_M \quad (20)$$

$$Q\{-\beta_1\, D_1 P_M^{\beta 1-1} + \beta_2(B_2 - D_2) P_M^{\beta 2-1}\} + (1-T)/\delta = 0 \quad (21)$$

$$Q\{-A_1 P_H^{\beta 1} + B_2\, P_H^{\beta 2} + P_H(1-T)/\delta - C\,(1-T)/r\} = I \quad (22)$$

$$Q\{-\beta_1\, A_1 P_H^{\beta 1-1} + \beta_2 B_2 P_H^{\beta 2-1}\} + (1-T)/\delta = 0 \quad (23)$$

$$Q\{-(D_1 - \mathrm{A}_1) P_S^{\beta 1} + D_2\, P_S^{\beta 2}\} - M/r = -\, E_S \quad (24)$$

$$Q\{-\beta_1(D_1 - \mathrm{A}_1) P_S^{\beta 1-1} + \beta_2 D_2 P_S^{\beta 2-1}\} = 0 \quad (25)$$

In the case where the manager has the flexibility of shutting down the mine or where the mothballing option is not economically valuable, we use the equation (26), (27), (28), and (29).

$$Q\{-A_1P_H^{\beta1} + B_2\, P_H^{\beta2} + P_H(1-T\,)/\delta - C\,(1-T\,)/r\} = I \tag{26}$$

$$Q\{-\beta_1\, A_1P_H^{\beta1-1} + \beta_2B_2P_H^{\beta2-1}\,\} + 1(1-T\,)/\delta = 0 \tag{27}$$

$$Q\{-A_1P_S^{\beta1} + B_2\, P_S^{\beta2} + P_S(1-T\,)/\delta - C\,(1-T\,)/r\} = -E \tag{28}$$

$$Q\{-\beta_1\, A_1P_S^{\beta1-1} + \beta_2B_2P_S^{\beta2-1}\} + 1(1-T\,)/\delta = 0 \tag{29}$$

where $Q$ is the annual copper production quantity, $C$ is the flow operating cost, $T$ is the income tax rate, $I$ is the capital investment cost of developing the greenfield copper mine, $R$ the is capital investment cost of reactivating the mothballed mine, $E_M$ is the capital cost of mothballing operating mine, $E_S$ is the capital cost of scrapping down the mothballed mine, $E$ is the capital cost of scrapping the operating mine, $M$ is the flow expense of maintaining mothballed mine, $P_M$ is the threshold price of mothballing the operating copper mine, $P_R$ is the threshold price of reactivating mothballed copper mine, $P_S$ is the threshold price of scrapping the mothballed or operating copper mine, $P_H$ is the threshold price of investing greenfield mine.

Parameters constructed for valuation of the Tintaya copper mine were derived from historical data and Morrison & Knudsen Corp.'s fair discretion.

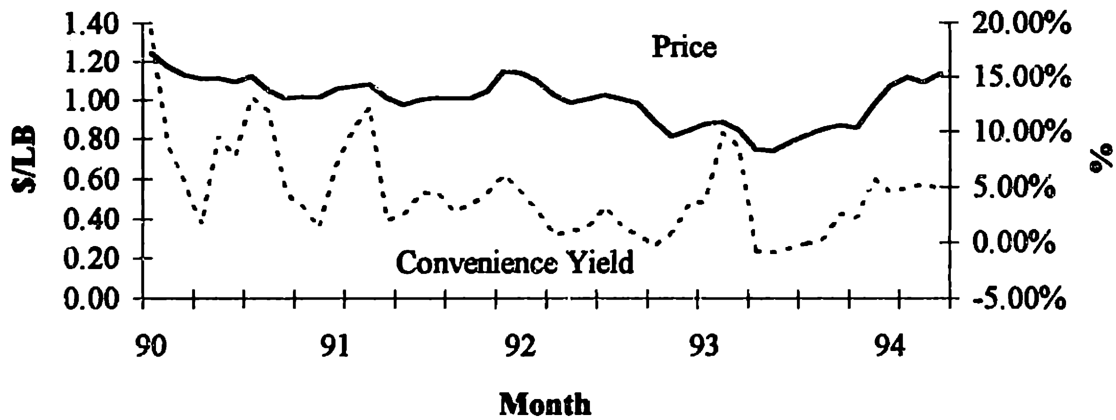
We use monthly average LME grade A settlement prices, which are mostly used as a base price for international physical copper transaction, as spot price in analysis.

The average of monthly settlement prices in September 1994 was US$ 1.14/lb. We use this figure as the current spot price.

The annualized standard deviation of the rate of change in monthly average of settlement prices for 4 years, from October 1990 till September 1994, was about 18 percent (Appendix A). This period is probably representative of the type of period that market participants might have expected to occur from 1995 on. It includes periods of crisis, as well as periods of relative tranquillity (Figure 2).

London Interbank Offer Rate (LIBOR) for one year in September 1994 was 6%. The annualized U.S.A CPI growth rate in the same month was 4.77%. We assume a 1% spread over LIBOR in order to fund for a spanning portfolio. So we assume 3.00% as the riskfree rate in this analysis (Appendix A).

## Figure 2. Copper Price & Convenience Yield




The convenience yield of copper, derived from the equation of $\frac{F(\tau_2)}{F(\tau_1)} = e^{[(r-\delta)(\tau_2-\tau_1)]}$, was 4.51% during the same period.[17] We assume 4.5% to be the convenience yield per year for this analysis.

We assume an expected development investment cost of about US$ 400 million for construction of an equivalent size green field copper mine now.[18] The estimated investment cost of developing Montanore copper mine, producing 20,000 tons of ore per day, is about US$ 250 million.[19] The Management of Diamante copper mine estimates US$ 250 million for constructing the mine, producing 50,000 tons of ore per day.[20] About US$

[17] Nalin Kulatilaka (Financial Management, Autumn 1993, page 275)
[18] Generally the quantity of ore treated per unit period is used for comparing the scale of mine.
[19] Metals Week March 6 1995, Vol. 66, No 10 Page 14
[20] Metals Week July 19 1993 Page 7

450 million was estimated by management to develop Radomiro Tomic copper mine, capable of producing 150,000 MT of cathode per year.[21] The management of La Candelaria estimated US$ 550 million for constructing the mine, producing 50,000 tons of ore per day.[22] In consideration of US dollar purchasing power's devaluation from 1986 to 1994, it is fair to estimate US$ 400 million for the current equivalent greenfield mine construction from its development capital cost of US$ 325 million.

US$ 200 million, which is half of greenfield investment, is assumed to be the sunk capital cost for reactivating the mothballed copper mine. Actually it is difficult to figure out the fair cost of reactivating an idle mine. It depends on how long it has been in mothball state, whether it is located in a rainy jungle area or dry area, and whether it has been carefully maintained without rust on capital equipment or water dampening. The reactivating cost increases as the mothball period increase. In 1991, they estimated about US$ 100 million for reactivating Kilembe copper mine in Uganda, capable of producing 16,000 MT of copper per year[23], after 12 years of being shut down. In 1995, CRA of Australia estimated about US$ 450 million for reactivating Bougainville copper mine in Papua New Guinea after 5 years of being shut down, which had been producing 166,000 MT of copper per year.[24] As Papua New Guinea is rainy jungle country, the open

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[21] The Mining Journal April 30, 1993, Pg. 305
[22] Mining Magazine January, 1994, Pg. 26
[23] The Mining Journal, May 17, 1991; Pg. 371
[24] The Mining Journal December 8, 1995, Pg. 432

pit was severely damaged, and required of a huge amount of reactivating capital investment. We neglect the marketing cost for re-entering the market on this analysis because the mine is assumed to be able to sell the product to metal exchange with normal transaction costs as a marginal market player.

A total of US$ 18 million is assumed for complete shutdown (abandoning) of the sunk capital cost. About US$ 6 to 8 million is estimated for environmental reclamation required by the government and about US$ 5 to 10 million is estimated for compensation to employees. Among this total cost, about US$ 17 million is required at the initial shutdown stage (mothball) and only a balance of US$ 1 million is required for a complete shutdown from mothballed mine. We neglect the penalty from breaching the long term contract with the customer, which is a general practice in metal mine for stabilizing the sale of the product, because the mine can buy an equivalent product from an exchange or spot market and deliver to the customer. The possible price difference between long term contract and spot purchase, which is termed as the premium difference, is neglected because there is no consistent relation between the two premiums and the difference is not so big.

The cost of care for maintaining the mine is not high. US$ 3 million of flow expenses per year is thought to be enough even in the worst case scenario.

The arithmetic average of mine production for 21 years is about 145 million lbs. per year. This simplification is not extreme for a long life mine with stable production. Average production cost of US$ 0.76/lb from Table 2 is used as the real operating cost.

We assume the following parameters in valuation of Tintaya Copper Mine:

Table 4. **The parameters into the Valuation Equation**

| | |
|---|---|
| Riskless Interest Rate ($r$): | 3% |
| Convenience Yield ($\delta$) : | 4.5% |
| Standard Deviation of Copper Price Change ($\sigma$) : | 18% |
| Investment Cost ($I$) : | US$ 400 million |
| Reactivating Cost ($R$): | US$ 200 million |
| Mothball Cost ($E_M$) : | US$ 17 million |
| Scrapping Cost of Mothballed Mine ($E_S$) : | US$ 1 million |
| Maintenance Cost of Mothballed Mine ($M$) : | US$ 3 million |
| Spot Copper Price ($P$): | US$ 1.14/lb |
| Production Cost ($C$): | US$ 0.76/lb |
| Copper Production ($Q$) : | 145 million lbs/Year |
| Mine Life : | 21 years |
| Income Tax | 35.6% |

With the above parameters,[25] we got the following threshold prices and value of mine numerically by Micro - Soft Excel Solver program.

$P_M$: US$ 0.54/lb

$P_R$: US$ 1.22/lb

$P_S$: US$ 0.53/lb

$P_H$: US$ 1.52/lb

$V_1(P)$: US$ 786 million

By exercising the flexibility option, the manager of Tintaya can decrease his loss by mothballing the mine and consequently enhancing the whole mine value, if the spot price falls down below US$ 0.54/lb, and stop the further loss by abandoning the mine permanently, if the spot price falls down below US$ 0.53/lb. The manager has to wait till the spot price goes up above US$ 1.22/lb before reactivating the mine once it is shut down.

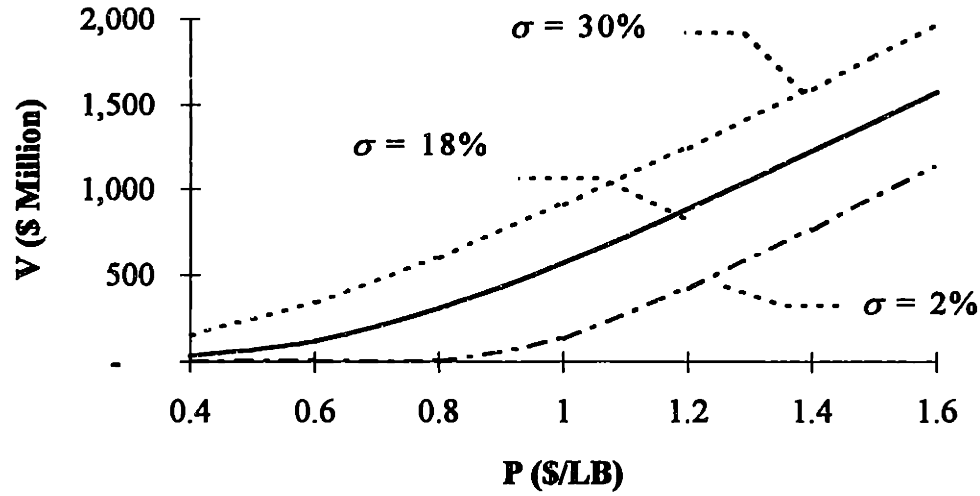
[25] To keep the analysis simple, we will ignore the fact that the life of Tintaya mine is limited by about 21 years. This is not too extreme an assumption, since the cash flow after 20 years is negligible. Dixit and Pindyck (page 224)

## 3.4. Results

### *3.4.1.* Value of Mine

Given the 18% volatility of the copper price, the Tintaya copper mine is worth US$ 786 million, which is about 3.4 times the US$ 233.5 million from optimistic DCF valuation. The value of the flexibility option endowed to the manager is worth approximately US$ 550 million given the copper price volatility of 18% per annum and the convenience yield (opportunity cost of holding the option) of 4.5%. The volatility of the copper price enhances the value of mine over the case that the copper price is zero volatile. For example, at a volatility of 2%, the Tintaya copper mine value is US$ 326 million. Figures 3, 4, and 5 demonstrate how the value of the operating mine changes sensitively in response to changes in volatility, convenience yield of the copper price, or riskless rate. The decrease in convenience yield, the opportunity cost of holding an option alive, leads to increases in option value. Increases in standard deviation, or volatility, of copper prices leads to increases in option value. Increasing volatility from $\sigma = 2\ \%$ to $\sigma = 18\%$ increases the mine value 2.4 times. As we see from figures 6, 7, and 8, other parameters, investment capital cost, reactivating capital cost, and mothballing capital cost, do not affect the value of the mine significantly.

## Figure 3. Mine Value as a Function of Volatility




## Figure 4. Mine Value as a Function of Convenience Yield

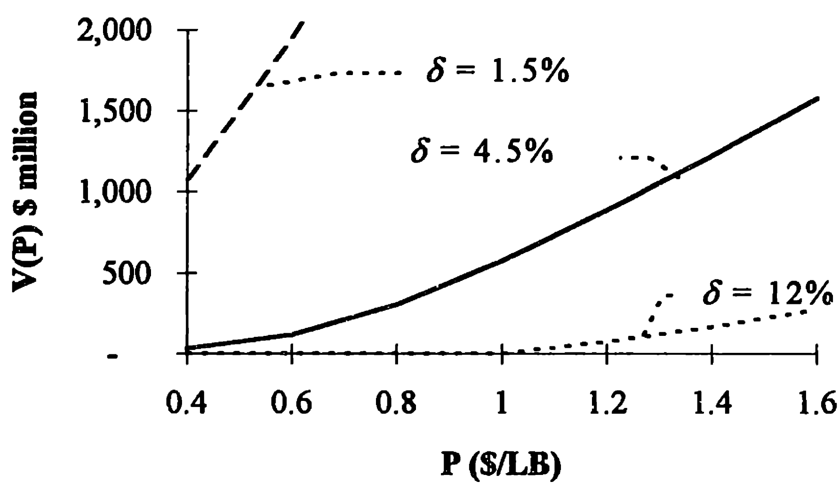


**Figure 5. Mine Value as a Function of Riskless Rate**

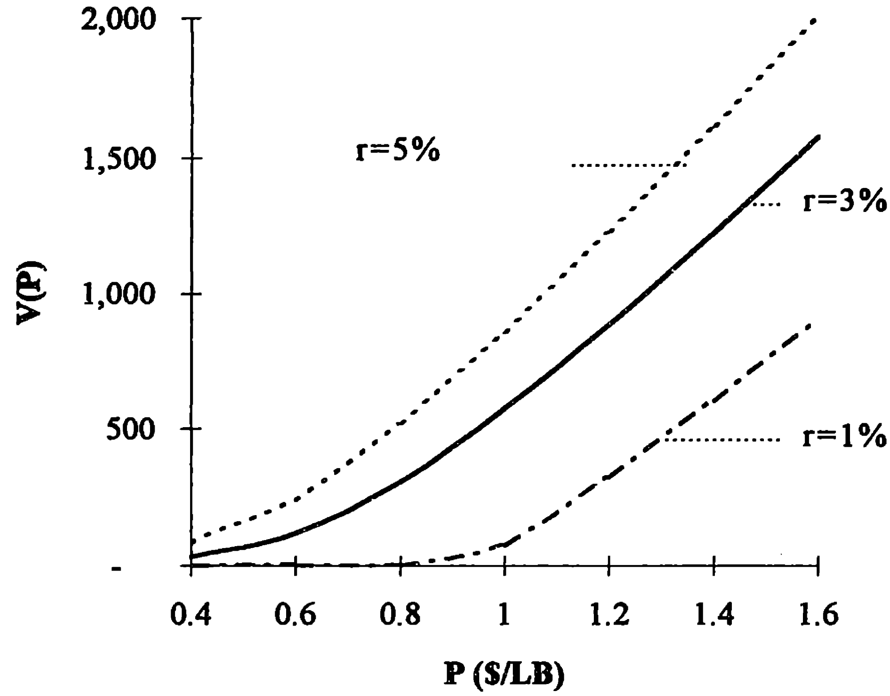



**Figure 6. Mine Value as a Function of Reactivation Cost**

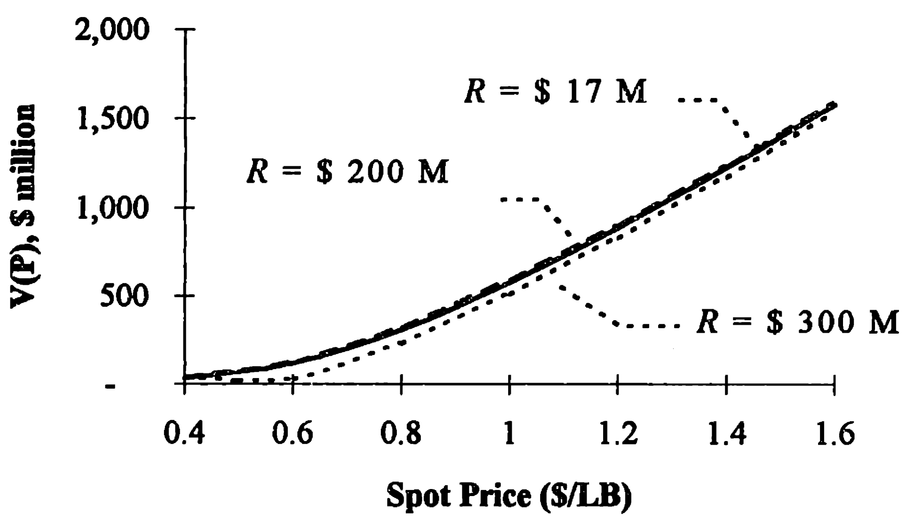


**Figure 7. Mine Value as a Function of Mothballing Cost**

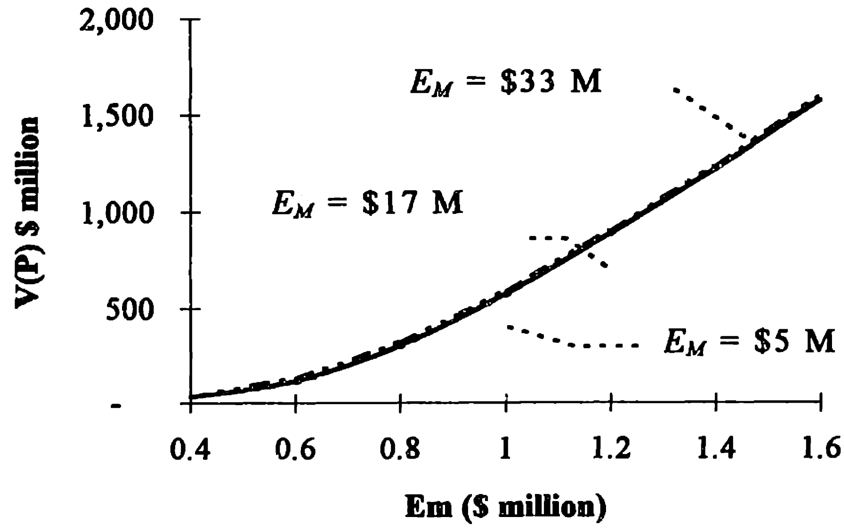
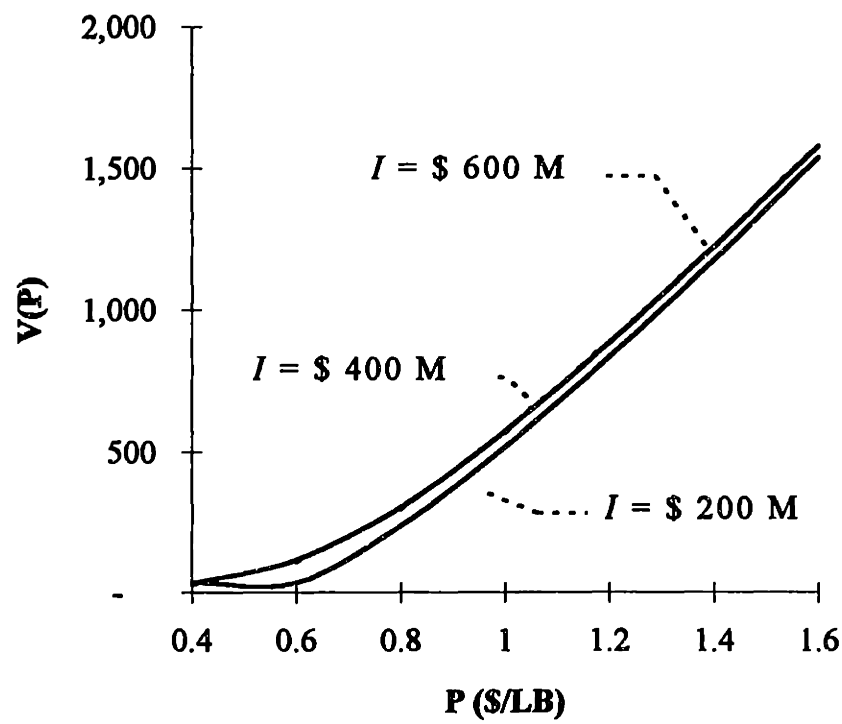



**Figure 8. Mine Value as a Function of Investment Cost**



### 3.4.2. Threshold Prices (Optimal timing to exercise the flexibility option)

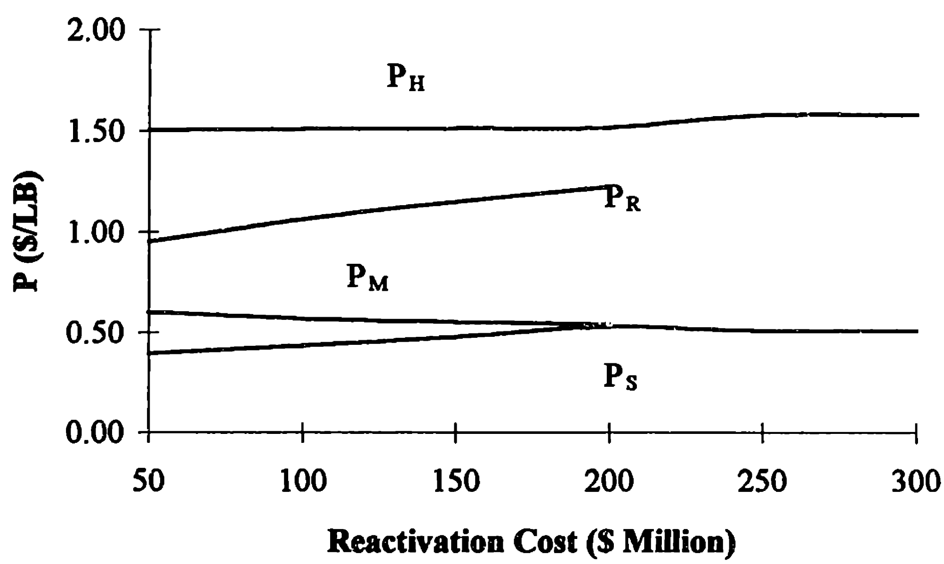
Obviously these critical threshold prices depend on the assumed cost of entry, mothballing, reactivating the mine, and maintenance. But volatility and convenience yield of copper prices affect these thresholds the most.

If the mine is already opened and operating, it is not optimal to close it down until the copper price drops to US\$0.54/lb, which is far less than production cost US\$0.76/lb.

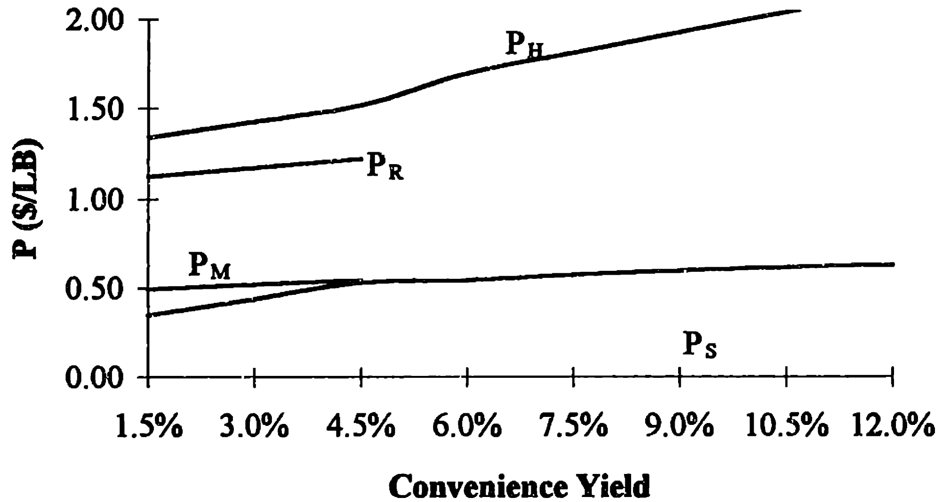
$P_M$ (threshold to mothball the operating mine) increases almost linearly as the volatility of the copper price decreases. As volatility decreases below 18%, the $P_M$ converges with $P_S$ (threshold to scrap -down of the mine) (Fig. 13). And it decreases slowly as the reactivating cost increases and converges into scrapping thresholds (Fig. 9). As the reactivating cost increases beyond US\$ 200 million, $P_M$ converges into $P_S$ and the mothball option becomes uneconomical. The sensitivities on other parameters, convenience yield, and mothball cost are dull. If volatility were lower than 18%, the reactivating cost were US\$ 200 million, or the opportunity cost of holding the option (convenience yield) were higher than 4.5% per annum, the mothballing and reactivating option would not have sensible economic value. So the manager is thought to have only the option of operating and abandoning the mine.

$P_R$ (threshold to reactivate the mothballed mine) also increases sensitively as the volatility or reactivating cost increases, but more slowly as the convenience yield increases of (Fig. 13, 9, and 10). It increases from US\$ 1.22/lb at $\sigma$ = 18% to US\$ 1.44/lb at $\sigma$ = 30%. $P_R$ increases from US\$ 0.95/lb when $R$ = US\$ 50 million to US\$ 1.22/lb at $R$ = US\$ 200 million.

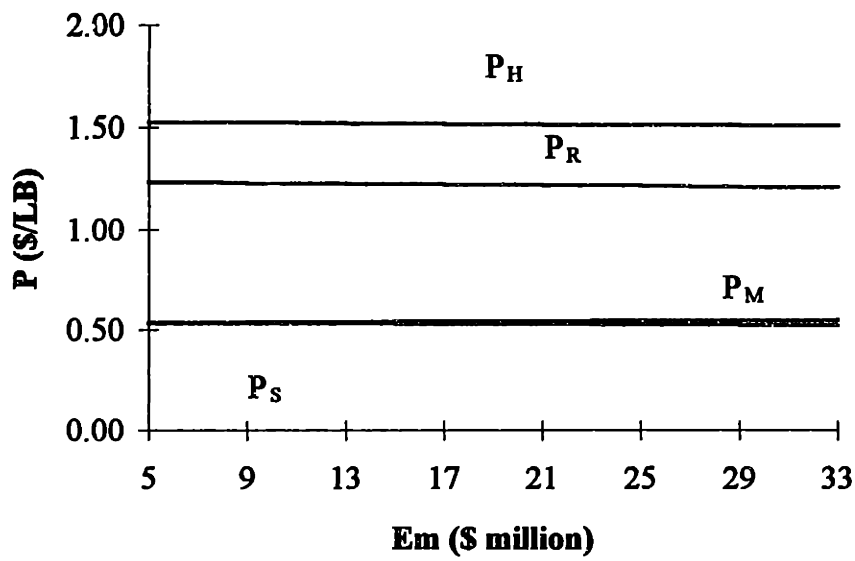
**Figure 9. Thresholds per Reactivation Cost**



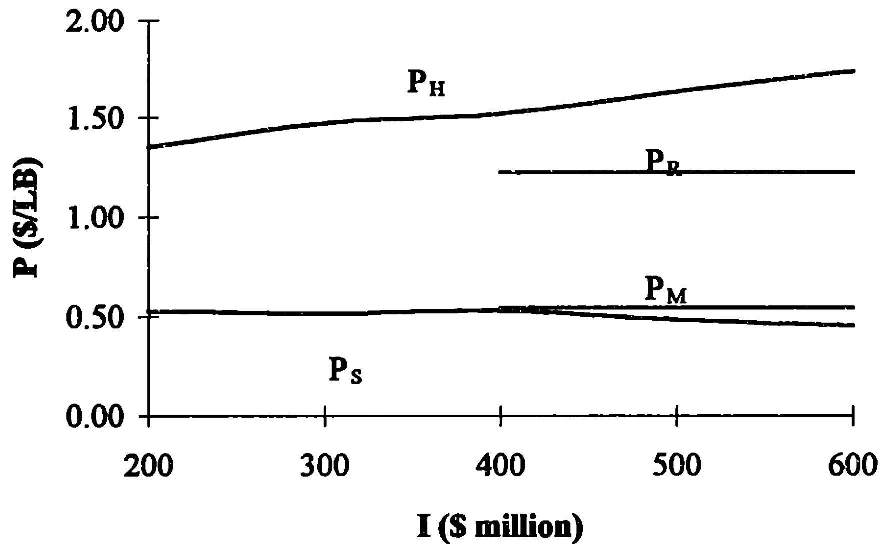
## Figure 10. Thresholds per Convenience Yield



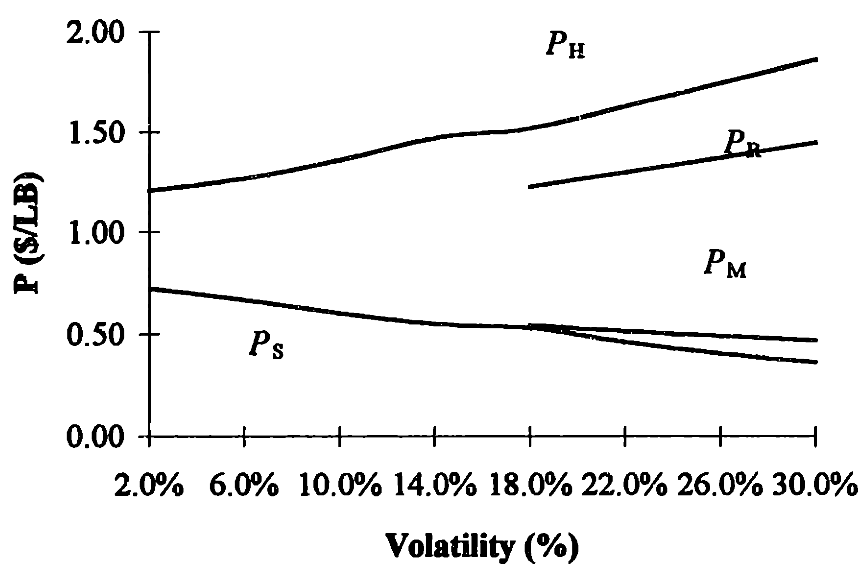

## Figure 11. Thresholds per Mothballing Cost



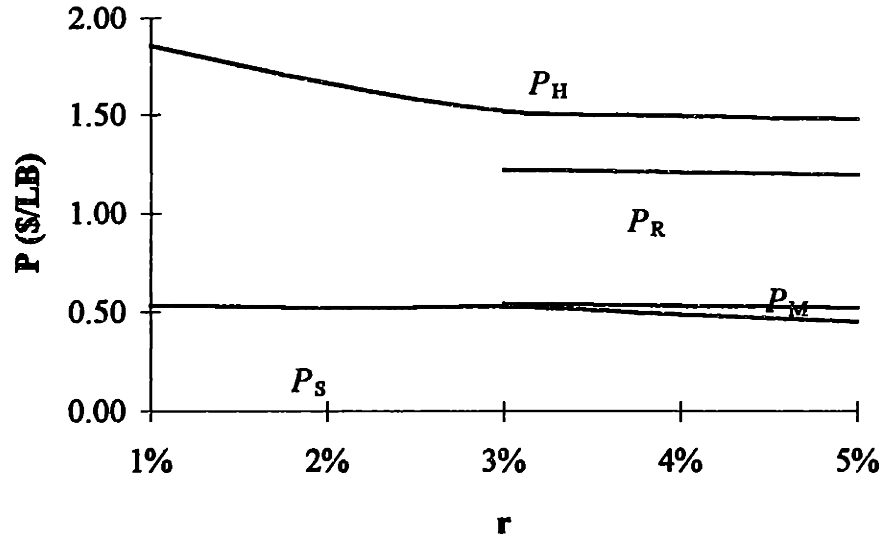
## Figure 12. Thresholds per Investment Cost




## Figure 13. Thresholds per Volatility



## Figure 14. Thresholds per Riskless Rate




$P_S$ (threshold to scrapping down mothballed mine or abandoning operating mine) decreases as volatility increases and increases as the convenience yield and reactivating cost increase (Fig. 13, 10). Other parameters do not affect the level of $P_S$.

$P_H$ (threshold to development of greenfield mine) increases almost linearly, parallel with increases of volatility, convenience yield, or investment cost (Fig. 13, 10, 12). It increases from US\$ 1.20/lb at $\sigma$ = 2% (near zero) to US\$ 1.86/lb at $\sigma$ = 30% when other parameters are the same. It increases from US\$ 1.35/lb at $I$ = US\$ 200 million to US\$ 1.73/lb at $I$ = US\$ 600 million when other parameters are the same. It increases from US\$ 1.34/lb at $\delta$ = 1.5% to US\$ 2.15/lb at $\delta$ = 12%. As intuitively understood, $P_H$ does not react sensibly on mothball capital cost or reactivating cost. Having the option on hand, it is more profitable to hold than to exercise the option.

Table 5. **Relation between parameters and value of mine and thresholds**

| Parameters | Value of Mine | $P_M$ | $P_R$ | $P_S$ | $P_H$ |
|---|---|---|---|---|---|
| Volatility | ++ | - | ++ | - | ++ |
| Convenience Yield | +++ | + | + | + | ++ |
| Riskless rate | ++ | 0 | 0 | 0 | -- |
| Investment Cost | + | 0 | 0 | 0 | + |
| Reactivation Cost | 0 | - | + | + | 0 |
| Scrapping Cost | 0 | 0 | 0 | 0 | 0 |

+++: The most positive correlation. ++: High positive correlation
o: Non significant correlation
--: High negative correlation. -: Negative correlation

The thresholds for entry, and reactivating from an idle position are far higher than the intuitively known, while thresholds for mothballing, and scrapping from active position are far lower than the intuitively known. This implies that calls on nondividened-paying assets are "worth more alive than dead." The right of the American call holder to exercise early is irrelevant because it will never pay to exercise early.[26]

[26] Bodie, Kane, & Marcus: Page 672

### 3.5. Comparison with NPV

Contingent claim analysis identifies the value of the flexibility option under price uncertainty which is denoted by volatility. So the volatility and convenience yield (opportunity cost to hold option) are the key factors on deciding the project value. As shown above, the fair value of Tintaya copper mine from contingent claim analysis, is US$ 786 million, which is larger than the value from the conventional NPV method. This difference from the NPV results from value of the option, the manager's capability of flexible operating decision depends on exogenous stochastic copper price. In NPV valuation, the positive cash flow is severely discounted when it is analyzed over a very long time frame, very common in mine projects. As a result, the volatility or risk is not properly valued.

The threshold derived from contingent claim analysis provides the manager with a clear guide on the optimal decision rule under price uncertainty.

## 4. Summary

If future prices (cash flows) are uncertain, the NPV analysis must be modified. The discount rate must be adjusted for a risk premium. In perfectly integrated capital markets, where all investors have full access to all assets and can efficiently diversify, all investors would agree on the risk adjusted interest rate appropriate for discounting each cash flow stream. In practice, however, capital market segmentation may cause different investors to view risk attributes differently.

The flexible managerial decision results in a valuable asymmetry in future cash flows. When metal price follows a lower path than operating cost, mothball option minimizes the operating loss down to care-maintenance cost only. Abandonment option minimizes the project loss only to one time abandonment sunk capital cost. The expected cash flow of the mine project with managerial real option is actually a function of metal price volatility, even when that volatility is symmetric around the mean price.

The ability that options impart to truncate unfavorable outcomes creates asymmetries that make NPV calculated from a "mean scenario" a downward-biased estimate of the true value. Options may arise naturally, as when the manager exercises investment timing, abandonment, or temporary mothball options.

The investment timing option (thresholds to investment and reactivation)

Even if the project NPV is positive today, it still may be better to wait to invest. This is because waiting enables the manager to respond to new market evolvement. The discounted expected NPV of a project deferred one year may exceed the NPV of a mine committed today. The closest competitor to any project is generally the same project deferred for some period. Waiting to invest is the most ubiquitous of all real options.

The abandonment option ( threshold to abandon)

If conditions turn out for the worse, the manager may abandon the project before the end of its economic life by incurring the abandonment sunk capital cost that is lower than the loss of the ongoing project. The option to abandon is likely a put option to sell the project for, at worst, the abandonment cost. The abandonment option may make the project attractive even if the NPV, based on the expected cash flow, is negative. The abandonment option limits the downside exposure of the project: the worst outcome is the project's abandonment cost. The firm retains all upside potental if market conditions improve. Hence the project resembles a call option where the holder has limited liability, but still profits from any positive market movements.

The mothball option (threshold to mothball)

When the mine is in operation and metal output prices fall unexpectedly, the management has the option to temporarily mothball the mine operations to avoid operating losses. Once prices recover sufficiently, the mine can again be made operational. The cash flow to the mine in each period is max { price-variable cost, 0}, similar to a call option with the exercise price equal to the variable cost. This option to temporarily mothball a mine has the effect of truncating the lower tail of the cash flow distribution. Again, the manager of the mine has a claim similar to a call option on metal. If metal prices rise, the firm profits. If they fall, losses can be limited by the option not to engage in active operation.

Options give project managers the ability to choose in the future the best operating mode. By truncating away bad states of nature, these options introduce asymmetries implying that (contrary to common practice) project cash flows cannot be calculated in a single "mean" scenario, even if underlying probability distributions are symmetric.

These options easily can have substantial value and can tip the balance between project acceptance and rejection. The options occur commonly in the context of investment timing decisions, mothball, and abandonment decisions. It is most important to incorporate options analysis into project valuation when the ultimate best operating mode in the future is difficult to forecast today; uncertainty is sufficiently great that differences in profits across possible operating modes are substantial.

The contingent claim analysis has advantages over the currently used discounted cash flow method. First, it provides a guide for the optimal timing of the decision rule (thresholds). Second, it provides important insights for value of flexibility under uncertainty.

Specific limitations of the option valuation model include the assumptions that the metal to be exploited is a known amount, that costs are known, that interest rates and convenience yields are nonstochastic, and that the manager can freely exercise options at his own discretion without any limitations. Using the historical convenience yield and volatility also limit the specification's feasibility.

The option value of the manager's flexibility would be decreased by limitations. The covenants of project financing may not allow the manager's decision to shut down the mine though the product price is lower than the variable operating cost. The government might also prevent the mine from suspending its operation when hard currency obtained from the exporting of the metal is a major source of the country's hard currency earnings.

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Appendix A.

**Copper Price's Volatility, Convenience Yield, and Riskless Rate**

Annualized STDEV of Cu Price Return Rate (Oct. 90 - Sep. 94): 18%

Convenience Yield/Year (Oct. 90 - Sep. 94) : 4.51%

Riskless Rate (Jul. 94 - Sep. 94) : 3.01%

Source of Data

Copper Price: LME Copper Grade A Official Price

CPI: Monthly Labor Review, US Department of Labor Bureau of Labor Statistics

LIBOR: International Financial Statistics, IMF

| Month | Copper Price | | | CPI Growth Rate/Month | Libor + 1% | Yearly Real Riskless Rate | Convenience Yield Yearly |
|---|---|---|---|---|---|---|---|
| | Settlement | | 3 Month | | | | |
| | $/lb | Return | $/lb | | | | |
| Oct-90 | 1.24 | -9.54% | 1.21 | 0.60% | 9.19% | 1.69% | 19.50% |
| Nov-90 | 1.17 | -5.74% | 1.17 | 0.22% | 9.03% | 6.30% | 8.76% |
| Dec-90 | 1.13 | -3.89% | 1.14 | 0.00% | 8.68% | 8.68% | 5.36% |
| Jan-91 | 1.11 | -1.48% | 1.13 | 0.60% | 8.43% | 0.99% | 1.62% |
| Feb-91 | 1.11 | 0.03% | 1.11 | 0.15% | 7.92% | 6.12% | 9.50% |
| Mar-91 | 1.09 | -1.62% | 1.09 | 0.15% | 7.72% | 5.92% | 7.68% |
| Apr-91 | 1.12 | 2.59% | 1.11 | 0.15% | 7.82% | 6.03% | 13.10% |
| May-91 | 1.05 | -6.72% | 1.03 | 0.30% | 7.28% | 3.67% | 11.70% |
| Jun-91 | 1.01 | -3.76% | 1.02 | 0.29% | 7.89% | 4.29% | 4.61% |
| Jul-91 | 1.01 | 0.77% | 1.03 | 0.15% | 7.82% | 6.04% | 3.26% |
| Aug-91 | 1.01 | -0.14% | 1.03 | 0.29% | 7.18% | 3.59% | 1.34% |
| Sep-91 | 1.05 | 4.10% | 1.05 | 0.44% | 6.95% | 1.54% | 6.98% |
| Oct-91 | 1.07 | 1.67% | 1.06 | 0.15% | 6.62% | 4.86% | 10.21% |
| Nov-91 | 1.08 | 0.73% | 1.06 | 0.29% | 6.20% | 2.64% | 12.18% |
| Dec-91 | 1.01 | -6.63% | 1.02 | 0.07% | 5.65% | 4.78% | 2.03% |
| Jan-92 | 0.97 | -3.65% | 0.98 | 0.15% | 5.48% | 3.72% | 2.50% |

| Month | Copper Price | | | CPI Growth | Libor + 1% | Yearly Real | Convenience |
|---|---|---|---|---|---|---|---|
| | Settlement | | 3 Month | Rate/Month | | Riskless | Yield |
| | $/lb | Return | $/lb | | | Rate | Yearly |
| Feb-92 | 1.00 | 2.94% | 1.00 | 0.36% | 5.64% | 1.20% | 4.37% |
| Mar-92 | 1.01 | 1.00% | 1.01 | 0.51% | 6.07% | -0.18% | 4.64% |
| Apr-92 | 1.00 | -0.54% | 1.01 | 0.14% | 5.68% | 3.94% | 2.80% |
| May-92 | 1.01 | 0.05% | 1.01 | 0.14% | 5.59% | 3.85% | 3.43% |
| Jun-92 | 1.04 | 3.66% | 1.04 | 0.36% | 5.33% | 0.94% | 4.46% |
| Jul-92 | 1.14 | 9.80% | 1.14 | 0.21% | 4.52% | 1.92% | 6.11% |
| Aug-92 | 1.14 | -0.36% | 1.14 | 0.28% | 4.74% | 1.26% | 4.58% |
| Sep-92 | 1.10 | -3.50% | 1.10 | 0.28% | 4.47% | 1.00% | 2.82% |
| Oct-92 | 1.02 | -7.05% | 1.03 | 0.35% | 4.68% | 0.34% | 0.62% |
| Nov-92 | 0.98 | -4.17% | 0.99 | 0.14% | 5.10% | 3.39% | 1.05% |
| Dec-92 | 1.00 | 2.36% | 1.01 | -0.07% | 5.11% | 5.95% | 1.33% |
| Jan-93 | 1.02 | 2.04% | 1.03 | 0.49% | 4.81% | -1.29% | 3.22% |
| Feb-93 | 1.00 | -2.04% | 1.01 | 0.35% | 4.50% | 0.20% | 1.43% |
| Mar-93 | 0.98 | -2.62% | 0.99 | 0.35% | 4.60% | 0.32% | 0.72% |
| Apr-93 | 0.89 | -9.20% | 0.90 | 0.28% | 4.23% | 0.83% | -0.25% |
| May-93 | 0.81 | -8.19% | 0.82 | 0.14% | 4.67% | 2.99% | 0.90% |
| Jun-93 | 0.84 | 3.46% | 0.84 | 0.14% | 4.84% | 3.16% | 3.55% |
| Jul-93 | 0.87 | 3.77% | 0.88 | 0.00% | 4.75% | 4.75% | 3.71% |
| Aug-93 | 0.88 | 1.09% | 0.87 | 0.28% | 4.67% | 1.29% | 9.84% |
| Sep-93 | 0.84 | -4.42% | 0.84 | 0.21% | 4.54% | 2.02% | 8.64% |
| Oct-93 | 0.75 | -11.6% | 0.76 | 0.41% | 4.56% | -0.53% | -0.75% |
| Nov-93 | 0.74 | -1.03% | 0.75 | 0.07% | 4.77% | 3.94% | -0.84% |
| Dec-93 | 0.78 | 5.77% | 0.79 | 0.00% | 4.80% | 4.80% | -0.50% |
| Jan-94 | 0.82 | 4.78% | 0.83 | 0.27% | 4.76% | 1.41% | -0.10% |
| Feb-94 | 0.85 | 3.38% | 0.86 | 0.34% | 5.06% | 0.87% | 0.32% |
| Mar-94 | 0.87 | 2.60% | 0.88 | 0.34% | 5.61% | 1.44% | 2.59% |
| Apr-94 | 0.85 | -1.73% | 0.86 | 0.14% | 6.13% | 4.49% | 2.27% |
| May-94 | 0.98 | 14.28% | 0.98 | 0.07% | 6.61% | 5.79% | 5.84% |
| Jun-94 | 1.07 | 9.93% | 1.08 | 0.34% | 6.77% | 2.62% | 4.53% |
| Jul-94 | 1.12 | 3.98% | 1.12 | 0.27% | 6.79% | 3.49% | 4.89% |
| Aug-94 | 1.09 | -2.11% | 1.10 | 0.40% | 6.78% | 1.81% | 5.32% |
| Sep-94 | 1.14 | 4.14% | 1.14 | 0.27% | 7.00% | 3.73% | 4.77% |

## Appendix B. Thresholds on Parameters

Unit: US$/lb

| σ | 2% | 6% | 10% | 14% | 18% | 22% | 26% | 30% |
|---|---|---|---|---|---|---|---|---|
| Ph | 1.20 | 1.26 | 1.36 | 1.46 | 1.52 | 1.63 | 1.74 | 1.86 |
| Pr | | | | | 1.22 | 1.29 | 1.37 | 1.44 |
| Pm | | | | | 0.54 | 0.51 | 0.49 | 0.47 |
| Ps | 0.72 | 0.67 | 0.60 | 0.55 | 0.53 | 0.46 | 0.40 | 0.36 |

| δ | 2% | 3% | 5% | 6% | 8% | 9% | 11% | 12% |
|---|---|---|---|---|---|---|---|---|
| Ph | 1.34 | 1.43 | 1.52 | 1.69 | 1.81 | 1.92 | 2.04 | 2.15 |
| Pr | 1.12 | 1.17 | 1.22 | | | | | |
| Pm | 0.49 | 0.52 | 0.54 | | | | | |
| Ps | 0.35 | 0.44 | 0.53 | 0.54 | 0.57 | 0.59 | 0.61 | 0.63 |

| R ($ million) | 50 | 100 | 150 | 200 | 250 | 300 |
|---|---|---|---|---|---|---|
| Ph | 1.50 | 1.51 | 1.52 | 1.52 | 1.58 | 1.58 |
| Pr | 0.95 | 1.06 | 1.15 | 1.22 | | |
| Pm | 0.60 | 0.57 | 0.55 | 0.54 | | |
| Ps | 0.39 | 0.43 | 0.48 | 0.53 | 0.51 | 0.51 |

| r | 1% | 2% | 3% | 4% | 5% |
|---|---|---|---|---|---|
| Ph | 1.86 | 1.66 | 1.52 | 1.49 | 1.48 |
| Pr | | | 1.22 | 1.21 | 1.20 |
| Pm | | | 0.54 | 0.53 | 0.52 |
| Ps | 0.53 | 0.52 | 0.53 | 0.49 | 0.45 |

| Em ($ million) | 5 | 9 | 13 | 17 | 21 | 25 | 29 | 33 |
|---|---|---|---|---|---|---|---|---|
| Ph | 1.52 | 1.52 | 1.52 | 1.52 | 1.51 | 1.51 | 1.51 | 1.50 |
| Pr | 1.23 | 1.23 | 1.23 | 1.22 | 1.22 | 1.21 | 1.21 | 1.21 |
| Pm | 0.53 | 0.54 | 0.54 | 0.54 | 0.54 | 0.54 | 0.55 | 0.55 |
| Ps | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.52 | 0.52 | 0.52 |

| I ($ million) | 200 | 300 | 400 | 500 | 600 |
|---|---|---|---|---|---|
| Ph | 1.35 | 1.47 | 1.52 | 1.63 | 1.73 |
| Pr | | | 1.22 | 1.22 | 1.22 |
| Pm | | | 0.54 | 0.54 | 0.54 |
| Ps | 0.52 | 0.51 | 0.53 | 0.48 | 0.45 |

**Appendix C. Value of developed mine**

Unit: US$ million

| σ | 2% | 6% | 10% | 14% | 18% | 22% | 26% | 30% |
|---|---|---|---|---|---|---|---|---|
| $0.4/lb | - | - | - | - | 31 | 63 | 102 | 146 |
| $0.6/lb | - | - | - | - | 115 | 186 | 262 | 337 |
| $0.8/lb | - | - | 43 | 131 | 304 | 405 | 504 | 598 |
| $1/lb | 129 | 193 | 287 | 394 | 572 | 688 | 800 | 905 |
| $1.2/lb | 420 | 490 | 592 | 706 | 883 | 1,007 | 1,126 | 1,239 |
| $1.4/lb | 759 | 828 | 930 | 1,047 | 1,220 | 1,348 | 1,472 | 1,589 |
| $1.6/lb | 1,124 | 1,190 | 1,289 | 1,405 | 1,574 | 1,704 | 1,831 | 1,951 |
| $1.14/lb | 326 | 395 | 496 | 609 | 786 | 908 | 1,026 | 1,136 |

| δ | 1.5% | 3.0% | 4.5% | 6.0% | 7.5% | 9.0% | 10.5% | 12.0% |
|---|---|---|---|---|---|---|---|---|
| $0.4/lb | 1,068 | 174 | 31 | - | - | - | - | - |
| $0.6/lb | 1,951 | 426 | 115 | - | - | - | - | - |
| $0.8/lb | 3,012 | 828 | 304 | 55 | - | - | - | - |
| $1/lb | 4,155 | 1,316 | 572 | 232 | 112 | 51 | 16 | - |
| $1.2/lb | 5,337 | 1,848 | 883 | 445 | 266 | 170 | 112 | 74 |
| $1.4/lb | 6,539 | 2,406 | 1,220 | 680 | 440 | 307 | 224 | 168 |
| $1.6/lb | 7,753 | 2,979 | 1,574 | 930 | 629 | 456 | 347 | 272 |
| $1.14/lb | 4,980 | 1,685 | 786 | 378 | 217 | 132 | 81 | 48 |

| Em ($ million) | 5 | 9 | 13 | 17 | 21 | 25 | 29 | 33 |
|---|---|---|---|---|---|---|---|---|
| $0.4/lb | 31 | 31 | 31 | 31 | 31 | 31 | 31 | 31 |
| $0.6/lb | 104 | 108 | 111 | 115 | 119 | 123 | 127 | 131 |
| $0.8/lb | 294 | 297 | 300 | 304 | 307 | 310 | 313 | 316 |
| $1/lb | 564 | 567 | 569 | 572 | 575 | 577 | 580 | 583 |
| $1.2/lb | 876 | 878 | 881 | 883 | 885 | 888 | 890 | 893 |
| $1.4/lb | 1,214 | 1,216 | 1,218 | 1,220 | 1,222 | 1,224 | 1,226 | 1,229 |
| $1.6/lb | 1,568 | 1,570 | 1,572 | 1,574 | 1,576 | 1,578 | 1,580 | 1,582 |
| $1.14/lb | 779 | 782 | 784 | 786 | 789 | 791 | 794 | 796 |

| r | 1% | 3% | 5% |
|---|---|---|---|
| $0.4/lb | - | 31 | 83 |
| $0.6/lb | - | 115 | 238 |
| $0.8/lb | - | 304 | 513 |
| $1/lb | 74 | 572 | 853 |
| $1.2/lb | 320 | 883 | 1,222 |
| $1.4/lb | 597 | 1,220 | 1,607 |
| $1.6/lb | 895 | 1,574 | 2,001 |
| $1.14/lb | 242 | 786 | 1,109 |

| R ($ million) | 50 | 100 | 150 | 200 | 250 | 300 |
|---|---|---|---|---|---|---|
| $0.6/lb | 114 | 121 | 117 | 115 | 22 | 22 |
| $0.8/lb | 317 | 309 | 305 | 304 | 228 | 228 |
| $1/lb | 583 | 576 | 573 | 572 | 507 | 507 |
| $1.2/lb | 893 | 887 | 884 | 883 | 826 | 826 |
| $1.4/lb | 1,229 | 1,223 | 1,221 | 1,220 | 1,169 | 1,169 |
| $1.6/lb | 1,582 | 1,577 | 1,575 | 1,574 | 1,527 | 1,527 |
| $1.14/lb | 797 | 790 | 787 | 786 | 727 | 727 |

| I ($ million) | 200 | 300 | 400 | 500 | 600 |
|---|---|---|---|---|---|
| $0.4/lb | 36 | 32 | 31 | 28 | 26 |
| $0.6/lb | 34 | 27 | 115 | 111 | 108 |
| $0.8/lb | 237 | 232 | 304 | 300 | 298 |
| $1/lb | 515 | 510 | 572 | 569 | 567 |
| $1.2/lb | 833 | 829 | 883 | 880 | 878 |
| $1.4/lb | 1,175 | 1,172 | 1,220 | 1,218 | 1,216 |
| $1.6/lb | 1,533 | 1,530 | 1,574 | 1,572 | 1,570 |
| $1.14/lb | 735 | 730 | 786 | 784 | 782 |